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## **Abstract:**

In the paper we test for the different reactions of stock markets to the current financial crisis. We focus on Central European stock markets, namely the Czech, Polish and Hungarian ones, and compare them to the German and U.S. benchmark stock markets. Using wavelet analysis, we decompose a time series into frequency components called scales and measure their energy contribution. The energy of a scale is proportional to its wavelet variance. The decompositions of the tested stock markets show changes in the energies on the scales during the current financial crisis. The results indicate that each of the tested stock markets reacted differently to the current financial crisis. More important, Central European stock markets seem to have strongly different behaviour during the crisis.

**Keywords:** wavelet analysis, multiresolution analysis, Central European stock markets, financial crisis

**JEL:** C14, C22, G15

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## **1. Introduction**

The current stock market crisis offers applied researchers new possibilities for testing stock market behaviour from different perspectives. In this short paper we take advantage of this opportunity and use wavelet analysis to see if it is able to uncover more information about the stock markets.

Wavelet analysis is a powerful mathematical tool for signal processing. Although it has recently shown diverse applications in many fields of research, it has received little attention in the econometric analysis of financial data. The few authors dealing with this area of research are Vuorenmaa (2005), Vacha and Vosvrda (2007) and Gallegati and Gallegati (2007). In particular, the discrete wavelet transform is very powerful in decomposing time series into an orthogonal set of components associated with both time and scales (frequencies). Examining the relationship between high-frequency and low-frequency fluctuations in the stock returns of different countries, we can investigate different behaviour that cannot be extracted using common econometric analysis. Moreover, the application of wavelet multiresolution analysis allows us to see even more deeply into the market structure. The main purpose of this paper is to use this analysis to compare Central European stock markets, represented by the Prague (PX), Budapest (BUX), Warsaw (WIG20) and German (DAX30) indices, and the U.S. (S&P 500) market. We will compare decomposed signals of these markets during the current financial crisis. Moreover, we will compare the evolution of their energies on scales in time, which will allow us to see possible differences in behaviour.

Wavelet decomposition reveals a higher energy contribution at lower frequencies, i.e. higher scales, during the most turbulent two-week period. This is caused by a higher proportion of periodic behaviour (in the market data), which may indicate lower market efficiency in this short period.

The paper is organized as follows. We begin with a methodology description presenting wavelet analysis. An empirical analysis of the Central European stock market data and results and conclusions follow.

## **2. Wavelet Analysis**

Wavelets are small waves that begin at a finite point in time and die out at a later finite point in time. This feature makes wavelike functions ideal for local approximation and time-scale decomposition of the time series under investigation. Time-scale decomposition helps us to

recognize relationships between economic variables on a disaggregate (scale) level rather than on an aggregate level. Unlike Fourier analysis, wavelets are suitable for detecting regime shifts, discontinuities and frequency changes. These features make wavelets a powerful tool for investigating the financial markets during the current financial crisis. For a more comprehensive analysis of the topic see Gencay et al. (2002), Abramovich et al. (1999) and Percival and Walden (2000).

There are two wavelets which form a pair in a wavelet family: father wavelets  $\varphi(\cdot)$  and mother wavelets  $\psi(\cdot)$ . The father wavelet (scaling function) integrates to unity and is used for the trend components; on the other hand, the mother wavelet integrates to zero and is suitable for detection of all deviations from the trend. The mother wavelet is compressed or dilated in a time domain to generate cycles to fit the actual time series. The formal definition of wavelets is

$$\varphi_{j,k}(t) = 2^{-\frac{j}{2}} \varphi\left(\frac{t-2^j k}{2^j}\right), \quad \psi_{j,k}(t) = 2^{-\frac{j}{2}} \psi\left(\frac{t-2^j k}{2^j}\right), \quad (1)$$

where  $j$  is the scale (or dilatation) and  $k$  is the translation (or shift). Commonly, many types of wavelets can possibly be used, including the Haar wavelet, the Mexican hat, the Morlet wavelet and the Daubechies wavelet. For a more detailed discussion see Gencay et al. (2002) or Percival and Walden (2000).

Let  $x(t)$ , ( $t=1,2,3,\dots,N$ ) be a time series of length  $N$  in  $L^2(R)$ . Time series  $x(t)$  can be built up as a sequence of projections onto father and mother wavelets indexed by both  $j$ , the scale, and  $k$ , the number of translations of the wavelet for any given scale, which is assumed to be dyadic. The wavelet coefficients are approximated by integrals

$$s_{j,k} \approx \int_{-\infty}^{\infty} x(t) \varphi_{j,k}(t) dt, \quad d_{j,k} \approx \int_{-\infty}^{\infty} x(t) \psi_{j,k}(t) dt, \quad (2)$$

$j = 1, 2, \dots, J$ , where  $J$  is the maximum scale. An important feature of wavelet analysis is the possibility to decompose a time series into its constituent multiresolution components. The multiresolution analysis (MRA) of a time series  $x(t)$  in  $L^2(R)$  is given by the following formula:

$$x(t) = S_J + D_J + D_{J-1} + D_{J-2} + \dots + D_1, \quad (3)$$

where

$$S_J = \sum_k s_{J,k} \varphi_{J,k}(t), \quad (4)$$

$$D_j = \sum_k d_{j,k} \psi_{j,k}(t), \quad j = 1, 2, \dots, J,$$

where the basis functions  $\varphi_{j,k}(t)$  and  $\psi_{j,k}(t)$  are assumed to be orthogonal. The sequence of terms  $S_J$ , called smooth, and wavelet details  $D_J, D_{J-1}, \dots, D_1$  represents a set of signal components that provide representations of the signal at the different resolution levels.

## 2.1. Wavelet Variance

The wavelet variance is a concise alternative to the power spectrum based on the Fourier transform and is often easier to interpret than the frequency-based spectrum (Percival and Walden, 2000). To compute the wavelet variance we use the multiresolution components

from the time series decomposition. Such analysis helps us to track the evolution of the energy contribution at various scales related to traders' investment horizons.

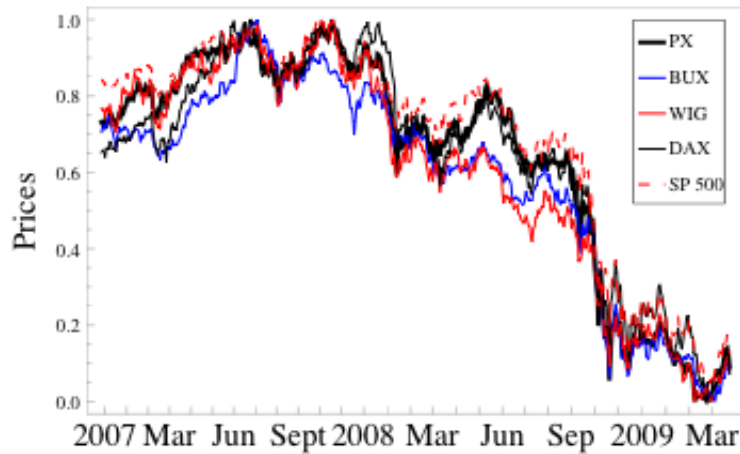
$$\|x\|^2 = \sum_{j=1}^J \|D_j\|^2 + \|S_j\|^2, \quad (5)$$

where the “energy” term means the sum of the squared coefficients of a vector, i.e.  $\sum_{t=0}^{N-1} x_t^2 = \|x\|^2$ . In our analysis we focus mainly on wavelet detail vectors  $D_1$ ,  $D_2$  and  $D_3$ , which represent the highest frequencies and energies of the examined time series, because these three scales (the highest frequencies) have a major energy contribution. For the MRA we use six-scale decomposition ( $J=6$ ) with the Daubechies “*db8*” wavelet filter.

### 3. Results

In this section we will apply the wavelet analysis methodology described in the previous text to a real-world data set consisting of Central European, German and U.S. stock market indices. We use a sample of 512 daily prices from 20 December 2006 to 31 April 2009 from the value-weighted PX, BUX, WIG20, DAX30 and S&P500 indices, representing an approximation of the Prague, Budapest, Warsaw, German and U.S. stock markets. Graph 1 shows the normalized prices of all indices from the sample. The prices are normalized to the interval [0,1]. The reader may have noticed that the sample includes the current financial crisis of 2008. As these markets have different holidays and trading schedules, we use a dummy variable for such days so we can exclude them from the analysis in order to be sure that each observation corresponds to the same day in the whole sample of all five tested indices. After matching the daily observations we transform the prices into continuously compounded index returns.

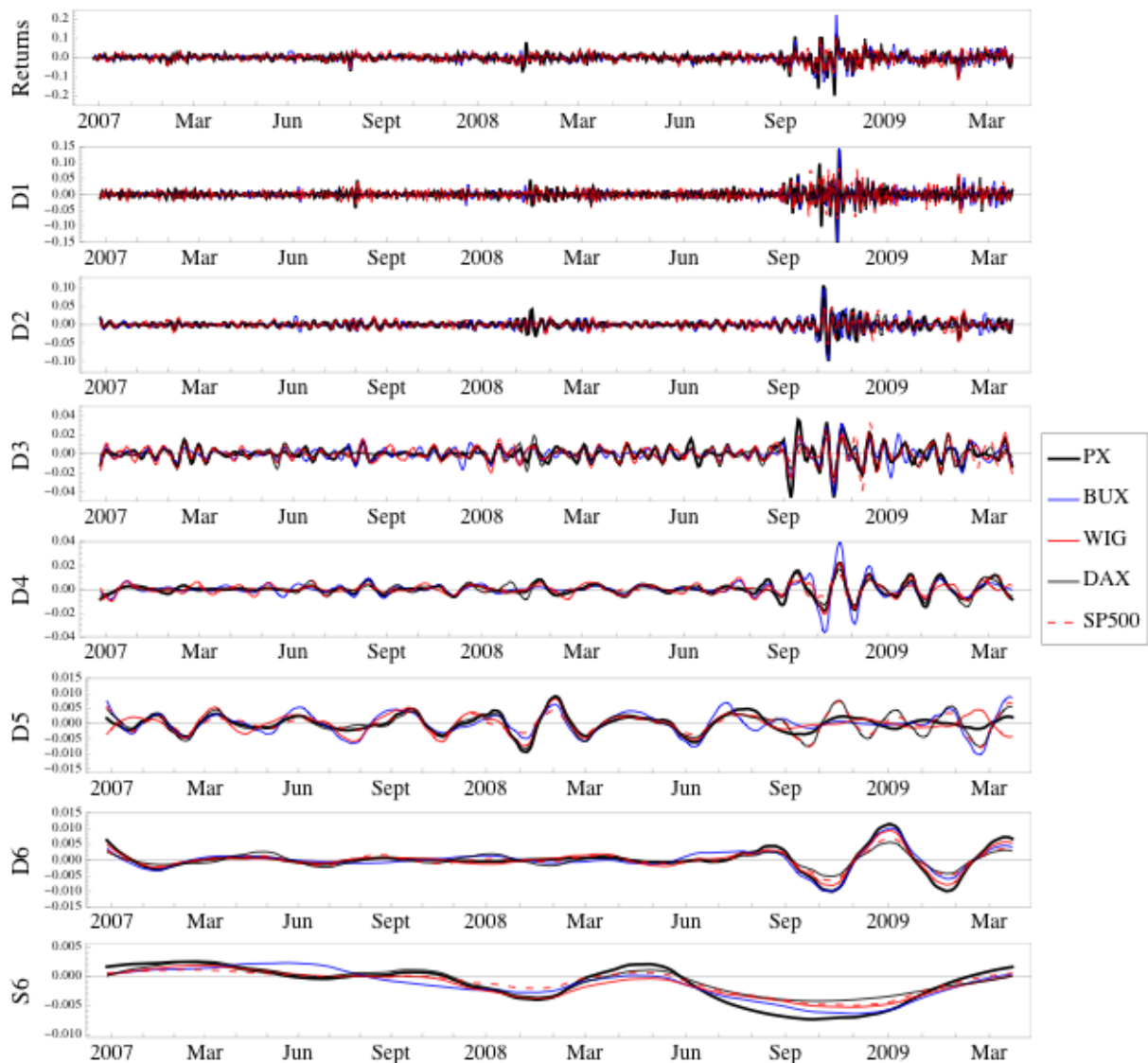
We begin the analysis with discrete wavelet decomposition of all time series and multiresolution analysis (MRA) using the Wavelet package for Mathematica “WM.M” written by Ian McLeod. Graph 2 depicts the returns and the wavelet MRA of all five market indices. We use the six-scale decomposition ( $J=6$ ) from Equation 3 with the Daubechies “*db8*” wavelet filter. As we would like to compare all five indices, we use one illustrative plot. The reader should note the different *y-axis* ranges for all the decomposed signals, as it is crucial to understand that the higher frequencies  $D_1$  and  $D_2$  represent much more energy than the lower ones. Interestingly, the volatility of all signals substantially increased during the end of 2008 during the biggest drops on all stock markets. We thus pick the greatest 2-week loss of the whole tested period in October 2008, more precisely the period from 6 October 2008 to 21 October 2008, and compare the contribution of each wavelet energy scale to the signal with the whole sample period. The increased volatility of all frequencies during this period indicates a very uncommon situation for all stock markets, as all energies during this period seem to contribute to the signal much more than in other periods. We will thus follow with an analysis of wavelet variance, which we use for analysing the energies at different levels.



*Graph 1: Prices of all series normalized to the interval [0,1]*

For the analysis of energies, we only compare  $D_1$ ,  $D_2$  and  $D_3$ , as they contribute most of the variance of the signal. Lower frequencies are moreover long-term frequencies and have a deniable contribution to the signal. Graph 3 shows the sum of the energies for the wavelet detail vectors for the whole tested period. During the whole period the highest frequency  $D_1$  clearly dominates for all countries. There are also significant differences between the markets. The US benchmark S&P500 index has the highest percentage of  $D_1$ , followed by WIG20 and PX.

Graph 4 depicts the sum of the energies for the wavelet detail vectors  $D_1$ ,  $D_2$  and  $D_3$  for all five examined market indices for the specific two-week large drop in October 2008 that we have chosen because of the largest variance across all frequencies. The main feature is a higher percentage of energy on the scale represented by  $D_2$ . For BUX, DAX30 and PX the energy on  $D_2$  was dominant in the two-week crash period. WIG20 and S&P500 have  $D_1$  dominant even in the short two-week period, but  $D_2$  contributes a high percentage of the variance as well. This is a surprising result, especially in comparison with the whole period.

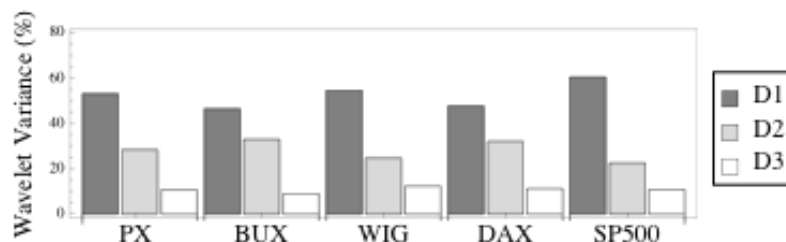


Graph 2: MRA of all series for whole tested period.

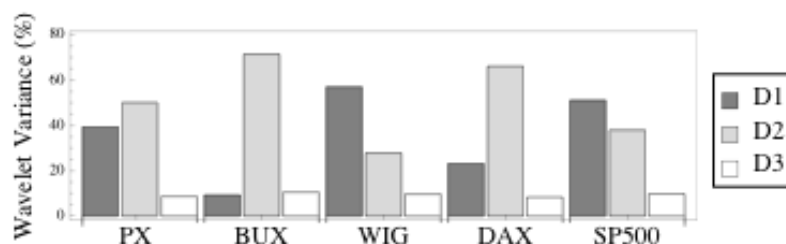
Hence Graphs 3 and 4 give a clear comparison of market behaviour during the whole examined period and the short period of major market collapse. The highest frequency  $D_1$ , representing short-term variations, makes the most important contribution to the overall variance of the series during the whole period (53%, 46%, 54%, 47% and 60% for PX, BUX, WIG20, DAX30 and S&P500 respectively), while  $D_2$ , which accounts for variations over a time scale of 4 days ( $2^2$ ), has lower explanatory power (28%, 33%, 24%, 32% and 22% for PX, BUX, WIG20, DAX30 and S&P500 resp.), and  $D_3$  (10%, 9%, 12%, 11% and 10% for PX, BUX, WIG20, DAX30 and S&P500 respectively),  $D_4$ ,  $D_5$ ,  $D_6$  and  $S_6$  account for the rest of the energy. This indicates that stock market movement is driven mainly by short-term fluctuations during the crisis. In contrast, the major two-week drop during October 2008 shows very different behaviour, as  $D_1$  represents the highest variance only for WIG20 and S&P500. More precisely, it is 39%, 9%, 57%, 23% and 51% for PX, BUX, WIG20, DAX30 and S&P500 respectively, while the energy of  $D_2$  is much stronger for PX and especially for BUX and DAX30 (50%, 71%, 27%, 66% and 37% for PX, BUX, WIG20, DAX30 and S&P500 respectively).

Thus, during the period of the largest two-week drop, the stock markets' behaviour changed significantly. The lower-frequency component  $D_2$  plays a more important role for all the CEE countries except WIG20, as well as for DAX30, thus the market is mostly driven by four-day fluctuations. The individual markets' reactions are also very different during this period. We can conclude that S&P500 seems to be the most efficient, as the highest-frequency component explains most of the variance also during this period of large drops. On the other hand,  $D_2$  also plays important role in comparison with the whole period. In this manner WIG20 seems to behave similarly to S&P500 and holds its efficiency also during the two-week period. The PX and BUX markets do not seem to hold their efficiency during this short period, as the highest energy contribution comes from the four-day frequency. DAX30 is also surprising, as it also behaves strongly inefficiently, unlike its counterpart S&P500. For the sake of clarity we should mention that  $D_1$  makes a major energy contribution for all the other two-week periods.

We have to remind the reader that these differences between the markets may also have been caused to some extent by the different structures of the tested indices. S&P500 has a very broad base of 500 stocks, while the other indices contain different industry stocks, thus the debate about efficiency should be addressed with caution, as more rigorous analysis needs to be carried out for stronger conclusions. This, however, is not the purpose of this short paper. Overall, the higher contribution of  $D_2$  is probably caused by the very pessimistic mood on all world markets, as short-term expectations were negative.



*Graph 3: Wavelet variance of  $D_1$ ,  $D_2$  and  $D_3$  for all tested countries for whole tested period*



*Graph 4: Wavelet variance of  $D_1$ ,  $D_2$  and  $D_3$  for all tested countries for 6 October 2008–21 October 2008*

The last part of our analysis is devoted to testing the differences in energy contributions across the markets. From Graph 3 we concluded that the short-term variations represented by frequency  $D_1$  play the most important role for all stock markets. Still, the question of whether the stock markets are moving together during the crisis at all frequency levels remains to be answered.

For this we use variance equality tests, which evaluate the null hypothesis that the variances in all subgroups are equal against the alternative that at least one subgroup has a different variance. For a general discussion of variance testing see Conover et al. (1981). More precisely we use the Levene test, which is based on an analysis of the variance of the absolute

difference from the mean. The F-statistic for the Levene test has an approximate F-distribution with numerator degrees of freedom and denominator degrees of freedom under the null hypothesis of equal variances in each subgroup (Levene, 1960).

The Levene test strongly rejects ( $p < 0.01$ ) the null hypotheses of equal variances of  $D_1$  frequencies in all the tested markets, and also strongly rejects the null hypotheses of equal variances for all other frequencies. This result tells us that the various frequencies across the different countries are not the same. In other words, if we decompose the tested stock markets using MRA and compare them in the means of the frequencies, we arrive at the result that they have significantly different variances, hence they have significantly different energy contributions. This also rigorously proves our expectation from Graph 3, where we could see differences between frequencies across countries.

#### 4. Conclusion

In this short paper we applied wavelet analysis to compare the Prague, Budapest, Warsaw, German and U.S. stock markets. We used multiresolution analysis to decompose the tested stock market indices into different frequency components. Moreover, we computed the variances of all frequencies, which represent energy contributions, and we used them to compare the markets.

Looking at the wavelet multiresolution analysis of all five stock market returns we can immediately see increased variance at the end of 2008 across all frequency components. This corresponds to the largest two-week drop in the tested period. We thus compute the variances of all frequencies for the whole period as well as for this short two-week period of large consecutive losses and we find significant differences. For the whole period the highest frequency  $D_1$  accounts for the highest energy in all five stock markets, while the energy at the lower four-day  $D_2$  frequency is much lower. The same analysis on the short two-week period of large consecutive losses during October 2008 shows very different results. For the PX, BUX and DAX30 markets, frequency  $D_2$  has the highest energy, which indicates that these stock markets became highly inefficient during this period. For the WIG20 and S&P500 returns,  $D_1$  still accounts for the highest energy, but frequency  $D_2$  also remains at high levels. Moreover, we found differences in the various energies between the compared stock markets.

Hence, we showed that the behaviour of the five tested stock markets differs across various decomposition levels during the current financial crisis. Wavelet analysis allowed us to test the decomposed series and see exactly the contributions of each scale to the energies of the markets. This short paper is a pilot study of the wavelet energies of Central European stock markets.

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