Analysis of the 2009 European Parliament elections in the Czech Republic

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- analysis of the powers of the parties in the European Parliament (2004-20009) - generalized power indices (Aleskerov 2006) based on parties' preferences
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- analysis of the electoral system (probability to enter the scrutiny, probability to influence the elections outcome by a single voter)
- introducing three possible behavioral structures for the voters: rational, semi-rational and non-rational

The European Parliament

- European Parliament - the only directly elected body of EU
- 736 members from 27 member states elected for 5 years
- has some legislative and budgetary power (can amend or reject legislation)
- five main decision proscedures: Codecision, Assent, Consultation, Cooperation and Conciliation procedures
- 7 big parties in (2004-2009)
- the European Parliament decides proposals by absolute majority

The European Parliament

- codecision: Codecision procedure is the main legislative procedure by which directives and regulations are adopted. The Council of the European Union and the European Parliament jointly adopt legislation based on a proposal by the European Commission. Both Parliament and Council (the latter acting by a qualified majority) are required to agree on an identical text before a proposal can be adopted.
- assent: Under assent, the Council can adopt legislation based on a proposal by the European Commission after obtaining the consent of Parliament. Thus Parliament has the legal power to accept or reject any proposal but no legal mechanism exists for proposing amendments.

The European Parliament

- consultation: Under the consultation procedure the Council, acting either unanimously or by a qualified majority depending on the policy area concerned, can adopt legislation based on a proposal by the European Commission after consulting the European Parliament. It is applied in all the cases where no other procedure is specified.
- cooperation: Under the procedure the Council can, with the support of Parliament and acting on a proposal by the Commission, adopt a legislative proposal by a qualified majority, but the Council can also overrule a rejection of the particular proposed law by the Parliament by adopting a proposal unanimously.

The European Parliament

- conciliation: The aim of the conciliation procedure is to achieve agreement between the European Parliament and the Council. It takes place in a ‘Conciliation Committee’ consisting of representatives of the two institutions.

The parties in the EP (2004-2009)

- (UEN): The Union for Europe of the Nations Group (27 MEPs)
- (ALDE): Group of the Alliance of Liberals and Democrats for Europe (88 MEPs)
- (PSE): Socialist Group in the European Parliament (200 MEPs)
- (Greens): Group of the Greens/European Free Alliance (42 MEPs)
- (GUE): Confederal Group of the European United Left (41 MEPs)
- (PPE): Group of the European People's Party (Christian Democrats) and Eu- ropean Democrats (268 MEPs)
- (IND): Independence/Democracy Group (37 MEPs)

The extent of matching votes

	(UEN)	(ALDE)	(PSE)	(IND)	(Greens)	(GUE)	(PPE)
(UEN)	100%	70.9%	63.4%	54.3%	48.7%	45.2%	81.31%
(ALDE)	70.9%	100%	75.4%	45.3%	61.9%	51.5%	77.1%
(PSE)	63.4%	75.4%	100%	39.7%	69.8%	62.0%	69.7%
(IND)	54.3%	45.3%	39.7%	100%	38.7%	40.9%	50.7%
(Greens)	48.7%	61.9%	69.8%	38.7%	100%	74.0%	49.8%
(GUE)	45.2%	51.5%	62.0%	40.9%	74.0%	100%	41.4%
(PPE)	81.3%	77.1%	69.7%	50.7%	49.8%	41.4%	100%

Generalized power indices

- based on the extent of matching votes we can derive linear ordering representing preferences for each of the seven parties
- we assume each party act homogeneously (it is not always the case)
- we employ generalized relative Banzhaf-Penrose power index for party i:

$$\pi_i^{GPB} = \frac{\chi_i}{\sum_j \chi_j},$$

where

$$\chi_i := \sum_{\omega} f\left(\{i\}, \omega\right) \cdot I\left[i \text{ is in a swing in } \omega\right]$$

and the sum goes over all possible coalitions ω, where f is an intensity function.

Generalized power indices

- in (Aleskerov 2006), there are many possible intensity functions proposed, we use just 6 of them (3 ordinal and 3 cardinal)

Notation	Intensity function $f(\{i\}, \omega)$	Type
(a)	$\frac{\sum_{j\in\omega} P_{ij}}{\lvert\omega\rvert}$	ordinal
(b)	$\frac{\sum_{j\in\omega} P_{ji}}{\lvert\omega\rvert}$	ordinal
(c)	$\frac{\left(\sum_{j\in\omega} P_{ij}+\sum_{j\in\omega} P_{ji}\right)}{2\lvert\omega\rvert}$	ordinal
(d)	$\frac{\sum_{j\in\omega} p_{ij}}{\lvert\omega\rvert}$	cardinal
(e)	$\frac{\sum_{j\in\omega} p_{ji}}{\lvert\omega\rvert}$	cardinal
(f)	$\frac{\left(\sum_{j\in\omega} p_{ij}+\sum_{j\in\omega} p_{ji}\right)}{2\lvert\omega\rvert}$	cardinal

Generalized power indices

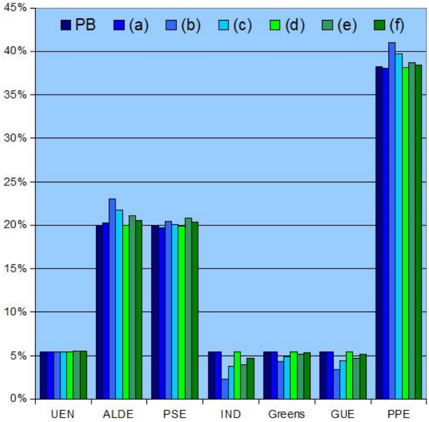


Figure: The generalized power indices of the parties in the European Parliament (2004-2009)

2009 elections in the EU

- the electoral systems over the EU are different
- there are different numbers of districts (one in the Czech Republic, more in France)
- there are different thresholds for entering the scrutiny (most states do not have zero threshold, but in the Cyech Republic, Denmark, France, Lithuania, Latvia, Hungary, Germany, Poland and Slovakia has 5%, in Austria and Sweden it is 4% and in Greece, it is 3%)

2009 elections in the Czech Republic

- one stage, one district electoral system
- if the party obtains more than 5% of all submitted votes, it enters the scrutiny
- for the parties which entered scrutiny modified d'Hondt method is used to assign mandates
- there are 22 seats in the EP for the Czech Republic

Modified d'Hondt method

- we denote $v_j^{SCRUTINY}$ the number of votes obtained by j-th party in the scrutiny
- we diviede $v_j^{SCRUTINY}$ by the elements of an increasing sequence of positive integer numbers up to 22 for each j
- the results of all the divisions we order in decreasing order
- we assign the 22 mandates to the parties according to this ordering (the party obtains a mandate if and only if the nominator of the given ratio corresponds to the number of votes the party obtained in the elections)

Probability to enter the scrutiny

- we assume the multinomial distribution of number of votes assigned to the parties and of the non-participating voters
- we take the parameters of the multinomial distribution for example from the pre-election surveys
- the aim is not to build a predictive model, but to study the probability of entering the scrutiny:

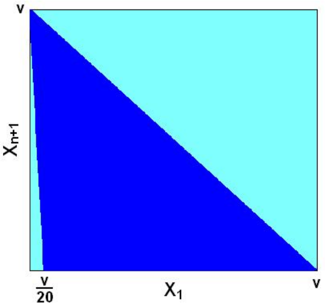
$$P\left(\frac{X_i}{v - X_{n+1}} > 0.05\right),$$

where v is the total number of all voters (no matter if participating or not), X_i is the number of voters voting for party i and X_{n+1} is the number of non-participating voters and n is the number of parties

Probability to enter the scrutiny

According to the assumption of multinomial probability, we have:

$$P\left(X_i = k_i, X_{n+1}\right) = \frac{v!}{k_1!k_{n+1}!(v-k_i-k_{n+1})!} p_i^{k_i} p_{n+1}^{k_{n+1}} \left(1 - p_i - p_{n+1}\right)^{v-k_i-k_{n+1}},$$



kde $v \geq k_i + k_{n+1}$

Probability to enter the scrutiny

- a priori probability (using assumption about equal parameters of the multinomial distribution)

$$(X_1, \ldots, X_n, X_{n+1}) \sim M\left(n; \underbrace{\frac{1}{n+1}, \ldots, \frac{1}{n+1}, \frac{1}{n+1}}_{n+1}\right),$$

  where $k-$th element of this vector represents the number of votes the party k obtains in the elections, for all $k \in \{1, \ldots, n\}$ and the number of voters which refuse to vote for any party for $k = n + 1$.

Probability to enter the scrutiny

- a posteriori probability (using assumption about estimated propensities of voters to vote for distinct parties) $(X_1, \ldots, X_n, X_{n+1}) \sim M(n; p_1, \ldots, p_n, p_{n+1})$, where $k-$th element of this vector represents the number of votes the party k obtains in the elections, for all $k \in \{1, \ldots, n\}$ and the number of voters which refuse to vote for any party for $k = n + 1$.

Probability to enter the scrutiny

- computational problems with exact probabilities (it is very time demanding)
- have to use generalized binomial coefficients, the Beta function:

$$\sum_{i=0}^{s} \binom{n}{i} p^i (1-p)^{n-i} = F_{2(n-s),2(s+1)}\left[\frac{(s+1)(1-p)}{p(n-s)}\right],$$

where $F_{a,b}[x]$ is the value of the Fisher-Snedecor distribution functon with a and b degrees of freedom in the point x.

Probability to enter the scrutiny

$$\sum_{k_1=0}^{\left\lfloor \frac{v}{20} \right\rfloor} \sum_{k_{n+1}=0}^{v-20k_1} P\left(X_1 = k_1, X_{n+1} = k_{n+1}\right) =$$

$$\sum_{k_1=0}^{\left\lfloor \frac{v}{20} \right\rfloor} \left(\begin{array}{c} v \\ k_1 \end{array} \right) p_1^{k_1} \left(1 - p_1\right)^{v-k_1} \cdot$$

$$F_{2(19k_1),2(v-20k_1+1)} \left(\frac{\left(v - 20k_1 + 1\right)\left(1 - \frac{p_{n+1}}{1-p_1}\right)}{\frac{p_{n+1}}{1-p_1}\left(19k_1\right)} \right) =$$

$$\sum_{k_1=1}^{\left\lfloor \frac{v}{20} \right\rfloor} \left[F_{2(v-k_1),2(k_1+1)} \left(\frac{\left(k_1 + 1\right)\left(1 - p_1\right)}{p_1 \left(v - k_1\right)} \right) - \right.$$

$$\left. F_{2(v-k_1+1),2k_1} \left(\frac{k_1 \left(1 - p_1\right)}{p_1 \left(v - k_1 + 1\right)} \right) \right] \cdot$$

$$F_{2(19k_1),2(v-20k_1+1)} \left(\frac{\left(v - 20k_1 + 1\right)\left(1 - \frac{p_{n+1}}{1-p_1}\right)}{\frac{p_{n+1}}{1-p_1}\left(19k_1\right)} \right) + \left(1 - p_1\right)^v$$

Probability to enter the scrutiny

Občanská demokratická strana	$p_1 = 0.0867$	Strana důstojného života	$p_{13} = 0.0003$
Česká strana sociálně demokratická	$p_2 = 0.0834$	Evropská demokratická strana	$p_{14} = 0.0045$
Komunistická strana Čech a Moravy	$p_3 = 0.0402$	Dělnická strana	$p_{15} = 0.0024$
KDU-ČSL	$p_4 = 0.0171$	Strana svobodných občanů	$p_{16} = 0.0033$
Strana zelených	$p_5 = 0.0069$	Moravané	$p_{17} = 0.0021$
Nezávislí	$p_6 = 0.0117$	Starostové a nezávislí	$p_{18} = 0.0036$
Libertas.cz	$p_7 = 0.0027$	Česká strana národně socialistická	$p_{19} = 0.0030$
Věci veřejné	$p_8 = 0.0054$	Volte Pravý Blok	$p_{20} = 0.0018$
Zelení	$p_9 = 0.0036$	Strana demokracie a svobody	$p_{21} = 0.0015$
Suverenita	$p_{10} = 0.0105$	Koruna Česká	$p_{22} = 0.0006$
Demokratická SZ	$p_{11} = 0.0039$	Nejen hasiči a živnostníci s učiteli	$p_{23} = 0.0003$
SNK-ED	$p_{12} = 0.0030$	Strana svobodných demokratů	$p_{24} = 0.0015$
No party	$p_{25} = 0.7000$		

Based on $SC\&C$ pre-election survey.

Probability to enter the scrutiny

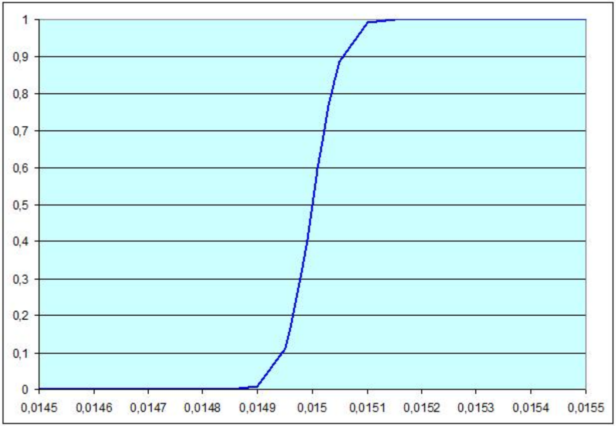


Figure: The probability to enter the scrutiny as a function of the party's share on the total number of votes

Probability to influence entering scrutiny

Description	Probability expression
Probability that party i enters the scrutiny.	$P_{(I)} := P\left(\frac{X_i}{v-X_{n+1}} > 0.05\right)$
Probability that party i enters the scrutiny when given voter votes for it and does not enter the scrutiny when he or she votes for party $j \neq i$.	$P_{(II)} := P\left(\frac{X_i+1}{v-X_{n+1}} > 0.05 \wedge \frac{X_i}{v-X_{n+1}} \leq 0.05\right)$
Probability that party i enters the scrutiny when given voter votes for it and does not enter the scrutiny when he or she does not vote at all.	$P_{(III)} := P\left(\frac{X_i+1}{v-X_{n+1}+1} > 0.05 \wedge \frac{X_i}{v-X_{n+1}} \leq 0.05\right)$
Probability that given voter gets to a situation where he can change the set of parties entering the scrutiny on his or her own.	$P_{(IV)} := P\left(\vee_{i=1}^{n}\left[\left(\frac{X_i}{v-1-X_{n+1}} \leq 0.05 \wedge \frac{X_i+1}{v-X_{n+1}} > 0.05\right) \vee \vee_{j=1, j\neq i}^{n}\left(\frac{X_j}{v-1-X_{n+1}} > 0.05 \wedge \frac{X_j}{v-X_{n+1}} \leq 0.05\right)\right]\right)$

Probability to influence entering scrutiny

- influential situation is such a situation where a single voter can change the set of parties which enter the scrutiny on his or her own
- there are two types of influential situations:

1. direct (single voter introduces a party into the scrutiny by voting for it)
2. indirect (single voter avoids a party to enter the scrutiny by voting for another party)

Probability to influence entering scrutiny

- the number of distinct outcomes single voter can face can be computed using the formula

$$D_{out} = \binom{v+n}{n},$$

  where v is the number of voters and n is the number of parties.

- the number of distinct outcomes, where the voter is influntial can be computed using the formula

$$\sum_{k=0}^{v-2}\sum_{i=1}^{n-1}(-1)^{i+1}\binom{n}{i}\binom{v-1-k-i\left\lfloor\frac{v-k}{20}\right\rfloor+n-1-i}{n-1-i}+1,$$

  where v is the number of voters and n is the number of parties.

Probability to influence entering scrutiny

- using the above formulas, we compute the ratio of the number of influential outcomes and all possible outcomes:

$$0.0000221813 = \frac{5.466 \cdot 10^{137}}{2.464 \cdot 10^{142}}$$

- the number of such influential situations, where a single voter can influence entering scrutiny for one given party can be computed using formula:

$$\sum_{k=0}^{v-1} \binom{v - 1 - k + n - 2 - \left\lfloor \frac{v-k}{20} \right\rfloor}{n - 2}.$$

The game of elections

- stochastic single round simultaneous move game of two players (single voter, all the other voters)
- assuming cost C of participating in the elections
- assuming individual additional gains from each distinct party entering the scrutiny $G_1, \ldots, G_n$, where each gain can be either positive, zero or negative
- we denote Ω_i the set of all outcomes, where the single voter can make the party enter the scrutiny on his or her own by voting for this party
- we denote Θ_i the set of all outcomes, where the single voter can make the party not enter the scrutiny on his or her own by voting for another party

Rational voter

The rational voter support the party for which he or she has the highest expected gain and so his expected pay-off is given by

$$EP_R = \max\left\{ \max_{i=1,\ldots,24}\left\{\left(P(\Omega_i)\cdot G_{1,i} + \sum_{j=1,j\neq i}^{24} P\left(\Theta_j\right) G_{2,j}\right) - C\right\}, 0\right\}.$$

Whenever $EP_R > 0$, then the rational voter votes for some of the parties, otherwise he or she would not.

Semi-rational voter

The semi-rational voter choses the supported party for which he or she maximizes the gain given just by making a party to enter the scrutiny. His or her expected pay-off is given by

$$EP_{SR} = \max\left\{\max_{i=1,\ldots,24}\left\{\left(P(\Omega_i)\cdot G_{1,i}\right)-C\right\} + \sum_{j=1,j\neq i_{max}}^{24} P\left(\Theta_j\right) G_{2,j}, 0\right\},$$

where $i_{max} = \arg\max_{i=1,\ldots,24}\left\{\left(P(\Omega_i)\cdot G_{1,i}\right)-C\right\}$. Whenever $EP_{SR} > 0$, then the semi-rational voter votes for some of the parties, otherwise he or she would not.

Non-rational voter

The non-rational voter choses the supported party randomly and so his expected pay-off is given by the probability distribution of his or her support. However if we assume the usual uniform distribution over all the possible actions the voter has, it is given by

$$EP_{NR} = \sum_{i=1}^{24} \left[\frac{1}{25} \left(P(\Omega_i) \cdot G_{1,i} + \sum_{j=1, j \neq i}^{24} P\left(\Theta_j\right) \cdot G_{2,j} \right) \right] - \frac{24}{25} C.$$

Whenever $EP_{NR} > 0$, then the non-rational voter votes for some of the parties, otherwise he or she would not.

Conclusion

- The paper is preliminary and not completed, however the theoretical part has been done. It is now being used for computations and further conclusions.
- It is clear, that $EP_R \geq EP_{SR} \geq EP_{NR}$.
- We will compute the expected pay-offs for the representative voters and will generalize their decisions based on their expected pay-off functions to be able to state, in what extent the voters act rationally.
- This paper introduces some methods of computing very small probabilities in relatively short time. When using the classical methods, it would not be possible to obtain the probabilities within a reasonable time period.