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DOCTORAL THESIS

Alternative Yield Curve Modelling Approach: Regional Models

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

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Abstract

In this thesis, we focus on thorough yield curve modelling. We build on extended classical Nelson-Siegel model, which we further develop to accommodate unobserved regional common factors and principal components. We centre our discussion on central European currencies' yield curves: CZK, HUF, PLN and SKK.

We propose two novel models to capture regional dynamics; one based purely on state space formulation and the other relying also on principal components of the regional yield curves. Moreover, we supplement the models with two application examples in risk management and structural break detection.

The main contribution of this thesis is a creation of a complete framework that enables us to analyse yield curves, to design risk scenarios and to detect structural breaks of various types.

JEL Classification	C51, C53, G17
Keywords	Dynamic Factor Model, Kalman Filter, Nelson-Siegel, State Space, Regional Yield Curve, Principal Component Analysis

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Abstrakt

Tato diplomová práce představuje několik modelů výnosových křivek. Vycházíme z dynamického modelu Nelson-Siegel, který jsme dále rozšířili pro modelování regionálních latentních faktorů a hlavních komponentů. Naši analýzu převážně zaměříme na středoevropské výnosové křivky denominované v těchto měnách: CZK, HUF, PLN a SKK.

V této práci představujeme dva původní modely, které zachycují regionální dynamiku: první založen na "state space framework" a druhý navíc využívá metody hlavních komponent regionálních výnosových křivek. Práce dále obsahuje dvě praktické aplikace modelů na řízení rizik a na detekci strukturálních změn.

Hlavním přinosem této diplomové práce je vytvoření komplexního rámce, který umožňuje analýzu výnosových křivek, přípravu krizových scénářů a detekci strukturálních změn.

Klasifikace JEL	C51, C53, G17
Klíčová slova	model s dynamickými faktory, Kalmanův filter, Nelson-Siegel, state space, regionální výnosová křivka, metoda hlavních komponent

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Acronyms

coeff. coefficient

DNS Dynamic Nelson-Siegel model

est. estimate

PCA Principal Component Analysis

PCRM Principal Component Regional Model

RCFM Regional Common Factor Model

SE Standard Errors

SSF State Space Framework/Formulation

Chapter 1

Introduction

Time value of money is a main input for economic decision-making of all companies. Since ancient times people have known that lending money for various time-spans has a different price linked to the length of a loan. In modern times, we refer to these as interest rates with different tenors depending when the loan matures. A set of different interest rates observed at a point in time is called a yield curve or a term structure.

Yield curve dynamics heavily affect financial institutions' decision-making thus good understanding of underlying driving forces is essential. Its importance has been increasing, since marking-to-market practice has become a common standard. These days, nearly whole banks' or insurance companies' value is based on market bases, which makes the exposure to changes in term structure one of the main factors determining the value of financial institution's assets and liabilities, therefore its solvency.

The recent financial crisis has shown how quickly can a solvent institution become insolvent. Moreover, throughout the crisis the investor's risk aversion increased significantly, which drained liquidity in the market and force monetary authorities to keep lowering interest rates. The resulting sharp drop in short-term rates shuffled with financial institutions' balance sheets.

Furthermore, as financial integration proceeds we can expect country specific dynamics to gradually diminish. Therefore, we need a framework capable of modelling several yield curves of different countries' at once. With such a framework, we are able to analyse dynamics and possible structural breaks.

This thesis presents such a framework. We mostly build on works of Diebold et al. (2006; 2008) who introduced a dynamised version of classical Nelson-Siegel model and discovered a global yield curve. We proceed notionally in a similar fashion and introduce two novel regional models.

The thesis is organised in five chapters. We begin our thesis with a survey of related literature, in which we review theoretical and statistical background.

Since our models rely on state space framework, in chapter 3 we proceed with introduction of such models and derivation of Kalman filter. Consequently, we show how to construct a likelihood function for these models and present diagnostics. In addition, we describe the data-sets used in this thesis.

Chapter 4 introduces gradually all the models. We start with the basic dynamic Nelson-Siegel model as proposed by Diebold et al. (2006). We describe the way to put such a model into state space formulation to exploit its properties. Moreover, this chapter introduces two novel, regional models.

Continuing with the regional models, we first perform the principal component analysis on the stacked currencies' yield curve to confirm our intuition that regional yield curve exists. While performing the principal component analysis, we also extract the first two principal components, which are needed for one of the regional models.

The first of the two novel models is purely based on unobserved latent factors and uses the strength of Kalman filter to extract them. This model is thus called the Regional Common Factor model. The second novel model includes the extracted principal components as regressors into the state space framework; we refer to it as the Principal Component Regional model. This idea simplifies the estimation and has a few advantages over the previous model, which we mention further in the thesis.

The thesis further introduces two practical application of the models. The first application focuses on a stress scenario design relevant to interest rate risk management. The forecasting properties of the state space framework allows us to easily construct forecasts on a give horizon and at a desired confidence level. Through combination of point and interval forecasts of the extracted factors, we are able to compute stressed curves. These stressed curves can be further used in economic capital allocation.

The second practical application deserves more attention in general economics, because it allows to detect structural breaks within the extracted factors. This goes in two step procedure; first plotting standardised residuals and consequently estimating the model again with included dummy variable in place of possible shock. As the dummy variable can take several forms, we not only estimate significance and magnitude of the break, but additionally we can see how the structural breaks can be modelled. Chapter 6 then concludes.

Chapter 2

Related Literature

This section shortly introduces the related literature on yield curve modelling and theoretical background on state space framework and estimation methods, which the thesis is altogether based on. Similarly, as the thesis contains three notional parts we relate literature separately to all of these. We are mostly concerned with regional models of yield curve; though, estimation methods and practical usage of the models are also of keen interest.

Factor models Nelson and Siegel (1987) published a seminal paper in 1987. This paper introduced a parsimonious way to model the whole yield curve with only three parameters. This approach is able to reproduce all stylised facts about possible yield curve shape: monotonic, humped or S-shaped (Nelson and Siegel 1987, p. 474).

Moreover, the introduced framework arises as a solution to a second-order differential equation with real and unequal roots (Nelson and Siegel 1987, p. 475), which gives a straight-forward inference of the model. That is why the Nelson-Siegel model relies on exponential components that take different values for various maturities.

Further developments are introduced in Litterman and Scheinkman (1991). Setting off to improve hedging of bond portfolios, they propose a hedging technique based on bond-return-factor analysis. The main novelty of this paper was the identification of three main factors explaining up to 97% of variance of returns. These three factors are called: level, steepness (slope) and curvature. Unlike in this thesis, these factors are computed as mimicking portfolios fulfilling certain demand of factor loadings, whereas we apply principal component analysis.

Nelson-Siegel Model in its original form allows for straightforward extrapolation and smoothing of yield curve at one point in time. Despite profound literature on term structure modelling with no-arbitrage approach bases, Diebold and Li (2006) state that there had not been paid much attention to practical question of forecasting the yield curves. Diebold and Li (2006) propose an exponential component framework based on the classical model of Nelson and Siegel (1987) to answer this need. Diebold and Li (2006) reformulate the functional form of the exponential components and used ordinary least squares regression to fit the Nelson-Siegel curve to every year of their sample resulting in series of three Nelson-Siegel coefficients. The innovative approach is to use these series as factors driving the yield curve. Diebold and Li (2006) fit to these series an autoregressive model of order 1 and use the natural forecasting abilities of AR(1) to produce encouragingly good forecasts.

Diebold et al. (2006) introduces the Nelson-Siegel model into state space framework thus allowing for clear inference and one step estimation. The paper shows how the forecasting results can be further improved, if we include macroeconomic variables as regressors into the state space framework.

Last but not least paper dealing with Nelson-Siegel yield curve modelling from statistical perspective, Diebold et al. (2008), proposes an idea of Global yield curve, which affects country specific yield curves and which country specific yield curves can load on. The main result is that there is an unobserved Global yield curve.

Building on innovations of Diebold and Li (2006), Diebold et al. (2006) and Diebold et al. (2008); Šopov (2009) proposed an approach to model yield curve stress scenarios. Since AR(1) specification for factors and state space formulation allows for easy forecasting, Šopov (2009) combines interval and point forecasts to build through Nelson-Siegel model a stressed yield curves and concludes that this approach gives encouraging results compared to benchmark approaches.

No-arbitrage Models In spite of concentrating mostly on modelling in this thesis, there are extensions to Nelson-Siegel framework allowing for no-arbitrage estimation so that the model can be used for pricing. Diebold et al. (2006) stress the poor forecasting abilities of no-arbitrage models of term structure that are predominantly used for pricing and show that the Nelson-Siegel model is not a member of affine class. Christensen et al. (2009) examine 4-factor Nelson-Siegel model extended by Svensson (1995), which still cannot fulfil no-arbitrage conditions. Finally, they introduce 5-factor extension, which works better in longer maturities and most importantly is an affine class model, thus arbitrage free. Moreover, such a model brings reasonable improvement in forecasting. These results are further developed in Christensen et al. (2006).

Estimation The estimation of such a complex model developed in this thesis relies on methods developed to extract signal from noisy measurements and mostly applied in engineering and physics. Kalman (1960) introduces a very efficient way of computing mean and variance of a noisy signal. Since the state space formulation has a state equation and an observation equation, it allows easy application of Kalman filter. The parameters are either known or have to be estimated; the maximum likelihood is an convenient choice.

In this thesis, we mostly use Durbin and Koopman (2001) and Harvey (2002) as our theoretical background. The former book gives a coherent treatment of state space models and Kalman filter and its application to economic time-series models. The book serves also as a background text to support SsfPack 2.2 (Koopman et al. 1998), which is a package for Object-Oriented Matrix Programming Language Ox[1] (Doornik 2007). The latter book extensively treats structural time-series application in both univariate and multivariate cases. Recently, Jungbacker and Koopman (2008) further improved efficiency of Kalman filter and likelihood evaluation.

[1] The package and Ox compiler are free for academic purposes.

Chapter 3

Methodology

In this chapter, we introduce the data, which we use to estimate the parameters, and the needed methodology, which our models are based on. To get the feel for the data, section 3.1 shows plots of the data and gives some intuition on the common movements in the samples. Consequently, we introduce derivation of Kalman filter, present some results in parameter estimation using Maximum likelihood estimator and related diagnostics.

3.1 Data

In this paper, we use two different data-sets to estimate coefficients of our models, one based on swap rates and the other based on zero rates. The former data needs to be recalculated into zero rates in order to match the type of rates between the two data-sets.

We start with the first date-set by collecting historical swap rates from Bloomberg for three currency zones, which represent strong world currencies; Euro (EUR), United States dollar (USD) and Pound Sterling (GBP). In Table 3.1 we present the sample details and in Table 3.2 we indicate the available maturities.

We denote S_n the n-year swap rate, so that we can extract the discount factors. By setting $D_0 = 1$ and $D_1 = \frac{1}{1+S_1}$, we use the relation in equation (3.1) to compute the discount factors. The zero yield curve rates are computed as shown in equation (3.2).

$$D_n = \frac{1 - S_n \sum_{i=1}^{n-1} D_i}{1 + S_n}, \tag{3.1}$$

where D_n is a n-year discount factor.

$$y_n = \frac{1}{\sqrt[n]{D_n}} - 1 \tag{3.2}$$

Table 3.1: Data-set 1 Details

Currency	start date	end date	tenors p	sample size n
EUR	31-Aug-1999	30-Jun-2008	13	119
USD	30-Jun-1996	30-Jun-2008	11	157
GBP	30-Sep-1999	30-Jun-2008	12	118

Source: Author's calculations

Table 3.2: Maturities of Data-set 1

	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y	12Y	15Y	20Y	25Y	30Y	40Y
EUR	X	X	X	X	X	X	X	X	X	X	X	X	X	
USD	X	X	X	X	X	X	X	X		X	X		X	
GBP	X	X	X	X	X	X	X	X		X	X		X	X

Source: Author's calculations

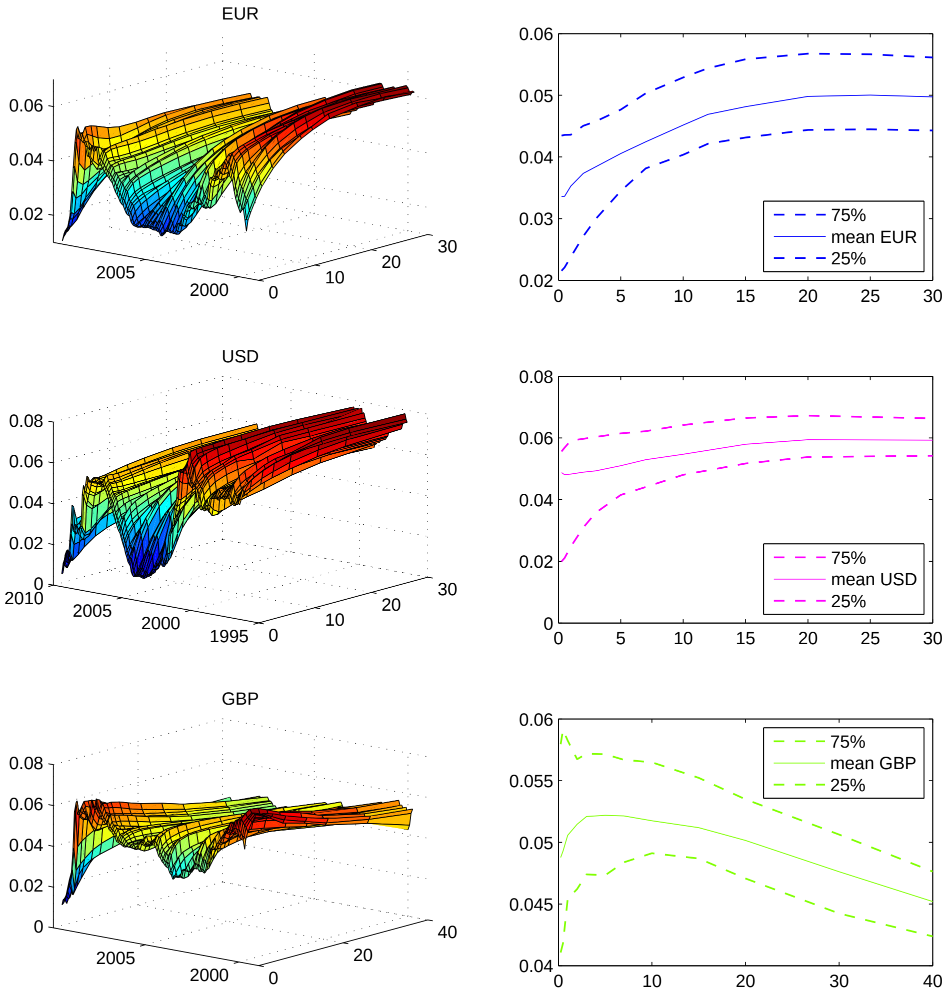
The final zero curves of these currencies are shown in Figure 3.1, where we also present *mean curve* as a series of means of each maturity rate, *first quartile* and *third quartile* computed as quartiles for each maturity rate.

We observe that the zero curves inherit some similarities with the common global factor hypothesis of Diebold et al. (2008). Especially between the years 2003 and 2005, we see a global drop in short-term rates followed by global increase through 2006, though these movements are much weaker in case of GBP. There are also some clear country specific movements; e.g. on average the downward sloping GBP zero curve or the drop of USD short-term rates after 2006. Finally, the dataset contains the large drop in the third quarter of 2008 and in 2009 as a consequence of outbreak of the financial crisis.

The other data-set forms the basis for the regional analysis. We obtained the zero rates from Thomson-Reuters for four central European currencies: Czech Crown (CZK), Hungarian Forint (HUF), Polish Zloty (PLN) and Slovak Crown (SKK).

Due to limited data availability we restrict our analysis to shorter maturities, as the very long ones are not available with sufficient history and liquidity. The actual maturities for all four regional currencies are listed in Table 3.4.

Figure 3.1: Zero Yield Curves – Evolution over Time



x-axis denotes maturities τ in years; y-axis denotes interest rates

Clearly, the data provide no information on interest rates with maturities longer than 10 years so we can expect the dynamics to be slightly different to previous data-set. The differences are to some extent unimportant, because it shows how the model can accommodate various data and simply can capture dynamics of the sample fed in. Furthermore, the data-set is shorter as the liquidity was poor in years before 2000. The sample length and number of maturities is shown in Table 3.3.

Looking at Figure 3.2, we can see some similarities in the evolution of countries' yield curves over time. We have to bear in mind different lengths of data sample and scale, e.g. parameters for the SKK model are further estimated using lower number of observations, since the SKK sample ends in December 2008. At first sight, the Czech crown yield curve seems to be well behaved and

Table 3.3: Data-set 2 details

Currency	start date	end date	tenors p	sample size n
CZK	31-Mar-1999	31-Nov-2009	12	117
HUF	31-Mar-2002	31-Nov-2009	12	93
PLN	31-Oct-2000	31-Nov-2009	12	110
SKK	30-Sep-2003	12-Dec-2008	12	64

Source: Author's calculations

is similar to those of strong currencies. If the history of the data allows, we observe periods of high rates around the year 2000, which were especially high in Polish Zloty reaching over 15%. In Figure 3.3, the Hungarian Forint yield curve shows relatively high short-end volatility as well as high short-term rates, which is a sign that Forint and Hungarian economy are vulnerable to tempory shocks.

The same figure shows many of common types of yield curve; upward sloping in case of CZK, downward sloping in case of HUF and almost flat in case of PLN and SKK.

Table 3.4: Maturities of Data-set 2

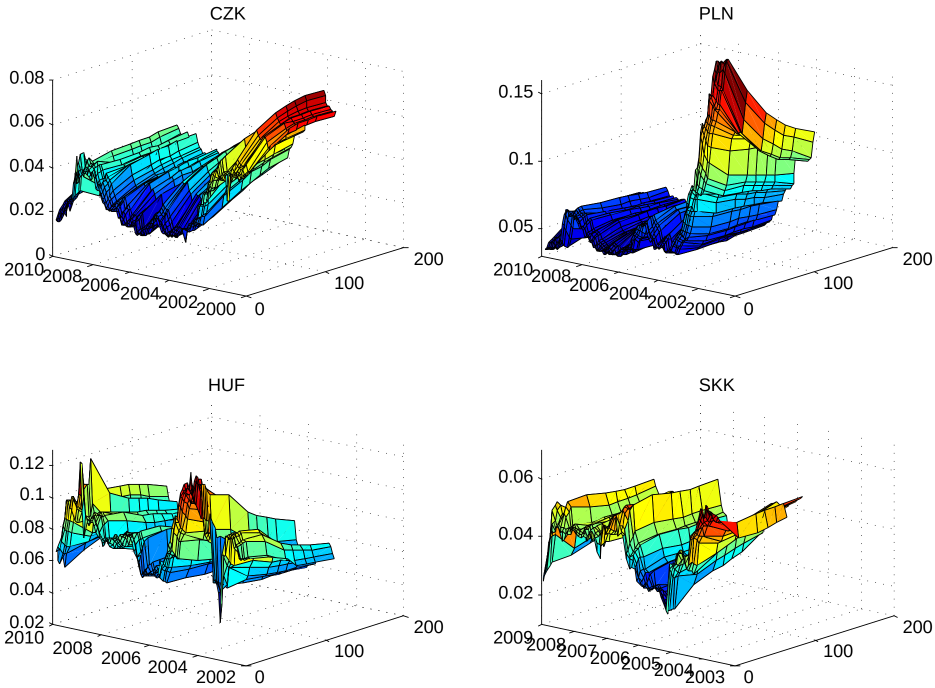
	1M	2M	3M	6M	9M	1Y	3Y	5Y	6Y	7Y	8Y	10Y
CZK	X	X	X	X	X	X	X	X	X	X	X	X
HUF	X	X	X	X	X	X	X	X	X	X	X	X
PLN	X	X	X	X	X	X	X	X	X	X	X	X
SKK	X	X	X	X	X	X	X	X	X	X	X	X

Source: Author's calculations

3.2 State Space Formulation

This thesis purely relies on state space formulation, which is a general framework able to accommodate various specifications of time-series models. Since seminal paper by Kalman (1960), state space framework has been a powerful tool to analyse time series. Unfortunately, it was not of much interest to econometricians but rather to engineers. Recently, it has been gaining deserved popularity for its strength. Unlike the black-box approach given by Box-Jekings

Figure 3.2: Regional Zero Yield Curves – Evolution over Time



x-axis denotes maturities τ in months; y-axis denotes interest rates

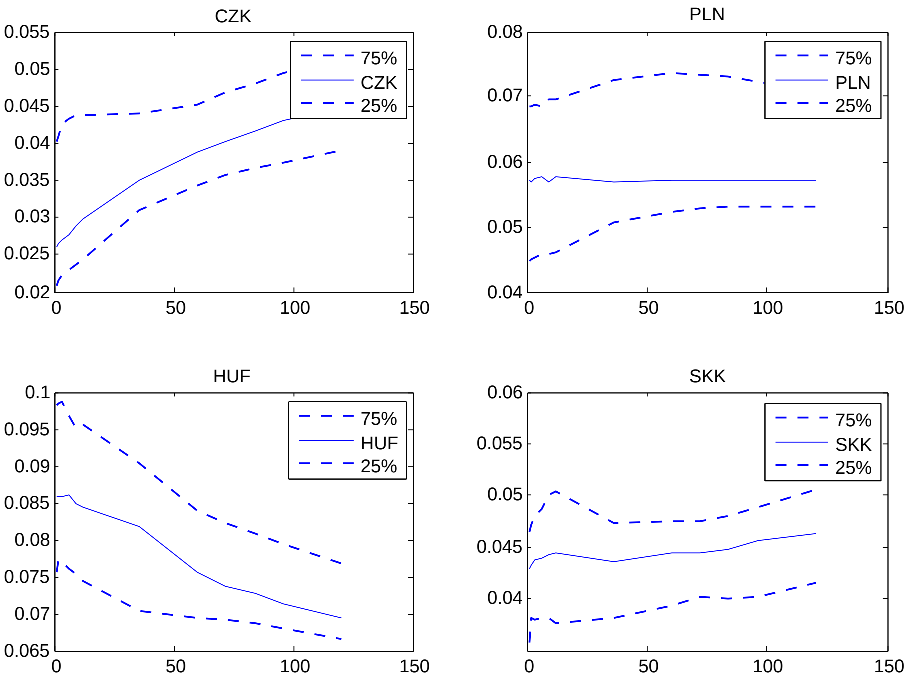
methodology, the state space framework models time-series in structural way explicitly (Durbin and Koopman 2001, p. 52).

For the purposes of this thesis; initially, we introduce the derivation of Kalman filter in section 3.2.1. To be able to perform diagnostic checks and further use the models for forecasting, we present Kalman Smoother and the forecasting routine in section 3.2.2. Section 3.2.3 then shows the likelihood function for state space models, diagnostics and structural break detection methods, which we use in the empirical part of the thesis in Chapter 4.

3.2.1 Kalman Filter Derivation

We use the state space framework with notation given in equations (3.3) throughout this thesis. The following derivation is based on Durbin and Koopman (2001) who give an excellent treatment of state space models, related issues

Figure 3.3: Regional Zero Yield Curves – Empirical Quartiles



x-axis denotes maturities τ in months; y-axis denotes interest rates

and provides tip for practical implementation.

$$
\begin{aligned}
y_t &= Z_t\alpha_t + \varepsilon_t, & \varepsilon &\sim \mathrm{N}(0, H_t), \\
\alpha_{t+1} &= T_t\alpha_t + R_t + \eta_t, & \eta &\sim \mathrm{N}(0, Q_t), \quad t = 1, \dots, n, \\
& & \alpha_1 &\sim \mathrm{N}(a_1, P_1), \qquad (3.3)
\end{aligned}
$$

where y_t is a vector of observation of yield curve in time $t = 1, \dots, n$. Moreover, we assume all matrices Z, T, R, H, Q time-invariant, hence we drop the time index below where possible. Note that the Principal Component Regional model needs some element of the transition matrix T to be time-varying, otherwise T is time-invariant.

Let Y_{t-1} denote all information available at time t; the set of past observation vectors $Y_{t-1} = \{y_1, y_2, \dots, y_{t-1}\}$. The idea is simple and stands on basic multivariate regression theorem in appendix on page I, we start at time $t = 1$ and recursively derive the distributions for α_t and y_t; moreover, as these are normally distributed, it is sufficient to derive mean and variance (Durbin and Koopman 2001). We endeavour to show that $p(y_t|\alpha_1, \dots, \alpha_t, Y_t) = p(y_t|\alpha_t)$ and $p(\alpha_{t+1}|\alpha_1, \dots, \alpha_t, Y_t) = p(\alpha_{t+1}|\alpha_t)$.

Furthermore, let the initial state for α_1 be $\mathrm{N}(a_1, P_1)$ and known. We seek conditional distributions for α_{t+1} given Y_t for $t = 1, \dots, n$. From the normality of all distributions follows that the conditional distributions of subset of variables given other subset of variables are also normally distributed (Durbin and Koopman 2001, p. 65). Hence the desired distributions are fully determined by knowing $a_{t+1} = \mathrm{E}(\alpha_{t+1}|Y_t)$ and $P_{t+1} = \mathrm{Var}(\alpha_{t+1}|Y_t)$. We substitute for α_{t+1} from equation (3.3),

$$\begin{aligned}
\alpha_{t+1} &= \mathrm{E}(T\alpha_t + R\eta_t|Y_t) = \\
&= \mathrm{E}(T\alpha_t|Y_t) + \mathrm{E}(R\eta_t|Y_t) = T\mathrm{E}(\alpha_t|Y_t), && (3.4) \\
P_{t+1} &= \mathrm{Var}(T\alpha_t + R\eta_t|Y_t) = \\
&= \mathrm{Var}(T\alpha_t|Y_t) + \mathrm{Var}(R\eta_t|Y_t) = T\mathrm{Var}(\alpha_t|Y_t)T' + RQR', && (3.5)
\end{aligned}$$

for $t = 1, \dots, n$. We denote one-step forecast error of y_t given Y_{t-1}

$$v_t = y_t - \mathrm{E}(y_t|Y_{t-1}) = y_t - \mathrm{E}(Z\alpha_t + \varepsilon_t|Y_{t-1}) = y_t - Z\alpha_t.$$

When Y_{t-1} and v_t are fixed then Y is fixed and *vice versa* (Durbin and Koopman 2001, p. 66); hence $\mathrm{E}(\alpha_t|Y_t) = \mathrm{E}(\alpha_t|Y_{t-1}, v_t)$. By contrast, $\mathrm{E}(v_t|Y_{t-1}) = \mathrm{E}(y_t - Z\alpha_t|Y_{t-1}) = \mathrm{E}(Z\alpha_t + \varepsilon_t - Z\alpha_{t-1}) = 0$, thus $\mathrm{E}(v_t) = 0$ and $\mathrm{Cov}(y_j, v_t) = \mathrm{E}[y_j \mathrm{E}(v_t|Y_{t-1})'] = 0$ for $j = 1, \dots, t-1$. The next step is done by applying the regression lemma A.1.1 in appendix on page I.

$$\begin{aligned}
\mathrm{E}(\alpha_t|Y_t) &= \mathrm{E}(\alpha_t|Y_{t-1}, v_t) = \\
&= \mathrm{E}(\alpha_t|Y_{t-1}) + \mathrm{Cov}(\alpha_t, v_t)[\mathrm{Var}(v_t)]^{-1} = \\
&= a_t + M_t F_t^{-1} v_t, && (3.6)
\end{aligned}$$

where $M_t = \mathrm{Cov}(\alpha_t, v_t)$, $F_t = \mathrm{Var}(v_t)$ and $a_t = \mathrm{E}(\alpha_t|Y_{t-1})$ is defined above. We proceed as follows

$$\begin{aligned}
M_t &= \mathrm{Cov}(\alpha_t, v_t) = \mathrm{E}\left[\mathrm{E}\{\alpha_t(Z\alpha_t + \varepsilon_t - Za_t)'|Y_{t-1}\}\right] = \\
&= \mathrm{E}\left[\mathrm{E}\{\alpha_t(\alpha_t - a_t)'Z'|Y_{t-1}\}\right] = P_t Z'
\end{aligned}$$

and

$$F_t = \mathrm{Var}(Z\alpha_t + \varepsilon_t - Za_t)) = ZP_tZ' + H.$$

Durbin and Koopman (2001) assume F_t to be nonsingular; generally this can be relaxed as it is shown further in Durbin and Koopman (2001), but for our model we assume it holds. We substitute in (3.4) and (3.6), which gives

$$\begin{aligned} a_{t+1} &= T\alpha_t + TM_tF_t^{-1}v_t = \\ &= Ta_t + K_tv_t, \qquad\qquad t = 1, \ldots, n, \end{aligned}$$

where

$$K_t = TM_tF_t^{-1} = TP_tZ'F_t^{-1}.$$

Now we apply the regression lemma A.1.1 again.

$$\begin{aligned} \mathrm{Var}(\alpha_t|Y_t) &= \mathrm{Var}(\alpha_t|Y_{t-1}, v_t) \\ &= \mathrm{Var}(\alpha_t|Y_{t-1} - \mathrm{Cov}(\alpha_t, v_t)[\mathrm{Var}(v_t)]^{-1}\mathrm{Cov}(\alpha_t, v_t)' \\ &= P_t - M_tF_t^{-1}M' \\ &= P_t - P_tZ'F_t^{-1}ZP_t'. \end{aligned}$$

Substituting in (3.5) gives

$$P_{t+1} = TP_tL_t' + R_tQ_tR_t', \quad t = 1, \ldots, n,$$

where $L_t = T - K_tZ$. The predictive Kalman filter is summarised in equations (3.7)

$$\begin{aligned} v_t &= y_t - Z\alpha_t, & F_t &= ZP_tZ' + H \\ K_t &= TP_tZ'F_t^{-1}, & L_t &= T - K_tZ, & t &= 1, \ldots, n, \\ a_{t+1} &= Ta_t + K_tv_t, & P_{t+1} &= TP_tL' + RQR', \end{aligned} \tag{3.7}$$

dimensions of these matrices are given in Table 3.5. The derivation of the contemporaneous filter is analogous and can be also found in Durbin and Koopman (2001). For completeness, equations (3.8) present the contemporaneous filter without derivation.

$$\begin{aligned} v_t &= y_t - Z\alpha_t, & F_t &= ZP_tZ' + H, \\ & & M_t &= P_tZ', & t &= 1, \ldots, n, \\ a_{t|t} &= a_t + M_tF_t^{-1}v_t, & P_{t|t} &= P_t - M_tF_t^{-1}M_t', \\ a_{t+1} &= Ta_{t|t}, & P_{t+1} &= TP_{t|t}L' + RQR'. \end{aligned} \tag{3.8}$$

For standard error estimation and other details see Durbin and Koopman (2001).

3.2.2 Smoothing & Forecasting

The universality of Kalman filter can be shown by the smoothing recursion and the forecasting abilities that are organic to state space models and that extend the possible applications.

Kalman Smoother The Kalman filter is a forward recursion evaluating one-step ahead estimators, whereas the associated moment smoothing algorithm is a backward recursion, which evaluates the mean and variance of specific conditional distributions given the data set Y_n. We are interested in calculating condition mean $\hat{\alpha}_t = \mathrm{E}(\alpha_t|Y_n)$ and conditional variance $V_t = \mathrm{Var}(\alpha_t|Y_n)$ given all information. The derivation relies only on the regression lemma A.1.1 and can be found in (Durbin and Koopman 2001, p. 70). Equation (3.9) presents only the final result known as Kalman smoother.

$$\begin{aligned} r_{t-1} &= Z'F_t^{-1}v_t + L_t'r_t, & N_{t-1} &= Z'F_t^{-1}Z + L_t'N_tL_t, \\ \hat{\alpha}_t &= a_t + P_tr_{t-1}, & V_t &= P_t - P_tN_{t-1}P_t,, \end{aligned} \tag{3.9}$$

for $t = n, \ldots, 1$ and initialised with $r_n = 0$ and $N_n = 0$, because there are no innovation at the end of the sample. Moreover, we assume $\alpha_1 \sim \mathrm{N}(a_1, P_1)$.

Furthermore, we need to compute the smoothed disturbances $\hat{\varepsilon}_t = \mathrm{E}(\varepsilon_t|Y_n)$ and $\hat{\eta}_t = \mathrm{E}(\eta_t|Y_n)$ that are use for parameter estimation and diagnostics. The backward recursion for disturbances is presented in equations (3.10).

$$\begin{aligned} u_t &= F_t^{-1}v_t - K_t'r_t, & D_t &= F_t^{-1} + K_t'N_tK_t, \\ \hat{\varepsilon}_t &= Hu_t, & \mathrm{Var}(\varepsilon_t|Y_n) &= H - HD_tH, \\ \hat{\eta}_t &= QR'u_t, & \mathrm{Var}(\eta_t|Y_n) &= Q - QR'D_tRQ, \end{aligned} \tag{3.10}$$

for $t = n, \ldots, 1$ and again initialised with $r_n = 0$ and $N_n = 0$.

Forecasting Knowing $y_1, \ldots, y_n$, we want to compute forecast y_{n+j} for $j = 1, \ldots, J$ given Y_n, which has the minimum square error matrix. We choose estimate $\bar{y}_{n+j}$ such as $\bar{F}_{n+j} = \mathrm{E}\left[(\bar{y}_{n+j} - y_{n+j})(\bar{y}_{n+j} - y_{n+j})'|Y_n\right]$ is minimal. The exact derivation can be found in (Durbin and Koopman 2001, p. 94).

The forecasting simplifies to continuation of Kalman filter as if the remaining observations were missing.

$$\bar{a}_{t+j+1} = T\bar{a}_{t+j}, \qquad \bar{P}_{t+j+1} = T\bar{P}_{t+j}L' + RQR', \tag{3.11}$$

for $t = 1, \ldots, n$, and $v_{n+j} = 0$ and $K_{n+j} = 0$, because there is no new information availble at time $n + j$.

3.2.3 Likelihood, Goodness-of-fit & Diagnostics

Likelihood Evaluation Once we know the parameters, Kalman filter provides very efficient and fast method of extracting the latent factors. Clearly, we do not know the parameters of the models *ex ante*, therefore we need to find them. There are not many methods that can deal with various space state models, so we employ with Maximum Likelihood estimation. Since the models are Gaussian and linear in parameters, we can easily build the log-likelihood function for the state space model as in equation (3.12) taken from Koopman et al. (1998, p. 138):

$$\log L(\psi) = \log p(y_1, \ldots, y_n; \psi) = -\frac{np}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{n}\left(\log|F_t| + v_t' F_t^{-1} v_t\right), \tag{3.12}$$

where ψ is a vector of parameters. The matrices v_t and F_t are computed by equations (3.7) and (3.8). There is no available closed-form solution for optimisation problem of such complex models; therefore, the log-likelihood function needs to be maximised with numerical methods. We use a very powerful method called BFGS based on the well-known modified Newton's method (Brinkhuis and Tikhomirov 2005, p. 300) and named after its inventors Broyden–Fletcher–Goldfarb–Shanno method. This method is available in the Ox package (Doornik 2007) as *MaxBFGS* function.

The goodness-of-fit measures and diagnostic tests can be obtained using *standard* tools in univariate case with only little adjustments, whereas in case of multivariate models our set of tools becomes quite limited. We focus on selection criteria and Likelihood ratio test when specifying the models and we look at parameter t-tests and structural break detection.

Selection Criteria The well-known and popular selection criteria AIC (Akaike Selection Criterion) and BIC (Bayes Selection Criterion) can be applied to

Table 3.5: Kalman Filter Dimensions

vector	size	matrix	size
y_t	p×1	Z	p×m
α_t	m×1	T	m×m
ε_t	p×1	H	p×p
η_t	r×1	R	m×r
y_t	p×1	Q	r×r
		P_t	m×m
v_t	p×1	F_t	p×p
a_t	m×1	K_t	m×1
		L_t	m×m

p is a number of maturities, m is a number of factors and r is assumed to be equal to m.

Source: Durbin and Koopman (2001)

state space models in a modified form. The modifications account for multivariate nature of the models. Let $\hat{\psi}$ denote the estimate of the vector of parameters, then the criteria are presented in equation (3.13) (Harvey 2002, p. 270).

$$\text{AIC} = -2\log L(\hat{\psi}) + 2(k+d), \quad \text{BIC} = -2\log L(\hat{\psi}) + (k+d)\log n, \tag{3.13}$$

where k, d and n represent the number of parameters to be estimated, number of non-stationary elements in state vector and number of observations, respectively. The aim is to find a specification in such a way to minimise the selection criteria.

Likelihood Ratio This test is built up on the property that large sample maximum likelihood estimates of parameter vector $\hat{\psi}$ is normally distributed as in equation (3.14) (Durbin and Koopman 2001, p. 150).

$$\psi \sim \text{N}(\psi, \Omega), \tag{3.14}$$

where Ω is the Hessian of the log-likelihood function.

$$\Omega = \left[-\frac{\partial^2 \log L}{\partial\psi\partial'\psi}\right]^{-1}. \tag{3.15}$$

Since the ratio of the squares of two random variables with standard normal distribution has χ^2 distribution with two degrees of freedom. Therefore, the ratio of the squares of the $\log L$ has χ^2 distribution with degrees of freedom equal to $df_2 - df_1$, where df_i is number of degrees of freedom for the i-th competing model, which represents number of parameters that needs to be fixed to obtain the simpler model from the more complicated one. The LR statistics has under null hypothesis that the restricted model fits better χ^2 distribution as in equation (3.16).

$$LR = -2 \log \left(\frac{L(\hat{\psi}_1)}{L(\hat{\psi}_2)} \right) \sim \chi^2_{(df_1 - df_2)}, \tag{3.16}$$

where $L(\hat{\psi}_1)$ denotes the maximised value of likelihood function of the restricted model and $L(\hat{\psi}_2)$ of the unrestricted. The crucial assumption for Likelihood ratio test is that we consider nested models, in other words such models that only by setting some parameters to 0 in one of them we get the other. Furthermore, throughout this thesis the standard errors are based on the Hessian of log-likelihood function.

t-test & Structural Breaks The structural breaks can be detected via so-called auxiliary residuals that essentially take form of standardised residuals associated with both observation and state equations, which we compute as in equation (3.17) and (3.18). Furthermore equation (3.19) suggests that matrix B is a diagonal matrix.

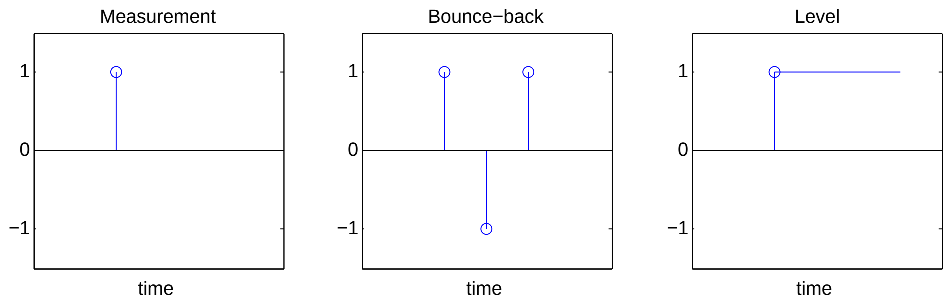
$$\hat{\varepsilon}_t^s = \frac{\hat{\varepsilon}_t}{\sqrt{B_t^\varepsilon}} \qquad [\mathrm{Var}(\hat{\varepsilon})]^{-1} = B_t^{\varepsilon\prime} B_t^\varepsilon \tag{3.17}$$

$$\hat{\eta}_t^s = \frac{\hat{\eta}_t}{\sqrt{B_t^\eta}} \qquad [\mathrm{Var}(\hat{\eta})]^{-1} = B_t^{\eta\prime} B_t^\eta \tag{3.18}$$

$$P_t^{-1} = B_t{}' B_t \tag{3.19}$$

Plotting $\hat{\varepsilon}_t^s$ and $\hat{\eta}_t^s$, we can visually assess presence of outliers and structural breaks in latent factors, respectively. Relatively large values indicate an outlier in case of $\hat{\varepsilon}_t^s$ and a structural break in case of $\hat{\eta}_t^s$ (Doornik 2007, pp. 35–37). As pointed out in Doornik (2007), de Jong and Penzer (2000) discusses a general approach to detecting structural breaks in state space models. Based on this approach, we consequently include impulse intervention into the model. The impulse interventions simply take form of a dummy variable and we denote its coefficient as β_A further in the thesis. In the simplest case, we include a variable equal to zero everywhere but at time t, where we visually detected a possible structural break. Such a dummy would measure the break magnitude, thus we call it a measurement impulse dummy (de Jong and Penzer 2000, p. 6). We can also impose different structure to study various breaks such as bounce-back[1] or level dummies, whose structure is plotted in Figure 3.4.

Figure 3.4: Impulse Dummies



Significance of such a dummy coefficient can be tested using the t-test based on equation (3.20).

$$\hat{\varepsilon}^s_{i,t} = \frac{\hat{\varepsilon}_{i,t}}{\sqrt{B^{\varepsilon}_{ii,t}}} \sim t_\alpha(n-1), \tag{3.20}$$

where $\hat{\varepsilon}_{i,t}$ denotes the i-th element of vector $\hat{\varepsilon}$, $B^{\varepsilon}_{ii,t}$ denotes the i-th diagonal element of matrix B^{η}_t and $t_\alpha(n-1)$ represents the α quantile of student's t-distribution with $n-1$ degrees of freedom (Doornik 2007, p. 37).

3.3 Principal Component Analysis

In this section, we shortly review a popular method to reduce dimension of data. This method also allows us to extract principal component, which is an orthogonal projection of the original data that maximises component variance. The basic idea to choose a number (lesser than number of original time-series) components with largest variance in such a way that these chosen components explain certain amount of variance in the data.

[1]Including only one positive and one negative dummy is called a switch (de Jong and Penzer 2000, p. 6)

The approach presented in this thesis is based on Tsay (2002), where one can find additional information not presented here. The principal component analysis is performed on covariance or correlation matrix of the returns. Let $\mathbf{r}$ denote a matrix of return series $\mathbf{r} = (r_1, \ldots, r_k)'$, where k is a number of series. Denote the covariance matrix $\mathbf{\Sigma}_r$ and let $\mathbf{w}_i = (w_{i1}, \ldots, w_{ik})'$ be a k-dimensional vector, where $k = 1, \ldots, k$. Then

$$y_i = \mathbf{w}_i'\mathbf{r} = \sum_{j=1}^{k} w_{ij} r_j$$

is linear combination of returns $\mathbf{r}$. Multiplying the vector $\mathbf{w}_i'$ by a constant does not change the proportions of $\mathbf{w}_i'$, we can standardise the vector $\mathbf{w}_i'$ so that $\mathbf{w}_i'\mathbf{w}_j = 1$.

Knowing the properties of a linear combination of random variables, we have

$$\mathrm{Var}(y_i) = \mathbf{w}_i'\mathbf{\Sigma}_r\mathbf{w}_i, \quad i = 1, \ldots, k \tag{3.21}$$

$$\mathrm{Cov}(y_i, y_j) = \mathbf{w}_i'\mathbf{\Sigma}_r\mathbf{w}_j, \quad i, j = 1, \ldots, k. \tag{3.22}$$

What PCA does is essentially spectral-decomposition of the covariance matrix. Since the covariance matrix $\mathbf{\Sigma}_r$ is non-negative definite, it has a spectral decomposition. The result is a diagonal covariance matrix of the of the matrix of the new linear combination and a matrix of eigenvectors. In other words, we are looking for linear combinations $\mathbf{w}_i$ such that y_i and y_j are uncorrelated for $i \neq j$ and the variances of y_i are as large as possible. Tsay (2002, p. 421) sums this up as followings:

1. The first principal component of $\mathbf{r}$ is the linear combination $y_1 = \mathbf{w}_1'\mathbf{r}$ that maximises $\mathrm{Var}(y_1)$ subject to the constraint $\mathbf{w}_1'\mathbf{w}_1 = 1$.

2. The second principal component of $\mathbf{r}$ is the linear combination $y_2 = \mathbf{w}_2'\mathbf{r}$ that maximises $\mathrm{Var}(y_2)$ subject to the constraint $\mathbf{w}_2'\mathbf{w}_2 = 1$ and $\mathrm{Cov}(y_1, y_2) = 0$.

3. The first principal component of $\mathbf{r}$ is the linear combination $y_i = \mathbf{w}_i'\mathbf{r}$ that maximises $\mathrm{Var}(y_i)$ subject to the constraint $\mathbf{w}_i'\mathbf{w}_i = 1$ and $\mathrm{Cov}(y_i, y_j)$ for $j = 1, \ldots, i-1$.

Let $(\lambda_1, \mathbf{e}_1), \ldots, (\lambda_k, \mathbf{e}_k)$ be the eigenvalue-eigenvector pairs of $\mathbf{\Sigma}_r$ in descending order. Then Tsay (2002, p. 422) presents the following results in equations (3.21) and (3.22).

$$\mathrm{Var}(y_i) = \mathbf{e}_i' \boldsymbol{\Sigma}_r \mathbf{e}_i = \lambda_i, \quad i = 1, \ldots, k \tag{3.23}$$

$$\mathrm{Cov}(y_i, y_j) = \mathbf{e}_i' \boldsymbol{\Sigma}_r \mathbf{e}_j = 0, \quad i \neq j. \tag{3.24}$$

In addition, it holds

$$\sum_{i=1}^{k} \mathrm{Var}(r_i) = \mathrm{tr}(\boldsymbol{\Sigma}_r) = \sum_{i=1}^{k} \lambda_i = \sum_{i=1}^{k} \mathrm{Var}(y_i). \tag{3.25}$$

The equation (3.25) implies how we can evaluate how much a single component adds to the total variance in equation (3.26).

$$\frac{\mathrm{Var}(y_i)}{\sum_{i=1}^{k} \mathrm{Var}(y_i)} = \frac{\lambda_i}{\lambda_1 + \cdots + \lambda_k} \tag{3.26}$$

Hence, we have the tools to perform the PCA.

For additional discussion on PCA, we refer to Tsay (2002). For an application to yield curve modelling see, amongst others, Litterman and Scheinkman (1991), who were the first to interpret the three principal components as level, slope and curvature of yield curve, and Rodrigues (1997) gives a methodology to design stress scenarios. Finally, Diebold et al. (2008) uses PCA to extract global yield curve components. We apply this notion in section 4.2.2 to regional yield curve.

Chapter 3 showed necessary statistics and econometrics so the we can proceed to introduce the core and original models of the thesis in Chapter 4 and their estimation in Chapter 5

Chapter 4

The Model

4.1 Nelson-Siegel Model

4.1.1 Classical Nelson-Siegel Model

This section builds the dynamic version of Nelson and Siegel (1987) model as developed in Diebold et al. (2006). Nelson and Siegel (1987) study term structure models and propose static model based on three exponential components. Having parsimony in mind, Nelson and Siegel (1987) put forward a model with only 3+1 parameters to capture the entire yield curve. This setting can reproduce all stylised facts about yield curve shape and behaviour such as: monotonic, humped and S-shaped. Over the time this model has become well-known; mainly as a yield curve smoothing tool (Diebold and Li 2006).

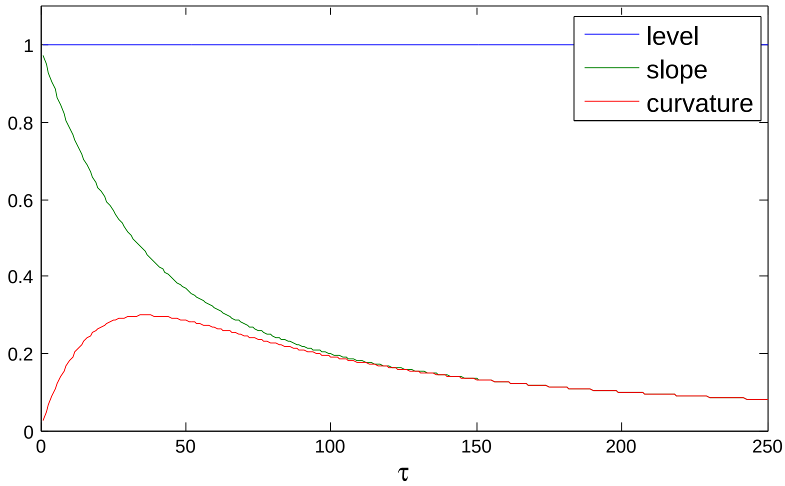
We use the specification given in Diebold et al. (2006), who restate the classical Nelson-Siegel model as in equation (4.1) and emphasize the advantage that the corresponding coefficient can be interpreted as level, slope and curvature without any further rotation.

$$y(\tau) = \beta_1 + \beta_2\Big(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\Big) + \beta_3\Big(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\Big), \tag{4.1}$$

where τ is the maturity in months and the parameter λ specifies where loadings on β_3 reach its peak. The factor loadings for various τ and fixed $\lambda = 0.0609$, which correspond to $\tau = 29.88$, are in Figure 4.1. In this framework, the level factor β_1 loads equally on all maturities and can be directly interpreted in corresponding interest rates. The slope factor β_2 loads mostly on short maturities with exponentially decaying influence on longer-term rates. The coefficient β_2

represents slope and its rate of decay is determined by the coefficient λ[1]. Lastly, the curvature factor β_3 loadings reach maximum in mid-term maturities, which is determined by λ. Importantly, these factor loadings determine the shape of the designed yield curve shocks.

Figure 4.1: Factor Loadings



Moreover, this framework is being further developed; e.g. Christensen et al. (2009) modified the model to become arbitrage-free. Recently, Jungbacker et al. (2009) implemented findings in efficient estimation of state space models of Jungbacker and Koopman (2008) in order to model term structure in similar fashion to Diebold and Li (2006) and Diebold et al. (2006).

4.1.2 Dynamic Nelson-Siegel & State Space Framework

In this section we present the dynamic Nelson-Siegel model specified and estimated using state space framework as presented in Diebold et al. (2006). This framework allows for clear statistical inference (Diebold et al. 2006) and produces correct standard errors, which can further be used for testing purposes and optionally for constructing yield curve shocks.

The state space formulation used in this paper is stated in equation (4.2),

[1] More details on λ computation can be found in section 4.4

which is essentially a factor model with vector AR process for factors.

$$\begin{aligned} y_t &= Z_t\alpha_t + \varepsilon_t, & \varepsilon &\sim \mathrm{N}(0, H_t), \\ \alpha_{t+1} &= T_t\alpha_t + R_t + \eta_t, & \eta &\sim \mathrm{N}(0, Q_t), \quad t = 1, \ldots, n, \\ & & \alpha_1 &\sim \mathrm{N}(a_1, P_1), \end{aligned} \tag{4.2}$$

where $y_t = (y_t(\tau_1), \ldots, y_t(\tau_p))'$ is a $p \times 1$ vector of yields of p different maturities, α_t a 3×1 vector of common factors, Z_t is a $p \times 3$ matrix of factor loadings, T_t is a 3×3 transition matrix and R_t is assumed to be a 3×3 diagonal matrix.

We denote the parameters of the Nelson-Siegel yield curve model as l_t, s_t and c_t, level, slope and curvature, respectively. Furthermore, we drop the time indices for time-invariant matrices to make the notation simpler and denote the common factors as l_t, s_t and c_t for level, slope and curvature factor respectively. The resulting model is in equations (4.3) and (4.4).

$$\begin{pmatrix} l_{t+1} - \mu_l \\ s_{t+1} - \mu_s \\ c_{t+1} - \mu_c \end{pmatrix} = \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}_{T} \begin{pmatrix} l_t - \mu_l \\ s_t - \mu_s \\ c_t - \mu_c \end{pmatrix} + \eta_t, \quad \eta \sim \mathrm{N}(0, Q) \tag{4.3}$$

We assume R to be a (3×3) unit matrix, yet we estimate also off-diagonal elements of state covariance matrix Q to capture possible correlation of disturbances η_t. Note that unlike principal components, the latent factors can be correlated in this specification. The observation equation is given in equation (4.4).

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \end{pmatrix}}_{Z} \begin{pmatrix} l_t - \mu_l \\ s_t - \mu_s \\ c_t - \mu_c \end{pmatrix} +$$

$$+ \varepsilon_t, \quad \varepsilon \sim \mathrm{N}(0, H), \tag{4.4}$$

where H is assumed to be diagonal, which implies uncorrelated disturbances in observation matrix. Diebold et al. (2006) stresses that the assumption of uncorrelated yields for different maturities is quite common. Moreover, it forces unobserved factors to capture the covariance between different maturities. The elements of the factor loadings matrix Z are functions of maturity τ and parameter λ, which we fix as specified in section 4.4.

We estimated the model using Kalman filter[2], which delivers mean square error estimates of each component in α_{t+1} even if the observations are not normally distributed, given the linearity of estimators (Koopman et al. 1998, p. 67). The unknown parameters needed for the recursion are estimated using Maximum likelihood estimation as described in section 3.2.3 or Durbin and Koopman (2001). In total we need to estimate 18+p parameters; 3 means in state equations, 9 elements in state transition matrix T, 3 diagonal elements and 3 off-diagonal elements in covariance matrix Q and p number of diagonal elements of in covariance matrix H for p number of maturities of yields. That means 31, 29 and 30 parameters for EUR, USD and GBP model, respectively, and 30 for CZK, HUF, PLN and SKK models. The implementation is described in section 4.4.[3]

We refer to this model as 'Dynamic Nelson-Siegel' model or we simply use abbreviation the DNS.

4.2 Regional Models

In this section we build the regional models: Regional Common Factor Model model and Principal Component Regional model. The former model is inspired by Diebold et al. (2008), yet we introduce some changes. Unlike single country models, the regional models combine several single currency models together to allow to extract regional common factors. Most importantly, we estimate the regional model at once, which is possible with additional restrictions on parameters. The latter model is built on the idea of the Regional Common Factor model. In contrast the Principal Component Regional model, we use extract the principal components of the countries' yield curves to include them in the model as regressors. Such a technique is, to the author's best knowledge, novel and possible to be estimated due to the state space formulation of the model.

[2]See section 3.2 or Durbin and Koopman (2001) for details

[3]The time needed to estimate each model did not exceed 80 seconds on an average computer. These times can be further improved by most recent findings in Jungbacker and Koopman (2008). As a result of the estimation, we extract the series of factors $l_{t|t-1}, s_{t|t-1}$ and $c_{t|t-1}$.

4.2.1 Regional Common Factor Model

Regional Common Factor Model follows the logic of Diebold et al. (2008) in a way it implicitly models the global / regional yield curve. We refer to our models as 'regional', because we use neighbouring currencies of data-set 2. Note that 'global' yield curve is not observed and luckily not needed to be modellled. The model as such essentially combines Dynamic Nelson-Siegel models for all four countries of the Central European region and models them as one model, which provides a clear inference thus can be estimated at once by Maximum likelihood estimator and Kalman filter as presented in section 3.2.

We begin by stacking the currency yield curves $y_{\text{currency},t(\tau_i)}$ in one (48×1) column vector $Y_t(\tau_i)$ as it can be seen in equation (4.5). The crucial part is to correctly build the (48×14) loadings matrix $\mathbb{Z}$ that translates the factors into yields. The new loadings matrix contains original currencies' loadings matrices on the diagonal and two columns of zeros, which means that the regional latent factors do not load on the yields directly, but though currency specific latent factors. This specification estimates 14 latent factors, summarised as α_t; for the implementation purposes, the model implicitly extracts 26 latent factors, since the factor means μ_{currency} are modelled as latent factors as well in order to ease the estimation. Equation (4.5) presents the observation equation, which is obviously just an ordinary observation equation in state space formulation, which we derived in section 3.2.

$$\underbrace{\begin{pmatrix} y_{CZK,t}(\tau_1) \\ y_{CZK,t}(\tau_2) \\ \vdots \\ y_{HUF,t}(\tau_1) \\ y_{HUF,t}(\tau_2) \\ \vdots \\ y_{PLN,t}(\tau_1) \\ y_{PLN,t}(\tau_2) \\ \vdots \\ y_{SKK,t}(\tau_1) \\ y_{SKK,t}(\tau_2) \\ \vdots \end{pmatrix}}_{Y_t(\tau)} = \underbrace{\left(\begin{array}{cccc|cc} Z & 0 & 0 & 0 & 0 & 0 \\ 0 & Z & 0 & 0 & 0 & 0 \\ 0 & 0 & Z & 0 & 0 & 0 \\ 0 & 0 & 0 & Z & 0 & 0 \end{array}\right)}_{\mathbb{Z}} \underbrace{\begin{pmatrix} l_{CZK,t} - \mu_{CZK} \\ s_{CZK,t} - \mu_{CZK} \\ c_{CZK,t} - \mu_{CZK} \\ l_{HUF,t} - \mu_{HUF} \\ s_{HUF,t} - \mu_{HUF} \\ c_{HUF,t} - \mu_{HUF} \\ l_{PLN,t} - \mu_{PLN} \\ s_{PLN,t} - \mu_{PLN} \\ c_{PLN,t} - \mu_{PLN} \\ l_{SKK,t} - \mu_{SKK} \\ s_{SKK,t} - \mu_{SKK} \\ c_{SKK,t} - \mu_{SKK} \\ \hline L_t \\ S_t \end{pmatrix}}_{\alpha_t} +$$

$$+ \varepsilon_t, \quad \varepsilon \sim \mathrm{N}(0, H), \tag{4.5}$$

Since we not only put together four times the DNS model, somehow larger changes are made to the state matrix. In order to simplify the notation of the state equation (4.9), we introduce a set of substitutions presented in equations (4.6), (4.7) and (4.8).

$$\alpha_t = \left(\begin{array}{c} \alpha_{CZE,t} \\ \alpha_{HUF,t} \\ \alpha_{PLN,t} \\ \alpha_{SKK,t} \\ \hline L_t \\ S_t \end{array} \right) = \left(\begin{array}{c} l_{CZK,t} - \mu_{CZK} \\ s_{CZK,t} - \mu_{CZK} \\ c_{CZK,t} - \mu_{CZK} \\ l_{HUF,t} - \mu_{HUF} \\ s_{HUF,t} - \mu_{HUF} \\ c_{HUF,t} - \mu_{HUF} \\ l_{PLN,t} - \mu_{PLN} \\ s_{PLN,t} - \mu_{PLN} \\ c_{PLN,t} - \mu_{PLN} \\ l_{SKK,t} - \mu_{SKK} \\ s_{SKK,t} - \mu_{SKK} \\ c_{SKK,t} - \mu_{SKK} \\ \hline L_t \\ S_t \end{array} \right) \tag{4.6}$$

In order to shorten the notation, equation (4.6) puts together the latent factors of each currency, equation (4.7) then summarises each currency's transition matrix and stresses that we estimate only diagonal elements. This simplification is rather technical and eases the optimisation of the log-likelihood function.

$$T_{\text{currency}} = \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix} \tag{4.7}$$

The state equation of this model needs to be adjusted in order to allow currency latent factors to load on regional factors. This is done by matrix Ψ_{currency}, which enters the full model transition matrix $\mathbb{T}$; equation (4.8) shows that we do not allow currency's level factor to load on regional slope factor and *vise versa*. There may be a natural relationship between the regional level and the country specific slope factors and *vise versa*, which we may miss. This drawback is addressed in the Principal Component Regional model in section 4.2.2.

Moreover, the coefficients of matrices Ψ_{currency} are restricted to be positive, which solves the possible identification problems. Note that identification is the main issue in these models; though, we do not need to worry too much about specification thanks to most-likely 'true' loading matrix Z.

The matrices Ψ_{currency} suggest that we do not assume any curvature factor to load on regional level or slope factors. The dynamics of the regional latent factor captures transition matrix ρ, in equation (4.8).

$$\Psi_{\text{currency}} = \begin{pmatrix} \psi_1 & 0 \\ 0 & \psi_2 \\ 0 & 0 \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \tag{4.8}$$

The state equation (4.9) containing all the substitutions explains the currency factors' dynamics similarly to the Dynamic Nelsol-Siegel model; as AR(1) processes. Furthermore, it also captures the regional factors' dynamics as vector AR(1) process, thus allowing for possible cross relations in regional factors.

$$\begin{pmatrix} \alpha_{CZE,t+1} \\ \alpha_{HUF,t+1} \\ \alpha_{PLN,t+1} \\ \alpha_{SKK,t+1} \\ \hline L_{t+1} \\ S_{t+1} \end{pmatrix} = \underbrace{\left(\begin{array}{cccc|cc} T_{CZE} & 0 & 0 & 0 & \multicolumn{2}{c}{\Psi_{CZE}} \\ 0 & T_{HUF} & 0 & 0 & \multicolumn{2}{c}{\Psi_{HUF}} \\ 0 & 0 & T_{PLN} & 0 & \multicolumn{2}{c}{\Psi_{PLN}} \\ 0 & 0 & 0 & T_{SKK} & \multicolumn{2}{c}{\Psi_{SKK}} \\ \hline 0 & 0 & 0 & 0 & \rho_{11} & \rho_{12} \\ 0 & 0 & 0 & 0 & \rho_{21} & \rho_{22} \end{array}\right)}_{\mathbb{T}} \begin{pmatrix} \alpha_{CZE,t} \\ \alpha_{HUF,t} \\ \alpha_{PLN,t} \\ \alpha_{SKK,t} \\ \hline L_t \\ S_t \end{pmatrix} +$$

$$+ R\eta_t, \quad \eta \sim \mathrm{N}\left(0, \begin{pmatrix} Q & 0 \\ 0 & \mathbb{I}_{(2\times 2)} \end{pmatrix}\right) \tag{4.9}$$

We assume R to be (14×14) unit matrix and we do not estimate the off-diagonal elements of state covariance matrix Q to keep a reasonable complexity of the model. Moreover, estimating the off-diagonal elements may lead to identification problems and to badly-behaved likelihood function.

Note the structure of the matrix $\mathbb{T}$, which in fact determines the whole dynamics of the model and shows the innovative methods of this thesis. Once again, in order to be able to identify the model, we introduce another restrictions on parameters. We assume the regional factors' covariance matrix to be (2) unit matrix. The pleasant side effect is that the magnitude of coefficients of matrices Ψ_{currency} can be directly interpreted as how much certain currency's factors load on regional factors.

4.2.2 Principal Component Regional Model

Principal Component Regional model further extends the Regional Common Factor model of section 4.2.1. This model also follows the logic of Diebold et al. (2008) in a way it implicitly models the global / regional yield curve, which as it is stated above cannot be observed. Unlike the Regional Common Factor model, the Principal Component Regional model does not extract the regional factors as latent factors, rather the model includes two other series to act as regional level and slope factors. These series are obtained as principal components extracted from all four yield curves using the Principal Component Analysis presented in section 3.3.

The main idea is to include the extracted regional principal components into the model as if these were explanatory variables in ordinary linear regression model. Diebold et al. (2008) use principal component analysis to obtain starting paths for Monte Carlo Markov Chain evaluation of likelihood function, whereas we include the paths directly into the state equation (4.14).

The observation equation (4.10) looks similar to the one of the Regional Common Factor model, yet with more latent factors in the vector α_t. The loadings matrix $\mathbb{Z}$ suggests that the regional factors do not load on the yields directly; moreover, the factors act as regression coefficients, i.e. they are time invariant.

$$\underbrace{\begin{pmatrix} y_{CZK,t}(\tau_1) \\ y_{CZK,t}(\tau_2) \\ \vdots \\ y_{HUF,t}(\tau_1) \\ y_{HUF,t}(\tau_2) \\ \vdots \\ y_{PLN,t}(\tau_1) \\ y_{PLN,t}(\tau_2) \\ \vdots \\ y_{SKK,t}(\tau_1) \\ y_{SKK,t}(\tau_2) \\ \vdots \end{pmatrix}}_{Y_t(\tau_i)} = \underbrace{\left(\begin{array}{cccc|c} Z & 0 & 0 & 0 & \mathbf{0}_{(12\times 8)} \\ 0 & Z & 0 & 0 & \mathbf{0}_{(12\times 8)} \\ 0 & 0 & Z & 0 & \mathbf{0}_{(12\times 8)} \\ 0 & 0 & 0 & Z & \mathbf{0}_{(12\times 8)} \end{array}\right)}_{\mathbb{Z}} \underbrace{\left(\begin{array}{c} l_{CZK,t} - \mu_{CZK} \\ s_{CZK,t} - \mu_{CZK} \\ c_{CZK,t} - \mu_{CZK} \\ l_{HUF,t} - \mu_{HUF} \\ s_{HUF,t} - \mu_{HUF} \\ c_{HUF,t} - \mu_{HUF} \\ l_{PLN,t} - \mu_{PLN} \\ s_{PLN,t} - \mu_{PLN} \\ c_{PLN,t} - \mu_{PLN} \\ l_{SKK,t} - \mu_{SKK} \\ s_{SKK,t} - \mu_{SKK} \\ c_{SKK,t} - \mu_{SKK} \\ \hline L_{CZK} \\ S_{CZK} \\ L_{HUF} \\ S_{HUF} \\ L_{PLN} \\ S_{PLN} \\ L_{SKK} \\ S_{SKK} \end{array}\right)}_{\alpha_t} +$$

$$+ \varepsilon_t, \quad \varepsilon \sim \mathrm{N}(0, H), \tag{4.10}$$

where H is assumed to be diagonal matrix and $\mathbf{0}_{(12\times 8)}$ denotes (12×8) zero matrix.

We again exploit the variability of State Space framework and incorporate this regression in such a way that the latent level factors L_{currency} and the latent slope factors S_{currency} become regression coefficients. We specify it in the following way; we assume the transition matrix $\mathbb{T}$ to have time-varying coefficients; furthermore, we assume regional latent factors to have a zero variance and be time-invariant, and most importantly the matrices $\Phi_{\text{currency},t}$ to carry the principal component paths. Hence, the matrices $\Phi_{\text{currency},t}$ are the time-varying elements in the matrix $\mathbb{T}$.

In order to simplify the notation of the state equation (4.14), we introduce a set of substitutions presented in equations (4.11), (4.12) and (4.13).

$$\begin{pmatrix} \alpha_{CZE,t} \\ \alpha_{HUF,t} \\ \alpha_{PLN,t} \\ \alpha_{SKK,t} \\ \hline L \\ S \end{pmatrix} = \begin{pmatrix} l_{CZK,t} - \mu_{CZK} \\ s_{CZK,t} - \mu_{CZK} \\ c_{CZK,t} - \mu_{CZK} \\ l_{HUF,t} - \mu_{HUF} \\ s_{HUF,t} - \mu_{HUF} \\ c_{HUF,t} - \mu_{HUF} \\ l_{PLN,t} - \mu_{PLN} \\ s_{PLN,t} - \mu_{PLN} \\ c_{PLN,t} - \mu_{PLN} \\ l_{SKK,t} - \mu_{SKK} \\ s_{SKK,t} - \mu_{SKK} \\ c_{SKK,t} - \mu_{SKK} \\ \hline L_{CZK} \\ S_{CZK} \\ L_{HUF} \\ S_{HUF} \\ L_{PLN} \\ S_{PLN} \\ L_{SKK} \\ S_{SKK} \end{pmatrix} \tag{4.11}$$

Note that the shortened matrix in equation (4.11) does not mean the same as the original one, because it shuffles order of elements. We choose this shorthand notation and bear in mind the correct order of elements.

$$T_{\text{currency}} = \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix} \tag{4.12}$$

The substitution for currency's transition matrix is the same as in previous case. The crucial matrix $\Phi_{\text{currency},t}$ is presented in equation (4.13), where the $\phi_{1,t}$ and $\phi_{2,t}$ are not coefficients in common sense, but they carry (all matrices the same) previously estimated principal components, the level and the slope principal components, respectively. Importantly, these coefficients occur on different places of the matrices $\Phi_{\text{currency},t}$. Therefore we can say that the matrices $\Phi_{\text{currency},t}$ have also a selection function to make sure that the regression coefficients L and S load on the correct country specific latent factors.

$$\Phi_{\mathrm{CZK},t} = \begin{pmatrix} \phi_{1,t} & 0 & \mathbf{0}_{(1\times6)} \\ 0 & \phi_{2,t} & \mathbf{0}_{(1\times6)} \\ 0 & 0 & \mathbf{0}_{(1\times6)} \end{pmatrix} \quad \Phi_{\mathrm{HUF},t} = \begin{pmatrix} \mathbf{0}_{(1\times2)} & \phi_{1,t} & 0 & \mathbf{0}_{(1\times4)} \\ \mathbf{0}_{(1\times2)} & 0 & \phi_{2,t} & \mathbf{0}_{(1\times4)} \\ \mathbf{0}_{(1\times2)} & 0 & 0 & \mathbf{0}_{(1\times4)} \end{pmatrix}$$
$$\Phi_{\mathrm{SKK},t} = \begin{pmatrix} \mathbf{0}_{(1\times6)} & \phi_{1,t} & 0 \\ \mathbf{0}_{(1\times6)} & 0 & \phi_{2,t} \\ \mathbf{0}_{(1\times6)} & 0 & 0 \end{pmatrix} \quad \Phi_{\mathrm{PLN},t} = \begin{pmatrix} \mathbf{0}_{(1\times4)} & \phi_{1,t} & 0 & \mathbf{0}_{(1\times2)} \\ \mathbf{0}_{(1\times4)} & 0 & \phi_{2,t} & \mathbf{0}_{(1\times2)} \\ \mathbf{0}_{(1\times4)} & 0 & 0 & \mathbf{0}_{(1\times2)} \end{pmatrix} \tag{4.13}$$

$$\begin{pmatrix} \alpha_{CZE,t+1} \\ \alpha_{HUF,t+1} \\ \alpha_{PLN,t+1} \\ \alpha_{SKK,t+1} \\ \hline L \\ S \end{pmatrix} = \underbrace{\left(\begin{array}{cccc|cc} T_{CZE} & 0 & 0 & 0 & \Phi_{CZE,t} & \\ 0 & T_{HUF} & 0 & 0 & \Phi_{HUF,t} & \\ 0 & 0 & T_{PLN} & 0 & \Phi_{PLN,t} & \\ 0 & 0 & 0 & T_{SKK} & \Phi_{SKK,t} & \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right)}_{\mathbb{T}} \begin{pmatrix} \alpha_{CZE,t} \\ \alpha_{HUF,t} \\ \alpha_{PLN,t} \\ \alpha_{SKK,t} \\ \hline L \\ S \end{pmatrix} +$$
$$+ R\eta_t, \quad \eta \sim \mathrm{N}\left(0, \begin{pmatrix} Q & 0 \\ 0 & \mathbf{0}_{(8\times8)} \end{pmatrix}\right) \tag{4.14}$$

We assume R to be (20×20) unit matrix and we do not estimate the off-diagonal elements of state covariance matrix Q to capture decrease complexity of the model. Moreover, estimating the off-diagonal elements may lead to identification problems and will lead to badly-behaved likelihood function.

4.3 Dynamic Nelson-Siegel Shocks

This section builds on novel approach to construct stress scenarios of Šopov (2009). The Dynamic Nelson-Siegel model and its state space formulation allows us to construct the 12-month forecast with ease, which is shown in equation (4.15).

$$\bar{\alpha}_{t+h+1} = T\mathrm{E}(\alpha_{t+h}|y) = T\bar{\alpha}_{t+h}, \tag{4.15}$$

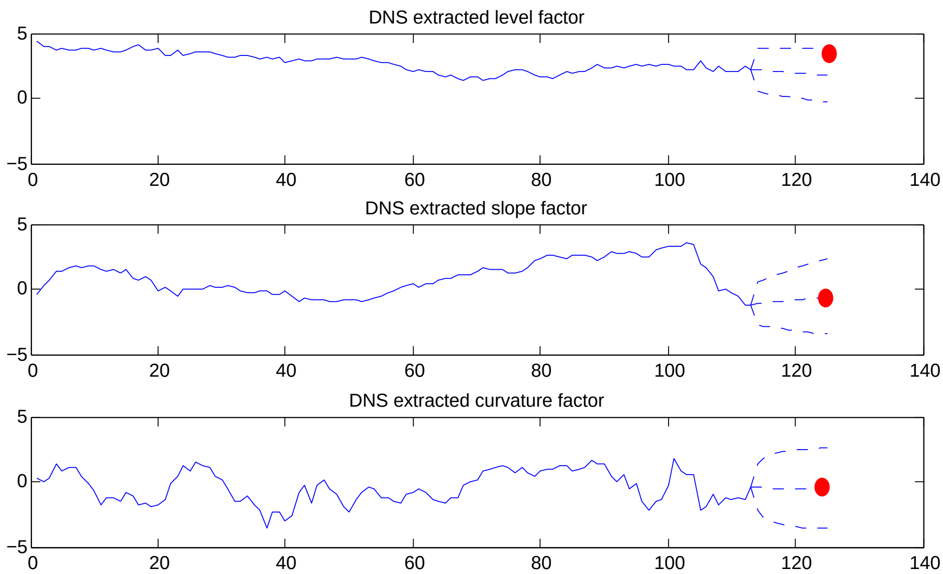
where $\bar{\alpha}_{t+h+1}$ represents the $h+1$-ahead forecast and $\mathrm{E}(\alpha_{t+h}|y)$ is the expected value of α_{t+h} given all observed information y. Hence the forecasting is essentially just continuation of Kalman recursion as if these were missing data (Koopman et al. 1998, p.94).

The shocks are then constructed similarly to building forecast intervals. The novel method presented in Šopov (2009) uses point forecasts for two factors and bounds of interval forecasts for the third factor. For example denoting $l_{t+h} + \mathrm{SE}(l_{t+h})\Phi^{-1}(1-p)$ as l^{+}_{t+h}, where l_{t+h} stands for h-period ahead forecast made in time t and $\mathrm{SE}(l_{t+h})$ represents its standard errors. Consequently, the shocked yield curve corresponding to the level shock is given by equation (4.16).

$$\begin{pmatrix} y^{+}_{level}(\tau_1) \\ y^{+}_{level}(\tau_2) \\ \vdots \end{pmatrix} = Z \begin{pmatrix} l^{+}_{t} - \mu_l \\ s_t - \mu_s \\ c_t - \mu_c \end{pmatrix} \tag{4.16}$$

Figure 4.2 presents the notion of choosing two point forecasts and one bound of interval forecasts to construct the shocked curve. The points used in equation (4.16) are marked with a red dot.

Figure 4.2: Yield Curve Shock Construction Method



4.4 Implementation

The estimation of the models introduced in this paper is a numerically challenging task, the proposed single country models imply optimisation over up to 31 variables.

To estimated the optimal parameter λ, which determines the loadings matrices Z, we use Matlab's default optimisation function *fminsearch* and following steps

1. fixing λ at 0.06 and fitting sequentially Nelson-Siegel curve with ordinary least squares,

2. taking OLS estimates for $\hat{\beta}_{i,t}$'s and λ_{t-1}[4] as starting values for the optimisation.

3. from the obtained series of sequentially optimal parameters λ, we take the mean, which will be used further in the thesis.

Since the DNS models are numerically more complicated to estimate, we perform the computation in econometric package Ox[5] using SsfPack 2.2[6].

Numerical optimisation is not a straight-forward method to solve arbitrary problem. Particularly, when optimising over large number of parameters, some reasonable starting values for the optimisation become crucial in order to estimate the model successfully. To estimate the 30 parameters and 6 latent factors[7], which each currency models has, we proceed with the following steps:

1. estimation of the model with diagonal transition matrix T,

2. setting the estimated coefficients of the restricted model as starting values for the optimisation.

This procedure improved the estimation times dramatically from hundreds of seconds to only tens of seconds.

[4]In time $t = 1$ we simply take $\lambda_1 = 0.06$.

[5]Ox is an object-oriented matrix programming language. We use Ox version 5.00 (Doornik 2007).

[6]SsfPack is an Ox package specialised for estimation of state space models. This solution has got two advantages: it offers complete set of function to work with state space models such as likelihood evaluation, and secondly the SsfPack is programmed in C, so it delivers very fast computation thus even larger models are feasible. For details see Koopman et al. (1998)

[7]We specified the models in such a way that the mean coefficient are modelled as latent factors with no dynamics and zero variance.

The estimation of the two regional models is even more complicated, yet we proceed in a similar way. The number of parameters would become rather high, if we estimated full models as in separate cases, so we restricted the transition matrices T to be diagonal.

1. estimation of the currency models with diagonal transition matrix T,
2. setting the estimated coefficients of the restricted model as starting values for the optimisation and setting the unknown coefficients close to 0,
3. using single currency estimates of covariance matrix $\hat{H}$ as 'true' values and not optimising over these parameters. This saves estimation of 48 parameters and considerably decreases the estimation time.

These procedures make sure that the optimisation converges. Furthermore, these two improvements are essential for practical implementation in industry and in further development of these models.

Chapter 5

Empirical Results

This chapter presents the estimation results of all the variations of Dynamic Nelson-Siegel model presented in chapter 4. The latent factors were extracted with Kalman filter and the model parameters were estimated with Maximum likelihood presented in section 3.2. The implementation details are described in section 4.4.

5.1 Dynamic Nelson-Siegel

5.1.1 Principal Component Analysis

In theory, the result of Litterman and Scheinkman (1991) suggests that yields with different maturities are driven by three common factors. Therefore by applying the principal component analysis on multivariate time-series of yield curves, we expect to obtain three components, which are supposed to explain over 80% of all variation in the data. These first three components are known as level, slope and curvature components, respectively. Usually, the level component explains majority of variation of yields (Diebold and Li 2006).

As we expected, our analysis confirms the results of Litterman and Scheinkman (1991) and as we can see in Table 5.1, the first three principal components do explain overwhelming majority of variations in the sample. In Table 5.1, there are relative and cumulative percentages of variation[1] explained by each component for each currency.

The level component delivers close to 93% of the variation in case of Czech Crown and Hungarian Forint, slightly above 91% in case of Slovak Crown and

[1] These percentages are computed by equation (3.25).

over 98% in case of Polish Zloty. The remaining two components are thus relatively lower in case of Zloty compared to other three currency zones. Czech Crown shows the highest percentage explained by the slope component suggesting more stable long-end of the curve over time as there is not much variation left for the curvature components. The curvature components are responsible for inflexion of the yield curve. High level of curvature changes can be observed in Figure 3.2 d) in case of Slovak Crown. The very same figure also helps to explain the large portion of variation explained by the first principal component of Polish yield curve, it's the large drop in yields, which occurred in 2002.

Table 5.1: Principal Component Analysis by Currencies

	PC1	PC2	PC3
CZK percentage	92.92%	6.32%	0.54%
CZK cumulative	92.92%	99.25%	99.79%
HUF percentage	92.97%	5.09%	1.38%
HUF cumulative.	92.97%	98.06%	99.44%
PLN percentage	98.37%	1.44%	0.15%
PLN cumulative	98.37%	99.82%	99.97%
SKK percentage	91.23%	5.13%	2.77%
SKK cumulative	91.23%	96.37%	99.14%

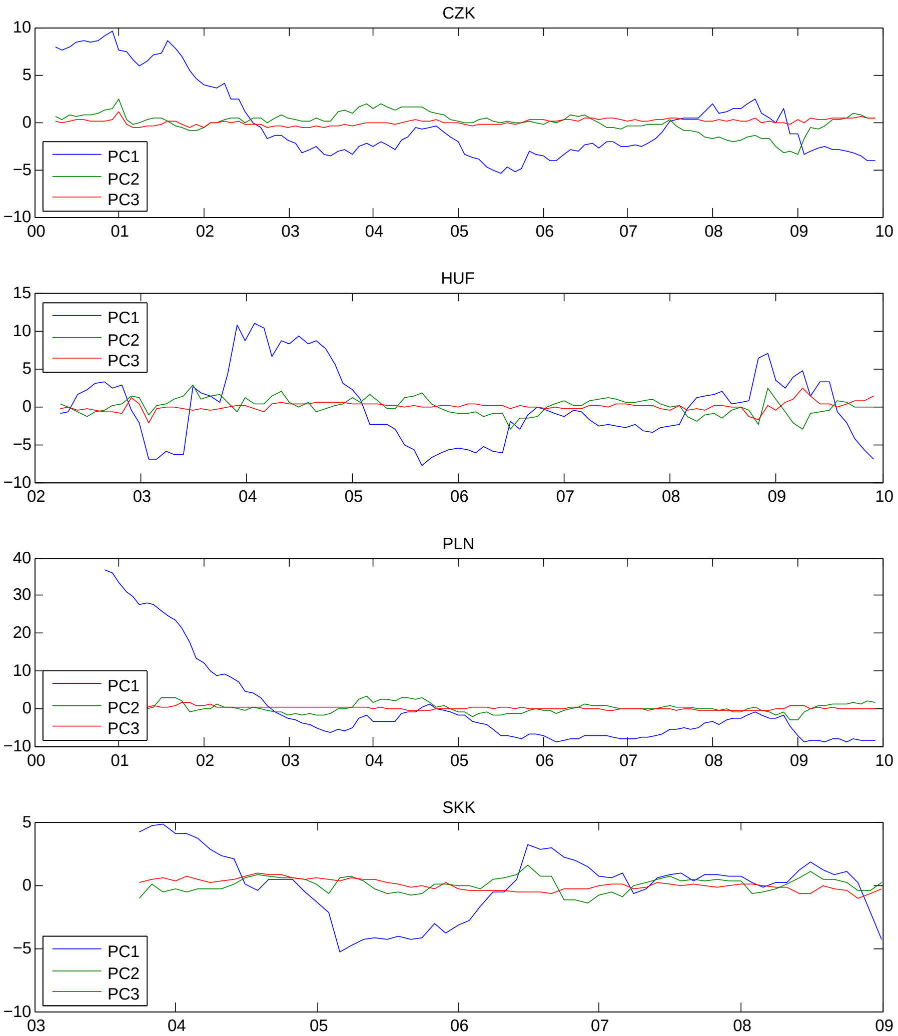
Source: Author's calculations

In Figure 5.1, we can see some similarities of the paths of the level components of Czech Crown and Polish Zloty. The latter one shows higher amplitude. All the four currency zones experienced large drop in yields at the end of 2008 and throughout the first half of 2009 caused by the spreading financial crisis.

5.1.2 Dynamic Nelson-Siegel Parameters

The estimated parameters and their standard errors are presented for clarity in matrices corresponding to the model formulation in equations (4.3) and (4.4). The estimates for CZK are in equations (5.1) and (5.2); for HUF in equations (5.3) and (5.4); for PLN in equations (5.5) and (5.6); and the results for SKK are in equations (5.7) and (5.8). The estimated diagonal elements of $\hat{H}$ matrices are presented as columns in Table 5.2. The last parameter of the Nelson-Siegel model λ takes these values: $\lambda_{CZK} = 0.0801$, $\lambda_{HUF} = 0.0891$, $\lambda_{PLN} = 0.0775$,

Figure 5.1: Principal Components of Yield Curves



Source: Author's calculations

$\lambda_{SKK} = 0.0702$.

$$\text{CZK}: \quad \hat{\mu} = \begin{pmatrix} -2.946 \\ (0.389) \\ 1.983 \\ (0.083) \\ 2.221 \\ (0.070) \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} 0.982 & 0.005 & -0.007 \\ (0.009) & (0.024) & (0.014) \\ 0.007 & 0.894 & 0.067 \\ (0.021) & (0.039) & (0.022) \\ -0.027 & 0.076 & 0.865 \\ (0.038) & (0.082) & (0.049) \end{pmatrix} \tag{5.1}$$

Studying closely the results, we see that diagonal elements of the $\hat{T}$ matrices dominate the other elements. Almost all factors seems quite persistent. The exceptions, yet still over 0.8, are the third diagonal elements, which determines the dynamics of curvature factor, in HUF currency zone and the second, which determines the dynamics of slope factor, in SKK currency zone. The diagonal elements are significantly different from 0 at 5% level of significance.

The first element on the diagonal mostly determining the dynamics of level factor is close to one in all currency zones, which indicates possibility non-stationary process. In fact, this would not be problem in state space framework, because it can handle non-stationary processes.

$$\text{CZK}: \quad \hat{Q} = \begin{pmatrix} 0.049 & -0.049 & 0.041 \\ -0.049 & 0.111 & -0.083 \\ 0.041 & -0.083 & 0.505 \end{pmatrix} \tag{5.2}$$

Lets look at each currency separately. In equation (5.2), we see an interesting structure of the CZK factor covariance matrix, where the slope factor is negatively correlated with the other two. Note that the magnitude of variances cannot be directly interpreted, because the level factor possesses larger loadings on all maturities, equal to 1, thus its variance seems relatively lower in comparison to the other two factors, although effectively capturing larger portion of variation in the data.

$$\text{HUF}: \quad \hat{\mu} = \begin{pmatrix} -2.088 \\ (0.461) \\ 0.009 \\ (0.201) \\ -1.070 \\ (0.236) \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} 0.997 & 0.050 & -0.042 \\ (0.016) & (0.023) & (0.017) \\ 0.016 & 0.806 & 0.132 \\ (0.044) & (0.052) & (0.037) \\ 0.061 & 0.152 & 0.690 \\ (0.084) & (0.120) & (0.089) \end{pmatrix} \tag{5.3}$$

In equation (5.3), we see that Forint's level factor coefficient in transition matrix T is close to one, which would mean it is driven by random walk process. Generally, all the level driving elements are not significantly different from 1.

The values of other two diagonal coefficients are evidence of AR(1) process that drives the slope and curvature factors. The estimate of covariance matrix $\hat{Q}$ in equation (5.4) exhibits the same structure as the one of CZK; the negatively correlated slope factor with the other two.

$$\text{HUF}: \quad \hat{Q} = \begin{pmatrix} 0.112 & -0.063 & 0.336 \\ -0.063 & 0.540 & -0.029 \\ 0.336 & -0.029 & 2.881 \end{pmatrix} \tag{5.4}$$

The parameters of the model for Polish yield curve, in equation (5.5), are to some extend affected by the large drop of the polish zero rates at the beginning of our sample. In comparison with the other three models, the Polish model has got relatively lower and negative mean parameters. We can see results of these differences in Figure 5.2, where we plotted the curves based only on the estimated mean parameters $\hat{\mu}$ as if the factors were equal to 0. The PLN curve is downward sloping and positioned at higher yields than the others.

$$\text{PLN}: \quad \hat{\mu} = \begin{pmatrix} -3.891 \\ (0.194) \\ -3.937 \\ (0.228) \\ -5.092 \\ (0.266) \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} 0.961 & 0.012 & -0.018 \\ (0.017) & (0.020) & (0.018) \\ 0.010 & 0.859 & 0.130 \\ (0.025) & (0.024) & (0.022) \\ -0.145 & -0.009 & 0.934 \\ (0.036) & (0.043) & (0.041) \end{pmatrix} \tag{5.5}$$

In equation (5.6), we see the same structure of covariance matrix $\hat{Q}$ as in case of CZK and HUF. Comparing the magnitude of the level variance, we see that the CZK zone seems to have the most stable yield curve and the Zloty's yields are by contrast the most varying. These results are confirmed by graphical analysis of the yield curve plots in Figure 3.2.

$$\text{PLN}: \quad \hat{Q} = \begin{pmatrix} 0.180 & -0.178 & 0.081 \\ -0.178 & 0.239 & -0.133 \\ 0.081 & -0.133 & 0.755 \end{pmatrix} \tag{5.6}$$

The Slovak Crown model exhibits similar parameter characteristics to the Czech

Crown model. In equation (5.7), we see a negative level factor mean, a relatively lower positive slope factor mean and a positive curvature mean, which leads to very similar mean yield curve in Figure 5.2.

$$\text{SKK}: \quad \hat{\mu} = \begin{pmatrix} -1.742 \\ (0.429) \\ 0.934 \\ (0.061) \\ 2.476 \\ (0.279) \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} 0.976 & 0.037 & 0.025 \\ (0.019) & (0.049) & (0.017) \\ 0.016 & 0.775 & 0.050 \\ (0.054) & (0.076) & (0.027) \\ 0.016 & 0.104 & 0.883 \\ (0.064) & (0.184) & (0.067) \end{pmatrix} \tag{5.7}$$

Unlike in other currency zones, the SKK covariance matrix $\hat{Q}$ has a different structure. In equation (5.8), we see that the curvature factor is negatively correlated with the other two. This suggests that an increase in the level factor is accompanied by a decrease in the curvature factor, which curbs the movements of mid-term rates and emphasises the shifts of short- and long-term rates. In other words, the upward shift goes together with inflexion and *vice versa*.

$$\text{SKK}: \quad \hat{Q} = \begin{pmatrix} 0.062 & -0.054 & -0.022 \\ -0.054 & 0.136 & -0.018 \\ -0.022 & -0.018 & 0.881 \end{pmatrix} \tag{5.8}$$

The estimates of covariance matrices $\hat{H}$ shows at which maturities the model corresponds the best to fitted yield curves. In section A.3 of Appendix, we present the estimates of the model based on currencies of data-set 1; for EUR, GBP and USD, which show similar results in terms of persistence of the factors and structure of the covariance matrices as in Diebold et al. (2006) and most importantly the parameter characteristics are almost identical to our regional currency models. The similarities are: the coefficients on the diagonal of transition matrices are close to 1 signalling high persistence in the process, the GBP model shows the same structure of the factor covariance matrix $\hat{Q}$ to the SKK model and the USD model possesses the same structure as the other three models. Table 5.3 presents the Likelihood ratio test for diagonality of covariance matrices $\hat{Q}$. We restrict the matrices $\hat{Q}$ to be diagonal and estimate the model once again to obtain the maximised value of the likelihood function of restricted models. We see that in all cases the extra three parameters in matrix $\hat{Q}$ add sufficient amount of log-likelihood, thus we can reject the null hypothesis at 5% level of significance in favour for the unrestricted models.

Table 5.2: Estimates of Diagonal Elemenets of $\hat{H}$

τ	$\hat{H}_{\text{CZK}}$	$\hat{H}_{\text{HUF}}$	$\hat{H}_{\text{PLN}}$	$\hat{H}_{\text{SKK}}$
1M	0.022	0.147	0.026	0.028
2M	0.009	0.040	0.005	0.004
3M	0.002	0.337	0.002	0.001
6M	0.013	0.064	0.007	0.034
9M	0.031	0.136	0.014	0.065
1Y	0.036	0.215	0.021	0.084
3Y	0.036	0.038	0.005	0.008
5Y	0.011	0.003	0.007	0.000
6Y	0.003	0.000	0.002	0.593
7Y	0.877	0.102	0.960	0.000
8Y	0.003	0.001	0.005	0.001
10Y	0.028	0.026	0.061	0.013

Source: Author's calculations

Slightly less significant results of the HUF and the SKK model caused the Bayes information criterion to prefer the restricted model, i.e. the criterion is lower for the restricted model; as it can be seen in Table 5.4.

Table 5.3: Diagonality of Q – Likelihood Ratio Test

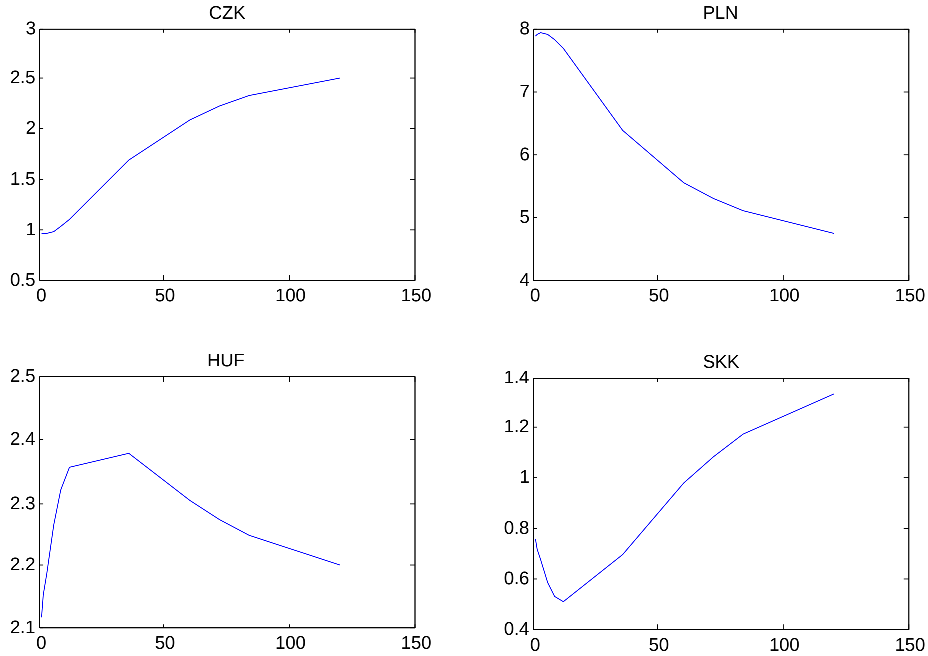
Currency	LR	p-value
CZK	31.171	+0.00
HUF	12.657	0.0054
PLN	59.043	+0.00
SKK	9.0492	0.0286
$\chi^2_3(0.975)$	9.3484	

Source: Author's calculations

5.2 Regional Models – Estimation

In this section, we estimate the regional models, yet we need to confirm our intuition first by performing the principal component analysis on the combined data-set. This data-set includes the four regional currency yield curves. We adjusted the sample length to fit the shortest sub-sample available, which is the SKK sample starting 30-Sep-2003 and ending 31-Dec-2008, when the Slovak

Figure 5.2: Regional Mean Yield Curves



x-axis denotes maturities τ in months; y-axis denotes yield rates in %

Republic accepted the common currency Euro. Hence, we work with sample of $n = 64$ observation and $p = 48$ maturities.

5.2.1 Principal Component Analysis – Regional Level

The results from the principal component analysis confirmed our intuition that the four regional currencies are driven by common factors. In Table 5.5, we show the results up to the fourth principal component. The first principal component (level) explains less variation of the data than in one currency only analysis. The second and the third component (slope and curvature, respectively)

Table 5.4: Selection Criteria

Currency	AIC	AIC_{res}	BIC	BIC_{res}
CZK	-871.93	-843.18	-579.77	-577.57
HUF	-149.00	-138.76	143.16	126.84
PLN	-521.43	-464.81	-229.27	-199.20
SKK	-405.88	-399.24	-113.71	-133.64

Source: Author's calculations

each explains over additional 10% of the variation in the data. Interestingly, the fourth principal component explains additional 5% of the variation, which can still be considered. We presume that this component gathers some regional specifics not significant or observable on country level.

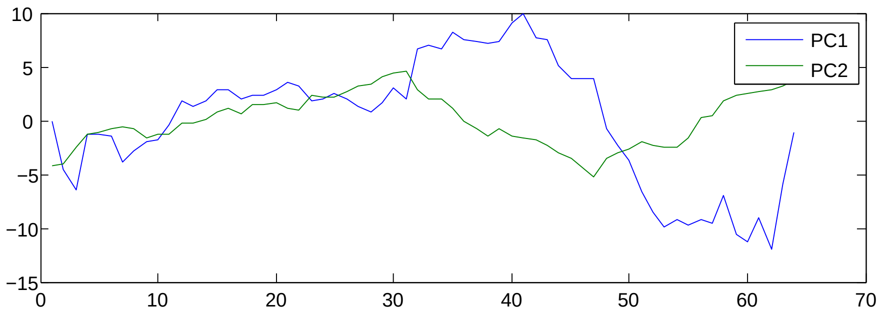
Table 5.5: Regional Principal Component Analysis

	PC1	PC2	PC3	PC4
Percentage	68.03%	11.89%	10.99%	5.65%
Cumulative	68.03%	79.92%	90.91%	95.56%

Source: Author's calculations

The Regional Common Factor Model, the way we specify it, requires two principal components as regressors. Therefore we include two strongest principal components known as the level and the slope components. These together capture near 80% of variation of the data. The paths of the two components are plotted in Figure 5.3.

Figure 5.3: Regional Principal Components



We use these results in both regional models. The Regional Common Factor Model is built with two regional common factors – level and slope, similarly the Principal Component Regional Model uses the paths of the two principal components as explanatory variables, which each currency sub-model can load on.

5.2.2 Regional Common Factor Model Parameters

The Regional Common Factor Model is estimated with method described in chapter 3. The estimation of the whole model takes about 8 minutes on an average computer, which may seem to be quite a long time, yet recall that the sample has 3072 data-entries and we estimate 37 parameters and extract 26 latent factors[2]. Moreover, due to implementation improvements presented in section 4.4, these times have been effectively decreased.

The resulting estimates of elements of transition matrix $\mathbb{T}$ and corresponding standard errors are in Table 5.6. For reader's convenience we put the diagonal elements into columns. The estimated means of currency specific factors and the estimates of currency loadings on regional factors is in Table 5.7.

Table 5.6: Estimates of Diagonal Elemenets of Transition Matrix $\hat{T}$

	CZK		HUF		PLN		SKK	
coeff.	est.	SE	est.	SE	est.	SE	est.	SE
a_{11}	1.005	0.006	1.003	0.007	0.996	0.009	0.991	0.011
a_{22}	0.944*	0.028	0.949	0.037	0.964	0.041	0.780*	0.083
a_{33}	0.891	0.067	0.832	0.102	0.952	0.044	0.818*	0.076

$*$ denotes element significantly different from 1 at 5% significance level.

Source: Author's calculations

As we can see in Table 5.6, when removing the off-diagonal elements from currency sub-models and introducing the regional common factor for level and slope, all diagonal elements increased. Most of the diagonal elements is not significantly different from 1, which suggest two things: the model can be estimated with restricting the diagonal elements to 1, which would mean assuming that the factors follow a random walk, and the regional common factors capture the 'predictable' dynamics.

The only exceptions are the slope AR(1) coefficient in the CZK sub-model and the slope and the curvature AR(1) coefficient in the SKK sub-model. The Czech Crown does not load on the regional slope factor at all, which keeps the currency slope AR(1) dynamics in place. In case of the SKK sub-model we see rather low loadings, which possibly still keeps the AR(1) process for currency specific factors.

The single currency model sign structure change in the regional model in case of Forint sub-model and most noticeably in case of Zloty sub-model. The

[2] 2×12 diagonal elements and variances of transition matrix $\mathbb{T}$ plus 4 elements of factor transition matrix ρ plus 1 corresponding covariance coefficient + 8 coefficients of matrices Φ = 37 parameters; 12 currency specific latent factors + 12 currency specific factors means modelled as latent factor with 0 variance + 2 regional factors = 26 latent factors.

Table 5.7: Estimates of Means $\hat{\mu}$ and Matrices $\hat{\Psi}$

	CZK		HUF		PLN		SKK	
coeff.	est.	SE	est.	SE	est.	SE	est.	SE
μ_l	-0.716	0.490	-2.123	0.497	-2.166	0.496	-1.477	0.483
μ_s	2.024	0.175	-0.879	0.413	1.296	0.370	0.393	0.046
μ_c	2.576	0.321	-0.201	0.405	1.898	0.453	0.368	0.256
ψ_1	0.027	0.930	0.196	0.196	0.395*	0.090	0.122	0.273
ψ_2	0.000	48.063	0.150	0.532	0.420*	0.091	0.030	1.685

* denotes element significantly different from 0 at 5% significance level.

Source: Author's calculations

latter sub-model estimate suggests change of sign of the slope and curvature mean. These changes can be explained while looking at the bottom part of Table 5.7, where we can see how much each currency's yield curve loads on the regional latent factors. To address the changes in Zloty sub-model; the PLN factors load the most on the regional level and slope factors. Moreover, these two coefficients are significantly different from 0.

The Czech Crown does not load on the regional slope factor at all and because the principal component analysis revealed important slope component, we conclude there are country specific slope driving forces. The CZK sub-model also has the lowest coefficient ϕ_1 and also the country specific means differ only slightly from the currency specific models.

Although lower than the PLN coefficients, the HUF sub-model loads relatively a lot compared to the CZK sub-model. The HUF and SKK sub-models follow similar pattern to the CZK model, with higher loadings on regional level factor.

Table 5.8: Regional Latent Factor Transition Matrix $\hat{\rho}$

	$\hat{\rho}$			$\widehat{\text{Cov}}(\rho)$	
ρ_{11}	-0.300	ρ_{12}	-0.150	1.	-0.841
	(0.257)		(0.261)		
ρ_{21}	0.620	ρ_{22}	0.458	-0.841	1.
	(0.259)		(0.264)		

The extracted regional factors are plotted in Figure A.2. We see that the

Table 5.9: Estimates of Diagonal Elemenets of $\hat{Q}$

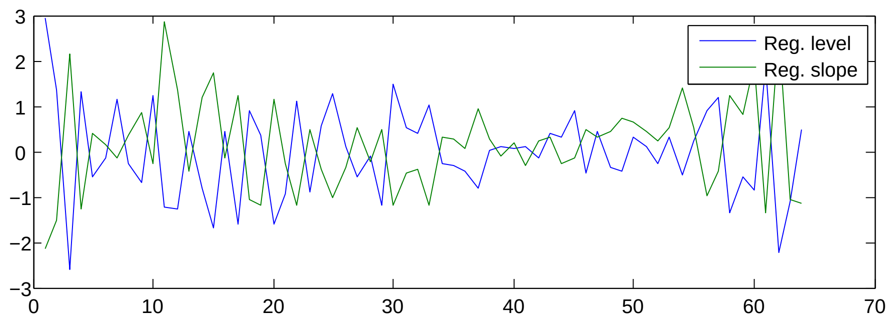
$\hat{Q}_{\text{CZK}}$	$\hat{Q}_{\text{HUF}}$	$\hat{Q}_{\text{PLN}}$	$\hat{Q}_{\text{SKK}}$
0.041	0.067	0.636	0.063
0.051	0.377	0.016	0.156
0.667	3.155	0.639	1.077

Source: Author's calculations

factors are implicitly forced to have a zero mean and are highly negatively correlated; see Table 5.8. Table 5.9 shows the estimated variances of currency specific factors associated with currency sub-models. Generally speaking, by introducing two regional common factors the currency specific factors' variance decreased. On the other hand, the variance of the curvature factor increased in all sub-models but the PLN one. The PLN sub-model experienced increase in the level factor variance and large decrease in the slope factor variance. Hence we conclude that the extracted regional factors are mostly influenced by Polish Zloty, whose impact is mostly on the regional curvature factor, which makes other currency load less on this regional factor and with higher errors.

To complete the analysis, we plotted the extracted latent factors for each currency zone in Figure A.2. When comparing graphically the extracted factors based on the DNS models and the Regional Common Factor model, we can see that the introduction of the regional factors absorbed some of the variance of the factors and that these currency specific level and slope factors of the regional model seem much smoother.

Figure 5.4: Regional Common Factors



5.2.3 Principal Component Regional Model Parameters

The Principal Component Regional Model is estimated in similar fashion as previous models with one difference: initially, we need to perform the principal component analysis to obtain the two principal component paths, which we use as regressors that represent the regional level and slope components. Despite the more elaborate notion behind this model, the estimation time is significantly lower; less than a minute. This fast estimation is due to lower number of parameters and orthogonal regressors. Moreover, we estimate only 24 parameters but 32 latent factors this time.[3]

Table 5.10 presents the estimates of the diagonal transition matrices. Nearly all the coefficient are not significantly different from 1 at 5% significance level. This result confirms similar result of the Regional Common Factor model; the model suggests that the introduction of regressors based on the first two principal component leads to factors, which are driven by random walk. The only exception are, the same as in the previous model, the coefficients driving the CZE slope factor and the SKK slope and curvature factors. These two exceptions signal country specific dynamics, whose predictability can be captured by AR(1) one specification and further used, p.e. forecasting. In other words, the regional components may absorb the predictability of present in the single currency models.

Table 5.10: Estimates of Diagonal Elemenets of Transition Matrix $\hat{T}$

	CZK		HUF		PLN		SKK	
coeff.	est.	SE	est.	SE	est.	SE	est.	SE
a_{11}	1.003	0.007	0.997	0.008	0.984	0.013	0.985	0.012
a_{22}	0.919*	0.033	0.948	0.037	0.984	0.063	0.784*	0.080
a_{33}	1.002	0.033	0.832	0.101	0.951	0.045	0.825*	0.078

$*$ denotes element significantly different from 1 at 5% significance level.

Source: Author's calculations

Since the principal components differ from the regional factors of the previous model, we can expect the results to differ as well. The mean coefficients

[3] 2×12 diagonal elements and variances of transition matrix $\mathbb{T}$ similar to the previous model = 24 parameters; 12 currency specific latent factors + 12 currency specific factors means modelled as latent factors with 0 variance + 8 latent factors with zero variance acting as currency loading coefficient on the two extracted principal components = 32 latent factors.

are not much of our interest, because the principal components do not have to have zero means, thus the currency specific mean coefficients have to balance this.

Nevertheless, the loadings coefficients or the regression coefficients in this case, are all significantly different from zero. This is probably caused by the fact that principal components are notionally the ideal regressor one can hope for: orthogonal to each other and capturing exhausting amount of variation in the data.

Table 5.11: Estimates of Means $\hat{\mu}$ and Currency Loadings L, S

	CZK		HUF		PLN		SKK	
coeff.	est.	SE	est.	SE	est.	SE	est.	SE
μ_l	-0.336	0.497	-2.150	0.500	-2.216	0.477	-1.479	0.465
μ_s	1.877	0.120	-0.875	0.411	1.297	0.478	0.383	0.049
μ_c	-0.140	0.520	-0.126	0.409	1.887	0.451	0.836	0.427
L	-0.156	0.003	-0.208	0.005	-0.137	0.008	0.184	0.038
S	-0.017	0.001	-0.196	0.010	0.183	0.004	-0.079	0.002

Source: Author's calculations

In Table 5.12 we can present the estimated of variances currency specific factors associated with the sub-models. Since the sub-models can load on the regional principal components also with negative sign the impacts on the factors' variance is ambiguous. The PLN sub-model shows general decrease in factor variation, whereas the variances of the SKK sub-model even increased. This leads us to similar conclusion as in the case of the RCFM. The Zloty yield curve is driven most in line with the regional yield curve, therefore the introduction of regional principal components decreased the Zloty sub-model variances. By contrast, the SKK yield curve is probably least bond to the regional yield curve.

Table 5.12: Estimates of Diagonal Elemenets of $\hat{Q}$

$\hat{Q}_{\text{CZK}}$	$\hat{Q}_{\text{HUF}}$	$\hat{Q}_{\text{PLN}}$	$\hat{Q}_{\text{SKK}}$
0.055	0.095	0.153	0.084
0.061	0.380	0.161	0.158
0.228	3.272	0.652	1.335

Source: Author's calculations

The results are notionally identical to the previous case confirming that the two specifications are relevant to yield curve modelling: low loadings in the CZK sub-model and high loadings in the HUF and the PLN sub-model.

In Table 5.11, we notice that the CZK sub-model has the lowest absolute magnitude of parameters compared to the other sub-model parameters. Introducing the regional principal components caused the HUF sub-model to load more on these components than on the regional common factors of the previous models. Furthermore, the HUF sub-model has both loading coefficients L_{HUF} and S_{HUF} negative as well as the CZK sub-model. We can thus expect both yield curves to behave in a similar fashion, yet Forint yield curve with higher amplitude.

Unfortunately, due to identification issues of the Regional Common Factor model and consequent restrictions to obtain positive loadings parameters, we cannot compare the signs of parameters between the models.

5.3 Application

In this section we present two possible applications of the models. To show the variability of the models and the state space formulation, we show one risk management example suitable for managing interest rate risk and one example applicable in general economics for analysing various time-series data. The former example introduces a straight-forward way how to design yield curve stress scenarios, which may be used to compute regulatory or economic capital. The latter shows how to detect structural breaks within the data.

5.3.1 Stress Scenarios Application

The interest rate risk management relies on well-estimated models, which we obtain with high quality data. For this reason, we proceed with EUR, GBP and USD zero rates due to higher liquidity of interest rate swaps denominated in these currencies. We exploit the outstanding forecasting properties of the DNS model found in Diebold and Li (2006) and Diebold et al. (2006) in combination with the stress scenario design approach of Šopov (2009).

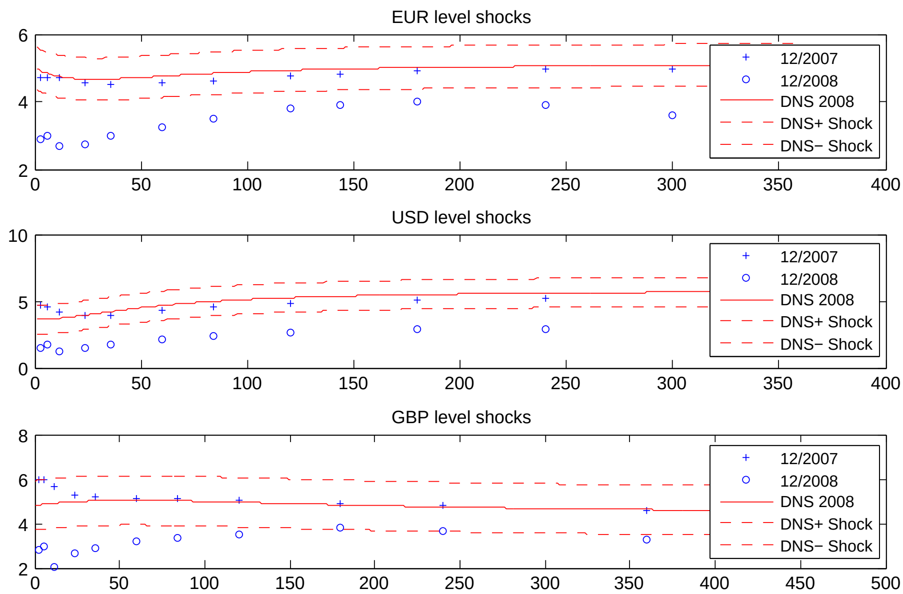
The estimates of EUR, GBP and USD models are presented in appendix A.3. Using these estimates, the forecasting algorithm of section 3.2.2 and stress curve design approach shown in section 4.3, we calculated the stressed curves as of December 2007 for the next 12 months at 99% confidence level.

The resulting stressed yield curves are plotted in Figures 5.5, 5.6 and 5.7. Each figure shows level, slope or curvature shocks for each currency zone in comparison with yield curves as of December 2007 and realised yield curve as of December 2008. The solid line denotes 12-month ahead forecast computed by equation (4.15). Clearly, the forecast could not capture such a massive drop experienced in 2008. The most successful is the forecast computed for the GBP zone, which predicted change in slope, or in other words predicted a moderate drop in short-term rates. The forecast for the USD zone also predicted a way of change of the short-term rates, yet by smaller amount.

We see how the shape of the stressed yield curves is fully determined by the Nelson-Siegel factor loadings. This leads to widest level shock and differently shaped slope and curvature shock. The level shock affects all maturities equally, the slope shock results in short-term rate movements and the curvature shock determines from shorter-term rates to mid-term rates, after which it starts to decay.

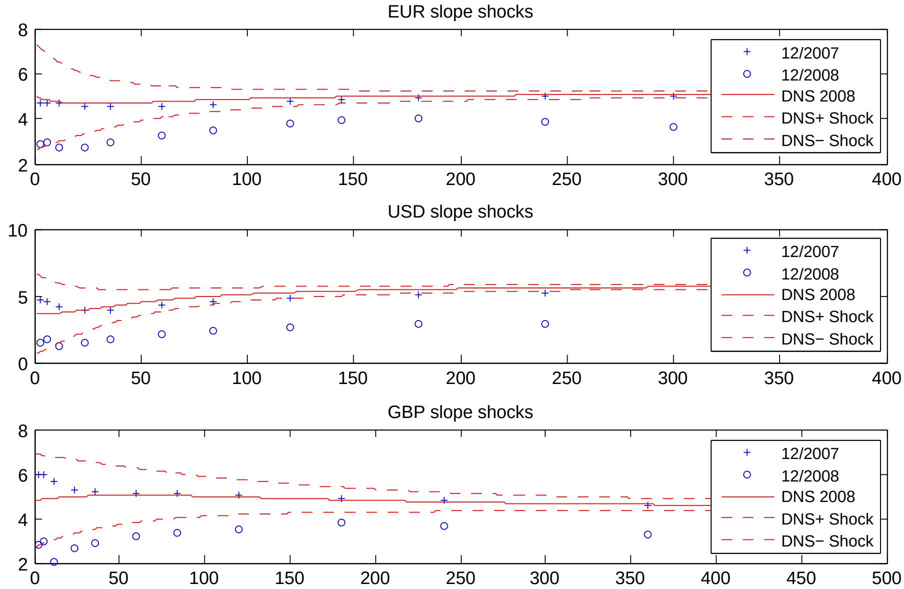
Considering the year 2008 to be an extreme year, these shocks were able to capture most of the price change of stylised portfolios as shown in Šopov (2009).

Figure 5.5: Level Shocks



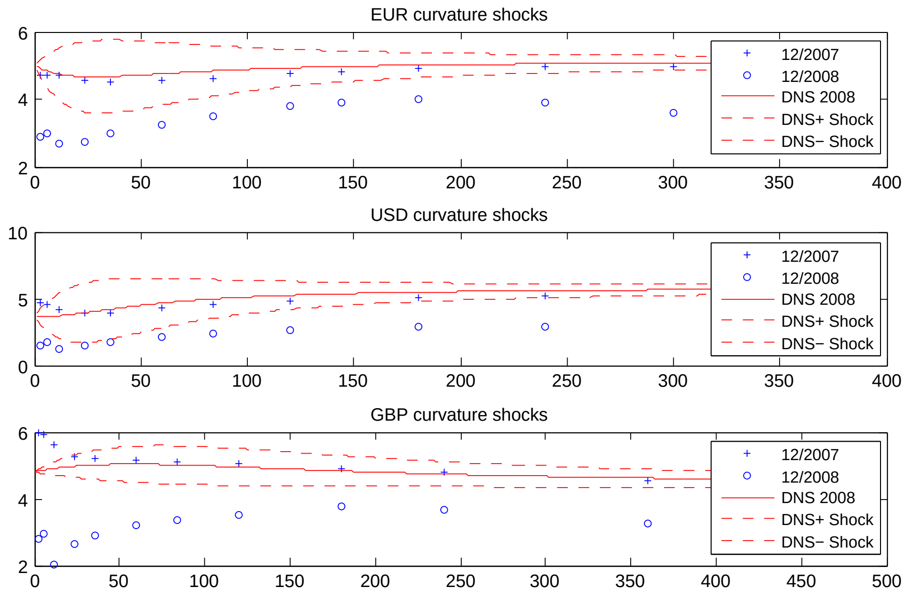
x-axis denotes maturities τ in months; y-axis denotes yield rates in %

Figure 5.6: Slope Shocks



x-axis denotes maturities τ in months; y-axis denotes yield rates in %

Figure 5.7: Curvature Shocks



x-axis denotes maturities τ in months; y-axis denotes yield rates in %

5.3.2 Structural Break Detection

This section introduces another application based on the abilities of state space framework. We show how to detect structural breaks in our single currency models of section 4.1.2. Using the method proposed in section 3.2.3, we first estimate the model and construct the auxiliary residuals of equations (3.17) and (3.18). The plots of the auxiliary residuals will uncover possible structural breaks.

We give examples of all regional currencies of data-set 2 and one currency model of data-set 1, which is the US dollar. Furthermore, we select two currencies for closer analysis, which would be CZK and for its high liquidity USD.

As discussed in section 3.2.3, we begin our analysis by computing and plotting the auxiliary residuals.[4] The plots of auxiliary residuals for CZK and USD are in Figure 5.8 and Figure 5.9 and for HUF, PLN and SKK in Figure A.3, Figure A.4 and Figure A.5.

By visual analysis we see that there are some relatively more extreme values in all of these plots. We selecte one suspicious point in time for all currencies and summarise them in the second columns of Table 5.13 and Table 5.14. In these tables, we see that we mostly selected the points of possible structural breaks associated with the slope factor. There is a very good explanation for this, this phenomenon is caused by the Nelson-Siegel loadings determining what factors drive what interest rates. The short-term rates are those most sensitive to shocks and because the slope factor loads mostly on these rates, it is very vulnerable to be affected. More persistent structural changes slowly affecting the whole yield curve would be reflected in the level factor fluctuations.

After selecting the possible breaks, we can proceed to include a dummy regressor in each single currency model. Estimating such models gives us the information about the size of the break and its significance. We chose the measurement impulse dummy for the HUF, PLN and SKK models, the switch impulse dummy for the CZK model and the bounce-back impulse dummy for the USD model.

Measurement Dummy The motivations is to show usage of different dummies giving different interpretations. In case of the HUF and PLN model we focused on the possible structural breaks associated with the outbreak of the crisis in September 2008[5]. In Table 5.13 we see that the dummy coefficient β_A

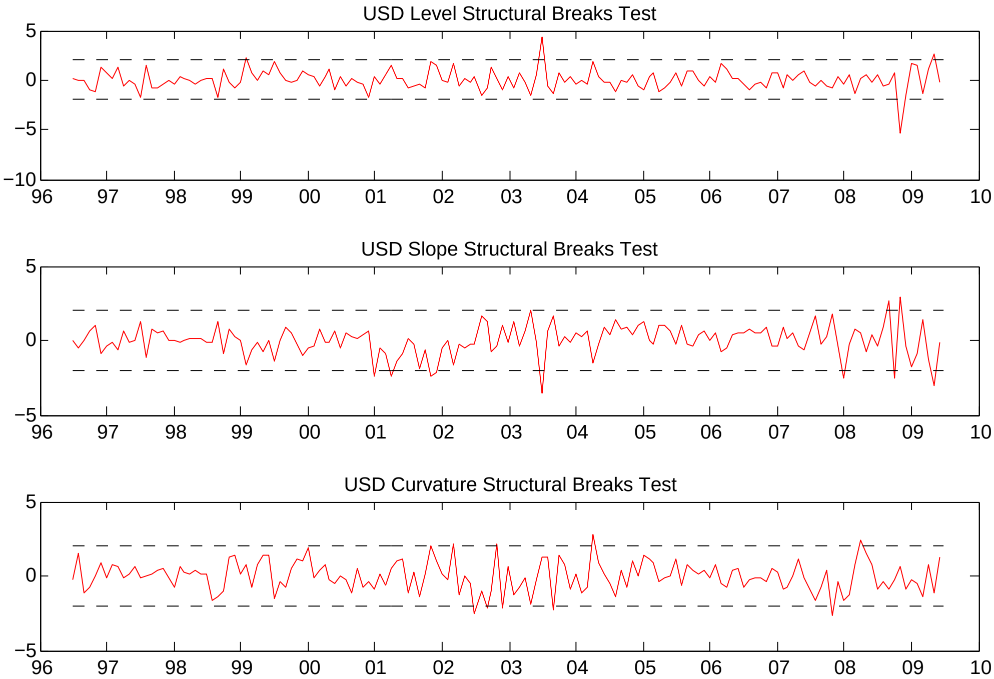
[4] Note that this computation can be done along with the estimation

[5] Lehman failed 15^{th} September 2008.

Figure 5.8: Auxiliary Residuals – CZK

x-axis denotes time in years

Figure 5.9: Auxiliary Residuals – USD



x-axis denotes time in years

takes large absolute values in both models, which means a sharp unexpected increase in Forint slope factor and a large unexpected drop in Zloty curvature factor. Both occurred in the by the end of the month of the crisis outbreak. The structural break detected in the SKK currency zone affected level factor of the whole yield curve by -0.597 and as we can see in Figure 3.2, it correctly matches with the time when the whole SKK yield curve started to rise. All β_A coefficients are significant at 1% level.

Table 5.13: Impulse Interventions Coefficient Significance

Type	Date	β_A	SE	t-stat	df	$t(0.95)$	p-val.
CZK Slope	Dec-2000	-1.107	0.018	-8.294	116	1.658	+0.00
HUF Slope	Sep-2008	2.180	0.292	4.032	92	1.662	+0.00
PLN Curv.	Dec-2008	-2.096	0.446	-3.139	109	1.659	0.001
SKK Level	Aug-2006	-0.597	0.021	-4.141	63	1.669	+0.00

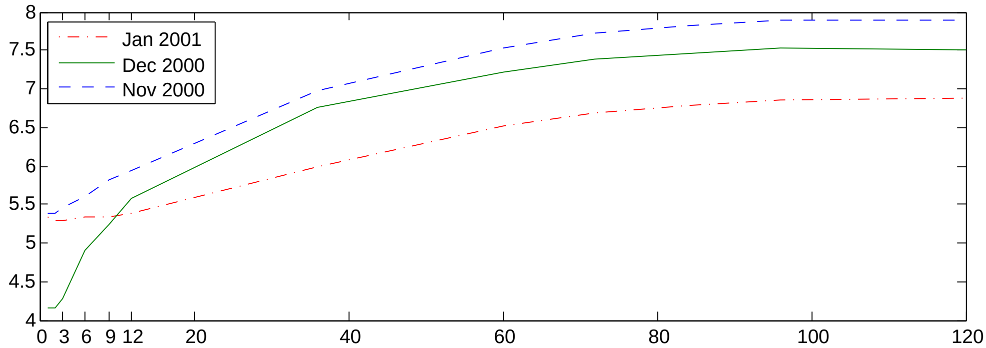
Source: Author's calculations

Switch & Bounce-back Dummies These two types of impulse response dummies can be used to describe better the dynamics of the structural breaks. One can combine different structures to analyse possible spill-overs between factors over time.

As noticed above, the CZK model is estimated with a switch dummy variable that has one at time t of suspicious break and -1 at time $t+1$. The estimation of the model including the dummy regressor delivered a significant estimate of the coefficient $\beta_A = -1.107$ precisely finding a temporary drop in the CZK short-term rates; see the first row of Table 5.13. The drop and the *switch* effect is plotted in Figure 5.10, where the yield curve as of November 2000 started to drop, yet the short-term rates dropped more and went up again in January of 2001, which shuffled with the yield curve's slope. This happened when the CZK crow reached its minimum USD exchange rate of 40.18 in last quarter of 2000.

Based on our visual analysis of auxiliary residuals of late 2008 in Figure 5.9, we decide to employ bounce-back impulse response dummy for USD model. The bounce-back structure can be seen best in slope factor residuals. Table 5.14 reports the results of three separate estimations, which specifies the bounce-back dummy each time for a different factor. In line with our visual analysis, we obtain highly significant non-zero estimate of coefficient β_A based on slope

Figure 5.10: Switch Effect – CZK



x-axis denotes maturities τ in months; y-axis denotes yield rates in %

residuals. We can reject the bounce-back type of structural break for the USD level factor, but we receive quite significant estimate of coefficient β_A at 10% level for the curvature factor.

Table 5.14: Impulse Response Significance – USD

Factor	Date	β_A	SE	t-stat	p-value
Level	Sep-2008	-0.064	0.016	-0.502	0.308
Slope	Sep-2008	0.608	0.029	3.569	0.000
Curvature	Sep-2008	0.562	0.170	1.363	0.087
$t_{157}(0.95)$				1.655	

Source: Author's calculations

5.4 Concluding Remarks

In this section we presented estimates of all models developed in Chapter 4, empirical results and we drew several conclusions. The simplest models were estimated for central European currencies for the first time and we conclude that the dynamic Nelson-Siegel model proposed by Diebold et al. (2006) is suitable for modelling these yield curves.

The single currency models brought encouraging results consistent with results of Diebold and Li (2006); Diebold et al. (2006). Consequently, we proceed to estimation of the two novel and more elaborate regional models introduced in section 4.2.1 and 4.2.2. Initially, we examine whether there exists a regional yield curve. The principal component analysis confirmed our intuition based on findings of Diebold et al. (2008). There is one dominant principal component, which we refer to as level component, two less dominant referred to as slope and curvature components; moreover, there is a fourth component, which captures comparable variation. As a detour we presume the fourth component to carry information observed as noise on currency level, but as a fourth most dominant component on regional level. We extract the paths of the first two components—level and slope components—to base our further analysis on.

The innovative Regional Common Factor model proves to be an adequate for modelling regional yield curve and the model manages to extract regional level and slope factors. Since these factors are forced to have zero mean, they seem to be negatively correlated distortions. Their introduction into the model, decreases variance of the currency specific level and slope factors. Moreover, it seems that these regional factors capture so much of the information present in the data that most of the currency specific diagonal elements of transition matrices T are not significantly different from one. This leads us to a conclusion, that the only *predictable* movements are determined on regional level. Interestingly, the Polish Zloty yield curve is attached to the regional factors more than other currencies. By contrast, the Slovak Crown yield curve inherits its own dynamics.

The second innovative model is the Principal Component Regional model, which directly exploits the first two principal components as regressors. This novel extension has two advantages: it speeds up the estimation and delivers very significant regression coefficients. This models confirms our findings based on the Regional Common Factor model and moreover it adds information about signs of loading coefficients. Similarly, most of the currency specific diagonal elements of transition matrices T are not different from 1; the only exceptions are the CZK coefficient associated with the slope factor and the SKK coefficients driving dynamics of the slope and the curvature parameters. The latter also supports the conclusion that the SKK yield curve is most detached from the regional yield curve.

Furthermore, we find that the CZK, the HUF and the PLN level loading coefficient is negative, which either means that the yield curve level moves in opposite direction to the regional one, or that the regional level component moderates the currency specific ones. In case of the curvature loadings, we find similar pattern with only difference and that is the PLN has the only positive coefficient in stead of the SKK one.

Finally, we presented two possible applications of the Nelson-Siegel and state space framework: one risk management example and one example applicable in general economics for analysing various time-series data. The former example provides an easy way to construct yield curve stress scenarios. The key idea is to exploit the three factor structure of the Nelson-Siegel model and straightforward forecasting abilities of the state space models. To construct the shock associated with a factor, we simply compute point forecasts for the other two factors for a desired horizon and interval forecast at desired level of confidence for the factor we are interested in. By using the Nelson-Siegel loadings matrix, the two point forecasts and one bound of the interval forecast we obtain shocked curve for given horizon and at a given confidence level.

The later example introduces a general approach to detect and model structural breaks. We show how to analyse various types of impulse intervention dummies as: measurement, switch and bounce-back dummy. After graphical analysis of auxiliary residuals, we select suspicious points and include the impulse interventions as dummy regressor into the Nelson-Siegel model. The significance of the dummy coefficient then suggests whether it should be rejected or not. We conclude that the HUF and the PLN yield curves both experienced structural break in the month of the crisis outbreak—September 2008. We found a significant switch type break in CZK yield curve when the Czech Crown was reaching the bottom of CZK 40.18 per USD at the end of 2000. Finally, we identified bounce-back type of structural break in USD yield curve in September 2008 and two consecutive months, again the months of Lehman fail.

Chapter 6

Conclusion

In this thesis we focus on and analyse dynamics of yield curves of various currency zones. We set off to modify Dynamic Nelson-Siegel approach to capture driving factors behind regional yield curves and by using state space framework and principal component analysis we endeavour to extract regional common factors. After setting up the framework and estimating the models we show two possible practical applications.

The dynamic Nelson-Siegel model proved to be an adequate model to describe dynamics in single country yield curve. The key idea is to model parameters of classical Nelson-Siegel one-period model as latent factors, thus looking at the model as if it was a factor model. In combination with state space formulation, we can take the yield curve as multivariate time-series and the unobserved factors as state equation. Therefore, we obtain a clear inference and a way to estimate the model in one step through Maximum likelihood and Kalman filter.

Furthermore, we propose two innovative regional models. These two models are, to the author's best knowledge, novel and not presented anywhere else. Our intuition is based on findings of Diebold et al. (2008) who discovered global yield curve by analysing zero rates of USD, GBP, DM and JPY. Hence, we expect to find similar common dynamics in currencies' yield curves geographically even closer: CZK, UHF, PLN and SKK.

We model the regional yield curve as a stacked currencies' yield curves in one step. The first proposed regional model relies on Kalman filter to extract the regional factors, thus we refer to it as to the Regional Common Factor model. The second proposed model includes regional principal components as regressors, thus we call it the Principal Component Regional model. One of the innovations of this thesis is to impose restrictions on parameters of the Regional Common Factor model to counter the identification issues. We restrict parameters determining how much each currency's yield curve loads on the regional factors to be positive. This device allows us to estimate the parameters and extract several unobserved latent factors: each currency's level, slope, curvature factor plus regional level and slope factors.

Despite its notional complexity, the Principal Component Regional model is easier to estimate than the Regional Common Factor model. By including the extracted first two principal components, we obtain two high quality regressors, which can be seamlessly included into the state space framework. Similarly, this model extracts currencies' unobserved latent factors, yet it does not extract the regional factors. It rather estimates loadings parameters associated with the regressors.

Since most of the models are quite complex and/or novel, we cannot rely on of-the-shelf econometric software. We estimate the models with custom made software programmed in OxDoornik (2007) extended with SsfPack 2.2 Koopman et al. (1998).

Initially, we can observe similar yield curve dynamics of the regional currencies and yield curves in more liquid currency zones of EUR, GBP and USD. Moreover, almost all diagonal coefficients of transition matrices representing the main dynamics of latent factors seem to be quite persistent with values close to 1. This suggests that possible random walk specification for the latent factors may be considered thus opening a new topic for further research.

The regional models bring more information how much currencies' sub-models load on regional factors or principal components. We find that the PLN yield curve loads the most on the regional factors and these loading coefficients are significant, whereas the CZK level factor loads the least and the slope factor does not load on regional factors at all. We conclude that the CZK slope factor possesses its own dynamics corresponding to country specific features.

The Principal Component Regional model brings similar conclusions. The CZK has a minute slope loadings coefficient. Since the loadings coefficient are not restricted to be positive in this model, we can observe their signs. The CZK and the HUF sub-models load with negative sign on the first two principal components. Interestingly, when negative coefficients are allowed, the HUF sub-model has the largest magnitude followed by the PLN coefficients. The SKK yield curve is the only one to load on the first principal component—level component—with positive sign and the PLN yield curve is the only one to load on the second principal component—slope component—with positive sign.

The first practical application shows how to use the established framework in interest rate risk management. Using the forecasting abilities of state space models, we computed 12-month stress scenarios at 99% level. The main idea is to combine interval forecasts with point forecasts to obtain shocked curves for one year horizon. This procedure is performed on models for more liquid markets: EUR, GBP and USD.

This thesis further applies currently known structural breaks detection tools on the single currency regional models to identify critical moments in evolution of each currency yield curves. This results in a coefficient associated with the dummy variable. The magnitude and significance of the coefficient give us additional information about the structural breaks.

In conclusion, the main contribution of this thesis is a creation of a complete framework that enables us to analyse yield curves, to design risk scenarios and to detect structural breaks of various types. Importantly, all of the applications rely on established econometric and statistic methods, which we, in addition, present along with statistical background needed to understand and be able to use the models.

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Appendix A

Appendix

A.1 Regression Lemma

The lemma and its proof can be found in Durbin and Koopman (2001, p. 37)

Regression Lemma A.1.1. *Assume* x, y, z *to be random vectors of arbitrary orders with means* μ_p *and covarance matrices* $\Sigma_{pq} = \mathrm{E}[(p-\mu_p)(q-\mu_q)']$ *for* $p, q = x, y, z$ *with* $\mu_z = 0$ *and* $\Sigma_{yz} = 0$. *Then*

$$\mathrm{E}(x|y,z) = \mathrm{E}(x|y) + \Sigma_{xz}\Sigma_{zz}^{-1}z, \tag{A.1}$$

$$\mathrm{Var}(x|y,z) = \mathrm{Var}(x|y) + \Sigma_{xz}\Sigma_{zz}^{-1}\Sigma_{xz}'. \tag{A.2}$$

♠

A.2 Dynamic Nelson-Siegel – Global Models

$$\text{EUR}: \quad \hat{\mu} = \begin{pmatrix} -2.268 \\ (0.467) \\ 2.365 \\ (0.294) \\ 2.539 \\ (0.038) \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} 0.992 & 0.017 & 0.017 \\ (0.007) & (0.017) & (0.016) \\ 0.007 & 0.962 & 0.067 \\ (0.015) & (0.026) & (0.024) \\ 0.001 & 0.018 & 0.883 \\ (0.026) & (0.054) & (0.054) \end{pmatrix} \tag{A.3}$$

$$\text{EUR}: \quad \hat{Q} = \begin{pmatrix} 0.026 & -0.029 & 0.002 \\ -0.029 & 0.058 & 0.003 \\ 0.002 & 0.003 & 0.316 \end{pmatrix} \tag{A.4}$$

$$\text{USD}: \quad \hat{\mu} = \begin{pmatrix} -3.134 \\ (0.416) \\ 1.412 \\ (0.338) \\ 2.24 \\ (0.21) \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} 0.991 & 0.016 & 0.013 \\ (0.008) & (0.012) & (0.014) \\ 0.009 & 0.942 & 0.067 \\ (0.015) & (0.021) & (0.034) \\ 0.003 & 0.019 & 0.893 \\ (0.014) & (0.063) & (0.052) \end{pmatrix} \tag{A.5}$$

$$\text{USD}: \quad \hat{Q} = \begin{pmatrix} 0.079 & -0.088 & 0.056 \\ -0.088 & 0.149 & -0.116 \\ 0.056 & -0.116 & 0.613 \end{pmatrix} \tag{A.6}$$

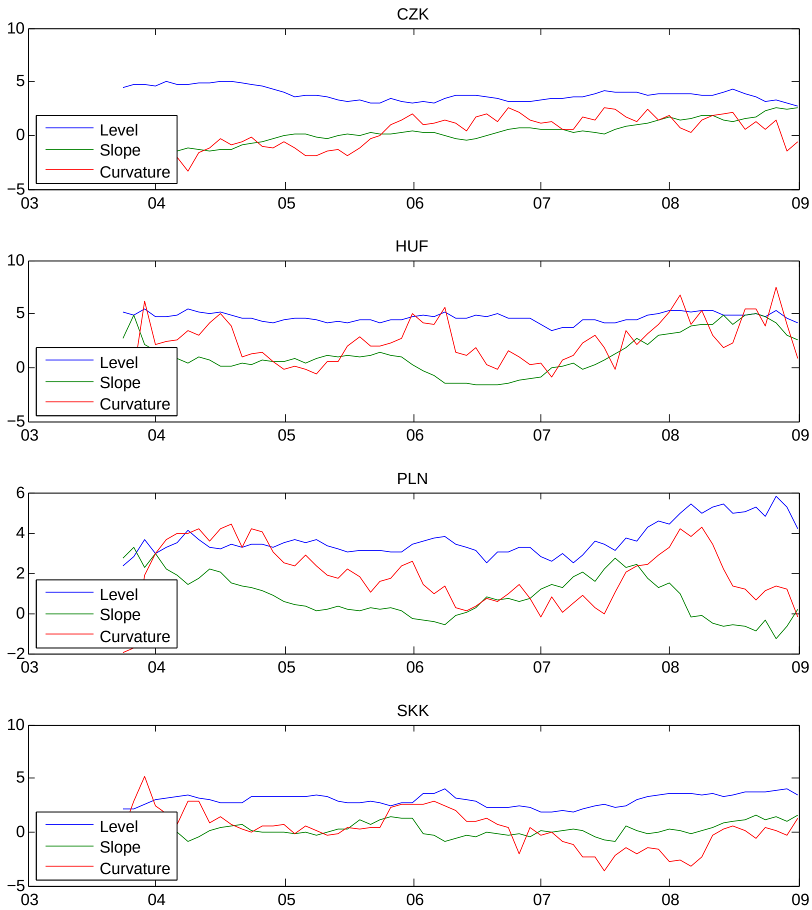
$$\text{GBP}: \quad \hat{\mu} = \begin{pmatrix} -2.808 \\ (0.168) \\ -0.731 \\ (0.121) \\ 0.041 \\ (0.239) \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} 0.959 & -0.006 & 0.034 \\ (0.01) & (0.003) & (0.006) \\ -0.099 & 0.928 & 0.067 \\ (0.082) & (0.041) & (0.052) \\ 0.421 & 0.126 & 0.616 \\ (0.138) & (0.06) & (0.086) \end{pmatrix} \tag{A.7}$$

$$\text{GBP}: \quad \hat{Q} = \begin{pmatrix} 0.011 & -0.001 & -0.002 \\ -0.001 & 0.071 & -0.005 \\ -0.002 & -0.005 & 0.207 \end{pmatrix} \tag{A.8}$$

A.3 Dynamic Nelson-Siegel – Regional Models

A.4 Detecting Structural Breaks – Auxiliary Figures

Figure A.1: Currency's Latent Factors – RCFM



Source: Author's calculations

Figure A.2: Currency's Latent Factors – PCRM

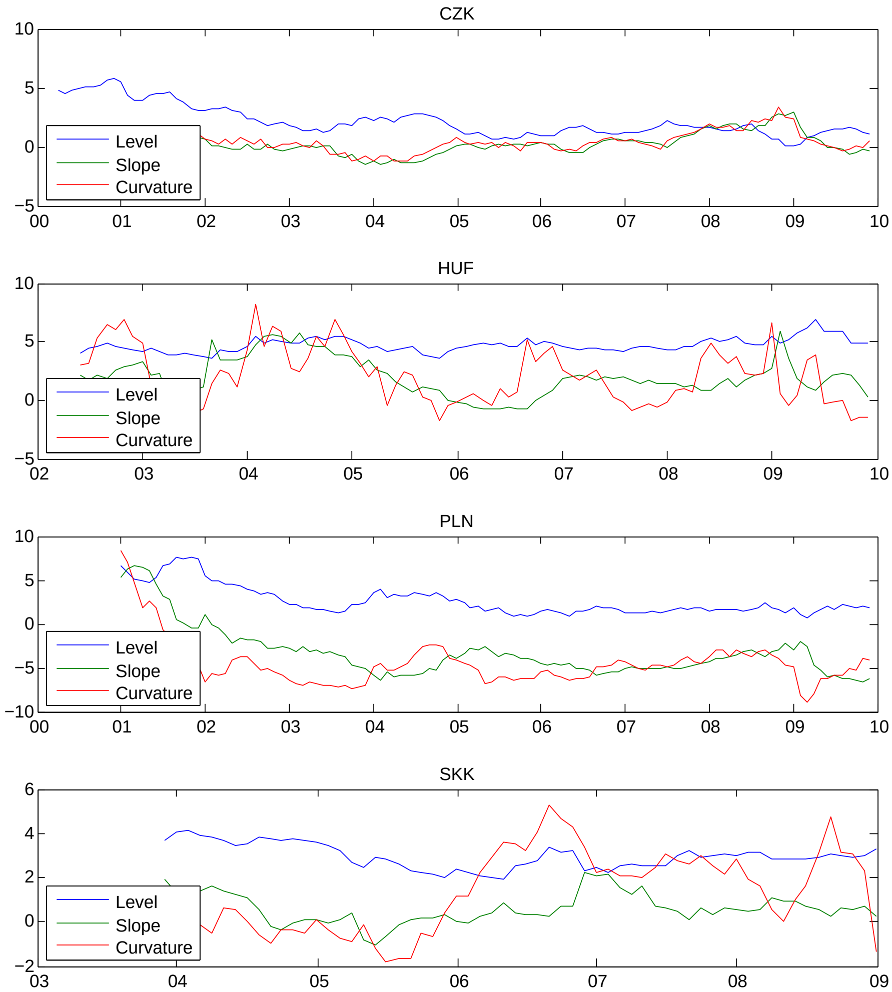
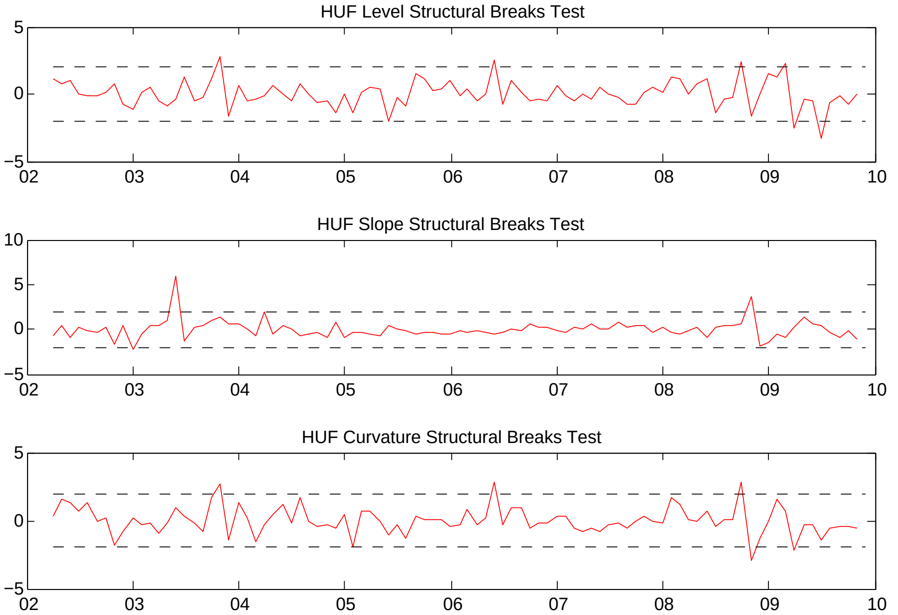
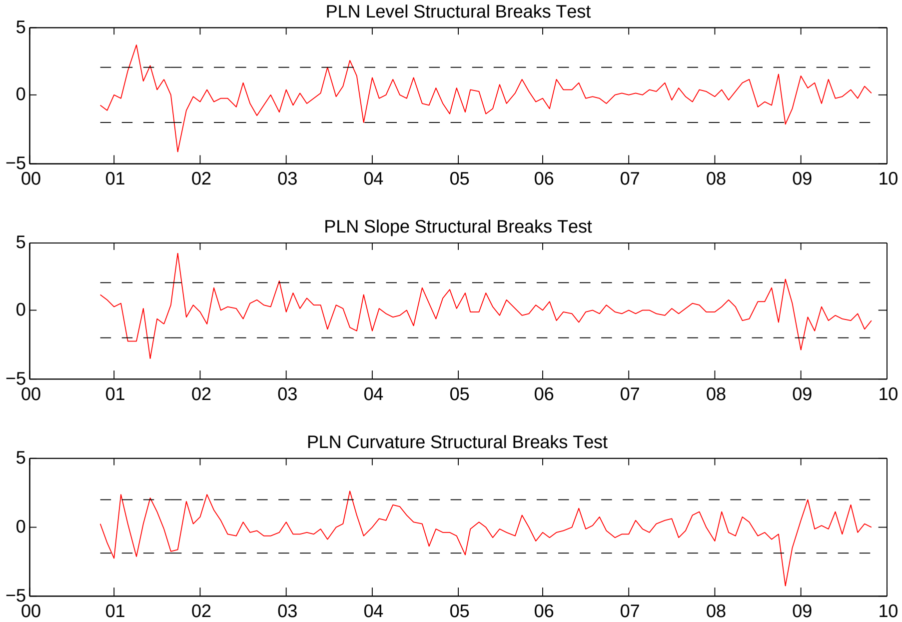


Figure A.3: Auxiliary Residuals – HUF



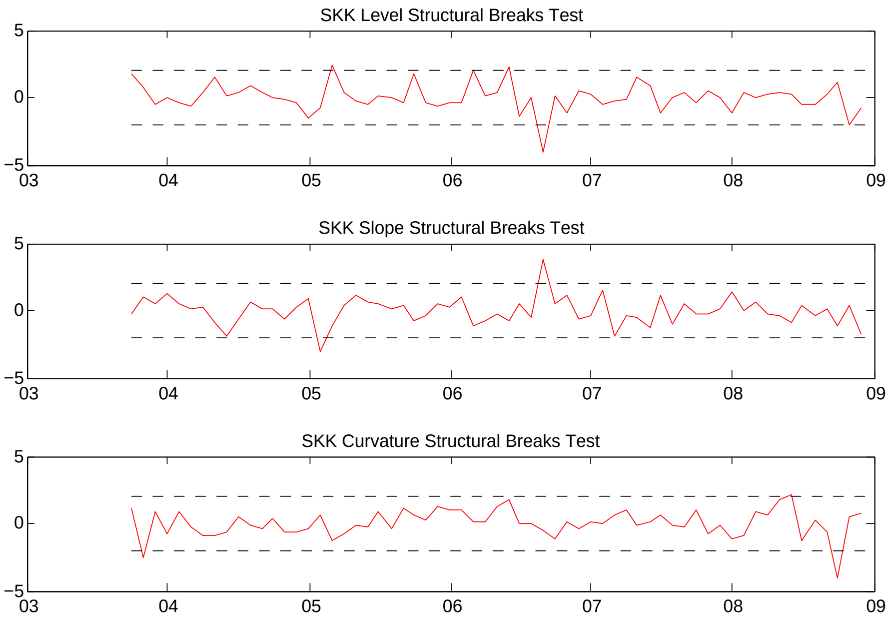
x-axis denotes time in years

Figure A.4: Auxiliary Residuals – PLN



x-axis denotes time in years

Figure A.5: Auxiliary Residuals – SKK



x-axis denotes time in years