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Tragedy of the Commons

Bachelor Thesis

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Declaration of Authorship

Hereby I declare that I compiled this thesis independently, using only the listed sources.

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Signature

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Abstract

This thesis focuses on the situation known as the Tragedy of the Commons, in which conflicting individual and group interests result in overappropriation of a common-pool resource. It examines the setting focusing on theoretical models, combining the approaches of game theory and maximization. It analyzes the effects of different aspects of the situation, such as game structure, game payoffs, repetitions and individual and group characteristics, first in the setting where no interactions among players are possible, and then in the situation in which players are allowed to communicate and decide on some kind of management of the resource. In accord with many recent studies, this thesis shows that state management of a resource is not the only solution to the dilemma. Moreover, when interactions among players are allowed, users may be able to design their own rules, thus avoiding the 'tragic' outcome. Communication and the ability to design sanctioning mechanism is shown to have a major impact on the success of these self-management rules.

Keywords: tragedy of the commons, game theory, contest

Abstrakt

Tato práce se zabývá situací známou jako Tragedy of the Commons, ve které konflikt mezi zájmy jedince a skupiny vyústí v přehnané využívání společného zdroje. Zaměřuje se na teoretické modely, přičemž kombinuje nástroje teorie her a maximalizace. Práce analyzuje různé aspekty této situace, mimo jiné strukturu hry, výplatní matice, opakování a charakteristiku jednotlivců i celé skupiny, a to nejprve v situaci, kdy není připuštěna žádná interakce mezi hráči a poté v situaci, kdy hráči mají možnost komunikovat a případně se dohodnout na způsobu spravování zdroje. V souladu s mnoha současnými studiemi ukazujeme, že státní správa zdroje není jediným řešením daného dilematu. Navíc, v situaci kdy je umožněn kontakt, hráči mohou být schopni si vytvořit vlastní pravidla hry a tak se vyhnout 'tragédii'. Komunikace a možnost vytvoření donucovacích prostředků se ukázaly jako zasadní faktory určující úspěch takto vytvořených pravidel.

Klíčová slova: tragedy of the commons, teorie her, soutěž

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Chapter 1

Introduction

Since the days of Aristotle at least, it has been observed that what is held in common is often neglected. Aristotle himself wrote: "What is common to the greatest number gets the least amount of care. People pay more attention to what is their own: they care less for what is common" (Aristotle 1261b32). More recently it was Garrett Hardin's famous article "The Tragedy of the Commons" (1968) that draw closer attention to this phenomenon. The main idea is that when property rights are not strictly defined, resources will be used more that what would be the optimal level. In his article, Hardin described the logic behind his tragedy on a simple example: imagine a pasture open to all. The pasture is a common-pool resource as anyone can use it according to his wish without restrictions. Each herdsman keeps his cattle to graze on this pasture. Now the argument is that each will try to keep as many cattle on the commons as possible. Hardin argues that this scheme may work well as long as tribal wars and disease keep the number of men and beast below the capacity of the land. However, once social stability prevails, rational behaviour of herdsmen lead to tragedy. From the point of view of each herdsmen, it is rational to add one more animal to his herd as he receives all the benefits, and only bears part of the costs as the negative impact of overgrazing are shared among all of them. After that, it is rational to add another one. And another. Moreover, as this is a conclusion reached by each of the herdsmen, the pasture will inevitably be overused. The tragedy lies in the fact that individual following their best self-interests together arrive at an outcome that is not optimal. The origin of the tragedy is in the fact that increased appropriation of one person increases costs of appropriation to all the other players; in other words the origin lies in the presence of negative externalities. In this article, the word 'tragedy' is used in a sense of something inevitable that cannot be avoided within the model itself. The solution lies according to Hardin in abandoning the commons by the way of either government or private property management. In his model, Hardin does not allow for the possibility of cooperation among players, he assumes that "social mechanisms to control self-interests (communication, trust) are lacking or ineffective"(Dietz *et al.* 2002, p. 5).

The Tragedy of the Commons obviously does not limit itself only to land, it comprises any use of a common-pool resources (CPR). Common-pool resources, or just common resources, can be both natural and man-made, but are always defined by two key attributes. The first one is that the cost of exclusion of each individual from the use of the resource is very high. This attribute they share with public goods. The second attribute is high subtractability of the resource, which is a feature shared with private goods. The subtractability is the key factor in understanding the origins of the tragedy: all the units used by an individual cannot be used by anyone else. Apart from pastures as mentioned by Hardin, the most common examples range from small fisheries, irrigation systems to the whole oceans or global air pollution.

The Tragedy of the commons is a part of a broader variety of problems known as social dilemmas. Social dilemmas are defined as situations in which an individual receives higher payoff for socially defecting choice, but all the individuals would be better of if everyone followed the joint strategy (Dawes 1980). The most well-known example is public goods dilemma. In situations such as pollution control or radio broadcast, everyone would benefit from the provision of a public good, but as the contribution is costly, everyone would prefer the others to pay the cost. Once the public good is provided, no-one can be excluded from the benefits, and thus everyone will have incentives to free-ride. We can see that the provision of public-good is the mirror problem of the common resource appropriation. However, there are other problems even in relation to common resources alone. These can be for example provision or assignment problems (Ostrom 1999). Moreover, common resources need maintenance so that they could be used properly. Such maintenance requires costly effort and other contributions of the users, while the benefits from proper maintenance are shared among all the individuals, both those who contribute and those who do not. In that sense maintenance is a public good, and thus its provision presents another public goods dilemma. The prediction of the theory is that common resources will be both overappropriated and underprovided.

However, an important distinction needs to be made between an open access, as used by Hardin, and common (or group) access, which is nowadays assumed in most of the models. A truly open access means that anyone from anywhere can come and use the resource. Contemporary researchers use the term common access (thus common resources) to suggest that there may be quite a large number of people who may want to use the resource, but this number is limited. When speaking about local commons, it makes sense to establish some limit on the number of resource users, as it is hard to imagine people going very far to graze their cattle (to use Hardin's example). Therefore, geographical location usually works automatically to limit the number of users. However, in certain types of common resources, the number of users may be if not unlimited, then definitely far too high. The most straightforward example would be commons covering large areas such as air or ocean pollution. This is not explicitly dealt with in this thesis, as most of the models focus on local commons.

Throughout the thesis, when we talk about efficiency, a common economic principle of Pareto efficiency is used to be able to achieve some kind of comparison. An allocation, or distribution of levels of appropriation in particular, is called 'Pareto optimal', or 'Pareto efficient' when there is no other allocation such that someone may be better off without making someone else necessarily worse off. A 'Pareto improving' distribution is such a change in distributions that improves the payoff of at least one person while harm no-one; thus such a distribution is closer to the 'Pareto efficient' outcome.

In the decades following the publication of Hardin's article, growing number of studies appeared on this topic by scientists from variety of fields: game theory, economics, psychology, anthropology and political science. However, there is a substantial controversy about how to deal with the 'tragedy'. As the appropriators seem to be in the dilemma from which they cannot escape by their own means, the most wide-spread solution is the change in the property rights. Two property regimes are suggested: private property and higher authority management. If rights to the use of a common resource are privately owned, then the market forces will ensure efficient appropriation. The main criticism of this solution is that in many common resource situations it is impossible to assign property rights. This could result from the unclear boundaries of the resource, difficulty in division, or, mainly when larger areas are involved, there is no authority that could undertake such assignment. On the other hand, in external authority could provide the users with artificial rules limiting the use of the resource. Moreover, when economies of scale, or particular technological knowledge is required, this authority should be central government.

However, more recent literature, both theoretical and empirical, is trying to disprove the assumption that these are the only solutions to the dilemma. In particular, what has been challenged is the inability of the resource users to design their own rules affecting the use of the resource. The importance and considerable progress of this approach is illustrated by the fact that one of the leading researchers in this field, Elinor Ostrom, was awarded this year's Nobel Prize in Economics as the first woman in history for "her analysis of the economic governance, especially the commons" (Nobelprize.org). Her main contribution is the systematic research providing evidence of the ability of the users to design and sustain rules governing the common resource.

Ostrom says that: "A substantial gap exists between the theoretical prediction that self-interested individuals will have extreme difficulty in coordinating collective action and the reality that such cooperative behavior is widespread, although far from inevitable" (2000b, p. 138). Cooperation in this sense in meant not only as actions resulting from binding agreements among players, but as any strategy that increases overall efficiency. In this thesis, we examine the theoretical achievements trying to fill this gap. We focus on theoretical models providing evidence that, under certain circumstances, common resource users need not be trapped in the dilemma. Tools of game theory and optimalization theory are both used throughout the thesis to show which factors determine the efficiency of the outcomes. By that we try to show that government management is not the only possible solution to the Tragedy of the Commons, as had been thought previously. The thesis is divided into two main parts. The following chapter deals with the situation where players are not able or willing to interact and engage in communication. In that it follows the original framework as described by Hardin. After the basic representation of the dilemma is presented, we proceed to show how payoff structure can change the incentives of the players and thus influence the resulting level of appropriation. The following sections describe how the structure changes when more time periods are taken into account, and when the possibility of resource depletion is present. The remaining sections examines the effect of player characteristics.

The third chapter relaxed the assumption of no interactions among players, and deals with the situation when players do engage in communication to try to agree on joint strategies, and when they are able to agree on enforcement of such rules. In the first part we show how the basic games changes with the simple possibility of monitoring. The second part deals with the problems and factors influencing the design of rules. The next section presents a model in which assumptions about inequality aversion are incorporated, and shows how such change increases the success in prediction real-life outcomes. The last section links the common resource problem to contest, and examines how possibility of contest may increase the incentives of the players to design appropriation rules.

Chapter 2

Unmanaged Common-Pool Resources

Hardin (1968) describes a situation in which there is no interaction among individuals sharing access to a common-pool resource. In such setting, when a common-pool resource does not have clearly assigned property right, so that it would be either in private hands or managed by government, it is supposed that the resource will be overused. It has been often argued that such a setting is far too restrictive and does not correspond to real-life situations, in which individuals are often able to interact. However, even within this original setting, many researchers tried to show that tragedy in not an inevitable outcome of a common-pool resource use. In this chapter, we are going to examine such assertions, combining game-theoretic approach with optimization theory, sometimes also using more verbal approaches. The first section focuses on the basic representation of the conflict between individual's and group's interests as the game of Prisoner's Dilemma. The next sections focuses on how different game payoffs may change the incentive structure, and thus increase efficiency of the outcome. The following sections then present different modifications to the basic Commons model to examine under what conditions the solution to the suggested game is not so 'tragic' by focusing on different aspects. The third section focuses on the effects of heterogeneity, in particular the effects of wealth distribution among players. The fourth section introduces time aspect into our analysis. In a simple extension to the basic model it is shown that the threat of resource depletion and growth of the common-pool resource between time periods alters to certain extent the expected outcomes. The following section extends this model by adding the possibility of exiting rather than staying and appropriating the resource. The last but one section then extends the time aspect from two periods to finitely and non-finitely repeated games. The last section uses evolutionary approach to show how multiple player types may emerge in the society, and how equilibrium emerges under such conditions.

2.1 Prisoner's Dilemma

Like some of the other social dilemmas, the Tragedy of the Commons is often modelled by the game of Prisoner's Dilemma to show the conflict between individual and group rationality, ie. the actions that would be rational from the point of view of an individual, and from the point of view of the group as a whole. Prisoner's dilemma, developed by a mathematician A.W.Tucker (1983) is a game in which all players are supposed to be rational and have complete information. It describes a situation in which two suspects are being interrogated by the police, each in his separate cell, so that they cannot communicate. Each of them can either confess or deny the crimes they are accused of. As there is not enough evidence, if neither of them confesses, they can only be charged with other minor offenses worth one year of imprisonment. If both of them confess, they are sentenced to ten years, and if one of them confesses and the other denies, the traitor is set free, while the other is sentenced to 14 years. The situation is illustrated in table 2.1.

		Player 2	
		Confess	Deny
Player 1	Confess	10,10	0,14
	Deny	14,0	1,1

Table 2.1: Prisoner's Dilemma

The situation is symmetric, and both suspects have dominant strategy to confess. Thus, the Nash equilibrium will be *(confess, confess)*. [1] From the table it is clear that the equilibrium payoffs are worse than if they both denied - therein lies the dilemma. For each suspect it is preferable to confess, as they cannot predict the behaviour of the other one. Even if communication is allowed, in the absence of a mechanism that would enforce both suspects to adhere to their agreement, absence of trust would make them both defect. As in the Tragedy of the Commons, when each of the players pursues his or her self interest, the joint payoff is worse than if everyone would follow the joint strategy. Moreover, in our particular setting, the overall length of years of imprisonment is the highest of all four possible outcomes, although, as will be shown in the next section, this need not be always the case.

In real life, the situation may be more complicated in two ways. First, number of players can be larger than two. Second, in situation like the decision about the level of investment to public good or level of appropriation of the common-pool resource, the question is not only whether to invest or not, but also how much to invest. This general social dilemma can be represented by the following equation. The logic is the same as in the two-players, two-strategies situation. Each of the n players has an option on how much to invest (or withdraw from the common-pool resource) represented by a variable $x \in [0, 1]$ (Beckenkamp 2006, p. 340). The payoff of a subject is given by:

$$\pi_i(x_i, X_{-i}) = b - cx_i + q_i f(X) \qquad \text{where} \quad \sum q_i = 1 \tag{2.1}$$

[1] The strategy combination s^* is a Nash equilibrium iif no player has an incentive to deviate from his strategy given that the other players do not deviate. So that:

$$\forall i, \quad \pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i^{'}, s_{-i}^*), \quad \forall s_i^{'}.$$

x_i**:** the level of 'cooperation' in a sense that it is the best behaviour from the point of view of the whole group; previously $x = 0$ meant confessing, $x = 1$ meant denying

X_{-i}**:** the sum of the decisions of all the other $n - 1$ subjects

$\pi_i(x_i, X_{-i})$**:** payoff that the ith player gets

$f(X)$**:** a function determining the total payoff depending on the strategies of all the players

cx**:** the cost of decision x

q**:** proportioning rule (what part of the total payoff does a particular individual get)

b**:** a baseline payoff that the subject receives ('showup-fee') which may or may not be part of the model

For simplicity, a linear payoff function is often assumed. However, in the common resource setting, the shape of the function has a crucial role in most of the predicted outcomes. A function with diminishing returns to scale is usually assumed for at least some level of appropriation, as will be discussed later. Moreover, in this setting we assume that the proportional share is fixed and independent on the investment. In particular, we assume that each player's share is $q_i = \frac{1}{n}$. In the Tragedy of the Commons, the proportioning rule can be different, though. Usually, we assume that $q_i = \frac{x_i}{X}$.

In order to establish a situation in which the conflict between individual's interest and group interest is present (n-player PD), the following conditions must be fulfilled. Firstly, the value of c must be greater than 0 because it presents the cost of cooperative decisions, and is multiplied by -1 in the general formula $U(x)$. Secondly, the value of f must be smaller than nc to ensure that, are else being equal, the resulting payoff from an individual's cooperation is less than if he does not cooperate. Finally, if f is greater than c then the complete cooperation yields a better total payoff than complete defection. Therefore, the equation must satisfy all the following conditions:

$$0 < c < f < nc \tag{2.2}$$

Supposing $q_i = \frac{1}{n}$ and a linear payoff function, the situation is represented in the following equations:

$$\begin{array}{llll} x_i^a = 0 & i = 1, \ldots n & \pi_i^a = b & \sum \pi^a = nb \\ x_i^b = 1 & i = 1, \ldots n & \pi_i^b = b - c + \frac{1}{n} f(n) & \sum \pi^b = nb - nc + f(n) \\ & & \sum \pi^a < \sum \pi^b & \end{array}$$

From that it is clear that under imposed conditions, complete 'cooperation' yields better total payoff than complete defection. The last part of the equation 2.2 must be fulfilled so that, all else being equal, the resulting payoff from an individual' s cooperation is less than if he does not cooperate. Supposing all the others cooperate, the situation looks like this:

$$
\begin{array}{ll}
x_i^a = 0 & \pi_i^a = b \\
x_i^b = 1 & \pi_i^b = b - c + \frac{1}{n} f(X) \\
\pi_i^a > \pi_i^b &
\end{array}
$$

Thus, neither the ability to choose the level of cooperation (ie. x can be any value from the interval $[0, 1]$), nor the number of players changes the predicted level of cooperation. From the point of view of all players, it is best not to cooperate so that $x_i = 0$ for all $i = 1 \dots n$.

So far, we have assumed symmetrical proportioning rule. However, Beckenkamp (2006) shows that when asymmetrical proportioning rule is introduced, under certain conditions, a player whose proportion is high enough may have the dominant strategy to cooperate. Thus, from his point of view, there is no dilemma any more. The effects of different proportioning rules will be discussed in Chapter 2.

Based on the model of Prisoner's dilemma, the expected outcomes of many of the social dilemmas are not efficient. Most of the criticism focuses on the fact that the assumptions of this non-cooperative setting are too restrictive. However, many economists have been trying to show that the tragic outcome is not inevitable even within the Hardin's non-cooperative framework. Firstly, we can examine the structure of the game, ie. the payoffs of the game and the possible repetitions of the game. Secondly, group characteristics and heterogeneity among individual members (their wealth levels, exit options) may be examined. Third, the economic assumption of rational individuals may be challenged to show how multiple types of players may emerge. These possibilities will be examined in the following sections.

2.2 Game Payoffs

In Prisoner's Dilemma, the payoff structure of the game leads to a dominant strategy of each player, and the only existing Nash equilibrium does not reach Pareto optimum. However, in their book *Rules, Games and Common-Pool Resources* (1994), Ostrom *et al.* show that the payoff structure may be changed so as to replace the initial game of PD by another type of game. In table 2.2 there is a general payoff structure of a two-person game. Possible outcomes are represented by letters a, b, c and d, which denote either payoff or utility. Each of the players must decide what strategy to choose; this depends not only on the values (or rather relative values) of a, b, c and d, but also on the choice of the other player.

		Player 2: Strategy 1	Player 2: Strategy 2
Player 1	Strategy 1	a,a	b,c
	Strategy 2	c,b	d,d

Table 2.2: Two-Player Game
Source: Ostrom *et al.* (1994, p. 54)

In PD game illustrated previously, the numbers stood for the years of imprisonment, ie. it was a negative variable, while in this table they represent positive utility.

If the PD situation was to be transformed to utility values, the numbers would satisfy the following inequalities: $b < d, c > a$ and $a > d$. However, Ostrom *et al.* point out that "there is no compelling theoretical reason for any particular game to be a Prisoner's Dilemma" (Ostrom *et al.* 1994, p. 56). In other words, the conditions a PD game must fulfill are rather restrictive, and not all the problems are likely to have this incentive structure. In connection to the Tragedy of the Commons, many sub-problems may be represented as having the same incentive structure, but there are other suboptimal or optimal outcomes that may result from different incentive structure.

Thus, there is no reason why the payoff structure should satisfy particular set of inequalities rather than any other. Therefore, if the relationship among the payoff parameters is different, the resulting game will be different as well. Maintaining the assumption that $a > d$, there are two other possible games. If $c > a$ and $b > d$, the resulting game is called Chicken; if $a > c$ and $d > b$, the resulting game is called Assurance. Both of those games have multiple equilibria, and even though the possible outcomes are still not Pareto efficient, they do not at least provide the worst possible outcome.

In the Tragedy of the Commons, one of the main features of the dilemma is that individuals will ignore the impact of their behavior on the other players while appropriating a common-pool resource. This means that one player's increased appropriation reduces the yield obtained by other users for some (or any) given level of appropriation (Ostrom *et al.* 1994, p. 10). For example, if a common-pool resource is a fishery, then the appropriation level means the amount of fish caught by any individual fisher. Then, if one fisher increases his appropriation level, the average yield of all the other fishers resulting from their fishing activities decreases. In other words, appropriators who consider only their returns and do not take into account impact they have on the other appropriators create a negative externality which leads to overappropriation.

In connection to the games discussed previously, Ostrom *et al.*(1994) provide a detailed model of games played by the appropriators of a certain common resource. In this simplified version, there are again two players with two strategies. Each of them has one unit of productive input (labor, capital), with the units called tokens, which they can invest either in a safe outside opportunity and receive payoff w[2], or invest in the common-pool resource. Let x_i denote the investment of player i in the CPR. Then $x_i = 0$ means taking the outside opportunity while $x_i = 1$ means appropriating the resource. Total output from the CPR is a function of appropriation levels of both players $f(\sum x_i)$, and a player's share of CPR output is assumed to be proportional to his input into the CPR (ie. 0 when he takes the outside opportunity and $(x_i/\sum x_i)f(\sum x_i)$ when he appropriates the CPR). The game is represented in table 2.3

The solution of the game depends on the relationship among w,$f(1)$ and $\frac{f(2)}{2}$. The main question is what is the shape of the function f, which depends mainly on the character of the common-pool resource itself and the nature of its appropriation. Let's assume that the CPR is subject to strictly diminishing returns to scale so that $\frac{f(2)}{2} < f(1)$ for all levels of appropriation. This means that at any level of

[2]this could represent for example working outside the game for a wage w

appropriation, if we increase the appropriation level twice, the resulting output from the resource will increase as well but less than twice. Usually, common-pool resources are supposed to fulfill this condition for at least some level of appropriation. In a fishery, up to a certain number of fishers, each of them can catch as many fish as he wishes. However, once the capacity of the fishery is reached, then the more fish one of the fishers catches, the less there is left for the other fishers. This condition captures the negative externality an individual imposes on the other players if he increases his appropriation level.

Under this condition, if it is more profitable to invest the first token into the CPR, so that $f(1) > w$, the outcome then depends on the relationship between half share in the output of the CPR ($\frac{f(2)}{2}$) and the outside opportunity w. Suppose the returns on the CPR are sharply diminishing, so that $\frac{f(2)}{2} < w$. Than the game has two equilibria each of the form: one player invests in the CPR, the other stays out. In this case the resulting game is a game of Chicken. Moreover, the outcome of this game is the best possible outcome from the point of view of the whole group, as the total payoff is the largest of the possible payoffs ($f(1) + w > 2w > f(2)$). On the other hand, the second case in which $\frac{f(2)}{2} > w$ results in a game with a unique Nash equilibrium where both players appropriate the CPR; this is the situation described in Prisoner's Dilemma. Each player has a dominant strategy, but even under these circumstances, the outcome may not be tragic at all. Whether there exists a tragedy (and its extent) is determined by the specific shape of the function f and the value of w, *ie.* on the relative values of $f(2)$, $f(1) + w$ and $2w$.

On the other hand, if the CPR is a concave function (ie. it is subject to increasing returns to scale), then there is no dilemma in the decision about the level of appropriation. Doubling of the appropriation level results in more than double increase in the output of the CPR. Then the strategy of both players to appropriate the CPR yields the largest possible payoffs. In reality, the nature of the CPR if often represented by more complex functions. Depending on the overall level of appropriation, shape of the function can change accordingly. Apart from constant, diminishing and increasing returns to scale, the overall output from the CPR may begin to decrease beyond a certain level of appropriation. This can be illustrated by the previous example of a fishery. Let's denote a catch of one fisher A. If there are two fishers, then with large enough fishery, each of the fishers can catch A. They may even catch more, as they can share information about the best spots. Let's assume that the capacity of the fishery is reached when there are three fishers. Each of them still can catch A, but when the fourth fisher is added, the catch of each

		Player 2	
		Strategy 1 (x=1)	Strategy 2 (x=0)
Player 1	Strategy 1	f(2)/2, f(2)/2	f(1),w
	Strategy 2	w,f(1)	w,w

Table 2.3: Appropriation Externality
Source: Ostrom *et al.* (1994, p. 58)

fisher starts to decrease, and each of them catches one fourth of the capacity of the fishery. Each additional fisher further diminishes the catch of all the other fishers. However, it can be said that a fishery is not strictly bounded. Then, it is possible that the decrease in each fisher's catch is not as large as suggested, but they are forced to go farther and farther away from the shore. Thus, even though the overall outcome increases slowly, each individual's catch still decreases. Furthermore, at some point there may be simply too many boats, the fishers may start to be in each other's way, and even the overall outcome starts to decrease.

In this setting we do not consider the time aspect of the appropriation of the CPR. If fishers catch too many fish one day, there may be less or even none available the following day. The question of depletion of the resource is considered in section 2.4.

In conclusion, in this section we showed that the PD situation in only one of the possible outcomes of two-player games with two strategies. The most important factor in the outcome of the game is the nature of the common-pool resource that is being appropriated, and the function that represents it. The dilemma occurs only under special conditions.

2.3 Heterogeneity Among Players

In real life, people are not all alike. In fact, both their inner qualities and circumstances they find themselves in are source of many difference. It has been shown that the characteristics of individuals in a group have an effect on the predicted outcome of the game. In particular, what has been studied is the effect of the level of heterogeneity among users on the efficiency of the outcome. As Bardhan and Dayton-Johnson (2002) point out, there may be many dimensions of heterogeneity: social, cultural and of course economic. Even within the category of economic inequality, there are still many types. Players may differ in their wealth, goals, time preferences[3] and preferences in general. In addition, as the relationship may often be interrelated, it's not always easy to separate them. For example, wealth of a player can influence his ability to make an effort, the ability to profit from a resource (ie. the technology he or she can use for appropriation), and it is also connected to the social position this player holds and his interest in the resource (Faysse 2005). There are two opposite effects of heterogeneity: the first one, as proposed by Olson (1965), is that an increase in heterogeneity will make richer users internalize more the consequences of their actions, and thus they will decrease their appropriation level. On the other hand, this will further diminish the already small propensity of poor users to take into account the impact on the resource and other players that they may cause. Thus, the overall effect depends on which of these opposite forces prevails. The four basic possible relations between the increase in the parameter of heterogeneity and the individual payoffs are shown in figure 2.1.

To decide what shape does a heterogeneity function have, we will consider one type of the economic heterogeneity - heterogeneity in wealth distribution. Olson (1965), in his pioneering study on collective action problems, suggests that , in pub-

[3]The individual's relationship between present and future consumption

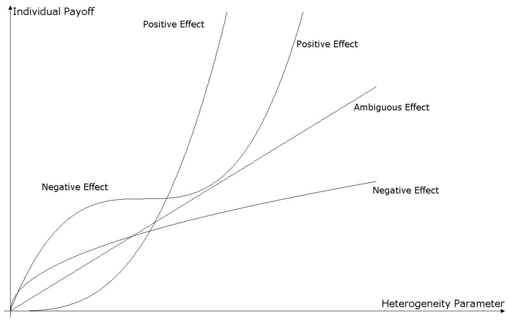


Figure 2.1: Possible relationships between an increase in heterogeneity and individual payoffs with indicated effect on collective efficiency
Source: Faysse (2005, p. 244)

lic goods situations, inequality increases efficiency because the greater the interest of any single individual in the resource, the more likely will that player get such a significant proportion of the total benefits from the provision of the public good that he will provide it even if he had to pay all the costs himself[4]. However, Olson considered only public goods, so the main question is whether such an effect can be found in commons setting as well? In addition, even if the more efficient appropriation is achieved, will the overall effect be Pareto improving? The possible results of the common-pool resource appropriation in connection to the distribution of wealth among players was analyzed for example by Baland and Platteau (1997). The main argument of their paper is that under special circumstances, inequality in wealth distribution can have a positive impact on the game outcome and thus extend the validity of Olson's proposition also to commons resources.

In their model, they assume n individuals who exploit a resource. The output function depends on the sum of the levels of appropriation by each of them:

$$f = f(\sum_{j=1}^{n} x_j)$$

where $f'' < 0$ so that the function is concave, so that it embodies diminishing returns to scale. If we assume zero cost, the payoff for i-th player is:

$$\pi_i = \frac{x_i}{\sum_{j=1}^{n} x_j} f(\sum_{j=1}^{n} x_j) \tag{2.3}$$

Each player maximizes his profit by taking as fixed the level of appropriation or effort of other players. Thus, the resulting Nash equilibrium is given by the n following conditions:

$$\frac{\partial \pi_i}{\partial x_i} = \frac{\sum_{k \neq i} x_k}{(\sum_{j=1}^{n} x_j)^2} f(\sum_{j=1}^{n} x_j) + \frac{x_i}{\sum_{j=1}^{n} x_j} f'(\sum_{j=1}^{n} x_j) = 0 \tag{2.4}$$

[4]In the common resource setting this would mean a decrease in his appropriation even if he is the only one to do it.

If we denote the Nash equilibrium appropriation levels x_j^N for all individuals, then the total amount of appropriation is given by $\sum_j x_j^N$. To satisfy the equation 2.4, marginal productivity of the total amount of appropriation must be negative, ie. $f'(\sum_j x_j^N) < 0$. Thus, from the perspective of each player, even if the total income is falling at this level of appropriation, still increased appropriation increases the player's share in total income. This situation of over-exploitation is the situation of the Tragedy of the Commons, while in socially optimal equilibrium, the marginal productivity of aggregate appropriation should be equal to zero. We can derive the social optimum by differentiating the sum of all the payoff functions.

Baland and Platteau then analyze the effects of an external constraint limiting the amount of effort some of the players can exert in the appropriation of the resource. The authors consider limitations arising form imperfect financial markets, in which information problems can prevent poor individuals from obtaining the credit necessary to exert as much effort as they would like. This difference among individuals in their access to credit mainly reflects the initial distribution of wealth. This assumption is quite reasonable especially in poor countries where access to capital is extremely difficult (Dayton-Johnson and Bardhan 2002, p. 580).

For simplification, in the model the authors suppose that one unit of credit allows a user to exert exactly one unit of effort. We can say that in the previous situation, credit constraints were such that each user could appropriate the Nash equilibrium level, but no more; thus, the total amount of available credit is just equal to the total amount of appropriation under unconstrained Nash equilibrium.

Now, we change the distribution of credit constraints so that some $(n-m)$ users must reduce their individual amount of appropriation from x_k^N to $\overline{x_k}$ (type k), while the rest of the players (m) have more credit available (type r). The total amount of credit is assumed to remain constant.

The users of type r will not respond to the changed credit availability by increasing their appropriation by the same amount the users of type k are forced to reduce theirs. Intuitively, the equilibrium level of type r players will be higher than it was under the initial wealth distribution, while obviously players of type k are forced to reduce their equilibrium level. Furthermore, the aggregate level of appropriation will decrease. This conclusion can be drawn from the equation 2.4. If the players of type r increase their level of appropriation so that the aggregate level remains unchanged, the first-order condition of this player is now violated: if we keep $\sum_j x_j$ constant, the derivative of the payoff function of user r is negative (because positive term f is multiplied by $\sum_{j \neq r}^n x_j$ which is smaller, while negative term f' is multiplied by larger x_r), thus inducing player r to reduce his appropriation level below what would be required to keep the overall level constant. Therefore, the aggregate level of appropriation will be lower than in the initial situation.

Moreover, by repeating the argument, a further reduction in credit imposed on type k users accompanied by corresponding increase in credit for type r players, will be followed by another fall in total appropriation of the resource. The same result can be achieved by applying disequalizing changes within the class of type r players.

To sum up, according to Baland and Platteau, the more unequal the distribution of credit constraints in the appropriation game, the more efficient the use of the common resource. Moreover, under some conditions, the disequalizing change can be Pareto-improving. There are two opposite effects of this change: on the one hand, it decreases the share of each constrained user in the total output. On the other hand, the total level of appropriation decreases, and thus the total income increases. The resulting effect depends on the relative strength of each of the opposite effects. The Pareto-improving possibility can be illustrated in the following example. We consider an appropriation game with four players and with the following production function:

$$f = \sum_{j=1}^{n} x_j - b(\sum_{j=1}^{n} x_j)^2$$

The payoff of player i is then:

$$\pi_i = \frac{x_i}{\sum x_j}\Big(\sum x_j - b(\sum x_j)^2\Big)$$

By derivating the payoff function with respect to x_i, $(i = 1, 2, 3, 4)$, and solving the resulting equations we get the Nash equilibrium appropriation and profit levels (see the Appendix for a detailed derivation of the results):

$$\begin{array}{c} x_1^N = x_2^N = x_3^N = x_4^N = \frac{1}{5b} \\ \pi_1^N = \pi_2^N = \pi_3^N = \pi_4^N = \frac{1}{25b} \end{array}$$

We now consider a credit constraint on players 2,3 and 4 such that their individual appropriation levels fall from $\frac{1}{5b}$ to a. The equilibrium level of appropriation of player 1 as a result of reduced appropriation levels by the other players can be derived from 2.4 by substituting $3a + e_i$ for $\sum e_j$:

$$x_1^* = \frac{1 - 3ab}{2b} > \frac{1}{5b}$$

This amount is smaller than the credit now available to player 1 which is (from the assumption that total credit remains constant):

$$\frac{1}{5b} + 3\Big(\frac{1}{5b} - a\Big) = \frac{\frac{4}{5} - 3ab}{b} \quad \Big(> \frac{1 - 3ab}{2b}\Big)$$

Now the payoff of the three constrained players is

$$\pi_k^* = \frac{a}{2} - \frac{3a^2b}{2}, \quad \text{for} \quad k = 2, 3, 4$$

we can compare this payoff to the initial one:

$$\pi_k^* > \pi_k^N \Leftrightarrow \frac{1}{5b} > a > \frac{2}{15b} \qquad \text{for} \quad k = 2, 3, 4$$

Obviously, there is a range of values of a for which the disequalizing change is Pareto-improving. Moreover, the higher the number of users who are constrained, the more likely it is that the effect of increased total income will prevail over the effect of reduced share.

However, as the authors state further in the text, these results may be of less practical importance than suggested. Empirical literature provides evidence of societies in which all individuals enjoy equal access to local common-pool resource, thus rendering heterogeneity unfeasible. This can be caused in three ways. First, if the appropriating technology is simple, everybody has similar ability to appropriate. Second, access to properties of a certain community may serve to validate membership, thus any inequality among individuals would be considered illegitimate. Third, egalitarian access to local CPR often constitutes an important component to socially protect the deprived sections of rural communities. On the whole, even under unequal wealth distribution, the distribution of access to CPR is often equal.

Moreover, Barnard and Dayton-Johnson (2002) argue that the relationship between wealth distribution and level of appropriation is in fact U-shaped. According to them, higher efficiency is possible at very high and very low levels of appropriation, while for a middle range of inequality, it is not possible. Their model has several crucial assumptions, though, that account for different results. First, unlike Baland and Platteau, they consider a linear output function. Moreover, they consider multiple periods, so that the negative externalities players impose on one another do not occur within periods, but rather across the two periods.

Another criticism suggested by Baland *et al.* (2007) is that inequality is studies in terms of the constraints on the contributions of some of the players. While this presents a useful tool for some of the setting, it does not offer a more general view. Bardhan *et al.* propose a different model, in which both private and collective goods are used in production function of a final good. Assuming increasing and concave function like Baland and Platteau, the wealth level of each individual is projected to the contribution to the collective action[5] in a different way, though. Payoff of the ith individual is given by the production function minus the cost of effort put into the collective action:

$$\pi_i = f(w_i, z_i) - x_i \qquad \text{where} \qquad z_i = bx_i + c\sum_j x_j \qquad \text{for} \quad j = 1, \ldots, n$$

w_i is the initial endogenously given wealth level of an individual. This wealth is purely private good, and each individual must decide on the level he uses. However, as this good is supposed to be non-tradeable, the invested level will be equal to the wealth endowment. On the other hand, z_i is a collective good that has either positive or negative externalities. In the case of commons resources the parameters must fulfill the following conditions:

$$b > 0, \quad c < 0$$

Also, both private and public goods are supposed to be complements throughout the model. Obviously, the level of appropriation under there circumstances is always positive for all the individuals. what is then examined by the authors in not the appropriation levels resulting from the wealth redistribution, but rather the effect

[5]The authors consider general model with both positive and negative externalities, thus incorporating both the case of commons and public goods in one model. Contribution in this case refers to public goods, for the case of common resources the term appropriation level would apply.

on the total profits. In Baland and Platteau it was shown that disequalizing change increases the overall efficiency of the resource use, and thus increases overall payoffs. In this model, according to the authors, what wealth distribution maximizes the overall payoff depends on the cost c, ie. the extent of the negative externality players impose on one another. For c small enough (in absolute terms), payoffs are maximized at perfect equality. However, costs of negative externalities (a convex function of wealth distribution) dominate over the concave function representing the sum of individuals' payoffs ignoring the externalities, and thus higher payoffs are achieved by higher inequality.

We have seen that the assumptions adopted in each of the models have a crucial role in determining results. On the whole, however, we showed that under certain conditions, an increase in wealth inequality can decrease level of appropriation of the CPR, and thus ensure a more efficient outcome. Furthermore, in some cases it is able to produce a Pareto improving outcome. Also, the extent of the negative externality is important in deciding the effect of wealth distribution.

2.4 Threat of Resource Depletion

In none of the previous sections did we take into account the possibility of resource depletion. However, in many of the setting this threat is acute as many of the resources are not renewed as fast as they are appropriated. The number of fish in a lake is a limited, and the more fishers catch one day the less there is of them left to reproduce.

To be able to examine the possibility of resource depletion, we consider a model with more than one time period. Dayton-Johnson and Bardhan (2002) present a two-period, two-player model that examines the Nash equilibria of the game under the threat of resource depletion. A two-player model is proposed because it enables transparent results, and it also abstracts from the effects of group size. In this model, the negative externalities players impose on each other do not occur within the period itself through increased appropriating costs as is usual in other models. Rather, one players' high appropriation level in the first period affects the other's incentives through payoffs in the second period.

There are two players who share access to a common-pool resource. Each is endowed with wealth e_i. However, this endowment is not spent in any of the periods, so that any proportion of it can be used in any of the two periods. In that sense, it functions mainly as a constraint on the level of appropriation. The authors use an example of fishery to make the model more transparent. The common resource, a fishery, has capacity F. In each of the two periods, each fisher chooses to spend some portion of his endowment on fishing, ie. a certain level of appropriation $x_i^t \leq e_i$. Each player's yield is a function of his level of appropriation, unless the total level of appropriation exceeds capacity of the resource; in that case each player's yield is proportional to the total amount of effort invested:

$$f_i^1 = \begin{cases} x_i^1 & x_1^1 + x_2^1 \leq F \\ \frac{x_i^1}{x_1^1+x_2^1}F & x_1^1 + x_2^1 > F \end{cases} \tag{2.5}$$

Where F is a stock of fish available, and x_i^1 is the level of appropriation of the ith player in the first period. Player's utility then depends on the total appropriation in both periods.

Unlike Baland and Platteau (1997) in the previous model, they consider a linear output function, so that a given increase in effort always leads to the same increase in common resource output. This simplification is allowed by the fact that negative externalities which are a sign of the Tragedy of the Commons need not be present in this model within one of the periods. As was said above, in this model negative externalities are imposed on the other player in the following period.

Between periods the resource is supposed to grow at rate $g \geq 0$; thus, in the second period the capacity of the resource is $G(F - f_1^1 - f_2^1)$ where $G = 1 + g$. The assumption that makes the threat of depletion acute is that the total initial wealth endowment of both players (and, therefore, the total possible level of appropriation) is high enough so that it is possible for them to appropriate the full capacity of the stock. In other words, assuming there was no appropriation in the first period, players' wealth endowment makes it possible to appropriate all the resource available in the second period:

$$e_1 + e_2 \geq GF$$

As GF is the highest level that is available for appropriation from a resource under any appropriation strategy at any of the periods, this condition is obviously sufficient to ensure that players are able to appropriate the whole resource in any of the time periods. In our example, this means that together the fishers are able to deplete the entire stock of fish in any of the two periods.

In this model, as in the one presented in the previous section, we do not consider cost arising from the decision on the level of appropriation. That is not consistent with what happens in reality, as players always need to invest time and effort to the appropriation and often even direct investments for obtaining technology needed for appropriation. Also, even if there were no other costs, there are always opportunity costs, because both time and effort may be used for different activities yielding certain payoff. This omission can be explained by the attempt to obtain results as straightforward as possible. However, we need to be aware of the implications(limitations) such omission produces. Also, the model purposely abstracts from the consideration of discount rates so that the incentives for conservation are clearer. In reality, a positive discount rate would have to be subtracted from G. Therefore, if the discount rate is higher than G, the first-period depletion of the resource is optimal.

Because there are only two periods considered, in the second period both players will always appropriate to maximum $x_i^2 = e_i$, and the payoff of each of the players will be

$$f_i^2 = \frac{e_i}{E} G(F - f_1^1 - f_2^1)$$

where $E = e_1 + e_2$. Thus, the question is what will their behaviour be like in the first period. If there was not an assumption that $g \geq 0$, the resource capacity would decrease in the second period, and there would be no dilemma: depletion of the resource in the first period would be both an equilibrium and an optimum. However, under the conditions imposed, the optimum is to restrain from appropriating in the first period (this strategy is called conservation by Dayton-Johnson and Bardhan), and then deplete the resource in the second period. In this model, the decisions made in the second period are not influenced by the decisions made in the first period; the strategy is always to deplete the resource, whatever the decisions of the first period. In that sense, e_1^2 and e_2^2 are strategically neutral to the actual amount of the resource available in the second period $(F - f_1^1 - f_2^1)$. As there are only two periods considered, the model does not allow for complicated punishment strategies between the players[6], but as the authors state "it is sufficient to capture the fundamental dilemma of resource conservation: namely, when is it reasonable to forgo current-period consumption in return for higher next-period gains" (Dayton-Johnson and Bardhan 2002, p. 582).

As the resource will always be depleted in the second period, from now on we will only consider the decisions in the first period. In this setting, each player chooses his or her effort level x_i; if $x_j \in [0, F - x_i]$, then the resource is not depleted in period 1, while if $x_j \in [F - x_i, e_j]$ the resource is depleted. There are no constraints on the relationship among $0, F - x_i$ and e_j, so any of these intervals can be empty. The following lemmas establish the characteristics of the player's best-response function. The proofs can be found in the Appendix.

Lemma 1 *If, for player i, the interval $[F - x_j, e_i]$ is non-empty, then i's optimal choice is $x_i = e_i$.*

This lemma says that, under the situation in which the resource is depleted in the first period, the optimal response by the player i is to appropriate at the level of his wealth endowment.

On the other hand, if $x_i \in [0, F - x_j]$, the payoff of player i is linear in x_i, and it is expressed by the following equation.

$$\pi_i = x_i + \frac{e_i}{E} G(F - x_i - x_j)$$

From this equation we can express the slope of the payoff function as $s_i = 1 - \frac{e_i}{E} G$. We assume such e_i, E and G that $s_i \neq 0$ for $i = 1, 2$. However, as there were no other restrictions, the slope s_i can be both positive and negative, depending on the relative values of e_i, E and G. Positive slope says that player i's payoff increases with higher levels of appropriation, while negative slope means payoff decreasing with increased level of appropriation. For a negative slope, either the initial wealth distribution has to be very unequal, or the resource growth is very high, or both. For example, if, initially, wealth was equally distributed, for the slope to be negative the resource would have to grow at rate of more than 100% $(G \geq 2)$.

The following two lemmas consider the cases when $s_i > 0$ and $s_i < 0$ respectively:

Lemma 2 *If $s_i > 0$, then e_i is the unique best response to any action x_j taken by player j.*

[6]The strategies players can adopt when a game is repeated are discussed in section 2.6

Lemma 3 *Let* $s_i < 0$.

a) If $e_i \leq F - x_j$, *then* i*'s best response is* $x_i = 0$.

b) If $e_i > F - x_j$, *the best response is* $0, e_i$ *or both* 0 *and* e_i *as*

$$\frac{e_i}{E} G(F - x_j)$$

is greater than, less than, or equal to

$$\frac{e_i}{e_i + x_j} F$$

respectively.

Thus, if the slope of the player i's payoff function is positive, his best response to any action j might take is to appropriate at the level of his wealth endowment. On the other hand, if $s_i < 0$, the situation is more complicated, and we need to separately consider the possible relative values of e_i and $F - x_j$.

Lemma 4 *There are only four possible equilibria:* $(0,0), (e_1,0), (0,e_2)$ *and* (e_1,e_2).

The set of Nash equilibria to this game is characterized in the following proposition which draws on the previous lemmas. For proof see the Appendix.

Proposition 1 *The Nash equilibria of the game are as follows:*

(i) $s_1 > 0, s_2 > 0$: (e_1, e_2) *is the only equilibrium, and it is a dominant-strategy equilibrium.*

(ii) $s_1 > 0, s_2 < 0$: $(e_1, 0)$ *is an equilibrium* $\Leftrightarrow$ *either* $e_2 \leq F - e_1$ *or* $e_2 > F - e_1$ *and* $(G-1)F \geq Ge_1$. (e_1, e_2) *is an equilibrium* $\Leftrightarrow$ $e_2 > F - e_1$ *and* $(G-1)G \leq Ge_1$.

(iii) $s_i < 0, s_2 < 0$: $(0,0)$ *is always an equilibrium.* (e_1, e_2) *is an equilibrium* $\Leftrightarrow$ $(G-1)F \leq Ge_i$, $i = 1, 2$.

From the proposition it is clear that complete depletion of the resource is a possible equilibrium in all of the above cases. On the other hand, under certain conditions, the first-best efficiency equilibrium emerges. The conditions $s_i < 0$ for $i = 1, 2$ can be rewritten as $e_i > \frac{E}{G}$ for $i = 1, 2$. Thus, $\frac{E}{G}$ is a threshold amount of wealth above which the players will conserve, conditional on the other's conservation. Alternatively, we can interpret this condition as defining a minimal regeneration rate G such that mutual conservation is possible in equilibrium. In our two-person model under equal wealth distribution the condition is $G > 2$, so the stock of fish must grow at a rate higher than 100%. If this condition is fulfilled, then there exists a sufficiently high level of F such that full depletion is no longer an equilibrium outcome under any wealth distribution. The condition on the level of growth can be derived generally. Re-writing the previous condition $e_i \geq \frac{E}{G}$ and substituting $e_1 + e_2$ for E we get for the first player:

$$G \geq \frac{e_1 + e_2}{e_1}$$

Now expressing $e_1 = \sigma E$ and $e_2 = (1 - \sigma)E$ we get:

$$G \geq \frac{\sigma E + (1 - \sigma)E}{\sigma E} = \frac{1}{\sigma}$$

Similarly, for the second player we get the condition:

$$G \geq \frac{1}{1 - \sigma}$$

Thus, the condition which defines a minimal regeneration rate allowing mutual conservation is:

$$G \geq max\{\frac{1}{\sigma}, \frac{1}{1 - \sigma}\}$$

This condition is clearly the same as the one given previously; assuming $\sigma = (1 - \sigma) = \frac{1}{2}$ we get $G \geq 2$.

Proposition 1 also indicates the parameter combinations under which there are multiple equilibria. It can be shown that whenever $e_i > \frac{E}{G}$ for $i = 1, 2$, both full depletion and full conservation are equilibrium outcomes in the first period. This follows from the case (iii) of the Proposition. If $e_i > \frac{E}{G}$, then the condition $s_i < 0$ for $i = 1, 2$ and the condition of case (iii) is met. The commons-dilemma assumption $E \geq GF$ implies that $E \geq (G - 1)F$. Meanwhile, the condition $e_i > \frac{E}{G}$ can be rewritten as $E < Ge_i$. Putting those two conditions together we get a condition for full depletion equilibrium $(G - 1)F \leq Ge_i$.

In conclusion, it is clear that presented extension to the basic common-pool resource model in effect supports the conclusions of the basic model, although it suggests that growth of a resource together with consideration of more than one time period may improve the expected outcomes of the game. Even though full depletion of a resource in the first period is always an equilibrium, there are conditions under which full conservation is possible. Also, in case of multiple possible equilibria, the question of coordinating players on the more efficient outcome arises. Thus, a dilemma is transformed into a coordination game. Success of solving coordination games depends on many factors, one of which can be the possibility of face-to-face communication. The effect of communication and other possible factors will be discussed in the second chapter.

2.5 Exit Options

As Dayton-Johnson and Bardhan (2002) state in their article, if agents have more opportunities to move their appropriating technology elsewhere, they are much less concerned about conservation of the resource in a given location (Dayton-Johnson and Bardhan 2002, p. 589). This would imply that the richer or larger users will be more likely to defect. On the other hand, other authors claim that it is the poorer or smaller users who exercise exit options (Bergeret and Ribot 1990).

In the previous section, the basic model proposed by Dayton-Johnson and Bardhan was described. In this section, we will examine the effects of wealth inequality when the option of exiting rather than appropriating the resource in the second period is present. The possibility of exiting in the first period is not considered because if players could exit in the first period, the best response of the remaining player would be to conserve. Thus, a player can stay and conserve, stay and degrade or degrade and then leave. Obviously, if one of the players exits in the second period, the other player receives the entire catch of that period. The authors assume that the exit options depend on a player's endowment level, so it's given by the function $\psi(e_i)$; in that sense 'exit option' may refer to investing or deploying one's capacity in another sector. Also, both users are supposed to have the same exit options function: if they have the same endowment, the value of their respective exit option will be the same. Now, for a full conservation to be an equilibrium, player i's share on the second period catch (under the assumption that j's first period appropriation level is zero) must be higher than the value of deviating (ie. maximum appropriation in the first period and then exiting in the second period):

$$\frac{e_i}{e_1 + e_2} GF \geq min\{e_i, F\} + \psi(e_i) \tag{2.6}$$

Generally, the result depends on the nature of $\psi(e_i)$ function. In other words, outcomes will be different for concave and convex relationship between wealth endowment and exit options. This relationship can be often determined empirically. Possibly, beyond some wealth level, the relationship could be linear. At lower levels of wealth, though, the relationship would probably be convex because of borrowing constraints.

First, we will examine the case where $\psi(e_i)$ is a non-decreasing concave function such that $\psi(0) = 0$. Dayton-Johnson and Bardhan restrict the attention to those cases where there exists some wealth distribution for which full conservation is not an equilibrium, ie. they focus on those cases where 'distribution matters' (Dayton-Johnson and Bardhan 2002, p. 591). Further restriction focuses on cases where the maximum possible payoff from the second period appropriation is greater than maximum appropriation in the first period plus the value of exiting in the second period. This can be written as:

$$GF \geq F + \psi(e_i) \tag{2.7}$$

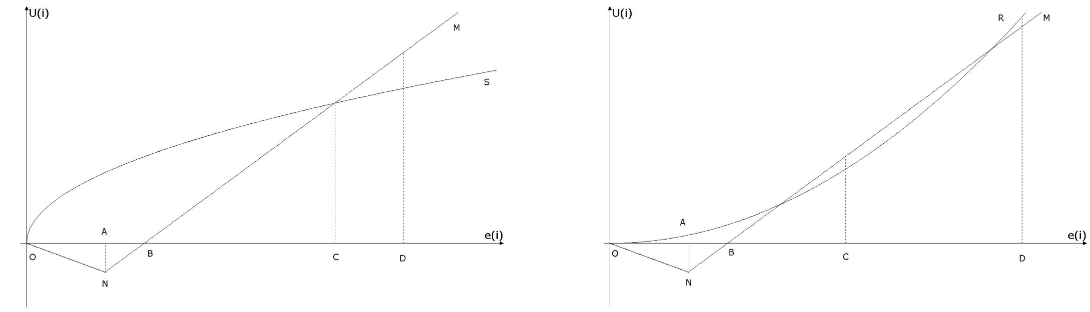
From this condition, the authors state that

> [...] Whenever full conservation is an equilibrium with a perfectly equal distribution of wealth, there always exists a less equal distribution of wealth such that full conservation is not an equilibrium. In this case, equality is more conductive to conservation. (Dayton-Johnson and Bardhan 2002, p. 591)

Moreover, under these circumstances, it is the poorer player who benefits from playing the exit strategy.

Secondly, the authors examine the case where $\psi(e_i)$ is a convex function increasing in wealth. They state that, under previous conditions, if there exists a particular wealth distribution under which full conservation is an equilibrium outcome, then under perfectly equal wealth distribution full conservation is no longer an equilibrium.

The effect of both convex and concave ψ functions can be best illustrated in Figure 2.2. The curve OS (OR respectively) represents the concave (convex) ψ function. The line ONM in both figures represents the remaining terms of equation 2.6, namely $\frac{e_i}{e_1+e_2}GF - min\{e_i, F\}$. Point A in the figures corresponds to F. Point B in the figure represents a point where the right-hand side of the equation changes from negative to positive that is where $e_i = \frac{e_1+e_2}{G}$. Suppose that both players were initially endowed with wealth D. Then, the wealth is redistributed away from player i until his wealth lies to the left of C. In figure 2.2a, OS lies above ONM, so the player i will prefer an exit option, and thus full conservation is no longer an equilibrium. In figure 2.2b, the situation is more complicated, as there are two intersections. Thus, if the distribution of wealth is perfectly unequal (one user has D, and the other has zero), the user with D will prefer the exit option, as the curve OR is above ONM. If wealth is distributed more equally, full conservation is an equilibrium outcome. However, with further decrease in wealth of one of the players (to the range of A), then again full conservation is not an equilibrium outcome.



(a)
Concave

(b)
Convex

Figure 2.2: Concave and Convex Exit Option Function
Source: Dayton-Johnson and Bardhan (2002, p. 592-593)

In reality, though, exit options of each of the players are often different. This could be caused for example by different technology each of them uses for appropriation. If the technology used is very specific, it can be neither moved nor used for other activities. Dayton-Johnson and Bardhan consider the case when only one of the players has the option to exit. In this situation, whether a full conservation is an equilibrium depends on the identity of the player who gains by wealth redistribution. The case in which exit options are a concave function of wealth can be again depicted in figure 2.2a. Suppose it represents the situation of player 1 only, while player 2 does not have exit options; thus, his only concern is whether or not to conserve in period 1. Then, conditional on player 1's conservation, player 2 will always conserve if his or her wealth lies anywhere to the right of point B. If the wealth distribution was equal for both players at level D, then full conservation would be an equilibrium outcome like it was in the situation where both players had exit options. The difference arises when the endowment of the second player decreases to any level between B and C. Under these circumstances, full conservation would still be the equilibrium. However, further reduction in player 2's wealth to the left from point B would make him appropriate in the first period, and thus full conservation would no longer be an equilibrium. Thus, the unequalizing wealth distribution must be larger to cause full conservation to cease to be an equilibrium.

In the situation when the relationship between exit option and endowment is convex, the situation is again more complex and not easy to determine, as there are multiple crossing of the lines. Moreover, changes in the appropriation technology may further complicate the matter, as convex or concave relationship instead of a linear one may further increase number of crossing of the lines.

In conclusion, if a user's exit options is a concave function of wealth, an increases in inequality up to a certain point will reduce conservation. This is because the poorer user will optimally choose not to conserve. As his wealth declines, the gain from conserving falls more rapidly than the gain from exercising his exit options (because the former is a linear function of wealth while the latter is a concave function of wealth). Thus, he gets higher return if he stays on the commons and degrades it. On the other hand, if the relationship between exit options and wealth is convex, the effect is indeterminate and depends on the wealth level. So increased inequality might both enhance or damage conservation. Thus, we can see that the relationship between exit options and wealth has an important effect on the conservation of the resource in this model. The main contribution of this model is that in real situations, each of the individuals who appropriate the resource usually has many other possibilities of how to spend his time and effort. Therefore, we can see that the possibility of effective resource appropriation is complicated by multiple opportunities. On the other hand, in more local resources or less developed areas when individuals are dependent on the resource, such possibilities may not be present. However, with increased development and globalization of the world, this complication to the basic common resource representation cannot be omitted.

2.6 Repetitions of the Game

Although there are games that are played only once, in real-life situations involving common-pool resources, decisions about the level of appropriation are taken repeatedly. Repeated games are far more complicated than one-shot games, so in this section we will describe only some elements of this broad subject.

Ostrom *et al.* (1994) describe how the number of strategies quickly increases with the number of repetitions. In one-shot Prisoner's Dilemma, a strategy is a decision about how to act; there are two pure strategies each of the players can choose from. In a PD game repeated twice, number of strategies increases to 32. In the second round, each of the players must decide what action to take taking into account all the possible outcomes of the first round. So, a possible strategy for player one can look like this:

Strategy I

First round: *play s*

Second round: *play s if the first round outcome was (s,s)*
play s if the first round outcome was (s,t)
play s if the first round outcome was (t,s)
play s if the first round outcome was (t,t)

We can see that even though the strategy requires a player to play s in the first round, the strategy must still specify how to act in case he doesn't. In this strategy, a player should play s under all circumstances. There are 5 occurrences of 'play s' action, each of which could be replaced by 'play t'. Thus, in this game repeated twice, there are $2^5 = 32$ possible strategies.

Another complication is that games can be repeated either finitely (the number of rounds of the game is known), or infinitely. The easiest way to solve finitely repeated games is to start by understanding the last period of the game, where the future is shortest (Rasmusen 2000, p. 156). In the last round of Prisoner's Dilemma, the situation is the same as in the one-shot game, so the strategy for both players is to confess. Next, we consider the last but one round of the game. Neither of the players will deny because they both know that in the last period they will confess no matter what happens in this round; so the strategy is to confess as well. We can use the same reasoning for the last but two round and similarly, by continuing backward induction, for all the previous rounds of the game. Therefore, repetition alone does not induce cooperative behaviour. However, as Rasmusen (2000) points out, the strategy of always confessing is not a dominant strategy as it is in the one-shot game. In repeated game this strategy is not the best response to various suboptimal strategies such as *(Deny until the other player Confesses, then Deny for the rest of the game).*

On the other hand, when the number of rounds is not known, ie. when the game is repeated infinitely, then we cannot use backward induction like we did in the finitely repeated games situation. As in other infinitely repeated games, in infinitely repeated Prisoner's Dilemma there are multiple equilibria, and the main problem is to decide which of them will be played. Thus, we can find an equilibria in which strategies of each of the players induce them to cooperate. One such strategy is called Grim Strategy, because each of the players starts by choosing *Deny*, but once any player chooses *Confess*, they choose *Confess* forever.

Grim Strategy

1. Start by choosing Deny.
2. Continue to choose Deny unless some player has chosen Confess, in which case choose Confess forever.

If player 1 uses this strategy, then if player 2 'cooperates', he will receive the high *(Deny, Deny)* payoff forever. However, if player 2 confesses, he will receive the higher *(Deny, Confess)* payoff once, but then the first player will switch to *Deny*, so the best player 2 can hope for is the *(Confess, Confess)* payoff. Thus, even in infinitely repeated game, cooperation is not immediate (Rasmusen 2000, p. 158). Moreover, not every strategy that punishes confessing is perfect. An example given by Rasmusen is the Tit-for-Tat strategy.

Tit-for-Tat
1. Start by choosing Deny.
2. Thereafter, in period n choose the action that the other player chose in period (n-1).

If player 1 uses the Tit-for-Tat strategy, then player 2 does not have an incentive to confess first because if he cooperates his payoff is higher. The result of initial punishment by player 1 of player's 2 decision to confess is an alternation of *(Confess, Deny)* and *(Deny, Confess)*. Thus, as the payoffs from this alternation are lower than the payoffs from *(Deny,Deny)*, player 1 would rather ignore player 2's first confession. Therefore, this strategy cannot enforce cooperation.

From this it is clear that the distinction between finitely and infinitely repeated games is a crucial one. In an infinitely repeated game, there is no Last Period, and the probability with which the game ends in each period (p) is 0. As Rasmusen points out, though, even the assumption that $p > 0$ does not make a drastic difference as long as it does not become too large. If $p > 0$, the expected number of repetitions is finite, but the game still behaves like an infinite game because the expected number of future repetitions is always large, no matter how many rounds have already occurred. On the other hand, if we know that the game will end at some uncertain date before T, the game is finite because, as time passes, the maximum length of time still to run shrinks to zero. Therefore, we must be careful in distinguishing finitely and infinitely repeated games, as the equilibria differ significantly in consequence.

In conclusion, the analysis of repeated games is complicated. In this section, we only touched upon the most important distinction to show the difference between finitely and infinitely repeated games and some possible strategies players can adopt. As can be seen, when a game of Prisoner's Dilemma is repeated finitely, the predicted outcome does not change, it is still *(Confess, Confess)* in all the individual rounds. On the other hand, outcome of an infinitely repeated PD game is very difficult to predict, as there are many possible strategies each of the players can adopt.

2.7 Types of Players

The initial assumption that all individuals are selfish, and their decisions are based solely on maximization of their profits does not always correspond to what we can see around us. While this assumption has been proved quite robust in predicting the outcomes of competitive market situations, its application to collective action situations has been less successful. One of the main findings is that different people, when facing a particular situation, act in different ways. Among the possible suggestions that could explain this is that the world contains multiple types of individuals. Some of them may be selfish as assumed previously, but others are more willing to initiate reciprocity to achieve the benefits of coordinated action (Ostrom 2000b).

In her article, Ostrom (2000b) assumes the existence of two other types of players apart form the so-called 'rational egoists' who only pursue their material interests. The first type are 'conditional cooperators' who are "willing to initiate cooperative action when they estimate others will reciprocate, and to repeat these actions as long as sufficient proportion of the others involved reciprocates" (Ostrom 2000b, p. 142). Thanks to them, relatively high levels of contributions[7] in the initial rounds of Prisoner's Dilemma games were observed. Their initial contribution may also encourage some rational egoists to contribute as well, so as to obtain higher returns in the early rounds of the game (Kreps *et al.* 1982). Tolerance for free riding among conditional cooperators may vary, though. Gradually, without communication or some institutional mechanism, they begin to reduce their own contributions because of increased free riding. Ostrom says that:"Conditional cooperators are apparently a substantial proportion of the population, given the large number of one-shot and finitely repeated experiments with initial cooperation rates ranging from 40 to 60 percent" (2000b, p. 142).

The second type are 'willing punishers' who, if given an opportunity, are willing to punish presumed free riders through either verbal means or costly material punishment. Willing punishers are supposed to also become willing rewarders if they are allowed to reward those who contribute more than the minimal level.[8] However, could those norm-using types emerge and survive in a world of rational egoists?

Before we try to answer this question, let's have a look at one of the most famous models using the evolutionary game theory - Hawk-Dove Game. Evolutionary theories provide useful ways of modeling the emergence and survival of multiple types of players in a population. In this game, we have a population of birds, each of which can behave either as an aggressive Hawk, or a pacifist Dove. When 2 randomly chosen birds from the whole population meet, they 'fight' over a resource (this can be for example food) with value $v = 2$. If they both fight, the loser incurs cost $c = 4$. Thus, the expected payoff when 2 Hawks meet is $0.5 \cdot 2 + 0.5 \cdot (-4) = -1$. When two Doves meet, they do not fight, but they split the resource in half. When a Dove meets a Hawk, the Dove flees with no payoff, while the Hawk receives the whole resource. The situation is represented in table 2.4.

		Bird Two	
		Hawk	Dove
Bird One	Hawk	-1,-1	2,0
	Dove	0,2	1,1

Table 2.4: Hawk-Dove Game
Source: Rasmusen (2000, p. 180)

There are two asymmetric pure strategy Nash equilibria $(2, 0)$ and $(0, 2)$. Thus, Hawks would invariably get more resource, and thus Dove would slowly disappear from the population. However, from the point of view of the Evolutionarily Stable Strategy, [9] neither Hawks nor Doves can take completely over the population, as both population of Hawks and population of Doves can be invaded by the other type;

[7]In the CPR setting this means low levels of appropriation

[8]Again, in the CPR setting this means appropriating less than what was agreed upon

[9]A strategy s^* is Evolutionarily Stable Strategy (ESS) if, using the notation $\pi_i(s_i, s_{-i})$ for player i's payoff when his opponent uses strategy s_{-i}, for every other strategy s' either

$$\pi_i(s^*, s^*) > \pi_i(s^{'}, s^*)$$

ie. there is no evolutionary stable pure strategy, because if a population consisted entirely of Hawks, a Dove could invade it a receive a one-round payoff of 0 compared to -1 that Hawk receives against itself and vice versa. On the other hand, there may be mixed strategy ESS. If there are h Hawks in the population, and $(1-h)$ Doves, then the expected payoffs of Hawks and Doves are:

$$P_h = h(-1) + (1-h)2 = 2 - 3h$$
$$P_d = h(0) + (1-h)1 = 1 - h$$

From that the mixed strategy ESS is h=0.5. In other words, in a stable situation, the population will consist of 50% of Doves and 50% of Hawks. Alternatively, we can say that the probability of encountering a Hawk or a Dove is 0.5. This equilibrium is stable in a sense that if there were more Doves, the expected payoff of a Hawk would be higher than that of a Dove, and thus the number of Hawks in the population will keep increasing until the equilibrium state is reached. Obviously, the resulting proportion of Hawks and Doves in population depends on the relative values of v and c. The higher the cost of loosing (c) the less Hawks there would be in the population.

This simple evolutionary process can be applied to a world where there are two types of players: rational egoists and conditional cooperators.[10] In the evolutionary approach individuals inherit strategies and do not change them during their life time. As we saw, those employing the more successful strategies for an environment are supposed to reproduce at a higher rate. After many iterations, the more successful strategies come to prominence in the population (Axelrod 1986). This approach can explain why, in repeated games where there are multiple equilibria, the more successful ones are reached.

The different strategies are supposed to be learned in form of social norms[11]. Which norms in particular are learned, though, varies across cultures, families, and with exposure to diverse social norms expressed within various types of situations (Ostrom 2000b). Several possibilities on how such evolution of different norms may have occurred are mentioned by Sethi and Somanathan (2004). One of them supposes that people with preferences for reciprocity will behave reciprocally when meeting with each other, while they will behave selfishly when they meet rational egoists. As in the model of Hawks and Doves, pairs of players meet and interact over a resource. As long as reciprocal preferences cannot be perfectly imitated, the first type will receive higher payoffs from cooperation that more than offsets losses from encountering the second type.

In addition, an indirect evolutionary approach analyzes how such achieved norms could evolve and adapt: players receive objective payoffs, but to decide on an action they transform these payoffs into 'intrinsic preferences' - they add a subjective

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or

$$\pi_i(s^*, s^*) = \pi_i(s', s^*) \qquad \text{and} \qquad \pi_i(s^*, s') > \pi_i(s', s')$$

The first condition says that a population using s^* cannot by invaded by a player using s'. The second set of conditions says that s' does well against s^*, but badly against itself, so if more than one individual tried to invade a population using s^* by using s', the invaders would fail.

[10] the third type was omitted for simplification

[11] Social norms are shared understandings about actions that are obligatory, permitted or forbidden (Crawford and Ostrom 1995).

change parameter to actions that are consistent or not consistent with their norms. In other words, their utility is not dependent only on the objective payoff, but also on how well their decisions conform with their social norms. Therefore, even though individuals start with a predisposition to act in a certain way, their preferences are allowed to adapt (Ostrom 2000b). In collective action situations, players would start with their individual preferences based on predisposition towards reciprocity and trust. Then, during the game once they have learned about the likely behaviour of the other participants, they would shift their behavior in view of that and their received payoffs. However, just as positive experience can enhance cooperation, so can negative experience reduce it. This approach explains how, in situations where mutual trust needs to be established for more effective outcomes, a mixture of norm-users and rational egoists emerge. Each of these two types would behave differently in the same setting according to the degree to which they value conformance to the norms. Rational egoists would have intrinsic payoff the same as objective payoffs, while conditional cooperators would have an additional parameter that adds value to the objective payoff when reciprocating trust with trustworthiness. In their interaction, different types will gain different objective returns. The resulting relative proportion of each of those two types in the population will then depend on the level of information available to the players.

With complete information about player types, conditional cooperators will more frequently receive the higher payoff in Prisoner's Dilemma-like games, while rational egoists will consistently receive lower payoffs since the others will not trust them. Thus, only the trustworthy type would survive in an evolutionary process, as new entrants are more likely to adopt the more successful strategy. Moreover, less successful individuals would be likely to adopt the more successful strategy as well. The assumption of complete information is very strict, though, and unlikely to be met in real-life situations. On the other hand, if there are no information about player types, only rational egoists will survive as they will receive high payoff at the expense of conditional cooperators. If information about the proportion of trustworthy population is known, but there is no information about the specific player, conditional cooperators will trust as long as the expected return of meeting another trustworthy player (and thus receiving the higher payoff) exceeds the payoff obtained when neither player trusts the other one. We saw how in such situation an equilibrium level of both types emerges. Moreover, if there is a noisy signal[12] about a player's type that is more accurate then random, trustworthy types will survive as a proportion of the population.

A step in this direction is a model proposed by Falk *et al.* (2002). We present this model in the following chapter that deals with the self-management of the commons. The reason for this is that even though the behavioral adjustments proposed by this model can be applied to the setting of this chapter (when there is no possibility of interactions among players) as well, it does not seem to significantly change the predicted outcomes. On the other hand, when applied to situations where interactions among players are possible, thus allowing for communication and sanctioning, the proposed model seems to estimate much better the predicted behaviour.

[12] This can result for example from seeing one another or face-to-face communication

Chapter 3

Self-Managed Common-Pool Resources

In the previous chapter we tried to show that even under Hardin's original non-cooperative assumption, the Tragedy of the Commons is not the necessary outcome of common-pool resource appropriation. The areas examined show that under special circumstances, the outcomes may be more efficient. However, such circumstances are still too specific to explain what we observe around us. In this chapter, we will consider how, when granted contact, players may be able to manage their use of CPR to achieve a more efficient outcome: they are often able to estimate an optimal level of appropriation, agree on strategies of behaviour and stick to those rules.

The first section of this chapter describes how, when the possibility of monitoring the use of the resource is present, the game and game outcomes change. However, as players are still unable to communicate, the increase in efficiency depends on the particular incentive structure. Therefore, the second section deals with the question how communication may result in the design of rules for the management of the resource. It examines both theoretical and empirical suggestions on which variables are crucial for the successful design of rules.

The third part presents a model that incorporates inequality aversion to individual's utility function to show how such assumption about incentives improves the predictive ability of players' behaviour and levels of appropriation. The model is first introduced on the basic commons framework, and then the utility function is modified to show how efficiency may increase when communication and sanctioning are allowed.

The last part of this chapter deals with contest; it shows how Tragedy of the Commons may be viewed as a contest with particular type of prize that depends on the effort levels of the players. It also shows how under certain circumstances, contest may increase incentives for the design of rules.

3.1 Monitoring Game

In this section, we will consider a game with two players, where one of them is able to monitor the actions of the other player as suggested by Ostrom *et al.* (2002).

Moreover, if that player detects appropriation level that is too high, he can punish the other player. So, the position each of the players holds is different. Even though the players do not communicate, this possibility of one of the players to monitor the actions taken by the other player do have an effect on the outcome of the game, and thus it manages the appropriation level in some way.

		Player 2	
		Monitor	Not Monitor
Player 1	Take Share	0,-C	0,0
	Take More	B-p(F+B) -B-C+p(F+B)	B,-B

Table 3.1: Monitoring Game
Source: Ostrom *et al.* (1994, p. 64)

In this game, player 1 can either take his share of a resource, or he can take more than his share. What is the appropriate share of player 1 is decided outside our model, and we do not consider it in this setting. Player 2 can either monitor or not monitor what action the first player takes. The ability to monitor can be caused for example by the physical position of each player in connection to the resource. Ostrom *et al.* give an example in which player at the head of the resource system has access to the water flow before player at the tail. Player at the tail, in consequence, has an opportunity to monitor what player at the head does.

In this game, payoffs are calibrated to a $(0, 0)$ benchmark, which is maximum possible total payoff, so when player 1 takes his share and player 2 does not monitor, payoff of each player is 0. If player 1 takes more than his share, he gets a positive benefit B which can be the value of the additional resource he got. If at the same time player 2 is not monitoring, he loses the benefit of that water, so his payoff is $-B$. However, player 2 has an opportunity to monitor what strategy player 1 takes at a cost c. Thus, if player 1 takes only his share, but player 2 decides to monitor, the payoff to player 1 is 0 and to player 2 is $-c$. Moreover, there is an imperfect detection technology available to player 2, so even when he decides to monitor player 1, the probability that he detects excess taking by player 1 is $p < 1$. When player 2 detects excessive appropriation of player 1, he gets all the water back plus a bonus F for successful monitoring. Meanwhile, player 1 apart from giving back the excess water, he has to pay a fine F for getting caught. We suppose that the bonus the player that monitored receives and the fine the caught player must pay are of the same value; what one of the players pays the other receives. On the other hand, with probability $1 - p$ player 2 does not detect excessive appropriation by player 1. In this case, there is no other consequence than that player 2 pays the cost of monitoring. So the expected payoff for player 1 in this situation is $E(P_1) = p(-F) + (1 - p)B = B - p(F + B)$. For player 2 the expected payoff is $E(P_2) = p(M - C) + (1 - p)(-B - C) = pM - (C + (1 - p)B)$. This game is represented in table 3.1.

In our model, we do not consider who makes player 1 pay the fine for excessive appropriation, and how does it get to player 2. Supposedly, the same circumstances that endowed player 2 with the ability to monitor also make sure these 'rules' are complied with. For example, if both of the players are members of a certain group which owns the resource, such rules may have been used in that group for generations, and thus we do not need to consider their development in our particular setting.

There are four possible games depending on the payoff parameters. In all the individual games, when player 1 takes his share, player 2 will always not monitor, so the strategy pair *(take share, monitor)* can never be an equilibrium. Moreover, when player 2 does not monitor, player 1 will take more than his share, so the strategy pair *(take share, not monitor)* is never an equilibrium. Now, the four possible games are as follows:

a) If $B > p(F+B)$ and $C < p(F+B)$, then player 1 has dominant strategy to take more than his share, so in the equilibrium player 1 takes more than his share, and player 2 monitors.

b) If $B > p(F+B)$ and $C > p(F+B)$, then again player 1 has dominant strategy to take more than his share, player 2 has dominant strategy not to monitor, so the equilibrium is $(take \quad more, don't \quad monitor)$. The difference from situation *a*) is either an increased cost of monitoring, or decreased probability of detection.

c) If $B < p(F+B)$ and $C > p(F+B)$, then player 2 has dominant strategy not to monitor, and in the equilibrium player 1 takes more than his share, which is the same as in situation *b*). If we added to our initial assumptions a constraint on the relative values of B and C such that $B > C$, this would no longer be a possible outcome.

d) If $B < p(F+B)$ and $C < p(F+B)$, then there is no pure strategy equilibrium. However, we can compute a mixed strategy equilibrium.[1]

To compute a mixed strategy equilibrium, we denote m the probability of monitoring by player 2 (so that $(1-m)$ is the probability of not monitoring), and t the probability of player 1 to take his share. Now in equilibrium, the probability of monitoring by player 2 must be just high enough so that player 1 is indifferent between taking his share and taking more than his share. If player 1 takes more than his share, then with probability m he does get monitored, and his payoff is $B-P(F+B)$; with probability $1-m$ he does not get monitored, and his expected payoff is B. On the other hand, if player 1 takes only his share, his payoff is 0. In equilibrium, the expected value of taking more than his share $(m(B-p(F+B))+(1-m)B)$ must be equal to the value of taking only his share (0) so that:

$$m(B-p(F+B))+(1-m)B=0$$

The solution to this equation, m^* is the equilibrium probability of monitoring by player 2. Solving the equation we have $m^* = \frac{B}{p(F+B)}$. We can see that the probability

[1] A mixed strategy of a player assigns each of the possible actions certain probability with which the action will be played.

decreases with increased F; this confirms out intuition, the higher the fine player 1 has to pay when he is caught taking more than his share, the smaller the probability of getting caught needs to be so that he is indifferent between taking his share and taking more. Also, re-writing the equilibrium to $m^* = \frac{1}{p(1+\frac{F}{B})}$ we can see that the equilibrium probability of successful monitoring decreases with increased relative value of $\frac{F}{B}$. In other words, relative increase of B to F increases the probability, as player 1 has higher incentives to take more, so there needs to be higher probability of his getting caught to make him indifferent between taking his share and taking more.

At the same time, in equilibrium the probability of player 1 taking more than his share must be just high enough so that player 2 is indifferent between monitoring and not monitoring. If player 2 does not monitor, then with probability t player 1 takes his share, leaving player 2 with zero payoff; and with probability $1 - t$ player 1 takes more than his share, and player 2 gets $-B$. Thus, the expected payoff of not monitoring is $0t + (1 - t)(-B) = (1 - t)(-B)$. On the other hand, if player 2 does monitor, then with probability t player 1 takes his share, and player 2 gets the payoff $-C$; with probability $(1 - t)$ player 1 takes more than his share, and player 2 gets $pF - (C + (1 - p)B)$. Thus, the expected payoff of monitoring is $t(-C) + (1 - t)(pF - (C + (1 - p)B))$. In equilibrium, the expected payoff from monitoring and not monitoring must be equal so that:

$$t(-C) + (1 - t)(pF - (C + (1 - p)B)) = (1 - t)(-B)$$

Solving the equation we get the equilibrium level $t^* = 1 - \frac{C}{p(F+B)}$. Thus, an asymmetric mixed strategy equilibrium is (m^*, t^*). We can see that t^* is decreasing in C, which supports our expectations: the higher the cost of monitoring, the less likely is player 1 to only take his share. Also, the higher the F or B, the higher is t^*. Obviously, as both these parameters express the benefit of successful monitoring, the higher they are, the higher the equilibrium probability of monitoring.

Thus, it is clear that there is always some tendency to take more water than one's share. In situations $a), b), c)$ player 1 always takes more than his share, and in situation $d)$ he takes more than his share with positive probability. It is clear then that players physical position at the head of a resource is a strategic advantage, and obviously the possibility of monitoring alone is not enough to decisively increase efficiency. On the other hand, the ability of a player to monitor, and more importantly to sanction excessive appropriation, can have a positive effect under certain conditions on cost of monitoring and probability of detection.

However, it is not clear why only one of the players should be able to appropriate the resource when there is free access within a certain group. Also, the second player may have the ability to monitor as well as the first player; for example because of the geographic proximity of the players while appropriating the resource, fishers can see how much fish another fisher caught. Thus, we can extend the proposed model so that each of the two players has four strategies - a combination of taking the share, or taking more, and monitoring or not monitoring. The benefits and costs to players are the same as in the previous model, so that when none of them decides to monitor and both only take their share of the resource, each of them receives a benchmark payoff 0. The only assumption added is that the benefit of excessive appropriation

Player 2

Player 1	Take Share No Monit.	Take Share Monitoring	Take More No Monit.	Take More Monitoring
Take Share No Monit.	0,0	0,-C	-B,B	-B,B-C
Take Share Monitoring	-C,0	-C,-C	-B-C+p(F+B), B-p(F+B)	-B-C+p(F+B), B-C-p(F+B)
Take More No Monitoring	B,-B	B-p(F+B), -B-C+p(F+B)	0,0	-p(F+B), -C+p(F+B)
Take More Monitoring	B-C,-B	B-C-p(F+B), -B-C+p(F+B)	-C+p(F+B), -p(F+B)	-C,-C

Table 3.2: Extended Monitoring Game

is higher than the cost of monitoring ($B > C$). The situation is represented in table 3.2.

Let's have a look at the situations that were not present in the previous model. When player 1 takes only his share and does not monitor, and player 2 takes more than his share and does monitor, then player 1's expected payoff is $-B$. on the other hand, player 2's payoff is the difference $B - C$. If player 1 takes only his share but monitors, and player 2 takes more than his share and does not monitor, then player 1's payoff is $pF + (1 - p)(-B) = -B - C + p(F + B)$, and player 2's payoff is $p(-F) + (1 - p)B = B - p(F + B)$. In the situation when both of them monitor, but only player 2 takes more than his share, then player 1's payoff is $-p(F + B)$, and player 2's payoff is $-C + p(F + B)$. When both of them take more, but none monitors, then the payoffs are $(0, 0)$, while if one of them monitors then the monitoring player gets $-C + p(F) + (1 - p)(-B)$, and the other gets $p(-F) + (1 - p)B$. At last, when both take more and both monitor, the payoff to both is $-C + p(F) + (1 - p)(-B) + p(-F) + (1 - p)B = -C$. We can see that the overall payoff in the different situations in either 0, $-C$ or $-2C$ depending on how many players monitor.

The way we defined it, this game can be seen as two separate games put together. In the first game, player 1 decides whether to take his share or take more, while player 2 decides whether or not to monitor; this is the game described previously. Simultaneously, the same game with switched roles is played. In this second game, it is player 1 who has the ability to monitor, while player 2 can take his share or more. We can prove this very easily: let's take a look at the situation in which player 1 plays *(take more, do not monitor)*, while player 2 plays *(take share, monitor)*. We can divide these decisions in two games; in the first player 1 plays *(take more)* and player 2 *(monitor)*, and in the second game player 2 plays *(take share)*, while player 1 plays *(do not monitor)*. If we have a look at the payoffs from the first game (in table 3.1), we get for the first player $B - p(F + B)$ and for the second player $-B - C + p(F + B)$. In the second game we get $(0, 0)$. If we add up these payoffs we get $B - p(F + B), -B - C + p(F + B)$. We can compare these results to the payoffs of the extended game in table 3.2, and see that they are the same. Similarly, we could proceed with all the different combinations.

Because the extended game consists of two mirror simple games, we can use this to write players' expected payoff functions in a more transparent way. Player 1's expected payoff is:

$$\begin{aligned} E(\pi_1) &= (1 - t_1)[m_2(B - p(F + B)) + (1 - m_2)B] + t_1[m_2(0) + (1 - m_2))] + \\ &+ m_1[t_2(-C) + (1 - t_2)(-B - C + p(F + B))] + \\ &+ (1 - m_1)[t_2(0) + (1 - t_2)(-B)] \end{aligned}$$

Where m_1, m_2 denote the probabilities of monitoring by individual players, and t_1, t_2 denote the probabilities of taking share. The equation is obviously divided into two individual games, as discussed previously; payoffs from each of the situations are multiplied by the probabilities with which such situation arises. Similarly, we could write the expected payoff function for the second player.

Now, depending on the relative values of the parameters, players will either have a pure strategy, or a mixed strategy. If we take into account only the first of the two individual games (first two terms of the equation), it can be seen as a best response function of player 1 to player 2's choice of m_2. We can rewrite it so that:

$$\begin{aligned} BRF &= t_1[m_2 p(F+B) - B] + [m_2(B - p(F+B)) + (1-m_2)B] = \\ &= \delta_1(m_2) + [m_2(B - p(F+B)) + (1-m_2)B] \end{aligned}$$

There are two possibilities to consider. First, the function may be maximized at one of the endpoints - if $\delta_1(m_2) > 0$, then $t_1^* = 0$, and if $\delta_1(m_2) < 0$ then $t_1^* = 1$. Second, for $\delta_1(m_2) = 0$ the expected payoff is constant with respect to t_1, so that $t_1^* \in [0, 1]$. In our situation, setting $\delta_1(m_2) = 0$ gives a condition:

$$m_2^* = \frac{B}{p(F+B)}$$

Correspondingly, we can consider the second individual game expressed in the expected payoff function and we arrive at $\delta_1(t_2)$ and resulting condition:

$$\delta_1(t_2) = m_1[-C - t_c p(F+B) + p(F+B)]$$
$$t_2^* = 1 - \tfrac{C}{p(F+B)}$$

It is possible for the values of m_2 and t_2 not to be an element of $[0, 1]$; in that case the whole interval lies on one side of the m_2^* or t_2^* values. In other words, $\delta_1(m_2)$ and $\delta_1(t_2)$ will have the same sign throughout the whole interval $[0, 1]$. Therefore, player 1 will have the same response for every action player 2 might take. Because B, C and $p(F + B)$ are all positive, there are only two conditions under which this situation arises:

$$B > p(F+B) \qquad \text{and} \qquad C > p(F+B)$$

When these conditions hold, player 1 has a dominant strategy in which $t_1^* = 0$ and $m_1^* = 0$, so that he will take more than his share and will not monitor. Because the situation is symmetric, the same reasoning could be used for the second player. Therefore, the pure strategy Nash equilibrium will be $(0, 0)$. This corresponds to situation *b)* in the simple monitoring game.

Also, when only $B > p(F + B)$, but $C < p(F + B)$, then the dominant strategy is to take more then share for both of the players. However, there is no dominant strategy deciding whether or not to monitor. However, in the analysis of the simple 2×2 game we saw that the best response of the other player is to monitor, so that under these conditions, both players will take more than their share and both will monitor, so the equilibrium payoffs will be $(-C, -C)$. On the other hand, conditions $C > p(F + B)$ and $B < p(F + B)$ imply that $C > B$, which is a contradiction to our additional assumption. The last case is that both $B, C < p(F + B)$. In this case, there is no pure strategy equilibrium, and only mixed strategy equilibrium is possible. We have already computed the equilibrium levels of m^* and t^*. Therefore, each of the players will play *(take share, do not monitor)* with probability $t^*(1-m^*)$, *(take share, monitor)* with probability $t^* \cdot m^*$, *(take more, do not monitor)* with probability $(1-t^*)(1-m^*)$ and *(take more, monitor)* with probability $(1-t^*)m^*$.

Obviously, as in the simple game, m^* is decreasing in F, so the higher the fine a player that is caught pays, the smaller the probability of monitoring. On the other hand, t^* is increasing in F. This supports our intuition: higher fine induces a player to take only his share with higher probability, while it decreases the probability of monitoring needed for the other player to be indifferent between how much he takes from the resource. Also, m^* is increasing in B, so that the higher the benefit of successful monitoring, the higher the incentives to monitor. t^* is decreasing in C because higher costs of monitoring dissuade the other player from only taking his share. The effects of p are straightforward as well. Higher probability of successful monitoring decreases the equilibrium probability of monitoring that is needed for the other player to be indifferent between his choices. Also, the higher the probability of success in detection of excessive appropriation, the higher the probability of taking only one's share as it is more risky to take more.

We defined the extended monitoring game as consisting of two entirely independent games, which simplified our analysis of the equilibria. However, the question is whether such supposition is probable. It is easy to imagine a situation, in which a third variable influences both the probability of monitoring and taking more than share. Such variable could be for example general tendency for activity of passivity of an individual. Therefore, the decisions would no longer be independent.

In conclusion, we have seen that presence of monitoring has some effect on the resulting outcomes, even though it depends very much on the particular set of parameters. Therefore, although it is one of the possible self-management rules regulating the use of a resource, monitoring alone is not enough to explain the observed behaviour in commons situations.

3.2 Design of Rules

Because of the character of the dilemma, it has been thought that the only way out of the "tragedy" is to engage an external authority to take over a common-pool resource. Moreover, in cases where technical knowledge and economies of scale are involved, this external authority is taken to be the government. However, in the last years, effort of many researcher has been focused on challenging this view. They often draw on considerable empirical evidence suggesting that appropriators do create and enforce their own rules (see for example Ostrom 1990, Baland and Platteau 1996). Initially, the research has focused mainly on situations where the positions individuals take are symmetric. However, more recent studies deal with more unequal distribution of economic or political assets, or their physical relationship. The success in crafting such rules depends highly on the particular setting. Therefore, set of conditions enhancing and reducing the probability needs to be developed (Ostrom and Gardner 1993, p. 95).

When trying to design their rules, players need to overcome three social dilemmas (Ostrom and Jensen 2001):

1. The first is the dilemma about the level of appropriation, the "Tragedy of the Commons".
2. The second is that of spending time and effort to create a new set of rules that would bring benefit to all users of a CPR whether they contribute or not. These rules are public goods, so this is a public-good dilemma. The prediction is that the rules will not emerge.
3. The third is that of monitoring and imposing agreed sanctions on deviators. Those are again costly activities, but they generate benefits for all the users of a resource. In that sense, this is a public-good dilemma as well.

Neither of these three dilemmas is predicted to be overcome by rational payoff-maximizing players in the given setting. However, despite all the complications presented by the theory, players have been proved to be able to agree on their own strategies and on creating and enforcing their own set of rules. Ostrom states that

> Once communication is allowed, subjects spend time and effort assessing each other's trustworthiness and reaching agreements about the strategies they should jointly take to achieve this best outcome. Further, individuals in a laboratory setting are willing to monitor each other and invest in costly sanctions in order to punish those who overappropriate as well as in devising specific rules that they themselves enforce on each other.
> Ostrom (2000a, p. 38)

So, players somehow do overcome all three dilemmas, although the resulting arrangement need not be always optimal. However, Seabright points out that: "[...] the very existence of an optimal collective management policy cannot be taken for granted" (1993, p. 115). Thus, there may not be an efficient management strategy. This could result for example from the situation, in which the costs of designing rules is too high, so that it exceeds the benefits from more efficient appropriation. There are three types of costs that need to be taken into account: costs of time and effort spent on devising and agreeing upon the new rules, short-term costs of adopting those new rules and long-time costs connected to maintaining and monitoring the new system over time (Ostrom 2000a, p. 39). These costs as well as benefits arising from the new system can differ from individual to individual and from situation to situation. If the resource is not salient enough, then the benefits arising from it do not provide incentives enough for spending time and effort on design of rules. The differences in possibility, success or failure of establishing new rules, therefore, stem from the differences in cost and benefit distribution in particular setting. In this section, however, we assume that an efficient management exists, and that it is known to all the players.

An important distinction is the scope of the possible tragedy. The two opposite poles of the scale is local versus global commons problem. Obviously, in both the basic structure is the same. However, in global common-resource appropriation, the possibility of communication may be much more limited if not altogether impossible as it involves great number of people. Even if only few players actually decide on the actions (usually governments), there is no higher authority that could intervene. From that point of view, the ability to design rules is even more prominent. On the other hand, when a resource is managed locally, individuals know each other and often the actions taken are observable by the others. Also, this offers the possibility of building reputation. Moreover, there may be relationships among players that influence their willingness to impose negative externalities on each other. Thus, appropriation may be lower than expected even when no regulation is in place. Throughout this section, the focus is on local management, although some of the points may be applicable to global common resources as well.

The first and most important question is what would make players spend time and effort on design of management rules? One of the reasons, as discussed in the previous chapter, is the existence of the type of players who are motivated by reciprocity in addition to their payoff. Another explanation is that even though players still pursue their own interests, it is important to distinguish long-term and short-term interests of players, which may differ. The pursuit of short-term interests may harm the long-term ones as it affects the reaction of the others in the future interactions (Seabright 1993, p. 117.). This approach draws on repeated game theory, which has been analyzed in the previous chapter. Yet another explanation may be the role of traditions and experience. Design of rules is more likely to occur and more likely to be successful in a setting where either the resource has already been managed in the past, or the people have experience in resource management, or both. However, this factor may be of more influence in the absence of communication, so it may not be relevant for local management.

Once set of rules is agreed on, in the absence of trust, each of the members has incentives to defect and thus increase his payoff. However, if trust can be established, and each of the players could expect the others to behave according to the rules in all individual games, the incentives to defect before the others do it would disappear. The experience of past cooperation may create trust, which in turn increases probability of future cooperation.

It has been observed that the likelihood of creating a set of rules and the success with which they will be carried out depends on many different variables. Most of the variables analyzed in the previous chapter (particular payoff distribution, exit option, resource depletion, types of players) have an impact on the design of rules.

Another such variable not mentioned previously is size of the group. However, the effect of group size on the level of collective action[2] is not agreed on. On the one hand, Olson (1965) argued that smaller groups have higher likelihood of solving collective action problems in general. This can be caused by higher opportunity in smaller groups for more frequent interactions, and thus the possibility to build reputation. Ostrom and Poteete (2004) say that: “The expectation of future interactions increases the value of reputations for co-operative behaviour. Moreover, frequent interaction facilitates mutual monitoring” (p439). Both reputation building and monitoring then foster higher levels of trust which in turn creates conditions for more successful collective action. On the other hand, empirical evidence seems to suggest the opposite. Marwell and Oliver (1993) claim that “a significant body

[2]Collective action means the pursuit of a common goal by more than one person. Thus, in our analysis it represents overcoming any or all of the three levels of dilemma in design of rules

of empirical research [...] finds that the size of a group is positively related to its level of collective action" (p. 38).

One of the problems researchers encounter when trying to analyze the effects of group size is that it is a key factor determining many other variables, such as the production technology, its degree of excludability and the level of heterogeneity of the group. Furthermore, it can be related to the costs of invention of the new rules. If the costs remain relatively constant, then an increase in the number of individuals in the group brings additional resources that can be drawn upon, and thus it increases the probability of creation of the new rules (Ostrom 2000a). In other words, they are more likely to gain necessary means to cover the costs as they are more likely to have enough individuals who will contribute (Oliver and Marwell 1988). However, when costs increase with the size of the group, as is actually often the case, then increased number of users decreases the probability, which supports the Olson hypothesis.

Another variable is heterogeneity of group members. The question of the effect of heterogeneity has proved even more complicated. The main reason is that there are many different types of heterogeneity that are often closely related, and thus cannot be examined separately. Homogeneity can facilitate collective action through two channels. First, individuals sharing certain characteristics with the other group members may find the others more predictable which could enhance mutual trust. Second, "common traits suggest common interests" (Ostrom and Poteete 2004, p. 441). In other words, if people are more homogeneous, they are more likely to have similar interests concerning the use of a resource. Therefore, they are more likely to find an acceptable set of rules. On the other hand, when individuals use a resource in different ways based on their socio-economic characteristics, the likelihood of collective action decreases (Cooke 2000). Another problem with the effect of heterogeneity is that socio-economic heterogeneity among resource users may be connected with different degree of access to, and control over a local common-pool resource (Adhikari and Lovett 2006).

Moreover, there is a connection between the group size and heterogeneity among individuals: generally, heterogeneity is thought to increase with group size. In addition, as a new member may add diversity to one or more dimensions, it is possible heterogeneity increases faster than group size. Unfortunately, as the group size increases, so does the demand for the use of a resource. Therefore, collective action is even more important in those groups (Ostrom and Poteete 2004).

As mentioned earlier, design of rules is not limited to the common-resource setting. Therefore, to identify the most important parameters in each particular setting, empirical studies have been carried out. In his article, Agrawal (2000) draws on three empirical studies (Ostrom 1990, Wade 1994 and Baland and Platteau 1996) trying to identify the variables that affect the success in creation of sustainable self-management rules by the particular users of a resource in commons setting. As each of the studies was carried out differently and with different scope of data tested, the results cannot be compared directly. However, it is possible to identify those variables that were generally important for all three authors.

One of the most important variables is that the higher the dependence of the group members on the resource, the higher the possibility of the emergence of some management, and thus efficient appropriation. This confirms out intuition - if individuals depend on the resource in their everyday lives, as can often be the case in local resources, they cannot risk being vulnerable to fluctuating payoffs from the resource use and even resource depletion due to unregulated appropriation. This is emphasized if the boundaries of the resource and its users are well-defined, and there is a geographic overlap between users' residence and resource location.

As for the resource characteristics, relatively small size and predictability of its use were identified as having an effect on the design of rules. To this, Agrawal adds the possibilities of of storage of benefits from the resource and low mobility. For example, a cattle herds managed as common property can be stored to some extent, but they are mobile. On the other hand, grazing lands are stationary, but the degree of storage is limited. Both immobility and possibility of storage increase predictability in the resource use. The higher the predictability, the higher willingness to design effective use.

Constraints on group characteristics include small size, shared norms, past successful experience, interdependence among group members and appropriate leadership. We have already talked about how shared norms and interdependence among players can increase trust, which increases the likelihood of resource management.

An area that is missing from the three presented studies is the question of population and market pressures on resource use (Agrawal 2002, p. 56). Usually, higher integration with markets is seen as having an adverse impact on the management of a resource. Increased connection increases cash exchanges, and the level of appropriation increases so that the cash income would increase. Moreover, better connection can introduce new technological means available to exploit a resource. This can have an effect on the benefit-cost ratios, and therefore undermine the sustainability of agreed rules.

On the whole, the question of the design of self-management rules for the resource use incorporates the analysis of overcoming the multiple level dilemma. While rational agents are not expected to engage in collective action, theories incorporating reciprocity and trust in the incentive models are better at explaining empirical evidence showing that these dilemmas are overcome and self-management rules are invented and carried out by communities appropriating a common-resource. However, many variables have been identified as influencing the probability of such organization.

3.3 Inequality Aversion

In the model, Falk *et al.* (2002) draw on other studies stemming from the assumption that a player's utility depends not only on his or her payoff, but also on the payoffs of the other players. This premise is in contrast to the economic theory that works with person's utility function depending only on his or her own payoff. Therefore, even though this model still assumes rational individuals, it allows for interdependent preferences of individual players. In the model, "fairness is modelled as 'inequity aversion' " (Falk *et al.* 2002, p. 160). In other words, individuals prefer outcomes that are equitable. Moreover, according to the results of other studies, many players seem to have aversion towards both disadvantageous and advantageous inequality (Falk *et al.* 2002, p. 160), though admittedly the latter is significantly weaker.

Therefore, individuals' utility is a function of both absolute and relative payoffs (both theirs and other players'). However, as people are not homogeneous, the relative importance of those can often vary. Formally, the utility function of player i can be expressed as

$$U_i = \pi_i - \frac{\alpha_i}{n-1} \sum_{j,\pi_j>\pi_i} (\pi_j - \pi_i) - \frac{\beta_i}{n-1} \sum_{j,\pi_i>\pi_j} (\pi_i - \pi_j) \quad (3.1)$$

where $\alpha_i \geq \beta_i \geq 0$ and $\beta_i \geq 1$.

In this equation, π_i denotes the payoff of player i, the second term measures the utility loss from disadvantageous inequality, and the third term measures the loss from advantageous inequality. α is larger than β because the loss from disadvantageous inequality is supposed to be greater than that from advantageous inequality. Therefore, for a given payoff x_i, the utility of a player i is at maximum when $x_i = x_j$. Requirement $\beta_i \geq 0$ rules out the existence of individuals who like to be better off than others. Requirement $\beta_i < 1$ is necessary, because $\beta_i = 1$ means that player i is willing to throw away \$1 to reduce his advantage relative to player j, which does not seem plausible. On the other hand, there is no reason to put an upper bound on α_i. For $\alpha_i = 4$, player i is willing to give up \$1 to reduce the payoff of the other player by \$1.25. In the following subsections we will first examine how this utility function changes the predicted outcomes of the standard commons game, and then we will take into account those situations in which either sanctioning or communication is possible.

3.3.1 Standard Common-Pool Resource Game

In this part we will calculate the Selfish Nash Equilibria (SNE) in the standard commons game. As in some of the models described previously, each player has endowment w. Each of the n players decides independently how much he wants to appropriate from a common-pool resource; the decision is denoted x_i. This decision imposes cost c per unit, but it also yields a revenue which is dependent not only on the x_i but on the decisions of other players as well. The total revenue of all players is given by $f(\sum x_j)$. We assume that for lower levels of appropriation this function is increasing in $\sum x_j$, but above a certain level the function is decreasing. Each player receives a revenue from the total revenue based on his relative share in total appropriation $\frac{x_i}{\sum x_j}$. So the payoff of player i is given by

$$\pi_i = w - cx_i + \frac{x_i}{\sum x_j} f(\sum x_j) \quad (3.2)$$

We substitute for the function $f(\sum x_j) = u \sum x_j - v(\sum x_j)^2$. This function satisfies the condition that it is increasing up to certain level of overall appropriation, and then it is decreasing. Thus, it captures the negative externality players impose on each other when they keep increasing their level of appropriation. Then, after some modifications we get $\pi_i = w + (u - c)x_i - vx_i(\sum x_j)$. For simplification, we can re-write that as $\pi_i = w + ax_i - bx_i(\sum x_j)$. Individual's optimal level of appropriation is found as a solution to the following equation (the full derivation as well as all the following proofs can be found in Appendix).

$$\frac{\partial \pi_i}{\partial x_i} = a - 2bx_i - b\sum_{j\neq i} x_j = 0 \tag{3.3}$$

If the Nash equilibrium is symmetric, the individual appropriation level is found by solving the given set of equations as $x_i^* = \frac{a}{b(n+1)}$. This is the equilibrium if everybody ignores the negative externality of their decisions they impose on the other players. It is useful to compare the Selfish Nash Equilibrium to the social optimum that is, in a symmetric case, $x_i = \frac{a}{2nb}$. Obviously, the socially optimal level of appropriation is lower than the SNE level.

This result depends not only on the shape of function f we choose, but also on the proportioning rule we decide on. Throughout the previous chapter, the model always used the same one in which the level of player i's share is directly proportional to his share in the total appropriation level. However, there are other possibilities. The share does not have to be linked to a player i's appropriation at all (for example $q_i = \frac{1}{n}$ for $\forall i$), or it can depend on the level of appropriation in some other way.

The relationship between decisions of selfish and inequality-averse person can be best illustrated in figure 3.1. This figure shows best response behavior of player i based on the behavior of the other players. The thin line represents the optimal behavior of selfish player: he appropriates less the more the others appropriate. The Selfish Nash Equilibrium prevails at the point when the best response behaviour intersects the diagonal. The bold line on the other hand show the best response behaviour of inequity averse player; it was derived from the inequality-averse utility function using the values $\alpha = 4$ and $\beta = 0.6$.

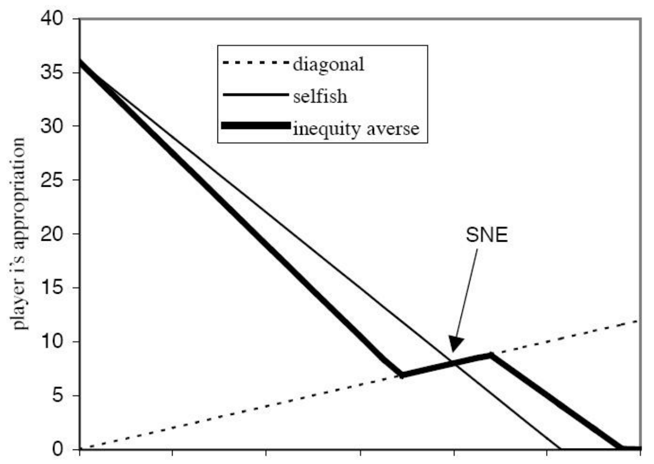

Figure 3.1: Best Response Behaviour
Source: Falk *et al.* (2002, p. 166)

There are several attributes worth noticing. First, when other players appropriate less than in the SNE, the best response is to reciprocate the ‘nice’ behavior of the other players by not exploiting their kindness and appropriating less than what is his material self-interest. Secondly, if the other players appropriate more than in the SNE, the best response is to appropriate more as well. Thus, a player imposes negative externalities on the other players as a revenge, and thus he is able to reduce the payoff differences. Third, the part of inequality averse player’s best response function that lies on the diagonal represents the area where symmetric equilibria may exist. The equilibria to the left from the SNE are the ones that increase overall efficiency. Fourth, when the other players do not appropriate at all, the best response of both selfish and inequality averse player is the same. This is because at that point a player’s decision does not affect the other players’ payoffs.

3.3.2 Inequality Aversion in the Standard Game

In this section we try to analyze the impact of the presence of inequality averse preferences on the standard common-pool resource game outcome, for both symmetric and asymmetric equilibria. Thus, we can compare the results for a standard model of the commons and altered model incorporating the aversion towards inequality.

First let us consider the symmetric equilibrium. Up to a certain level ($\sum x_j < \frac{a}{b}$) of overall appropriation, the players who choose the higher appropriation level also have higher payoffs:

$$\pi_j - \pi_i = (w + ax_j - bx_j \sum x_k) - (w + ax_i - bx_i \sum x_k) = (a - b \sum x_k)(x_j - x_i)$$

Suppose all players $j \neq i$ chose to appropriate $x_j = \hat{x} \leq x_{SNE}$. Now we only need to show that $x_i^* = \hat{x}$ is a local optimum because U_i is concave, and thus the best reply is unique. The optimal level of appropriation x_i^* obviously will not be lower than $\hat{x}$ because in this case player i could increase his material payoff as well as reduce inequality by increasing his level of appropriation. Moreover, player i has no incentive to increase his appropriation level above $\hat{x}$ if the derivative from above of his utility function must equal less than zero:

$$\begin{aligned} 0 \geq \frac{\partial U_i}{\partial x_i^+} &= \frac{\partial}{\partial x_i}(\pi_i - \frac{\beta_i}{n-1} \sum_{j, \pi_i < \pi_j} (\pi_i - \pi_j)) \\ &= \frac{\partial}{\partial x_i}(\pi_i - \beta_i(\pi_i - \pi_j)) \\ &= \frac{\partial}{\partial x_i}(\pi_i(1 - \beta_i) + \beta_i \pi_j) \\ &= (a - bx_i - b \sum x_j)(1 - \beta_i) - \beta_i b x_i \end{aligned}$$

Thus, the critical condition for x_i^* is:

$$x_i^* \geq \frac{a(1 - \beta_i)}{b(1 + n(1 - \beta_i))} \tag{3.4}$$

Now, we assume that all players $i \neq j$ appropriate $x_j = \hat{x} > x_{SNE}$. This time the condition is:

$$\begin{aligned} 0 &< \frac{\partial U_i}{\partial x_i^-} = \frac{\partial}{\partial x_i}(\pi_i - \frac{\alpha_i}{n-1} \sum_{j,\pi_j<\pi_i} (\pi_j - \pi_i)) \\ &= \frac{\partial}{\partial x_i}(\pi_i - \alpha_i(\pi_j - \pi_i)) \\ &= \frac{\partial}{\partial x_i}(\pi_i(1+\alpha_i) - \alpha_i \pi_j) \\ &= (a - bx_i - b\sum x_j)(1+\alpha_i) + \alpha_i b x_i \end{aligned}$$

Thus, the critical condition is:

$$x_i^* \leq \frac{a(1+\alpha_i)}{b(1+n(1+\alpha_i))} \tag{3.5}$$

Now, putting both conditions together we get an interval of $\hat{x}$. The right-hand side of the first condition 3.4 is decreasing in β_i. For the equation to hold for all i, it must hold for the highest of the possible values, thus it must hold for the lowest β_i. On the other hand, the second condition 3.5 is increasing in α_i. For this condition to be satisfied for all i, it must hold for the lowest α_i. So, there is a symmetric equilibrium in which each subject chooses an appropriation level $x_i^* = \hat{x}$ if $\hat{x}$ is in the interval:

$$\left[\frac{a(1-\min(\beta_i))}{b(1+n(1-\min(\beta_i)))}, \frac{a(1+\min(\alpha_i))}{b(1+n(1+\min(\alpha_i)))}\right]$$

It is clear that if both the smallest α_i and β_i are zero (ie. if there is just one egoistic player), the only equilibrium will be Selfish Nash Equilibrium. The presence of only one such player is enough to induce all other players to act selfishly regardless of how inequality averse they may be. Even though there is possibility to move in both directions from the SNE equilibrium, of particular interest are those lying "to the left" of SNE for which the smallest β_i is greater than zero. Those are equilibria in which players appropriate less than in SNE, and thus they are more efficient.

So far, we have considered only the symmetric equilibria. However, the following Proposition deals with the possibility of asymmetric equilibria:

Proposition 2 (Asymmetric Equilibria with Inequality-Averse Subjects)

(i) If there are at least k players with $\frac{\beta_i}{\alpha_i} > \frac{n-k}{k-1}$ then there is an equilibrium with less appropriation than in the SNE. In this equilibrium at least k players choose the same appropriation $\hat{x} < x_{SNE}$; the other players j choose higher appropriation levels.

(ii) If there is no k such that there are at least k players with $\frac{\beta_i}{\alpha_i} > \frac{n-k}{k-1}$ then there is no equilibrium with less appropriation than in the SNE.

Corollary: *If there are $\frac{n}{2}$ or more selfish players, there is no equilibrium with less appropriation than in the SNE.*

Therefore, if there is no such k that there are at least k players with $\frac{\beta_i}{\alpha_i} \geq \frac{n-k}{k-1}$, there is no equilibrium with less appropriation than in the Selfish Nash Equilibrium.[3]

So, there is an equilibrium with less appropriation than in the Selfish Nash Equilibrium if there are at least k players with $\frac{\beta i}{\alpha_i} > \frac{n-k}{k-1}$. There will be k players with the same appropriation level $\hat{x} < x_{SNE}$ and the rest of the players will choose higher appropriation levels. If there are not k such players, there is no equilibrium with lower levels of appropriation. Moreover, as $0 < \beta_i < 1$ and $\alpha_i > \beta_i$, the expression $\frac{\beta_i}{\alpha_i}$ is between zero and one. Therefore, k needs to be higher than $\frac{n}{2}$. Otherwise, if $k \leq \frac{n}{2}$, the only equilibrium will be SNE. The more people who are nonselfish, the weaker is the requirement. Thus, it takes either many subjects with moderate $\frac{\beta_i}{\alpha_i}$ or fewer subjects (but still more than half) with high $\frac{\beta_i}{\alpha_i}$ to achieve a more efficient outcome than the SNE.

In conclusion, in the standard game the requirements for the possibility of a more efficient outcome seem quite strict. In a symmetric case, only one selfish person can prevent improved efficiency. In the asymmetric case more efficient outcomes are possible even in the presence of selfish players. However, there still need to be more than half of the non-selfish players with quite high utility loss from advantageous inequality compared to the utility loss from disadvantageous inequality. Moreover, as the authors point out, we expect that the aversion of disadvantageous inequality is far stronger than that of advantageous inequality, so the requirements will still be hard to fulfill. As we mentioned previously, in standard commons game this prediction was found to be true. On average, the Selfish Nash Equilibrium describes the behavior of players quite well. The situation is changed dramatically, though, once we allow for the players to create informal rules.

3.3.3 The Effects of Sanctioning Opportunities

In this setting, subjects play the standard game, and after each round of play, all players receive data on all individual appropriation decisions. Then, each player can decide to sanction any other player at a certain cost. Thus, any player can deduct p_{ij} points from player j's payoff, but there is a cost cp_{ij} for the sanctioner. c is assumed to be a positive number smaller than 1. If we assume selfish preferences, the predicted equilibrium is the SNE. When sanctioning is costly and each person's utility depends only on his material payoff, sanctioning is equivalent to throwing away money. Moreover, rational individuals know that no one will sanction because of the backward induction reasoning.

However, this prediction is in contradiction with observed behavior of individuals. Ostrom *et al.* (1994) for example point out that significantly more sanctioning occurs than predicted. However, when allowing for inequality averse preferences, this evidence can be explained. The model presented in previous sections predicts that players with sufficiently strong preferences for equity will punish defectors. As a consequence of this discipline device, selfish players have an incentive to act

[3]Proof can be found in the Appendix.

more cooperatively. Thus, more efficient appropriation level are more likely. Precise conditions for the existence of equilibria with levels of appropriation below the SNE are given in the following proposition (Proof can be found in the Appendix):

Proposition 3 (Equilibria with Sanctioning Possibilities) *Suppose there is a number $k \leq n$ such that for all players $i \leq k$, the utility parameters α_i and β_i satisfy*

$$c < \frac{\alpha_i}{(1+\alpha_i)(n-k)+(k-1)(1-\beta_i)} = \frac{\alpha_i}{(n-1)(1+\alpha_i)-(k-1)(\alpha_i+\beta_i)} \equiv \hat{c}.$$

We call the players who satisfy this condition "conditionally cooperative enforcers" (CCEs). Suppose further that all players $i > k$ obey the condition $\alpha_i = \beta_i = 0$ (they are selfish). We define $\beta_{min} = min_{i \leq k}\beta_i$ as the smallest β_i among the CCEs. Then there is an equilibrium that can be characterized as follows:

(i) All players choose $x \in \left[\frac{a(1-\beta_{min})}{b(1+n(1-\beta_{min}))}, x_{SNE}\right]$.

(ii) If each player does so, there is no sanctioning in the second stage.

(iii) If one player chooses a higher appropriation level, then this player is sanctioned equally by all CCEs. The sanctioning equalizes the payoffs of those who sanction and the deviator.

(iv) If more than one player does not play x, an equilibrium of the sanctioning subgame is played.

This proposition presents a critical condition for an equilibrium with lower appropriation levels than in Selfish Nash Equilibrium. It says that the cost of sanctioning must be below a certain threshold level $\hat{c}$ which is defined by the parameters α_i and β_i, and which is increasing in α_i, β_i and k.

$\hat{c}$ is increasing in α_i because the larger the parameter α_i, the higher disutility a player experiences from disadvantageous inequality. Thus, the player is willing to punish the deviator (who appropriates more and thus earns more) even if the sanctioning costs are high. The reason that $\hat{c}$ is increasing in β_i is that a player with strong aversion against advantageous inequality will experience a strong disutility toward the other CCEs if he himself does not sanction a deviator. In other words, he will feel solidarity towards other players who sanction. Finally, higher k means that there are more punishers, thus the desire to be in solidarity with those who punish increases as well, and that means that there will be more punishment.

In comparison to the situation without sanctioning, the probability of achieving more efficient equilibria is much higher as the conditions for α_i and β_i are looser. Therefore, for a given distribution of selfish and inequality averse players, it may be impossible to reach a more efficient equilibrium when sanctioning is impossible, while there may be such equilibrium when sanctioning is practiced. Thus, this model explains why sanctioning improves achieved equilibrium levels of appropriation. Moreover, it also predicts that the players who are sanctioned are the defectors, which is in accordance to the observed experimental data.

3.3.4 The Effects of Communication

So far, the individual players remained anonymous. In reality, however, people are often able to communicate, or at least "identify" each other. However, as long as the resulting agreements are non-binding, the predictions about the behavior remain unchanged. In the framework of selfish players, no promise can be actually expected to be fulfilled. Thus, the opportunity to communicate is supposed to be irrelevant for the predicted outcomes. However, empirical evidence often contradicts this prediction. As presented in Ostrom *et al.*, the possibility to communicate significantly increases efficiency.

Communication is a broad term, though. In some experiments, subjects are really allowed to communicate, they talk face-to-face, see each other's facial expression. In other experiments, players are not actually allowed to talk, they simply identify each other. In yet other experiments, they do not meet at all, they communicate only via computers. Therefore, the observed effects of communication may vary in dependence on the type of communication allowed. Falk *et al.* consider two main effects communication can bring about: coordination and the expression of approval and disapproval.

Firstly, communication may have an important effect when there are multiple equilibria. In this situation, communication may help players coordinate their behavior and thus achieve more efficient outcome. If a standard commons game has a form of Prisoner's Dilemma, as shown in Chapter 1, there is only one equilibrium, the *(defect, defect)* one. However, in the presence of inequality averse preferences, standard game has no longer the characteristics of the Prisoner's Dilemma as shown in table 3.3. No player has a dominant strategy, there are two equilibria (the efficient one *(cooperate, cooperate)* and the inefficient one *(defect, defect)*) because in this situation players don't like to cheat on each other. Thus, this is a coordination game in which any of those two equilibria can be played. As neither of the players has a dominant strategy, they both face a strategic uncertainty. In this situation, communication can have a positive impact as it can help the players to coordinate their choices on more efficient equilibria.

		Player 2: Cooperate = low appropriation	Player 2: Defect = high appropriation
Player 1	Cooperate	10,10	0,9
Player 1	Defect	9,0	5,5

Table 3.3: CPR Game with Inequality Averse Preferences
Source: Falk *et al.* (2002, p. 174)

Secondly, it has been observed that efficiency may be influenced by the anticipation of social rewards of punishments, triggered by positive or negative emotions such as approval or disapproval. When players are allowed to communicate, they can convey their approval or disapproval towards other players. For this type of communication to have a positive effect on the outcome, two assumptions must be met. At first, some of the players must care about approval or disapproval, and they have to change their behavior accordingly in expectation of them. Furthermore, there must be subjects who express approval and disapproval. This is important because it is usually not costless to express approval or disapproval. Therefore, preferences as they are assumed in the previous model can explain why communication used to express approval or disapproval may have an impact on cooperative behavior.

In a laboratory experiment carried out by Ostrom *et al.* (1994), this setting was created to test the effects of communication on the levels of investment in the CPR. The subjects used in these experiments were university students, mainly economics students. In each round of the game, students were given certain amount of tokens which they could invest in two markets. Each token invested in Market 1 yielded a constant rate of output and each unit of output yielded a constant return. Each token invested in Market 2 (the CPR) yielded a rate of output per token dependent upon the total number of tokens invested by the entire group. The rate of output at each level of group investment was described both in functional and tabular form. Also, subjects were told that they would receive a level of output from Market 2 that was equivalent to their share of invested tokens in all tokens invested. Moreover, each unit of output from Market 2 yielded a fixed rate of return. Students also knew the total number of players in the group, total group tokens and that endowments were identical. They knew that the game would be repeated several times within assigned two hours, so they knew there is a relatively short finite horizon, though how many times exactly they did not know. There were 8 subjects in each of the experiments, who received either 10 (low) or 25 (high) endowment. The parameters u and v were 23 and -0.25, so that the production function had a form of $23\sum x_j - 0.25(\sum x_j)^2$. Return from Market 1 investment was set to 0.05, and return from Market 2 was set to 0.01.

Given the parameters in the case of low endowment, the predicted individual contribution is $x_i = \frac{18}{0.25(8+1)} = 8$. This can be easily derived from the equation given in section 3.3.1. For given production function, endowment $e = 10$ and $c = 5$ we get the payoff function as $\pi_i = 10 + 18x_i - 0.25x_i \sum x_j$ (Falk *et al.* 2002, p. 164). This is the total investment of 64 tokens which is much higher than the optimal group level of investment of 36 tokens. (Ostrom *et al.* 1994, p. 114) As the equilibrium level is independent on the endowment, the same prediction hold for the second type of the experiment with $e = 25$ as well. At the Nash equilibrium, players can earn approximately 39 percent of maximum net yield from the common resource.

The baseline experiment carried out without the possibility of communication shows that generally the outcomes predicted by the previously described model do hold, though on an individual level the results were more efficient than the Nash Equilibrium. In the second part of the experiment, players were either allowed to communicate, or they were able to sanction each other after each round of the game, or both. The predicted investment is still 8 tokens, as communication is considered 'cheap talk' and thus should have no effect. Also, rational players are not expected to sanction other players as this incurs cost to themselves.

In table 3.4, the results for all the suggested experiments are given; the numbers represent the achieved net yield as a percentage of maximum net yield. The first line represents the baseline experiment for comparison. The second line represents a setting, when sanctioning was possible without communication. In the third line, players were given the opportunity for discussion after round 10. In the following two settings, players were allowed to communicate and to decide on whether they want to use the possibility of sanctioning, or not.

Experimental Design	Average Percent Net Yield	Average Percent Net Yield (minus fees and fines)	Defection Rate (Percentage)
Baseline	21	na	na
Sanctioning	37	9	na
Sanctioning + one-shot communication	85	67	1
One-shot communication + no sanctioning chosen	56	na	42
One-shot communication + sanctioning chosen	93	90	4

Table 3.4: Aggregate Results, 25-Token Endowment
Source: Ostrom *et al.* (1994, p. 192)

The results show that without communication, sanctioning reduces average net yield, as players overuse the sanctioning mechanism. However, when communication is possible, average net yields increase to 85 (67 without costs of sanctioning) percent. Moreover, when the decision to adopt the sanctioning mechanism is up to the players, groups that adopt it are better off than groups that decide not to. In conclusion, when subjects use the opportunity to communicate to agree on a joint strategy and chose their sanctioning mechanism, the resulting payoffs are far closer to the optimum level.

In conclusion, in combination with the previously assumed preferences, communication can have a positive impact on behavior such that it enhances the more efficient outcomes. These effects are particularly strong when face-to-face communication is possible. However, communication alone may not be sufficient to overcome the appropriation dilemma. Thus, the sanctioning opportunity is important to increase the achieved efficiency.

3.4 Contest and the Commons

"A contest is a situation in which individuals or groups compete with one another by expending effort to win the prize" (Baik 2001, p. 363). The most common examples include lobbying, R&D investments and election campaigns. However, in this section we will show that under certain circumstances, Tragedy of the Commons may be viewed as a kind of contest as well. We will have a look at the link between the contest and common resource appropriation to show that contest may increase incentives for self-management of the common resource.

In the basic form, payoff function of player i in a contest can be expressed by the following function:

$$\begin{aligned}\pi_i(e_1, e_2, \ldots, e_i, \ldots, e_n) &= p_i(e_1, e_2, \ldots, e_i, \ldots, e_n) \\ &\times \; V_i(e_1, e_2, \ldots, e_i, \ldots, e_n) - C_i(e_i)\end{aligned}$$

where e_i denotes the level of effort or investment made by player i, C_i is a cost of player i's decision, and V_i is the value of the prize which may or may not be a function of efforts made by individual players. In Commons setting, it represents the payoff from the resource, which depends on the sum of all the appropriation levels. The exact relationship may vary, but, as we have seen, overall efforts often at first increase the total payoff from the resource (although often at decreasing rate) but after certain level of appropriation, the payoff begins to fall. On the other hand, in contests the value of the prize if usually constant; even when it is allowed to change it is supposed to increase with increased overall effort of contestants. p_i is a function expressing the probability that player i wins the contest, which is determined by actions taken by all the players. Another interpretation that is more suitable in common resource setting is that p_i denotes the fraction of the prize obtained by player i. This function is called the *Contest Success Function*, and in our previous models, it usually had form $p_i = \frac{e_i}{\sum_j e_j}$.

There are many different functions that could be used. Generally, we can express the contest success function as:

$$\begin{aligned} p_i = \frac{\phi_i(e_i)}{\sum_j \phi(e_j)} \qquad & if \sum_j e_j > 0 \\ p_i = \frac{1}{n} \qquad & otherwise \end{aligned}$$

where ϕ_i is and increasing function of the effort level. In contest situations, probability of winning the prize when no one participates in the contest (ie. $\sum_j e_j = 0$) is $\frac{1}{n}$. However, in resource appropriation, this in not suitable supposition, as with no effort invested in the appropriation, a player does not receive anything. Thus, this part of the general form of the Contest Success Function has to be altered to suit our situation, so that if $\sum_j e_j = 0$, then $\pi_i = 0$. Thus, we can say that the situation of the common resource appropriation is a contest with a specific type of prize and with altered probability of winning when no one participates.

One of the possible *Contest Success Functions*, as suggested by Tullock (1980) is $\phi(e_i) = e_i^{\epsilon}$. If $\epsilon = 1$, we have the form used throughout this thesis. This form is the most often used, as it keeps the assumption that each individual's share depends on his or her effort level, but it is still simple enough so that it allows clear results. There are other possibilities, though. If $\epsilon = 0$, then the share an ith player receives would be independent on his investment. In our setting it would mean that his share is independent on his level of appropriation which does not seem feasible.

An interesting link between contest and common-pool resource appropriation was presented by Nitzan and Ueda (2009). They propose a 2 stage model in which several groups compete over an access to a common-pool resource. Such contest is possible for example in conflicts between countries over an area that contains a common-pool resource (eg. fishery, gas or water). In the first stage, individual group members decide on their effort levels to ensure the victory of the contest to gain the prize. In the second period, the winning group gains access to the resource, and its members can appropriate it. Crucial assumption of the model is that the access of the winning group's members to the resource is not restricted in any way. Thus, in the second stage, such situation leads to overappropriation of the resource. In the second part or the article, the authors examine how altered appropriation regulation of the resource may change the probability of a group to win the resource in the first stage. In other words, what they examine is whether, under certain conditions, groups that adopt some kind of self-management of the resource may increase their probability of winning the resource. Moreover, the effect of group size on the probability of winning the prize is examined.

In their model, individual player's effort in stage 1 is determined in anticipation of the future level of appropriation. Thus, we will first have a look at the level of appropriation, and how it changes with number of members in a group. As in other models, player's payoff depends on his own appropriation level as well as other players' appropriation levels. The situation is symmetric for all the players, each individual's equilibrium level of appropriation is higher than would be reasonable from the point of view of the whole group; these are the conditions leading to the 'tragedy'. Now, suppose there are k members in a particular group. If this group wins the resource, each of the members will appropriate his equilibrium level, so the overall level of appropriation will be the sum of all the individual appropriations $\sum_k x_k^*$ (we denote that $X^*(k)$). The proposition of the article is that if there are more members in the group, the overall level of appropriation will be higher. However, as the share of each of the members in the larger group will be lower, the overall appropriation level will be higher, but not proportionally. Thus, while the overall appropriation of a group increases with higher number of members, the appropriation of each individual member decreases. Nitzan and Ueda (2009) derive this conclusion generally for any appropriation function in the case of symmetric equilibrium. However, it is possible to show the logic on a specific function; we can use the results derived in the model on inequality aversion from the previous section. In that model, equilibrium level of appropriation for any of the individuals, under the condition of concave resource function, was derived as $x^* = \frac{a}{b(n+1)}$; this is the situation without management of the resource. We can see that for $h > n$ and all other parameters being equal:

$$x^*(n) > x^*(h) \qquad n < h$$

where n and h are the numbers of players in the two groups. In other words, the larger the group, the smaller an individual's appropriation level. However, the overall appropriation level of a group with more members is higher:

$$X^*(n) < X^*(h) \qquad n < h$$

Where $X^*(n) = n \cdot x^*(n)$. This calculation is supposed to be done by each of the members before the game starts. In the first stage, m groups compete among each other for the obtaining of the resource. The probability of a group winning the contest is given by:

$$p_i = \frac{E_i}{\sum_{t=1}^{m} E_t}$$

where E_i (E_t) is the total amount of effort made by the members of group i (t respectively). E_i and E_t depend on the expected payoff from the appropriation of the resource in case group i or t wins. Thus, each member of each group maximizes his own utility given by the payoff from obtaining and appropriating the resource minus the cost of effort invested. The equilibrium level of effort in the first stage by a member of group i is given by:

$$e^* = \arg\max_e u_i = \arg\max_e \frac{e + e_i^{-j}}{e + e_i^{-j} + E_{-i}} x^*(n) - c(e) \tag{3.6}$$

where e denotes the equilibrium effort level, e_i^{-j} is the sum of the efforts made by the other members of group i, and E_{-i} is the total effort made by all the other players. As we said earlier, in larger groups there is smaller appropriation by each of the members of the group. As a result, each of the members exerts less effort in the first stage of the game. On the other hand, larger group implies smaller marginal cost for each of the members which enhances effort level. Thus, the relative values of these opposite incentives decide whether larger groups have lower of higher probability of winning the prize. In our particular situation, we can rewrite the equation 3.6 as:

$$e^* = \arg\max_e \frac{ne}{E} x^*(n) - c(e)$$

where again E denotes the sum of the effort levels by all the players of all the goups. Because the situation is symmetric for each member of a particular group, we can rewrite the numerator as ne. In other words, members of one group would exert the same effort in the first stage of the game (supposing the same cost function for each of them). Because members of a different-sized group have different value of the prize ($x^*(h)$, their effort level will be different, but the same for each of them. Deriving the equation with respect to e, and modifying it we get the equilibrium level of effort as:

$$\frac{n(E - e^*)}{E^2} x^*(n) = c_n^{'}(e)$$

from that we get:

$$e^* = \sqrt{\frac{nE_{-j}x^*(n)}{c_n^{'}(e)}} - E_{-j}$$

Where E_{-j} is $E - e^*$. Effort levels of other players are seen as fixed, so the parameters that change with different number of group members is n, $x^*(n)$ and $c_n^{'}(e)$. We suppose such a cost function that in larger groups, the costs paid by each member are lower. In other words, the denominator decreases in n, the numerator is increasing in n. On the other hand, as we have shown, $x^*(n)$ decreases in n. Therefore, the equilibrium effort level is either increasing or decreasing in n, depending on which of these effects prevail.

The important question is whether some kind of self-management of the resource within the group may enhance its prospects of winning. With no self-management, probabilities of winning for each of the group are distributed in certain way. Obviously, if only one of the groups has the ability (or possibility) of self-management in the second stage, it is thus able to reach more (or even fully) optimal level of appropriation and, effectively, payoff. Thus, the equilibrium level of effort in the first stage for each member of this group would increase as well, which would lead to an increase in the group's probability of winning the resource relative to the other groups. If all of the groups have such ability, however, we can show that such change is more favourable for larger groups. The efficient level of appropriation of the resource by the winning group is $\frac{a}{2b}$ (which we denote as $X_M^*(n)$) as has been derived in previous section. Thus, for each of the members, the optimal level of appropriation is $\frac{a}{2bn}$ (ie. $x_M^*(n)$). Larger groups have more members, so the appropriation level of each of them is lower then in smaller groups ($h > n$ implies that $x_M^*(n) > x_M^*(h)$). However, it can be shown that the difference in the resulting levels of appropriation is larger for large groups:

$$[x_M^*(n) - x^*(n)] < [x_M^*(h) - x^*(h)]$$

Simple modification shows that this inequality holds for any $h > n$ from $n > 3$.[4] Thus, the increase in efficiency resulting from self-management of the resource is higher for larger groups. This means higher increase in effort levels in the first stage, which in turn increases the probability of winning relatively to a smaller group. This conclusion may be important when trying to identify the incentives that lead to an initiation of self-management in a group.

In conclusion, we have shown that the common resource setting can, under certain conditions, be viewed as a contest with variable rent. Moreover, a link between contest, common resources and self-management incentives can be found.

[4]For $h > n + 1$ it holds from $n > 1$.

Chapter 4

Conclusion

This thesis presented an overview of the different theoretical approaches to the Tragedy of the Commons. The main focus was on the idea that unlike the grim predictions of Hardin (1968), common resource users are not necessarily trapped in the dilemma between their personal interests and joint interests. In that sense, we tried to provide the theoretical basis for what has been often proved by empirical evidence.

The first chapter dealt with the situation described by Hardin, in which players are not able to engage in communication, and any form of willing cooperation is precluded. We have shown that different aspects of the game may be altered so as to change the incentives of the players, and thus provide the opportunity of more efficient outcomes. The payoff structure of the basic Commons representation, Prisoner's Dilemma, was shown to be only one of the possible games in two-player two-strategy situation. In the third chapter we added the possibility of monitoring to the appropriation problem to show how this simple extension may alter the outcomes. Moreover, we extended this game by giving both of the players the option to combine two appropriation levels with the decision on monitoring. We then found the pure strategy and mixed strategy equilibria to show the effect on the predicted outcomes.

Also, we examined the effect of player characteristics. Heterogeneity among players, and wealth distribution in particular, was proved to be able to have a positive impact on the overall efficiency. Moreover, when the possibility of resource depletion is present and acute, players may decrease their appropriation levels; this effect would be the more important the more the players are dependent on the resource. On the other hand, exit options present a difficulty in trying to arrive at a more efficient payoff because having an outside opportunity decreases players' incentives to take into account the negative externalities they impose on one another in both the current period and all the future periods as well. The third chapter then extended the question of player characteristics on the problem of the design of self-management rules. In itself, design of rules is a complex problem that incorporates many areas of economics and other social sciences.

An important idea suggested that evolutionary theory may be used to explain the emergence of multiple player types in the population, as well as equilibrium levels of different types surviving along each other. This suggestion was then used in a model by Falk *et al.* (2002), which alters the utility function in such a way that it does not only reflect the objective payoff, but it also incorporates aversion towards inequally distributed payoffs among players. This model is then used in a situation when communication, sanctioning or both are allowed, and it is shown that its predictions are far closer to the empirical observations.

In addition, we presented a link between the common resource situation and contest. Commons, under certain circumstances, may be viewed as a contest with a specific type of prize that is dependent on the effort levels by individual contestants. Moreover, we presented a model that shows how contest among groups over a common resource may increase incentives of the group members to design rules for resource management.

The models presented in the thesis were aimed at showing that government management of common-pool resources is not a necessity, as has been believed for a long time. The tragic outcomes may be avoided either by outside characteristics of the situation, or by the players themselves. The important task at hand for the future would be to develop such theoretical framework that reconciles and explains in full the observed behavior of common resource users.

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Appendix A

Chapter 2

A.1 Derivation of Equilibrium Effort and Payoff Levels

We have a following payoff function for player i:

$$\pi_i = \frac{x_i}{\sum x_j}(\sum x_j - b(\sum x_j)^2)$$

We differentiate the payoff function with respect to x_i:

$$\frac{\partial \pi_i}{\partial x_i} = \frac{\sum_{j \neq i} x_j}{(\sum x_j)^2}(\sum x_j - b(\sum x_j)^2) + \frac{x_i}{(\sum x_j)(1 - 2b(\sum x_j))} \tag{A.1}$$

We set the equation equal to zero:

$$0 = \frac{\sum_{j \neq i} x_j}{\sum x_j} - b \sum_{j \neq i} x_i + \frac{x_i}{\sum x_j} - 2bx_i$$

The first and the third term together are equal to 1, so we have:

$$1 = b(\sum_{j \neq i} x_j + 2x_i)$$

Now we use the fact that the equilibrium is symmetric as the situation of each of the players is the same:

$$1 = b(3x_i + 2x_i) = 5bx_i x_i = \frac{1}{5b}$$

Substituting x_i to the payoff function we get:

$$\pi_i = \frac{x_i}{\sum x_j}(\sum x_j - b(\sum x_j)^2) = \frac{\frac{1}{5b}}{\frac{4}{5b}}(\frac{4}{5b} - b(\frac{4}{5b})^2)\pi_i = \frac{1}{25b}$$

For level of appropriation of player 1 under unequal wealth distribution we can use equation A.1. We substitute $\sum x_j$ by $3a + e_1$ and thus we get:

$$0 = \frac{3a}{(3a + e_1)^2}((3a + e_1) - b(3a + e_1)^2) + \frac{e_1}{3a + e_1}(1 - 2b(3a + e_1))x_1 = \frac{1 - 3ab}{2b}$$

A.2 Proofs of Lemma 1-4

Proof of Lemma 1 For all $x_i \in [F - x_j, e_i]$, the resource will be depleted in period 1. Thus, player i can increase his payoff only by increasing his share of the catch which is strictly increasing in x_i. Thus, he should appropriate at the highest possible level of x_i which is e_i.

Proof of Lemma 2 Because s_i is positive, the maximum of the intersection $[0, F - x_j] \cap [0, e_i]$ is $x_i = min\{e_i, F - x_j\}$. Even if $min\{F - x_j, e_i\} = F - x_j$, then, by Lemma 1, $x_i = e_i$ is still strictly better, given that i's share increases in x_i.

Proof of Lemma 3

(a) Because $s_i < 0$, the optimal choice over $[0, F - x_j]$ is 0. If $e_i \leq F - x_j$, the upper interval $[F - x_j, e_i]$ is empty and need not be considered.

(b) If $e_i > F - x_j$, then 0 must be compared to e_i, which is the maximum on $[F - x_j, e_i]$. If $x_i = 0$, then i's payoff is given by $\frac{e_i}{E}G(F - x_j)$, while if $x_i = e_i$, i's payoff is given by $\frac{e_i}{e+x_j}F$. A comparison of these payoffs then gives part (b) of the Lemma.

Proof of Lemma 4 As long as $s_i \neq 0$ for $i = 1, 2$, Lemmas 1 to 3 establish that the only responses for player i are 0 and e_i.

A.3 Proof of Proposition 1

Case (i) follows from Lemma 2 applied to both players. If $s_i > 0$, then for player i, e_i is the unique best response to any action chosen by player j, and likewise for player j. Thus, (e_1, e_2) is a unique, dominant-strategy equilibrium.

Case (ii) is a consequence of applying Lemma 2 to player 1 and Lemma 3 to player 2. First, Lemma 2 implies that e_i is player 1's unique dominant strategy, so there cannot be a $(0, 0)$ equilibrium. Thus, we need to show that player 2 will choose $x_2 = 0$ or $x_2 = e_2$. The condition $(G - 1)F \geq Ge_1$ is equivalent to stating that $\frac{e_i}{E}G(F - a_j) > \frac{e_i}{e_i+x_j}F$ where $i = 2$ and $x_i = e_i$. Then player 2's best response is $x_2 = 0$ by Lemma 3. On the other hand, player 2 will choose $x_2 = e_2$ iff $(g - 1)G \leq Ge_1$.

Case (iii) need to be considered separately for the different equilibria. First, let us consider the $(0, 0)$ equilibrium. Player i's best response to $x_j = 0$ is $a_i = 0$ if either $e_i \leq F$ (Lemma 3, case (a)) or $e_i < F$ and $\frac{e_i}{E}GF \geq F$ (Lemma 3, case (b)). Given the assumption that $s_i < 0$ for $i = 1, 2$, then if $e_i \leq F$, $(0, 0)$ is an equilibrium. If, on the other hand, for at least one of the fishers $e_j > F$, then the best response to $x_i = 0$ is $x_j = 0$ as long as $\frac{e_j}{E}GF \geq F$ which can be simplified to $e_j > \frac{E}{G}$. The last condition is equivalent to the assumed slope condition $s_j < 0$, so it is always fulfilled.

The condition for the (e_1, e_2) equilibrium follows from Lemma 3 as well. e_1 is the best response to e_2 if and only if $e_1 > F - e_2$ and

$$\frac{e_1}{E} G(F - e_2) \leq \frac{e_1}{E} F$$

This follows from Lemma 3 case (b) after substituting $e_i = e_1$ and $x_j = e_2$. This condition can be simplified to $G(F - e_2) \leq F$ which is equivalent to the second inequality of the Proposition.

It remains to be shown that there are no equilibria of the form $(e_1, 0)$. If this was an equilibrium, then, from Lemma 3 it follows that $e_1 > F$ and

$$\frac{e_1}{E} GF \leq \frac{e_1}{e_1} F = F$$

Thus, $e_1 \leq \frac{E}{G}$ which implies that $s_i \geq 0$ which is a contradiction.

Appendix B

Chapter 3

B.1 Derivation of the Selfish Nash Equilibrium

To find the Selfish Nash Equilibrium, we use the differentiated payoff function:

$$\frac{\partial \pi_i}{\partial x_i} = a - 2bx_i - b\sum_{j\neq i} x_j = 0 \tag{B.1}$$

First, supposing that all $x_i^* > 0$ we re-write the function as $2bx_i^* = a - b\sum_{j\neq i} x_j$. Then we sum the terms for all i and get $2b\sum x_i^* = na - b(n-1)\sum x_i^*$. From that we have $\sum x_i^* = \frac{na}{b(n+1)}$. Now substituting this sum in B.1 we get

$$2bx_i^* = a - b(\frac{na}{b(n+1)})$$

And thus

$$x_i^* = \frac{a}{b(n+1)}$$

Now we consider that there are n_0 players who choose $x_i^* = 0$. Then the appropriation level for all the other players must be equal to $\frac{a}{b(n-n_0+1)}$. Now expressing the best response of any of the n_0 players, we get a positive number, which is a contradiction to the original assumption.

B.2 Derivation of Social Optimum

To derive the social optimum, we use a sum of the payoff function of all individuals. Thus:

$$\frac{\partial \sum \pi_i}{\partial x_i} = an - 2nb\sum_j x_j$$

From that by solving the equation we get the socially optimal level of appropriation as $\sum x_j = \frac{a}{2b}$.

B.3 Proof of the Symmetric Equilibria

Most of the Symmetric equilibria was derived in the text. However, it still remains to be shown that there is no equilibrium above the overall appropriation level $\sum x_j > \frac{a}{b}$. We put $x_j = \hat{x} > \frac{a}{nb}$. The critical condition is now $\frac{\partial U_i}{\partial x_i^-}(x) \geq 0$. As a decrease in the level of appropriation now generates inequality in favour of player i, we get the following condition:

$$\begin{aligned}
0 \;&\leq\; \frac{\partial U_i}{x_i^-} = \frac{\partial}{\partial x_i}\left(\pi_i - \frac{\beta_i}{n-1}\sum_{j,\pi_i>\pi_j}(\pi_i - \pi_j)\right) \\
&=\; (a - 2bx_i - b(n-1)x^*)(1-\beta_i) - \beta_i b x_i \\
&\leq\; \left(a - 2bx_i - b(n-1)\frac{a}{nb}\right)(1-\beta_i) - \beta_i b x_i \\
&=\; \left(\frac{a}{nb} - 2x_i\right)b(1-\beta_i) - \beta_i b x_i \\
&\leq\; (\hat{x} - 2x_i)b(1-\beta_i) - \beta_i b x_i
\end{aligned}$$

The last term is negative if x_i is close to $\hat{x}$ because $\beta_i < 1$. Therefore, there are no equilibria with $x^* > \frac{a}{nb}$.

B.4 Proof of Proposition 2

First we prove (ii): Let's assume there is an equilibrium with $x_i^* < x_{SNE}$. We can reorder the players so that $x_1^* \leq x_2^* \leq \ldots \leq x_n^*$. Now let k be the higher index for which $x_1^* = x_k^*$, and let's suppose $i \leq k$. In an equilibrium, we have $\frac{\partial U_i}{\partial x_i^+} \leq 0$. Moreover, we modify the utility function using the $\pi_j - \pi_i = (a - b\sum x_k)(x_j - x_i)$ shown earlier. So we have:

$$\begin{aligned}
0 \;&\geq\; \frac{\partial}{x_i}\left(\pi_i - \frac{(a - b\sum x_k)}{n-1}\left(\beta_i(k-1)(x_i - x_1^*) + \alpha_i\sum_{j>k}(s_k^* - x_i)\right)\right) \\
&=\; \frac{\partial \pi_i}{\partial x_i^+} - \frac{(a - b\sum x_k)}{n-1}(\beta_i(k-1) - \alpha_i(n-k)) \\
&+\; \frac{b}{n-1}\left(\beta_i(k-1)(x_i - x_1^*) + \alpha_i\sum_{j>k}(x_j^* - x_i)\right) \\
&\geq\; -\frac{(a - b\sum x_k)}{n-1}(\beta_i(k-1) - \alpha_i(n-k))
\end{aligned}$$

$$0 \;\geq\; \frac{\partial}{x_i}\left(\pi_i - \frac{(a - b\sum x_k)}{n-1}\left(\beta_i(k-1)(x_i - x_1^*) + \alpha_i\sum_{j>k}(s_k^* - x_i)\right)\right)$$

$$
\begin{aligned}
&= \frac{\partial \pi_i}{\partial x_i^+} - \frac{(a - b\sum x_k)}{n-1}(\beta_i(k-1) - \alpha_i(n-k)) \\
&+ \frac{b}{n-1}\Big(\beta_i(k-1)(x_i - x_1^*) + \alpha_i \sum_{j>k}(x_j^* - x_i)\Big)
\end{aligned}
$$

Taking into account only the second term we have

$$
0 \geq -\frac{(a - b\sum x_k)}{n-1}(\beta_i(k-1) - \alpha_i(n-k))
$$

And thus

$$
\beta_i(k-1) - \alpha_i(n-k) \geq 0
$$

$$
\frac{\beta_i}{\alpha_i} \geq \frac{n-k}{k-1} \tag{B.2}
$$

Now we get to the proof of (i): Without loss of generality we can assume that for i between 1 and k we have $\frac{\beta_i}{\alpha_i} > \frac{n-k}{k-1}$. As $\frac{\beta_i}{\alpha_i} < 1$, this implies that $k > \frac{n}{2}$. Now we will show that there is an equilibrium with $x_1 = x_2 = \ldots = x_k < x_{SNE}$. For $x \in [0, x_{SNE}]$ we define the strategy combination $s(x)$ as: fixed $s(x)_i = x$ for $i \leq k$ and for $j > k$ we choose $s(x)_j$ as the best joint reply. Thus, $s(x)_j$ is a part of a Nash equilibrium in the $(n-k)$-player game resulting from the fixed choices of x by the first k players. The best reply can never be smaller than x because in that case both the material payoff could be increased and the inequality disutility could be decreased by increasing the appropriation level. If we find $\hat{x}$ such that $\frac{\partial U_i}{\partial x_i^+}(s(\hat{x}) \leq 0$ for $i \leq k$, then $(\hat{x}, \ldots, \hat{x}, s(\hat{x})_{k+1}, s(\hat{x})_n)$ is the equilibrium. So:

$$
\begin{aligned}
\frac{\partial U_i}{\partial x_i^+}(s(\hat{x})) &= \frac{\partial \pi_i}{\partial x_i} - \frac{(a - b\sum s(\hat{x})_j)}{n-1}(\beta_i(k-1) - \alpha_i(n-k)) \\
&+ \frac{b}{n-1}\Big(\beta_i(k-1)(x_i - \hat{x}) + \alpha_i \sum_{j>k}(s(\hat{x})_j - x_i)\Big) \\
&= \frac{\partial \pi_i}{\partial x_i} - \frac{(a - b\sum s(\hat{x})_j)}{n-1}(\beta_i(k-1) - \alpha_i(n-k)) \\
&+ \frac{b}{n-1}\Big(\alpha_i \sum_{j>k}(s(\hat{x})_j - x_i)\Big)
\end{aligned}
$$

From that we get:

$$
\lim_{\hat{x} \to x_{NE}} \frac{\partial U_i}{\partial x_i^+}(s(\hat{x})) = \frac{(a - b\sum s(\hat{x})_j)}{n-1}(\beta_i(k-1) - \alpha_i(n-k)) < 0
$$

Thus, for some $\hat{x}$ near x_{SNE} we get $\frac{\partial U_i}{\partial x_i^+}(s(\hat{x})) < 0$. The strategy combination $s(\hat{x})$ is the desired equilibrium.

B.5 Proof of Proposition 3

Firstly, note that the condition $x \in \left[\frac{a(1-\beta_{min})}{b(1+n(1-\beta_{min}))}, x_{SNE}\right]$ guarantees that x maximizes the utility of a CCE player if all other players choose x. Secondly, we call a player a deviator if he chooses an appropriation level $x^{'}$ that results in a higher payoff in the stage before punishment takes place compared to x. Thus, if there is a single deviator, the payoffs of the other players are smaller than they would be in the situation with no deviator.

If punishment is executed, then the selfish players have no incentive to deviate from x, because it results in equal payoffs for both the CCEs and the deviator, and the payoff is smaller than in the stage before punishment. Now it remains to show that no CCE has an incentive to change the punishment strategy if a deviator chooses $x^{'}$. Let p be the optimal punishment level, π_p be the payoff after punishment for both the CCEs and the deviator. Let π_s be the payoff of the deviators. Obviously, a CCE player has no incentive to choose a higher level of punishment, as that would increase inequity with respect to all players and reduce the material payoff. So let $w > 0$ and assume that a CCE player chooses a punishment level of $p - w$. His utility function is then:

$$
\begin{aligned}
U_i &= \pi_p + cw - \frac{(n-k-1)\alpha_i}{n-1}(\pi_s - (\pi_p + cw)) - \frac{\alpha_i}{n-1}(\pi_p + w - (\pi_p + cw)) \\
&- \frac{(k-1)\beta_i}{n-1}(\pi_p + cw - \pi_p)
\end{aligned}
$$

This is a linear function in w. For player i to have no incentive to deviate from the optimal punishment level p the first derivative with respect to w needs to be negative:

$$
\begin{aligned}
0 &\geq \frac{\partial U_i}{\partial w} = c + c\frac{(n-k-1)\alpha_i}{n-1} - (1-c)\frac{\alpha_i}{n-1} - c\frac{(k-1)\beta_i}{n-1} \\
&\Leftrightarrow c[(n-1) + (n-k-1)\alpha_i + \alpha_i - (k-1)\beta_i] \leq \alpha_i \\
&\Leftrightarrow c \leq \frac{\alpha_i}{(n-1)(1+\alpha_i) - (k-1)(\alpha_i + \beta_i}
\end{aligned}
$$

Appendix C

Bachelor Thesis Proposal

Author: **Andrea Pospíšilová**
Supervisor: **PhDr. Martin Gregor, PhD**
Academic Year: **2009/2010**
Proposed Topic: **Tragedy of the Commons**

Topic Characteristics: Od roku 1968, kdy byl publikován článek Garretta Hardina, se fenomén Tragedy of Commons rozšířil do mnoha oblastí. V první části své práce bych se ráda zaměřila na teoretické modely vycházející z teorie her, faktory které určují pravidla hry a některá navrhnutá řešení. Ve druhé části se zaměřím na empirická data podporující a vyvracející tuto teorii, a na současný vývoj tohoto tématu.

Outline:
1. Úvod
2. Teoretické modely
    2.1. Kolektivní vězňovo dilema
    2.2. CC-PP Game
3.Faktory ovlivňující hru
    3.1.Motivace
    3.2.Instituce
    3.3.Strategie
4.Možná řešení tragédie
    4.1.Role státu
    4.2.Netechnická řešení
5.Empirická evidence
    5.1.Podporující TOC
    5.2.Vyvracející TOC
6. Závěr

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Datum zadání: únor 2009

Podpisy konzultanta a studenta:

V Praze dne