Institute of Economic Studies, Faculty of Social Sciences
Charles University in Prague

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Radovan Parrák
Jakub Seidler

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Institute of Economic Studies,
Faculty of Social Sciences,
Charles University in Prague

[UK FSV – IES]

Opletalova 26
CZ-110 00, Prague
E-mail : ies@fsv.cuni.cz
http://ies.fsv.cuni.cz

Institut ekonomických studií
Fakulta sociálních věd
Univerzita Karlova v Praze

Opletalova 26
110 00 Praha 1

E-mail : ies@fsv.cuni.cz
http://ies.fsv.cuni.cz

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Mean-Variance & Mean-VaR Portfolio Selection: A Simulation Based Comparison in the Czech Crisis Environment

Radovan Parrák[#]
Jakub Seidler*

[#] IES, Charles University Prague
E-mail: rado.parrak@gmail.com

*Czech National Bank and
IES, Charles University Prague

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Abstract:

This paper focuses on two methods for optimum portfolio selection. We compare Mean-Variance method with Mean-VaR method by the means of investment simulation, based on Czech financial market data from turbulent market periods of the year 2007 and the year 2008. We compare both strategies, basing on measurements of relative and absolute profitability of both strategies in crisis periods. The results indicate that both strategies were relatively profitable in both simulation periods. As a consequence of our results, it seems that it is worth to adhering investment decisions to outputs of optimisation algorithms of both methods. Moreover, we consider Mean-VaR strategy to be safer in turbulent times.

Keywords: portfolio optimization, investment strategy, Mean-Variance, Mean-Var

JEL: C52, G01, G11

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The findings, interpretations and conclusions expressed in this paper are entirely those of the authors and do not represent the views of any of the above-mentioned institutions.

Introduction

In this paper, we put two models for optimum portfolio selection in direct confrontation. Two portfolio selection methods were transformed into investment strategy simulations with the aim of their direct comparison based on a real historical data from the Czech environment. We compared the Markowitz's Mean-Variance theory with method using modern approach to risk measure – Mean-VaR. Many studies deal with comparing these two approaches, e.g. Baptista and Gordon [2002], or Campbell et al. [2001]. However, in comparison to mentioned studies, this paper focuses on testing these strategies in the financial crisis environment.

We worked with data set consisting of 510 consecutive day returns[1] of 13 stock titles traded at Prague Stock Exchange (PSE). Both portfolio selection strategies were examined under harsh simulation environment conditions of high volatility and low variety of relevant stocks.[2] This setting corresponds to the impacts of financial crisis on small number of stocks traded on PSE. Portfolio selection methods were transformed into different investment strategies and empirically tested – both strategies proved to be safe and profitable (at least relatively i.e. in comparison to PSE performance).

The structure of the paper is following. After short description of relevant literature, following chapters presents main building blocks in which the problem of portfolio optimization takes place and focuses on describing both optimization methods as well as investment strategies. The third chapter describes simulation strategies and used data sample. Finally, results are provided in the last chapter.

1. Relevant Literature

Mean-Variance theory founded by Markowitz [1952] is based on presumption that distribution of portfolio returns is normal and can be successfully described by two moments – mean and variance. Mean-Variance theory was further developed by Sharpe (e.g. Sharpe [1966], [1994], [2000]). Sharpe [1964] developed Capital Asset Pricing Model (CAPM), which is still used by practitioners and serves as a cornerstone for a variety of portfolio selection computer optimizers. Further, Sharpe [1966] brought a theoretical concept for picking the portfolio which yields the highest return over the unit of risk. This paper used this method in our simulations as representative of Mean-Variance framework.

After proposal of Mean-Variance theory, the question about suitability of variance as a risk parameter was raised (e.g. Markowitz [1991], Campbell et al. [2001]). Since variance is symmetrical, it does not consider the direction of the price movement. Thus, optimizing the variance can prevent investor from losses in same manner as from gains. Moreover, Roll [1977, 1978, and 1979] firstly pointed out other weaknesses of the theory. This evidence forced academics to search for more appropriate risk measures. For instance, Markowitz [1991], Fishburn [1977], Bawa [1977] proposed mean-lower partial moment approach, Yitzhaki [1982]; Shalit, Yitzhaki [1984] proposed mean-Gini portfolio selection model, Konno and Yamazaki [1991] proposed Mean-Absolute Deviation (MAD) approach, Uryasev [2000] Mean-VaR (Mean-CVaR) type models, etc.

As long as we presume that portfolio returns of our data sample are not normally distributed and simultaneously agree with the restrictive character of the variance as a risk parameter, we choose Mean-VaR method to be the "counterpart strategy" to the Mean-Variance in this paper. Mean-VaR strategy uses VaR as a parameter of risk instead of variance. Concept of Mean-Variance was proposed by Campbell, et al. [2001]. Although this concept perfectly fitted Arzac's and Bawa's [1977] framework, complications occurred on technical side of the problem. Artzner et al. [1999] pointed out the non-coherency of VaR. This would make the process of finding minimum of the VaR function extremely difficult. Because of that, we worked with more "user-friendly" Mean-CVaR approach developed by Uryasev [2000].

[1] From 19.12.2006 till 30.12.2008.

[2] In sense of their presence on the market with sufficient liquidity.

2. Models' Building Blocks

To set up the models environment, let assume *n* risky assets and one risk-free asset. Structure of optimum portfolio is determined by risk-return trade-off and optimum portfolio characterized by optimum combination of risky and risk-free assets is searched.

We assume no transaction costs, no short sales, perfect liquidity and divisibility of securities and the assumption of uncorrelated risky and risk-free assets as Sharpe [2000]. In addition, we used assumption about imperfect correlation of risky securities, which leads to rounded efficient frontier (implying unique optimum portfolio).[3]

- Mean-Variance

Within a frame of Mean- Variance portfolio selection, we followed algorithm described in Markowitz [1991] and Sharpe [2000]. Let us consider *n* risky assets, $x_1,..., x$ for investment. Having historical data of returns for each asset and *K* periods, optimum portfolio can be find by solving following problem[4]:

$$\min \quad \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}$$
$$s.t. \quad r_p(w) \geq \mu \wedge \mathrm{w}_i \geq 0 \wedge \mathbf{w}^T \mathbf{e} = 1$$

where w is vector of portfolio weights, e is column vector of ones and Σ is variance-covariance matrix of returns. The term $r_p(w)$ stands for portfolio return and portfolio variance is represented by:

$$\text{variance}_p(w) = \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}$$

Portfolio risk is then derived:

$$\rho_p(w) = \sqrt{\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}}$$

And by varying μ, the efficient frontier (different optimum $r_p - \rho_p$ pairs) is obtained.

We use approach described in Sharpe [2000] in order to determine the best portfolio on the efficient frontier – optimum portfolio with highest performance ratio.

Sharpe [1966, 1994] firstly proposed performance ratio called reward-to-variability ratio.[5] Since Sharpe ratio is designed for mean-variance framework, it fits to our needs. The definition of Sharpe ratio based on Sharpe [1994] is as follows [6]:

$$SR(w) := \frac{\overline{d}}{\sigma_d} = \frac{r_p(w) - r_f}{\sigma_d}$$

where d-bar is "expected differential return", i.e. the difference between expected return of the portfolio combined from risky assets and expected risk-free return. Term σ_d is expected standard deviation of *d*.[7] Thus, performance ratio is the ratio between expected return of a portfolio and its risk. More formally, it has following form:

$$PR(w) := \frac{\mu(\mathbf{w}^T \mathbf{E(r)})}{\rho(\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w})}$$

where μ(.) is expected return of a portfolio and ρ(.) is portfolio risk.

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3 See Sharpe [2000] or Markowitz [1991] for deeper introduction of the Mean-Variance concepts (e.g. efficient frontier, risky security, risk-free security, etc.).

4 Problem can be also stated as expected return maximization problem, subject to variance constraint.

5 Reward-to-variability is original name of this ratio, but nowadays term "Sharpe Ratio" is used more frequently in research papers.

6 In this paper we will work with *Ex ante* Sharpe ratio.

7 For detials, see Sharpe [1994].

We used Sharpe ratio maximisation as a tool for finding optimum combination of risky and riskless securities. The algorithm was executed in two stages as follows:

First Stage – by SR(w) maximization, we obtained unique efficient portfolio of risky assets which yielded the highest expected return per unit of risk. This is possible since the risk-free rate of return is known.[8]

Second Stage – optimum risky portfolio is combined with desired weight of risk-free security. According to Sharpe [2000, p. 67]:*"Inclusion of riskless security makes part or all of the efficient border of the r_p-ρ_p region linear"*. More precisely, the r_p-ρ_p efficient border will start at the value of pure interest rate and will rise linearly, through the point of optimum combination of risky securities continue behind it. This is caused by the relationship between expected return of the entire portfolio and its variance, described in Sharpe [1966]:

$$E_p = r_f + SR(w) \cdot \sigma_p ,$$

where $SR(w)$[9] is maximum Sharpe ratio of the risky assets portfolio[10] and r_f is risk-free rate. As Sharpe [2000, p. 70] quotes, the existence of a tangent portfolio has another advantage, since *"The investor need only decide how much to borrow or lend. There is but one appropriate combination of risky securities in which to invest the remainder of his funds...*"

We consider agent's awareness of desired volume of risk-free security in portfolio as a non-limiting assumption. Therefore, we use this assumption (as a representative of agent's risk aversion) instead of presuming agents awareness of his utility function (which is also a possibility).

Stoyanov et al. [2007] presented that the problem of maximization of Sharpe ratio can be transferred into two equal problems[11]:

(SR A):

$$\begin{aligned} \max_{(z,t)} \quad & \mathbf{z}^t \mathbf{E}(\mathbf{r}) - t \cdot r_f \\ s.t. \quad & (-t, \mathbf{z})^T \mathbf{\Sigma_1} (-t, \mathbf{z}) \le 1 \\ & \mathbf{z}^T \mathbf{e} = t \\ & t \ge 0 \end{aligned}$$

(SR B):

$$\begin{aligned} \min_{(z,t)} \quad & (-t, \mathbf{z})^T \mathbf{\Sigma_1} (-t, \mathbf{z}) \\ s.t. \quad & \mathbf{z}^t \mathbf{E}(\mathbf{r}) - t \cdot r_f = 1 \\ & \mathbf{z}^T \mathbf{e} = t \\ & t \ge 0 \end{aligned} \; ;$$

Where $\mathbf{\Sigma_1}$ is matrix in form:

$$\mathbf{\Sigma_1} = \begin{bmatrix} \sigma_f^2 & \sigma_{f,i} \\ \sigma_{f,i} & \Sigma \end{bmatrix}$$

After solving one of the previous mathematical programming problem, we combine optimum portfolio of risky securities with riskless security. This locate demanded portfolio somewhere on the tangent

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8 Risk-free rate of return has to be lower than expected return of some attainable combination of risky asset, in order to obtain relevant results (i.e. investment under uncertainty is wise only in case that expected return is higher than return from risk-free security).

9 Sharpe ratio is the slope of the r_p-ρ_p efficient line.

10 Portfolio with highest Sharpe ratio is used to be called tangent or Markowitz portfolio.

11 Equal in sense of obtaining same solution. Moreover, Stojanov, et al. [2007] proved, that:"*The performance measure optimization problem is equivalent to problems SR A, SR B in sense that if the pair (z_A^*,t_A^*) is solution to Problem (SR A) and (z_B^*,t_B^*) is solution to Problem (SR B), then $w^* = z_A^* / t_A^* = z_B^* / t_B^*$ solves the Problem (*maximize Sharpe Ratio*)...*".

line to efficient frontier of risky securities. Since every point on the straight tangent line is a convex combination of optimum combination of risky securities and riskless security, the final portfolio will have following structure: $w_{entire\ portfolio} = \lambda w_f + (1-\lambda)w_p$, where λ is the desired fraction of riskless security in entire portfolio.

- Mean-VaR (Mean-CVaR)

We mainly build on Jorion's [2001] VaR framework and work by Palmquist, Uryasev [2002], Rockafellar and Uryasev [2000] and Stojanov et al. [2007].

The advantage of VaR concept is not relying heavily on normal distribution of returns. Since Wang [2000] proved that mean-VaR efficient set is not mean-variance efficient set and vice versa, we are confident about different structure of mean-variance and mean-VaR portfolios, and therefore is worth to compare these two approaches.[12] Moreover, Wang [2000] presented that VaR is not limiting the possible gains. More precisely, since standard deviation is symmetrical risk measure, by its minimization we are penalizing ourselves from possible gains too. On the contrary, VaR is a measure of downside risk, which in case of skewed distributions can significantly differ from upside risk.

As pointed out by Gaivoronski and Pflug [2005], Mean-VaR is theoretically good strategy to find an optimum portfolio of risky assets, nevertheless computationally extremely demanding (in sense of finding optimum mean-VaR portfolio).[13] Since VaR is not a coherent measure of risk, according to Artzner, et al. [1999], it can cause high losses when finding optimum portfolio based on VaR.[14] This is caused mainly by the fact that VaR is not subadditive. Therefore, VaR based optimization discourages from diversification. More detailed description of coherency is provided by Artzner, et al. [1999].

Due to the VaR's coherency problem, we use CVaR as sufficient substitute of VaR in our Mean-VaR (Mean-CVaR) framework. By the definition, CVaR is conditional expectation of losses above VaR. More formally:

$$CVaR_{\alpha}(X) = E\left(X \middle| X \geq VaR_{\alpha}(X)\right)$$

where X is some random cost variable.

As presented in Rockafellar and Uryasev [2000] and later in Palmquist, Uryasev [2002], CVaR[15] is coherent measure of risk. Therefore it has all the desirable characteristics which are needed for solving the linear programming problem.[16] Moreover, Palmquist and Uryasev [2002, p. 3] also reported, that ”*... the minimization of CVaR also leads to near optimal solutions in VaR terms because VaR never exceeds CVaR* “.

First of all, we propose the function which minimizes CVaR and VaR in the same time. Later, we will implement this function in performance ratio maximization method similar to Sharpe ratio. This method will represent a Mean-VaR (or Mean-CVaR) approach.

Before we get closer to the CVaR function, we shortly describe scenarios generation. As mentioned before, Mean-VaR (or Mean-CVaR) does not rely on presumption of normal distribution of returns. Thus, the method can better estimate the distribution function of the returns. For the estimating of returns distribution, we used method of historical scenario generation, also presented in Palmquist and Uryasev [2002].[17] We used historical returns of all stocks included and the length of the data string

[12] Also VaR and variance are independent; except the case of multivariate normal distribution.

[13] Mainly because of occurrence of many local minimums of the VaR function. Gaivoronski and Pflug then came out with concept of smoothed VaR (SVaR), which is computionaly very demanded and can be substituted by CVaR with sufficient accuracy.

[14] VaR is coherent only when assumption of normal distribution of returns holds. VaR is then a multiple of standard deviation, as mentioned earlier in the text.

[15] In literature also called Expected shortfall or Expected Tail loss, (e.g. Acerbi and Tasche [2002]).

[16] It happens to be linear programming problem after implementing algorithms developed in Rockafellar and Uryasev [2000].

[17] Obviously, there are more sophisticated methods for scenario generations relying on some presumption about known property of distribution or Monte Carlo simulations.

was dependent on the time period for which we intended to create scenarios. In the concrete, in our simulation we were optimizing portfolio for one month (i.e. 20 days). As the result, we generated 20 scenarios for future returns. First of all, we created 20 possible scenarios for future prices of the stocks, based on the historical data:

$$y_{i,j} = q_i \frac{p_i^{t_j+\Delta t}}{p_i^{t_j}},$$

where $y_{i,j}$ is end-of-period price of stock i, $j \in \{1,....,20\}$ (i.e. number of days), q_i is current price of stock i, p_i^t is historical price of stock i in time t and Δt is the period (20 days). Afterwards, we computed scenarios for future returns:

$$r_i^j = \ln\left(\frac{y_{i,j}}{q_i}\right)$$

After this process, we obtained 20 predictions of future returns for every security.

We use the function which minimizes CVaR presented in Rockafellar and Uryasev [2000]:

$$\tilde{F}_\alpha(\mathbf{w},\delta) = \delta + \frac{1}{K(1-\alpha)}\sum_{j=1}^{K}[-\mathbf{w}^T\mathbf{r_j} - \delta]^+,$$ [18]

Where **w** is vector of weights, δ is parameter (and optimum δ^* is optimum VaR), K is number of days in period (i.e. number of scenarios) and $\mathbf{r_j}$ is random vector of future portfolio returns.

After implementing auxiliary variables, the problem will reduce to:

$$\tilde{F}_\alpha(w,\delta) = \delta + \frac{1}{K(1-\alpha)}\sum_{j=1}^{K}u_j$$

$$and$$

$$u_j \geq 0; \mathbf{w}^T\mathbf{r}_j + \delta + u_k \geq 0, \forall j \in \{1,...,K\}$$

Then according to Palmquist and Uryasev [2002], the entire linear programming problem has form[19]:

$$\begin{aligned}
\min_{w,\delta} \quad & \sum_{i=1}^{n} -E[r_i]w_i \\
s.t. \quad & \delta + \frac{1}{K(1-\alpha)}\sum_{j=1}^{K}u_j \leq \omega \cdot W^0 \\
& \mathbf{w}^T\mathbf{r_j} + \delta + u_j \geq 0, \forall j \in \{1,...,K\}, \\
& w_i \geq 0 \\
& \mathbf{w}^T\mathbf{e} = 1
\end{aligned}$$

where ω is a percentage of the wealth which is allowed for risk exposure and W^0 is the value of agent's wealth in current period. By solving the minimization problem for various levels of risk exposure, one can arrive to the Mean-VaR efficient frontier.

- STARR Ratio and its Maximization

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[18] It is important to note, that we expect that occurrence of each scenario of return has the same probability.

[19] The original relation from Palmquist and Uryasev [2002] was slightly modified by us to serve better our purposes.

To build a comparable Mean-VaR strategy, we have to implement previous relations to the performance ratio. Once we succeeded, we will use similar algorithm to the one used in Mean-Variance section (i.e. maximising performance ratio to find optimum combination of risky securities, then combining optimum risky portfolio with riskless security). We built on Stoyanov, et al. [2007] and Biglova, et al. [2004] – we are looking for a performance ratio which has in its denominator risk measure based on VaR (CVaR) instead of variance.

This ratio was firstly presented by Martin, et al. [2003] and was called STARR ratio [20] (Stable Tail Adjusted Return Ratio), defined as:

$$STARR(w) = \frac{r_p(w) - r_f}{ETL_\alpha(\mathbf{w}^T\mathbf{r} - r_f)},$$

Where $ETL_\alpha(.)$ is Expected Tail Loss – CVaR.[21]

Based on Stoyanov, et al. [2007], maximisation of *STARR(w)* can be transferred into problem:

(STARR A)[22]

$$\begin{aligned}
&\max_{z,t,u,\delta} && \mathbf{z}^T\mathbf{E(r)} - tE(r_f) \\
&s.t. && \delta + \frac{1}{K(1-\alpha)}\sum_{j=1}^{K} u_j \le 1 \\
& && -\mathbf{z}^T\mathbf{r^j} + tr_f - \delta \le u_j \\
& && \mathbf{z}^T\mathbf{e} = t \\
& && z_i \ge 0, \forall i \in \{1,....,n\} \\
& && t \ge 0,\ u_j \ge 0, j = 1,2,....,K
\end{aligned}$$

where ***E(r)*** is the vector of expected returns, obtained from the generated scenarios as:

$$E(r_i) = K^{-1}\sum_{j=1}^{K} y_{ij}$$

When the risky portfolio which maximizes STARR is found, we combine this portfolio with desired weight of risk-free security in the entire portfolio. It is important to notice, that δ (after carrying out the minimization) is minimized (optimum) VaR. δ is in percentage terms (moreover, in percentage terms of risky portfolio).Thus, if we want to arrive to VaR of the whole portfolio, it must be derived from:

$VaR_{entire} = W^0\delta(1-\lambda)$, where W^0 is current portfolio value and λ is fraction of riskless security in portfolio.

3. Simulations And Data Sample

Let define "investment strategy" by which we understand gradual and repetitive portfolio optimization routine (by means of either Mean-Variance or Mean-VaR method) in predefined moments in certain time period.

As mentioned before, we worked with time horizon of 509 days which included two investment periods. Each of investment periods took six months (120 days). First period started on July 10, 2010 and ended on December 28, 2007. First period supposed to be a representative of pre-crisis market times with slightly higher market volatility. Second period started on July 9, 2008, ended on December

[20] Term "STARR ratio" is pleonasm in fact, but in order to stay compact with other research paper we will be using it in this form.
[21] According to Stoyanov, Rachev, Fabozzi [2007].
[22] According to Stoyanov, et. al. [2007], there is also second form of stating the mathematical programming problem.

31, 2008 and was representative of crisis times with significant market volatility. Rest of the data served as historical, needed especially for estimating means and variances (alternatively VaRs).[23] Both of investment periods consist of six consecutive rounds. Each round has 20 days.

For better and more intuitive strategies description, we created one agent for each strategy. Agent A is a representative of Mean-Variance strategy and agent B represents Mean-VaR strategy. For both agents, we tested three levels of risk aversion measured in terms of percentage of initial wealth devoted to investment in risky assets. We arbitrary (and without prejudice to the generality) set these levels:

Level 1:	λ= 20%	(i.e. he lends 20% of his initial wealth)
Level 2:	λ= 0%	(i.e. he does not lend nor borrow)
Level 3:	λ=-20%	(i.e. he borrows funds worth 20% of his initial wealth)

Therefore, agent A(1) is creating his portfolio with a help of Mean-Variance method and allocate 20% of his initial wealth into risk-free asset and the rest in the risky assets. In general, we can assume, that in case of profitable portfolio selection, higher risk exposure (in terms of risk-free asset, i.e. level 3) generate higher profit, otherwise, safer strategy (i.e. level 1) generates lower losses.

At the first day of each period optimum portfolio is found. First of all, both agents borrow (or lend) desired level of funds – riskless allocation. Then, the rest of their funds are allocated among risky securities. In this step, agents are basing their decisions in accordance to the developed theory. Agents hold the same positions of risky securities for 20 days (one round). At the end of each round all the risky assets are sold at market price. In the next round they can again optimize risky assets allocation, taking into the considerations performances of stock returns from previous rounds.[24] Weight of risk-free asset in the portfolio stays unchanged for the whole period (six rounds). After one period, money (riskless asset) is withdrawn from the bank with an interest. The simulation is evaluated after every period.

Initial wealth of all agents at the beginning of every period was set to be one million CZK. The value of 6 month PRIBOR rate was 3.09% p.a. in the first period and 4.29% p.a. in the second period.[25] Agents can reallocate their funds during the investment period invested in risky securities, but are not allowed to do the same with a risk-free asset. This measure was put in action mainly due to the high transaction costs in reality (e.g. term deposit or loan).[26]

- Compatibility (Comparability) vs. Statistical Significance Trade-off

Before the first round of simulations, we have 137 data of daily returns available.[27] Since one investment round took one month (i.e. Agent A is optimizing for one month horizon), daily returns need to be transformed into over month returns in order to compute means and variance-covariance matrix for monthly returns. By doing so, we would obtain only 7 historical over-month returns before the first investment round, for each of 13 stocks.[28] In the next step, we would compute over-month mean return of each stock from return distribution defined by only seven historical observations (similar applies for variance-covariance matrix)[29].Since seven observations is an insufficient number for construction of returns distribution, we optimize for the over-day horizon, keeping portfolio unchanged for 20 days (one round).[30] Thus, Agent A compute expected over-day return from 137 daily

[23] In case of Mean-Variance method, for every round all the previous (historical) data from dataset were used for computation of means and variances (i.e. in second period, historical data, data from first period of simulation and data between end of first period and start of second period were used).

[24] Optimisation is dependent also on agent's wealth after particular round, derived from portfolio value from previous period.

[25] This values were obtain via ARAD system - http://www.cnb.cz/cnb/STAT.ARADY_PKG.STROM_KOREN.

[26] despite of using non transaction costs assumption in this paper.

[27] From 19.12.2006 to 9.7.2007.

[28] Moreover only 6 months would consist of 20 days.

[29] Due to lack of historical data.

[30] Even in last investment round (28.11.– 30.12.2008), there would be only 24 historical over-month returns available.

return observations for each of 13 risky securities and compile their variance-covariance matrix. Then, he computes one-day PRIBOR from the referential PRIBOR.

Arising from the character of approach, Agent B did not face problems with data string. Thus, he optimized the portfolio for one round (month) horizon as follows. After generating 20 possible scenarios of future returns for each stock, he implemented them in the process of solving mathematical programming problem STARR A. By arriving to solution, Agent B finds optimum portfolio weights and allocates according to them his funds. From now on, both routines are proceeding as described above.[31]

- Data Sample

This paper focuses on the Czech environment. Due to the fact, that PX index embodies the performance of Prague Stock Exchange, we constructed PX index portfolio (i.e. all funds invested in PX index), which served as a benchmark for comparing of relative profitability of both Mean-Variance and Mean-VaR strategy. Explicit discussion of time series related to relevant stocks and PX index are provided in Appendix C.

We worked with 13 risky securities from Official Market. More specifically, stocks were mainly from main market (SPAD), but for reasons which will be explained later, we also included few stocks from Free Market (KOBOS). Due to the fact that PSE is relatively young stock exchange in comparison to Western Europe or Northern American stock exchanges, it was harder to use "high quality" data – in sense of data series longitude and liquidity of particular stocks. In most of the cases, SPAD stocks were rather limiting component of this study, due to the fact, there are only 9 SPAD market instruments quoted before 2006. On the other hand, SPAD stocks are the most liquid segment of PSE market, therefore we implement as many of them as possible. Since we want to focus our study on financial crisis times, we accepted certain trade-off between number of observations (implying efficiency of some statistical concepts), variety of stocks and their liquidity.[32]

Finally, 9 SPAD stocks were chosen. The chosen stocks and dates of their initial public offerings (IPO) are summarized in Table 1.

NAME	IPO	Market
CETV	27.6.2005	Main
ČEZ	5.6.1998	Main
ECM	7.12.2006	Main
ORCO	1.2.2005	Main
PEGAS NONWOVENS	18.12.2006	Main
PHILIP MORRIS ČR[33]	9.10.2000	Main
TELEFÓNICA O2	4.6.1998	Main
UNIPETROL	29.5.1998	Main
ZENTIVA	28.6.2004	Main

Table 1: SPAD stocks

Source: PSE, www.finance.cz, www.iPoint.cz

The rest of the stocks were traded via KOBOS system.[34] These stocks are either from Main Market (but not in SPAD Pražská Energetika) or from Free Market (JČ PAPÍRNY VĚTRNÍ, PARAMO, TOMA). During the examined period, they possessed low volatility (did not react to turbulent changes on market so flexibly)[35] what resulted in their inclusion into the list of stocks used in this paper. In reality, one of the biggest disadvantages of KOBOS system is illiquidity of stocks traded. As long as we wanted to make a use of relative short term price stability and did not want to include lot of

[31] The mathematical programming problem of maximizing Sharpe ratio was solved in Excel Solver. Variance-Covariance matrix, efficient frontier was obtained by means of VBA (Visual Basic for Applications) routines, whose codes are provided in Appendix A.

[32] Moreover, we decided to work only for selected non-financial companies listed on PSE.

[33] Traded on Free Market, according to PSE fact book..

[34] For more information about KOBOS market see www.pse.cz

[35] For price and returns development see appendix C.

illiquidity in our portfolio selection problem, we worked only with 4 KOBOS stocks, which are described in Table 2.

NAME	MARKET
Pražská Energetika	MAIN
JČ Papírny Větrní	FREE
PARAMO	FREE
TOMA	FREE

Table 2: KOBOS stocks

Source: PSE, www.finance.cz, www.iPoint.cz

As a riskless security we use six month PRIBOR. The reason why six month rate as referential was taken is the fact that the one period of simulation process will take exactly six months. This enables us to provide the precise results of returns from investments in riskless assets after testing period.[36] The values of PRIBOR for particular period of simulation are provided in Table 3:

PRIBOR (6 month)	1st period	2nd period
Rate per annum	3.09%	4.29%
Over-month rate	0.26%	0.36%
Over-day rate	0.01%	0.02%

Table 3: Risk-free asset

Source: Author's computations

After explaining theoretical background of Mean-Variance and Mean-VaR portfolio selection strategies and data the sample, we will empirically used abovementioned concepts.

4. Empirical Testing and Results

As mentioned above, Markowitz's Mean-Variance framework presumes normality (log-normality[37]). Although Markowitz [2000] argues that:"*Normal distributions or other two-parameters families of probability distributions were not part of the Markowitz [1959]*[38] *justification for mean and variance. Nowhere in chapters 10-13 of that book is "normal" of "Gaussian" or "two-parameter family" mentioned...*".But the theory itself imply this assumption. Since the Mean-Variance theory uses variance as risk measure, which is symmetrical in fact, the underlying distribution of the returns ought to be (in order to be compatible) also symmetrical. Otherwise, making a use of Mean-Variance theory should lead to inefficient allocation of assets. Nevertheless, not satisfying of this presumption is not inconsistent with "*Capabilities and Assumptions of the Model*" chapter in Markowitz [2000].

As far as Sharpe ratio is arising from Mean-Variance framework, we shall arrive to inefficient portfolio when working with not normally distributed returns. This was pointed out by Biglova, et al. [2004]," *Although this ratio is fully compatible with normally distributed returns (or, in general with elliptical returns), the Sharpe ratio will lead to incorrect investment decisions when returns present kurtosis and/or skewness*."[39] Therefore, the normality tests could serve as serious indicators of upcoming results from simulations.

Therefore we have tested all (13) of the returns of the stocks for normality. We have carried out both tests for normality and subsequently for log-normality of returns. Our tests took into the consideration time period before first period of testing (137 days) as well as whole period (509 days).[40]

[36] In contrast to other papers which use as long terms as possible (e.g. 5 or even 10 year LIBOR etc.).
[37] Concept of log-normality is also used in litarature, for instance Palmquist and Uryasev [2002].
[38] We used newer edition of this monograph cited as Markowitz [1990].
[39] Page 2. For details about this critique see for instance Leland [1999] or Ortobelli, et al. [2003].
[40] For normality testing we used several tests (in GRETL software) to make sure that it is worth to adhere to our results (i.e. Doornik-Hansen test, Shapiro-Wilk test, Lillieforse test and Jarque-Berra test).

The results of normality tests for data including the whole period were rather unambigous. For every stock we rejected the null hypothesis of the normality test (i.e. H0: data are normally distributed) at 99% confidence level.[41] These results are evidenced well-known property of financial data series, i.e. returns are usually not normally distributed. In many cases, skewness and/or kurtosis of sample distribution disrupts circa Gaussian shape of distribution function. In addition to the skewness and kurtosis, another peculiar property of risky assets returns has been discovered and is true for our data sample-“fat tails”. Since both problems are true for our data sample, we suppose that, by using Mean-VaR strategy one should end up better off, due to the fact that VaR is not symmetrical measure of risk. Based on theses results, we assume that within a frame of certain time period observed in our paper, Mean-VaR strategy shall perform better (e.g. creates larger profit/smaller loss) than Mean-Variance strategy.

We have also tested for normality returns of date earlier than 10.7.2007. This was done due to the fact that only these data were available to agents before first day of initial simulation round. Therefore, they were only able to construct their optimum portfolios given available data. The results of tests are not included due to space limitation, but are available upon the request. We have to admit that these results were not so explicit in comparison to results of whole period testing, but were still define enough to premise that Mean-Variance framework will be less effective.

- Results

In order to make comparing of both strategies clearer, we will deal with the first and the second period separately.

First period – was characterized by higher volatility of stock returns, but rather stagnating trend of PSE.[42] We assume inefficiency of Mean-Variance strategy due to proved non-normality of returns. When it comes to Mean-VaR analysis, and its historical simulation approach to estimation of future returns, problems can occur in periods preceded by periods of relatively high appreciation of minority of stocks (and consecutive depreciation in next period), but stable performance of whole stock market. This is caused by the fact that Mean-VaR strategy reacts on changes more flexibly, estimates future returns on different basis (working with particular scenarios – days, rather than particular securities) and from shorter data string.[43]

Figure 2 depicts performance of risky part of portfolio of Mean-Variance and Mean-VaR strategy for the first period. Additionally, there is a benchmark portfolio (constructed purely from PX index) available for comparison of relative profits. We chose (without limiting generality) exposure level 1 (i.e. 80% of initial wealth invested in risky part of portfolio). Returns from other exposure levels, would be in the same relationship, only shifted upward or downward depending on initial level of investments in risky portfolio. Figure 1 also depicts the value of entire portfolio (i.e. riskless security value in particular point in time is also included).[44]

[41] P-values and Q-Q plots are not included because of space limitations, but are available upon request.

[42] See Appendix

[43] This can be solved by concerning longer data string, and would lead to even more computationally demanding process. Thus, it is not problem of method itself, rather problem of problem of computational capabilities and extent of this paper.

[44] Figures for exposure level 2 and 3, with the rest of information about portfolios in particular rounds are provided in Appendix D.

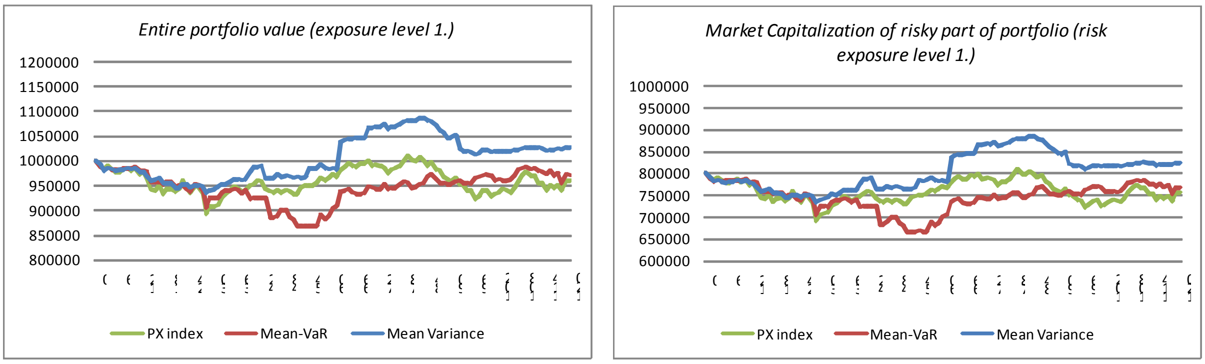

Figure 1: Portfolio returns – 1st period

Source: Author's computations

Arising from the results of the first period, the both strategies were relatively profitable, i.e. it was worth to optimize the portfolio, because it yielded higher return (lower loss) than stock exchange average (measured in terms of PX index). Agent A profited even in absolute values – 26 571 CZK (circa 2.7% profit), in period when PX lost 4.1% of its value. Agent B lost circa 3% of his initial wealth, which is less than stock exchange average loss.

Agent A was more profitable at the exposure level 2 and even more profitable at exposure level 3, which is in line with our presumption, that when profiting, more risky strategies will generally generate higher profits. Same was true for Agent B – the higher the risk exposure was, the higher loss was generated. The results of period one (for the last day of period) are concluded in Table 4:

Level of exposure	Value of	Agent A	Agent B	PX index portfolio
1 (λ=0.2)	*risk-free part*	*203090*	*203090*	*203090*
	risky part	*823481.8*	*766477.5*	*755880.1*
	entire portfolio	*1026571.8*	*969567.5*	*958970.1*
	portfolio return	*2.66%*	*-3.04%*	*-4.10%*
2 (λ=0)	*risk-free part*	*0*	*0*	*0*
	risky part	*1029352.3*	*958096.9*	*944850.2*
	entire portfolio	*1029352.3*	*958096.9*	*944850.2*
	portfolio return	*2.94%*	*-4.19%*	*-5.51%*
3 (λ=-0.2)	*risk-free part*	*-203090*	*-203090*	*-203090*
	risky part	*1235222.7*	*1149716.2*	*1133820.2*
	entire portfolio	*1032132.7*	*946626.20*	*930730.2*
	Portfolio return	*2.94%*	*-5.34%*	*-6.93%*

Table 4: Simulation Results – 1st period

Source: Author's computations

Information contained in Table 4 describes the main results of the first period of the simulation. In the following lines we offer some explanatory notes on breaking points of development of Agents A's and Agent B's portfolio values.

According to the Figure 1, performance of both strategies was in line with market performance in the first 20 days (first two rounds of simulation). Then, in the second round of the first period Agent B's portfolio lost in value, but on the contrary, PX portfolio and Agent B's portfolio still appreciated. The main difference in Agent A's and B's portfolio weights were in portfolio diversification, especially in investment in JČ Papírny stock as depicted by Figure 2:

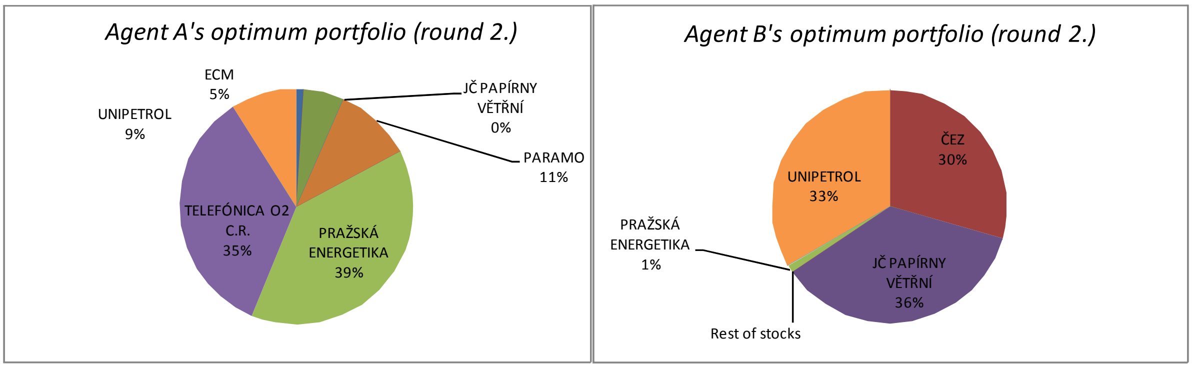

Figure 2: Portfolio structure in 2nd round

Source: Author's computations

Historical simulation predicted good future prospects of JČ Papírny stocks due to short term appreciation. It is important to note, that by short term we mean more than 20 days (more than one round from which the distribution of future returns was predicted). As mentioned before, these mistakes can be avoided by extending the range of historical simulation window, which would lead to technically and computationally more demanding algorithm, based on the same theoretical basis. Anyway, Mean-Variance framework was more efficient in this point of simulation. Agent A was not strongly influenced by short term appreciation of JČ Papírny stock, since it did not influence significantly distribution of entire distribution of historical returns, thus he did not invest into it. This conservatism of Mean-Variance approach appeared to be good decision this time, since JČ Papírny fell back at its pre-period value in round 2.

Agent B's portfolio continued in plunging, attaining lowest values in next period (around day 50, half of round 3). This was again caused by investment into JČ Papírny stock. The reason, why Agent B, who is using Mean-VaR framework, chose again to invest in the same stock in which he lost circa 2% of portfolio value last month is straightforward – JČ Papírny stock had relatively high expected return for round 3 (in comparison to the rest of stock). Additionally, its expected losses were balanced by combination with PARAMO stock (which had also relatively high expected return).[45] Nevertheless, this solution appeared to have "depleting" effect on portfolio again. By this move, Agent B lost another 25000 CZK (i.e. circa 2.5% of initial value), even though VaR_{95} was estimated to be 5865 CZK (i.e. agent will not lose more than 5865 in 95% of cases). Again, Agent A's portfolio remained on the same level, since he did not invest in JČ Papírny stocks in such extensive manner.

Described steps of the both agents made circa 83 000 CZK gap between the two of them. In the next round (fourth), both strategies were profiting, following global trend of PSE. Agent B's portfolio continued in appreciating trend till the end of the first period. Even though Agent B's portfolio performed better than PX index (from 5th round), it ended up in red numbers.

Agent A was above the PSE average for the whole observed period, but his portfolio started to depreciate at the beginning of 5th period (in contrast to Agent B) and followed this trend till the end of period.

As the results show, both strategies had proved that it is essential to optimize the portfolio in order to obtain better results than market average. On the other hand, our hypothesis did not hold, since Mean-Variance strategy (Agent A) performed better than Mean-VaR strategy (Agent B) in period characterized by higher relative volatility. Here it is important to remind that relative failure of Mean-VaR strategy was caused mainly by algorithm of data generating rather than theory itself. Since that, we believe that this attribute will turn out to be advantage in next period, which is characterized by extreme (short term) volatility and depreciating global stock exchange trend.

[45] Mean-VaR portfolio selection method ,in practice, chooses such portfolio that in every of 20 scenarios (days), expected losses of some stocks are balanced by expected returns of another ones. Additionally, expected losses are minimized as much as possible.

Second Period – results of the second period indicate (as we were expecting) that Agent B's strategy coped with the turbulent environment in considerably better manner. Figuer 3 is depicting performance of both strategies throughout period 2.

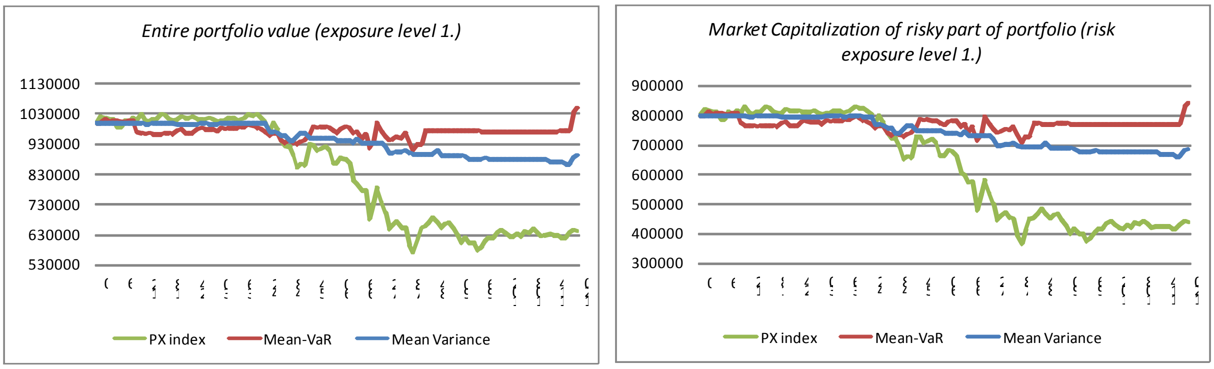

Figure 3: Portfolio returns – 2nd period

Source: Author's computations

Generally speaking, in the rounds with the biggest depreciation of PX index, when the expected returns of all of the stocks were low (several also negative), both agents focused on the investments into the stocks with the lowest volatilities (KOBOS stocks, most of the time). Again, both agents were relatively profitable, Agent B was profitable even in absolute terms (almost 5% profit). These profits are even more remarkable, when one takes into account the fact that PX benchmark portfolio lost almost 36% (risky part 45%) of its initial value. This implies that both strategies were sufficiently safe even in the turbulent market environment. Table 5 is offering an overview of the second period.

Level of exposure	Value of	Agent A	Agent B	PX index portfolio
1 (λ=0.2)	***risk-free part***	*204290*	*204290*	*204290*
	risky part	*686808.6*	*842467.2*	*436194.7*
	entire portfolio	*891098.6*	*1046757.2*	*640484.7*
	portfolio return	*-10.89%*	*4.67%*	*-35.95%*
2 (λ=0)	***risk-free part***	*0*	*0*	*0*
	risky part	*858510.7*	*1053084*	*545243*
	entire portfolio	*858510.7*	*1053084*	*545243*
	portfolio return	*-14.15%*	*5.31%*	*-45.48%*
3 (λ=-0.2)	***risk-free part***	*-204290*	*-204290*	*-204290*
	risky part	*1030212.9*	*1263700.8*	*654292*
	entire portfolio	*825922.9*	*1059410.8*	*450002*
	portfolio return	*-17.41%*	*5.94%*	*-55%*

Table 5: Simulation results – 2nd period

Source: Author's computations

Let us focus on the differences in optimum portfolios of both agents, which led to difference in their final profits. Similarly to the first period, development of both portfolios was in line with PX index until PX index started to depreciate in value. Due to lack of stocks with non-negative expected returns or returns higher than risk-free asset return, Agent A focused mainly on KOBOS stocks (Paramo, Pražská energetika, Toma). During the whole second period, he was constructing portfolio from mentioned stocks, by changing the relative weights. This does not imply insufficiency of options for including SPAD stocks into the optimum portfolio, due to their non-appreciating trend during observed rounds. Rather the appreciations were only short termed (1–2 rounds) and they did not significantly influence the distribution of returns of particular stocks, implying that Agent A did not react to these changes.

In this sense, Mean-VaR framework (Agent B) reacted more appropriately to the stock price changes, which subsequently led to the higher profit. On one hand, Agent B formed some of his portfolios mainly out of KOBOS stocks for their low price variability (round 5 and 6). On the other hand, he managed to build also mixed (SPAD-KOBOS) portfolios (round 1, 2, 3 and 4), because of seemingly appreciating trend of SPAD stocks (even short term). This diversification appeared to work for the Agent B specifically in round three, which was breaking point for each of three portfolios. Now let us describe round 3 in more detail. As mentioned before, until round 3 (and some days within) each of three portfolios had roughly the same value. But during this period, PX index started to tumble and dropped rapidly in the next two periods. For the reasons mentioned before, also arising from the theoretical background, Agent A and B constructed completely different portfolios. Figure 4 depicts portfolios of both agents in round 3.

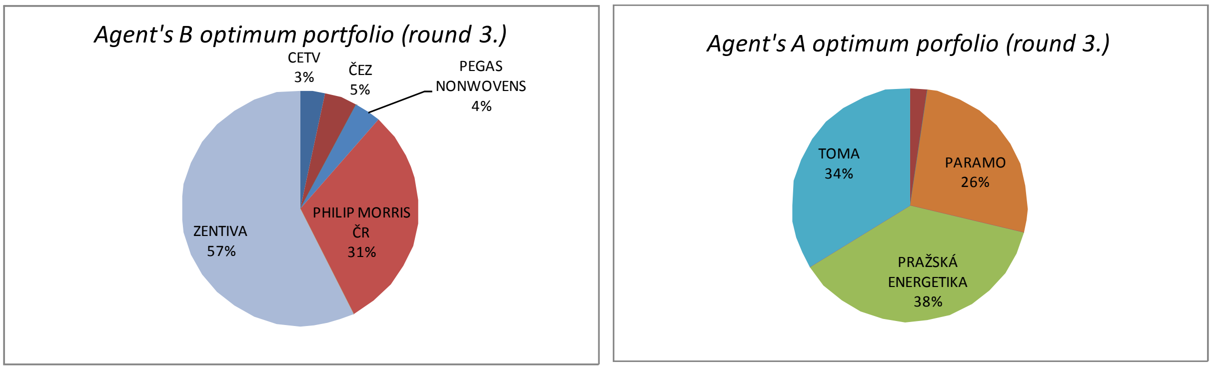

Figure 4: Portfolio structure in 3rd round

Source: Author's computations

Different portfolio composition secured that Agent B lost around 14 000 CZK, which was less than loss of Agent A (circa 57 000 CZK). It is important to point out, that PX benchmark portfolio lost circa 159 000 CZK, which proves the trend of PSE of that period.

The question may arise, why more than half of Agent B's optimum portfolio is composed mainly from Zentiva stocks, even if their expected return arising from historical simulation is -1.21% per day and there are stocks with higher average return. The answer is straightforward. Since Agent B is using Mean-VaR framework (obtaining entry data from historical simulations) and he is not considering return distribution of each stock (based on mean and variance) separately, rather he is thinking of all possible future periods[46] ,deciding on past experiences and choosing a portfolio which would perform best in all of them.[47]

From this point both agents generated roughly the same profits/losses, while PX index (consequently PX benchmark portfolio) continued in depreciating trend. In the next rounds, Agent B constructed portfolios mainly from KOBOS stocks, also with lack of diversification (caused by few stocks and scenarios expecting higher profit from risky securities than risk-free asset).

This part of empirical testing confirmed our hypothesis regarding the superiority of Mean-VaR (Mean-CVaR) strategy in particular environment. As mentioned before, we believe that more remarkable results can be obtained, by making a use more scenarios, more sophisticated method or combination of the two, even in such volatile periods.

5. Conclusion

The aim of this paper was to test the profitability of two investment strategies in turbulent (crisis) times by comparing their profitability, basing on two different approaches for optimum portfolio selection. Mean-Variance and Mean-VaR portfolio selection methods were compared with a use of Czech market stocks in period from 18.12.2006 to 30.12.2008. Both methods were choosing from

[46] Next 20 days in our simulations.

[47] For the reasons explained in this paragraph, none of the stocks chosen by Agent A could be chosen by agent B, since their expected returns were 0 (for all days and each of three stocks).

SPAD, KOBOS and riskless stocks in order to compile optimum portfolio. In consequence, we were measuring their relative profitability (i.e. in comparison to PSE performance) and absolute profitability (i.e. profit/loss in absolute terms).

To our surprise, both strategies were relatively profitable in both simulation periods, i.e. first period characterized mainly by higher volatility of stock exchange and second period characterized by extreme market volatility accompanied by plunging trend of PSE.

Arising from our tests of normality of returns, we supposed Mean-VaR strategy to perform better in both periods (i.e. moderate volatility as well as the market turmoil). Different was true for the first period, when Mean-Variance profited (almost 3%) and Mean-VaR strategy was only relatively profitable. Nevertheless, in the second period Mean-VaR strategy performed better with almost 5% profit in comparison to circa 11% loss of Mean-Variance (36% loss of PX benchmark portfolio[48]).

As a consequence of our results, it seems that it is worth to adhering investment decisions to outputs of optimisation algorithms of both methods. Moreover, we consider Mean-VaR strategy to be safer in turbulent times.

However, there is still room for further research. Firstly, paper was based on simple scenario generation procedure which could be substituted for more sophisticated methods as Monte Carlo simulations, etc. Furthermore, the same simulation framework can be done with a use of multi-period optimisation routine (i.e. with a help of Markov processes), instead of single period optimisation in several steps. Even abovementioned limitations are present, the main value added of this paper is based on the fact that two single period optimum portfolio selection methods were transformed into strategies and further tested in crisis environment, which differs from other research papers in volatility, stock exchange history and variety of stocks.

[48] For λ=0,2

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Appendices

Appendix A

VBA code for efficient frontier:

```
Private Sub CommandButton1_Click()
Dim i As Long
For i = 3 To 60
SolverReset
SolverOk SetCell:=Cells(i, 95).Address, MaxMinVal:=2, ValueOf:="0", ByChange:=Range("$CZ$" & i, "$DL$" & i)
SolverAdd CellRef:=Cells(i, 117).Address, Relation:=2, FormulaText:="1"
SolverAdd CellRef:=Range("$CZ$" & i, "$DL$" & i), Relation:=3, FormulaText:="0"
SolverOk SetCell:=Cells(i, 95).Address, MaxMinVal:=2, ValueOf:="0", ByChange:=Range("$CZ$" & i, "$DL$" & i)
SolverSolve UserFinish:=True
SolverFinish KeepFinal = "1"
Next i
End Sub
```

VBA code for Variance/Covariance matrix:

```
Function VarCovar(rng As Range) As Variant
Dim i As Integer
Dim j As Integer
Dim numCols As Integer
numCols = rng.Columns.Count
numRows = rng.Rows.Count
Dim matrix() As Double
ReDim matrix(numCols - 1, numCols - 1)
For i = 1 To numCols
For j = 1 To numCols
matrix(i - 1, j - 1) = Application.WorksheetFunction.Covar(rng.Columns(i), rng.Columns(j)) * numRows / (numRows - 1)
Next j
Next i
VarCovar = matrix
End Function
```

Appendix B

For optimization in Matlab we used Linprog function, which solves linear programing problem in a form:

$$\min_x f^T x$$

such that:

$$\mathbf{A} \cdot x \leq b$$

$$\mathbf{Aeq} \cdot x = beq$$

$$lb \leq x \leq ub$$

f,x,b,beq,lb and *ub* are vectors, and **A** and **Aeq** are matrices.

Code:

```
f=[c]
A=[r1;r2;r3;r4;r5;r6;r7;r8;r9;r10;r11;r12;r13;r14;r15;r16;r17;r18;r19;r20]
Aeq=[jednotky;Er]
b=[zeros(14,1);-1000000;zeros(20,1)]
Beq=[0;1]
b=[zeros(20,1)]
lb=[zeros(14,1);-1000000;zeros(20,1)]
[x,fval,exitflaq,output]=linprog(f,A,b,Aeq,Beq,lb)
```

where *c* is vector in form:

$c = (x_1,....,x_{13}, t, \delta, u_1,....,u_{20})^T$

r1,....,r20 are vectors of 20 historical days, where every one of them is composed from returns of each stock, i.e. :

$ri = (r_{i1},....,r_{i13})^T, \forall i \in \{1,...,20\}$

Appendix C

Development of PX50 Index in 2006-2008

Arising from the Figure 5, PX index fell at circa 50% (in the end of 2008) of its value from the end of 2006. Performance of the PX index copies the development of the Czech economy. It has rather horizontal but volatile trend in 2007 (starting to drop in the last months) and declining, even more volatile trend in 2008.

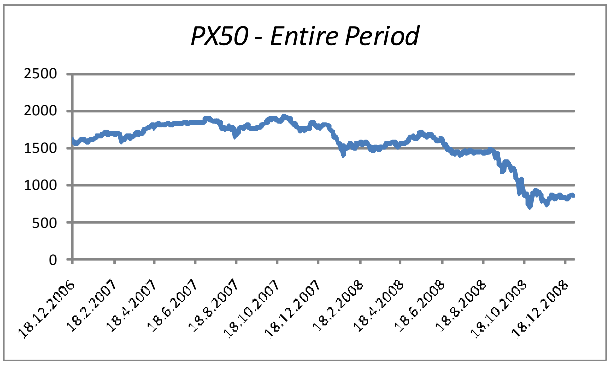

Figure 5: PX50 – Entire period

Source: www.pse.cz, author's computations

First Period (10.7.2007 – 28.12.2007)

We chose this period to represent an environment with moderate market volatility (slightly higher than in non-crisis times), indicating possible future volatile times. As a sign of unstable times could serve the fact that the volatility of returns (measured in variance terms) of PX50 had been higher in only seven cases (only once from October 2002) in almost entire history of PX index.[49] To avoid a criticism of unambiguousness of measurement (i.e. volatility in the long term tends to be lower than in short term), we divided the period from 1.1.1997 to 9.7.2007 into 21 blocks of data strings of daily returns. Each block consisted of 120 consecutive daily returns (6 months – same length as testing period) of PX50. Particular volatilities of returns are depicted by Figure 6.

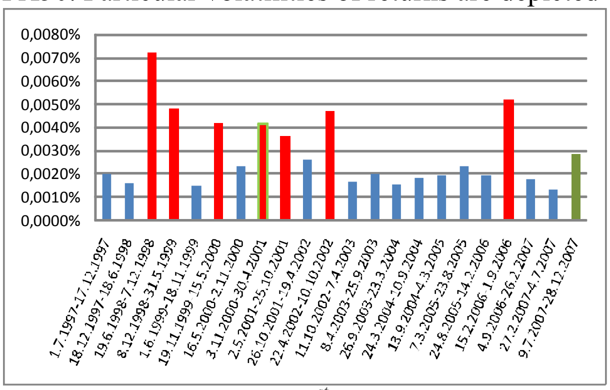

Figure 6: Returns volatility – 1st period

Source: www.pse.cz, author's calculations

Red columns are those periods where variance exceeded the reference variance. Reference variance (green) is variance of PX50 index in first period of our case study (i.e. from 10.7.2007 to 28.12.2007). Performance of PX50 index in this period is depicted in Figure 7.

[49] We were able to download historical data string of PX50 index, beginning at 1.1.1997.

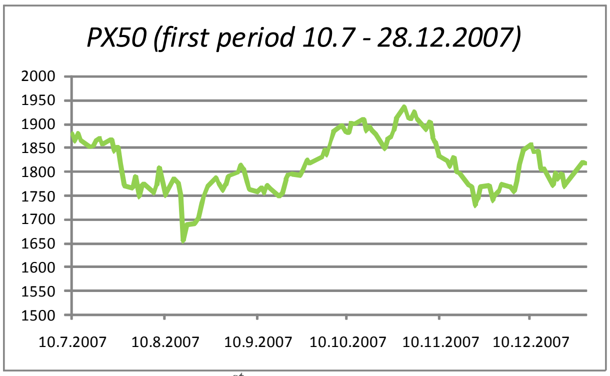

Figure 7: PX index – 1st period
Source: www.pse.cz, author's calculations

Second Period (9.7.2008 – 30.12.2008)

This period was chosen as a representative of crisis environment. It is characterized by enormous volatility and declining stock market trend. During this period, PX50 fell to 58% (858.2) of its initial value (1455.2 – begging of the period). There are two clear reasons why we are able to claim that this period is a good example of crisis environment:

- It is resulting from the macroeconomic background
- There had not been more volatile period on PSE yet.[50]

To justify the second reason, we add another two blocks of data strings[51] to that we already used in first period. Then, we calculated variances of these PX returns for year 2008 and put them in to similar chart to first section. Figure 8 captures the results. Dark green column is variance of the reference period (period 2.) and bright green column depicts variance PX50 in the first period.

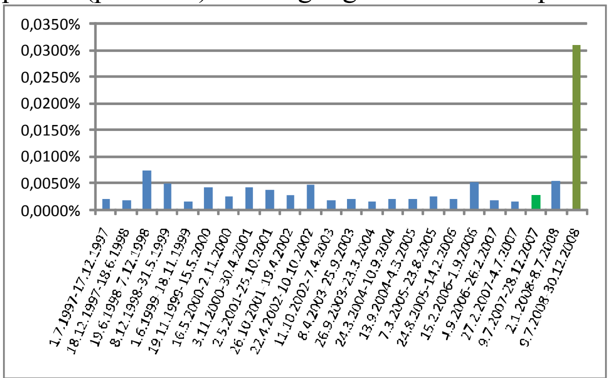

Figure 8: Returns volatility – 2nd period
Source: www.pse.cz, author's calculations

Finally, chart containing performance of PX50 index in period 2.

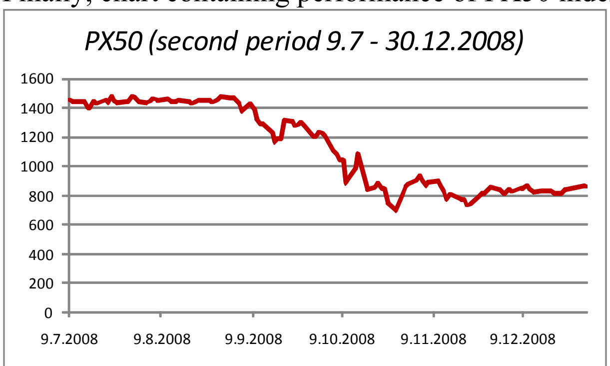

Figure 9: PX Index – 2nd period
Source: www.pse.cz, author's calculations

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[50] From 1.1.1997 to 31.12.2008.
[51] From 1.1.2008 to 8.7.2008 and referential period from 9.7.2008 to 31.12.2008.

Appendix D

Risky portfolio weights of risky securities in particular periods:

period	Security	1.						2.					
round		1.	2.	3.	4.	5.	6.	1.	2.	3.	4.	5.	6.
Mean-Variance	CETV	13.2%	0.9%	1.8%	2.9%	10.4%	1.5%	0%	0%	0%	0%	0%	0%
	ČEZ	0%	0%	0%	0%	0%	3.1%	5.6%	2.3%	2.4%	0%	0%	0%
	ECM	9.2%	5.6%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	JČ PAP.	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	ORCO	0%	0%	0.%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	PARAMO	20.9%	10.6%	17.9%	19.1%	19.1%	20.6%	25.2%	26.3%	26.3%	28.8%	13.2%	13.2%
	PEGAS	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	P. MORRIS	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	P. ENERG.	22.6%	38.9%	39.1%	56.3%	42.5%	30.4%	36.2%	37.5%	37.4%	34.5%	39.5%	39.5%
	O2	26.4%	34.9%	37.8%	5%	6.8%	6.3%	0%	0%	0%	0%	0%	0%
	TOMA	2.97%	0%	0%	0%	8.3%	25.1%	33%	33.9%	33.8%	36.7%	47.4%	47.4%
	UNIPETROL	4.73%	8.9%	2.3%	16.5%	12.9%	13%	0%	0%	0%	0%	0%	0%
	ZENTIVA	0.%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Mean-VaR	CETV	3.3%	0%	0%	0%	0.43%	0%	0%	0%	3.33%	0%	0.%	0%
	ČEZ	9.9%	29.5%	0%	0%	3.7%	47.1%	0%	0%	4.55%	0%	0%	0%
	ECM	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	JČ PAP.	4%	36.8%	37.5%	16.7%	0%	0%	0%	0%	0%	19.1%	3.7%	0%
	ORCO	0%	0%	0%	7.4%	0%	0%	0%	0%	0%	0%	0%	0%
	PARAMO	15.8%	0%	62%	22.5%	14.9%	0.2%	0%	0%	0%	0%	0%	0%
	PEGAS	0.1%	0%	0%	0%	1%	0%	0%	0%	3.6%	0%	0%	0%
	P. MORRIS	18.8%	0%	0%	4.8%	0%	0.1%	0%	0%	31%	58.9%	0%	0%
	P. ENERG.	30%	1%	0.5%	33.4%	0%	0%	0.2%	0%	0%	0%	0%	0%
	O2	0%	0%	0%	0%	0%	0%	0%	100%	0%	0%	0%	0%
	TOMA	4.2%	0%	0%	0.1%	52.7%	52%	0.2%	0%	0%	0%	94.5%	100%
	UNIPETROL	5%	33.4%	0%	15.4%	27.2%	0.6%	0%	0%	0%	0%	0.%	0%
	ZENTIVA	8.6%	0%	0%	0.04%	0%	0%	99.6%	0%	57.4%	21.9%	1.7%	0%

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Univerzita Karlova v Praze, Fakulta sociálních věd
Institut ekonomických studií [UK FSV – IES] Praha 1, Opletalova 26
E-mail : ies@fsv.cuni.cz http://ies.fsv.cuni.cz