Comparison of Capital Adequacy Requirements to Market Risks According Internal Models and Standardized Method

Dissertation

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Proclamation:
I declare that I completed Dissertation “Comparison of Capital Adequacy Requirements to Market Risks According Internal Models and Standardized Method” myself and that I used only literature listed in the chapter References.

Prague, 22 May 2005
I would like to thank Mgr. Petr Franěk, Ph.D. for his help in the area of internal models performance assessment.
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1 List of abbreviations

The following abbreviations are used:

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<tr>
<th>Abbreviation</th>
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<tr>
<td>ALM</td>
<td>Assets and Liabilities Management</td>
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<tr>
<td>BIS</td>
<td>Bank for International Settlements</td>
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<tr>
<td>ČNB</td>
<td>Czech central bank, Česká národní banka</td>
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<td>IR</td>
<td>Interest Rate</td>
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<td>VaR</td>
<td>Value at Risk</td>
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2 Introduction

Banks perform intermediary and payment functions that distinguish them from other businesses. Their core product is intermediation that means to intermediate liquidity between economic subjects with a liquidity surplus and economic subjects with a deficit of liquidity. Subjects with surplus liquidity make deposits and subjects with a deficit of liquidity borrow from the bank.

The role of financial intermediary causes that banks face a number of risks atypical of non-financial firms. Therefore, financial risk measurement and management is significantly more important for banks than for other companies. Hence, one of the core banks’ activities is managing the risks arising from on- and off-balance sheet assets and liabilities.

Credit risk, i.e. the risk that a borrower defaults on a bank loan, is the oldest risk banks faced because of the lending side of the intermediary function. However, as banks become more complex organisations offering fee-based financial services and relatively new financial instruments, other types of financial risk have become more important.

2.1 Risk and shareholder value added

The objective of all banks is to maximize profit and shareholder value added and risk management is central to the achievement of this goal. Shareholder value added is defined as earnings in excess of a “minimum required return” on capital. The minimum required return is risk-free rate plus a risk premium for the bank that varies depending on the riskiness of the bank’s activities. Therefore, the less risky the bank business is while having the same profit, the more shareholder value is added.
Banks measure their profitability as a return on capital, while capital is adjusted for the riskiness in order to receive reasonable profitability measure. The more risky is the bank’s business, the more capital the bank should hold in order to be able to cover possible future losses.

2.2 Types of risk in banking

Significance of various risks the banks face differs for each bank category (i.e. it differs for investment bank and retail bank). However, the majority of risks are common to all banks. These are:

- Credit risk
- Liquidity risk
- Solvency risk
- Operational risk
- Market risk
- Interest rate risk

2.2.1 Credit risk

Credit risk is the risk that borrowers do not repay their loan on time or that they default on repayment. In both cases present value of the bank’s assets decreases. For the majority of banks, credit risk is the most significant since losses incurred as a result of credit risk are the highest.

Counterparty risk is another name for credit risk; it is used for the risk that counterparty defaults on the terms of a contract on the financial markets.

2.2.2 Liquidity risk

Liquidity risk is the risk that the bank will not be able to meet its obligations when they come due without unacceptable losses. Liquidity risk includes the inability to manage planned or unplanned decrease in funding sources.
If the bank does not manage properly its liquidity risk then at one point might happen that it is not able to fulfil the demand of its depositors for the withdrawal of cash. In this case the bank has to borrow the liquidity for inadequately high interest rates and its costs increase which at the end causes decrease of the bank’s profit.

2.2.3 Solvency risk

Solvency is the capacity to meet external liabilities in full by realising assets at current values. It can be expressed by a value calculation, as the opposite to liquidity which is a cash flow phenomenon (see Cade, 1997). How will inadequate liquidity transform into inadequate solvency? In the case of excessive demand for liquidity from its depositors, a bank is forced to borrow for inadequately high interest rate and to sell its assets which both leads to diminishing of the bank’s value (by increasing the costs of deposits and decreasing the value of assets).

2.2.4 Operational risk

Operational risk means the risk that the bank will experience loss as a result of operational incidents. These can be for example thefts, system errors, etc. BIS defines operational risk as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This definition includes legal risk, i.e. the risk of exposures from fines, penalties, or punitive damages resulting from supervisory actions or private settlements. (BIS, International Convergence of Capital Measurement and Capital standards, 2004).

Operational risk has recently moved to the centre of banks’ and regulators’ attention and new quantitative methods on the basis of statistical analysis are used to measure it. New capital adequacy regulations require banks to calculate operational risk and allocate
necessary capital to it. Therefore, capital adequacy is now beginning to be calculated also with respect to operational risk. However, lack of necessary input data make bankers consider reliability and usefulness of statistical models for operational risk calculation. Qualitative risk assessment might prove as useful as quantitative methods.

2.2.5 Interest rate risk

Interest rate risk is the risk that a bank’s profit will be reduced as a result of interest rate movements. Interest rate risk is further elaborated in chapter 2.4 Interest rate risk.

2.2.6 Market risk

Market risks can affect bank’s profit and loss as a result of changes in market prices. Market risk is relevant to banking book and trading book but its measurement and management might differ in each book. The methods used for the calculation of market risk are well described and generally accepted. The most sophisticated methods for any non-credit risk measurement have been developed for market risk. Market risks are:

- Exchange rate risk
- Equity price risk
- Commodity price risk

As banks started to undertake more non-traditional business and as the trading book become larger and more important, the necessity for precise and complex market risk measurement emerged. This was an impulse for banks to begin with statistical models development.
Risk classifications differ according individual authors and we could name several other banking risks; however the aim of this chapter is to give a basic overview.

2.3 Difference between banking book and trading book

Management, measurement, reporting and in the last couple of years the capital requirement related to various financial risks differ for each part of the bank: for banking book and for trading book.

2.3.1 Banking book

Banking book includes products that are held to maturity and as a long term investments. The major part of these products is comprised from deposits and loans to retail or corporate clients. However, banks can invest in other instruments as well and these can be shares, commodities, etc. Products held in banking book usually do not have efficient secondary market and therefore value calculation for these products usually cannot be based on publicly quoted prices.

Major risk that bank experience from banking book is credit risk, i.e. the risk that borrower will not be able to repay its obligations. Other risks of banking book include interest rate risk, currency risk, equity price risk and commodity risk (in the case equities and commodities are held as a long-term investments).

2.3.2 Trading book

Instruments categorized into trading book are usually bought for speculation. For example if securities are held in trading portfolio, these are not intended to be held until maturity and will be very likely sold before.

Major risks that occur in trading book are: credit risk – referred to as a specific or counterparty risk for the purposes of capital adequacy
calculation, interest rate risk, currency risk, commodity risk and equity price risk.

Capital requirements are - according international and Czech banking regulation (BIS 1999 and ČNB, 2002/Vyhláška 333) – calculated for individual risks and bank portfolios as follows:

- **Credit risk capital charge** - for banking book and trading book (specific risk)
- **Exchange rate risk capital charge** - for banking book and trading book
- **Equity price risk capital charge** - for trading book
- **Interest rate risk capital charge** - for trading book
- **Commodity risk capital charge** - for banking book and trading book

Two types of risks are excluded from capital charges on banking book: equity price risk and interest rate risk.

Exclusion of equity price capital charge in banking book can be supported by an assumption that banks hold all securities (i.e. bonds and equities) on this book as a long term investments and until maturity. Therefore, changes in market value of these instruments are not important as far as all other banking assets and liabilities are not valued in present value terms.

However, exclusion of capital charge to interest rate risk in banking book has more complex explanation and I will further discuss it in chapter 5.5 Capital adequacy to interest rate risk of banking book.

### 2.4 Interest rate risk

Interest rate risk affects bank portfolios in two different ways. First, it influences net interest income that is large source of bank’s profit. This risk can be called Earning risk and it stands for the risk that as a
result of repricing schedule of bank’s asset and liabilities the bank will experience decrease in net interest income or it will, in the worst case, have negative interest income.

This can be illustrated on the following example. Lets assume that a bank holds only one deposit maturing in one year with interest rate of X% (where X = current market interest rate) and only one loan with monthly fixing of interest rate. Lets assume that the current interest rate of this loan is (X+1) %. And finally let’s assume that market interest rates will rapidly fall by 2% and then will stay the same for the whole year. After one month the loan will reprice and its interest rate will be (X-1) % (where X = original market interest rate before interest rate fall) but interest rate for the deposit will be fixed on the level of X% for the whole year. Therefore for the rest of the year the bank will realize negative net interest income on the level of –1%.

If the bank’s repricing mismatch is set up so that it correctly anticipates future interest rate movements the bank can realize profit if such anticipated interest rate happens. In order to be able to speculate on this part of risk it is necessary for the bank to accurately forecast future interest rates and to modify repricing characteristics of its portfolio so that the future interest rate changes will lead to increase in net interest income. Neither interest rate forecast nor the portfolio modifications are the main objectives of this thesis.

Second, interest rate risk influences the present value of bank’s asset and liabilities. The present value represents value of future cash flows and is the major factor in assessing the bank’s value (Principles for the management of interest rate risk, September 1997). This type of interest rate risk can be called Economic Value Risk.
Both ways of interest risk influence are present in both parts of the bank: in banking book as well as in trading book but the importance of both types of influences on the bank’s value differ between the two portfolios. This is due to the different intension for holding banking book products and trading book products; banking book products are usually held by the bank until maturity and their present value is only informative because their resale is very difficult and nearly impossible. On the other hand all products held in trading book are meant to be instruments that can be sold in short term and their present values are very important indicator for determining their market value. Together with different accent on the two ways market risk influences Banking book and Trading book methods used for interest risk measurement and management differ.

2.5 Types of interest rates movements

Interest rates might move in several different ways that cause different interest rate risks. All the movements are related to a yield curve.

Yield curve is given by a set of points corresponding to standardly quoted maturities. It usually starts with overnight interest rate and goes to very long maturities such as several years. Usually, yield curve has positive slope (i.e. long term interest rates are higher then short term interest rates) and a concave shape.

2.5.1 Parallel movement of yield curve

Parallel movement of a yield curve means such movement that will affect all the yield curve points with the same intensity, e.g. a 1% fall or rise in all rates. In such a case, yield curve will have the same shape after the adjustment as it had before but all its interest rates points will be higher or lower then originally.
2.5.2 **Yield curve (twist) risk**

Above described uniform change of interest rates is unlikely. In a real life individual interest rates do not change by absolutely the same amount and each point of yield curve is adjusted to the new market perspective by different percentage. In such a case the slope of a yield curve changes. Sometimes such a change in the shape of yield curve might lead even to reverting its slope. Yield curve twist usually worsens the effects of parallel movement, i.e. interest income is affected usually more when on the top of parallel interest yield curve movements its twist occurs.

2.5.3 **Basis risk**

This risk arises when yield curves for different products or instruments do not move in parallel, i.e. that relationships between rates of individual products and their differentials may change. Correlation between interest rates of different products is not necessarily the same during the life of these products. If a bank uses different products for hedging from the products hedged then even portfolio hedged so that its assets and liabilities reprice or mature in the same time might experience loss due to the imperfect correlation of interest rates on liability side with interest rate on asset, i.e. due to the basis risk.

2.5.4 **Option risk**

Another risk that has begun to be more relevant is option risk. Major characteristic of option is that it gives the holder right, but not the obligation, to buy or sell financial instrument or to alter the cash flow of financial instrument. Originally the word option was used only for stand alone financial instruments that could be either standardized and traded on the official markets or they could be over-the-counter contracts. Moreover, options can be embedded in other financial
instruments. These embedded options are more important for non-trading activities of banks, i.e. in banking book. They include various products such as bonds with call or put provisions, loans that give borrowers right to prepay balances and various types of non-maturity deposit instruments which give depositors right to withdraw funds at any time. These instruments with optionality features can pose significant risk to banks, since the option held are usually exercised to the advantage of holder and disadvantage of the seller, which is in this case always bank.
3 Basic concept of interest rate risk measurement

Interest rate risk can be measured by several methods. These methods differ by the period at which they were used and by detailed purpose. Basic concepts of interest rate risk measurement are:

- GAP analysis,
- Duration,
- Value at Risk (VaR).

GAP analysis is the simplest and oldest method and VaR and simulations are the most up-to-date methods. Moreover, different methods are currently used for slightly different purpose. As was already mentioned, banking and trading portfolios differ substantially and so do the requirements on risk measurement. Trading portfolio is meant to incorporate instruments that might be resold or re-acquired in short term and the most important measure is then the portfolio’s present value. For this concept only VaR is suitable method because it incorporates present value as the key aspect and because it works with short time periods.

3.1 Original methods of risk measurement

3.1.1 GAP analysis

Influence of interest rate risk on portfolio of instruments can be seen from original methods used for interest rate (IR) measurement. First of these methods was GAP analysis.

3.1.1.1 GAP calculation

In this approach, assets and liabilities are sorted according their re-pricing dates into several time buckets. Re-pricing date equals to maturity date for fix rate instruments and it equals to the first day when product’s interest rate will be re-set for floating rate instruments.
In each of these buckets the difference between asset re-pricing volume and liabilities re-pricing volume is calculated and this difference is called interest rate gap. It expresses the volume of asset or liabilities that are un-hedged and that might cause decrease in interest income in the case interest rates move.

Moreover, usually cumulative interest rate GAP is calculated as well. It expresses overall sensitivity on interest rates with neglecting where exactly interest mismatch appears. Example of GAP analysis can be seen in the table below.

Positive GAP expresses the situation where volume of assets re-pricing exceeds volume of liabilities re-pricing, negative GAP express the situation where volume of liabilities re-pricing exceeds volume of assets re-pricing. In the case volume of assets re-pricing equal the volume of liabilities re-pricing the bank's portfolio is in equilibrium in given time bucket.

Positive GAP denotes asset-sensitive situation in which majority of assets will re-price before liabilities when interest rates change. Net interest income should therefore show an early increase if interest rates rise and decrease when interest rates fall. The larger is the GAP; the grater is interest rate risk.

3.1.1.2 Use of GAP analysis in risk management

GAP analysis must be always done separately for each currency the bank is holding portfolio in. After the analysis completion, risk management can use GAP method to set limits on interest rate GAP in each bucket or limits on cumulative GAP in each bucket. Moreover, GAP analysis can be used as a basis for hedging the portfolio by
constructing counterbalancing transactions. Counterbalancing transactions can be constructed by:

a) Duration matching of assets and liabilities, i.e. acquiring the same amount of assets and the same amount of liabilities with the same duration,

b) Swapping interest payments in accordance with balance sheet characteristics,

c) Using interest rate futures or options (compare Sinkey, 1996, p. 482).

Duration matching is relatively the least expensive way of hedging interest rate risk but it can be done only gradually by switching the bank’s portfolio characteristics. Bank can either change its funding strategy, e.g. lengthen or shorten the term of new deposits for the retail or wholesale clients, or bank can change re-pricing characteristics on the lending side, e.g. put more or less emphasis on fixed versus floating rate loans, or a bank can switch out some types of investment into others (compare Cade, 1997, p. 153). Since all such changes depend on client’s products characteristic changes and since these must be supported by change in bank’s offering policy, at the end all such changes take longer then is acceptable time period for efficient interest rate risk management.

Swapping interest payments as well as using interest rate futures and options is more expensive but using this hedging (at least on developed markets) is relatively easy thanks to wide offer of variety of products. Trades can be completed in short time and hedging might be in place soon after the interest risk profile in given period is known.
3.1.1.3 Problems with GAP analysis

Time buckets used in GAP analysis must be set up so that they are relatively narrow to capture majority of mismatches but on the other hand time buckets must be set relatively wide to enable measurement as such. Final buckets setting is then compromise between precision and manageability.

After appropriate time buckets setting there will be mismatches remaining in each bucket and these will not be only un-hedged but these will be un-noticed (Compare Heffernan, 1996). Coming back to the example above lets say that there is one loan re-pricing in one month and only one deposit re-pricing in three months. According GAP analysis only 50 CZK is not hedged but if interest rates rise then first re-priced product will be the loan with notional 50 CZK and only after two months deposit in the notional of 100 CZK will be re-priced leaving the bank exposed to interest rate risk on the 50 CZK loan during two months.

Large limitation of GAP analysis is its basic assumption that all interest rates will move together in the same direction by the same amount. It works with an assumption that only parallel yield curve movements appear and it has nothing to say on yield curve risk or basis risk.

Another problem with GAP analysis is to properly position some of bank’s products into given time buckets. Between those are typically current accounts and overdrafts that have contractual maturity different from their real maturity, bank capital and other products. Selection of right time buckets requires setting many assumptions based on internal bank’s research.
3.1.2 Duration analysis

Second method used for determining interest rate risk is duration analysis.

Duration provides very essential calculation for interest rate elasticity; it gives the information about how the present value of a financial instrument or portfolio changes if interest rates change for a given percentage (e.g. for 1%). As such it is first order derivative of the function describing relationship between interest rates level and portfolio value.

Duration measures the impact on shareholders’ equity if a risk-free rate for all maturities rises or falls. The longer the duration (absolute value), the greater the interest rate risk.

It can be said that duration represents the measure of average life of an asset or liability which can differ from its maturity. This method aggregates the present value of all future cash flows (both principal and interest) within a portfolio for given currency, then weights them by their respective periods to maturity. The total of weighted values divided by present values gives a single number representing the duration of the portfolio. It is normally expressed in years. It is weighted average life of the individual cash flows in the portfolio, where the weighting factors are the present values of those cash flows.

3.1.2.1 Advantages and disadvantages of duration analysis

Advantage of GAP analysis is that it gives one single number that is easy to understand and that expresses overall portfolio sensitivity to interest rate risk. As such it provides relatively good basis for hedging; portfolio can be hedged by opposite position in a single instrument with the same duration.
The main weakness of duration analysis is its simplicity. The single number can underestimate mismatches within narrower periods that offset one another in the aggregate. Moreover, duration as well as GAP analysis does not address the yield curve risk or basis risk assuming that all the interest rates move together and by the same amount. Further, preparation of duration analysis is in the case of large portfolios very demanding on the volume of data input.

3.1.3 Convexity

When calculating duration of portfolio one must bear in mind that duration could precisely estimate interest rate risk only in the case of linear relationship between interest rate changes and change in asset value. Since duration is first order derivative it can be used as the only measure only in the case of linear relationship between interest rates and portfolio value (which is very rare) or only for prediction of portfolio value in the case of very small interest rate changes. If neither of these holds then convexity of portfolio must be taken into account. Convexity is second order derivative of the function describing relationship between interest rates level and portfolio value.

The greater the convexity of the interest rate and asset value relationship the less useful is the simple duration measure. Hence, the use of duration to predict interest rate sensitivity should be either limited to small changes of the interest rate or convexity of relationship must be taken into account.

3.1.4 Duration GAP analysis

This form of analysis puts together both GAP and duration analysis. All the items of bank portfolio that are interest rate sensitive are placed in time bands based on the date of maturity or repricing of the
instrument. The position in each time band is netted and the net position is weighted by an estimate of its duration. Duration of the net position can be measured as weighted average of durations of instruments in given time band or it can be calculated for each time band according cash flows of each instrument. As mentioned before, duration measures the price sensitivity of interest rate instruments with different maturities to changes in interest rates.

Result of duration GAP analysis can be then used the same way as duration, i.e. as a simple measure of interest rate risk for bank management information. Moreover, mismatches in each time band can be used as a basis for portfolio immunization. Alternative way how to complete duration GAP analysis is to calculate duration of all products in portfolio first and then place them into time bands according their duration and to their maturity. Resulting positions in each time band can be then summarized to get single and simple interest rate measures or it can be used as the source for immunization of interest rate risk in each time band.

3.2 Value at Risk models

The most recent method for measuring interest rate risk was developed as a part of broader market risk measurement framework.  

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1 Portfolio is immunized against given risk when it is perfectly hedged against it. No market event then can cause loss to such a portfolio.
The method was named Value at Risk and its aim is to express total market risk as a single number, i.e. to summarize the expected maximum loss over a target horizon within a given confidence interval. VaR framework was developed by investment bank J. P. Morgan and it spread into large global banks so that before 1990 the majority of these banks used it for determining their market risk. However, in the same period VaR was not broadly used by smaller and local foreign banks. More extensive use of VaR models in Czech banks started only at the end of the ninetieth of the last century. Banks use “pure” VaR models to calculate interest rate risk only for trading book; similar methods for calculating interest rate risk in banking book will be discussed in chapter 3.5 Measurement of interest rate risk in banking book.

3.2.1 Value at risk calculation

Value at risk or expected loss is calculated from volatilities of returns on individual market risk, present value of individual cash flows and correlations between market risks factors. VaR can have several forms and these differ by means of calculating volatilities of market risk factors.

There are three steps in calculating VaR:

• Value estimate for individual instruments,
• Risk factor identification,
• Risk estimate.

3.2.1.1 Value estimate

First of all, the value of instruments of given portfolio must be calculated. Market value of instruments is taken as a basis for simple products like publicly traded shares with known and reliable market value. This activity can be more complex if there is no secondary market for given product and their market value must be estimated.
Second, products having more than one cash flow and derivatives must be decomposed into individual cash flows and each of these cash flows must be valued as if it was a stand-alone product, i.e. each of these cash flows should be discounted by appropriate interest rate.

Third, all product values should be converted into reporting base currency by spot exchange rate.

Another complication comes with products that do not have linear pay-off, i.e. with options and option-like products. These must be valued by option pricing models, see chapter 3.2.6 Option risk in VaR.

3.2.1.2 Risk factors identification

Next step is to link all products with underlying risk factors. An instrument can be exposed to more than one risk factor. For example Czech based bank investing into US government bonds is exposed to the interest rate risk of US government bond and to the CZK/USD exchange rate risk.

As I mentioned above, VaR is used for calculating interest rate risk of trading portfolio. And since instruments in trading book are held with the intention to speculate on the price movements and since these instruments are not held to maturity, present value of these instruments is the key indicator of portfolio value. Interest rate changes affect present value; therefore interest rate risk must be calculated for all instruments with future cash flows, of which present value will be affected by interest rate changes.

For example futures on US equities held by a Czech bank bear equity price risk - since their value can change if price of given share changes, they bear exchange rate risk - since the amount received in USD when futures will mature will have to be converted into CZK and finally futures on equities bear interest rate risk since cash flows from
them will be realized in some point in the future and if USD interest rates change then present value of future cash flow changes as well.

Futures on US equities in trading book of a Czech bank therefore have three underlying risk factors.

3.2.1.3 Risk estimate

Third step is to estimate risk of portfolio. Except above mentioned inputs we need to have additional information about volatilities of individual risk factors (in above mentioned example these are volatility of US equities underlying futures, volatility of CZK/USD exchange rate and volatility of relevant US interest rate for the same maturity as futures) and information about correlation between all risk factors (i.e. in the example correlation between given US share and CZK/USD exchange rate, between given US share and appropriate US interest rate and between CZK/USD exchange rate and appropriate US interest rate).

If the value of portfolio depends on multiple risk factors then the potential change in its value is a function of the combination of each risk factor volatility and each correlation between all pair of risk factors.

3.2.1.4 VaR calculation example

VaR calculation can be easily demonstrated on the following example. Czech bank is holding five year US government bond in nominal value 10 million USD. The only cash flow that will be realized during the life of a bond is the final cash flow coming exactly in 5 years. The standard deviation of daily returns on USD/CZK exchange rate is 0.78% and standard deviation of daily returns on us 5 year interest rate is 0.62. Estimated correlation between the returns on USD/CZK exchange rate and 5 year interest rate of government bond is 0.24; current USD/CZK exchange rate is 30 CZK/USD. In order to
calculate maximum expected loss with 95% probability we first calculate exposures to individual risks. Value of the position in CZK is 300 millions, and assuming normal distribution 95% probability corresponds to 1.65 of standard deviation. Exposure to exchange rate risk only can be then expressed in CZK millions as:

\[ 300 \times 1.65 \times 0.0078 = 3.86 \]

Exposure to interest rate risk only can be then expressed in CZK millions as:

\[ 300 \times 1.65 \times 0.0062 = 3.07 \]

Since the two risks are not perfectly correlated, total risk of portfolio is not just a sum of the two risks. Total portfolio risk can be calculated as:

\[ \sqrt{(3.86)^2 + (3.07)^2 + (2 \times 0.24 \times 3.86 \times 3.07)} = 5.48 \]

Total portfolio risk is then CZK 5.48 millions.

3.2.2 Differences in VaR models

VaR model is usually used for the whole portfolio of assets. Its main advantage is that it calculates with correlations between individual risk factors, one of them being interest rates. So the VaR described below is not applicable only for interest rate risk measurement but also for other market risks. However, there is no use in describing VaR only for interest rate risk, since its aspects are similar to other market risks.
VaR framework has several modifications that differ mainly in one aspect - in the way how they estimate potential market movements. There are several basic types of potential market risk estimates:

- Variance-covariance method uses estimated volatilities and correlations to calculate VaR. This method calculates VaR only for one single point of time (i.e. for one day to which we want to know value at risk) and therefore only one set of risk factor volatilities and correlations is used for the calculation. Volatilities are usually for this purpose assumed to be the same as they were in the past, i.e. this model assumes that distribution of past returns can provide reasonable forecast of future return over given horizon. This method is also sometimes referred to also as delta – normal method.

- Historical Simulation method uses historical return distributions for portfolio revaluation. Portfolio is usually valued under number of historical time windows defined by users.

- Monte Carlo Simulation method uses statistical parameters and defined stochastic processes to determine portfolio returns. Both historical and Monte Carlo simulations are computationally intensive and usually require considerable amount of time.

3.2.3 Variance-covariance method

The variance-covariance method assumes that all asset returns are normally distributed. As the portfolio is a linear combination of normal variables, portfolio returns are also normally distributed. Portfolio return can be expressed as:

\[
R_{P,T+1} = \sum_{i=1}^{N} w_i R_{i,T+1},
\]

where \( R_P \) is portfolio return, \( w_i \) is weight of product i in portfolio and \( R_i \) is return of product i.

Risk is generated by a combination of linear exposures to many factors that are assumed to be normally distributed and by the
forecast of the covariance matrix\(^2\) \(\Sigma_{t+1}\). Portfolio variance can be expressed as:

\[ V(R_{P,T+1}) = w'_T \sum_{T+1} w_T, \]

where \(V(R_{P,T+1})\) is portfolio variance and \(\sum_{T+1}\) is forecast of the covariance matrix.

This method can accommodate a large number of assets and is simple to implement.

Two methods of estimating variance-covariance matrix (\(\Sigma_{t+1}\)) can be used. First one is based solely on historical data; even though the data can be given different weights according the historical period it appeared in. This method is further used for calculating model parameters in chapter 6 Project: Capital requirement calculation for sample portfolios. Second method for estimating variance-covariance matrix is to use implied volatilities\(^3\) that can be calculated from traded option products.

Implied volatilities can be more precise then historical volatilities but unfortunately there is not usually sufficient option market for all

\(^2\) Covariance matrix has the information about variances of returns on individual risk factors and information about correlations between each pair of risk factors.

\(^3\) Implied volatility can be calculated for underlying assets with corresponding options from option prices. Option price is a function of several factors, one of which is price volatility of underlying asset. However, option price is created on public market and it can therefore indicate what underlying asset price corresponds to a given option price, all other parameters being the same. Implied volatility is market price volatility of given asset calculated from option price of that asset.
products held in the portfolio. In such a case combination of both methods can be used.

Disadvantage of variance-covariance method is mainly the inability to deal with event risk. This refers to the possibility of unusual or extreme circumstances such as stock market crashes or exchange rate collapses. The problem is that event risk does not occur frequently enough to be adequately represented by a probability distribution based on recent historical data. But this is a general shortcoming of all methods using historical series.

Second and related problem is existence of “fat tail” that appears often in the distribution of returns on financial assets. These fat tails can cause real problems in VaR estimates because VaR tries to capture the portfolio return behaviour in the left tail. If fat tails exist then model based on normal distribution underestimates the number of outliers and value at risk. This shortcoming must be overcome by additional analysis of fat tail risk together with calculation of VaR based on normal distribution.

Third disadvantage of this method is the way how it deals with non-linear positions. Without incorporating method of measuring the option risk, variance-covariance VaR is not able to precisely measure the risk of portfolio formed at least partly by options.
Methods of dealing with option price risk are explained in higher detail in chapter 3.2.6 Option risk in VaR.

3.2.4 **Historical simulation method**

Under the historical simulation current weights of individual products in portfolio are applied to historical price changes. Portfolio valuation is then calculated for number of time windows. Value at risk is then obtained from the entire distribution of hypothetical returns.

This method is relatively simple to implement if historical data have been collected for daily marking to market. This data can be later used to estimate VaR. As in variance-covariance method, the choice of the sample period reflects the trade-off between using longer and shorter sample sizes. Longer intervals increase the accuracy of estimates but could use irrelevant data, thereby missing important changes of underlying changes in returns evolution.

Another advantage of the method is that it deals directly with the choice of horizon for measuring VaR. Returns are measured simply over intervals that correspond to the length of the horizon. For example, if the task is to calculate 10 days VaR, then portfolio value

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4 Value at risk calculates only the maximum possible loss which can be found on the left hand side of histogram. Right side of histogram represents maximum profit.
changes are calculated for number of 10 days periods in the let’s say past two years.
And last but not least, by relying on actual prices, the method allows nonlinearities and non-normal distributions. The method captures gamma and Vega risk and correlations and it does not rely on assumptions about valuation models or the underlying stochastic structure of the market. It can deal with “fat tails”. All factors that make variance-covariance method less reliable, i.e. nonlinearity and “fat tails” are already incorporated in historical market price movements. Therefore if we apply historical prices and their real changes as these happened in the past then all the factors such as “fat tails” of returns distributions or changes of option delta are already included in the calculation.
Disadvantage of the model is that it uses only one path and that it assumes that the past represents the immediate future properly. Moreover, the historical simulation will miss situations with temporarily high volatility. And finally, the method puts the same weight on all observations in the window, including old data values. The measured value at risk can then be biased towards the less relevant old values and can change significantly when old data are excluded.

3.2.5 Monte Carlo simulation

Monte Carlo method is similar to the historical simulation, except that hypothetical changes in prices of assets in portfolio are created by random draws from a stochastic process and not from historical changes in prices.
The method has two phases. In the first phase the risk manager specifies a stochastic process for financial variables and other parameters. These are for example volatilities and correlations and can be derived from historical or option data. In the second phase,
price paths according the stochastic model are generated for all risk parameters and for all chosen combinations of parameters. For one or more time horizons specified the portfolio is marked to market using full valuation. Each of these scenario values are then used to compile distribution returns, from which VaR figure can be measured. When comparing this method to other methods, Monte Carlo simulation is probably the most powerful and the most complex to compute value at risk. It can incorporate wide range of risks including non-linear price risks, volatility risk (i.e. risk that volatilities are not constant over time), fat tails and extreme scenarios.

The biggest disadvantage of this model is its computational cost. If 100 sample paths are generated for portfolio of 100 assets then the total number of valuations amounts to 10,000. When there are too many assets in the portfolio and when its valuation is complex then the method becomes too difficult to implement and to use on daily basis. Moreover, it is also the most expensive to implement in terms of system and intellectual development and it is quite difficult to be developed from scratch.

Another disadvantage is that the model relies on a specific stochastic model for the underlying risk factors (i.e. it does not rely on historical risk parameters). This is reasonable so far as the model risk is being tested. To check whether the parameters derived from the parameters models can be considered as usable, simulation results should be complemented by sensitivity analysis.

3.2.6 Option risk in VaR

Basic difference between the models is their approach to valuation of options. First group of models, i.e. variance-covariance method, is based on local valuation of options; second group of models is based on full valuation. This classification reflects trade-off between relatively simple correlations handling in the delta-normal approach
and relatively more difficult complex handling of non-linear products. The delta-normal approach is much easier to implement and use than other methods; that is why it was expanded so it was able to deal with non-linear positions by incorporating simplified methodology called the Greeks.

3.2.6.1 Delta normal method

Delta normal method used for calculating VaR for options within variance-covariance method uses so called Greek letters that characterise options.

Greek letters (Greeks) describe sensitivity of option price on underlying factors. Option delta represents option price sensitivity on changes in prices of underlying asset value, option gamma represents option price sensitivity on changes in option delta, option vega represents option price sensitivity on changes in volatility of underlying instrument price, option theta represents option sensitivity on passage of time and option rho represents option sensitivity on changes in interest rates.

Within delta normal method, options are represented by their delta-equivalents. But as is known, option value does not depend only on its delta, i.e. on the first price derivatives, but also on its gamma, second order price derivative. So linear approximation to option values is valid only for very narrow range of underlying spot prices and it should be used only in the case that options do not comprise large portion of portfolio that is valued.

Local (delta normal) valuation is easy because it can use normally distributed variables that are put together to create portfolio. Portfolio is then linear combination of individual assets and delta-normal method is linear. The potential loss in value $V$ of portfolio is computed as

$$\Delta V_0 = \beta_0 \cdot \Delta S,$$
where $\beta$ is the portfolio sensitivity to changes in prices evaluated at the current position $V_0$, $V_0$ is the value of current position and $\Delta S$ is potential change in prices.

The main benefit of this method is that it requires computing the portfolio value only once, at the current position $V_0$, which depends on current prices $S_0$. Hence, the delta-normal method is ideally suited to large portfolios exposed to many risk factors.

With options in the portfolio the delta approach suffers from several problems, of which the most important are:

• The worst loss may not be obtained for two extreme realizations of the underlying spot rate.
• The portfolio delta might be different for up and down movements$^5$.

The conclusion is that in general, it is not sufficient to evaluate the portfolio at the two extremes but also all intermediate values must be checked. The full valuation approach therefore requires computing the value of the portfolio for different levels of prices:

$$\Delta V = V(S_t) - V(S_0)$$

---

$^5$ Option delta changes according changes in underlying asset price and option gamma and therefore option delta can differ for up and for down movements.
Computationally this approach may be quite demanding, since it requires marking to market the whole portfolio over a large number of realizations of underlying random variables.

3.2.6.2 Delta-Gamma Approximations

In principle, to make Delta approach more suitable for options portfolio, one should add terms to capture gamma and vega risks, which are additional terms in Taylor expansions:

$$ dc = \Delta ds + \frac{1}{2} \Gamma dS^2 + \Lambda d\sigma ... $$

where $\Delta$, $\Gamma$ and $\Lambda$ are net values for the total portfolio of options that are all written on the same underlying asset.

On the basis of this method, Amendment to the capital accord to incorporate market risks issued by BIS in 1996 recommends that at the minimum risk measurement should incorporate option behaviour through a nonlinear approximation approach involving higher-order risk factor sensitivities.

In summary each of above mentioned methods can be used in different environment:

- If there are not many options in a large portfolio, then variance-covariance method with delta normal can be used with the advantage of being relatively computationally non-demanding.
- Increased precision with still relatively low computational costs can be reached by using Delta-gamma methods (the Greeks) for portfolios with substantial portion of options.
- To reach the precise value at risk number for portfolios with substantial portion of options, Monte Carlo simulation should be used.

There is one important feature of models dealing with non-linearity. With linear models, daily VaR can be easily adjusted for longer time horizons by simply multiplying the daily VaR by square root of time.
factor. However, this approach assumes that positions are constant, that daily returns are independent and identically distributed and that optionality in the portfolio is negligible. Therefore, the approach cannot be used for portfolios with substantial options segment and full valuation method must be implemented over the desired horizon instead.

3.3 Method comparison – model selection for capital adequacy calculations

As was seen above, there are many types of VaR models. These are usually used for the whole portfolio, so that the bank can make use of their main advantage of incorporating all the risk factors and their relationships in the form of variance-covariance matrix. All the models have the benefit of supporting the management with one number that describes overall risk position.

\[ \sigma_i = \sqrt{\text{var}_i}, \quad \text{var}_i = n \times \text{var}_1, \]
\[ \sigma_2 = \sqrt{n \times \text{var}_1} = \sqrt{n} \times \sigma_1. \]

When calculating VaR for multiple of period for which standard deviations are known then calculated VaR must be multiplied by square root of time.

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\[ \text{VaR is expressed in a number of standard deviations. Standard deviation is square root of variance that is additive. It can be expressed as follows:} \]
\[ \sigma_i = \sqrt{\text{var}_i}, \quad \text{var}_2 = n \times \text{var}_1, \]
\[ \sigma_2 = \sqrt{n \times \text{var}_1} = \sqrt{n} \times \sigma_1. \]
Each of the models has its advantages and disadvantages and can be used under different circumstances. Banks may choose which method is the best suitable to each of them and then to employ it both for internal risk assessment and management and externally as a basis for capital adequacy calculations. However, each model has to be regularly checked and its real performance must be measured, for example by method suggested lower. Only by regular checking the bank can be confident that it systematically does not underestimates or overestimates the market risk.

In order to keep model comparison simple and to focus on capital adequacy calculations, the local valuation VaR was used in the exercise below.

### 3.4 Stress testing

Stress (sometimes called Scenario analysis) testing is a complement to “standard” value at risk calculation. It cannot replace it but it can evaluate the worst-case effect of large movements in key variables. It consists of subjectively specified scenarios of interest that are then applied to current portfolio and the change of value of this portfolio is then calculated. The portfolio return can be expressed as:

\[
R_{P,S} = \sum_{i=1}^{N} w_{i,S} R_{i,S},
\]

where \(R_{P,S}\) is return of portfolio in scenario \(S\), \(w_{i,S}\) are weights of individual assets in portfolio and \(R_{i,S}\) is return for individual asset in given scenario.

The preferred scenarios might for example calculate with 100 basis points parallel movement of yield curve (up or down) or it can calculate with 50 basis point twist of yield curve. The calculation is useful only if appropriate level of extreme changes is used: there is no point to calculating stress tests to the portfolio by applying such
changes in underlying factors that occur often. Usually, many various values of portfolio are calculated using various stress conditions. After that probability $p_s$ for each scenario $S$ is set up and distribution of “stress returns” is created. Finally, stress VaR is calculated on given confidence level.

The aim of this method is to cover situations completely absent from the historical data. However, even if used only for this purpose, the model has several disadvantages. The main drawback is that the model internally ignores correlations. Users define the stress scenarios so that large movements of several risk factors were included in the scenario but with no knowledge about correlations of these factors. It might happen that the worst value of portfolio will result from risk factor $x$ increase and the worst value of portfolio will result from risk factor $y$ decrease. However, to be able to find the worst possible effects of factors $x$ and $y$ together, one must have the knowledge about their correlation.

3.4.1 \textit{VaR – Model verification}

As VaR is used for risk quantification and for capital adequacy calculations, bank management must be assured that VaR estimates are justifiable, i.e. that VaR does not underestimate or overestimate risk. Since VaR is calculated on specified confidence level, for
example 95%, then we expect that the real losses would be higher than those estimated by VaR model on the same confidence level, i.e. in 5% of all concurrenties. Small difference from this rate could occur in the case of bad luck, but if there is significantly more losses higher then estimated then regulators cannot accept the model for capital adequacy calculations and bank management should reconsider its use.

The easiest method for verification of model accuracy is to monitor “failure rate”\(^7\) that express the proportion of times VaR is exceeded in a given sample. Let’s assume that a bank calculates daily VaR on 99% level for total of T days. For these T days, real losses exceed VaR calculated for given day in number of N days. Regulator will ask whether N is too small or too large. In order to ensure that VaR model is not significantly overestimating or underestimating losses the standard tests can be used. Kupiec (1995) developed confidence regions for such tests for different days of history and different confidence levels. For each of these combinations it states the interval of possible values of N, in which no statement about model accuracy can be said. For example, with two years data (T = 510 working days) on the 99% confidence level, we would calculate N:

\(^7\) Compare with Joiron, 1997, p. 196
\[ N = p \times T \]
\[ N = 1\% \times 510 = 5 \]

On the basis of this calculation, we would expect that real losses will exceed calculated VaR in 5 days. However, according the Kupiec’s confidence regions, we would not be able to reject the model in the interval \( 1 < N < 11 \). Value over or equal to 11 would indicate that the model underestimates possible losses, value under or equal to 1 would indicate that the model is too conservative and that losses occur in reality much less then the model predicts. The confidence intervals, as can be seen in the following table, shrinks as the sample size increases, which means that with more data it is possible to reject the model more easily if it is incorrectly set-up. However, form small parameters of percentile level it becomes increasingly difficult to confirm model deviations. Detection of systematic biases becomes increasingly difficult for low values of percentile level because these correspond to very rare events. This explains why some banks prefer to chose a higher value percentile level (for example 5%). Then they are able to observe a sufficient number of model deviations to validate the model\(^8\).

\[^8\] Joiron, 1997, p93
Table 1: Nonrejection Regions

<table>
<thead>
<tr>
<th>Probability level</th>
<th>Nonrejection Regions for Number of Failures N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T=255 days</td>
</tr>
<tr>
<td>1%</td>
<td>N &lt; 7</td>
</tr>
<tr>
<td>2.50%</td>
<td>2 &lt; N &lt; 12</td>
</tr>
<tr>
<td>5%</td>
<td>6 &lt; N &lt; 21</td>
</tr>
<tr>
<td>7.50%</td>
<td>11 &lt; N &lt; 28</td>
</tr>
<tr>
<td>10%</td>
<td>16 &lt; N &lt; 36</td>
</tr>
</tbody>
</table>

Joiron, 1997, p.95

3.5 Measurement of interest rate risk in banking book

Measurement of interest rate in banking book is different from those of trading book because of the special products inherent in the banking portfolio. Between these typically are demand deposits and mortgages. Both these product types contain option properties. There is option included to mortgage that allows clients to repay the whole amount or part of it in the time of repricing. Regarding demand deposits their construction would lead to the conclusion that all of them reprice instantly after the market interest rates change. However, characteristics that are demonstrated by demand deposits are quite different. There is always delay in repricing of these deposits but in the case repricing takes place all demand deposits are repriced, applying that to existing as well as planned new deposits.

Nevertheless, the banking book is different from trading book mainly in the time horizon it is interested in. Products are held to maturity and portfolio performance is usually measured over longer horizon, for example one year. In this case simple VaR measurement is not sufficient because it would not take into consideration the changes in bank portfolio. Local or full valuation VaR calculates portfolio riskiness only for products currently held. This is correct as far as the
VaR horizon is relatively small. However, if risk should be measured for longer horizon, then assumptions regarding future portfolio development should be taken. And this is when more complex simulation takes place. These generate scenarios on the basis of assumptions taken about portfolio replication and new products selling. For example on the basis of past trends and future interest rate projections portfolio replication can be set up so that the total balance sheet will grow slightly and that term deposits structure will shift from short term deposits to longer term deposits. These scenarios should be specified and portfolio future value should be calculated for each of them. Simulation analysis is then the way to deal with interest rate risk in its entire complexity. It covers modelling the characteristics of all of the bank’s asset and liability products and it uses different scenarios of future repayment and re-pricing patterns and interest rate projections.

For portfolio value any of the VaR methods can be used, or alternatively other methods for interest rate risk calculation can be used. However, the first part depending on portfolio replication and portfolio embedded options is computationally demanding itself, not mentioning the complexity that is associated with VaR calculation. At the end measuring of interest rate risk or calculating future portfolio value is more difficult for banking book.
4 Interest rate risk management and hedging

Models discussed so far are used for measuring risk. Results of these models then serve as an input for internal risk management, for internal economic capital requirements\(^9\) calculation and for calculation of regulatory capital requirements.

Methods of hedging interest rate risk depend on the way of measuring interest rate risk. Since methods differ for banking book and trading book, interest rate risk management is different for the two as well.

4.1 Interest rate risk hedging in banking book

Hedging of interest rate risk in banking book means hedging either bank’s net interest income\(^{10}\) (NII) or hedging the bank’s present value and therefore its share price\(^{11}\).

These strategies are both compatible with maximising shareholders value but they are mutually exclusive. Bank can either hedge its NII or present value, but it cannot hedge both at one time.

Hedging NII means adjusting bank’s balance sheet so that any interest rate change would not cause decrease in NII. A bank can

\(^9\) In some cases banks want to calculate their internal capital requirements based on slightly different risk measures from the measures defined by regulators. In such case

\(^{10}\) Net interest income (NII) is the difference between interest revenues and interest costs.

\(^{11}\) Compare with Cade, 1997, p. 146
achieve this by modifying its assets and liabilities repricing schedule\textsuperscript{12} or it can use derivatives to reach required balance sheet repricing schedule. Use of derivatives is more flexible; adapting of balance sheet interest rate repricing structure might take long time.

Hedging of bank’s share value is based on the way that stock market reacts to interest rate changes. Stock value is based on the value of future earnings discounted back with appropriate rate\textsuperscript{13}. If interest rates increase then stock price will decrease because future cash flows are now discounted with higher rate. In order to keep stock price unaffected, bank’s NII should increase when interest rate increase so that it would offset share price decrease caused by higher discount factor.

In practice bank with positive GAP, i.e. bank with volume of assets repricing higher then volume of liabilities (see chapter 3.1.1 GAP analysis for more detailed definition of GAP), have an inbuilt tendency for NII to increase when rates go up and to decrease when rates go down. These banks have a predisposition to hedge their share price. In other two possible cases, i.e. if there is no GAP in repricing of bank’s assets and liabilities and in the case of negative GAP (i.e. if volume of liabilities repricing exceeds volume of assets

\textsuperscript{12} Repricing means fixing of interest rate for floating rate products and replacing matured fixed rate products.
repricing) a bank should select hedging NII. In such a case, banks with zero GAP are already hedged, which means that in any change of interest rates bank’s NII does not change, while banks with negative GAP (i.e. liability – sensitive) have to make necessary steps in order to make its GAP positive or zero.

Both alternative strategies, i.e. stabilising net interest income or stabilising the bank’s economic value and share price, represent the boundaries of prudent bank behaviour with regard to interest rate risk. In practice these strategies mean keeping neutral interest repricing profile or moderately asset-sensitive position (i.e. position where gross interest income is more sensitive than gross interest costs). Compromise anywhere between these two opposite options is acceptable.

Another distinguishing feature of a banking book hedging is existence of a large volume of liabilities with very short contract maturity. These are current accounts that are repayable on demand. Real maturity of products like current accounts is different then contractual; therefore a bank has to take several assumptions regarding repricing volumes and time of repricing. Hedging of interest rate risk is based on calculated risk measure but the measure itself is based on several assumptions that might not hold in the future. Managing and hedging

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13 Appropriate discount rate reflects general interest rate level and riskiness of given
of interest rate risk in banking book therefore must be based on deep analysis of balance sheet behaviour that brings solid basis for necessary assumptions.

4.2 **Interest rate risk hedging in trading book**

As I mentioned earlier, value of trading book is expressed solely in present value terms and risk of trading book is expressed practically in all banks in term of VaR. VaR is calculated for the portfolio as a whole and not for its individual products groups. Major advantage of such an approach is the possibility of natural hedging between individual products and market risk categories.

Hedging of trading book portfolio therefore concentrates on overall risk reduction and not on hedging of individual risks. Its purpose is to reduce VaR so that it would be compatible with bank’s strategy.

VaR reduction can be done by using minimum-variance technique\(^4\) that is based on portfolio theory saying that weighted portfolio of two individual assets that are not perfectly positively correlated will have variance lower then weighted average variance of the two assets. Hedging involves taking two negatively correlated positions (for example a long position in one asset a short position in another positively correlated asset). Typically, standard deviation of hedged

\(^{4}\) See Harris, 2004, p. 2
portfolio can be substantially reduced. Reduction in portfolio variance can substantially decrease portfolio VaR. The extent to which VaR will be reduced depends on normality of portfolio return distribution. If portfolio shows skewness and kurtosis then decrease in portfolio standard deviation achieved by hedging might cause increase in portfolio skewness and kurtosis and subsequently to decrease in VaR lower then would correspond to decrease in standard deviation if portfolio returns were normally distributed.

Therefore, while hedging portfolio VaR, bank must model effect of hedging on portfolio so that effects of possible increase in skewness and kurtosis were known in advance and expected decrease in VaR could be estimated properly. Moreover, costs of hedging must be considered together with VaR decrease.

How do banks recognize that their VaR is too high? Every bank devotes part of its effort aimed to proper risk management to setting up set of limits mandatory for its business. Usually, trading book has several such limits, all of them expressed as VaR. Banks set VaR limits on trading book as a whole, on individual portfolios or desks. When these limits are exceeded then bank takes appropriate steps given by its internal regulations. One of these steps is hedging VaR.
5 Capital adequacy requirement to interest rate risk

Capital generally serves as a cushion that should absorb bank's losses. And since the probability of loss and its severity depends on the bank's riskiness, it follows that the level of capital bank holds should also reflect its overall riskiness. Even though this approach looks as very simple, it gets more complex when it comes to precise and exact riskiness measurement and comparison between individual banks.

5.1 Capital adequacy overview

Capital adequacy directives have special position between all other banks regulations. Other bank directives, such as directive on prudent risk management, set rules for banking behaviour but do not specify charges or penalties that will be imposed in the case banks do not comply with these regulations. On the opposite, capital adequacy directives aim to set clear rules that exactly specify the amount of capital banks must hold.

Since unexpected losses that would threaten bank's stability can be experienced more likely by the bank with the more risky business then by the bank that has less risky portfolio, capital cushion to absorb these losses should be higher in banks with more risky portfolio.

Together with opening of financial markets in different parts of the world the problem of one capital adequacy measure for all international banks emerged. With the need for unified capital measure also the need for aligning the capital held by a bank with its riskiness arose. Without this measure similar banks could hold different amounts of capital with a direct impact on their costs and thus their efficiency. If there was no capital adequacy regulation then
riskier banks could hold less capital relatively to their risk profile and thus look more profitable.

The Basel Committee on Banking Supervision\textsuperscript{15} worked during the period ending in 1988 on the task to establish guidelines for common capital adequacy measure for all international banks. Important objective of this effort was to close gaps in international supervisory coverage to ensure that no foreign establishment should escape supervision; and that supervision should be adequate.

The most important recommendations of the Committee were its papers concerned with capital adequacy. The aim of the Committee was not only to establish common capital measure but also establish such measure that would take into account banks’ riskiness.

The first risk-related capital adequacy recommendation the Committee issued concentrated on credit risk, which is the major risk banks are exposed to.

The first internationally recognized risk related capital adequacy framework was published by the Bank for International Settlement in 1988. Member states committed themselves to incorporate this recommendation into national legislations. The capital adequacy

\begin{footnote}
\textsuperscript{15} Basel Committee on Banking Supervision works as a part of Bank for International Settlements. It was formed by the central-bank Governors of the Group of Ten countries. Each country is represented not only by its central bank but also by the authority responsible for the prudential supervision of banking business. Conclusions of the
\end{footnote}
standards set in the recommendation should have been obligatory only for internationally active banks. However, some of the national regulators adopted more strict local directives that required compliancy with minimum capital requirements also for local banks. The ratio dealt only with credit risk and could be calculated as:

\[ CA = \frac{RWA}{Capital}, \]

where CA stands for capital adequacy, RWA stands for Risk weighted assets and C stands for capital. It was agreed that the minimum capital ration should not be lower in international and later in European banks then 8%.

5.2 Capital adequacy to market risks

5.2.1 History of capital charges to market risks

Risk adjusted capital measure was fully suitable for banks oriented on traditional bank products: loans and deposits. However, many banks begun to move their business towards market operations and trading activities. As the volume of these activities increased, banking book became more important as a source of bank’s profit. Together with this change, banks had to look for improved methods of market risks.
measurement and for the better way of calculating economic capital level that would be necessary as a cushion for market losses. International regulators in Bank for International Settlement soon became interested as well and the result was published as an Amendment to the Capital Accord to Incorporate Market Risks in 1996. This document prescribed that banks measure and apply capital charges in respect of their market risks in addition to credit risk. Market risk was defined as the risk of losses in on and off-balance sheet positions arising from movements in market prices. Capital charges should have been newly calculated for:

- Interest rate risk in the trading book,
- Equity price risk in the trading book,
- Exchange rate risk in the whole bank,
- Commodity risk in the whole bank.

Internationally active banks were required to measure capital and apply capital charges according this Amendment from the end of 1997 if national legislations did not require earlier term. In the Czech Republic the first capital adequacy regulation, that incorporated capital charges to market risks, came into effect at the end of 1998 and banks were required to calculate capital charges for market risk since the beginning of 1999.

All three sets of regulations (i.e. BIS Recommendations, EU Directives and ČNB regulations) are very similar and differ only in minor details. The major aspects of these regulations are summarized in the following chapter (5.2.2).

5.2.2 Market risk capital charges calculation

For all the market risks named above (i.e. interest rate risk, equity price risk, exchange rate risk and commodity risk) banks can choose between two types of calculation methods: standardised measurement method and internal models approach. It is possible to
combine the two methods but one risk must be always completely calculated by single method.

Standardised methodology uses “building blocks” approach in which capital charges for individual risks are first calculated separately and then added together.

Methodology slightly differs for each type of risk but the approach is similar for all types of risks. First all the positions in given risk factor must be identified (i.e. all positions in one currency for exchange rate risk, all positions in one commodity for commodity risk, etc.), then net position and gross position in given risk factor are calculated as absolute value of the sum of all positions and sum of all positions expressed in absolute value respectively, in given risk factor. Capital charge is then calculated as a percentage of either netted position or percentage of gross positions or as a combination of both. Capital charges for individual risk factors slightly differ between BIS and EU methodology; generally EU Directive\textsuperscript{16} allows banks to hold less capital then BIS Recommendation. Regarding Czech regulation, it can be noted that ČNB while creating capital adequacy provision usually chose more prudent approach.

\textsuperscript{16} Directive 93/6/EEC
5.3 **Capital adequacy to interest rate risk of trading book**

Capital charge to interest rate risk is calculated for instruments that include all fixed-rate and floating-rate debt securities and instruments that behave like them.

The minimum capital requirement consists of two separately calculated charges, one applying to the specific risk of each security not depending on whether the position held is short or long and the other to the interest risk of the portfolio called general market risk where long and short positions can be offset.

5.3.1 **Capital charge to specific risk**

Specific risk includes the risk that individual debt or equity security moves by more or less than the general market in day-to-day trading and event risk where the price of security moves rapidly relative to general market, e.g. on take-over bid or some other stock event. Such events include the risk of default.

The capital charge for specific risk is designed to protect against an adverse movement in the price of individual security as an outcome of factors related to individual issuer. Offsetting is restricted in this risk to matched positions in the identical issue.

Capital charge to specific risk is different for each of the categories defined below:

<table>
<thead>
<tr>
<th>Residual maturity</th>
<th>Cap. charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government instruments</td>
<td>All</td>
</tr>
<tr>
<td>Qualifying instruments</td>
<td>&lt; 6 months</td>
</tr>
<tr>
<td></td>
<td>6 months to 24 months</td>
</tr>
<tr>
<td></td>
<td>&gt; 24 months</td>
</tr>
<tr>
<td>Other instruments</td>
<td>All</td>
</tr>
</tbody>
</table>

Government instruments include all instruments that are charged 0% in current capital adequacy for credit risk. Qualifying instruments
include according ČNB regulations debt instruments issued by the banks registered in OECD, EEA\textsuperscript{17} and several other countries.

5.3.2 \textit{Capital charge to general market risk}

The capital requirements for general market risk are built so that they would capture the risk of loss arising from changes in market interest rates. Choice between two methods of measuring interest rate risk is permitted in BIS, EU and ČNB regulations: Maturity method and Duration method. In each method, capital charge is the sum of four components:

- Capital charge to the matched position in each time band (the “vertical disallowance”)
- Capital charge to the matched positions across different time-bands (the “horizontal disallowance”)
- Capital charge to the net short or long position in the whole trading book
- Net charge for positions in options, where appropriate.

For capital charge calculations, instruments have to be separated according their currency and special maturity ladders should be used for each of them. Capital charge is then calculated for each currency and then summed up.

\textsuperscript{17} European Economic Area
5.3.2.1 Maturity method

In maturity method, long and short positions in debt securities and other sources of interest rate exposures are placed into maturity ladder comprising 13 time bands or 15 time bands in the case of low coupon instruments. Fixed rate instruments are allocated according their residual maturity; floating rate instruments are allocated according residual time to repricing. First all positions in each time band are weighted by factor designed to reflect price sensitivity of these positions to assumed interest rates changes. Individual supervisory authorities set not only weights but expected interest rate changes as well. Then positions in each time band are offset and single position in each time band is calculated. However, since there are instruments with different maturities in each time band, and these are offset, small capital charge is imposed (10%) on the smaller of the offsetting positions, either short or long, to reflect basis risk and GAP risk within the time band.

Detailed description of weights can be seen in the table below.

Table 2: Maturity method: time bands and weights

<table>
<thead>
<tr>
<th>Time-band</th>
<th>Zone</th>
<th>Coupon 3% or more Maturity</th>
<th>Coupon less than 3% Maturity</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>From</td>
<td>To</td>
<td>From</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1 month</td>
<td>1 month</td>
<td>1 month</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1 month</td>
<td>3 months</td>
<td>1 month</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3 months</td>
<td>6 months</td>
<td>3 months</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>6 months</td>
<td>12 months</td>
<td>6 months</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1 year</td>
<td>2 years</td>
<td>1 year</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>2 years</td>
<td>3 years</td>
<td>1.9 years</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>3 years</td>
<td>4 years</td>
<td>2.8 years</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>4 years</td>
<td>5 years</td>
<td>3.6 years</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>5 years</td>
<td>7 years</td>
<td>4.3 years</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>7 years</td>
<td>10 years</td>
<td>5.7 years</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>10 years</td>
<td>15 years</td>
<td>7.3 years</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>15 years</td>
<td>20 years</td>
<td>9.3 years</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
<td>20 years</td>
<td>10.6 years</td>
<td>12 years</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>12 years</td>
<td>20 years</td>
<td>8.00%</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>20 years</td>
<td>12.50%</td>
<td></td>
</tr>
</tbody>
</table>

BIS, 1996, p. 12
After the first netting, the result is one offset position in each time band and one residual position. Banks can then conduct two rounds of offsetting, first within the three zones and then between zones. In each step, offset position is penalized by capital charge that is the higher the offset positions are more distant from each other. At the end the bank gets one residual position for the whole trading book (and single currency). The position offset between Zone 1 and Zone 3 is subject to the highest capital charge; according ČNB and EU regulations it is 150%, however according BIS Recommendation it is 100%. At the end of offsetting, there is usually remaining interest rate position. This is position that cannot be offset with any other position and therefore bears the highest risk: if interest rates change then this part of balance sheet will reprice (for example asset part) but no opposite position will reprice (liabilities part). This residual position is subject to the capital charge of 100%.

Detailed capital charge can be seen in the table below:
Table 3: Capital charge for matched positions

<table>
<thead>
<tr>
<th>Time-band</th>
<th>Zone</th>
<th>Maturity (Coupon &gt; 3%)</th>
<th>Duration or Maturity (Coupon &lt; 3%)</th>
<th>Within zone</th>
<th>Between adjacent zones</th>
<th>Between zones 1 and 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1 month</td>
<td>1 month</td>
<td>40%</td>
<td>40%</td>
<td>100%</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1 month 3 months</td>
<td>1 month 3 months</td>
<td>30%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3 months 6 months</td>
<td>3 months 6 months</td>
<td>30%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>6 months 12 months</td>
<td>6 months 12 months</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1 year 2 years</td>
<td>1 year 1.9 years</td>
<td>30%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>2 years 3 years</td>
<td>1.9 years 2.8 years</td>
<td>30%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>3 years 4 years</td>
<td>2.8 years 3.6 years</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>4 years 5 years</td>
<td>3.6 years 4.3 years</td>
<td>30%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>5 years 7 years</td>
<td>4.3 years 5.7 years</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>7 years 10 years</td>
<td>5.7 years 7.3 years</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>10 years 15 years</td>
<td>7.3 years 9.3 years</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>15 years 20 years</td>
<td>9.3 years 10.6 years</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>20 years</td>
<td>10.6 years 12 years</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td>12 years 20 years</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td>20 years</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

BIS, 1996, p. 13

5.3.2.2 Duration method

Under the alternative duration method, banks can use more accurate method of measuring all of their general market risk by calculating the price sensitivity of each position separately (for detailed duration calculation see chapter 3.1.2 Duration analysis). In duration method, banks have to:

- First calculate the price sensitivity of each instrument. The sensitivity is multiple between its duration, present value and assumed change in interest rates. Assumed change in interest rates is between 0.6% and 1% depending on maturity.
- Place instruments into duration based ladder with 15 time bands
- Impose capital charge of 5% on netted positions in each time band in order to capture basis risk.
- Carry forward positions that were not offset and provide offsetting first within each zone and then between zones. Offset positions as well as final unmatched position are subject to the
same capital charge as in the maturities method above (table 3).

Table 4: Duration method: time bands and assumed changes in yield

<table>
<thead>
<tr>
<th>Time-band</th>
<th>Zone</th>
<th>Duration From</th>
<th>Duration To</th>
<th>Assumed change in yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1 month</td>
<td></td>
<td>1.00%</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1 month</td>
<td>3 months</td>
<td>1.00%</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3 months</td>
<td>6 months</td>
<td>1.00%</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>6 months</td>
<td>12 months</td>
<td>1.00%</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1 year</td>
<td>1.9 years</td>
<td>0.90%</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>1.9 years</td>
<td>2.8 years</td>
<td>0.80%</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>2.8 years</td>
<td>3.6 years</td>
<td>0.75%</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>3.6 years</td>
<td>4.3 years</td>
<td>0.75%</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>4.3 years</td>
<td>5.7 years</td>
<td>0.70%</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>5.7 years</td>
<td>7.3 years</td>
<td>0.65%</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>7.3 years</td>
<td>9.3 years</td>
<td>0.60%</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>9.3 years</td>
<td>10.6 years</td>
<td>0.60%</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
<td>10.6 years</td>
<td>12 years</td>
<td>0.60%</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>12 years</td>
<td>20 years</td>
<td>0.60%</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>20 years</td>
<td></td>
<td>0.60%</td>
</tr>
</tbody>
</table>

BIS, 1996, p. 14

5.3.3 Interest rate derivatives

The measurement system must include positions in all interest rate derivatives and off-balance sheet instruments of trading book which value reacts to changes in interest rates. These are mainly forward rate agreements (FRA), other forward contracts, bond futures, interest rate and cross-currency swaps and forward exchange rate positions. Options are treated separately and will be described in chapter 5.3.4 Interest rate options.

The derivatives must be converted into positions in relevant underlying instrument (being it deposit, loan or bond) and must be subject to specific and general market risk charges as describe in chapters above (5.3.1 Capital charge to specific risk and 5.3.2 Capital charge to general market risk). In order to calculate relevant position, banks must use present values of the principal amount of the underlying instrument.
5.3.3.1 Forward rates agreements (FRAs)

FRAs are treated as one long and one short position in government security. As a maturity of future or FRA is taken the period until delivery or exercise of the contract plus, if applicable, the life of the underlying instrument. For example short position in interest rate one month future starting in October and sold in July will be treated as a short position in government security starting in July with maturity of 3 months and long position in the same government security starting in July with maturity of 2 months.

5.3.3.2 Swaps

Swaps are treated as two notional positions in government securities with relevant maturities. For example, interest rate swap under which bank is paying floating rate that is fixed every 1 month and under that bank is receiving fixed rate in 1 year will be treated as a short position in a floating rate instrument of maturity equivalent to the period until next repricing, i.e. with maturity of 1 month, and as long position in fixed rate instrument with maturity equal to the swap maturity, i.e. 1 year. Each of cross currency swap legs is
5.3.4 **Interest rate options**

Measuring of risks associated with options is more complex than measurement of other banking risks (see for example chapter 3.2.6 Option risk in VaR) and therefore calculation of capital requirement is more complex. Calculation rules are similar for all types of options. Capital requirement has to be calculated for all exchange rate and commodity options within a bank, i.e. for all such options in trading book and in banking book. Capital requirement for interest rate options and equity options is calculated only for trading book.

Capital charge to interest rate options (as well as to other options) can be calculated by three basic methods allowed by BIS\(^\text{18}\):

- Simplified approach
- Delta-plus method
- Scenario approach

Moreover, regulation of ČNB\(^\text{19}\) allows use of the fourth method:

- Margin method

Simplified approach can be used only for portfolios of purchased options. Option and associated underlying are not subject to the standardized methodology but are subject to separately calculated

\[\text{References:}\]

\(^{18}\) BIS, Amendment to the capital accord to incorporate market risks, 1996

\(^{19}\) ČNB Regulation No. 333, 2002
capital charges that incorporate both general market risk and specific risk.

Both other methods, delta-plus and scenario approach require calculating capital charge also to option-specific risks. Under the delta-plus method delta equivalent of each position becomes part of the standardized methodology. Within this approach for example positions in interest rate options are included into standardized calculation of interest rate risk in the form of delta equivalents. Moreover, separate capital charges are then applied to gamma and vega risks of option positions.

The scenario approach uses simulation techniques to calculate changes in the value of option portfolio for the changes in the price level and volatilities of its associated underlying instruments. Under this approach, the general market risk charge is determined by the specified combination of underlying price and volatility changes that produces the largest loss. For the delta-plus method and scenario approach the specific risk capital charges are determined separately by multiplying the delta equivalent of each option by the specific risk weights given by standardized methodology for interest rate specific risk.

Margin method is allowed in the Czech regulatory framework and can be used for all options traded on recognized exchanges. Under this method capital requirement is equal to the margin required by given exchange.

5.4 Capital charge calculated according internal models

The use of internal models for capital requirement calculation is subject to the explicit approval of the bank’s supervisory authority. The supervisor will give its approval only if its conditions are fully met. These conditions include:

- Qualitative standards
• Specification of market risk factors
• Quantitative standards
• Adequate stress testing

The aim of qualitative standards is to ensure that a bank using internal model has sound risk management and reporting processes. Specifically, the bank must have organizational unit responsible for risk management system, independent on the trading unit and reporting directly to senior management. This unit should conduct regular back-testing program and internal model has to be integrated into day-to-day risk management process. Internal risk management processes must be clearly set up, must be regularly audited by the bank itself and must be regularly checked by external auditor.

If a bank wants to use internal model for calculating capital charge, it has to prove that set of market risk factors, i.e. set of market rates and prices that affect the value of trading positions, is appropriately specified. Regulators provide guidance on how detailed the set should be for individual risk categories.

The quantitative conditions a bank has to meet before it can use its internal model for capital charge calculation are mainly: daily VaR calculation, 99th percentile and 10 days horizon. Additionally, historical observation period used for calculation of volatilities and correlations must be at least one year and input data files are updated no less frequently than once in every three months.

Another set of condition that a bank must meet before it can receive approval of internal model are conditions related to stress testing. The bank has to have stress testing program in place and its stress scenarios need to cover a range of factors that can create extraordinary losses in trading portfolios.

If all these conditions are satisfied than a bank can calculate its capital charge to market risks according internal model. Capital
charge is then equal as the higher of its previous day’s VaR number (calculated with 10-days horizon and on 99% probability level) and an average of the daily VaR measures on each of the preceding 60 business days multiplied by multiplication factor. Multiplication factor is set individually by supervisory authorities, its minimum being 3 and maximum 4. Multiplication factor for individual bank then depends on “plus” factor that can range from 0 to 1 and that is set according specific national regulations. According ČNB\textsuperscript{20}, “plus” factor depends on the number of days at which actual losses were higher than losses predicted by VaR.

Table 5: Multiplication factor according ČNB

<table>
<thead>
<tr>
<th>Number of days with losses exceeding VaR in 250 days</th>
<th>ČNB Plus factor</th>
<th>Total multiplication factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 5</td>
<td>0.00</td>
<td>3.00</td>
</tr>
<tr>
<td>5</td>
<td>0.40</td>
<td>3.40</td>
</tr>
<tr>
<td>6</td>
<td>0.50</td>
<td>3.50</td>
</tr>
<tr>
<td>7</td>
<td>0.65</td>
<td>3.65</td>
</tr>
<tr>
<td>8</td>
<td>0.75</td>
<td>3.75</td>
</tr>
<tr>
<td>9</td>
<td>0.85</td>
<td>3.85</td>
</tr>
<tr>
<td>&gt;=10</td>
<td>1.00</td>
<td>4.00</td>
</tr>
</tbody>
</table>

ČNB, 2004, p. 66

\textsuperscript{20} ČNB Regulation No. 333, 2002, p. 51, §56
However, according the Kupiec’s confidence region (see chapter 3.4.1 VaR – Model verification), for the period of 255 days and 99% probability level, we can confirm that model underestimates losses when number of losses exceeding VaR is higher or equal to 7. ČNB sets in this case more strict rules, since it significantly increases “plus” factor even for lower number of days when losses are higher then VaR (i.e. for 5 and 6 days in 250 days). Further description of back-testing framework can be found in chapter 5.4.2 Back testing.

5.4.1 **Possible drawbacks of calculating capital charge according internal models**

As can be seen in a preceding chapter, supervisory authorities set very detailed requirements on internal model itself and on risk management internal processes as well, before they allow a bank to use internal model for calculating its capital charge. The process of internal model approval is time consuming and might take as long as several months. Subsequent inspection of the model, when approval has been already given, is necessary and takes also long time. However, periodic full recalculation of the capital charge in order to find out, whether given bank does not underestimate it, is nearly impossible for supervisory authority. The reasons are mainly: volume of data about bank’s positions and possible variability in model parameters calculation (i.e. in calculation of volatilities, correlations, etc.). Therefore, even if supervisory authority intends to check bank’s internal models regularly, it would not be able to do such a check too frequently simply because one check takes long period of time. On the contrary, internal models and their parameters change according market conditions and bank’s evolvement (in processes, in approach, etc.). Therefore, there is a risk that once approved internal model will not be able, after some changes in its parameters, to reliably estimate
extent of a market risk and relevant capital charge. In such a case, supervisory authority has very little chance to uncover the problem. The main instrument of supervisory policy is then check of back-testing results.

5.4.2 Back testing required by supervisory authorities

Back testing consists of periodic comparison of the daily bank’s value at risk measures with the subsequent daily profit or loss (i.e. trading outcome). The value at risk measures should be larger then all but a certain fraction of trading outcomes, where that fraction is determined by the confidence level of the value at risk measure. To perform back-testing, the bank counts the number of times that the risk measures were larger than the trading outcome. The fraction actually covered can then be compared with the intended level of coverage to judge the performance of the bank’s internal model.

However, taking the exact expected number of real trading outcomes that can exceed VaR measure can lead to falsely rejecting accurate models and vice versa, it can lead to accepting the inaccurate model. BIS sets detailed methodology and reasoning for the thresholds chosen to distinguish well performing models from those performing worse. However, BIS admits that capital requirement settings on the basis of back testing thresholds cannot be the only tool because of the possibility that model will be inaccurately marked as worse performing, therefore generating higher capital charge, even though the model is accurate and is performing well.

BIS recommends establishing of three zones and categorization of internal models into them. In the first (according BIS ‘green’) zone, the stress testing results fully corresponds with expectations and there is very low probability that accepting stress testing outcomes would lead to erroneously accepting inaccurate model. Second zone (marked by BIS as ‘yellow’) include outcomes of back testing that are
plausible both for accurate and inaccurate models. BIS has agreed on recommendation of additional capital charge to the models showing the results indicated in this zone; however, it recommends that this increase is not meant to be purely automatic. BIS advises to supervisory authorities to allow banks in the yellow zone to prove that their model is accurate and that not favorable results of back testing were caused by bad luck. If banks prove this, capital charge should not be, according BIS recommendation, increased. However, if a bank’s back testing results belong to the third, ‘red’ zone then it is very unlikely that supervisory authority would erroneously marked this model as accurate. Therefore, additional capital charge should be applied in such a bank with no possibility for a bank to prove that the back testing results were influenced by bad luck.

ČNB imposes slightly more strict standards on back testing results. It adopted the same scale for additional capital charge as BIS; however, ČNB does not allow any exception to them even in a ‘yellow’ zone. Moreover, ČNB sets strict rule regarding the stress testing of specific risk related to equities and bonds.

In the following table the three zones are defined based on the number of exceptions realized in a sample of 250 observations with assumption that the model should cover 99% of losses. Column ‘Increase of scaling factor’ indicates, what number will be added to
multiplication factor used when calculating capital requirement from value at risk measure\textsuperscript{21}. Cumulative probability is probability of obtaining given number or fewer exceptions in a sample of 250 observations when the true coverage of the model is 99\%. For example, the cumulative probability shown for 6 exceptions is the probability of obtaining between zero and 6 exceptions.

Table 6: Back testing exceptions and increase in scaling factor

<table>
<thead>
<tr>
<th>Zone</th>
<th>Number of exceptions</th>
<th>Increase in capital adequacy scaling factor</th>
<th>Cumulative probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green Zone</td>
<td>0</td>
<td>0.00</td>
<td>8.11%</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.00</td>
<td>28.58%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.00</td>
<td>54.32%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.00</td>
<td>75.81%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.00</td>
<td>89.22%</td>
</tr>
<tr>
<td>Yellow Zone</td>
<td>5</td>
<td>0.40</td>
<td>95.88%</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.50</td>
<td>98.63%</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.65</td>
<td>99.60%</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.75</td>
<td>99.89%</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.85</td>
<td>99.97%</td>
</tr>
<tr>
<td>Red Zone</td>
<td>10 or more</td>
<td>1.00</td>
<td>99.99%</td>
</tr>
</tbody>
</table>

BIS, 1996, p. 15

5.5 **Capital adequacy to interest rate risk of banking book**

Interest rate risk and equity price risk are the only risks of banking book that do not invoke specific capital charge. In the case of equities

\textsuperscript{21} Basic multiplication factor is 3. Based on back testing result, this factor can be increased
this is justified by the fact that banks should hold only long-term strategic investments in their banking books. But why there is no exactly specified capital charge to interest rate risk of banking book for example the Czech regulatory framework?

Interest rate risk affects banking book the same way as trading book: it can negatively affect both earnings and economic value of a bank. However, calculation of both these effects is, unlike in trading book, difficult in banking book. It requires setting of number of assumptions and can be monitored by external parties only with difficulty.

First, if a bank chooses to measure interest rate risk in terms of its impact on net interest income, then it has to make assumptions about the repricing dates\textsuperscript{22} of products in banking book. This assumption must be made for mortgages with embedded option allowing their holders to repay any time. Moreover, the bank has to take assumption regarding its internal interest rate flexibility, i.e. it has to estimate how long it usually takes to project change of market conditions for example to interest rates on current accounts. However, the second assumption can be estimated more exactly then the first one.

\textsuperscript{22}Repricing date is a date when floating rate instruments will fix interest rate for next period and it is a maturity date for fixed rate instruments.
Second, if a bank chooses to measure interest rate risk in terms of economic value change, it will face much larger problem of setting the right parameters of the model. Measuring economic value means either taking market prices of instruments or calculating value of instruments by discounted cash flows associated with them. Products in banking book are not sold before maturity and therefore there is usually no secondary market and no market price for these instruments, with an exception of asset securitization. The value of bank’s asset and liabilities in banking book must be therefore calculated by discounting future cash flows.

So the bank has to estimate, except above mentioned estimate of the percentage and timing of embedded options exercise, also cash flows associated with the products that have different contractual and real maturity. Typical examples are current accounts and overdrafts with contractual maturity 1 day but with real maturity spread in much longer period.

Setting right all these assumptions is a key task in banking book interest rate risk measurement. And the assumptions are the main reason why pure supervision of interest rate risk measurement is

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23 Asset securitization is issuing of securities backed by specific bank’s assets. Holder of asset backed security is entitled to receive interest payments during the life of security and notional amount at the maturity of security. Both interest payments and notional comes from the specific pool of banking assets. Hence, during asset securitization process bank’s
difficult and, therefore, why setting capital charges to interest rate risk is not for now common practise.

However, since capital charge has become calculated for wide range of major banking risks, we might guess that banking supervisory authorities will direct their effort to setting capital charge also to interest rate risk.

This direction in banking regulation can be supported by recent initiative of BIS that sets guidelines for banking book interest rate risk management\(^{24}\). As a part of its recommendation, it advises to national supervisors to increase capital charge in the case bank do not measure interest rate risk appropriately. So it looks inevitable that international supervisory authorities will specify detailed techniques of capital charge calculation in the future. However, they will have to cope with several problems caused by necessity to work with assumptions about client behavior. And still they have only limited chance to check the accuracy and appropriateness of banking book interest rate risk measurement.

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\(^{24}\) BIS, Principles for the management of interest rate risk, 2004
6 Project: Capital requirement calculation for sample portfolios

Basic task of the project is to use internal models for capital adequacy calculation in the Czech market and to compare the different capital requirements based on an internal model and on the standardized method. Part of the project is focused on back testing. As a whole, the project should give overview of basic models use, consequences of their use for capital adequacy calculation and bring this in a context with the models performance.

Project can be divided into four phases:

- Project preparation. It includes selection of portfolios and calculating market parameters.
- Calculation of cash flows and cash flows mapping.
- Capital adequacy calculation according to historical simulation and variance-covariance VaR
- Capital adequacy calculation according standard method.

6.1 Phase 1: Project preparation

In the first phase, I selected portfolios and calculated models parameters. Model parameters are volatilities, correlations and time frame for VaR calculations.

6.1.1 Portfolio selection

Analysis of capital adequacy requirements is based on two different portfolios. Each of the portfolios is balanced, that means that assets of each portfolio equal its liabilities. This approach should simulate trading book that has its own balance sheet separated from banking book.
Trading book sample portfolios are selected so that they would be exposed solely to interest rate risk. Other risks, such as exchange rate risk or option risk are eliminated. It could be reached by selecting instruments with the following characteristics:

- Instruments are denominated in CZK only.
- Government bonds, money market deposits and interest rate derivatives compose the portfolio.

Individual instruments that constitute portfolios were chosen from instruments traded on Czech market according above mentioned criteria. The only additional condition was that all instruments were required to have sufficient price quotations history.

Excluding some categories of instruments does not make the results less valuable since it would not contribute to the intended investigation goals. This would only increase requirements on data sets input (i.e. necessity to calculate coefficient of correlation between Czech government bonds and foreign bonds, between exchange rate and Czech government bonds, etc.). Moreover, it would not be then possible to apply the results solely to interest rate risk.

Both portfolios were constructed as of March 1st, 2004. I calculated VaR and capital requirements for the 10 subsequent working days. VaR, portfolio present value and capital requirements are calculated for the following days:

- 1.3.2004
- 2.3.2004
- 3.3.2004
- 4.3.2004
- 5.3.2004
- 8.3.2004
- 9.3.2004
6.1.2 Portfolios interest rate risk

For this project, I constructed two basic portfolios. Each of them was meant to have the extreme risk profile:

**Portfolio #1** with very high interest rate risk, i.e. portfolio constructed from assets with very long time to maturity and financed by short term bank loans.

**Portfolio #2** with very low interest rate risk, i.e. portfolio constructed from relatively short term assets financed by liabilities with approximately the same maturity.

Portfolio #1

Portfolio #1 consisted of bonds and small amount of interbank loans on the asset side and from short term interbank deposits on liabilities side. Detailed list of instruments in the Portfolio #1 can be seen from the table below. Abbreviation CZGBxx stands for Czech government bonds with various issue dates, various coupon dates and various maturity dates.

Table 7: Portfolio #1 - Assets

<table>
<thead>
<tr>
<th>Type of Instrument</th>
<th>Notional (in '000 CZK)</th>
<th>Maturity</th>
<th>Next coupon date</th>
<th>Coupon / Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bond</td>
<td>CZGB6.05 9</td>
<td>04 Govt</td>
<td>30</td>
<td>14.9.2004</td>
</tr>
<tr>
<td>Bond</td>
<td>CZGB6.75 2</td>
<td>05 Govt</td>
<td>40</td>
<td>18.2.2005</td>
</tr>
<tr>
<td>Bond</td>
<td>CZGB6.55 10</td>
<td>11 Govt</td>
<td>40</td>
<td>5.10.2011</td>
</tr>
<tr>
<td>Bond</td>
<td>CZGB3.7 6</td>
<td>13 Govt</td>
<td>45</td>
<td>16.6.2013</td>
</tr>
<tr>
<td>Bond</td>
<td>CZGB6.95 1</td>
<td>16 Govt</td>
<td>40</td>
<td>26.1.2016</td>
</tr>
<tr>
<td>Interbank loan</td>
<td>Pribor</td>
<td>5</td>
<td>1 day</td>
<td>2.3.2004</td>
</tr>
<tr>
<td>Total Assets</td>
<td></td>
<td>200</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 8: Portfolio #1 - Liabilities

<table>
<thead>
<tr>
<th>Liabilities</th>
<th>Type of Instrument</th>
<th>Notional (in '000 CZK)</th>
<th>Maturity</th>
<th>Next coupon date</th>
<th>Coupon / Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interbank deposit</td>
<td>Pribor</td>
<td>30</td>
<td>1 day</td>
<td>2.3.2004</td>
<td>1.990%</td>
</tr>
<tr>
<td>Interbank deposit</td>
<td>Pribor</td>
<td>30</td>
<td>8 days</td>
<td>9.3.2004</td>
<td>2.011%</td>
</tr>
<tr>
<td>Interbank deposit</td>
<td>Pribor</td>
<td>40</td>
<td>18 days</td>
<td>19.3.2004</td>
<td>2.022%</td>
</tr>
<tr>
<td>Interbank deposit</td>
<td>Pribor</td>
<td>100</td>
<td>1 month</td>
<td>1.4.2004</td>
<td>2.030%</td>
</tr>
<tr>
<td><strong>Total Liabilities</strong></td>
<td></td>
<td><strong>200</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As can be seen from the tables above, more than 60% of all assets are long term and will mature in more than 7 years. To the contrary, all liabilities are short term and will mature in 1 month or earlier.

Portfolio #2
Portfolio #2 consisted of bonds and a small amount of interbank loans on assets side and from short term interbank deposits on liabilities side. Details of the portfolio can be seen from the table below. Abbreviation CZGBxx stands for Czech government bonds with various issue dates, various coupon dates and various maturity dates. “Pribor” stands for loans and deposits on interbank market.

Table 9: Portfolio #2 - Assets

<table>
<thead>
<tr>
<th>Assets</th>
<th>Type of Instrument</th>
<th>Notional (in '000 CZK)</th>
<th>Maturity</th>
<th>Next coupon date</th>
<th>Coupon / Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond</td>
<td>CZGB6.05 9</td>
<td>04 Govt</td>
<td>95</td>
<td>14.9.2004</td>
<td>14.9.2004</td>
</tr>
<tr>
<td>Bond</td>
<td>CZGB6.75 2</td>
<td>05 Govt</td>
<td>100</td>
<td>18.2.2005</td>
<td>18.2.2005</td>
</tr>
<tr>
<td>Interbank Loan</td>
<td>Pribor</td>
<td>5</td>
<td>14 days</td>
<td></td>
<td>2.020%</td>
</tr>
<tr>
<td><strong>Total Assets</strong></td>
<td></td>
<td><strong>200</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 10: Portfolio #2 - Liabilities

<table>
<thead>
<tr>
<th>Liabilities</th>
<th>Type of Instrument</th>
<th>Notional (in '000 CZK)</th>
<th>Maturity</th>
<th>Next coupon date</th>
<th>Coupon / Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interbank Deposit</td>
<td>Pribor</td>
<td>100</td>
<td>6 months</td>
<td>1.9.2004</td>
<td>2.130%</td>
</tr>
<tr>
<td>Interbank Deposit</td>
<td>Pribor</td>
<td>100</td>
<td>1 year</td>
<td>1.3.2005</td>
<td>2.320%</td>
</tr>
<tr>
<td><strong>Total Liabilities</strong></td>
<td></td>
<td><strong>200</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
From the tables describing Portfolio #2 it can be seen, that there is not perfectly immunized\textsuperscript{25} against interest rate risk but that the interest rate mismatch is very low.

For the purpose of analysis, I assumed that market value of instruments is the same as their notional value.

Market values changed over the period in which portfolios were monitored. However, change in market value did not cause neither real cash in / cash out nor it had an impact on future cash flows. Since both methods of capital requirement calculation work only with the projected future cash flows then we do not have to pay attention to the changes in market values.

Since the time period of portfolios examination was extended beyond the maturity of some short term instrument until March 13, 2004, I had to make several assumptions about portfolio replication. Hence, I assumed that all maturing instruments will be replaced by the instrument with the same duration, i.e. 1 day interbank loan would be replaced on its maturity with 1 day loan from interbank market.

\textsuperscript{25} To immunize portfolio against risk factor means to hedge it perfectly. If portfolio is immunized then it cannot generate loss in the case of any random move of risk factor it is immunized against.
6.1.3 Parameters of the model

While preparing Value at Risk in order to calculate riskiness of specified portfolio and to detect capital allocation needed for this portfolio several basic assumptions must be made and basic parameters must be set. Between these are three main categories: calculation of volatilities and correlations used in the model, choice of probability distribution and time horizon.

6.1.3.1 Yield curve

The only external inputs to VaR model were historical CZK interest rates quotations and these were used for calculation of variance-covariance matrix. However, this step does not have to be executed for large number of risk factors, since variance covariance matrix can be obtained from commercial providers. It is solely each bank’s decision whether to use data sets provided or whether to construct them in-house. In developing markets, however, data sets must be usually collected in-house since these are not available commercially. Number of vertices used for cash flow decompositions in order to calculate capital adequacy requirement is not exactly specified by Czech central bank (ČNB); the only restriction is given on the lowest number of vertices used. ČNB requires that at least 6 vertices are used; however, the model must meet other criteria and between these are the necessity to identify all risk factors important for portfolio and the necessity to prove that model can accurately predict the risk. Both these requirements will very likely lead banks to the construction of risk yield curves with more then 6 vertices.

6.1.3.2 Volatilities calculation

Risk can be measured in terms of price changes. These changes can take variety of forms such as absolute price change, relative price change and log price change. When a price change is defined
relative to some initial price, it is known as return. Widespread RiskMetrics methodology as well as this project measures change in portfolio value in terms of log price changes also known as continuously compounded returns.

To be precise, relative price change (i.e. percent return) for single period horizon can be defined as:

$$ R_T = \frac{P_T - P_{T-1}}{P_{T-1}}, $$

where $P_T$ is price of instrument in time T, $P_{T-1}$ is price of instrument in time T-1 and $R_T$ is percent return.

If the gross return on a security is just $1 + R_T$, then the log price change (continuously compounded return) of security $- r_T$ is defined to be the natural logarithm of its gross return. That is:

$$ r_T = \ln(1 + R) $$

$$ r_T = \ln\left(\frac{P_T}{P_{T-1}}\right) $$

$$ r_T = (p_T - p_{T-1}), $$

where $p_T = \ln(P_T)$ is natural logarithm of $P_T$.

The main reason for working with returns rather then prices is that returns have more attractive statistical properties then prices. Further, relative returns and log price changes are often preferred to the absolute price changes because the later do not measure change in terms of given price level but rather the change independent on price level. For small changes in underlying prices, log and relative price changes should be similar to one another.

Returns for individual yield curve vertices used in the project were calculated as log price changes. These were used for volatility and correlation calculations.
6.1.3.3 Data sets used for volatility calculation

In order to preserve simplicity and transparency of capital adequacy calculation, Czech Pribor and swap interest rates were used for calculations in the model. However, the problems encountered during its use are similar to those we would experience if we the model was more complex and was exposed to more interest rate risks (e.g. interest rate risk of EUR instruments, etc.).

Interest rates chosen as a market rates for calculation basis parameters were rates given by PRIBOR yield curves for maturities up to 1 year and CZK swap yield curve for maturities starting from 3 years. All the points of yield curve that are quoted in 2004 were taken into account, i.e. 1D, 1W, 2W, 1M, 2M, 3M, 6M, 9M, 1Y, 3Y, 5Y, 7Y, 9Y, 11Y and 13Y. Yield curve for Czech interbank deposits (i.e. Pribor) changed in the past several times. New yield curve maturities were added on July 27, 1993 (2M and 9M) and swap yield curve was in its final form quoted only from August 25, 2004. On this date 11Y and 13Y maturities were quoted for the first time.

Setting yield curve for maturities starting with 1D differs from generally accepted methodology and provided market data sets, both by RiskMetrics. In this methodology, sets of volatilities for each vertex as well as correlations between vertices are provided only for vertices starting with 1M maturity. Since for the model used in this project I calculated both sets of parameters (volatilities and correlations), set of vertices used in the model differs from RiskMetrics methodology.

History taken for volatility estimation starts on 21 September 1999; this was compromise between attempt to use the longest history possible and attempt to eliminate periods of very high volatility in distant past. Moreover, there was not compete history available for the longest maturities. I calculated their standard deviation as well as the correlation between them and other points of yield curve from
time series starting 28 August 2000 in order to have full history for all yield curve points (vertices).
These rates were used to establish volatility (i.e. standard deviation) and correlation both for underlying risk factors of market deposits and government bonds.

6.1.3.4 Correlation
I calculated coefficient of correlation for the market interest rates. As was expected, time series of short and mid-term interest rates except 1D returns (i.e. 1W, 2W, 1M, 2M, 3M, 6M, 9M, 1Y) are strongly positively correlated. Returns on 1D rate however are correlated to other points of yield curve only slightly. This observation holds even if we exclude period of higher volatility and concentrate only on the most recent period (starting on 1 March 2002) even though this operation shows much higher correlation of 1D rates with other rates then other time bands. Details can be seen in the table below:

<table>
<thead>
<tr>
<th>Time series</th>
<th>Start</th>
<th>01.03.2002</th>
<th>01.12.1999</th>
<th>28.07.1993</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Maturity</th>
<th></th>
<th>7 dní</th>
<th>14 dní</th>
<th>1 měsíc</th>
<th>2 měsíce</th>
<th>3 měsíce</th>
<th>6 měsiců</th>
<th>9 měsiců</th>
<th>1 rok</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.66</td>
<td>0.67</td>
<td>0.63</td>
<td>0.65</td>
<td>0.59</td>
<td>0.49</td>
<td>0.46</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.25</td>
<td>0.25</td>
<td>0.22</td>
<td>0.21</td>
<td>0.20</td>
<td>0.13</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.22</td>
<td>0.16</td>
<td>0.12</td>
<td>0.11</td>
<td>0.06</td>
<td>0.05</td>
<td>0.11</td>
<td>0.06</td>
</tr>
</tbody>
</table>

However, for the maturities starting on 2 years and longer, there was very small and positive correlation and in several cases there was small and negative correlation.
Example of correlation matrix calculated for 2 March 2004 can be seen below:
For the purpose of capital adequacy calculation, Czech as well as international regulators do not require volatilities and correlations to be calculated daily but allow for data set reconstruction once in three month. However, if market condition changes, then data set used for variance-covariance matrix must be renewed more often and variance-covariance must be recalculated more often. For the period of model examination, I renewed data set every day. It means that variance-covariance matrix was recalculated for every day and if market conditions changed then it was included in VaR calculation.

### 6.2 Phase 2: Portfolio Cash Flows, Present value Calculation and Cash Flows Mapping

#### 6.2.1 Cash Flows Decomposition

To be able to assess interest rate risk by any of the method specified in the first part, all individual future cash flows must be identified. For portfolios such as introduced above it is relatively simple: all expected future cash outflows and inflows including coupon payments are listed together with the date of their occurrence.

Examined portfolios’ cash flows consist of maturity repayments of interbank loans, deposits or government bonds or from bond coupons.
6.2.2 Present value calculation

Since VaR is based on present values of individual cash flows, these had to be calculated as the first step. Linearly interpolated interest rates were used for CF calculation. Present values were mapped into individual points of yield curve according the method specified below. Present values of portfolios for monitored period can be observed in the tables below:

<table>
<thead>
<tr>
<th>Date</th>
<th>PV of Portfolio #1 in '000 CZK</th>
<th>PV of Portfolio #2 in '000 CZK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3.2004</td>
<td>17.668</td>
<td>9.012</td>
</tr>
<tr>
<td>2.3.2004</td>
<td>18.120</td>
<td>9.015</td>
</tr>
<tr>
<td>3.3.2004</td>
<td>18.184</td>
<td>9.015</td>
</tr>
<tr>
<td>4.3.2004</td>
<td>18.157</td>
<td>9.017</td>
</tr>
<tr>
<td>5.3.2004</td>
<td>18.946</td>
<td>9.018</td>
</tr>
<tr>
<td>8.3.2004</td>
<td>19.326</td>
<td>9.018</td>
</tr>
<tr>
<td>10.3.2004</td>
<td>19.412</td>
<td>9.019</td>
</tr>
<tr>
<td>11.3.2004</td>
<td>19.650</td>
<td>9.020</td>
</tr>
<tr>
<td>12.3.2004</td>
<td>19.859</td>
<td>9.020</td>
</tr>
</tbody>
</table>

Present value of Portfolio #1 is higher even though its positive cash flows will be realized later than cash inflows of Portfolio #2 and therefore their present value is lower then present value of cash flows realized earlier (in Portfolio #2). Present value of cash inflows resulted form notional redemption in Portfolio #1 is lower then present value of cash inflows caused by redemption of notional in Portfolio #2 and lower then present value of cash outflows in Portfolio #1. What makes present value of Portfolio #1 significantly higher then present value of Portfolio #2 are coupon payments; present value of expected incoming coupon payments only is 53,700 CZK and this compensate the negative present value from principal payments.

6.2.3 Cash Flow Mapping

Cash flows decomposition, as was described in Chapter 3.2.1.1 Value estimate, is sufficient for capital requirement calculation
according to standard methods but it is not enough for capital requirement calculation according VaR models using Variance-covariance method.

For VaR calculations, basic parameters are necessary. These are volatilities of individual risk factor returns, for example volatility of the return on EUR/USD exchange rate or volatility of 1 month Pribor rate return and correlations between individual returns. Returns on EUR/USD exchange rate or on 1 month Pribor can be easily calculated since this exchange rate is daily quoted on public markets. Correlation between these returns can than be easily calculated as well.

However, the problem with interest rate risk is that volatility of its instruments’ returns is not known for the majority of instruments in the portfolio, and it follows that correlation between these cannot be easily calculated. Let’s assume that we have only three deposits that form portfolio. Residual maturity is 8 days, 20 days and 40 days for the first, second and third deposit respectively. To calculate VaR we would then need to know volatility of 8 days, 20 days and 40 days deposit returns. Moreover, we would need to know the correlation between these returns, i.e. correlation between 8 days and 20 days return, correlation between 8 days and 40 days returns and correlation between 20 days and 40 days returns. To calculate volatilities for these maturities would not be easy since they are not quoted on any market. Even if we estimated their volatility, then whole calculation would become very complex and time consuming, since there can be thousands of individual cash flows with different residual maturity.

However, interest rate for standard maturities (given by various yield curves constructed for different money markets) are quoted daily and volatilities of individual interest rates returns can be then easily
calculated. For the Czech interbank market we can then work with volatilities known for the following maturities: 1 day, 1 week, 2 weeks, 1 month, 2 months, 3 months, 6 months, 9 months, 1 year, 3 years, 5 years, 7 years, 9 years, 11 years and 13 years. For all of those, daily quotations are available and can be used for log price changes.

6.2.3.1 Methods of Cash Flows Decomposition
In order to be able to calculate interest rate risk with given level of accuracy and to deal with the number of different residual maturity dates, two ways of calculation are possible. These are duration mapping and cash flow mapping.

In duration mapping method, duration of the whole portfolio is calculated\(^{26}\) and then correspondent volatility is calculated from volatilities of two adjacent yield curve points. Let's say that overall duration of portfolio is 0.54 years and that standard deviation of 6 month and 9 months interest rate returns are 0.69\% and 0.68\% respectively. Standard deviation of 0.54 years returns can be then calculated as linear interpolation between the two points, i.e.

\[
\sigma_{0.54} = \sigma_{0.5} - \left( \frac{\sigma_{0.5} - \sigma_{0.75}}{T_{0.75} - T_{0.5}} \right) \times (T_{0.54} - T_{0.5}) = 0.69\% - \left( \frac{0.69\% - 0.682\%}{0.75 - 0.5} \right) \times (0.54 - 0.5) = 0.691\% 
\]

\(^{26}\) Compare with Joiron (1997, p. 213)
This approach can be acceptable if only interest rate risk is present but in the case other risks have to be incorporated into VaR this approach must be abandoned. The problem comes with necessity to calculate correlation between returns on calculated point of yield curve based on portfolio duration and return of another risk.

Duration mapping’s advantage is its simplicity, however it works with assumption that all interest rates move in parallel by the same amount. As soon as this is not true the calculation does not estimate risk exposure right and another method must be used.

One of the more advanced methods maps the cash flows into “vertices” of yield curve by using duration of the whole portfolio or single cash flows. In this case we would not calculate volatility of given point of the yield curve according volatilities of adjacent volatilities, but we would map cash flow into the point of yield curve that is quoted with the help of duration. Let’s assume that portfolio consists of one cash flow of 100 thousands CZK with duration $D_p$ of 0.54 years and let’s define $x$ as a portion of cash flow mapped into the closest shorter interest rate on the yield curve. We can then state:

$$D_{CF} = xD_{lower} + (1-x)D_{higher}$$

$$x = \frac{D - D_{higher}}{D_{lower} - D_{higher}} = \frac{0.54 - 0.75}{0.5 - 0.75} = 84\%$$

This would lead to allocating 84 thousands CZK into the lower vertex, i.e. into 6 months interest rate, and 16 thousands CZK into the 9 month interest rates. This method is relatively simple but can lead to the different risk mapped cash flows profile then original portfolio risk. Duration approximation is correct only under condition that the volatility of each vertex is proportional to its duration. Since this does
not necessarily have to be true, this approach should be abandoned as well, if we want to estimate interest rate risk correctly.

More precise method was introduced by RiskMetrics\textsuperscript{27}. This method is based on maintaining the overall riskiness of portfolio by allocating cash flows according on the basis of linearly interpolated volatility. To find the proportion of cash flow that should be allocated to the lower vertex on the yield curve, mentioned equation should be modified as follows:

\textsuperscript{27} Risk Metrics – Technical Document
\[ \sigma_{CF}^2 = x^2 \sigma_{lower}^2 + (1-x)^2 \sigma_{higher}^2 + 2x(1+x) \rho \sigma_{lower} \sigma_{higher} \]

By solving this equation for x we get the portion of cash flow that should be allocated to each of the closest vertices. By this approach we make an assumption that volatility (in the form of variance) can be linearly interpolated. This might not hold true in all cases but it would cause the smallest error compared to other methods.

Only to make the list of methods complete the last approach should be mentioned. It uses the similar approach as the cash flow allocation according variance and can be expressed as:

\[ \sigma_{CF} = x \sigma_{lower} + (1-x) \sigma_{higher} \]

This approach again assumes that correlation between the vertices, i.e. between individual points of yield curve, equals to 1 and that yield curve will move only in parallel with no twists. Even this method is not suitable if we want to make the model accurate.

For the cash flow mapping in the project, mapping according volatilities with regard to coefficient of correlation was used.

**6.3 Phase 3: Capital requirement calculation according VaR**

Calculation of capital requirement according VaR can be divided into following steps:

- Cash flows decomposition
- Calculation of interest rates corresponding to decomposed cash flows
- Cash flows present value calculation
- Cash flows mapping into vertices of yield curve
- Total cash flows allocated into vertices calculations
- VaR calculation based on variance-covariance matrix calculated for specified day and comparison of VaR results for different market data sets
• VaR calculation based on historical method calculated for specified day

6.3.1 **Total of cash flows mapped into vertices**

Sum of all cash flows allocated into vertices for 2 March 2004 and both portfolios can be seen below. Cash flows were mapped by using the allocation method that preserves overall cash flow riskiness and correlations between risk factors were taken into account.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>PV of CF</th>
<th>St.deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 D</td>
<td>-25,000</td>
<td>4.474%</td>
</tr>
<tr>
<td>7 D</td>
<td>-29,984</td>
<td>0.804%</td>
</tr>
<tr>
<td>14 D</td>
<td>-37,458</td>
<td>0.791%</td>
</tr>
<tr>
<td>1 M</td>
<td>-102,468</td>
<td>0.762%</td>
</tr>
<tr>
<td>2 M</td>
<td>0.000</td>
<td>0.720%</td>
</tr>
<tr>
<td>3 M</td>
<td>1.581</td>
<td>0.716%</td>
</tr>
<tr>
<td>6 M</td>
<td>32.913</td>
<td>0.690%</td>
</tr>
<tr>
<td>9 M</td>
<td>4.595</td>
<td>0.682%</td>
</tr>
<tr>
<td>1 Y</td>
<td>44.718</td>
<td>0.722%</td>
</tr>
<tr>
<td>3 Y</td>
<td>10.403</td>
<td>1.610%</td>
</tr>
<tr>
<td>5 Y</td>
<td>19.025</td>
<td>1.722%</td>
</tr>
<tr>
<td>7 Y</td>
<td>33.121</td>
<td>1.257%</td>
</tr>
<tr>
<td>9 Y</td>
<td>18.691</td>
<td>1.039%</td>
</tr>
<tr>
<td>11 Y</td>
<td>45.285</td>
<td>1.475%</td>
</tr>
<tr>
<td>13 Y</td>
<td>2.698</td>
<td>1.469%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>18.120</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Vertex</th>
<th>PV of CF</th>
<th>St.deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 D</td>
<td>5,000</td>
<td>4.474%</td>
</tr>
<tr>
<td>7 D</td>
<td>0.000</td>
<td>0.804%</td>
</tr>
<tr>
<td>14 D</td>
<td>0.000</td>
<td>0.791%</td>
</tr>
<tr>
<td>1 M</td>
<td>0.000</td>
<td>0.762%</td>
</tr>
<tr>
<td>2 M</td>
<td>0.000</td>
<td>0.720%</td>
</tr>
<tr>
<td>3 M</td>
<td>0.000</td>
<td>0.716%</td>
</tr>
<tr>
<td>6 M</td>
<td>2.986</td>
<td>0.690%</td>
</tr>
<tr>
<td>9 M</td>
<td>3.906</td>
<td>0.682%</td>
</tr>
<tr>
<td>1 Y</td>
<td>-2.087</td>
<td>0.722%</td>
</tr>
<tr>
<td>3 Y</td>
<td>0.000</td>
<td>1.610%</td>
</tr>
<tr>
<td>5 Y</td>
<td>0.000</td>
<td>1.722%</td>
</tr>
<tr>
<td>7 Y</td>
<td>0.000</td>
<td>1.257%</td>
</tr>
<tr>
<td>9 Y</td>
<td>0.000</td>
<td>1.039%</td>
</tr>
<tr>
<td>11 Y</td>
<td>0.000</td>
<td>1.475%</td>
</tr>
<tr>
<td>13 Y</td>
<td>0.000</td>
<td>1.469%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>9.806</strong></td>
<td></td>
</tr>
</tbody>
</table>

**VaR Calculation for both portfolios**

After cash flow decomposition and mapping, portfolio variance can be calculated according formula:

\[
VaR = \sqrt{VRV^T}
\]

where \( V \) is vector of VaR estimates for individual instruments and \( R \) is the correlation matrix.

Since VaR has to be calculated on 99% confidence level and since we approximated real distribution by standard distribution, then VaR formula can be rewritten as:

\[
VaR = 2.37\sqrt{V_iRV_i^T}
\]
where $V_1$ is vector of present value and respective volatility multiple and $R$ is the correlation matrix.

VaR calculations for both portfolios can be seen in the table below together with present value calculation.

In the case portfolio is consisted of only two instruments, then VaR can be written as:

$$\text{VaR}_{portfolio} = 2.37\sqrt{\left(\sigma_1 PV_1\right)^2 + \left(\sigma_2 PV_2\right)^2 + 2\sigma_1 \sigma_2 PV_1 PV_2 \rho_{1,2}}$$

For capital requirement 10-days VaR on 99% confidence level is calculated.

### 6.3.2 Capital adequacy requirements according VaR and standard method

For both portfolios I calculated capital requirement based on 10 days VaR according historical simulation method and capital requirement for general market risk according maturity method and according duration method.

In order to be able to compare capital requirements and after analysis of VaR values for subsequent dates, I assumed that VaR for given day is equal to average of VaR in past 60 days. Capital requirement based on internal model is calculated as the higher from previous’ day VaR and average of the daily VaR measures on each of the preceding sixty business days multiplied by multiplication factor that can range from 3 to 4.

Analysis of VaR values showed relatively stable results that were changing from day to day by less than 1%. Between analyzed VaR measures, there was no outlier that would be higher then any of other VaR measures multiplied by 3.

Therefore, for the purpose of analysis of differences between capital adequacy requirement calculated by standardized method and by
internal method we can assume that previous’ day VaR measure can sufficiently substitute for average 60 days VaR.

I calculated Capital requirement according VaR for both extreme values of multiplication factors, i.e. the lowest capital requirement was equal to VaR for given day multiplied by 3 and the highest possible capital requirement was equal to VaR for given day multiplied by 4.

Capital requirements relative amount for each method differs for both portfolios.

Portfolio No. 1 with much higher interest rate risk shows the lowest capital requirement when using maturity method. Capital requirement calculated according VaR is similar to the capital requirement calculated by duration method; the lowest possible capital requirement calculated by VaR is lower than capital requirement calculated by duration method and the highest possible capital requirement according VaR is higher than capital requirement according duration method. In this case maturity method is the most favourable for the overall capital requirement calculation; we might suppose that it is the least sensitive way of measuring interest rate risk. Therefore, for the portfolios with high risk, maturity method can be a way how to reach the best results (after additional testing and only if we omit additional advantages of VaR method).

On the other hand, the lowest capital requirement for portfolio No. 2 can be reached by using VaR for calculation. Even if the VaR was multiplied by 4 (i.e. in the case its back-testing showed lower reliability, see chapter 5.4.2 Back testing required by supervisory authorities), capital requirement would be approximately 1/2 of the capital requirement calculated according duration method and approximately 1.3 of capital requirement according maturity method. It seems that capital requirement calculated according VaR is more sensitive to riskiness of portfolio than maturity method; it higher than
capital requirement in maturity method in the case portfolio is riskier and it is significantly lower than capital requirement according maturity method for portfolio with very low interest rate risk. Capital requirements for portfolio No. 1 are displayed in the tables below.

Table 11: Capital requirements for Portfolio #1 (in '000 CZK)

<table>
<thead>
<tr>
<th>Date</th>
<th>Minimum VaR capital requirement</th>
<th>Maximum VaR capital requirement</th>
<th>Maturity method capital requirement</th>
<th>Duration method capital requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3.2004</td>
<td>9.138</td>
<td>12.185</td>
<td>7.939</td>
<td>10.411</td>
</tr>
<tr>
<td>4.3.2004</td>
<td>9.064</td>
<td>12.085</td>
<td>7.940</td>
<td>10.411</td>
</tr>
<tr>
<td>5.3.2004</td>
<td>8.958</td>
<td>11.944</td>
<td>7.939</td>
<td>10.415</td>
</tr>
<tr>
<td>8.3.2004</td>
<td>8.904</td>
<td>11.872</td>
<td>7.939</td>
<td>10.410</td>
</tr>
<tr>
<td>10.3.2004</td>
<td>8.884</td>
<td>11.846</td>
<td>7.939</td>
<td>10.401</td>
</tr>
<tr>
<td>11.3.2004</td>
<td>8.839</td>
<td>11.786</td>
<td>7.939</td>
<td>10.400</td>
</tr>
<tr>
<td>12.3.2004</td>
<td>8.802</td>
<td>11.735</td>
<td>7.939</td>
<td>10.399</td>
</tr>
</tbody>
</table>

Table 12: Relative difference between capital requirements for Portfolio #1 (in ‘000 CZK)

<table>
<thead>
<tr>
<th>Date</th>
<th>Maturity method &amp; VaR</th>
<th>Duration method &amp; VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% Difference (Min VaR cap. req. - Maturity cap. req.)</td>
<td>% Difference (Max VaR cap. req. - Duration cap. req.)</td>
</tr>
<tr>
<td>1.3.2004</td>
<td>24%</td>
<td>66%</td>
</tr>
<tr>
<td>2.3.2004</td>
<td>15%</td>
<td>53%</td>
</tr>
<tr>
<td>3.3.2004</td>
<td>22%</td>
<td>63%</td>
</tr>
<tr>
<td>4.3.2004</td>
<td>14%</td>
<td>52%</td>
</tr>
<tr>
<td>5.3.2004</td>
<td>13%</td>
<td>50%</td>
</tr>
<tr>
<td>8.3.2004</td>
<td>12%</td>
<td>50%</td>
</tr>
<tr>
<td>9.3.2004</td>
<td>12%</td>
<td>49%</td>
</tr>
<tr>
<td>10.3.2004</td>
<td>12%</td>
<td>49%</td>
</tr>
<tr>
<td>11.3.2004</td>
<td>11%</td>
<td>48%</td>
</tr>
<tr>
<td>12.3.2004</td>
<td>11%</td>
<td>48%</td>
</tr>
</tbody>
</table>

Capital requirements for Portfolio No. 2 are displayed in the following tables.
Table 13: Capital requirements for Portfolio #2 (in '000 CZK)

<table>
<thead>
<tr>
<th>Date</th>
<th>Minimum VaR capital requirement</th>
<th>Maximum VaR capital requirement</th>
<th>Maturity method capital requirement</th>
<th>Duration method capital requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3.2004</td>
<td>0.040</td>
<td>0.053</td>
<td>0.175</td>
<td>0.122</td>
</tr>
<tr>
<td>2.3.2004</td>
<td>0.040</td>
<td>0.053</td>
<td>0.171</td>
<td>0.122</td>
</tr>
<tr>
<td>3.3.2004</td>
<td>0.039</td>
<td>0.052</td>
<td>0.171</td>
<td>0.121</td>
</tr>
<tr>
<td>4.3.2004</td>
<td>0.039</td>
<td>0.052</td>
<td>0.171</td>
<td>0.297</td>
</tr>
<tr>
<td>5.3.2004</td>
<td>0.039</td>
<td>0.052</td>
<td>0.171</td>
<td>0.294</td>
</tr>
<tr>
<td>8.3.2004</td>
<td>0.042</td>
<td>0.056</td>
<td>0.565</td>
<td>0.290</td>
</tr>
<tr>
<td>9.3.2004</td>
<td>0.044</td>
<td>0.059</td>
<td>0.565</td>
<td>0.288</td>
</tr>
<tr>
<td>10.3.2004</td>
<td>0.046</td>
<td>0.062</td>
<td>0.565</td>
<td>0.287</td>
</tr>
<tr>
<td>11.3.2004</td>
<td>0.048</td>
<td>0.064</td>
<td>0.565</td>
<td>0.286</td>
</tr>
<tr>
<td>12.3.2004</td>
<td>0.048</td>
<td>0.064</td>
<td>0.565</td>
<td>0.284</td>
</tr>
</tbody>
</table>

Table 14: Relative difference between capital requirements for Portfolio #2 (in '000 CZK)

<table>
<thead>
<tr>
<th>Date</th>
<th>Maturity method &amp; VaR</th>
<th>Duration method &amp; VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% Difference (Min VaR cap. req. - Maturity cap. req.)</td>
<td>% Difference (Max VaR cap. req. - Maturity cap. req.)</td>
</tr>
<tr>
<td>1.3.2004</td>
<td>-77%</td>
<td>-70%</td>
</tr>
<tr>
<td>2.3.2004</td>
<td>-77%</td>
<td>-69%</td>
</tr>
<tr>
<td>3.3.2004</td>
<td>-77%</td>
<td>-69%</td>
</tr>
<tr>
<td>4.3.2004</td>
<td>-77%</td>
<td>-70%</td>
</tr>
<tr>
<td>5.3.2004</td>
<td>-77%</td>
<td>-70%</td>
</tr>
<tr>
<td>8.3.2004</td>
<td>-93%</td>
<td>-89%</td>
</tr>
<tr>
<td>9.3.2004</td>
<td>-92%</td>
<td>-89%</td>
</tr>
<tr>
<td>10.3.2004</td>
<td>-92%</td>
<td>-89%</td>
</tr>
<tr>
<td>11.3.2004</td>
<td>-91%</td>
<td>-89%</td>
</tr>
<tr>
<td>12.3.2004</td>
<td>-91%</td>
<td>-89%</td>
</tr>
</tbody>
</table>

Capital requirement reflects in both portfolios their riskiness; even though portfolios’ assets and liabilities have the same notional value, capital requirement is significantly higher (from 14% to 230%) between each of them. The biggest difference between capital requirements for both portfolios is shown by VaR method, the smallest difference between capital requirements shows maturity method.

6.3.3 Change in capital adequacy requirement caused by market factors change

When calculating VaR according Variance-covariance method and corresponding capital requirement, the major factor determining...
capital adequacy requirement is the set of market volatilities and corresponding correlations.

Even though I used VaR - Historical simulation method to determine capital requirement for two examined portfolios, I calculated also VaR according Variance-covariance method with the intent to find out how capital requirement will change after changing the set of data used for calculating volatilities and correlations.

First, I calculated set of VaR measures for both portfolios by using set of market interest rates from the period starting 25.8.2000 and ending on the date of calculation (i.e. from 1.3.2004 to 12.3.2004). It was the longest period for which I could find quotations for all the point of yield curve.

After that I calculated VaR measures for both portfolios by using “old” set of data, simulating the situation when a bank would not change market data sets for two years and when a bank uses only 1 year history for volatility and correlation calculation. This is not allowed by any supervisory authority (a bank can use the same data set only for three months) but verification of the correct bank procedure is difficult.

The third evaluated case was simulation of the procedure fully supported by supervisory authorities, i.e. the case when data sets are not modified for 3 months and only 1 year history is used for volatility and correlation calculations.

After calculating Variance-covariance VaR for all three cases I compared resulting figures and calculated possible differences in capital requirement corresponding to each examined case.

When comparing final capital requirements, I took as a basis capital requirement based on the longest data set. Results then showed that shortening of historical period brings significant reduction in calculated volatilities and correlations and therefore also significant
reduction in capital requirement. The reduction ranged for Portfolio No. 1 from 2% to more than 40% in both alternative scenarios (i.e. short historic periods). The reduction of capital requirement for Portfolio No. 2 ranged from 30% to 70% in both alternative scenarios. Such relatively large difference in capital requirement shows importance of internal bank research that has to set up appropriate time period for data sets used in the model. But even after careful evaluation of period used for volatility and correlation calculations, banks have to re-examine regularly suitability of chosen time period and adjust it in the case market conditions change. Drawback of this approach is obvious: after regulators approve the model, it can change significantly only by choosing different time period for volatility and correlation calculation.
7 Results summarization

Evaluated portfolios have significantly different interest rate risk and significantly different capital requirements from each other according all the methods used. VaR method seems the most sensitive method from the three methods evaluated. VaR method reflects the most market risk; maturity method on the other hand is the least sensitive. VaR method for the portfolio with high interest rate risk does not bring important decrease in capital requirement. However, the decrease in capital requirement for portfolio with lower interest rate risk is significant.

Moreover, further capital requirements savings could be expected after adding more than one risk; since the only risk factors used for the analysis - interest rates - are positively correlated. We can assume that adding other risk factors would bring savings in capital requirement for the whole portfolio by taking advantage of risk diversification.

However, calculation brings several open questions and concerns related to use of VaR models for capital adequacy calculation. Both concerns relate to the ability of regulators to approve the model. First, approval is very much dependent on model parameters (i.e. on the time period for calculating volatilities and correlations) during the period of evaluation. After these parameters change, model and its ability to correctly measure capital requirement changes as well. However, national supervisors are in no position to re-examine each internal model every times its parameters change. Therefore, banks could in such a case use models that have low ability to correctly calculate capital requirements.

Second, even though regulators are approving model and know what data sets are used for model’s parameterization, check of model
appropriateness is difficult from several reasons. The main ones are:

low possibility to replicate bank portfolio in supervisors’ calculation tool due to the volume of data and complexity of calculations that can easily cause errors during calculations. Even in very small portfolios, results of VaR calculated according different methods (i.e. Variance-covariance and Historical simulation) can differ significantly. Correctness of capital charge calculation for large portfolios than becomes nearly impossible to verify.

The only way how to ensure that banks’ model appropriately measures risk and related capital requirement, is to check bank’s back-testing results. Back-testing is the only way how to ensure that models’ results correspond to risk undertaken by a bank and, therefore, the only way how to ensure that capital requirements calculated according internal models are calculated correctly.
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Právní předpisy ČR:
Opatření České národní banky č. 2 ze dne 3. června 2002 o kapitálové přiměřenosti bank a dalších pravidel obezřetného podnikání na individuálním základě
Opatření České národní banky č. 3 ze dne 28. června 1999 o kapitálové přiměřenosti bank zahrnující úvěrové a tržní riziko
Opatření ČNB č. 4 ze dne 30. července 2002 o řízení tržních rizik v bankách

Czech National Bank Regulation No. 333 to specify prudential behaviour of banks controlling other subjects on consolidated basis, July 3, 2002 (Vyhláška č. 333 České národní banky ze dne 3. července 2002, kterou se stanoví pravidla obezřetného podnikání ovládajících osob na konsolidovaném základě)