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Committed to Deficit: The Reverse Side of Fiscal Governance
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Committed to Deficit: The Reverse Side of Fiscal Governance

MARTIN GREGOR*

Abstract

Common wisdom dictates that fiscal governance (i.e., procedural fiscal rules) improves fiscal discipline. We rather find that selected fiscal constraints protect the coalitional status quo from logrolling. In effect, fiscal governance may deteriorate fiscal position.

In political economy with heterogeneous agents, we examine four procedural fiscal rules: limits on amendments in legislative committees, timing of a vote on the budget size, deficit targets, and spending level targets. We find that fiscal governance protects the budgetary contract of governing coalition from attractive compromises with the opposition. When parties are evenly distributed across single policy dimension, and minimum winning connected coalitions are equiprobable, this protection is shown to magnify volatility in taxes and spending. Moreover, the volatility may increase in more fragmented party systems. We conclude fiscal governance not always and not necessarily reduces fiscal costs of fragmentation.

Keywords: Fiscal Governance, Party Fragmentation

JEL Classification: D78, H61, H62

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1 Introduction

A large part of political economics examines fiscal outcomes in democratic political markets, with emphasis on deficit and spending biases (see Imbeau [9] for an up-to-date survey). The biases naturally motivate macroeconomists and public economists to design fiscal rules so as to eliminate accompanying inefficiencies. Among rules under consideration, there is a subset of procedural fiscal rules (a.k.a. fiscal governance), which are often advocated by practitioners as relatively uncontroversial. Contrary to, say, balance-budget requirement, procedural changes need not to overcome ex-ante political constraints, unless they involve redistribution of power on massive scale.

Fiscal governance comprises a very rich set of rules, ranging from power of Finance Minister to the presence of independent statistical forecasts in budgeting. The aboundness might be a weakness, however. Firstly, global fiscal governance indices suffer from measurement problems with a pending problem of weighting each criterion. Secondly, although the first-generation research by von Hagen [13] demonstrated significant impact of fiscal procedures on deficit and public debt, later studies conclude only “... that budget institutions affect fiscal policy outcomes, but the effect is quite small.” (de Haan et al. [3], p. 284)

Even more than empirics, the theoretical part of fiscal governance approach has been waiting for precision. Typically, it refers to common-pool resource (CPR) problem, which arises when policy-makers consider full benefits of their spending on their constituencies but only part of the tax burden. Followers of the CPR argument, such as Hallerberg [7], argue that any coordination, remediing this competitive negative externality game, costlessly improves on efficiency. More advanced modelling of budgeting by Dharmaphala [4] and Primo [11] nonetheless feature a virtual plethora of results; CPR is but one feature present in budgeting.

More to that, two basic modes of fiscal governance have been identified by Hallerberg and von Hagen [8]. In states where one-party government is the norm, centralization can be achieved by delegating strong agenda-setting powers to the finance minister (Delegation). Highly fragmented governing coalitions in contrary require commitments to fiscal targets negotiated among the coalition partners (Commitment). Fragmentation thus conditions effects of various procedural rules on budgetary outcomes.

In this paper, we attempt to shed new light on fiscal governance, complementing the traditional CPR approach. We analyze cases when budgeting does not feature tragedy of commons, but political contest of parties with different priorities on total budgets. The paper has a twofold aim—develop a framework for spatial modelling of procedural fiscal rules, and demonstrate
their potentially adverse effects.

We distinguish between two cases. In Basic Case, we provide with a full microeconomic foundation of political demand at the disadvantage of having only eight simple institutional configurations (Section 2). On the other hand, for Basic Case we are able to derive explicit results on how fiscal governance affects coalitional contract stability. Classic Case, based on classic specifications in spatial modelling (Euclidean preferences), is introduced in an ensuing section. Here, a richer institutional setting with fifteen configurations is achieved at the expense of implicit microeconomic foundings. We derive results of Classic Case in the general form and illustrate them in a particular setting. Section 4, embedded in Basic Case, finally explores how increased party fragmentation affects fiscal volatility. The last and the least Section 5 concludes.

2 Basic Case

2.1 Assumptions

2.1.1 Citizens

Assume a population of citizens \( C \) in an economy without production. (We need taxes to have zero distortionary effect, so it is sufficient to assume no production.) The population lives in two periods \( t \in \{1, 2\} \). In the beginning of each period \( t \), each individual \( c \in C \) is endowed with income \( y_{c,t} \). We shall denote the average income \( \hat{y}_t := E(y_t) \). In each period, incomes are taxed by flat tax \( \tau_t \in T := (0, 1) \) and the citizens use the after-tax profit only for private consumption in that period. Tax revenues of both periods cover production of public good in both periods \( g_t \in G := (0, 2) \), satisfying intertemporal budgetary balance requirement (BBR) with interest rate \( r \):

\[
g_1 + \frac{g_2}{1+r} = \tau_1 \hat{y}_1 + \frac{\tau_2 \hat{y}_2}{1+r}
\]

(1)

In other words, neither private saving nor lending is permitted; only government has access to financial markets.

The citizens have Cobb-Douglas utility function with private and public good consumption in the argument and individually propensity to consume private good \( \alpha_c \in (0, 1) \):

\[
u_{c,t} = ((1 - \tau_t) y_{c,t})^{\alpha_c} g_t^{1 - \alpha_c}
\]

(2)

The lifetime utility is a present value of discounted utilities over two periods, with individual discount rate \( \delta_c \). Individuals are of different age,
which reflects an additional variable $p_c$, defined as the probability of surviving the second period.\(^1\) Lifetime utility is accordingly:

$$U_c = u_{c,1} + p_c \delta_c u_{c,2}$$

In this framework, BBR in Equation (1) allows to have public deficit in the first period as long as it is ultimately balanced.

**Definition 1 (Relative deficit and spending)** Define $b \in B : = (-1,1)$ as the deficit relative to revenues, i.e. the proportion of second-period tax revenues used for the first-period public consumption:

$$b := \frac{g_1 - \tau_1 \hat{y}_1}{\tau_2 \hat{y}_2 / (1 + r)}$$

Spending relative to national income is to be defined as $\gamma := g_1 / \hat{y}_1$.

With this notation of budget deficit, we can express public good consumption in both periods:

$$g_1 = \tau_1 \hat{y}_1 + b \tau_2 \hat{y}_2 \quad \frac{1}{1 + r} 
\quad g_2 = (1 - b) \tau_2 \hat{y}_2 \quad \quad \quad (3)$$

In order to manage the analysis, let us impose reasonable simplifications.

**Definition 2 (Simplifications)** Hereafter, assume constant tax rate in both periods, homogenous discount rate in the population equal to banker’s discount rate, and endowments in both periods constant in real terms:

$$\delta := \delta_c = \frac{1}{1 + r} \quad \quad \quad y_{c,2} = (1 + r) y_{c,1} \quad \quad \quad \tau := \tau_1 = \tau_2 \quad \quad \quad (4)$$

As a result, the lifetime utility function simplifies as follows:

$$U_c(\alpha_c, p_c, \tau, b) = \frac{(1 - \tau)^{\alpha_c} y_c^{\alpha_c}}{\tau^{\alpha_c - 1} \hat{y}^{\alpha_c - 1}} \left[ (1 + b)^{1 - \alpha_c} + p_c (1 - b)^{1 - \alpha_c} \right] \quad \quad \quad (5)$$

\(^1\)Since individuals exit probabilistically (which can be interpreted as gradual exit), there is a legitimate question what happens to private assets left for private consumption but unconsumed. We assume they entirely disappear from the economy. (For instance, all bequests fall into an external pool such as foreign-aid charity). Otherwise we would have to specify how bequests are distributed, which arguably affects equilibrium tax and deficit levels. We put this effect aside for an extended version of the paper, although we expect that the deficit levels would only slightly increase in the anticipation of additional revenues.
By Definitions 1 and 2, $b = (g_1 - \tau \hat{y})/\tau \hat{y} = (\gamma - \tau)/\tau$ and $\gamma = (1 + b)\tau$. This helps us to understand that total tax revenues at present value $2\tau \hat{y}$ are divided into public consumption in both periods by the following ratio:

$$\frac{g_1}{\gamma} = \frac{(1 + b)\tau \hat{y}(1 + r)}{(1 - b)(1 + r)\tau \hat{y}} = \frac{1 + b}{1 - b} = \frac{\gamma}{2\tau - \gamma}$$

The utility function re-writes alternatively as:

$$U_c(\alpha_c, p_c, \tau, \gamma) = \left(1 - \frac{\tau}{\hat{y}}\right)^{\alpha_c} y_c^{\alpha_c} \left[\gamma^{1-\alpha_c} + p_c(2\tau - \gamma)^{1-\alpha_c}\right]$$

(6)

### 2.2 Bliss points

With utility function expressed in Equations (5) and (6), it is easy to derive individually optimal tax and deficit levels. These are, in other words, individually-specific bliss points.

**Proposition 1 (Bliss points in Basic Case)** The optimal tax, deficit, and spending levels for each individual $c \in C$ for unconstrained $\tau \in T$, $b \in B$ and $\gamma \in G$ are given as:

$$\tau^*_c = 1 - \alpha_c$$

$$b^*_c = \frac{1 - \alpha_c \sqrt{p_c}}{1 + \alpha_c \sqrt{p_c}}$$

$$\gamma^*_c = \frac{2(1 - \alpha_c)}{1 + \alpha_c \sqrt{p_c}}$$

(7)

Notice individuals differ in endowment $y_c$, preference for private consumption $\alpha_c$, and survival rate $p_c$. However, Cobb-Douglas utility function allows to disregard income effect on relative demand for private vs. public consumption. Proposition 1 indeed shows that changes in endowment don’t affect global political demand of individuals, so individuals can be classified by merely two variables, the relative preference for private goods $\alpha_c$ and the probability of survival $p_c$.

### 2.2.1 Parties

Suppose a set of parties $N$, where each party $i \in N$ is a set of successful policy-seeking citizen-candidates who are committed to a common policy platform. The platform is to be given and stable during the budget process, which allows us to avoid issues of party formation and (re)election. Moreover, the policy platform is defined as a utility function expressed in Equation (6), with bliss points described in Proposition 1 below. (This notion can be motivated, for example, by reference to a median citizen-candidate in each party.)
To reflect heterogenous voting power and specify coalition formation, we introduce $s_i$ as the number of citizen-candidates (i.e. seats in the legislature) of party $i$, where $s_i \in \mathbb{N}, s_i > 0$. Thus, each party can be described by $\alpha_i$, $p_i$ and $s_i$.

Any non-empty subset of parties $S \subseteq N$ we shall call a coalition. By $R(N)$, denote the set of all subsets of $N$, i.e. the set of all $S$. Given allocation of seats $s$ and required majority (quota) $q$ (where $\sum_{i \in N} s_i/2 < q \leq \sum_{i \in N} s_i$), we say a coalition $S$ is a winning coalition, if it gathers support of at least $q$ votes, i.e. $\sum_{j \in S} s_j \geq q$. Note that winning coalitions not necessarily need to be minimum-winning, nor connected.2

### 2.3 Constrained political optima

In order to solve budgetary games as defined below, we have to find optima of political parties under several constraints. First of all, we seek political optima constrained by fixed levels of $\tau$, $b$, and $\gamma$ (to be denoted as $\bar{\tau}$, $\bar{b}$, and $\bar{\gamma}$):

$$
\gamma^*_i(\bar{\gamma}) := \gamma_i^*(\alpha_i, p_i, \gamma)|_{\gamma=\bar{\gamma}} \quad b^*_i(\bar{b}) := b_i^*(\alpha_i, p_i, \tau)|_{\tau=\bar{b}} \\
\tau^*_i(\bar{\tau}) := \tau_i^*(\alpha_i, p_i, \gamma)|_{\gamma=\bar{\gamma}} \quad \tau^*_i(\bar{b}) := \tau_i^*(\alpha_i, p_i, b)|_{b=\bar{b}}
$$

For $\gamma^*_i(\bar{\tau})$, $b^*_i(\bar{\tau})$ and $\tau^*_i(\bar{b})$, we have algebraic solutions in explicit form, whereas for $\tau^*_i(\bar{\gamma})$ we arrived at one in implicit form.

**Proposition 2** For any fixed level of $\bar{\tau} \in T$, the optimal spending and deficit levels in Basic Case are:

$$
\gamma^*_i(\bar{\tau}) = \frac{2\bar{\tau}}{1 + \sqrt{\bar{p}_i}} \quad b^*_i(\bar{\tau}) = b_i^*(\tau)|_{\tau=\bar{\tau}} = 1 - \frac{\sqrt{\bar{p}_i}}{1 + \sqrt{\bar{p}_i}}
$$

For any fixed $\bar{\gamma} \in G$, the optimal tax in Basic Case satisfies:

$$
\tau^*_i(\bar{\gamma}) = 1 - \alpha_i + \frac{\alpha_i \bar{\gamma}}{2p_i} \left[p_i - \left(\frac{2\tau^*_i(\bar{\gamma}) - \bar{\gamma}}{\bar{\gamma}}\right)^{\alpha_i}\right] \quad \gamma \leq 2\tau
$$

For any fixed $\bar{b} \in B$, the optimal tax in Basic Case is constant:

$$
\tau^*_i(\bar{b}) = 1 - \alpha_i
$$

---

2To capture connectedness in brief, define a minimum convex set including all bliss points of coalition members. If that set includes no bliss point of any other party, we call the coalition connected.
2.4 Fiscal governance

Before the budget process is entirely specified, we need to describe budgetary rules. We draw on rules examined in a fiscal governance index by De Haan, Moessen and Volkerink [3] who re-constructed a seminal index by von Hagen [13]. Of the index, we concentrate on two items, namely the position of legislature and the presence of explicit fiscal targets. This is captured by two survey questions posed by De Haan et al. [3]:

5. Could you please indicate which one of the following is the best characterization of the position of the parliament:
   a. Possibility to propose amendments: unlimited/limited
   b. Are these amendments required to be offsetting: yes/no
   c. Can accepted amendments cause fall of the government: yes/no
   d. Are all expenditures passed in one vote: yes/chapter by chapter
   e. Is there a global vote on total budget size: final/initial

6. Could you please indicate whether the government is bound by some general constraint?
   □ None
   □ Public-debt-to-GDP ratio
   □ Public-debt-to-GDP ratio and deficit-to-GDP ratio
   □ Government-spending-to-GDP ratio or Golden Rule
   □ Government-spending-to-GDP ratio and deficit-to-GDP ratio

Clearly, the questions investigate on four procedural rules: Vote on budget size (initial/final), Spending level target (present/absent), Deficit level target (present/absent), and Amendments (any/offsetting/none). These become the rules subject to our concern.

2.5 Budget process

First, let Nature select a winning coalition \( S \in G \). The coalition members bargain over the budget irrespective of development in the legislature, so we may suppose an unconstrained Nash-bargaining solution (in Section 2.7.1, the coalition members begin to consider development in later stages when bargaining over coalitional contract):

\[
[\tau_S, \gamma_S] = \operatorname{argmax} \prod_{j \in S} U_j(\tau_S, \gamma_S) \quad \tau_S \in T \quad \gamma_S \in G, \gamma_S \leq 2\tau_S \quad (8)
\]

The budget process consists of the following steps (nodes):
1. **Fiscal targets.** The coalition $S$ may agree on two aggregate level limits which are enforceable—Spending level ($\gamma = \gamma_S$) and/or Deficit level ($b = b_S$, i.e. $\gamma/\tau = \gamma_S/\tau_S$) targets. Then, the budget goes to the legislature.

2. **Initial vote.** There may be an initial simple-majority vote on the budget size $\gamma$ (unless the spending level is already targeted).

3. **Amendments.** This procedure is left for Classic Case. In Basic Case, we consider only single public good $\gamma$ without any internal structure and possibility of amendments.

4. **Revenue side.** The Parliament votes on tax revenues $\tau$ by simple majority unless both the deficit level and spending level have been set (in that case, tax is set as a residual variable).

5. **Final vote on the budget size.** This node doesn’t realize if the vote has already taken place (see Node 2), or if the spending has been determined residually by deficit target and tax revenues (in Nodes 1 and 4).

We assume status quo (provisory budget, or caretaker government) is prohibitively costly for all participants, so a fully specified budget is always accepted in the last node.

In Basic Case, we combine three binary variables (Vote, Deficit target, Spending target), which results in eight decision-making configurations. Table 1 describes the order of variables to be agreed in each case. In brackets, residual variables are described.

<table>
<thead>
<tr>
<th>Vote/Target</th>
<th>None</th>
<th>Deficit</th>
<th>Spending</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>$\gamma; \tau(b)$</td>
<td>$b_S; \gamma(\tau)$</td>
<td>$\gamma_S; \tau(b)$</td>
<td>$\gamma_S, b_S(\tau_S)$</td>
</tr>
<tr>
<td>Final</td>
<td>$\tau; \gamma(b)$</td>
<td>$b_S; \tau(\gamma)$</td>
<td>$\gamma_S; \tau(b)$</td>
<td>$\gamma_S, b_S(\tau_S)$</td>
</tr>
</tbody>
</table>

### 2.6 Basic Case equilibria

Since we’ve got 8 finite extensive games, we apply backward induction and identify subgame-perfect equilibria. To solve each subcase, a concept of median party is crucial. We obtain that by analogy to the Median Voter Theorem by Black [2]. The theorem states that a median individual is always decisive in electoral stage if policy is located in single dimension and preferences are single-peaked. Although we have a two-dimensional space here
and parties of different size, we can get a median party provided that decisions on either of $\tau$ and $\gamma$ are made separately. This is, indeed, our case. The separation implies that a decision on one dimension is made after the other dimension is fixed. When fixing any first dimension, parties of course anticipate the subsequent decision constrained by the fixed dimension.

We need to analyze how median parties arise in single dimensions. First of all, recall simple majority of votes is required in all nodes of the budget process, so we use the quota $q_{SMV} := |\sum_{i \in N} s_i| + 1$. Consider first the subcase of the Initial vote/No target, when $\gamma$ is set in the first step, $\tau$ in the second step, and the deficit level $b$ is obtained residually (see also in Table 1). For $\gamma$ fixed in the first step for some $\bar{\gamma}$, we get constrained optima in the second step $\tau^*_i(\bar{\gamma})$ for each $i \in N$. We sort parties by the optimal tax constrained by fixed spending and look for a median player. Intuitively, median player has the property that no connected winning subset of parties can do without him or her. For simple majority voting (as in our budget process) such a player can be unique, or there can be two such players. If the player is unique, the legislature coordinates on his/her preferred tax. For two players we assume, without loss of generality, that the lower tax of the two is selected.

**Definition 3 (Medians constrained on fixed levels)** Let the median tax constrained on fixed spending $\tau^G_M : G \to T$ be a function of $\bar{\gamma} \in G$ such that:

$$\sum_{l \in N} s_l < q_{SMV} \land \exists i \in N : \tau^*_l(\bar{\gamma}) = \tau^G_M(\bar{\gamma}) \land s_i + \sum_{l \in N} s_l \geq q_{SMV}$$

The median tax constrained on fixed deficit $\tau^B_M : B \to T$ be a function of $\bar{b} \in B$ such that:

$$\sum_{l \in N} s_l < q_{SMV} \land \exists i \in N : \tau^*_l(\bar{b}) = \tau^B_M(\bar{b}) \land s_i + \sum_{l \in N} s_l \geq q_{SMV}$$

The median spending constrained on fixed tax $\gamma^T_M : T \to G$ be a function of $\bar{\tau} \in T$ such that:

$$\sum_{l \in N} s_l < q_{SMV} \land \exists i \in N : \gamma^*_l(\bar{\tau}) = \gamma^T_M(\bar{\tau}) \land s_i + \sum_{l \in N} s_l \geq q_{SMV}$$

Let us proceed with the example of Initial vote/No target. The importance of median function constrained on fixed level in the second step is anticipated as early as in the first step; in other words, players recognize that
the outcome in the second step cannot be conditioned by anything preceding, for such a promise/threat could not be enforced. (Note that the process ends after the second step, so nothing else but median tax constrained on spending can be credibly promised in the first step to those who decide on expenditures.) So, what is the equilibrium \( \gamma \) chosen in the first step? Since parties anticipate the second stage, they optimize on the set restricted as follows: \( \tau = \tau^G_M(\bar{\gamma}), \tau \in T, \gamma \in G, \gamma \leq 2\tau \). On that set, each party \( i \) has an optimum denoted as \([\tau^G_M(\gamma^T_i), \gamma^T_i]\). So, it would be the first-best for each party to ensure that \( \gamma^T_i \) gets elected in the first step, because a tax the legislature selects on the basis of this spending is the best given the restrictions. However, since \( \gamma^T_i \) are located in a single dimension, it must be again the median player who will be decisive.

This case illuminates how agents in early stages use constraints in order to optimize in next stages. Notice that the median spending is constrained; not on what has happened, but on what is anticipated to happen. By the same token, we derive solutions also for other institutional configurations. For this, we find parties’ optima on the set restricted as follows: \( \gamma = \gamma^T_M(\bar{\tau}), \tau \in T, \gamma \in G, \gamma \leq 2\tau \). We write them as \([\tau^G_i, \gamma^T_M(\tau^G_i)]\). Notation reflects the order of votes (GT for \( \gamma; \tau \) and TG for \( \tau; \gamma \)).

**Definition 4 (Medians constrained by anticipated responses)** Define \( \gamma^G_T \) as the median spending constrained on anticipated \( \tau^G_M \) (i.e. median tax constrained on fixed spending):

\[
\sum_{l \in N} s_i < q^{SMV} \land \exists i \in N : \gamma^T_i = \gamma^G_T \land s_i + \sum_{l \in N} s_i \geq q^{SMV}
\]

Let \( \tau^T_M \) be the median tax constrained on anticipated \( \gamma^T_M \) (i.e. median spending constrained on fixed tax):

\[
\sum_{l \in N} s_i < q^{SMV} \land \exists i \in N : \tau^G_i = \tau^T_M \land s_i + \sum_{l \in N} s_i \geq q^{SMV}
\]

**2.6.1 Median identity**

Note that a single median voter need not exist in an entire definition set when a median constrained on a fixed variable or a median constrained by anticipation is constructed. Simply, players in the early stages not only constrain sets for later stages, but also, at the same time, decide on who will be the next decisive player, which Figure 1 illustrates. On the figure, we have three functions of optimal spending constrained on fixed tax, each for one of
players A, B and C. The highlighted line is the function of median spending constrained on fixed tax. On the left side, we can see that for very low levels of $\tau$, it is B who is decisive, while for higher values it is A or C. For the highest values, it is again B who becomes the median player. On the right side, we get the other case when only one player (B) is decisive regardless of decision on $\tau$ in the preceding stage, so we may call such a median function to be ‘one-player’. For example, the median tax constrained on spending is to be called ‘one-player’ as long as $\exists i \in G, \forall \gamma \in \langle 0, 2 \rangle : \tau^*_{\gamma_i}(\gamma) = \tau^T_M(\gamma)$.

Otherwise, the median tax constrained on spending is a ‘many-player’ median function. By analogy, we would define one-player/many-player median tax constrained on deficit as well as one-player/many-player median spending constrained on tax. Nonetheless, the fact that median players are different adds nothing extra to the problem of finding the subgame-perfect Nash equilibrium strategy profile; it is but an interesting element in the game.

![Figure 1: Many-player median spending vs. One-player median spending](image)

2.6.2 Solutions in the general form

By backward induction, we obtain solution for any institutional configuration.

**Initial vote/No target** We have already described the solution in the preceding subsection to be $[\tau, \gamma] = [\tau^G_M(\gamma), \gamma^G_T(\tau)]$.

**Final vote/No target** In the second stage, we obtain $\gamma^T_M(\tau)$. Definition 3 has introduced tax demands for the first stage, constrained by anticipation of $\gamma^T_M(\tau)$, denoted as $\tau^T_M$. The solution is therefore $[\tau^T_M, \gamma^T_M(\tau^T_M)]$. 

10
**Initial & Final vote/Deficit target**  The deficit contractually set at $b_S$ is fixing the relation between $\gamma$ and $\tau$ such that $\gamma = \tau(1 + b_S)$. The solution is therefore $[\tau_M^B(b_S), (1 + b_S)\tau_M^B(b_S)]$.

**Initial & Final vote/Spending target**  Suffice to use the median tax constrained on fixed spending in order to arrive at the solution $[\tau_M^G(\gamma_S), \gamma_S]$.

**Initial & Final vote/Both targets**  Obviously, the solution is $[\tau_S, \gamma_S]$.

### 2.6.3 Solutions in the specific form

In Section 4 dealing with effect of fragmentation, we employ explicit solutions to be applied in Basic Case, thereby making use of explicit assumptions from Section 2.1. With the exception of Initial vote/No target, we can derive them explicitly. Before doing that, we have to identify two median parties. Denote $A = [\alpha_A, p_A, s_A]$ the party with median value of $\alpha$ and $P = [\alpha_P, p_P, s_P]$ the party with median value of $\sqrt{p_P}$:

$$\sum_{l \in N \quad \alpha_l < \alpha_A} s_l < q_{SMV}^* \wedge s_A + \sum_{l \in N \quad \alpha_l < \alpha_A} s_l \geq q_{SMV}^*$$

$$\sum_{l \in N \quad \alpha_{q\bar{p}}^* < \alpha_l < \alpha_P} s_l < q_{SMV}^* \wedge s_P + \sum_{l \in N \quad \alpha_{q\bar{p}}^* < \alpha_l < \alpha_P} s_l \geq q_{SMV}^*$$

These two median parties may be different but also identical (i.e. we may have a double-median party).

**Proposition 3**  Tax and spending in Basic Case are set as follows:

1. **Final vote/No target case:**

   $$[\tau, \gamma] = \left[1 - \alpha_A, \frac{2(1 - \alpha_A)}{1 + \alpha_S\sqrt{p_P}}\right]$$

2. **Initial & Final vote/Deficit target case:**

   $$[\tau, \gamma] = \left[1 - \alpha_A, \frac{\gamma_S(1 - \alpha_A)}{\tau_S}\right]$$

3. **Initial & Final vote/Spending target case:** [\tau, \gamma] = [\tau_M(\gamma_S), \gamma_S], where

   $$\tau_M^G(\gamma_S) = 1 - \alpha_S + \frac{\alpha_S\gamma_S}{2p_S} \left[p_S - \left(\frac{2\tau_M^G(\gamma_S) - \gamma_S}{\gamma_S}\right)^{\alpha_S}\right]$$

4. **Initial & Final vote/Both targets case:** $[\tau, \gamma] = [\tau_S, \gamma_S]$
2.6.4 Illustration

Outcomes put in Proposition 3 can be illustrated on four different coalitional contracts $S_1, \ldots, S_4$ in space $T \times G$. Figure 2 features the contracts as well as bliss points of median parties, A and P. Having coalitional contracts and median preferences, we are able to get solutions in each institutional configuration. For Final vote/No target subcase, we denote the solution as F (recall by Proposition 3, F is stable irrespective of coalition); it is the situation when median parties A and P can exploit their power the most. The solutions for Deficit target subcase are $D_1, \ldots, D_4$ and for Spending target subcase $T_1, \ldots, T_4$. When both targets are set, the solutions are the initial contracts $S_1, \ldots, S_4$.

The figure illuminates several facts. In the absence of fiscal governance, the median parties A and P can exploit their power and always establish F. When partial fiscal governance is present, solutions differ tremendously; the initial contract can be either utterly destroyed ($T_2$ vs. $S_2$), or only slightly compromised ($T_4$ vs. $S_4$). When fiscal governance is complete, coalitional contracts are perfectly stable and we get $S_1, \ldots, S_4$. The case $D_2$ reflects the intuition in the title of the paper: parties “commit to deficit” and the spending grows dramatically comparing to zero-governance mode F. By analogy, “commitment to spending effect” occurs in case of $T_2$, when deficit is close to deficit in F, but spending significantly grows.
2.7 Endogenous coalitional contract

In the beginning, we assumed coalitional bargaining over an unrestricted space $T \times G, \gamma \leq 2\tau$, which led to an unconstrained Nash-bargaining solution. However, the coalition parties know that in some configurations, their contract may be unsustainable. Why should they agree to a contract whose terms cannot be maintained? We shall distinguish between three cases. Firstly, if no part of the contract can be preserved, the contract is irrelevant (cheap talk), and coalitional agreement is irrelevant. Quite contrary, when parties commit to two targets, the contract is protected as the whole, so the unconstrained Nash-bargaining solution is stable.

The only analytically interesting extension emerges when the contract is partially unsustainable, which occurs when exactly one fiscal target (i.e. deficit or spending level) is subject to contract. We suggest here how to refine solutions of two cases on the grounds of the anticipation of partial unsustainability.

Initial & Final vote/Deficit target In the 2nd stage, $\tau^B_M(b_S)$ is selected. Hence:

$$b_S = \arg\max_{j \in S} \prod_{j \in S} U_j[\tau^B_M(b), (1 + b)\tau^B_M(b)]$$

Initial & Final vote/Spending target In the 2nd stage, $\tau^G_M(\gamma_S)$ is selected. Hence:

$$\gamma_S = \arg\max_{j \in S} \prod_{j \in S} U_j[\tau^G_M(\gamma), \gamma]$$

3 Classic Case

The next step in the analysis of fiscal governance is to introduce different types of expenditures, which allows to tackle the effect of amendment rules. Suppose we have two public goods (with expenditure ratio $x \in G$ and $w \in G$) and all public good expenditures $\gamma$ must be allocated into $x$ and $w$ so that $x + w = \gamma$.

To be able to analyze the system, we will assume Euclidean preferences in $T \times G^2$, and thereby refrain from Cobb-Douglas utility function introduced in (5). We follow spatial approach to political economy which typically assumes Euclidean preferences, specifically in space $T \times G$ (see Balassone and Giordano [1] as well as classic Ferejohn and Krehbiel [6]); because of that, we call the following model a Classic Case. The big disadvantage of Classic Case is that it is actually very difficult to realize how a realistic tax system
(e.g. one with proportional tax rate) and standard assumptions imposed on utility functions would result in circular preferences. For extensive discussion on spatial models encountering these limitations, see Milyo [10].

The main advantage of Euclidean preferences are algebraic properties which we intend to exploit here. For instance, $\gamma_G^M(\bar{\gamma})$ and $\gamma^T_M(\bar{\tau})$ are constants.

In this section, we extend the spatial literature by postulating preferences in three-dimensional space, which we ensure through the following assumptions:

$$\forall x = \bar{x} : U_i(\tau, \bar{x}, w) - U_i(\tau^*_i, \bar{x}, w^*_i) = (\tau - \tau^*_i)^2 + [w - (x^*_i + w^*_i - \bar{x})]^2$$
$$\forall \tau = \bar{\tau} : U_i(\bar{\tau}, x, w) - U_i(\bar{\tau}, x^*_i, w^*_i) = 2(x - x^*_i)^2 + 2(x - x^*_i)(w - w^*_i) + (w - w^*_i)^2$$

This satisfies that circular preferences are achieved in subspace $T \times G$ for any $\tau$ and in subspace $G^2$ for any $\tau$. Figures 3 and 4 reveal how Classic Case differs from Basic Case in subspace $T \times G$.

![Figure 3: Utility functions in Classic Case](image)

### 3.1 Institutional configurations

The division of budget into two public goods is a way to study amendment part of budgeting. If amendments are permitted, we assume that the first committee votes on $x$. Then, if there is no spending limit (imposed either
by targeted spending, initial vote or offsetting requirement), the second committee votes on \( w \). Otherwise, the second committee doesn’t vote. It is also possible that amendments are prohibited as such and no committee votes at all.

By combining three binary variables (Spending target, Deficit target, and Final vote) and one trinomial variable (Amendment), we study 24 institutional compositions instead of only 5 in Basic Case. However, in three cases when amendments are banned, total spending can differ from contracted \( \gamma_S \), and it is necessary to define how the budgetary structure adjusts. This unfortunately requires that amendment procedure must take place, so we skip these three cases as logically inconsistent. Furthermore, in some combinations we receive identical nodes, thus an identical game tree. Hence, we have in the end only 10 different node combinations (see Table 2).

Table 2: Decision-making nodes in Classic Case

<table>
<thead>
<tr>
<th>Amendment Target</th>
<th>Initial vote</th>
<th>Deficit</th>
<th>Spending</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>( \gamma; x(w); \tau )</td>
<td>( b_S; \gamma(\tau); x(w) )</td>
<td>( \gamma_S; x(w); \tau )</td>
<td>( b_S; \gamma_S(\tau); x(w) )</td>
</tr>
<tr>
<td>Any</td>
<td>( \gamma; x(w); \tau )</td>
<td>( b_S; \gamma(\tau); x(w) )</td>
<td>( \gamma_S; x(w); \tau )</td>
<td>( b_S; \gamma_S(\tau); x(w) )</td>
</tr>
</tbody>
</table>

Figure 4: Utility functions in Basic Case
Table 2: Decision-making nodes in Classic Case

<table>
<thead>
<tr>
<th>Final vote</th>
<th>Amendment</th>
<th>Polling</th>
<th>Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Any, Offset</td>
<td>$x; w(\gamma); \tau$</td>
<td>$b_S; x; w(\gamma, \tau)$</td>
<td>$\gamma_S; x(w); \tau$</td>
</tr>
<tr>
<td>Offsetting, None</td>
<td>$x; \tau; \gamma(w)$</td>
<td>$b_S; x; \tau(\gamma, w)$</td>
<td>$\gamma_S; x(w); \tau$</td>
</tr>
<tr>
<td>Offset</td>
<td>$\gamma_S(s, w_S); \tau$</td>
<td>$b_S; \gamma_S(\tau_S, s, x_S, w_S)$</td>
<td></td>
</tr>
</tbody>
</table>

Our interest rests with the properties of fiscal governance indices, so we construct a basic index $F \in \{0, \ldots, 5\}$, which is a sum of the Amendment part (0 for any, 1 for offsetting, 2 for none), Final vote part (0 for initial, 1 for final)\(^3\), and Targets (0 for none, 1 for deficit, 1 for spending, 2 for both). Table 3 comprises values of $F$ for each institutional case.

Table 3: Fiscal governance index ($F$)

<table>
<thead>
<tr>
<th>Vote</th>
<th>Amendment</th>
<th>None</th>
<th>Deficit</th>
<th>Spending</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>Offsetting</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Final</td>
<td>Offsetting</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>None</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

3.2 Classic Case equilibria

This section shows how to identify solutions for each mode of fiscal governance when the budget is structured. We derive general solutions regardless of properties of utility function assumed in Classic Case so as to have a tool applicable beyond the scope of this paper. Doing so, we must introduce several new variables; notice that their notation always reflects the order of nodes. For instance, when the order of node is $\gamma; x; \tau$, we use superscript $GXT$ (for budget deficit target, we shall use $B$, and for spending target $S$).

3.2.1 Solutions in the general form

Initial vote/No target/Any & Offsetting amendments (GXT) Solving backwards, we firstly derive the tax. Optimal $\tau$ constrained on fixed

\(^3\)In the case of Vote on budget size, we follow a non-intuitive result derived theoretically by Ferejohn and Krehbiel [6] and experimentally by Ehrhart et. al [5]: initial vote on global budget actually weakens the coalitional contract.
spending (as well as on fixed composition of spending) shall be found as $\tau^*_i|_{x=\bar{x},\bar{w}=\bar{w}}$, with median value $\tau^*_{\text{GXT}}(\bar{x},\bar{w})$. In the preceding (second) step, politicians set spending composition. They are constrained by fixed spending from the first step, and anticipated tax in the final step. Individual optima can be received by $x^*_i|_{\gamma=\bar{\gamma},\tau=\bar{\tau}^*_{\text{GXT}}}$, with median value $x^*_{\text{GXT}}(\bar{\gamma})$.

Finally, we derive how spending is set in the anticipation of $x^*_{\text{GXT}}(\bar{\gamma})$ and $\tau^*_{\text{GXT}}(\bar{x},\bar{w})$. We obtain individually optimal spending by optimizing $\gamma_i^*|_{x=x^*_{\text{GXT}},\tau=\tau^*_{\text{GXT}}}$, and the median is $\bar{\gamma}^*_{\text{GXT}}$. The solution we arrive at is $[\tau; \gamma; x] = [\tau^*_{\text{GXT}}(\bar{\gamma}^*_M, x^*_{\text{GXT}}); \gamma^*_{\text{GXT}}(\bar{x}, x^*_{\text{GXT}})]$.

**Final vote/No target/Any amendment (XGT)**

Obviously, $\tau^*_{\text{XGT}}(\bar{x}, \bar{w}) := \tau^*_{\text{GXT}}(\bar{x}, \bar{w})$. By analogy to the previous case, we derive $\gamma^*_{\text{XGT}}(\bar{x})$ and get the optimal value in the first step $x^*_{\text{XGT}}$.

The solution is $[\tau; \gamma; x] = [\tau^*_{\text{XGT}}(\bar{\gamma}^*_M, x^*_{\text{XGT}}); \gamma^*_{\text{XGT}}(x^*_{\text{XGT}}); x^*_{\text{XGT}}]$.

**Final vote/No target/Offsetting amendment (XTG)**

By analogy, we find $\gamma^*_{\text{XTG}}(\bar{x}, \bar{\tau})$ for the third step. In the second step, we look for $\tau^*_{\text{XTG}}(\bar{x})$, which allows to specify median value $x^*_{\text{XTG}}$ in the initial node. The solution is $[\tau; \gamma; x] = [\tau^*_{\text{XTG}}(x^*_{\text{XTG}}); \gamma^*_{\text{XTG}}(x^*_{\text{XTG}}); x^*_{\text{XTG}}]$.

**Initial vote/Deficit target/Any & Offsetting amendments (BGX)**

Two variables are under control of the legislature. In the last step, parties select $x^*_{\text{BGX}}(\bar{\gamma}, b_s)$. The median spending in the preceding node is $\gamma^*_{\text{BGX}}(b_s)$, and the solution writes $[\tau; \gamma; x] = [(1 + b_s)\gamma^*_{\text{BGX}}(b_s); \gamma^*_{\text{BGX}}(b_s); x^*_{\text{BGX}}(\bar{\gamma}^*_M, b_s)]$.

**Final vote/Deficit target/Any & Offsetting amendments (BXG)**

In the first step, parties choose $x$ anticipating selection of $\gamma$ in the second step. This case is very much similar to the previous case. We denote the median spending in the last step as $\gamma^*_{\text{BXG}}(\bar{x}, b_s)$. The median composition adopted in the previous step is $x^*_{\text{BXG}}(b_s)$. We get the solution as follows: $[\tau; \gamma; x] = [(1 + b_s)\gamma^*_{\text{BXG}}(b_s); \gamma^*_{\text{BXG}}(b_s); x^*_{\text{BXG}}]$.

**Initial & Final vote/Spending target/Any & Offsetting amendments (SXT)**

Like in the first two configurations, we seek for tax constrained by fixed spending and composition, thus $\tau^*_{\text{SXT}}(\bar{x}, \bar{w}) := \tau^*_{\text{SXT}}(\bar{x}, \bar{w})$. To find median decision on composition of spending, we simply insert $\gamma$ and have $x^*_{\text{SXT}} := x^*_{\text{GXT}}(\gamma_S)$, so the solution is $[\tau; \gamma; x] = [\tau^*_{\text{SXT}}(\gamma_S, x^*_{\text{SXT}}); \gamma_S; x^*_{\text{SXT}}]$.

**Initial & Final vote/Spending target/No amendment (ST)**

Obviously, the solution is $[\tau; \gamma; x] = [\tau^*_{\text{ST}}(\gamma_S; x_S); \gamma_S, x_S]$.  
Initial & Final vote/Both targets/Any & Offsetting amendments (BSX) The only variable set in the legislature is composition of spending, reflecting pre-determined $\gamma_S$ and $\tau_S$. We define median value simply as $x_M^{BSX}$ and achieve solution $[\tau; \gamma; x] = [\tau_S; \gamma_S; x_M^{BSX}]$.

Initial & Final vote/Both targets/No amendments (BS) The agreement of the coalition is preserved in total, and we receive $[\tau; \gamma; x] = [\tau_S; \gamma_S; x_S]$.

3.2.2 Illustration

Euclidean preference allow to provide the general solution in a very simple manner. This is to be illustrated on a special case. Suppose three parties (A, B, and C) are located in space $T \times G^2$ and Nature selects A+B as the winning coalition. Since majority of median functions constrained on fixed variables and median functions constrained by anticipation are constant, we derive solutions very easily. These are depicted in Figures 5–8 and summarized in Tables 4 and 5 as well as in Figure 9.

Table 4: Tax & Spending allocations

<table>
<thead>
<tr>
<th>Vote</th>
<th>Amendment</th>
<th>No target</th>
<th>Deficit</th>
<th>Spending</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Offset</td>
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<td>E</td>
<td>F</td>
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</tr>
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<td>E</td>
<td>F</td>
<td>AB</td>
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<tr>
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<td>D</td>
<td>E</td>
<td>F</td>
<td>AB</td>
<td></td>
</tr>
<tr>
<td>Final</td>
<td>Offset</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>AB</td>
</tr>
<tr>
<td>None</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>AB</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Budget structure

<table>
<thead>
<tr>
<th>Vote</th>
<th>Amendment</th>
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<th>Deficit</th>
<th>Spending</th>
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</thead>
<tbody>
<tr>
<td>Initial Offset</td>
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<td>E’</td>
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<td>AB’</td>
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<td>E’</td>
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<td>None</td>
<td>D’</td>
<td>E’</td>
<td>F’</td>
<td>AB’</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5: No target (GXT, XGT, XTG)

Figure 6: Deficit target (BGX, BXG)

Figure 7: Spending target (SXT, ST)
Figure 8: Both targets (BSX, BS)

Figure 9: Summary
Interestingly, the index of fiscal governance again relates to final outcomes. The higher index, the closer is the outcome to the coalitional contract AB (AB’). Table 6 demonstrates this with frequencies and expected values of fiscal governance for each outcome. In other words, we observe that fiscal governance protects the coalitional contract like it did in Basic Case.

Table 6: Frequencies

<table>
<thead>
<tr>
<th>F</th>
<th>E</th>
<th>D</th>
<th>E'</th>
<th>D'</th>
<th>F</th>
<th>E'</th>
<th>D'</th>
<th>F'</th>
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</tr>
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<td>1</td>
<td>2</td>
<td>2 \frac{1}{2}</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

4 Fragmentation

The effects of fragmentation on fiscal outcome have long been subject to intensive research, verifying mainly variants of CPR hypothesis (see Riccuiti [12] for perhaps the most recent contribution to the study of fiscal outcomes and fragmentation).

By CPR hypothesis, the more fragmented is the party system (measured by Herfindahl index), the stronger are incentives to exploit common pool, and worse fiscal outcomes. Moreover, highly fragmented systems suffer from a pronounced problem of collective action during the budget coordination, which makes the Finance Minister less reliable as a ‘non-partial’ budgetary coordinator. Hence, Hallerberg [8] recommends Commitment mode, including e.g. deficit and/or spending targets. To sum up, fiscal targets are quoted as the remedy exactly to the situations of fragmented party systems where fiscal costs are expected to abound.

In this section, we observe different results. In Proposition 3, we already found that fiscal targets increase fiscal volatility. From macroeconomic point of view, volatility is always costlier than stability, so despite reducing CPR incentives, fiscal targets keep extreme fiscal positions unchallenged, thereby amplify costs of fiscal volatility. Quite contrary, in the absence of targets, the budget rests in the position of median political agent regardless of fragmentation and coalition composition.

We deal with two specifications. In the first one, the higher number of parties, the lower standard deviation of coalitional position from the globally
median position. As a result, costs of fiscal volatility increase in fiscal governance but paradoxically decrease in fragmentation; both findings are in stark contrast to CPR hypothesis. In the second specification, the higher fragmentation, the bigger deviation of coalitional position from the global median. Here, fiscal volatility increases both in fiscal governance and fragmentation.

4.1 Fragmentation in $\alpha_i$ (Basic Case)

4.1.1 Assumptions

Assume each party (citizen-candidate) $i \in \{1, \ldots, n\}$ has preferences $[\alpha_i, p_i] = [\alpha_i, K^\alpha_i]$, where $K$ is constant and $a \in (0, 1)$. The citizen-candidates are drawn from the population such as to cover the dimension $\alpha$ in fixed distances, i.e. $\alpha_{i+1} - \alpha_i = \alpha_i - \alpha_{i-1}$. This corresponds, among others, to a case of equal distribution of $\alpha^*_i$ over space $(0, 1)$ and equal electoral support to all parties.

Assume also that only minimum-winning connected coalitions (MWC) can be formed. This might be justified by a small amount of rent distributed in an equal proportion to coalition members. The rent introduces an incentive to create small, namely minimum-winning coalitions. Furthermore, since the payment scheme is fixed, median members have reason to include parties which are connected in an existing coalition (here we can easily claim that a party $x$ is connected to a coalition $S$ if $\min_{i \in S}(\tau^*_i) \leq \tau^*_x \leq \max_{i \in S}(\tau^*_i)$).

We can represent preference of parties in a single dimension, because for preferences defined in our way, bliss points coordinates are functions of single variable $\alpha$:

$$
\tau^*_i = 1 - \alpha_i \\
b^*_i = \frac{1 - K}{1 + K} \\
\gamma^*_i = \frac{2\tau^*_i}{1 + K}
$$

For $n$ parties, we need to divide the set $(0, 1)$ into $n$ equal parts to satisfy the assumption of fixed distances. Doing so, we get preferences of party $i$ as $\alpha_i = (2i - 1)/2n$.

4.1.2 Coalitions

Consider simple-majority voting ($q = \sup(1/2)$). Since party preferences can be expressed in single dimension $\alpha$, we define coalitions by reference to this dimension. Define further the coalition size of $S$ as $\max_{i \in S}(\alpha_i) - \min_{i \in S}(\alpha_i)$.

**Odd number of parties** Any MWC has to include $(n + 1)/2$ parties to win a simple-majority vote. Therefore, coalition size is $(n - 1)/2n$ and
we have \( (n + 1)/2 \) MWCs. A median position within the coalition \( j \in \{1, \ldots, (n + 1)/2\} \) we write as \( \alpha_j^M = (4j + n - 3)/4n \).

**Even number of parties** Any MWC has to include \((n + 2)/2\) parties to win a simple-majority vote. Therefore, coalition size is \(1/2\) and we have \(n/2\) MWCs. A median position within the coalition \( j \in \{1, \ldots, n/2\} \) is \( \alpha_j^M = (4j + n - 2)/4n \).

**Proposition 4** When party system includes odd number of parties, an unconstrained coalitional contract of coalition \( j \in \{1, \ldots, (n + 1)/2\} \) is as follows:

\[
\tau_j = 1 - \alpha_j^M = \frac{3n + 3 - 4j}{4n} \quad \gamma_j = \frac{3n + 3 - 4j}{2n(1 + K)}
\]

When the party system is even-sized, a coalition \( j \in \{1, \ldots, n/2\} \) selects:

\[
\tau_j = 1 - \alpha_j^M = \frac{3n + 2 - 4j}{4n} \quad \gamma_j = \frac{3n + 2 - 4j}{2n(1 + K)}
\]

**4.1.3 Comparative statics**

First of all, we check that an increase in number of parties doesn’t affect the average position of the coalition, i.e. there is no bias involved with the fragmentation.

**Proposition 5** For party system of any size \(N\), the expected coalitional contract is \(E(\tau) = 1/2\).

Fragmentation brings a change to tax volatility, that we can measure by standard deviation of tax set in a coalitional contract.

**Proposition 6** For odd number of parties and \(n > 3\), tax volatility decreases in \(n\). For even number of parties and \(n > 2\), tax volatility grows in \(n\).

**4.2 Interpretation**

In traditional literature, fiscal targets serve to lower CPR incentives, and the need for targets becomes pronounced with higher fragmentation. Here we have a trade-off between CPR and volatility effect of fiscal governance for each level of fragmentation. In standard case (like in even-sized system we have introduced), higher fragmentation means higher volatility. Since higher fragmentation implies also the more pronounced CPR problem, we
cannot generally determine whether the optimal fiscal governance is stronger or weaker. In an alternative case (like the odd-sized system), volatility is smaller for highly fragmented system. Thus, the optimal fiscal governance is a stronger fiscal governance. This is in line with traditional literature on benefits of Commitment mode for highly fragmented systems. However, we must remember that we reached this result only for a very special case.

5 Conclusion

The budgets in games we have modelled do not primarily suffer from allocative inefficiencies of the common-pool type (CPR). The problem is the pure conflict of interest that cannot be overcome under given flat tax system and in the absence of compensations.

Fiscal governance appears to have two effects. In CPR, it works as a coordination device; in pure conflicts of fiscal preferences, it is a protection device. This ambiguity implies that strong fiscal governance both eliminates and magnifies fiscal costs of political fragmentation. The fact that multi-party governments find it easier to rely on Commitment mode, as Hallerberg [8] observes, not necessarily means motivation for efficiency, but for mutual protection.

To conclude, it may not be the case that the higher fiscal governance, the better fiscal policy.

References


A Proofs

Proof of Proposition 1 Part (i): Denote \( Q_\tau := y_c^{\alpha_c} \hat{y}^{1-\alpha_c}[(1 + \delta b)^{1-\alpha_c} + p_c \delta (1- b)^{1-\alpha_c}] \). The F.O.C. requires that in equilibrium:

\[
\frac{\partial U_c(\alpha_c, p_c)}{\partial \tau_c} = Q_\tau [(1 - \alpha_c)(1 - \tau_c)^{\alpha_c} \tau_c^{-\alpha_c} - \alpha_c (1 - \tau_c)^{\alpha_c-1} \tau_c^{1-\alpha_c}] = 0
\]

By eliminating strictly positive \( Q_\tau \), we get \( \alpha_c / (1 - \alpha_c) = (1 - \tau_c) / \tau_c \), which reduces to \( \tau_c^* = 1 - \alpha_c \). Secondly, we have to verify the second-order condition:

\[
\frac{\partial^2 U_c(\alpha_c, p_c)}{\partial \tau_c^2} = (\alpha_c - 1) \alpha_c (1 - \tau_c)^{\alpha_c-2} \tau_c^{-\alpha_c(1+\alpha_c)} (2 \tau_c^2 - 2 \tau_c + 1) < 0
\]
By assumptions, the first term is strictly negative ($\alpha < 1$), while the others are strictly positive (as to the last term, $\sqrt{D} = \sqrt{-4}$, so for all $\tau \in \mathbb{R}$ the term is strictly positive).

Part (ii): Denote $Q_b := (1 - \tau_c)^{\alpha_c} \tau_c^{1-\alpha_c} g_c \hat{y}^{1-\alpha_c}$. Again, suffice to apply F.O.C.:

$$
\frac{\partial U_i(\alpha_c, p_c)}{\partial b_c} = Q_b \delta (1 - \alpha_c) [(1 + \delta b_c)^{-\alpha_c} - p_c (1 - b_c)^{-\alpha_c}] = 0
$$

Now, since $Q_b > 0$, we obtain $b^*_c = (1 - \alpha c \sqrt{p_c})/(1 + \delta \sqrt{p_c})$. As usual, we verify the second-order condition:

$$
\frac{\partial^2 U_i(\alpha_c, p_c)}{\partial \tau_c^2} = -\alpha c \frac{1 + \delta}{1 + \delta \sqrt{p_c}} (\delta + \sqrt{p_c}) < 0
$$

The first term is strictly negative (by assumption, $\alpha > 0$), while the others are strictly positive ($\delta > 0$ by assumption).

Part (iii): Since $\gamma = \tau (1 + b)$, we simply insert $b^*$:

$$
\gamma^*_c = \tau^*_c (1 + b^*_c) = \frac{2(1 - \alpha_c)}{1 + \sqrt{p_c}}
$$

Proof of Proposition 2  Part (i): In order to get $\gamma^*_i(\bar{\tau})$, we apply F.O.C.:

$$
\frac{\partial U_i(\alpha_i, p_i, \bar{\tau})}{\partial \tau} = y^i_i \gamma^{1-\alpha_i} (1 - \bar{\tau})(1 - \alpha_i) [\gamma - \alpha_i - p(2\bar{\tau} - \gamma)^{-\alpha_i}] = 0
$$

Since $\alpha_i > 0$, we have $\bar{\tau} = \tau^* = 1 - \alpha_i < 1$, so the root $\tau = 1$ doesn’t apply. We put the last term equal zero and receive $\gamma^*_i(\bar{\tau}) = 2\bar{\tau}/(1 + \sqrt{p_i})$. At last, we check that the second-order condition holds:

$$
\frac{\partial^2 U_i(\alpha_i, p_i, \bar{\tau})}{\partial \gamma^2} = -\alpha_i \gamma^{-\alpha_i - 1} - p_i \alpha_i (2\tau - \gamma)^{\alpha_i - 1} < 0
$$

Very similarly, we derive $b^*_i(\bar{\tau})$:

$$
\frac{\partial U_i(\alpha_i, p_i, \bar{\tau})}{\partial b} = y^i_i \gamma^{1-\alpha_i} (1 - \alpha_i) [(1 + b)^{-\alpha_i} - p(1 - b)^{-\alpha_i}]
$$

By assumption, the first root $\alpha = 1$ is not available, so we get the only one from the last term. Since $\tau$ is not present in the term, we clearly have $b^*_i(\tau) = b^*_i$. The second condition is as follows:

$$
\frac{\partial^2 U_i(\alpha_i, p_i, \bar{\tau})}{\partial b^2} = -\alpha_i (1 + b)^{-\alpha_i - 1} - p_i \alpha_i (1 - b)^{-\alpha_i - 1} < 0
$$

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Part (ii): So far, we have not explicit solution of $\tau_i^*(\bar{\gamma})$.

Part (iii): Like in Proof of Proposition 1, Part (i), the F.O.C. necessary to derive $\tau_i^*(\bar{b})$ is given as:

$$\frac{\partial U_i(\alpha_i, p_i, \bar{b})}{\partial \tau} = Q_{\tau}[(1 - \alpha_i)(1 - \tau)^{\alpha_i - 1} - \alpha_i(1 - \tau)^{\alpha_i - 1}] = 0$$

The solution is then the same, namely $\tau_i^*(\bar{b}) = 1 - \alpha_i$. Also the second-order condition, already verified in Proof of Proposition 1, is identical. ■

Proof of Proposition 3 Part (i) (Final vote/No target): By backward induction, we start in the final (second) step. From Proposition 2, we can see $\gamma_i^*|_{\tau = \bar{\tau}}$ is a linear transformation of $\sqrt{\alpha_i p_i}$. A linear trasformation of a distribution doesn’t change the identity of the median, so we get $\gamma_M^T(\bar{\tau})$ is a one-player function of median player $P$.

Define $Q_{\tau} := 2/(1 + \alpha_i \sqrt{p_i})$ for simplicity. By entering $\gamma_M^T(\bar{\tau})$ into the utility function (6), we get $U_i = (1 - \tau)^{\alpha_i \tau - 1} - \alpha_i(1 - \tau)^{\alpha_i - 1}$, where by Proposition 1 the solution is $\tau_i^* = 1 - \alpha_i$. Obviously, the median $\tau_M^{TG}$ is given by median $\alpha_A$, hence $\tau_M^{TG} = 1 - \alpha_A$.

Part (ii) (Deficit target): By Proposition 2, we know $\gamma_i^*|_{b = \bar{b}} = 1 - \alpha_i$. Thus, $\tau_M^B = 1 - \alpha_A$. Since $b = b_S$ and $b = (\gamma - \tau)/\tau = \gamma/\tau - 1$, we have $\gamma/\tau = \gamma_S/\tau_S$. Accordingly, $\gamma_M^B = \gamma_S^T_M/\tau_S$.

Part (iii) (Spending target): Since $\gamma = \gamma_S$, we look for a median of $\tau_i^*|_{\gamma_S}$ as expressed in Proposition 2:

$$\tau_M^G(\gamma_S) = 1 - \alpha_S + \frac{\alpha_S \gamma_S}{2p_S} \left[ p_S - \left( \frac{2\tau_M^G(\gamma_S) - \gamma_S}{\gamma_S} \right)^{\alpha_S} \right]$$

Part (iv) (Both targets): An unconstrained Nash-bargaining outcome $[\tau_S, \gamma_S]$ is preserved here as the whole. ■

Proof of Proposition 4 Part (i). First of all, we obtain a coalitional contract for any coalition $S$. Nash-bargaining solution requires that $(\tau_S, b_S)$ satisfies:

$$(\tau_S, b_S) = \arg\max_{\tau_S} \prod_{i \in S} U_i(\tau_S, b_S)$$

Since $b$ is constant, suffice to seek $\tau_S$. We can rewrite:

$$\prod_{i \in S} U_i(\tau_S) = (1 - \tau_S)^{\sum_{i \in S} \alpha_i \tau_S^{1 - \alpha_i}} \prod_{i \in S} [(1 + b)^{1 - \alpha_i} + p_i(1 - b)^{1 - \alpha_i}].$$
By F.O.C., we get $\tau_S = 1 - \sum_{i \in S} \frac{\alpha_i}{n} = 1 - E_S(\alpha)$.

Part (ii). Consider the odd number first. In each coalition $j \in \{1, \ldots, \frac{n+1}{2}\}$, we have $(n + 1)/2$ parties. Each party $i$ within the coalition we can represent by $\alpha_i$ such that $i \in \{j, \ldots, j + \frac{n-1}{2}\}$. Before we express $E_j(\alpha_i)$, we use $\sum_{i=0}^{\alpha} = \frac{a(a+1)}{2}$ to compute the following auxiliary expression:

$$\sum_{i=j}^{j+\frac{n-1}{2}} i = \frac{j(n+1)}{2} + \sum_{i=0}^{\frac{n-1}{2}} i = \frac{(n+1)(4j + n - 1)}{8}$$

Now we easily derive $E_j(\alpha)$ and see that tax preferred by median position is the Nash-bargaining solution for the coalition.

$$E_j(\alpha) = \frac{2}{n+1} \left( \sum_{i=1}^{j+\frac{n-1}{2}} \frac{2i - 1}{2n} \right) = \frac{2}{n+1} \left( \sum_{i=1}^{j+\frac{n-1}{2}} i \right) - \frac{n+1}{4n} = \frac{4j + n - 3}{4n}$$

Part (iii). Consider the even-sized system. In each coalition $j \in \{1, \ldots, \frac{n}{2}\}$, we have $(n + 2)/2$ parties. Each party $i$ within the coalition we can represent by $\alpha_i$ such that $i \in \{j, \ldots, j + \frac{n}{2}\}$. Again, we compute an auxiliary expression:

$$\sum_{i=j}^{j+\frac{n}{2}} i = \frac{j(n+2)}{2} + \sum_{i=0}^{\frac{n}{2}} i = \frac{(n+2)(4j + n)}{8}$$

Now we easily derive $E_j(\alpha)$ and again confirm that tax preferred by median position is the Nash-bargaining solution for the coalition.

$$E_j(\alpha) = \frac{2}{n+2} \left( \sum_{i=1}^{j+\frac{n}{2}} \frac{2i - 1}{2n} \right) = \frac{2}{n+2} \left( \sum_{i=1}^{j+\frac{n}{2}} i \right) - \frac{n+2}{4n} = \frac{4j + n - 2}{4n}$$

In both cases, we get $E_j(\alpha) = \alpha_j^M$, and $\tau_j = \tau_j^M$. ■

**Proof of Proposition 5** Part (i). Let us express the expected value of $\tau$ for any $n$:

$$E(\tau) = 1 - E(\alpha^M) = 1 - \sum_{j=1}^{\frac{n+1}{2}} \frac{2(4j + n - 3)}{(n+1)4n} = \frac{1}{2}$$

Part (ii). Similarly, the expected value of $\tau$ for any $n$ writes as $E(\tau) = 1 - E(\alpha^M) = 1 - \sum_{j=1}^{\frac{n}{2}} \frac{2(4j + n - 2)}{n(4n)} = \frac{1}{2}$. ■
Proof of Proposition 6  We shall compute variance at first, given as
\[ \text{Var}(\tau) = E(\tau - E(\tau))^2 = E(\alpha - 1/2)^2. \]
In finding variance, we apply a rule \( \sum_{i=0}^{a} i^2 = a(a + 1/2)(a + 1)/3. \)

Part (i). For odd size, the variance is as follows:
\[
\text{Var}(\tau) = E \left( 4j - n - 3 \right)^2 = \frac{n^2 + 6n + 9}{16n^2} + \sum_{j=1}^{\frac{n+1}{2}} \frac{2j^2}{n^2(n+1)} - n \sum_{j=1}^{\frac{n+1}{2}} \frac{j}{n^2(n+1)} - 3 \sum_{j=1}^{\frac{n+1}{2}} \frac{j}{n^2(n+1)} = \frac{n^2 + 2n - 3}{48n^2}
\]
To check how volatility depends on fragmentation, we analyze the slope of standard deviation in \( n \), i.e the first derivative of square root of variance:
\[
\frac{\partial \sqrt{\text{Var}(\tau)}}{\partial n} = \frac{3 - n}{4n^2 \sqrt{3(n^2 + 2n - 3)}}
\]
For \( n > 3 \), the term is negative which means that fiscal volatility decreases in fragmentation.

Part (ii). For even size of the party system, we get:
\[
\text{Var}(\tau) = E \left( 4j - n - 2 \right)^2 = \frac{n^2 + 4n + 4}{16n^2} + \sum_{j=1}^{n} \frac{2j^2}{n^3} - n \sum_{j=1}^{n} \frac{j}{n^3} - 2 \sum_{j=1}^{n} \frac{j}{n^2} = \frac{n^2 - 4}{48n^2}
\]
Again, analyze the first derivative of square root of variance:
\[
\frac{\partial \sqrt{\text{Var}(\tau)}}{\partial n} = \frac{1}{\sqrt{3(n^2 - 4)}}
\]
For \( n > 2 \), the term is positive and we can state that fiscal volatility increases in fragmentation. \( \blacksquare \)
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