

An Experimental Test of Design Alternatives for Spectrum Auctions with Communication Channels

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MOTIVATION

- ▶ Simultaneous auctions as artificial mechanism solving allocation problems in many fields
 - ▶ airport time slots
 - ▶ delivery routes
 - ▶ allocation of **radio spectrum** for wireless communications
- ▶ Spectrum auctions particularly important
 - ▶ mechanism for construction of third millenia infrastructure
 - ▶ big impact on economy
 - ▶ high volumes of money in stakes

⇒ Vulnerable to non-competitive practices

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What do we know about collusion in simultaneous auctions?

- ▶ The effect of cooperation of agents in simultaneous auctions?
- ▶ What may prevent negative impact of coordinated strategies?
- ▶ Which auction format should the policymaker use?
- ▶ Little evidence on collusion in simultaneous auctions

PREVIEW

- ▶ Experiment on two simultaneous auction mechanisms **with & without** communication
 - ▶ Simultaneous Multi-Round Auction - SMR
 - ▶ Simultaneous Multi-Round Package Bidding Auction - SMRPB

	SMR	SMRPB
I.	no communication	no communication
	SMR	SMRPB
II.	communication all	communication all

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We found that

1. Package bidding does not bring higher efficiency than basic SMR

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We found that

1. Package bidding does not bring higher efficiency than basic SMR
2. Allowing for communication brought significantly higher efficiencies in both formats

LITERATURE REVIEW

SIMULTANEOUS AUCTIONS

- ▶ SMR developed in 90's for spectrum auctions
- ▶ Brunner et al. (2010) compare SMR and 3 combinatorial formats
 - ▶ efficiency higher in case of package bidding formats
- ▶ Bichler et al. (2013a) compare SMR and Combinatorial Clock (CC) auction in multiband model
 - ▶ number of possible combinations too high
 - ▶ simplicity of bidding language increases efficiency
- ▶ Bichler et al. (2013b) compare SMR and CC auction again
 - ▶ combinatorial format scored significantly worse in their environment
 - ▶ contradictory to Brunner et al. (2010)
- ▶ 2 contradictory even though parameter specific conclusions

COLLUSION IN AUCTIONS

- ▶ Collusion occur with communication present in auctions, Bidders tend toward collusive payoff-maximizing strategies (Kwasnica and Sherstyuk, 2013)
- ▶ Phillips et al. (2003) - 3 practices facilitating collusion
 1. knowledge about the number of units for sale
 2. familiarity through repeated interaction
 3. communication
- ▶ Lopomo et al. (2005) - collusion does effect auction efficiency
- ▶ Miralles (2010) examines a simple pre-play cheap talk
 - ▶ Full comparative cheap talk equilibrium exists for large number of objects
 - ▶ Partial cheap talk equilibrium with object split always exists

METHODOLOGY

AUCTION FORMATS

- ▶ Simultaneous Multi-Round Auction (SMR)
 - ▶ simple generalization of english auction for multiple objects
 - ▶ all goods offered at the same time
 - ▶ bids for **individual goods** submitted in sequential rounds
 - ▶ provisionall winner for each unit determined randomly
 - ▶ activity points - prevent "snake in the grass" strategy

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 - ▶ provisionall winner for each unit determined randomly
 - ▶ activity points - prevent "snake in the grass" strategy
- ▶ Simultaneous Multi-Round Package Bidding Auction (SMRPB)
 - ▶ combinatorial extention of SMR
 - ▶ bids submitted for desired **packages of goods**
 - ▶ provisionall winner for each package based on the highest price (same packages randomly)

COMMUNICATION

- ▶ Communication window
 - ▶ Prior to the whole auction for 2 minutes
 - ▶ Limited information available
 - ▶ Available also during the whole auction
 - ▶ Simple and self-enforced mechanism
 - ▶ No binding commitments for players

HYPOTHESES

- ▶ First set - NO COMMUNICATION
 - ▶ High degree of competition
 - ▶ Competitive prices
 - ▶ High public revenues
 - ▶ SMRPB higher efficiency than SMR
(high complementarities)

HYPOTHESES

- ▶ First set - NO COMMUNICATION
 - ▶ High degree of competition
 - ▶ Competitive prices
 - ▶ High public revenues
 - ▶ SMRPB higher efficiency than SMR
(high complementarities)

- ▶ Second set - COMMUNICATION
 - ▶ No or very little competition present
(in case of stable equilibrium)
 - ▶ Prices stay down at the upset level
(extreme case of fully coordinated strategies)
 - ▶ Shift in rent distributions from public to private subjects
 - ▶ An effect of communication on efficiency?
 - ▶ Does allowing for combinatorial bidding break collusion?

DESIGN

- ▶ 4 types of homogeneous goods
- ▶ 4 types of bidders
- ▶ Activity points - $1/4$ of total activity in experiment plus random bonus

DESIGN

- ▶ 4 types of homogeneous goods
- ▶ 4 types of bidders
- ▶ Activity points - $1/4$ of total activity in experiment plus random bonus
- ▶ Player valuations determined randomly
 - ▶ ex-ante symmetrical setting for each player type (draws made prior to the experiment only once)
 - ▶ CVC same for all players, no exact info, independent private signal on CVC

$$\text{signal} \in \langle \text{CVC} - \alpha; \text{CVC} + \alpha \rangle$$

- ▶ PVC private, full info

$$\text{PVC} \in \langle -\beta; +\beta \rangle$$

DESIGN CONTINUED

- ▶ Value complementarities linear $[1 + \alpha (K - 1)]$
 - ▶ K is sum of type of goods acquired
 - ▶ synergy factor $\alpha = 0.1$
- ▶ Final profit = total valuation acquired - total price paid
- ▶ Efficiency measurement
 - ▶ Goeree and Offerman (2002) approach to multiple objects
 - ▶ $e_i = \frac{PVC_{winner} - PVC_{min}}{PVC_{max} - PVC_{min}}; i \in 1, \dots, n$
 - ▶ average across n experimental goods: $E = \sum \frac{e_i}{n} \cdot 100\%$
 - ▶ normalized by the optimal allocation $-e_{optimal} = 0.9562$

▶ e optimal

PROCEDURE

- ▶ Laboratory of Experimental Economics in Prague
- ▶ 4 sessions; 1 session per treatment cell
- ▶ Between subject design
- ▶ 24 subjects per session, 96 participants in total
- ▶ 2 hours per session
- ▶ 3 auctions per session, 288 observations in total
- ▶ Experiment in Czech
- ▶ Expected average pay 500 CZK per subject

PROCEDURE CONTINUED

- ▶ Instruction procedures (complexity of required task)
 - ▶ Invitation five days prior to the experiment
 - ▶ Three days prior asked to fill in online questionnaire based on general instructions provided online
 - ▶ Whole procedure publicly known
 - ▶ Successful completion in advance over 95%

TASK

GOOD A		GOOD B		GOOD C		GOOD D		BIDDING BASKET		YOUR ACCOUNT	
price per unit	<input type="text"/>	price per unit	<input type="text"/>	price per unit	<input type="text"/>	price per unit	<input type="text"/>	GOOD A	<input type="text"/>	current activity	<input type="text"/>
goods in stock		goods in stock		goods in stock		goods in stock		GOOD B		LAST ROUND RESULTS	<input type="text"/>
activity per unit		activity per unit		activity per unit		activity per unit		GOOD C			
CVC Signal A		CVC Signal B		CVC Signal C		CVC Signal D		GOOD D			
PVC A		PVC B		PVC C		PVC D		GOOD A			
your valuation		your valuation		your valuation		your valuation		total valuation	GOOD B	GOOD C	
upset price		upset price		upset price		upset price		total price	GOOD C	GOOD D	
bid increment		bid increment		bid increment		bid increment		profit if won	total valuation	total price paid	
								activity at disposal	total profit made	total penalty in the auction	
								activity used			
								activity loss			
YOUR BID QUANTITY	<input type="text"/>	YOUR BID QUANTITY	<input type="text"/>	YOUR BID QUANTITY	<input type="text"/>	YOUR BID QUANTITY	<input type="text"/>	ERASE	SUBMIT		
-1	+1	-1	+1	-1	+1	-1	+1				
HISTORY						CHAT					

RESULTS

TREATMENTS WITHOUT COMMUNICATION

- ▶ Degree of competition high
- ▶ Prices reached competitive levels closely
- ▶ Combinatorial format scored generally worse in efficiency
⇒ contradictory to Brunner et al. (2010)
- ▶ Fewer goods sold at high prices
- ▶ High competition caused low revenues
(High prices could not compensate the losses)

TREATMENTS WITH COMMUNICATION

- ▶ Many cases of coordinated collusion appeared, no or very little competition present
- ▶ Prices remained down at the upset base
- ▶ Seller revenues and bidder surpluses increased (higher quantities sold more than compensated lower prices caused by insufficient competition)
- ▶ Allowing for combinatorial bidding in the SMRPB format generally breaks collusion
- ▶ Attempts to break collusive agreements resulted in overall decrease in efficiency
- ▶ Communication had positive impact on efficiency in both auction formats

QUALITATIVE ANALYSIS

- ▶ **SMR x SMRPB both NO COMMUNICATION**
 - ▶ Efficiencies statistically identical (p-value 0.4427)
 - ▶ Prices paid identical (p-value 0.9518)
 - ▶ Average final profits different!
SMR higher profits than SMRPB (p-value 0.0012)
 - ▶ Generally lower number of goods sold without communication channel
 - ▶ Package bidding further decrease total number of goods sold (lowering the efficiency)

QUALITATIVE ANALYSIS CONTINUED

▶ **NO COMMUNICATION** × **COMMUNICATION**

- ▶ Favours resolutely collusive treatments
- ▶ Efficiencies higher in communication treatments (p-value 0.000)
- ▶ Final profits higher in communication treatments (p-value 0.000)
- ▶ Total prices differ for SMR (p-value 0.000) but not for the SMRPB (p-value 0.1831)
- ▶ More goods sold in collusive treatments
⇒ higher efficiencies

QUALITATIVE ANALYSIS CONTINUED

- ▶ **SMR x SMRPB** both with **COMMUNICATION**
 - ▶ SMR better in efficiency (p-value 0.000)
 - ▶ Higher total prices in SMR (p-value 0.0031)
 - ▶ Allowing for combinatorial bidding often broke collusion (equilibrium usually unstable)

QUANTITATIVE ANALYSIS

- ▶ 2 model specifications of relative efficiency
- ▶ Standard OLS, robust standard errors, robust regression
- ▶ No observation omitted, same number in all models

$$e_r = \alpha + \beta_1 SMR + \beta_2 chat + \beta_3 \pi + \beta_4 q_a + \beta_5 q_b + \beta_6 q_c + \beta_7 q_d + \beta_8 vp_b^{diff} + \beta_9 vp_c^{diff} \\ + \gamma_1 time^{control} + \gamma_2 cr + \gamma_3 edu^{math} + \gamma_4 edu^{statistics} + \gamma_5 exp + \epsilon$$

$$e_r = \alpha + \beta_1 SMR + \beta_2 chat + \beta_3 \pi + \beta_4 q_a + \beta_5 q_b + \beta_6 q_c + \beta_7 q_d + \beta_8 vp_b^{diff} + \beta_9 vp_c^{diff} \\ + \beta_{10} pt^1 + \beta_{11} pt^2 + \beta_{12} pt^3 \\ + \gamma_1 time^{control} + \gamma_2 cr + \gamma_3 edu^{math} + \gamma_4 edu^{statistics} + \gamma_5 exp + \epsilon$$

THE MODEL RESULTS

VARs	(1)	(2)	(3)	(4)	(5)	(6)
	OLS	<i>without player types</i> robust std. e.	robust reg.	OLS	<i>with player types</i> robust std. e.	robust reg.
SMR	0.0616*** (0.0184)	0.0616*** (0.0200)	0.0357** (0.0181)	0.0497*** (0.0183)	0.0497** (0.0195)	0.0206 (0.0177)
chat	0.0599*** (0.0229)	0.0599*** (0.0214)	0.0501** (0.0225)	0.0290 (0.0240)	0.0290 (0.0218)	0.0171 (0.0231)
final profit	6.35e-05*** (2.10e-05)	6.35e-05*** (1.90e-05)	4.77e-05** (2.07e-05)	5.96e-05*** (2.09e-05)	5.96e-05*** (1.91e-05)	4.20e-05** (2.01e-05)
q_a	-0.00574 (0.0109)	-0.00574 (0.00977)	0.000479 (0.0107)	-0.0105 (0.0111)	-0.0105 (0.0103)	-0.00400 (0.0106)
q_b	0.0126*** (0.00225)	0.0126*** (0.00201)	0.0131*** (0.00221)	0.0154*** (0.00233)	0.0154*** (0.00214)	0.0159*** (0.00225)
q_c	0.0168** (0.00823)	0.0168*** (0.00620)	0.0174** (0.00808)	0.0200** (0.00812)	0.0200*** (0.00661)	0.0204*** (0.00782)
q_d	0.0141*** (0.00439)	0.0141*** (0.00537)	0.0174*** (0.00431)	0.00911** (0.00458)	0.00911* (0.00548)	0.0133*** (0.00441)
vp_b.diff	0.00117 (0.00138)	0.00117 (0.00156)	0.00264* (0.00136)	0.00398** (0.00156)	0.00398** (0.00163)	0.00571*** (0.00150)
vp_c.diff	0.000158 (0.000253)	0.000158 (0.000232)	0.000165 (0.000249)	-9.73e-05 (0.000258)	-9.73e-05 (0.000226)	-0.000109 (0.000248)
pt_1				-0.0870*** (0.0249)	-0.0870*** (0.0229)	-0.0861*** (0.0239)
pt_2				-0.0642*** (0.0238)	-0.0642*** (0.0231)	-0.0609*** (0.0230)
pt_3				-0.0129 (0.0208)	-0.0129 (0.0230)	-0.00747 (0.0200)

Robust standard errors in parentheses

$p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

THE MODEL RESULTS CONTINUED

VARs	(1)	(2)	(3)	(4)	(5)	(6)
	OLS	<i>without player types</i> robust std. e.	robust reg.	OLS	<i>with player types</i> robust std. e.	robust reg.
<i>control time</i>	-3.26e-05 (7.44e-05)	-3.26e-05 (8.38e-05)	-4.59e-05 (7.30e-05)	-3.97e-05 (7.30e-05)	-3.97e-05 (8.21e-05)	-4.05e-05 (7.03e-05)
<i>cr</i>	0.0260 (0.0176)	0.0260 (0.0184)	0.0257 (0.0173)	0.0194 (0.0174)	0.0194 (0.0183)	0.0186 (0.0168)
<i>edu math</i>	-0.0338** (0.0142)	-0.0338** (0.0131)	-0.0248* (0.0140)	-0.0383*** (0.0142)	-0.0383*** (0.0132)	-0.0276** (0.0136)
<i>edu statistics</i>	0.0317* (0.0188)	0.0317* (0.0165)	0.0243 (0.0184)	0.0388** (0.0186)	0.0388** (0.0164)	0.0281 (0.0179)
<i>experience</i>	-0.00105 (0.00211)	-0.00105 (0.00228)	0.000147 (0.00208)	-0.000136 (0.00209)	-0.000136 (0.00227)	0.00140 (0.00201)
<i>Constant</i>	0.166** (0.0718)	0.166*** (0.0600)	0.178** (0.0705)	0.231*** (0.0747)	0.231*** (0.0637)	0.239*** (0.0719)
<i>Observations</i>	288	288	288	288	288	288
<i>R-squared</i>	0.521	0.521	0.491	0.545	0.545	0.513

Robust standard errors in parentheses

$p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

MODEL EVALUATION

- ▶ Main differences in different model specifications
- ▶ Introduction of player type variables
 - ▶ shift of significance from auction formats and communication channels to player types
 - ▶ lot of variation caused by the parametric setting rather than experimental design
- ▶ Additional analysis - differences between treatments do not depend only on the player types specification

HYPOTHESES RESOLUTION

Table : I. set - hypotheses within formats - Results

	SMR	vs.	SMR ^{Coll}	SMR _{PB}	vs.	SMR _{PB} ^{Coll}
Total Prices	P_{SMR}	<***	P_{SMR}^{Coll}	$P_{SMR_{PB}}$	≤	$P_{SMR_{PB}}^{Coll}$
Efficiency	E_{SMR}	<***	E_{SMR}^{Coll}	$E_{SMR_{PB}}$	<***	$E_{SMR_{PB}}^{Coll}$
Revenues	R_{SMR}	<***	R_{SMR}^{Coll}	$R_{SMR_{PB}}$	<	$R_{SMR_{PB}}^{Coll}$
B. surpluses	S_{SMR}	<***	S_{SMR}^{Coll}	$S_{SMR_{PB}}$	<***	$S_{SMR_{PB}}^{Coll}$

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Efficiency	E_{SMR}	<***	E_{SMR}^{Coll}	$E_{SMR_{PB}}$	<***	$E_{SMR_{PB}}^{Coll}$
Revenues	R_{SMR}	<***	R_{SMR}^{Coll}	$R_{SMR_{PB}}$	<	$R_{SMR_{PB}}^{Coll}$
B. surpluses	S_{SMR}	<***	S_{SMR}^{Coll}	$S_{SMR_{PB}}$	<***	$S_{SMR_{PB}}^{Coll}$

Table : II. set - hypotheses between Formats - Results

	SMR	vs.	SMR _{PB}	SMR ^{Coll}	vs.	SMR _{PB} ^{Coll}
Total Prices	P_{SMR}	=	$P_{SMR_{PB}}$	P_{SMR}^{Coll}	>***	$P_{SMR_{PB}}^{Coll}$
Efficiency	E_{SMR}	≥**	$E_{SMR_{PB}}$	E_{SMR}^{Coll}	>***	$E_{SMR_{PB}}^{Coll}$
Revenues	R_{SMR}	=	$R_{SMR_{PB}}$	R_{SMR}^{Coll}	>***	$R_{SMR_{PB}}^{Coll}$
B. surpluses	S_{SMR}	>***	$S_{SMR_{PB}}$	S_{SMR}^{Coll}	=	$S_{SMR_{PB}}^{Coll}$

- ▶ SMR scored better in terms of efficiency than its combinatorial counterpart SMR_{PB} ▶ Original

CONCLUSIONS

- ▶ Package bidding format did not increase efficiency in our parametric setting
 - ▶ Does not bring higher efficiency than basic SMR (contradictory to Brunner et al. (2010))
- ▶ Basic treatments scored worse than with communication
 - ▶ competition drove prices to higher levels, weaker players fell behind
 - ▶ Fewer goods sold in basic treatments
⇒ substantially lower efficiencies, revenues and bidders surpluses
- ▶ Allowing for communication brought better results
 - ▶ Significantly higher efficiencies in both formats
 - ▶ Combinatorial bidding may break collusion
- ▶ Strength in simplicity of bidding languages favours SMR

THANK YOU FOR YOUR ATTENTION

Questions, comments?

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HYPOTHESES COMPARISONS

Table : I. set - hypotheses within formats

	SMR	vs.	SMR^{Coll}	SMRPB	vs.	$SMRPB^{Coll}$
Total Prices	P_{SMR}	>	P_{SMR}^{Coll}	P_{SMRPB}	>	P_{SMRPB}^{Coll}
Efficiency	E_{SMR}	??	E_{SMR}^{Coll}	E_{SMRPB}	??	E_{SMRPB}^{Coll}
Revenues	R_{SMR}	>	R_{SMR}^{Coll}	R_{SMRPB}	>	R_{SMRPB}^{Coll}
B. surpluses	S_{SMR}	<	S_{SMR}^{Coll}	S_{SMRPB}	<	S_{SMRPB}^{Coll}

Table : II. set - hypotheses between Formats

	SMR	vs.	SMRPB	SMR^{Coll}	vs.	$SMRPB^{Coll}$
Total Prices	P_{SMR}	<	P_{SMRPB}	P_{SMR}^{Coll}	<	P_{SMRPB}^{Coll}
Efficiency	E_{SMR}	<	E_{SMRPB}	E_{SMR}^{Coll}	??	E_{SMRPB}^{Coll}
Revenues	R_{SMR}	<	R_{SMRPB}	R_{SMR}^{Coll}	<	R_{SMRPB}^{Coll}
B. surpluses	S_{SMR}	>	S_{SMRPB}	S_{SMR}^{Coll}	>	S_{SMRPB}^{Coll}

PARAMETERS

Table : Common Value Component and private signal intervals

GOODS	CVC	CVC variance	CVC private signal intervals
A	1820	200	< 1620; 2020 >
B	50	5	< 45; 55 >
C	140	10	< 130; 150 >
D	50	5	< 45; 55 >

Table : Final parameters of the experiment

PLAYERS	GOODS				Activity
	A	B	C	D	
Blue	1886	53	138	48	27
Pink	1727	53	140	47	28
Red	1900	47	130	46	26
Green	1865	48	138	50	25

OPTIMAL ALLOCATION

		Player Types			
		Blue	Pink	Red	Green
A	quantity	1	0	2	1
	PVC	66	-93	80	45
	efficiency per unit	0.919	0	1	0.7976
B	quantity	10	14	0	0
	PVC	3	3	-3	-2
	efficiency per unit	1	1	0	0.1666
C	quantity	0	14	0	0
	PVC	-2	0	-10	-2
	efficiency per unit	0.8	1	0	0.8
D	quantity	0	0	0	9
	PVC	-2	-3	-4	0
	efficiency per unit	0.5	0.25	0	1
total activity		20	28	20	19
activity disposed		27	28	26	25
efficiency factor		10.919	28	2	9.76
total efficiency			0.95692		

- ▶ maximization of quantities subject to
 - ▶ efficiencies per unit
 - ▶ activity points at disposal for each player

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