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Foreign Exchange Interventions at the Zero Lower Bound in the Czech Economy: A DSGE Approach

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Abstract:
The paper contributes to understanding the economic dynamics at the zero lower bound and the exchange rate movements under different central bank intervention regimes. It provides a theoretical framework for modeling foreign exchange interventions at the ZLB within a dynamic general equilibrium model. We find a pronounced volatility of real and nominal macroeconomic variables in response to the domestic demand shock, the foreign demand and financial shocks and the terms-of-trade shock at the ZLB. This effects become severe in response to highly persistent shocks which leads to stronger reaction of variables and prolong period of binding constraint. The FX interventions have proven to be effective in mitigating deflationary pressures and recovering the economic activity in response to all examined shocks at the ZLB. In this sense, the central bank achieves the best performance by fixing the nominal exchange rate temporarily at the ZLB.

Keywords: zero lower bound, foreign exchange interventions, dynamic stochastic general equilibrium, Bayesian estimation, exchange rate and price dynamics
JEL: C11, E31, E43, E52, E58, F31

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1 Introduction

This paper presents a theoretical analysis of foreign exchange (FX) interventions at the zero lower bound (ZLB) in a standard New Keynesian dynamic stochastic general equilibrium (DSGE) model. Explicit modeling of FX dealers allows analyzing the transmission mechanism of interventions through a portfolio balance channel and an expectations channel. The paper contributes to understanding the dynamics of exchange rate movements under different central bank intervention regimes at the ZLB. The algorithm for simulation the model with ZLB enables to capture precautionary motives of economic agents. The intervention mechanism is introduced to work alongside a standard conventional reaction function of inflation targeting central bank to support it in achieving its main objective.

In the aftermath of the global financial crisis depressed economic conditions have required to employ accommodative monetary policy. The prolonged period of monetary easing has soon pushed policy rates to zero in many countries (and below it in some of them). As the central bank faces the limit of standard policy tools at the ZLB, it has resorted to unconventional measures. The attention is given to various types of liquidity provisions, large-scale asset-purchases, negative interest rates on deposits, and FX interventions.

The Czech National Bank (CNB) has also turned its attention to unconventional measures as it ran out of the space for managing the short-term interest rate in late 2012. At that time, the Czech economy was already in recession for four quarters with extremely weak domestic demand. But the main problem was a drop in consumer confidence in the first quarter of 2012 together with decrease in real wages. The pessimism of Czech households was the highest since 1999 and the CNB's forecast in the second quarter of 2013 suggested the inflation would turn negative at the beginning of 2014.

The ongoing recession together with the threat of disinflation led the Bank Board to discuss the use of additional policy tools for monetary easing. A very high degree of openness and a long-term liquidity surplus in banking sector predetermined using the exchange rate of the koruna. The use of quantitative easing (QE) in other economies has been supported by severe liquidity crisis and the instability of financial sector. But the Czech economy is in the opposite situation where a total amount of deposits is even higher than an amount of loans. In this case, QE would not be much effective. The CNB started to intervene on the FX market against the appreciation of the Czech koruna in late 2013. It commits to use a potentially unlimited amount of FX interventions to keep the nominal exchange rate above the threshold of 27 CZK/EUR. This provides a motivation to analyze the extent to which FX interventions are effective in mitigating deflationary pressures at the ZLB.

The rest of the paper is structured as follows. Section 2 discusses the use of exchange rate as a policy tool for escaping the ZLB. Section 3 describes the modeling strategy of FX dealers and the implementation of ZLB as an occasionally binding constraint. In section 4, the calibration and estimation of model parameters is performed for the Czech Republic. Finally, section 5 presents the comparison of different foreign exchange intervention regimes in response to domestic and foreign shocks. The core findings are summarized in section 6.

2 Exchange Rate as a Tool for Escaping Deflation

The exchange rate received a considerable attention in many open economies struggling with the zero lower bound and the threat of deflation. Already in the response to the Japanese experience in 1900s McCallum (2000) and Svensson (2003) propose the exchange rate as an effective way how to escape from the liquidity trap. From the theoretical point of view, the central bank may either manage the exchange rate through the Taylor-type policy rule with interest rate as a main instrument (dirty inflation targeting), or directly intervene in foreign exchange markets. In the second case the exchange rate becomes the instrument (instead of the interest rate) as well as operational target. While the first approach abstracts from potentially important transmission mechanism of FX interventions, the second one puts on the work also other transmission channels of monetary policy.

In addition, some authors propose to directly replace the interest rate in the central bank reaction function (e.g. McCallum, 2000). He suggests switching to the Taylor-type policy rule with the nominal exchange rate as the policy instrument whenever the economy is at the zero lower bound. With low inflation or output below its potential, the devaluation of currency should bring the inflation back on its target and close the output gap. This model does not include the UIP which can potentially break the intervention mechanism as the purchase of foreign exchange might not lead to the exchange rate depreciation at the ZLB (Christiano, 2000).

Svensson (2003) proposes in his "foolproof way" of escaping from the liquidity trap to temporary peg the currency at the substantially devalued exchange rate together with the future committing to the price-level target. The exchange rate peg is intended to be temporary and should be abandoned in favor of flexible price-level or inflation targeting when the price-level target is reached. Unlike McCallum (2000), Svensson (2003) does not rely on the portfolio balance channel as the central bank should be able to enforce the peg at a devalued rate using potentially unlimited amount of interventions.

More recent theoretical analysis of FX interventions at the ZLB is still limited. Following studies focus on their modeling just in normal times (without constrained interest rates). Bacchetta & van Wincoop (2006) provide a model where the exchange rate is closely related to order flows which represent a private information component of foreign exchange orders. This approach is supported by the empirical evidence which suggests most of the short-term and medium-term exchange rate fluctuations are caused by order flows. Vitale (2011) extends the model to allow for analysis of interventions effectiveness through the portfolio balance or expectations channel. Montoro & Ortiz (2013) follow Vitale (2011) using a standard New Keynesian DSGE model which allows to analyze the FX interventions together with the monetary policy.

Beneš *et al.* (2013) incorporate the financial sector into the DSGE model to analyze the impact of FX interventions through the portfolio balance channel. Within this framework authors examine the effect of different exchange rate regimes in combination with hybrid inflation targets. Their results suggest FX interventions can be helpful in the presence of foreign shocks, but may also hamper some necessary exchange rate adjustments. The effect of interventions through the portfolio balance channel is confirmed also by Herrera *et al.* (2013).

So far, three countries used FX interventions at the ZLB, including the Czech Republic, Japan and Switzerland. Moreover, the experience of other countries can be considered, e.g. Israel or Chile. The detailed discussion is provided in Lízal & Schwarz (2013) and Franta *et al.* (2014). To sum up, FX interventions have proven to be effective if the central bank is willing to clearly and credibly commit to unlimited purchases. Regarding the Czech empirical evidence, the most studies confirm their short-lasting weakly significant small effect (e.g. Disyatat & Galati, 2007; Geršl & Holub, 2006; Geršl, 2006; Égert & Komárek, 2006). This result may not be relevant in the situation when the central bank intervenes to keep exchange rate at substantially devalued level (with potentially unlimited amount of interventions). In this case, the effect of exchange rate on inflation is much more important, in particular the exchange rate pass-through to the inflation.

Existing estimates of the exchange rate shock transmission to the Czech inflation vary in a wide range between 0 and 80 percent, with a slight decreasing tendency in the last few years. In one of the most recent studies, Hájek & Horváth (2015) find significantly stronger exchange rate pass-through in the Czech economy compared to previous estimates. They confirm the pass-through at the ZLB is larger than in normal times. Binding constraint on some variable leads in general to more volatile response of remaining variables (see Portes, 1969). Intuitively, the exchange rate depreciation implies the increase in import prices which pushes up the inflation. Higher inflation together with zero nominal interest rate reduces gradually the real interest rate which supports economic growth. The channel of import prices and real interest rate work together (Lízal & Schwarz, 2013).

3 Model

The structure of the core model is standard.[1] Domestic economy is populated by a continuum of households, domestic firms, domestic retailers and foreign exchange dealers of mass 1. These economic agents act on average as representative while the deviation from such a behavior is normally distributed with zero mean. Therefore, it can be ignored. The agents are assumed to be rational. Any deviation from rational expectations is also normally distributed with zero mean, and therefore, it is not relevant for our purpose.

Each household enjoys a utility from the consumption of various goods produced by domestic and foreign firms, and receives disutility from working for domestic firms. Their income consists of wage, returns and profits distributed from firms and foreign exchange dealers. It can be divided between consumption and savings in domestic bonds. Domestic firms are producing non-traded goods (goods sold and consumed at the domestic market) and tradable goods (goods sold and consumed at the foreign market). Domestic retailers import foreign goods for which the law of one price holds at the dock. Due to the monopolistic competition these retailers have some degree of price setting power. Hence, the price of import can deviate from the law of one price in the short run. This feature is used to model the incomplete exchange rate pass-through (Monacelli, 2003). The structure of the world economy is similar to the domestic one.

3.1 Foreign Exchange Dealers

In general, the inclusion of risk-averse FX dealers generates a deviation from the traditional uncovered interest parity (UIP) condition as a result of their portfolio allocation decision. It is extended by a time-variant risk premium term which is a function of the central bank and foreign investors FX sale and purchase orders (Montoro & Ortiz, 2013).

[1] For technical details see Appendix A.

Each dealer d receives the same amount of sale and purchase orders in domestic bonds from households (ω_t^d) and central bank ($\omega_{CB,t}^d$), and in foreign bonds from foreign investors (ω_t^{d*}) and central bank ($\omega_{CB,t}^{d*}$).

The orders are exchange among dealers to satisfy following condition

$$\omega_t^d - \omega_{CB,t}^d + e_t(\omega_t^{d*} + \omega_{CB,t}^{d*}) = B_t^d + e_t B_t^{d*} \tag{1}$$

where B_t^d, B_t^{d*} are ex-post holdings of bonds in domestic and foreign currency by each dealer d.

Dealers maximize their constant absolute risk aversion utility function of following form

$$max - E_t^d e^{-\gamma \Omega_{t+1}^d} \tag{2}$$

Ω_{t+1}^d is total dealer's investment after returns

$$\begin{aligned}\Omega_{t+1}^d =&(1+i_t)B_t^d + (1+i_t^*)E_t^d e_{t+1} B_t^{d*} \\ =&(1+i_t)[\omega_t^d - \omega_{CB,t}^d + e_t(\omega_t^{d*} + \omega_{CB,t}^{d*})] \\ &+ (i_t^* - i_t + E_t^d e_{t+1} - e_t)B_t^{d*}\end{aligned} \tag{3}$$

where e_t is a logarithm of nominal exchange rate. It is also the only non-predetermined variable and assumed to be normally distributed. Then, the optimization with respect to B_t^{d*} yields

$$\begin{aligned}0 &= -\gamma(i_t^* - i_t + E_t^d e_{t+1} - e_t) + \gamma^2 B_t^{d*} \sigma_e^2 \\ B_t^{d*} &= \frac{i_t^* - i_t + E_t^d e_{t+1} - e_t}{\gamma \sigma_e^2}\end{aligned} \tag{4}$$

where $\sigma_e^2 = var(\Delta e_{t+1})$.

A market clearing condition for foreign bonds in the domestic market equals the sum of sale and purchase orders from the central bank and foreign investors in foreign bonds

$$\int_0^1 B_t^{d*} dd = \int_0^1 (\omega_t^{d*} + \omega_{CB,t}^{d*}) dd = \omega_t^* + \omega_{CB,t}^* \tag{5}$$

After substituting for (4) the modified UIP condition is computed

$$\bar{E}_t e_{t+1} - e_t = i_t - i_t^* + \gamma\sigma_e^2(\omega_t^* + \omega_{CB,t}^*) \tag{6}$$

where $\bar{E}_t$ is an average rational expectation across all dealers about the future nominal exchange rate. Assuming the information homogeneity, this expectation is the same for all dealers, i.e. $\bar{E}_t = E_t$.

The shortsighted and risk-averse dealers modify the standard UIP condition by adding a time-variant risk premium which depends on FX interventions and capital inflows/outflows. This intervention mechanism has a stabilizing effect. If the exchange rate deviates from its long-run level, the central bank starts intervening to bring it back to the steady state (or to the target). The risk premium term changes which works against the exchange rate movement. Interventions may influence the exchange rate through two transmission channels - the portfolio balance channel and the expectations channel. The effect of the portfolio balance channel is defined by the term $\gamma\sigma_e^2(\omega_t^* + \omega_{CB,t}^*)$ while the expectations channel works through expected future exchange rate $E_t e_{t+1}$. By intervening (change in $\omega_{CB,t}^*$) the central bank affects the ratio between domestic and foreign assets held by dealers, and hence, also the risk premium which these dealers require. The change in risk premium results in the exchange rate appreciation or depreciation.

3.2 Central Bank

The monetary authority uses two instruments, the short-term interest rate and FX interventions.

3.2.1 Foreign Exchange Interventions

Each period the central bank intervenes in the foreign exchange market through FX dealers which receive their purchase or sale orders in foreign bonds in exchange for domestic bonds. The central bank can always perform fully sterilized interventions. The central bank is not limited in its interventions and not obliged to distribute its profits to households. It decides on the level of foreign exchange interventions according to the following rule

$$\omega_{CB,t}^* = \chi_e(e_t - e_{t-1}) + \chi_{e^T}(e_t - e^T) + \chi_q q_t + \varepsilon_{cb,t}^* \tag{7}$$

where e^T is an operational exchange rate target and q_t is the deviation of the real exchange rate from its steady state level. The central bank can choose one of four different intervention regimes. If all parameters in (7) are equal to zero ($\chi_e = 0, \chi_{e^T} = 0, \chi_q = 0$) it performs unanticipated interventions. In the case of non-zero χ_e coefficient, the central bank intervenes to smooth nominal exchange rate movements, i.e. sells foreign bonds to prevent the depreciation and purchases foreign bonds to prevent the appreciation. The value of coefficient captures the intensity of this response. This type of regime (called "leaning-against-the-wind") can be easily used to model a managed float. Parameter χ_{e^T} describes to which extent the central bank intervenes in order to achieve a particular nominal exchange rate target. In this case ($\chi_{e^T} \to \infty$) the central bank uses an unlimited amount of interventions to mitigate any deviation from this target. In the last case ($\chi_q \neq 0$) the central bank attempts to correct a misalignment of the real exchange rate from its long-run (fundamental) values.

3.2.2 Interest Rate Rule

The central bank adopts the inflation target and follows a Taylor-type reaction function with weight assigned to deviation of inflation from this target one period ahead[2].

$$i_t = \bar{i} + \rho_i(i_{t-1} - \bar{i}) + \psi_\pi E_t(\pi_{t+1} - \pi_{t+1}^T) + \varepsilon_{i,t} \tag{8}$$

3.3 Implementation of the ZLB

The zero lower bound had been ignored almost completely before the experience of Japan in the 1990s, since many economists believed the constraint can bind only for a short period in time. The Japan's experience during the 1990s, as well as those in the EU and US in the aftermath of the crisis, forced researchers and policymakers to review their opinion. The basic issue the researchers face is the implementation of ZLB constraint in existing dynamic general equilibrium models. The current state of knowledge includes four main strands in the ZLB literature.

The first strand is based on the linearized equilibrium conditions except for the ZLB constraint (e.g. Eggertsson & Woodford, 2003). To obtain numerical results, they assume a two-state Markov chain with an absorbing state. Once the constraint is hit, there is the same fixed positive probability in each period the discount factor jumps to its long-run

[2] This policy rule is reasonably close to the CNB's reaction function (but still not entirely the same) where the future inflation also enters the rule.

level and the ZLB ceases to bind. Since then it will never be hit again which is restrictive assumption. Jung *et al.* (2005) and Guerrieri & Iacoviello (2014) extended the approach allowing the constraint to be hit again. The second strand of papers uses nonlinear perfect foresight solvers with "extended path" method (e.g. Coenen *et al.*, 2003; Braun & Körber, 2011). Solving under perfect foresight the precautionary motives of economic agents are omitted including those arising from the risk of hitting the ZLB. Since the model has to have returned to steady state up to machine precision by the final period considered, it requires a very large number of nonlinear equations to be solved. Hence, it tends to both be prohibitively slow, and unstable, with the algorithm frequently failing to find a solution to the equations. The third strand of papers is dealing with global approximations in a small scale New Keynesian model, using the Smolyak collocation method of Krueger *et al.* (2011) (see Fernández-Villaverde *et al.*, 2012). These models eliminate the disadvantages of previous ones as they successfully capture both the models nonlinearities and precautionary motives. Unfortunately, this methodology cannot be used in medium scale models.

We use the last approach which represents a compromise between the accuracy of global approximation methods, and the speed and scalability of linear ones. It relies on the idea of adding shocks to the bounded variable and is closely related to the work of Erceg & Linde (2010) and Holden & Paetz (2012). This method introduces anticipated news shock (called shadow price shock) to return the nominal interest rate to zero where disturbances cause negative rates. The ZLB on nominal interest rate binds as long as the shock which hits the economy causes the interest rate to be negative.[3] The algorithm basically ensures

$$i_t = max(0, i_t^T)$$

where i_t^T represents the central bank's operational target set by monetary policy rules.

This allows a rational expectations consistent simulation of a linearized DSGE model with an occasionally binding constraint. Shadow price shocks $\epsilon_{s,t}^{SP}$ are added to constrained equation

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(\bar{i} + \psi_\pi \pi_t + \psi_p p_t + \psi_e e_t) + \epsilon_{i,t} + \sum_{s=0}^{T^*-1} \epsilon_{s,t}^{SP}$$

The shock $\epsilon_{i,t}$ is known at t but does not materialize until $t + s$. The relative impulse response functions $m_{j,s}$ of all variables to the shadow price shock are found and stacked into a matrix $M = [m_{j,0}, m_{j,1}, ..., m_{j,T^*-1}]$. Then the impulse responses to a simultaneous

[3] For more technical details see Holden & Paetz (2012).

shock to $\epsilon_{i,t}$ of magnitude 1 and $\epsilon^{SP}_{s,t}$ of magnitude α_s are computed. The complete impulse response function to the shock $\epsilon_{i,t}$ with imposed constrained for variable x_j is given by the combination of the steady state of this variable, the impulse response to the shock $\epsilon_{i,t}$ and the multiplication between the magnitude of the news shocks and the matrix M_j, i.e. $\mu_j + v_j + M_j\alpha$ (Holden & Paetz, 2012). The main challenge is to find the value of α which can be done by solving the quadratic optimization problem[4]

$$\alpha^* := argmin_{\alpha \geq 0_{T^*} \mu^* + v^* + M^*\alpha \geq 0_{T^*}} \left[\alpha'(\mu^* + v^*) + \frac{1}{2}\alpha'(M^* + M^{*\prime})\alpha \right]$$

4 Model Parameters

Model parameters are partly calibrated and partly estimated. The calibration is based on stylized facts of the Czech data. Remaining parameters are estimated using Bayesian technique. The estimation is based on prior information which is updated by likelihood and results in a posterior distribution. For more details see Appendix B.

Parameters-setting is to a large extent a challenge, at least because of the data length in the Czech Republic. Just a short period of quarterly data from Q1/1997 to Q4/2014 is available. Moreover, these data are volatile, especially the inflation and the GDP in 1990s as the Czech economy went through its transition period. Despite this, the estimation may still contribute to better parameters-setting compared to the purely subjective choice of values.

During the estimation period, the CNB's inflation target is changed several times. Until 1997, the CNB was operating a fixed exchange rate regime which was abandoned in favor of inflation targeting. Initially, the CNB's inflation targets were set in terms of net inflation. In April 2001, a decision was made to switch to targeting consumer price index inflation. The target has decreased from 6 % in 1998 to current 2 %[5]. Following Ryšánek *et al.* (2011), it is controlled for this feature in estimation. The model target of inflation is linked to the data via measurement equation (9) and the target itself follows an AR(1) process as stated in equation (10).

[4]In practice it is solved using MATLAB function *quadprog*.

[5]For more details see `https://www.cnb.cz/en/monetary_policy/inflation_targeting.html`.

$$\pi^T_{data,t} = \pi^T_{model,t} \tag{9}$$

$$\pi^T_{model,t} = \rho^T \pi^T_{model,t} + \varepsilon^T_t \tag{10}$$

4.1 Calibrated Parameters

The range of data for both calibration and estimation is between 1997 and 2014. The steady state nominal interest rate $\bar{i}$ is computed as an average of 3-month PRIBOR. The discount factor is calibrated to be 0.990 which corresponds to an average annual nominal interest rate of 4 %[6]. This value is in line with commonly used values and research studies for the Czech Republic (e.g. Štork *et al.*, 2009). The foreign discount factor is set to 0.994 which is consistent with 2.6 % nominal interest rate (average of 3-month EURIBOR).

To obtain the degree of openness α we sum up import and export in each period and divide it by twice the GDP. Using quarterly data on real GDP, export and import of goods and services we average the ratio throughout all periods. The standard deviation of the depreciation rate σ_e is calibrated using quarterly data on the nominal effective exchange rate index constructed by the Czech National Bank. The elasticity of substitution for domestic and foreign goods is calibrated based on estimates from Ryšánek *et al.* (2011) and the value for foreign habit persistence based on Smets & Wouters (2004). The last coefficient, the absolute risk aversion of dealers γ, is set to 500 as advised by Montoro & Ortiz (2013). All parameters are summarized in Table 1.

4.2 Data and Prior Distribution

Following Justiniano & Preston (2010) and Bäuerle & Menz (2008), the data on output, inflation, short-term nominal interest rate and real exchange rate are used. The foreign economy is proxied with data on OECD countries, reflecting the share of the Czech export and import volumes. The domestic set of variables contains the real GDP, the annual consumer price inflation, the effective real exchange rate deflated by consumer prices and the 3-month PRIBOR. The foreign variables are the real GDP, the GDP deflator and the 3-month EURIBOR. As our theoretical model does not explicitly model a trend, we use output growth instead of output in levels. The first differences of real exchange rate are taken.

[6] $\beta = 1/(1+\bar{i})$ where $\bar{i}$ is a steady state nominal interest rate (see equation (22)).

The prior beliefs are obtained mostly from (Ryšánek *et al.*, 2011; Smets & Wouters, 2004; Altig *et al.*, 2010) and summarized in Appendix C. Gamma distribution with large tails was chosen for the inverse Frisch elasticity of the labor supply φ. The reason for such a loose distribution and high variance is a diversity in estimates provided in existing research studies (e.g. McDaniel & Balistreri, 2003). The beta distribution is assigned to most of the persistence parameters and to Calvo coefficients as they lie within 0 and 1. Where possible, the priors for persistence parameters and standard deviations of shocks are initialized following Ryšánek *et al.* (2011), Altig *et al.* (2010) or Smets & Wouters (2004). The prior distribution for Calvo parameters usually follows large values proposed in the literature. But there are some studies suggesting this approach may be misleading (e.g. Bils & Klenow, 2004). Some prior information on Calvo parameters can be obtained using the relationship $\theta = \frac{Q-1}{Q}$, where Q is the frequency of price changes in quarters. In the Czech National Bank Inflation Report II/2011 the average frequency of price changes was estimated as 10.7 months which corresponds to $\theta = 0.72$ (Collective, 2011). This value can be used as the prior mean for the domestic economy. Estimated open economy models exhibit usually large deviations from the law of one price, hence, we use the same (quite high) mean value for imported goods (Justiniano & Preston, 2010). The prior mean of Calvo parameter for the rest of the world θ^* is set to 0.75 as advised by Smets & Wouters (2004). This value corresponds to the contract of a one-year average length.

The price indexation on lagged inflation is initialized with 0.35 mean and standard deviation of 0.2 because its value usually falls within the range of 0 and 0.5 (e.g. Ryšánek *et al.*, 2011; Smets & Wouters, 2004). The habit persistence parameter is mostly estimated as very high (> 0.9) while the opposite is found by Smets & Wouters (2004) who obtained 0.59 for the EU and 0.69 for the US. To make a compromise the prior mean is set to 0.8 with the standard deviation of 0.15.

Table 1. Model Parameters

Calibration		
β	Domestic discount factor	0.990
β^*	Foreign discount factor	0.994
α	Trade openness	0.583
h^*	Foreign habit persistence	0.800
η	Elasticity of substitution for domestic goods	1.870
η^*	Elasticity of substitution for foreign goods	1.923
σ_e	Standard deviation - nominal ER depreciation rate	0.031
γ	Absolute risk aversion of FX dealers	500
ϑ	Portfolio-adjustment-cost parameter	1.000
Estimation		
σ	Domestic parameter of relative risk aversion	0.998
σ^*	Foreign parameter of relative risk aversion	1.609
φ	Domestic inverse labor supply elasticity	3.328
φ^*	Foreign inverse labor supply elasticity	1.276
h	Domestic habit persistence	0.738
δ_H	Indexation - domestic producers	0.232
δ_M	Indexation - importing retail firms	0.807
δ^*	Indexation - foreign economy/exporters	0.751
θ_H	Calvo-parameter - domestic producers	0.861
θ_M	Calvo-parameter - importing retail firms	0.854
θ^*	Calvo-parameter - foreign economy / exporters	0.902
ψ_π	Domestic inflation-targeting rule	1.445
ψ_π^*	Foreign inflation-targeting rule (inflation)	1.488
ψ_y^*	Foreign inflation-targeting rule (output gap)	0.082
ρ_i	Persistence - domestic monetary policy rule	0.836
ρ_g	Persistence - domestic preference shock	0.783
ρ_a	Persistence - domestic technology shock	0.256
ρ_s	Persistence - terms of trade shock	0.960
ρ_i^*	Persistence - foreign monetary policy rule	0.432
ρ_g^*	Persistence - foreign preference shock	0.963
ρ_a^*	Persistence - foreign technology shock	0.477
ρ_w^*	Persistence - capital-inflow shock	0.558
σ_i	Standard deviation - domestic interest rate shock	0.020
σ_g	Standard deviation - domestic preference shock	0.236
σ_a	Standard deviation - domestic technology shock	0.401
σ_s	Standard deviation - terms of trade shock	0.060
σ_i^*	Standard deviation - foreign interest rate shock	0.061
σ_g^*	Standard deviation - foreign preference shock	0.453
σ_a^*	Standard deviation - foreign inflation shock	0.122
σ_w^*	Standard deviation - capital-inflow shock	0.071

4.3 Posterior Distribution

The model is estimated using a Bayesian estimation routine in Dynare, software capable to handle rational expectations. After obtaining the posterior modes and approximation of the Hessian, the Metropolis-Hastings algorithm is used for the posterior sampling of parameters. In total one million draws is taken with two separate chains where 150 000 initial draws are burned in both cases. The acceptance ratio is set to 25 %. We control for convergence using CUMSUM (Cumulative Sum Control) plots. The convergence statistics and the results of estimation can be found in Appendix C.

Regarding the precision of estimation, the vast majority of parameters does not resemble the prior distribution. The posterior mean is quite close to the prior information just for the persistence parameter on lagged nominal interest rate in policy reaction function. All estimates give reasonable values which are in many cases in line with results in Ryšánek *et al.* (2011) or Smets & Wouters (2004). Results show high persistence of preference shock, both domestic and foreign, and terms-of-trade shock, and low persistence of domestic technology shock. Parameters of domestic monetary policy reaction function reached the expected values. The posterior mean of habit persistence parameter is estimated around 0.64 which is between the estimates for EU and US obtained by Smets & Wouters (2004). This value is not as high as expected but we can still suggest households care not just about the level of their consumption but also about its growth.

5 Foreign Exchange Interventions and the Severity of Zero Lower Bound

We study the economic dynamics and intervention mechanism under the zero lower bound in response to 5 different shocks, both domestic and foreign. On the supply side, the economy is hit by positive domestic technology shock, on the demand side by negative domestic and foreign preference shock. In addition, we examine the response to an unexpected inflow of capital (capital inflow shock) and a change in relative price of exports in terms of imports (terms-of-trade shock). It allows us to make a complex picture of the intervention mechanism and the effect of ZLB.

We compare five different situations. In the benchmark case, the nominal interest rate is not at the ZLB and central bank can use its conventional policy tool - short-term interest rate - to accommodate the shock. In four other cases the economy is at the ZLB, and the central bank can either do nothing or intervene at FX markets by choosing from three different intervention regimes, (i) the managed float, (ii) the fixed nominal exchange rate, and (iii) the real exchange rate rule. The parameterization of intervention rule for all cases is summarized in Table 2. Resulting impulse response functions can be found in Appendix D.

Table 2. Monetary Policy Regimes - Parameterization

		χ_e	χ_{e^T}	χ_q	
1	Conventional Times	0	0	0	Without ZLB
2	Inactive Central Bank	0	0	0	ZLB
3	Managed Float	20	0	0	
4	Fixed on the Target	0	∞	0	
5	Real Exchange Rate Rule	0	0	20	

First, we compare the situation without FX interventions at the ZLB and out of it (i.e. case 1 and 2). We find pronounced downward pressures on prices and more volatile reaction of majority of variables at the ZLB compared to conventional times in response to all shocks.[7] Regarding the positive technology shock, the economic expansion is muted. In normal times, the improvement in technology makes the production more efficient, increases output and lowers prices. The central bank reacts with decrease in nominal interest rate to get inflation back on the target. If the central bank cannot use this tool, prices fall sharply, and the nominal exchange rate appreciates. Due to stronger nominal exchange rate, initially positive response of export is much weaker and the overall economic expansion milder. This is consistent with empirical estimates. Gavin *et al.* (2013) find that in the response to the positive technology shock, the consumption and output are lower at the ZLB.

In case of the negative demand shock, households' preferences are shifted towards future consumption (households value future consumption more) and consumption initially decreases.[8] It creates downward pressures on both, domestic and import prices. The nominal exchange rate initially depreciates which together with lower domestic prices improves

[7]Pronounced volatility of real and nominal macroeconomic variables at the ZLB is in line with an extensive research including Coenen *et al.* (2003), Williams (2009), Bodenstein *et al.* (2009) and Amano & Shukayev (2012).

[8]As we assume additively separable preferences in consumption and labor, the utility from consumption does not affect the (dis)utility from working directly but just indirectly through labor supply curve. Cross-derivatives of utility function w.r.t. worked hours and consumption are zero.

the competitiveness and contributes to higher export. At the ZLB central bank cannot mitigate deflationary pressures which push up the real interest rate and results in even stronger negative response of domestic and import prices. The nominal exchange rate appreciates immediately which decreases the competitiveness of domestic export. As the result, export and output fall significantly.

Regarding foreign shocks and terms-of-trade shock, the downward pressures on prices are even more pronounced at the ZLB. Studying the response to foreign demand shock (foreign preference shock) is valuable for an open economy such as the Czech Republic. If the major trading partner is experiencing a recession, the real disposable income of its consumers fall and hence also demand for import (i.e. domestic export). It is followed by output shortage, nominal exchange rate appreciation and slight decline in prices. At the ZLB the initial appreciation and drop in export is much stronger which pushes down prices and dampens the economic activity much more.

The unexpected capital inflow results in the nominal exchange rate appreciation with the decline in import prices and the rise in import. At the same time the export worsens due to higher prices of domestic products at foreign markets. The fall in export together with the increase in import worsens the domestic net export and hence output. With the ZLB and inactive central bank, the stronger nominal appreciation of domestic currency together with increased exchange rate pass-through reinforces the impact on falling prices. The decline in net export is much deeper than the increase in domestic consumption which magnifies economic downturn.

A negative terms-of-trade shock puts severe downward pressures on prices through strong nominal appreciation and deterioration in competitiveness of domestic export at the ZLB. The net export declines significantly which is mirrored by output. In normal times this shock would be followed by a large interest rate cut which is not possible at the ZLB.

The severity of constrained rates is dependent on both, the size and the duration of the shock. With small or transitory shocks, the response of macroeconomic variables tends to be short and modest at the ZLB (Williams, 2014). On the other hand, if the shock is persistent, the effect of the ZLB may be more serious. This is apparent from the analysis of foreign preference shock with two different persistence parameters[9] (see Appendix D). Hence, the greatest threat comes with highly persistent shocks. Since the recovery after

[9] The pronounced response with higher persistence parameter is similar for all shocks.

banking and financial crises is often very slow (e.g. Jordá *et al.*, 2011; Reinhart & Rogoff, 2009), it could lead to prolonged periods of binding ZLB.

At this point we turn the attention to intervention regimes. In all cases, FX interventions help to mitigate deflationary pressures and recover economic activity, more or less successfully depending on specific regime. Fixing nominal exchange rate seems to be the most effective strategy among presented alternatives. This type of regime prevents the nominal appreciation and hence the deterioration in competitiveness of domestic export. This in turn improves net export and restores output. The real exchange rate rule proved to be the second best option. It gives similar results as fixing nominal exchange rate in a half of cases. Intervening to correct the real exchange rate misalignment from its fundamentals accommodates the shock nearly completely and performs even better than central bank in the benchmark case without constraint interest rate.

Both regimes perform very well in response to the capital inflow shock. The use of interventions almost completely counteracts the shock and leads to overall stabilization of the economy at the ZLB. This is possible because the shock affects economy through the same risk premium term as the central bank interventions. In the response to terms-of-trade shock the central bank is able to mitigate downward pressures on prices under both intervention regimes. But the volatility of domestic prices become pronounced upwards as the macroeconomic adjustment to this shock has to pass through prices instead of exchange rate (see equations (52) and (53)). Under the managed float regime the central bank intervenes just to smooth the nominal exchange rate movements between two consecutive periods, i.e. to smooth the speed of its adjustment and not to control for the particular level of nominal exchange rate (see Beneš *et al.*, 2013). Therefore it is less effective compared to remaining two regimes at the ZLB.

The adoption of real exchange rate rule might seem to be a better option than fixing nominal exchange rate as it does not require such a strong commitment. Nevertheless, in practice it will be almost impossible to target a precise level of the real exchange rate because it is very difficult to measure the real exchange rate misalignment correctly (Eden & Nguyen, 2012). For this reason, fixed nominal exchange rate has proven to be the most effective FX intervention regime at the ZLB when the economy is facing pronounced deflationary pressures, among presented alternatives.

6 Conclusions

The use of unconventional monetary policy tools may be reasonable when the central bank is not further able to lower the short-term interest rate in order to ease monetary conditions. The investigation of this situation together with the effectiveness of unconventional measures thus gains in importance as there is often a fear the economy might get on the deflationary equilibrium path (into the liquidity trap). Despite a growing number of empirical studies estimating the effect of these nonstandard measures, the analysis of their effectiveness in structural models is still limited.

This paper provides a theoretical framework for modeling FX interventions at the ZLB. It is motivated by the current Czech experience which gives rise to the question whether interventions really serve as an effective policy tool when the nominal interest rates are constrained at zero. The standard New Keynesian DSGE model is extended by implementation of FX dealers and occasionally binding constraint. The dealers optimize their portfolio allocation of foreign and domestic bonds in order to maximize their expected utility which allows analyzing the transmission of interventions through the portfolio balance channel and the expectations channel.

The paper contributes to understanding the economic dynamics at the ZLB and the exchange rate movements under different central bank intervention regimes. We find the increased volatility of real and nominal macroeconomic variables in response to the domestic demand shock, the foreign demand and financial shocks and the terms-of-trade shock at the ZLB. On the other hand, the effect of positive domestic technology shock is subdued and the economic expansion is muted. This effects become severe in response to highly persistent shocks which leads to stronger reaction of variables and prolong period of binding constraint.

The FX interventions have proven to be effective in further monetary easing at the ZLB. They help to mitigate deflationary pressures and recover economic activity in response to all examined shocks, more or less successfully depending on specific regime. In this sense, the central bank achieves the best performance by fixing the nominal exchange rate. The adoption of the real exchange rate rule (which corrects the misalignment of the real exchange rate from its fundamentals) also reduces the overall macroeconomic volatility and fall in prices significantly. In practice the real exchange rate misalignment is very difficult to measure and hence nearly impossible to target in real time.

This analysis opens the door to future research. The emphasis should be put on both the implementation of the ZLB algorithm and the model extension. In particular, some unpleasant properties of the ZLB constraint should be investigated in more details. Attention should be given to the existence of multiple equilibria, the invisibility of equilibria connected with the linearization of equilibrium conditions and the robustness of results obtained using the linearization in comparison to nonlinear equilibrium conditions.

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A Model

A.1 Domestic Households

In general, household preferences can be expressed by the following utility function

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{11}$$

where U_t is the utility at time t, E_t are rational expectations at time t, $U(.)$ denotes the utility function which is assumed to be continuous and twice differentiable in both, consumption and labor, with $U' > 0$ and $U'' < 0$, $\beta \in \langle 0, 1)$ is a discount factor, C_t is the consumption at time t and N_t is the labor supplied by representative agent at time t.

The specific utility function with constant relative risk aversion preferences is assumed while the labor and consumption enters the utility function separately

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \xi_{g,t} \left[\frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{12}$$

where $H_t = hC_{t-1}$ is a habit formation term which is given exogenously, $\xi_{g,t}$ is a preference shock, σ is a coefficient of relative risk aversion (measured as an inverse of the intertemporal elasticity of substitution (IES) in consumption) and φ is an inverse Frisch elasticity of labor supply. For $0 \leq \sigma < 1$, the IES is high and agents are quite willing to substitute consumption between periods. For $\sigma > 1$, the IES is low and agents do not like shifting their consumption in time. The Frisch elasticity captures the substitution effect between wage rate and labor supply, holding the marginal utility of wealth constant (Heer & Maussner, 2009).

For simplicity, households are able to save only in domestic bonds. Then the budget constraint of each representative agent can be expressed as follows

$$P_t C_t + D_t = D_{t-1}(1 + i_{t-1}) + \frac{\psi}{2}(D_t - \bar{D})^2 + W_t N_t + \Pi_{M,t} + \Pi_{H,t} + \Pi_{FX,t} + T_t \tag{13}$$

where D_t represents the quantity of one-period bonds denominated in domestic currency, P_t is a price level, W_t is a nominal wage, $\Pi_{X,t}$ denotes profits which are distributed from firms and foreign exchange dealers to households and T_t is a transfer from government. Moreover, the household faces portfolio adjustment costs $(\psi/2)(D_t - \bar{D})^2$ which provide a mechanism for closing the model, i.e. to determine a steady state (see Schmitt-Grohe & Uribe, 2003).

The households' consumption basket is composed of non-traded, domestically produced goods and imported, foreign goods. It follows a form of the Dixit-Stiglitz function

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} M_t^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{14}$$

where $C_{H,t}$ and M_t are indices of domestic and foreign goods consumption, respectively. Parameter η denotes the elasticity of substitution between these goods and α is the weight of foreign goods relative to the total consumption. It describes the trade openness of the economy. Both indices, $C_{H,t}$ and M_t, are given by the constant elasticity of substitution (CES) functions

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j) dj^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{15}$$

$$M_t = \left(\int_0^1 C_{i,t} di^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \tag{16}$$

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j) dj^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{17}$$

where $C_{i,t}$ is an index of consumption of goods imported from country i, ϵ is the elasticity of substitution between various goods produced domestically and γ is the elasticity of substitution between goods produced in different foreign countries.

The minimization of consumption expenditures $P_t C_t$ subjected to the consumption index (14) yields demand functions for non-traded goods (18) and import (19). Moreover, we assume a demand function for export 20 is of the same form as the demand function for import, just with different parameters

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{18}$$

$$M_t = \alpha\left(\frac{P_{M,t}}{P_t}\right)^{-\eta} C_t \tag{19}$$

$$X_t = \zeta\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} Y^*_t \tag{20}$$

where $P_{H,t}$, $P_{M,t}$ and $P^*_{H,t}$ are price levels of non-traded, imported and exported goods, respectively, and P^*_t is the world price. The aggregate price index P_t follows the form of the Dixit-Stiglitz function

$$P_t = \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{M,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{21}$$

Given these demand functions, households maximize their lifetime utility (12) subject to the budget constraint (13) by choosing the optimal amount of consumption, labor and investment. The combination of first order conditions yields following optimality conditions

$$\frac{\xi_{g,t}}{(C_t - hC_{t-1})^\sigma} = \beta E_t\left[\frac{\xi_{g,t+1}}{(C_{t+1} - hC_t)^\sigma}(1+i_t)\frac{P_t}{P_{t+1}}\right] \tag{22}$$

$$(C_t - hC_{t-1})^\sigma N_t^\varphi = \frac{W_t}{P_t} \tag{23}$$

Equation (22) represents the standard Euler equation and equation 23 describes the supply of labor.

A.2 Domestic Producers

Each firm produces differentiated good using the following linear technology with labor as a single input

$$y_t(i) = \xi_{a,t} N_t(i) \tag{24}$$

where $\xi_{a,t}$ represents technological innovation. The aggregate production function has the form of the Dixit-Stiglitz function

$$Y_t = \left[\int_0^1 y_t(i)^{\frac{\tau-1}{\tau}} di\right]^{\frac{\tau}{\tau-1}} \tag{25}$$

where τ is the elasticity of substitution between different varieties.

Firms face each period an exogenous probability $(1-\theta_H)$ of re-optimizing their prices. This probability is independent of the time when the firm last changes its price (Calvo, 1983). Following Bäuerle & Menz (2008), firms which cannot reset their prices at least adjust for inflation according to the following rule

$$P_{H,t}(i) = P_{H,t-1}(i)\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\delta_H} \tag{26}$$

where δ_H describes to which extent the firm reacts to the past inflation. All firms which re-optimize their prices in period t set the same price $P'_{H,t}$. The overall price index of domestic goods is as follows

$$P_{H,t} = \left\{ (1-\theta_H) {P'_{H,t}}^{1-\epsilon} + \theta_H \left[P_{H,t-1} \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\delta_H} \right]^{1-\epsilon} \right\}^{\frac{1}{1-\epsilon}} \tag{27}$$

Firms take into account the probability that they will not be able to re-optimize prices in the future. From the households' problem the form of the domestic demand for variety is known

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} C_{H,t} \tag{28}$$

We assume the foreign demand follows the same functional form. Hence, we can construct a demand curve for firm which resets its price in the period t and adjusts its price according to (25) in periods $t + k$

$$C_{H,t+\tau|t} = \left[\frac{P'_{H,t}}{P_{H,t+\tau}} \left(\frac{P_{H,t+\tau-1}}{P_{H,t-1}} \right)^{\delta_H} \right]^{-\epsilon} (C_{H,t+\tau} + X_{t+\tau}) \tag{29}$$

Domestic firms which can reset their prices in period t maximize the expected discounted profit given by equation (30) subjected to equation (29).

$$E_t \sum_{\tau=0}^{\infty} \theta_H^{\tau} Q_{t,t+\tau} C_{H,t+\tau|t} \left[P'_{H,t} \left(\frac{P_{H,t+\tau-1}}{P_{H,t-1}} \right)^{\delta_H} - P_{H,t+\tau} MC_{t+\tau} \right] \tag{30}$$

where $MC_t = W_t/(P_{H,t}\xi_{a,t})$ are real marginal cost, $Q_{t,t+\tau} = \beta^{\tau} \Lambda_{t+\tau}/\Lambda_t$ is a stochastic discount factor (pricing kernel) with $\Lambda_t = \xi_{g,t}(C_t - hC_{t-1})/P_t$. The stochastic discount factor measures the marginal rate of substitution between consumption at t and *t+1*.

The first order condition with respect to $P'_{H,t}$ represents the optimality condition for the price level of domestically produced and consumed goods

$$P'_{H,t} = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{\tau=0}^{\infty} \theta_H E_t[Q_{t,t+\tau} C_{H,t+\tau|t} P_{H,t+\tau} MC_{t+\tau}]}{\sum_{\tau=0}^{\infty} \theta_H E_t[Q_{t,t+\tau} C_{H,t+\tau|t} (P_{H,t+\tau-1}/P_{H,t-1})^{\delta_H}]} \tag{31}$$

Domestic goods are assumed to be sold at the law of one price at the foreign market $P_{H,t} = e_t P^*_{H,t}$. Hence, exporters face basically the same optimality condition where prices are just set in foreign currency.

A.3 Domestic Retailers

We assume the Calvo price setting with the probability θ_M and the indexation rule for firms which cannot reset their prices with parameter δ_M. Domestic retailers maximize the expected discounted profit subjected to the demand for this good

$$\max \qquad E_t \sum_{\tau=0}^{\infty} \theta_M^{\tau} Q_{t,t+\tau} M_{t+\tau|t} \left[P'_{M,t} \left(\frac{P_{M,t+\tau-1}}{P_{M,t-1}} \right)^{\delta_M} - e_{t+\tau} P^*_{M,t+\tau} \right] \tag{32}$$

$$s.t. \quad M_{t+\tau|t} = \left[\frac{P'_{M,t}}{P_{M,t+\tau}} \left(\frac{P_{M,t+\tau-1}}{P_{M,t-1}} \right)^{\delta_M} \right]^{-\epsilon} M_{t+\tau} \tag{33}$$

The maximization results in the following optimality condition for the price level of imported goods

$$P'_{M,t} = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{\tau=0}^{\infty} \theta_M E_t [Q_{t,t+\tau} M_{t+\tau|t} e_{t+\tau} P^*_{M,t+\tau}]}{\sum_{\tau=0}^{\infty} \theta_M E_t [Q_{t,t+\tau} M_{t+\tau|t} (P_{M,t+\tau-1}/P_{M,t-1})^{\delta_M}]} \tag{34}$$

A.4 Market Clearing

The market clearing condition equalizes the total domestic production of goods Y_t with the sum of non-traded goods $C_{H,t}$ and exported goods X_t.

$$Y_t = C_{H,t} + X_t \tag{35}$$

A.5 Additional SOE Relations

Another three expressions have to be defined to complete the model - the real exchange rate, the effective terms of trade and the law-of-one-price gap. The bilateral real exchange rate is defined as a ratio between countries' CPIs denominated in the same currency (Gali, 2009)

$$\mathcal{Q}_{i,t} = \frac{\mathcal{E}_{i,t} P_t^i}{P_t} \tag{36}$$

where $\mathcal{E}_{i,t}$ is the nominal exchange rate of country i for $i \in \langle 0, 1 \rangle$. The effective real exchange rate then follows

$$\mathcal{Q}_t = \frac{\mathcal{E}_t P_t^*}{P_t} = \left(\int_0^1 \mathcal{Q}_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \tag{37}$$

The bilateral terms of trade represents the price of country i's goods in terms of home goods for $i \in \langle 0, 1 \rangle$

$$S_{i,t} = \frac{P_{i,t}}{P_{H,t}} \tag{38}$$

The effective terms of trade is defined as follows

$$S_t = \frac{P_{H,t}}{P_{M,t}} = \left(\int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \tag{39}$$

The law-of-one-price gap refers to the ratio between world price denominated in domestic currency and the price of imported goods

$$\Psi_{M,t} = \frac{\mathcal{E}_t P_t^*}{P_{M,t}} \tag{40}$$

A.6 Log-linear Approximation to the Model

First, the Euler equation (22) is log-linearized. We should highlight two semi-standard things used in this process. As i_t is already in percent, we leave it in absolute deviation from the steady state $(i_t - \bar{i})$. Then we approximate the term $\bar{i}/(1+\bar{i}) = 1$ which is suitable if β is sufficiently high. Now the Euler equation can be linearized to

$$c_t - hc_{t-1} = E_t c_{t+1} - hc_t - \frac{1-h}{\sigma} \left[(i_t - E_t \pi_{t+1}) + \vartheta d_t + (\xi_{g,t} - E_t \xi_{g,t+1}) \right] \tag{41}$$

where $\vartheta = \psi \bar{D}$.

Before the log-linearization of the budget constraint (22) we adjust this relation a bit. Government transfers T_t are chosen to neutralize distortions from the monopolistic competition. The aggregate profits from FX dealers are equal to zero. Moreover, we follow Bäuerle & Menz (2008) in introduction of profits from the final goods producing firms $\Pi_{FG,t} = P_t C_t - P_{M,t} M_t - P_{H,t} C_{H,t}$. Then we can write

$$\Pi_{FG,t} + \Pi_{M,t} + \Pi_{H,t} = P_t C_t - W_t N_t - P_{M,t} M_t - P_{H,t} C_{H,t} \tag{42}$$

Substituting the relation into the budget constraint yields

$$D_t = (1 + i_{t-1})D_{t-1} + \frac{\psi}{2}(D_t - \bar{D})^2 - P_{M,t}M_t - P_{H,t}C_{H,t} \tag{43}$$

The linearization is now straightforward

$$\begin{aligned}(2\alpha - 1)[\bar{i}^{-1}\left(\beta^{-1}d_{t-1} - d_t\right) + i_{t-1}] = \\ =\alpha(p_{M,t} + m_t) + (1 - \alpha)(p_{H,t} + c_{H,t})\end{aligned} \tag{44}$$

Log-linearization of the market clearing condition requires some preliminaries. We assume $\bar{P} = \bar{P}_M = \bar{P}_H$ in the zero inflation steady state which implies $\bar{C}_H = (1 - \alpha)\bar{C}$, $\bar{M} = \alpha\bar{C}$ and $\bar{Y} = \bar{C}$. Then the linearization of the resource constraint (35) proceeds as follows

$$y_t = (1 - \alpha)c_{H,t} + \alpha x_t \tag{45}$$

The demand functions for export, import and domestic goods (18) - (20) are linearized quite easily, and they are expressed in equations (46) - (48).

$$x_t = -\eta^*(p_{H,t} - p_t^* - e_t) + y_t^* \tag{46}$$

$$m_t = -\eta(p_{M,t} - p_t) + c_t \tag{47}$$

$$c_{H,t} = -\eta(p_{H,t} - p_t) + c_t \tag{48}$$

The linearized CPI, terms of trade, law-of-one-price gap and real exchange rate are described in equations (53) - (50).

$$p_t = (1 - \alpha)p_{H,t} + \alpha p_{M,t} \tag{49}$$

$$s_t = p_{M,t} - p_{H,t} \tag{50}$$

$$\psi_{M,t} = e_t + p_t^* - p_{M,t} \tag{51}$$

$$q_t = e_t + p_t^* - p_t \tag{52}$$

By substituting the linearized terms-of-trade relation (50) into the aggregate price index (53) we get

$$p_t = p_{H,t} + \alpha s_t \tag{53}$$

The CPI inflation rate is defined as $\Pi_t = (P_t - P_{t-1})/P_{t-1}$, or in linearized form as

$$\pi_t = p_t - p_{t-1} \tag{54}$$

The price indices for domestic goods and import follow the similar definition

$$p_{H,t} = p_{H,t-1} + \pi_{H,t} \tag{55}$$

$$p_{M,t} = p_{M,t-1} + \pi_{M,t} \tag{56}$$

Now we focus on the log-linearization of optimal prices of domestic firms and domestic retailers. First, we divide both sides of equations (27) and (31) by $P_{H,t-1}$ to obtain the inflation rate

$$\Pi_{H,t}^{1-\epsilon} = \left[(1-\theta_H)\frac{P'_{H,t}}{P_{H,t-1}}^{1-\epsilon} + \theta_H \left(\Pi_{H,t-1}\right)^{\delta_H(1-\epsilon)}\right]$$

$$E_t \sum_{\tau=0}^{\infty} \theta_H Q_{t,t+\tau} C_{H,t+\tau|t} \Pi_{H,t-1,t+\tau-1}^{\delta_H} \frac{P'_{H,t}}{P_{H,t-1}} =$$

$$= \frac{\epsilon}{\epsilon-1} E_t \sum_{\tau=0}^{\infty} \theta_H Q_{t,t+\tau} C_{H,t+\tau|t} \Pi_{H,t-1,t+\tau} MC_{t+\tau}$$

Rearranging terms in the latter expression yields

$$0 = E_t \sum_{\tau=0}^{\infty} \theta_H Q_{t,t+\tau} C_{H,t+\tau|t} \left[\frac{P'_{H,t}}{P_{H,t-1}} \Pi_{H,t-1,t+\tau-1}^{\delta_H} - \frac{\epsilon}{\epsilon-1} MC_{t+\tau} \Pi_{H,t-1,t+\tau}\right] \tag{57}$$

In the zero inflation steady state following relationship holds $\bar{P_H} = \bar{P_H}'$. Therefore, the steady state level of real marginal cost is $\bar{MC} = (\epsilon-1)/\epsilon$. Moreover, the discount factor $Q_{t,t+\tau}$ is equal to β^τ. Now we can log-linearize equation (57)

$$0 = E_t \sum_{\tau=0}^{\infty} (\theta_H \beta)^\tau [p'_{H,t} - p_{H,t-1} + \delta_H(p_{H,t+\tau} - p_{H,t-1}) - (mc_{t+\tau} + p_{H,t+\tau} - p_{H,t-1})]$$

We need to rearrange the expression to obtain a gap between the optimal price and the price from the last period. This difference is equal to the weighted sum of expected future marginal cost and domestic inflation

$$p'_{H,t} - p_{H,t-1} = (1-\theta_H\beta)E_t\sum_{\tau=0}^{\infty}(\theta_H\beta)^{\tau}[(1-\delta_H\theta_H\beta)\pi_{H,t+\tau} + (1-\theta_H\beta)mc_{t+\tau}]$$

To get rid of the summation operator we use the first difference equation

$$p'_{H,t} - p_{H,t-1} = (1-\delta_H\theta_H\beta)\pi_{H,t} + (1-\theta_H\beta)mc_t + (\theta_H\beta)E_t[p'_{H,t+1} - p_{H,t}] \tag{58}$$

In the next step we log-linearize equation (27) and obtain

$$p'_{H,t} - p_{H,t-1} = \frac{\pi_{H,t}}{1-\theta_H} - \frac{\delta_H\theta_H}{1-\theta_H}\pi_{H,t-1} \tag{59}$$

Finally, equation (59) is substituted into the difference equation (58). By rearranging we obtain the Phillips curve for domestic goods

$$\pi_{H,t} - \delta_H\pi_{H,t-1} = \beta E_t[\pi_{H,t+1} - \delta_H\pi_{H,t}] + \kappa_H mc_t \tag{60}$$

where $\kappa_H = (1-\theta_H)(1-\theta_H\beta)/\theta_H$. In the following step the log-linear version of marginal cost has to be found. We substitute the optimality expression for the household labor supply (23) into the expression for marginal cost $MC_t = W_t/(P_{H,t}\xi_{a,t})$, and derive

$$MC_t = \frac{P_t(C_t - hC_t)^{\sigma}N_t^{\varphi}}{P_{H,t}\xi_{a,t}}$$

The log-linearization yields

$$mc_t = \varphi n_t + p_t - p_{H,t} - \xi_{a,t} + \sigma(1-h)^{-1}(c_t - hc_{t-1})$$

The substitution of terms of trade and production function gives us the final expression for marginal cost

$$mc_t = \varphi y_t - (1+\varphi)\xi_{a,t} + \alpha s_t + \frac{\sigma}{1-h}(c_t - hc_{t-1}) \tag{61}$$

The log-linearization of the optimal price setting of domestic retailers (34) is similar. The steady state level of the law-of-one-price gap is $\bar{\psi}_M = (\epsilon-1)/\epsilon$. Then we can obtain the following difference equation

$$p'_{M,t} - p_{M,t-1} = (1 - \delta_M \theta_M \beta)\pi_{M,t} + (1 - \theta_M \beta)\psi_{M,t} + (\theta_M \beta) E_t[p'_{M,t+1} - p_{M,t}]$$

The linearized price index for import has the same form as the price index for non-traded goods, just with different parameters

$$p'_{M,t} - p_{M,t-1} = \frac{\pi_{M,t}}{1 - \theta_M} - \frac{\delta_M \theta_M}{1 - \theta_M} \pi_{M,t-1}$$

The Phillips curve for imported goods is obtained by merging these two equations

$$\pi_{M,t} - \delta_M \pi_{M,t-1} = \beta E_t[\pi_{M,t+1} - \delta_M \pi_{M,t}] + \kappa_M \psi_{M,t} \tag{62}$$

where $\kappa_M = (1 - \theta_M)(1 - \theta_M \beta)/\theta_M$.

The modified uncovered interest rate parity is already linearized

$$i_t - i^*_t = (E_t e_{t+1} - e_t) - \gamma \sigma^2 (\omega^*_{CB,t} + \omega^*_t) \tag{63}$$

where the capital inflow follows an AR(1) process

$$\omega^*_t = \rho^*_\omega \omega^*_{t-1} + \varepsilon^*_{\omega,t} \tag{64}$$

As the foreign economy is of the similar structure, we present just the final set of log-linearized equations, i.e. the Euler equation, the Phillips curve, the marginal cost, the price index and the monetary policy rule

$$y^*_t - h^* y^*_{t-1} = E_t y^*_{t+1} - h^* y^*_t - \frac{1 - h^*}{\sigma^*}(i^*_t - E_t \pi^*_{t+1}) + (\xi^*_{g,t} - E_t \xi^*_{g,t+1}) \tag{65}$$

$$\pi^*_t - \delta^* \pi^*_{t-1} = \beta^* E_t[\pi^*_{t+1} - \delta^* \pi^*_t] + \kappa^* mc^*_t \tag{66}$$

$$mc^*_t = \varphi^* y^*_t + \frac{\sigma^*}{1 - h^*}(y^*_t - h^* y^*_{t-1}) - (1 + \varphi^*)\xi^*_{a,t} \tag{67}$$

$$p^*_t = p^*_{t-1} + \pi^*_t \tag{68}$$

$$i^*_t = \bar{i^*} + \rho^*_i (i^*_{t-1} - \bar{i^*}) + \psi^*_\pi (\pi^*_t - \pi^{*T}) + \psi^*_y (y^*_t - y^{*T}) + \varepsilon^*_{i,t} \tag{69}$$

where $\kappa^* = (1 - \theta^*)(1 - \theta^* \beta^*)/\theta^*$.

The preference and technology shock follows an AR(1) process

$$\xi_{g,t} = \rho_g \xi_{g,t-1} + \varepsilon_{g,t} \tag{70}$$

$$\xi_{a,t} = \rho_a \xi_{a,t-1} + \varepsilon_{a,t} \tag{71}$$

$$\xi^*_{g,t} = \rho^*_g \xi^*_{g,t-1} + \varepsilon^*_{g,t} \tag{72}$$

$$\xi^*_{a,t} = \rho^*_a \xi^*_{a,t-1} + \varepsilon^*_{a,t} \tag{73}$$

where $\left\{\varepsilon_{g,t}, \varepsilon^*_{g,t}, \varepsilon_{a,t}, \varepsilon^*_{a,t}\right\}$ together with $\left\{\varepsilon^*_{cb,t}, \varepsilon^*_{\omega,t}, \varepsilon_{i,t}, \varepsilon^*_{i,t}\right\}$ are $iid \sim N(0, \sigma^2_x)$.

B Bayesian Estimation

B.0.1 State Space Representation and Kalman Filter

Kalman filter serves as a convenient tool to evaluate the likelihood of the linearized DSGE model. It is used to solve a linear state space model with normally distributed errors. Therefore, if the initial state of the model and shocks are assumed to be normally distributed, the forecasts provided by the Kalman filter are optimal against observed variables and data-generating process of states. The state space model is defined by the system of transition and measurement equation

$$X_t = FX_{t-1} + GV_t \quad (74)$$

$$Y_t = HX_t + W_t \quad (75)$$

where X_t is a vector of unobserved state variables, Y_t is a vector of observed variables, V_t and W_t are vectors of error terms where $V_t \sim N(0, \Sigma_v)$ and $W_t \sim N(0, \Sigma_w)$. The coefficient matrices F,G and H are in most cases time-invariant and (together with Σ_v and Σ_w) in principle known, but they have to be estimated in practice. Assuming reasonable values for model parameters and both variances are available they can be summarized in the information set $\Theta = \{F, G, H, \Sigma_v, \Sigma_w\}$. The likelihood for given parameters is denoted by $f(Y_1, Y_2, ..., Y_T; \Theta)$. Now the Bayes' theorem can be used (Pichler, 2007)

$$\begin{aligned} f(Y_1, Y_2, ..., Y_T; \Theta) &= f(Y_1, \Theta) f(Y_2|Y_1, \Theta) f(Y_3|Y_2, Y_1, \Theta) ... f(Y_T|Y_{T-1}, ..., Y_1, \Theta) \\ &= \prod_{t=1}^{T} f(Y_t|Y^{t-1}, \Theta) \end{aligned}$$

where $Y^{t-1} = (y_1, y_2, ..., y_{t-1})$ for $t \geq 2$. The log-likelihood function is then given by

$$lnL(Y^T, \Theta) = \sum_{t=1}^{T} lnf(Y_t|Y^{t-1}, \Theta)$$

In the next step, Kalman filter is used to construct the likelihood function which is created recursively by generating the forecasts from the system of equations (74) and (75) and their update. In principle, the procedure follows four steps - (i) the initialization, (ii) the prediction, (iii) the correction and (iv) the likelihood construction (Pichler, 2007). Initial values can be estimated using MLE or set to the steady state values of the system.

Then the vector of state variables X_t is predicted based on the information set available in period $t-1$ (Θ_{t-1}), i.e. before observables Y_t are known

$$\begin{aligned} X_{t|t-1} &= E(X_t|\Theta_{t-1}) = FX_{t-1|t-1} \\ \Sigma^X_{t|t-1} &= var(X_t|\Theta_{t-1}) = F\Sigma^X_{t-1|t-1}P' + \Sigma_w \end{aligned}$$

where Σ^X is the covariance matrix of the vector of state variables. Further, $X_{t|t-1}$ can be used to obtain $Y_{t|t-1} = HX_{t|t-1}$ and to compute a prediction error $\eta_{t|t-1}$ with its variance $\Sigma^\eta_{t|t-1}$

$$\begin{aligned} \eta_{t|t-1} &= Y_t - Y_{t|t-1} = Y_t - HX_{t|t-1} \\ \Sigma^\eta_{t|t-1} &= H\Sigma^X_{t|t-1}H' + \Sigma_v \end{aligned}$$

The correction of forecasted $X_{t|t-1}$ and $\Sigma^X_{t|t-1}$ is based on the observed data Y_t and the Kalman formula (Kalman, 1960)

$$\begin{aligned} X_{t|t} &= X_{t|t-1} + K_t(Y_t - Y_{t|t-1}) = X_{t|t-1} + K_t(Y_t - HX_{t|t-1}) \\ \Sigma_{t|t} &= \Sigma^X_{t|t-1} - K_t(\Sigma_v + H\Sigma^X_{t|t-1}H')K_t' \end{aligned}$$

where

$$K_t = \Sigma^X_{t|t-1}H'(H\Sigma_{t|t-1}H' + \Sigma_v)^{-1}$$

Such an updating process refers simply to a linear combination of the old forecast $X_{t|t-1}$ and computed prediction error $\eta_{t|t-1}$. Following previous steps the densities $f(Y_t|Y^{t-1}, \Theta)$ can be derived recursively for $t = 1,2, \ldots, T$. Finally, the likelihood function is obtained

$$L(Y^T, \Theta) = \prod_{t=1}^{T} f(Y_t|Y^{t-1}, \Theta) \tag{76}$$

B.0.2 MCMC Algorithm

The log-likelihood obtained after the Kalman filter recursion together with the prior distribution gives the log-posterior kernel

$$ln(Y^T, \Theta) = lnL(Y^T, \Theta) + lnp(\Theta) \tag{77}$$

To find the posterior distribution equation (77) is maximized with respect to Θ using the Metropolis-Hastings algorithm.[10] This algorithm proceeds in four main steps (Griffoli, 2013)

1. First a starting point Θ^0 needs to be defined (usually a posterior mode). Then the algorithm loops over steps 2-4.
2. In the second step a proposal Θ^* from a jumping distribution J is drawn (i.e. the algorithm draws a candidate parameter from the Normal distribution with Θ^{t-1} mean)
$$J(\Theta^*|\Theta^{t-1}) = N(\Theta^{t-1}, c\Sigma_m) \tag{78}$$
where Σ_m is an inverse of the Hessian at the posterior mode and c is a scale factor. Too small scale parameter results in high acceptance ratio (see next step). Then the distribution converges very slowly to the posterior distribution and never visits tails of the distribution. Too large scale parameter implies low acceptance ratio which means the candidate parameters most likely emerge in regions of low probability density.
3. Next the acceptance ratio is computed
$$r = \frac{p(Y^T, \Theta^*)}{p(Y^T, \Theta^{t-1})} = \frac{(Y^T, \Theta^*)}{(Y^T, \Theta^{t-1})} \tag{79}$$
This ratio compares the value of the posterior kernel from the mean of the drawing distribution with the posterior kernel for the candidate parameter.
4. In the last step the algorithm accepts or discards the proposal Θ^* based on the following rule
$$\Theta^t = \begin{cases} \Theta^* & \text{with probability } min(r,1) \\ \Theta^{t-1} & \text{otherwise} \end{cases}$$
The algorithm discards candidate parameter if the acceptance ratio is lower than one, and goes back to the candidate from the last period.

Such a sequence of draws creates a Markov Chain with the unique stationary distribution. The algorithm should assure that the estimate is not influenced by initial draws or

[10] The precise posterior is only rarely feasible as it is a nonlinear and complicated function of deep parameters Θ. As we are interested just in the moments of the posterior distribution such as mean, median, variance or quantiles we can be derived it by a numerical approximation using draws from posteriors.

serial correlation. Despite this, it is recommended to discard a certain number of initial draws (Bäuerle & Menz, 2008).

C Bayesian Estimation - Results

Figure 1. Comparison of prior and posterior distributions

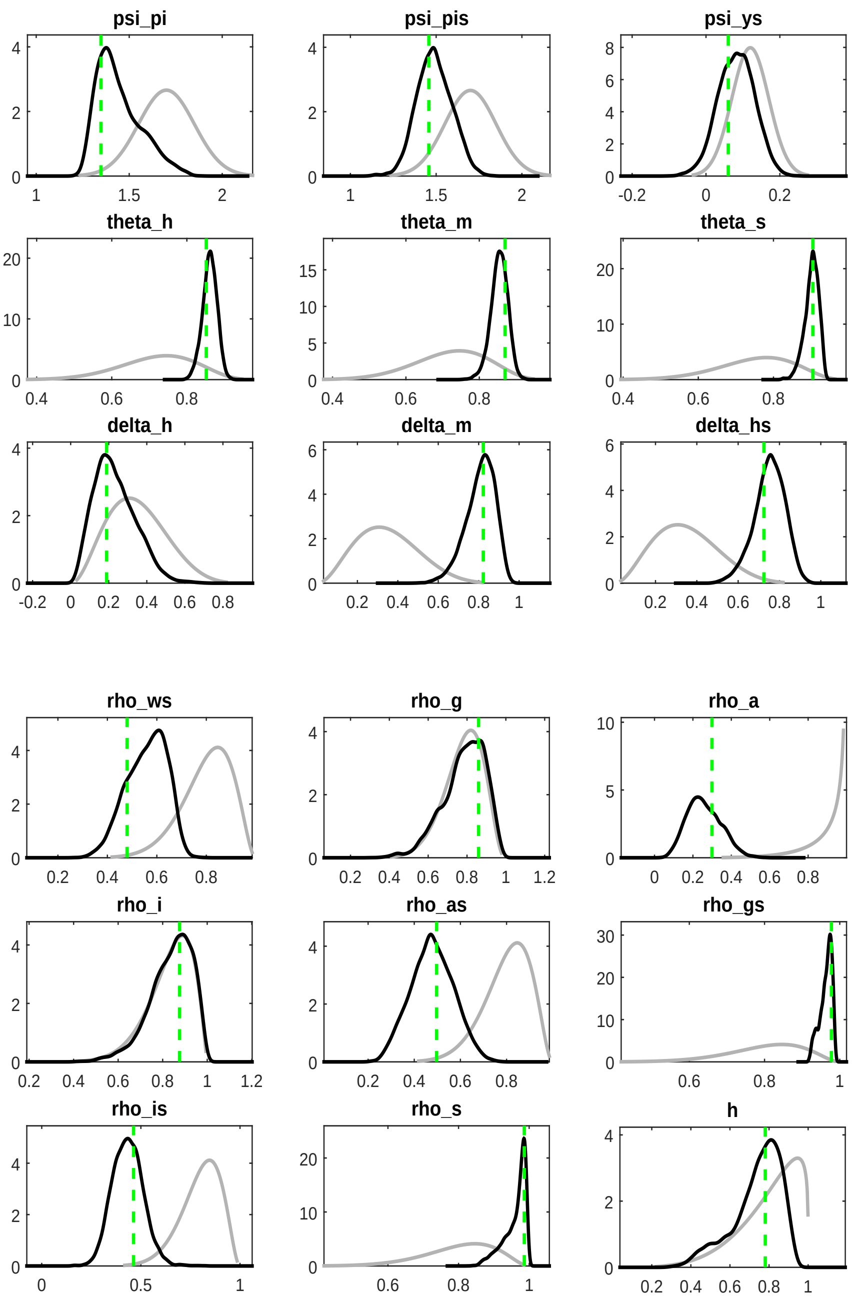

Figure 2. Comparison of prior and posterior distributions

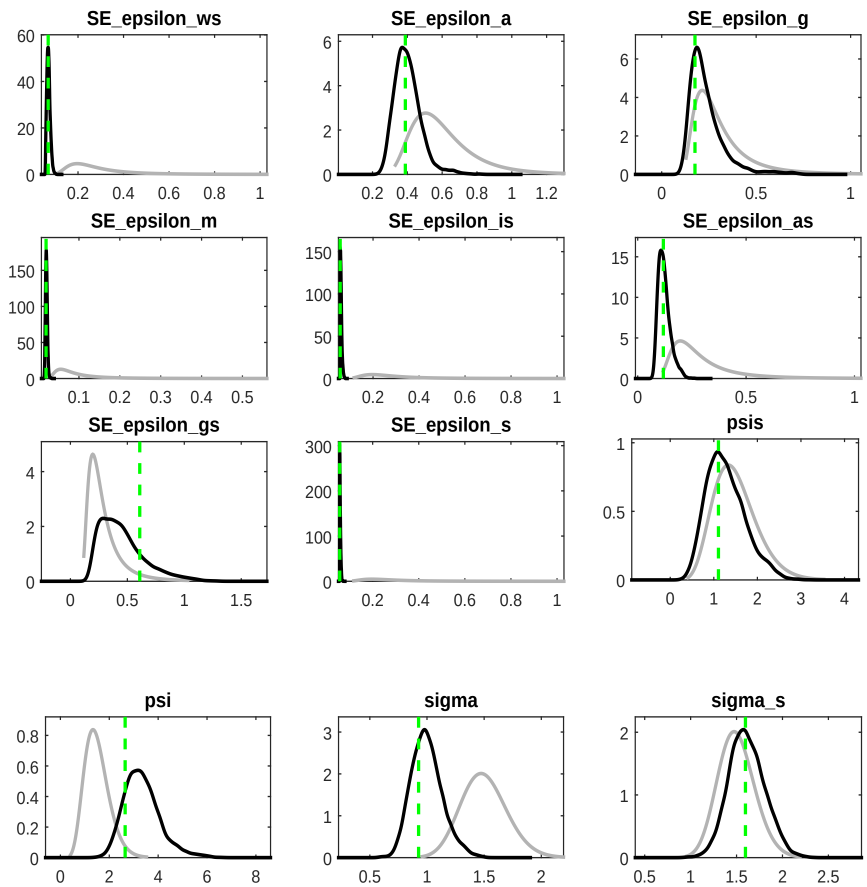

Figure 3. Convergence statistics of Metropolis-Hastings algorithm (500 000 draws of 2 parallel chains)

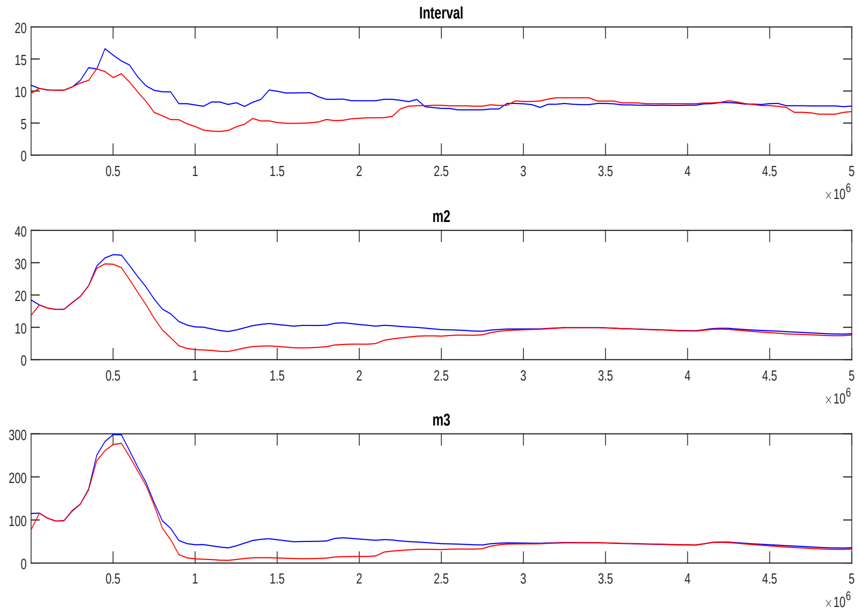

Table 3. Prior and posterior distribution of DSGE parameters

			Prior		Posterior	
Par.	Description	Distr.	Mean	StD	Mean	90% Int.
σ	Domestic parameter of relative risk aversion	Γ	1.50	0.20	0.9977	[0.7816, 1.2229]
σ^*	Foreign parameter of relative risk aversion	Γ	1.50	0.20	1.6093	[1.3089, 1.9485]
φ	Domestic inverse labor supply elasticity	Γ	1.50	0.50	3.3282	[2.1283, 4.4451]
φ^*	Foreign inverse labor supply elasticity	Γ	1.50	0.50	1.2755	[0.5424, 1.9515]
h	Domestic habit persistence	β	0.80	0.15	0.7375	[0.5390, 0.9248]
δ_H	Indexation - domestic producers	β	0.35	0.15	0.2323	[0.0561, 0.3971]
δ_M	Indexation - importing retail firms	β	0.35	0.15	0.8073	[0.6954, 0.9226]
δ^*	Indexation - foreign economy/exporters	β	0.35	0.15	0.7506	[0.6266, 0.8709]
θ_H	Calvo-parameter - domestic producers	β	0.72	0.10	0.8609	[0.8278, 0.8920]
θ_M	Calvo-parameter - importing retail firms	β	0.72	0.10	0.8544	[0.8171, 0.8919]
θ^*	Calvo-parameter - foreign economy / exporters	β	0.75	0.10	0.9015	[0.8729, 0.9323]
ψ_π	Domestic inflation-targeting rule	N	1.70	0.15	1.4446	[1.2678, 1.6369]
ψ_π^*	Foreign inflation-targeting rule (inflation)	N	1.70	0.15	1.4880	[1.3373, 1.6686]
ψ_y^*	Foreign inflation-targeting rule (output gap)	N	0.12	0.05	0.0816	[0.0034, 0.1632]
ρ_i	Persistence - domestic monetary policy rule	β	0.83	0.10	0.8357	[0.7020, 0.9834]
ρ_g	Persistence - domestic preference shock	β	0.78	0.10	0.7834	[0.6128 , 0.9469]
ρ_a	Persistence - domestic technology shock	β	0.93	0.10	0.2577	[0.1203, 0.4026]
ρ_s	Persistence - terms of trade shock	β	0.80	0.10	0.9601	[0.9102, 0.9971]
ρ_i^*	Persistence - foreign monetary policy rule	β	0.80	0.10	0.4322	[0.3067, 0.5559]
ρ_g^*	Persistence - foreign preference shock	β	0.80	0.10	0.9631	[0.9357, 0.9862]
ρ_a^*	Persistence - foreign technology shock	β	0.80	0.10	0.4770	[0.3220, 0.6294]
ρ_w^*	Persistence - capital-inflow shock	β	0.80	0.01	0.5575	[0.4293, 0.6840]
σ_i	Standard deviation - domestic interest rate shock	Inv-Γ	0.11	0.20	0.0204	[0.0166, 0.0239]
σ_g	Standard deviation - domestic preference shock	Inv-Γ	0.32	0.20	0.2361	[0.1184, 0.3577]
σ_a	Standard deviation - domestic technology shock	Inv-Γ	0.61	0.20	0.4012	[0.2801, 0.5067]

σ_s	Standard deviation - terms of trade shock	Inv-Γ	0.30	0.20	0.0596	[0.0571, 0.0627]
σ_i^*	Standard deviation - foreign interest rate shock	Inv-Γ	0.30	0.20	0.0614	[0.0571, 0.0662]
σ_g^*	Standard deviation - foreign preference shock	Inv-Γ	0.30	0.20	0.4525	[0.1768, 0.7407]
σ_a^*	Standard deviation - foreign inflation shock	Inv-Γ	0.30	0.20	0.1221	[0.0781, 0.1642]
σ_w^*	Standard deviation - capital-inflow shock	Inv-Γ	0.30	0.20	0.0705	[0.0578, 0.0811]

D Simulations

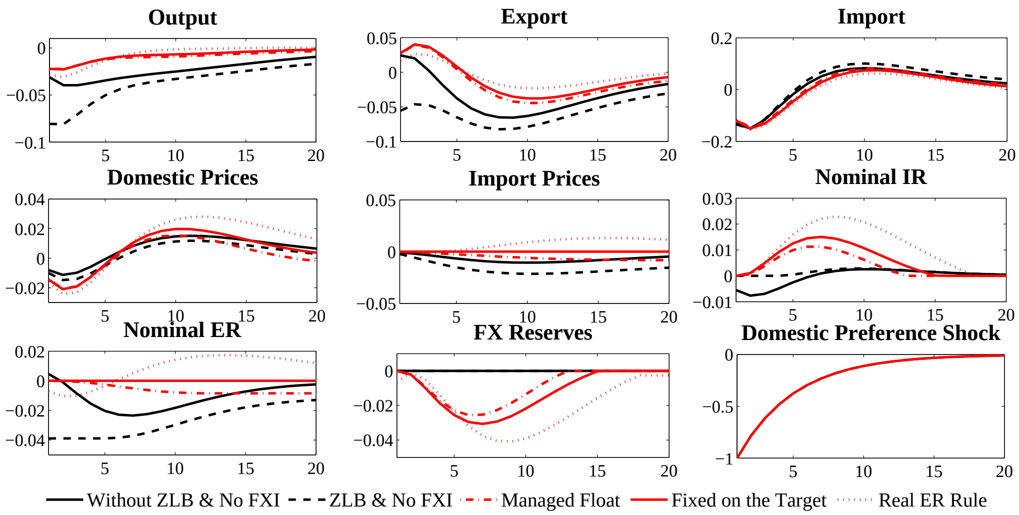

Figure 4. Domestic Preference Shock - Different FX Intervention Regimes

Quarterly impulse response functions after a shock of one standard deviation. Different FX intervention regimes are examined at the ZLB. Variables are measured in deviations from steady state (except the interest rate).

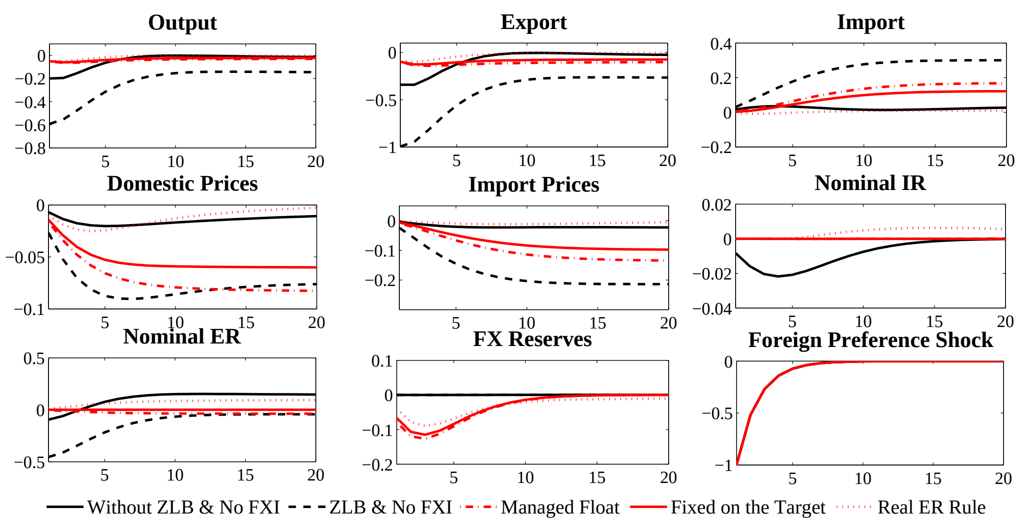

Figure 5. Foreign Preference Shock - Different FX Intervention Regimes

Quarterly impulse response functions after a shock of one standard deviation. Different FX intervention regimes are examined at the ZLB. Variables are measured in deviations from steady state (except the interest rate).

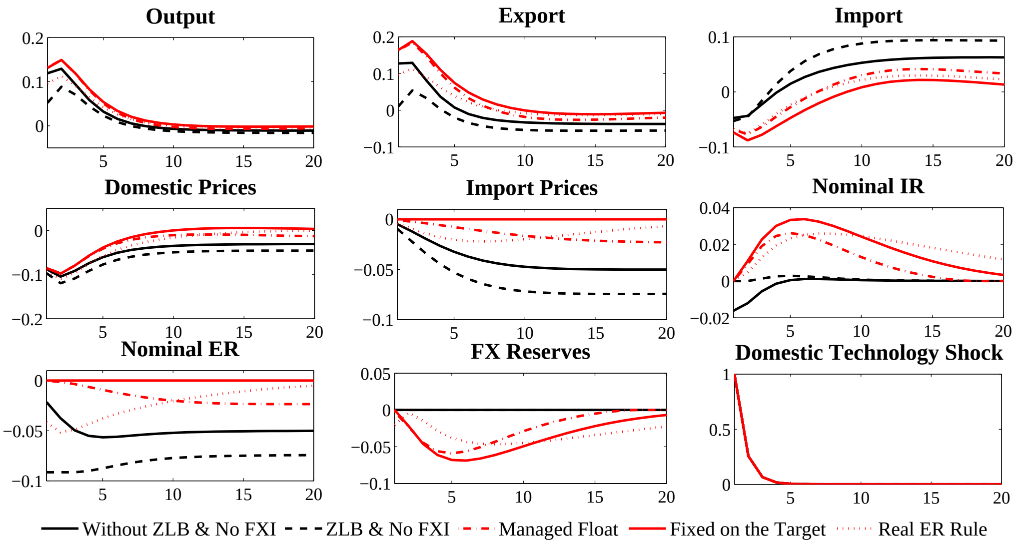

Figure 6. Domestic Technology Shock - Different FX Intervention Regimes

Quarterly impulse response functions after a shock of one standard deviation. Different FX intervention regimes are examined at the ZLB. Variables are measured in deviations from steady state (except the interest rate).

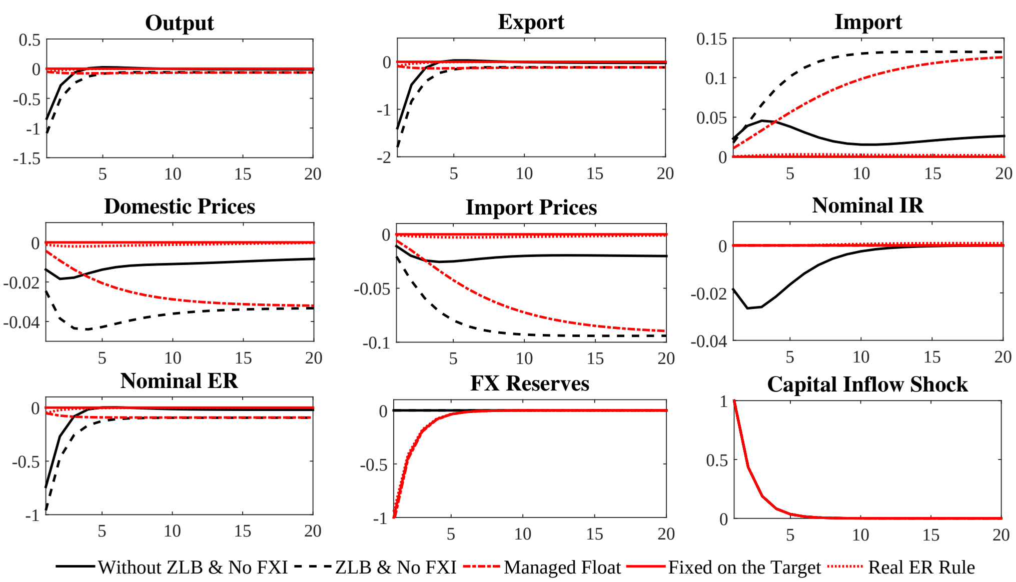

Figure 7. Capital Inflow Shock - Different FX Intervention Regimes

Quarterly impulse response functions after a shock of one standard deviation. Different FX intervention regimes are examined at the ZLB. Variables are measured in deviations from steady state (except the interest rate).

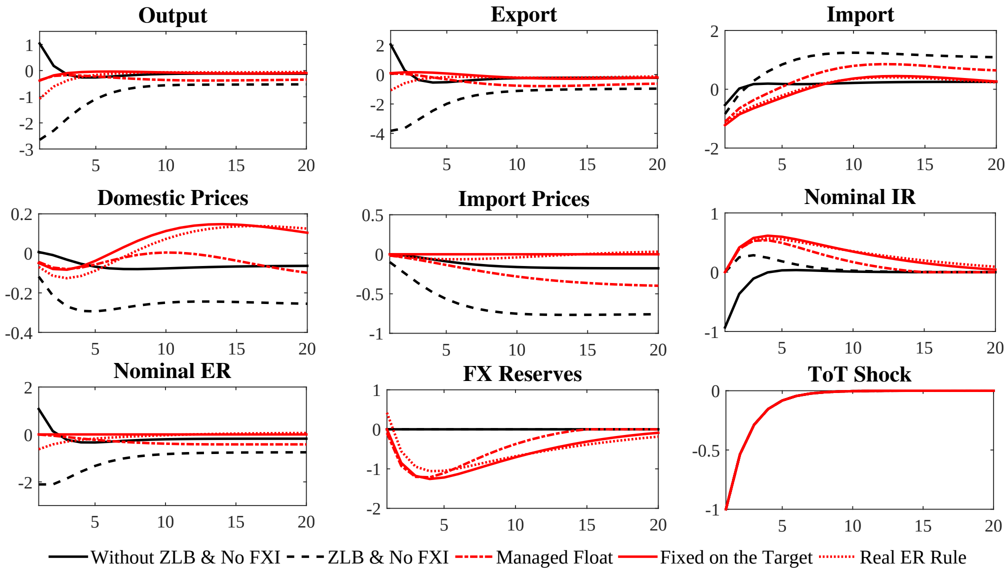

Figure 8. **Terms-of-trade Shock - Different FX Intervention Regimes**

Quarterly impulse response functions after a shock of one standard deviation. Different FX intervention regimes are examined at the ZLB. Variables are measured in deviations from steady state (except the interest rate).

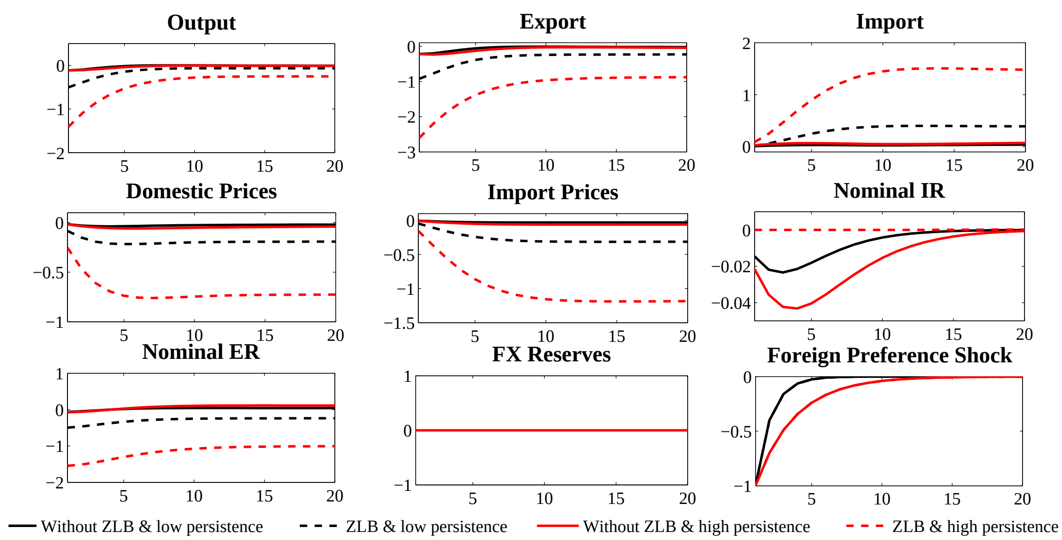

Figure 9. **Foreign Preference Shock - Different Persistence**

Quarterly impulse response functions after a shock of one standard deviation. Low persistence = 0.4, high persistence = 0.7. Variables are measured in deviations from steady state (except the interest rate).

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