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KAREL JANDA

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Comparative Statics of the Effects of Credit Guarantees and Subsidies in the Competitive Lending Market

KAREL JANDA*

Abstract

We compare the effects of government credit subsidies and guarantees on decreasing inefficiencies caused by principal-agent problems in the credit market in transition and posttransition economies. We show that guarantees and subsidies targeted to low-risk borrowers decrease efficiency, while those targeted to high-risk borrowers increase efficiency both in transition and in posttransition economies. Uniform nontargeted guarantees decrease the credit rationing or deadweight loss caused by collateral transfers. Uniform subsidies may be used to improve welfare in an economy subjected to credit rationing, but they do not have any effect on the size of collateral required in a posttransition economy.

Keywords: credit, guarantees, subsidies, transition.

JEL Classification: D82, G28, P31

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* Institute of Economic Studies, Charles University Prague, Opletalova 26, Prague 1, 110 00, Czech Republic and University of Economics, W.Churchill Sq. 4, Prague 3, 130 67, Czech Republic. E-mail: Karel-Janda@seznam.cz.
1. Introduction

An important component of economic transition since 1989 has been the establishment of a dynamic and efficient small- and medium-sized enterprise (SME) sector. The provision of start-up loan provisions for SMEs was a crucial precondition of such development. As opposed to large enterprises with a well-known credit history, the provision of loans to smaller, new enterprises was hindered by information asymmetry between lending agencies and aspiring entrepreneurs.

In order to entice lending institutions to provide capital to entrepreneurs lacking established credit records and/or sufficient collateral, transition-country governments established various credit-supporting institutions. Governmental agencies such as the Slovak Guarantee and Development Bank, the Czech-Moravian Guarantee and Development Bank, or the Czech Support and Guarantee Agricultural and Forestry Fund assisted the development of SMEs on their respective domestic markets. Their major activities consist in the provision of loan guarantees, direct loans, and interest-rate subsidies.

In this paper, we concentrate on the efficiency effects of loan guarantees and interest-rate subsidies in a situation marked by entrepreneurs’ private knowledge about their chances of success in the project for which they seek credit. We will look at this question from a historical perspective, comparing the situation during the initial phases of economic transition in the early nineties with the posttransition situation of the early twenty-first century.

We use the technical approach of the informational economy, widely used in analyses of credit markets under informational asymmetries since the seminal Stiglitz and Weiss (1981) paper. In particular, we utilize the screening role of collateral provisions and credit rationing in overcoming the adverse-selection effects of borrower private information. Major papers on the information economics of credit markets include Chan and Kanatas (1985), Bester (1985, 1987, 1994), and Besanko and Thakor (1987a, 1987b). In addition, Schmidt-Mohr (1997) analyzed different types of instruments for overcoming information asymmetries. These studies were primarily engaged in positive analyses of credit-market imperfections and, as such, were not concerned with government interventions toward alleviating these imperfections.

The basic idea of informational asymmetry and its alleviation through screening contracts in credit markets has been empirically tested too. For example, Capra, Fernandez, and Ramirez (2001) found that separating the role of collateral, as predicted by screening models, fits the credit market both based on real data and on experimental simulations. The
use of collateral and/or credit rationing as screening instruments in adverse-selection models was recently analyzed by Janda (2002, 2003). Stiglitz and Weiss’s (1981) approach was recently discussed and empirically applied by Minelli and Modica (2003) to regional policy in depressed regions of Italy, where structural adjustment and poverty-alleviation problems bear some resemblance to problems faced in transition economies such as Slovakia. A complex discussion of credit guarantees in the context of economic policy is provided by Gudger (1998) and Navajas (2001).

In our paper, we take the form of contract as a given. We consider the so-called standard debt contract as introduced by Townsend (1979) in the framework of the costly state verification problem. The appropriateness of the standard debt contract in the adverse-selection environment was first investigated by Innes (1993). Recently, Vauhkonen (2003) demonstrated that the standard debt contract is optimal as long as there is competition among lenders rather than a monopoly financier.

2. The Model

Our model provides an extension to the model introduced by Janda (2002). The model has two time periods, ex ante and ex post. While there were two classes of economic agents in the original model—lenders and borrowers (the latter also denoted as entrepreneurs here)—we add government as a third class of economic agent. The government is modeled as a benevolent body whose only concern is increased social efficiency and whose only role is to distribute exogenously determined lump-sum guarantees and interest-rate subsidies.

The role of lenders is to provide the financial funds required by borrowers to realize their projects. Risk-neutral lenders are engaged in Bertrand competition, leading to zero profits on lent sources. The supply of funds facing lenders is perfectly elastic, so that the lenders have available any demanded amount of funds under the unit cost of $\rho$.

There are two types of risk-neutral borrowers in this model, indexed as type 1 and type 2. The two types are distinguished by their probability of successfully realizing their project, denoted as $0 < \delta_1 < \delta_2 < 1$, and by their reservation utilities from not participating in the project, denoted as $h_1 < b_2$. A type-1 borrower is labeled as a high-risk borrower and a type-2 borrower as a low-risk borrower. The probability that the random borrower facing a lender is of type 1 is $\theta$, which is the proportion of type-1 borrowers in the total population of borrowers.
The borrower can either undertake one risky project, which yields \( y \) in the case of success and 0 in the case of a failure, or he may participate in some alternative activity, which yields an expected return of \( b_i, \ i \in \{1,2\} \). When the project is completed, the outcome of the project is freely observed by both borrower and lender. This means there is no costly state verification problem in this model.

In order to undertake the project, the borrower has to borrow a fixed amount of money from the lender. The size of this loan is normalized to 1. Each borrower is endowed with a nonstochastic endowment, \( W < \rho \), which will become available \emph{ex post} in the second period, regardless if the project will be undertaken or not and regardless of the outcome. This assumption means that the borrower’s own wealth is too low to finance his own project through a risk-free loan.

The flow of funds from lenders to borrowers and the repayment of these funds is governed by standard debt contracts. This means that in the case of the success of the project, the lender receives a constant repayment \( R \). In the case of failure, the lender receives the collateral \( C \). Each lender offers two types of contract. Each contract is a three-tuple, \((\pi_i, C_i, R_i), \ i \in \{1,2\}\), where \( \pi_i \) is the probability that the application of the borrower who chooses this contract will be satisfied and will receive credit; \( C_i \) is required collateral; and \( R_i \) is the interest factor \((1 + \text{the interest rate})\), which is equal to the required repayment because of our normalization of the loan size to 1.

The expected utility of a borrower of type \( i \) who applies for a contract designed for a borrower of a type \( j \) is, for notational simplicity, given as an incremental expected utility defined as \([\text{expected utility after applying for credit}] - [\text{utility in the case of nonparticipating}]\) that is:

\[
U_{ij} = \pi_j [\delta_j (y - R_j) - (1 - \delta_j)C_j + W] + \pi_j (1 - \pi_j)(W + b_j) - (W + b_j).
\]

The expression in curly brackets says the following: with the probability \( \pi_j \) the loan is granted, after which the borrower gains \( y - R_j \) in the event of success and loses collateral \( C_j \) in the event of failure. In the event of success, his endowment wealth \( W \) remains intact. In the event of failure, it is reduced by the collateral loss, \( C_j \). With the probability \((1 - \pi_j)\) the loan is not granted which means that the entrepreneur keeps his endowment wealth, \( W \), and engages in alternative activity, which gives him utility equivalent of \( b_j \). As such, the expression in curly brackets expresses the expected utility of the entrepreneur conditional on
requesting the loan before the decision of the lender (to loan or not) is announced. The subtracted term \((W + b_i)\) is the utility obtained by the entrepreneur in the case he does not apply for a loan.

The expression for the expected utility of the entrepreneur may be simplified as:

\[
U_{ij} = \pi_i \left[ \delta_i (y - R_j) - (1 - \delta_i)C_j - b_j \right].
\]

(1)

It should be noted that the expected utility formulation described above naturally leads to the inclusion of reservation utility term \(b_i\) into the expected utility function.

The lender’s valuation of collateral is given as \(\beta C_i\), where \(\beta \in (0;1)\). This means that any equilibrium involving collateral is not socially efficient, since the amount of \((1 - \beta)C_i\) is wasted. This waste of resources could be interpreted as any kind of dead weight of liquidation or losses caused by asset specificity.

We assume that each project is socially efficient, that is \(\delta_i y > b_i + \rho\). This implies that any equilibrium involving credit rationing with \(\pi_i < 1\) is not socially efficient.

The values of all parameters are known by borrowers, lenders, and government. The only informational asymmetry in the model is that \(ex\ ante\) lenders and government do not know the borrower type. Therefore, ours is an adverse-selection model.

The expected profit to a lender on one loan provided to a borrower of a type \(i\) is, under this asymmetric information, given as:

\[
B_i = \pi_i \left[ \delta_i R_i + (1 - \delta_i) \beta C_i - \rho \right].
\]

(2)

We assume that in the event a lender is indifferent between lending and not lending, he resolves this indifference in the favor of lending.

The government can attempt to reduce the inefficiencies created by the use of collateral and by credit rationing by two types of interventions. Under the lump-sum-guarantees program, the government guarantees the payment of an exogenously determined lump-sum \(g_i\) in the case of zero return from a project. The contracted collateral is thus passed to the government. The expected profit, Equation (2), is modified as:

\[
B_i = \pi_i \left[ \delta_i R_i + (1 - \delta_i)g_i - \rho \right].
\]

(3)
The other considered type of intervention is an interest-rate subsidy, $s$, which is paid only in the case of a project’s success, as opposed to guarantees paid in the event of failure. While the subsidy reduces the interest rate paid by a borrower, we can treat it analytically, just like an exogenous supplement to a repayment to a lender. The expected profit, expressed as Equation (2), is then modified as:

$$B_i = \pi_i [\delta_i (R_i + s_i) + (1 - \delta_i) \beta C_i - \rho].$$

The expected utility of a borrower, under both types of interventions, is still given by Equation (1) since the interventions influence the borrower’s utility only indirectly, through their impact on the lender’s profit.

We assume that the government implements one intervention program at a time. We also assume that the legislative status of the interventions is such that all loans provided to borrowers in a certain line of business are subjected to a given intervention, that is, all lenders lending in a given area participate in a government program. It is not possible for the government to reject a subsidy or a guarantee for loans provided by some lenders when giving subsidies or guarantees to other lenders offering the same contract. The participating lenders are obliged to use government support schemes when offered.

Assumptions about the probabilities of successfully completing a project in a given branch of a national economy, and regarding the opportunity costs of remaining in that branch of a national economy, which gave rise to our distinction of two market regimes, transition and posttransition, are formally expressed as: if $\frac{b_2}{b_1} \geq \frac{\delta_2}{\delta_1}$, then the model is in a transition-economy regime; otherwise, it is in a posttransition-economy regime.

This approach takes the relative chances of success in a given industry to be independent of the state of the transition. That is, we take the ratio $\frac{\delta_2}{\delta_1}$ as a constant, which is the same for both transition and posttransition economies. We assume success here to be related to personal factors, not to the state of the economy. On the other hand, we assume that opportunity costs are dependent on the state of the economy.

We assume that transition leads to a major stratification of society: from state-imposed equality to market-driven inequality. During transition, there are substantial possibilities for people to either become very rich or very poor, depending on their abilities. We expect that this process of social polarization is much stronger in a transitional than in a developed market economy. Therefore, we assume that during the transition the ratio $\frac{b_2}{b_1}$ is high. This
assumption captures the notion that very able people—people with entrepreneurial skills—will become wealthy \( (b_r) \) and will form the upper class. People lacking entrepreneurial skills \( (b_l) \) will remain as working class or they will become unemployed. The possibility of becoming very rich or very poor during transition is also connected with huge structural changes underway in the economy. Because the pretransition economy was artificially led by central planning, and in a condition of isolation from world markets, a transitional economy often behaves quite unexpectedly. We expect that people with good entrepreneurial skill are good at identifying which sectors will be profitable and viable in the future.

Once transition is realized, and as the economy stabilizes to the point of a “normal” posttransition market economy, the differences between opportunity costs narrow. The possibilities for sudden, “overnight” wealth are also gone. The structure of the economy is increasingly transparent and stabilized, so that it is largely evident which sectors are ascendant and which are descendent. Thus, we assume that in the posttransition period the ratio \( \frac{b_r}{b_l} \) is lower than during transition.

We define the dividing line between high relative differences in opportunity costs in the transition period and low relative differences in posttransition by the ratio \( \frac{\delta_r}{\delta_l} \). We thus assume that in posttransition period the relative difference in opportunity costs for good and bad entrepreneurs is not greater than their relative chances for success in the area in which they seek financing.

### 3. The Solution of the Model

In order to appreciate the effects of government interventions, we first introduce the properties of the model’s solution excluding government interventions, which are derived in detail by Janda (2002). We first mention the case when the lender has full information about the types of borrower; that is, when the lender is able to say whether the borrower is high or low risk \textit{ex ante}. Janda proves that credit provision under full information is efficient in this case since collateral is not used and credit rationing is not applied. Consequently, there would be no need for government intervention.

In the case when the risk level of the borrower is the private information of the applicant, Janda proves that asymmetric information leads to two kinds of inefficiency which call for government intervention: credit rationing in a transition economy, and, in a stabilized posttransition economy, it is the use of collateral that is accompanied by credit rationing if the
collateral wealth of a borrower is lower than the collateral required to provide credit to all applicants. In a posttransition economy, this collateral wealth could be so low that the credit market breaks down. The solution of the optimization problem under asymmetric information is provided in the Appendix. In the following two sections, we analyze two types of government interventions directed at alleviating these inefficiencies.

3.1. Lump-sum Guarantees

The lender under asymmetric information does not know ex ante the risk class of a borrower. Due to competition, each lender attempts to offer to each type of borrower the best conditions possible. Therefore, the maximization problem of a lender is given by:

\[
\max_{[\pi_1, R_1, C_1]} M = \theta U_{11} + (1-\theta)U_{22} \\
= \theta \pi_1 [\delta_1 (y - R_1) - (1-\delta_1)C_1 - b_1] + (1-\theta) \pi_2 [\delta_2 (y - R_2) - (1-\delta_2)C_2 - b_2]
\]

s.t.

\[
\begin{align*}
\pi_1 [\delta_1 (y - R_1) - (1-\delta_1)C_1 - b_1] &\geq \pi_2 [\delta_1 (y - R_1) - (1-\delta_1)C_1 - b_1], \\
(1-\theta) \pi_2 [\delta_2 (y - R_2) - (1-\delta_2)C_2 - b_2] &\geq \pi_1 [\delta_2 (y - R_2) - (1-\delta_2)C_2 - b_2], \\
U_{ii} &\geq 0, \\
0 &\leq \pi_i \leq 1, \\
0 &\leq C_i \leq W,
\end{align*}
\]

\[
\delta_i R_i + (1-\delta_i)g_i - \rho = 0, \quad i \in \{1, 2\}. 
\]

Equation (5) is a zero-profit condition for lenders, which explicitly prohibits cross-subsidization. This means that it is not possible for lenders to suffer a loss on a contract to one type of a borrower and to enjoy a positive profit on a contract to another type. Zero-profit constraint puts a bind on the competitive ability of the lender to offer the most attractive contract to the borrower.

Collateral is passed to the government. The government, in turn, guarantees the payment of an exogenously determined lump sum, \( g_i \), in the event of zero return from a project.
By solving this optimization problem along the lines of the solution provided in the Appendix, we find the following characteristics of the optimal solution. The contract for a high-risk borrower both in the transition and the posttransition economies is given by:

\[ C^*_1 = 0, \pi^*_1 = 1, R^*_i = \frac{\rho - (1 - \delta_i)g_i}{\delta_i}. \]  \hspace{1cm} (6)

While \( R^*_i \) for \( i = 2 \) in Equation (6) gives the interest-factor component of an equilibrium contract for a low-risk borrower, the remaining parts of the contracts are different in transition and posttransition economies with and without a binding collateral restriction.

If \( \frac{b_2}{b_1} \geq \frac{\delta_2}{\delta_1} \), then:

\[ C^*_2 = 0. \]  \hspace{1cm} (7)

\[ \pi^*_2 = \frac{\delta_1 y - \rho - b_1 + (1 - \delta_1)g_1}{\delta_2 y - \frac{\delta_1}{\delta_2} \rho - b_1 + \frac{\delta_2 - \delta_1}{\delta_2} g_2}. \]  \hspace{1cm} (8)

If \( \frac{b_2}{b_1} < \frac{\delta_2}{\delta_1} \), and collateral is unconstrained, then:

\[ C^*_2 = \frac{\rho(\delta_2 - \delta_1) + \delta_1(1 - \delta_2)g_2}{\delta_2(1 - \delta_1)} - g_1. \]  \hspace{1cm} (9)

\[ \pi^*_2 = 1. \]  \hspace{1cm} (10)

If \( \frac{b_2}{b_1} < \frac{\delta_2}{\delta_1} \), collateral is constrained, and collateral wealth \( W \) is such that (IC2) is satisfied, then:

\[ C^*_2 = W. \]  \hspace{1cm} (11)

\[ \pi^*_2 = \frac{\delta_1 y - \rho + (1 - \delta_1)g_1 - b_1}{\delta_1 y - \delta_1 \rho - \frac{\rho - (1 - \delta_2)g_2}{\delta_2} - b_1 - (1 - \delta_1)W}. \]  \hspace{1cm} (12)

As we mentioned previously, this solution of the model shows some inefficiencies, whose degree depends on the level of governmental intervention. In the cases of transition and posttransition economies with low levels of available collateral, the credit market is plagued by the credit rationing of low-risk borrowers. The extent of the credit rationing \( \pi^*_2 \) may be influenced by government support since \( \pi^*_2 \) depends on guarantees \( g_1 \) and \( g_2 \) both in
Equations (8) and (12). Similarly, the extent of the other possible inefficiency—the required collateral in a posttransition economy—may be regulated by the government’s choice of intervention parameters \( g_1 \) and \( g_2 \), which both enter Equation (9).

The optimal solution to the optimization problem above exhibits the following welfare properties:

The utility of a high-risk borrower supported by a lump-sum guarantee is given as:

\[
U_{11} = \delta_1 y - \rho - b_1 + (1 - \delta_1) g_1
\]

The utilities of low-risk borrowers are given according to the following three cases:

1. Transition economy:

\[
U_{22} = [\delta_2 y - \rho - b_2 + (1 - \delta_2) g_2] \frac{\delta_1 y - \rho - b_1 + (1 - \delta_1) g_1}{\delta_1 y - \frac{\delta_1}{\alpha_1} \rho - b_1 + \frac{\delta_1 (1 - \delta_1)}{\alpha_1} g_1}.
\]

2. Posttransition economy with unconstrained collateral:

\[
U_{22} = \delta_2 y - \rho - b_2 + (1 - \delta_2) \left[ \frac{(\delta_2 - \delta_1)}{\delta_2 (1 - \delta_1)} g_2 + g_1 - \frac{\delta_2 - \delta_1}{\delta_2 (1 - \delta_1)} \rho \right].
\]

3. Posttransition economy with binding collateral restriction and with wealth \( W \) such that (IC2) is satisfied.

\[
U_{22} = [\delta_2 y - \rho - b_2 + (1 - \delta_2) g_2 - (1 - \delta_2) W] \frac{\delta_1 y - \rho - b_1 + (1 - \delta_1) g_1}{\delta_1 y - \frac{\delta_1}{\alpha_1} \rho - b_1 + \frac{\delta_1 (1 - \delta_1)}{\alpha_1} g_2 - (1 - \delta_1) W}.
\]

From the solution of the lender’s optimization problem we are able to determine the main general qualitative effects of the government guarantees: guarantees to low-risk borrowers decrease efficiency, guarantees to high-risk borrowers relax an incentive-compatibility constraint and consequently increase an efficiency, and uniform non-targeted lump-sum guarantees are welfare-improving.

Below we show these efficiency results formally according to the three cases outlined above, and we provide the relevant economic intuition connected with each. We first deal with a transition economy.

If the economy is in transition regime \( \frac{\delta_1}{\alpha_1} \geq \frac{\delta_1}{\alpha_2} \), then:
This shows that lump-sum guarantees targeted to a high-risk borrower decrease the credit rationing of a low-risk borrower. This effect is caused by relaxing the incentive-compatibility constraint for the high-risk borrower. As long as the contract targeted to the high-risk borrower is made more attractive by a provision of government support, the incentive for the high-risk borrower to pretend to be low-risk borrower and to take a low-risk borrower’s contract is decreased. It means that instead of making a low-risk borrower’s contract unattractive to a high-risk borrower by imposing credit rationing, we may achieve the incentive compatibility by improving the terms of the high-risk contract through the use of a government guarantee.

\[
\left. \frac{\partial \pi^*_2}{\partial g_1} \right| _{g_2 = \text{const.}} = \frac{1 - \delta_i}{\delta_i y - \frac{\delta_i}{\delta_2} \rho - b_1 + \frac{\delta_i (1 - \delta_2)}{\delta_2} g_2} > 0.
\]

This means that lump-sum guarantees targeted to a low-risk borrower increase the credit rationing of a low-risk borrower. This is because the guarantees for a low-risk borrower make his contract more attractive. In order to satisfy the incentive constraint for a high-risk borrower, the low-risk borrower’s contract has to be made less desirable. This is achieved by increasing the credit rationing of the low-risk borrower.

These results show that the targeting of guarantees to low-risk borrowers is counterproductive. But it remains to be seen what happens when both types of borrower obtain the same guarantee. We need to establish which of the two incentive effects described above will be stronger.

\[
\left. \frac{\partial \pi^*_2}{\partial g} \right| _{g = g_1, g_2} = \frac{(1 - \delta_i)[\delta_i y - \frac{\delta_i}{\delta_2} \rho - b_1 + \frac{\delta_i (1 - \delta_2)}{\delta_2} g]}{[\delta_i y - \frac{\delta_i}{\delta_2} \rho - b_1 + \frac{\delta_i (1 - \delta_2)}{\delta_2} g]^2} - \frac{[\delta_i y - \frac{\delta_i}{\delta_2} \rho - b_1 + \frac{\delta_i (1 - \delta_2)}{\delta_2} g]}{[\delta_i y - \frac{\delta_i}{\delta_2} \rho - b_1 + \frac{\delta_i (1 - \delta_2)}{\delta_2} g]^2} - \frac{(\delta_2 - \delta_i)(\delta_i y - \delta_2 \rho - b_1)}{[\delta_i y - \frac{\delta_i}{\delta_2} \rho - b_1 + \frac{\delta_i (1 - \delta_2)}{\delta_2} g] \delta_2} > 0.
\]

This shows that the positive effect of improving the contract for the high-risk borrower is stronger than the negative effect of making the contract of the low-risk borrower more
attractive. Therefore, we conclude that nontargeted credit guarantees increase social efficiency since the credit rationing of a low-risk borrower is decreased.

Next, we will deal with targeted and uniform guarantees in posttransition economies. If the economy is in posttransition regime \((\frac{b_2}{\bar{b}} < \frac{\delta_2}{\bar{\delta}_2})\) and collateral is unconstrained, then:

\[
\frac{\partial C^*_2}{\partial g_1} \bigg|_{(g_2=\text{const.})} = -1.
\]

This means that lump-sum guarantees provided to a high-risk borrower decrease the collateral requirement for a low-risk borrower and, in this way, increase social efficiency. \(\frac{\partial C^*_2}{\partial g_2} \bigg|_{(g_1=\text{const.})} = \frac{\delta_1(1-\delta_2)}{\delta_2(1-\delta_1)} > 0.\)

From this we see that the lump-sum guarantees provided to a low-risk borrower increase the collateral requirement, which leads to decreased social efficiency.

\[
\frac{\partial C^*_2}{\partial g_2} \bigg|_{(g_1=g_2)} = -1 + \frac{\delta_1(1-\delta_2)}{\delta_2(1-\delta_1)} = \frac{-(\delta_2 - \delta_1)}{\delta_2(1-\delta_1)} < 0.
\]

This implies that uniform lump-sum guarantees have a positive social-efficiency effect since they lead to a lower collateral requirement. The incentive effects on collateral requirements are qualitatively the same as the incentive effects connected with credit-rationing requirements.

Finally, we consider the case of a posttransition economy with a low level of available collateral. If the economy is in a posttransition regime \((\frac{b_2}{\bar{b}} < \frac{\delta_2}{\bar{\delta}_2})\), collateral is constrained, and collateral wealth \(W\) is such that (IC2) is satisfied, then:

\[
\frac{\partial \pi^*_2}{\partial g_1} \bigg|_{(g_2=\text{const.})} = \frac{1-\delta_1}{\delta_1 y - \delta_1 \frac{\rho - (1-\delta_1) \delta_2}{\delta_2} - h_1(1-\delta_1)W} > 0.
\]

From this we see that lump-sum guarantees targeted to a high-risk borrower decrease the credit rationing of a low-risk borrower.
\[
\frac{\partial \pi^*_2}{\partial g_{2}} \bigg|_{(g_1=\text{const.})} = (\delta_i y - \rho + (1 - \delta_i) g_{1} - h_i)
\]
\[
- \frac{(1 - \delta_i)\delta_i}{\delta_y^2 \delta_z^2} [\delta_i y - \delta_i \frac{\sigma(1 - \delta_i)g_{2}}{\delta_z} - h_i - (1 - \delta_i)W]^2 < 0.
\]

Lump-sum guarantees targeted to a low-risk borrower are counterproductive since they increase the credit rationing of a low-risk borrower.

\[
\frac{\partial \pi^*_2}{\partial g} \bigg|_{(g=g_{1}=g_{2})} = (1 - \delta_i) \left[ \frac{(\delta_i - \delta_i^2)(\delta_i - \delta_i^2 + 2\rho - h_i)}{\delta_z^2 (1 - \delta_i^2)} - (1 - \delta_i)W \right]^2 > 0
\]
\[
\Leftrightarrow W < \frac{(\delta_i^2 - \delta_i)(\delta_i y - \delta_i \rho - h_i)}{\delta_z^2 (1 - \delta_i^2)}. \quad (13)
\]

Untargeted lump-sum guarantees decrease the volume of credit rationing of a low-risk borrower provided that the available collateral wealth is low enough that it satisfies Equation (13).

In the situation without government support, it could happen in the posttransition economy that the available collateral wealth would be too low to satisfy Equation (13). This would lead to the nonexistence of equilibrium. This extremely inefficient case of credit-market break up may be precluded by sufficiently high government intervention since the properly designed guarantee leads to the decrease of the collateral requirement.

### 3.2. Interest-rate Subsidies

This section closely follows the structure of the preceding section to facilitate easy comparisons of both types of government intervention.

The maximization problem is the same as in the case with lump-sum guarantees. The only change is in the zero-profit condition for lenders, where Equation (5) is replaced by:

\[
\delta_i (R_i + s_i) + (1 - \delta_i)\beta C_i - \rho = 0. \quad (14)
\]

The subsidy is paid only in the case of the project’s success, as opposed to guarantees, which are paid in the event of failure. The subsidy is just an exogenous supplement to a repayment to a lender.

In all cases, part of the equilibrium solution with interest-rate subsidies is given by:
\[ C^*_1 = 0, \pi^*_1 = 1, R^*_i = \frac{\rho - (1 - \delta_i)\beta C^*_i}{\delta_i} - s_i. \] (15)

The rest of the solution is given according to the following three cases.

If \( \frac{b_1}{\bar{y}} \geq \frac{\delta_1}{\bar{y}} \), then:

\[ C^*_1 = 0. \] (16)
\[ \pi^*_2 = \frac{\delta_2 y - \rho - b_1 + \delta_1 s_1}{\delta_2 y - \frac{\delta_1}{\delta_2} \rho - b_1 + \delta_1 s_2}. \] (17)

If \( \frac{b_1}{\bar{y}} < \frac{\delta_1}{\bar{y}} \) and collateral is unconstrained, then:

\[ C^*_2 = \frac{\rho(\delta_2 - \delta_1) - \delta_2 \delta_1 (s_1 - s_2)}{\delta_2 (1 - \delta_1) - \delta_1 (1 - \delta_2) \beta}. \] (18)
\[ \pi^*_2 = 1. \] (19)

If \( \frac{b_1}{\bar{y}} < \frac{\delta_1}{\bar{y}} \), collateral is constrained, and collateral wealth \( W \) is such that (IC2) is satisfied, then:

\[ C^*_2 = W. \] (20)
\[ \pi^*_2 = \frac{\delta_1 y - \rho - b_1 + \delta_1 s_1}{\delta_2 y - \frac{\delta_1}{\delta_2} \rho - b_1 - \left(\frac{\delta_1 (1 - \delta_1) - \delta_1 (1 - \delta_2) \beta}{\delta_2}\right) W + \delta_1 s_2}. \] (21)

The welfare properties of this optimal solution are given in the following paragraphs.

The utility of a high-risk borrower under the interest-rate-subsidies intervention is:

\[ U_{y_1} = \delta_1 y - \rho - b_1 + \delta_1 s_1. \]

The utilities of low-risk borrowers are given according to the following three cases.

1. Transition economy:

\[ U_{y_2} = (\delta_2 y - \rho - b_2 + \delta_2 s_2) \cdot \frac{\delta_1 y - \rho - b_1 + \delta_1 s_1}{\delta_2 y - \frac{\delta_1}{\delta_2} \rho - b_1 + \delta_1 s_2}. \]

2. Posttransition economy without collateral restriction:
\[ U_{22} = \delta_2 y - b_2 - \frac{\rho([\delta_2 - \delta_1 + \delta_2(1-\beta)] - \delta_2 \delta_1 (1-\delta_1)(1+\beta)(s_1 - s_2) + \delta_2 s_2}{\delta_2 (1-\delta_1) - \delta_1 (1-\delta_2) \beta}. \]

3. Posttransition economy with constrained collateral and with collateral wealth \( W \) such that (IC2) is satisfied:

\[ U_{22} = [\delta_2 y - \rho - b_2 + \delta_2 s_2 - (1-\delta_2)(1-\beta)W] \frac{\delta_1 y - \rho - b_1 + \delta_1 s_1}{\delta_1 y - \frac{\rho}{\delta_1} - b_1 + \delta_1 s_2 - (1-\delta_1)(1-\beta)W}. \]

The qualitative nature of the targeted interest-rate subsidies to low- or high-risk borrowers is similar to the case of guarantees.

The main qualitative features of the optimal solution of the optimization problem are that: The uniform subsidies in the transition economy and in the posttransition economy with a binding collateral decrease credit rationing. As opposed to guarantees, the uniform subsidies do not have any effect on the size of the collateral required in a posttransition economy.

The formal comparative statics and economic intuition on which these welfare results are based are presented here. We first consider the transition economy.

If the economy is in a transition regime \( (\frac{b_2}{s_2} \geq \frac{b_1}{s_1}) \), then:

\[ \frac{\partial \pi^*_2}{\partial s_1} \bigg|_{(s_2 = \text{const.})} = \frac{\delta_1}{\delta_1 y - \frac{\rho}{\delta_1} - b_1 + \delta_1 s_2} > 0. \]

This means that interest-rate subsidies targeted to a high-risk borrower decrease the size of the credit rationing of a low-risk borrower.

\[ \frac{\partial \pi^*_2}{\partial s_2} \bigg|_{(s_1 = \text{const.})} = \left[\delta_1 y - \rho - b_1 + \delta_1 s_1\right] \frac{-\delta_1}{\left[\delta_1 y - \frac{\rho}{\delta_1} - b_1 + \delta_1 s_2\right]^2} < 0. \]

Therefore, we observe that interest-rate subsidies to a low-risk borrower increase his credit rationing.

\[ \frac{\partial \pi^*_2}{\partial s} \bigg|_{(s_1 = s_2 = s)} = \frac{\delta_1}{\left[\delta_1 y - \frac{\rho}{\delta_1} - b_1 + \delta_1 s_2\right]^2} > 0. \]
The implementation of untargeted subsidies by the government leads to a decrease in the credit rationing of a low-risk borrower. This is because from the point of view of high-risk borrower the untargeted subsidy improves the desirability of both types of contracts, but the desirability of the low-risk contract is increased less than the desirability of high-risk contract. Therefore there is not needed so much credit rationing as a part of low-risk borrower’s contract for keeping high-risk borrower satisfied with the contract designed for him.

If the economy is in a posttransition regime \((\frac{b_1}{\delta_1} < \frac{b_2}{\delta_2})\) and collateral is unconstrained, then:

\[
\frac{\partial C_2^s}{\partial s_1} \bigg|_{(s_2 = \text{const.})} = \frac{-\delta_1 \delta_2}{\delta_2 (1-\delta_1) - \delta_1 (1-\delta_2) \beta} < 0,
\]

which means that interest-rate subsidies to a high-risk borrower decrease the collateral required of the low-risk borrower.

On the other hand, the subsidies provided to the low-risk borrower increase the collateral required of him.

We next consider a scenario in which the government provides untargeted interest-rate subsidies of uniform size for all borrowers, which leads to:

\[
\frac{\partial C_2^s}{\partial s} \bigg|_{(s_1 = s_2 = s_3)} = 0.
\]

As opposed to the other scenarios analyzed in this section, under this government policy we were surprised to observe that untargeted interest-rate subsidies are powerless since the uniform increase in subsides has no effect on the collateral requirement. This result is because the amount of collateral required is influenced by the subsidy differential between low- and high-risk borrowers. As long as this differential is zero, the subsidies provided to both types of borrower do not affect incentives.

Lastly, we consider the situation wherein the economy is in a posttransition regime \((\frac{b_1}{\delta_1} < \frac{b_2}{\delta_2})\), collateral is constrained, and collateral wealth \(W\) is such that (IC2) is satisfied.
Under these conditions, the three approaches to the targeting of subsidies considered in this paper generate the following results:

\[
\frac{\partial \pi^*_2}{\partial s_1} \bigg|_{(s_1=\text{const.})} = \frac{\delta_1}{\delta_2} \left( \rho - \frac{\delta_1}{\delta_2} \right) - \frac{\delta_1}{\delta_2} \beta \frac{\delta_1}{\delta_2} + \delta_1 s_2 > 0.
\]

The interest-rate subsidy targeted to a high-risk borrower decreases the credit rationing of a low-risk borrower.

\[
\frac{\partial \pi^*_2}{\partial s_2} \bigg|_{(s_1=\text{const.})} = \frac{(\delta_1 \rho - \delta_1 \beta \delta_1 + \delta_1 s_1)(-\delta_1)}{[\delta_1 \rho - \delta_1 \beta \delta_1 + \delta_1 s_1]^2} < 0.
\]

The subsidy targeted to a low-risk borrower increases the credit rationing of the targeted borrower and, consequently, it is not a desirable policy option.

And finally:

\[
\frac{\partial \pi^*_2}{\partial s} \bigg|_{(s=s_1=s_2)} = \frac{\delta_1 \left\{ \left[ \delta_1 \rho - \delta_1 \beta \delta_1 + \delta_1 s_1 \right] \right\}}{[\delta_1 \rho - \delta_1 \beta \delta_1 + \delta_1 s_1]^2} = \frac{\delta_1 \left\{ \frac{\delta_1 \rho}{\delta_2} - \frac{\delta_1 \beta}{\delta_2} \delta_1 + \delta_1 s_1 \right\}}{[\delta_1 \rho - \delta_1 \beta \delta_1 + \delta_1 s_1]^2} = \frac{\delta_1 \left\{ \rho (\delta_2 - \delta_1) - \delta_1 (1 - \delta_2) \beta \right\}}{\delta_2 (1 - \delta_2) - \delta_1 (1 - \delta_2) \beta} > 0
\]

which is the restriction on collateral, leading to the case \( \frac{\partial \pi^*_2}{\partial s} \bigg|_{(s=s_1=s_2)} > 0 \). Therefore, we conclude that uniform subsidies lead to a decrease in the credit rationing of a low-risk borrower in this case.

**4. Conclusions**

This paper introduced government interventions into a model of credit provision under asymmetric information. The aim of these modeled interventions was to alleviate the inefficiencies in credit markets caused by borrower private information. The inefficiencies are
credit rationing and the deadweight costs connected with the transfer of collateral in the event of the failure of a collateralized project. In accordance with empirically observed types of interventions, we concentrated on two kinds of government intervention: credit guarantees and interest-rate subsidies.

We compared these two intervention mechanisms in the framework of two regimes relevant for transition economies. First, we considered the transition environment of the early nineties, when major structural changes occurred both as regards the economy and the welfare of people. It was a time when some people with sharp entrepreneurial abilities became wealthy relatively quickly while others found that, lacking such abilities and due to structural changes, they were forced out of or into new employment situations and slipped down the social ladder. We also considered the posttransition environment of the years preceding the 2004 EU expansion, when the situation in the economy stabilized and the opportunities for people were not so radically different as in early transition.

We considered two ways of providing subsidies and guarantees. One is to provide uniform subsidies for all borrowers. This is a very attractive specification since it minimizes the discretion of the government and simplifies the provision of government support. We also considered government support targeted to different types of entrepreneurs, as revealed by their choice of contract as offered by lenders.

This assumption that the government is able to target support is quite in accord with the desire of many governments to fine-tune and better address policy. We would like to emphasize that the option of targeting credit support does not require more information on the side of government about the identities of the loan applicants as compared with the information available to the commercial lenders. It only requires the full rationality and full foresight of all involved players. We assume that the government is able to determine the optimal packages offered by commercial lenders. The government then targets its support by assigning it to an offered package, not to some "cheap talk" declaration by a borrower with regards to his type.

Based on such distinctions, we showed that the guarantees and subsidies targeted to low-risk borrowers decrease efficiency, while those targeted to high-risk borrowers increase efficiency both in transition and posttransition economies. Further, we proved that uniform, nontargeted guarantees improve welfare. We also proved that uniform subsidies may be used to improve welfare in an economy subjected to credit rationing, too, but we obtained the interesting result that they do not have any effect on the value of collateral required in a posttransition economy. Therefore, we conclude that guarantees and subsidies have, in the
majority of cases, qualitatively similar effects. Nevertheless, guarantees are a more robust instrument in situations where policymakers are not sure whether the economic environment corresponds more to a transition or posttransition economy.

This paper did not engage in a cost-benefit analysis of interest-rate subsidies and loan guarantees. It also did not consider the problems of determining the value of collateral for government or the quantification of collateral transfer costs connected with collateralized loans. In particular, we did not consider the government spending necessary for the guarantees in relation to the subsidies needed to achieve the same effect of decreasing the level of collateral or credit rationing by some fixed, predetermined amount. These are left for future research.
Appendix - The Solution of the Asymmetric Information Problem

The maximization problem of a lender is given by:

\[
\max_{\pi, R_1, R_2} M = \theta U_{11} + (1 - \theta) U_{22} \\
= \theta \pi_1 [\delta_1 (y - R_1) - (1 - \delta_1) C_1 - b_1] + \\
(1 - \theta) \pi_2 [\delta_2 (y - R_2) - (1 - \delta_2) C_2 - b_2]
\]

s.t.

\[
\begin{align*}
\pi_1 [\delta_1 (y - R_1) - (1 - \delta_1) C_1 - b_1] & \geq \pi_2 [\delta_2 (y - R_2) - (1 - \delta_2) C_2 - b_2], \quad (IC1) \\
\pi_2 [\delta_2 (y - R_2) - (1 - \delta_2) C_2 - b_2] & \geq \pi_1 [\delta_1 (y - R_1) - (1 - \delta_1) C_1 - b_1], \quad (IC2) \\
U_{i} & \geq 0, \quad (IRi) \\
0 & \leq \pi_i \leq 1, \\
0 & \leq C_i \leq W,
\end{align*}
\]

\[
\delta_1 R_i + (1 - \delta) \beta C_i - \rho = 0, \quad i \in \{1, 2\}. \tag{A1}
\]

Equation (A1) is a zero-profit condition for lenders that explicitly prohibits cross-subsidization. This means that it is not possible for lenders to suffer a loss on a contract to one type of borrower and to enjoy a positive profit on a contract to another type of borrower.

The solution to this problem consists of four possible cases, which we label as Cases A to D. The equilibrium contract for a high risk borrower in cases A, B, and C is identical with the high-risk-borrower’s contract under full information; that is:

\[
C_i^* = 0, \pi_i^* = 1, R_i^* = \frac{\rho}{\delta_i}.
\]

Case A:

This is the case of a transition economy defined by \( \frac{b_1}{\delta_1} \geq \frac{b_2}{\delta_2} \). An equilibrium contract of a low-risk borrower is given by:
\[ C^*_2 = 0. \]
\[ \pi^*_2 = \frac{\delta_i y - \rho - b_i}{\delta_i y - \rho - b_i} \]
\[ R^*_2 = \frac{\rho}{\delta_2}. \]

The equilibrium value of \( \pi^*_2 < 1 \) means that there is always credit rationing in the case of a competitive credit market under asymmetric information in a transition economy.

**Case B:**

This is the case of a stabilized posttransition economy defined by \( \frac{b}{\bar{y}} < \frac{b}{\bar{y}} \), with an additional provision that the collateral is unconstrained. The low-risk borrower’s contract is given by:

\[ C^*_2 = \frac{\rho(\delta_2 - \delta_i)}{(1 - \delta_i)\delta_2 - \delta_i(1 - \delta_2)\beta} > 0. \]
\[ \pi^*_2 = 1. \]
\[ R^*_2 = \frac{\rho}{\delta_2} - \frac{(1 - \delta_i)\beta C^*_2}{\delta_2} = \frac{\rho[(1 - \delta_i) + \beta(\delta_2 - 1)]}{(1 - \delta_i)\delta_2 - \delta_i(1 - \delta_2)\beta} > 0. \]

**Case C:**

This is the case of a stabilized posttransition economy defined by \( \frac{b}{\bar{y}} < \frac{b}{\bar{y}} \), with an additional provision that the collateral is constrained by the level of wealth which is of an intermediate level so that:

\[ W < \frac{\rho(\delta_2 - \delta_i)}{(1 - \delta_i)\delta_2 - \delta_i(1 - \delta_2)\beta}, \quad \text{(A2)} \]

\[ W \geq \frac{\delta_2[(\delta_i y - \frac{\rho}{\delta_2} - b_2)(\delta_i y - \frac{\rho}{\delta_2} - b_1) - (\delta_i y - \rho - b_i)(\delta_i y - \rho - b_2)]}{(\delta_i y - \frac{\rho}{\delta_2} - b_2)[(1 - \delta_i) - \delta_i(1 - \delta_2)\beta] - \delta_i(1 - \delta_2)(1 - \beta)(\delta_i y - \rho - b_i)}. \quad \text{(A3)} \]

In this case the low-risk borrower’s contract is given by:

\[ C^*_2 = W. \]
\[ \pi^*_2 = \frac{\delta_i y - \delta_i R^*_2 - b_i}{\delta_i y - \delta_i R^*_2 - b_i} - (1 - \delta_i)W \]
\[ = \frac{\delta_i y - \rho - b_i}{\delta_i y - \rho - b_i - (1 - \delta_i)W} \in (0, 1). \]
\[ R^*_2 = \frac{\rho - (1 - \delta_2)\beta W}{\delta_2} > 0. \]
Case D:
In the case of a stabilized economy with a very low level of wealth, which does not satisfy Equation (A3), a restriction, the credit market breaks down and no lending is realized, \( \pi_1^* = \pi_2^* = 0 \).

Proof:

We assume that \( \pi_i > 0 \) and that (IC2), (IR1), and (IR2) will not be violated by the solution. We will check these assumptions after we obtain the solution.

From the zero-profit condition (Equation [A1]), we substitute for \( R_i \) and we form the Lagrangian:

\[
\max L = \theta \pi_1 [\delta_i y - \rho - b_1 - (1 - \beta)(1 - \delta_i)C_i] + \]

\[
(1 - \theta) \pi_2 [\delta_2 y - \rho - b_2 - (1 - \beta)(1 - \delta_2)C_2] + \mu \pi_1 [\delta_i y - \rho - b_1 - (1 - \beta)(1 - \delta_i)C_i] - \]

\[
\pi_2 [\delta_2 y + \delta_i (1 - \delta_i) \beta C_2 - \rho - b_1 - (1 - \delta_i)C_2] + \]

\[
\tau_i C_1 + \tau_2 C_2 - \tau_3 (C_1 - W) - \tau_4 (C_2 - W) - \]

\[
\xi_i (\pi_1 - 1) - \xi_2 (\pi_2 - 1).
\]

Kuhn-Tucker conditions are FOC:

\[
\frac{\partial L}{\partial \pi_1} = (\theta + \mu) [\delta_i y - \rho - b_1 - (1 - \beta)(1 - \delta_i)C_i] - \xi_i = 0,
\]

\[
\frac{\partial L}{\partial \pi_2} = (1 - \theta) [\delta_2 y - \rho - b_2 - (1 - \beta)(1 - \delta_2)C_2] - \]

\[\mu [\delta_i y - \rho - \frac{[\delta_2 (1 - \delta_i) - \delta_i (1 - \delta_2) \beta]}{\delta_2} C_2] - \xi_2 = 0,
\]

\[
\frac{\partial L}{\partial C_1} = -\pi_i (1 - \beta)(1 - \delta_i) (\theta + \mu) + \tau_1 - \tau_3 = 0,
\]

\[
\frac{\partial L}{\partial C_2} = -\pi_2 [(1 - \theta)(1 - \beta)(1 - \delta_2) + \mu \frac{\delta_i (1 - \delta_i) \beta}{\delta_2} - (1 - \delta_i)] + \]

\[\tau_2 - \tau_4 = 0,
\]

and (IC1), \( 0 < \pi_i \leq 1, 0 \leq C_i \leq W \), complementary slackness conditions, and nonnegativity of multipliers.

First we show that \( C_i^* = 0 \).
Consider \( C_1 > 0 \Rightarrow \tau_1 = 0 \Rightarrow \tau_3 = -\pi_i(1-\beta)(1-\delta_i)(\theta + \mu) < 0 \Leftrightarrow \) a contradiction with the nonnegativity of a multiplier \( \tau_3 \).

Since \( \frac{\partial L}{\partial \pi_i} \bigg|_{\pi_i = C_i} = 0 \Rightarrow \xi_i = (\theta + \mu)[\delta_i y - \rho - b_i] > 0 \Leftrightarrow \) by a complementary slackness \( \pi_i^* = 1 \).

Substituting equilibrium values of \( \pi_i^*, C_i^* \), and \( R_i^* \) into the utility function of a high-risk borrower, we get:

\[
U_{1i} = \delta_i y - \rho - b_i > 0,
\]

which shows, that an individual rationality constraint (IR1) is satisfied.

Now we will discuss three possibilities (denoted as Cases A, B, C) with respect to an optimal choice of \( C_2 \).

**Case A.**

Consider \( C_2 = 0 \).

We know that \( \pi_2 < 1 \) since the full information solution is not incentive compatible with (IC1).

For \( \pi_2 \in (0,1) \), we have \( \xi_2 = 0 \). From \( \frac{\partial L}{\partial \pi_2} = 0 \), we get \( \mu > 0 \Rightarrow (IC1) \) is binding because of a complementary slackness condition \( \Rightarrow \pi_2 = \frac{\delta_i y - \rho - b_i}{\delta_i y - \frac{\delta_2 y - \rho - b_2}{\delta_2}} \).

We check the conditions under which (IC2) is satisfied:

\[
\left( \delta_2 y - \rho - b_2 \right) \frac{\delta_1 y - \rho - b_1}{\delta_1 y - \frac{\delta_i y - \rho - b_i}{\delta_i}} \geq \delta_2 y - \frac{\delta_2 y - \rho - b_2}{\delta_2} \mathrm{\rho} - b_2.
\]

\[
\left( \delta_2 y - b_2 \right) \left( \delta_i y - b_i \right) - \left( \delta_2 y - b_2 \right) \mathrm{\rho} - \delta_i y - \delta_2 y - b_2 \left( \delta_1 y - b_1 \right) -
\]

\[
\frac{\delta_2}{\delta_2} \mathrm{\rho} \left( \delta_2 y - b_2 \right) -
\]

\[
\frac{\delta_2}{\delta_1} \mathrm{\rho} \left( \delta_i y - \frac{\delta_1}{\delta_2} \mathrm{\rho} - b_i \right).
\]

\[
- \left( \delta_2 y - b_2 \right) - \left( \delta_1 y - \rho - b_i \right) \geq - \frac{\delta_1}{\delta_2} \left( \delta_2 y - b_2 \right) -
\]

\[
\frac{\delta_2}{\delta_1} \left( \delta_i y - \frac{\delta_1}{\delta_2} \mathrm{\rho} - b_i \right).
\]

\[
\delta_1 \delta_2 b_2 + \delta_i b_i \geq \delta_1^2 b_2 + \delta_i^2 b_i.
\]

\[
b_2 \delta_i \left( \delta_2 - \delta_i \right) \geq b_2 \delta_2 \left( \delta_2 - \delta_i \right).
\]

\[
\frac{b_2}{b_i} \geq \frac{\delta_2}{\delta_1},
\]

which agrees with our definition of a transition economy.

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If \( \frac{b_2}{\delta_1} \geq \frac{\delta_1}{\delta_2} \), then the solution is:

\[
\begin{align*}
C_1^* &= C_2^* = 0, \\
\pi_1^* &= 1, \\
\pi_2^* &= \frac{\delta_1 y - \rho - b_1}{\delta_1 y - \frac{\delta_1}{\delta_2} \rho - b_1}, \\
R_i^* &= \frac{\rho}{\delta_1}.
\end{align*}
\]

Substituting equilibrium values of \( \pi_1^*, C_1^* \), and \( R_2^* \) into the utility function of a low-risk borrower, we get:

\[
U_{22} = \frac{(\delta_1 y - \rho - b_1)}{(\delta_1 y - \frac{\delta_1}{\delta_2} \rho - b_1)}(\delta_2 y - \rho - b_2) > 0,
\]

which proves that the individual rationality constraint (IR2) is satisfied.

**Case B.**

Consider an interior solution with respect to \( C_2 \Rightarrow \tau_2 = \tau_4 = 0 \Rightarrow \frac{\partial}{\partial C_2} = (1 - \theta)(1 - \beta)(1 - \delta_1) + \mu \frac{\delta_1(1 - \delta_2)(1 - \delta_2)(1 - \delta_2)}{\delta_2} = 0 \\
\Rightarrow \mu = \frac{\delta_1(1 - \theta)(1 - \beta)(1 - \delta_2)}{\delta_2(1 - \delta_1) - \delta_1(1 - \delta_1) \beta} > 0. \quad (A4)

The expression

\[
D = \delta_2(1 - \delta_1) - \delta_1(1 - \delta_2) \beta > 0 \quad (A5)
\]

is positive since \( D \) is at a minimum for \( \beta = 1 \Rightarrow D_{\text{min}} = \delta_2 - \delta_1 > 0. \)

From Equation (A4), we see that (IC1) is binding \( \Rightarrow \)

\[
C_2 = \frac{\delta_2(\delta_1 y - b_1) - \frac{\delta_1(\delta_1 y - \rho - b_1)}{\delta_2}}{\delta_2(1 - \delta_1) - \delta_1(1 - \delta_1) \beta} \]

\[
= \frac{\pi_2[\delta_2(\delta_1 y - b_1) - \delta_1 \rho] - \delta_1(\delta_1 y - \rho - b_1)}{\pi_2[\delta_2(1 - \delta_1) - \delta_1(1 - \delta_1) \beta]} \quad (A6)
\]

We assume that \( \pi_2 = 1. \) To obtain the conditions under which this assertion is valid, we substitute \( C_2(\pi_2) \) from Equation (A6) into the utility function \( U_{22} \) and we find the conditions under which \( \frac{\partial U_{22}}{\partial \pi_2} > 0. \)
\[ U_{22} = \pi_2(\delta_2 y - \rho - b_2) - \pi_2 C_2(1 - \beta) (1 - \delta_2) \]
\[ = \pi_2(\delta_2 y - \rho - b_2) - \pi_2[\delta_2(\delta_2 y - b_2) - \delta_1(1 - \delta_2)\beta] \]
\[ \delta_2(1 - \delta_2) - \delta_1(1 - \delta_2)\beta \]

\[ \frac{\partial U_{22}}{\partial \pi_2} = \delta_2 y - \rho - b_2 - \frac{[\delta_2(\delta_2 y - b_2) - \delta_1(1 - \delta_2)\beta]}{\delta_2(1 - \delta_2) - \delta_1(1 - \delta_2)\beta} > 0 \Leftrightarrow \]

\[ (\delta_2 y - \rho - b_2)[\delta_2(1 - \delta_2) - \delta_1(1 - \delta_2)\beta] - \]
\[ (\delta_1 \delta_2 y - \delta_1 b_1 - \delta_1(1 - \delta_2)\beta) > 0. \quad (A7) \]

Expanding the left-hand side of Equation (A7) and collecting terms, we get:

\[ (\delta_2 - \delta_1)(y\delta_2 - \rho) + b_1 \delta_2(1 - \delta_2)(1 - \beta) \]
\[ + b_2 \beta \delta_1 - b_2 \beta \delta_1 \delta_2 - b_2 \delta_2 + b_2 \delta_1 \delta_2. \quad (A8) \]

Adding and subtracting \( \delta_1 b_2 \) to Equation (A8), we get:

\[ (\delta_2 - \delta_1)(y\delta_2 - \rho - b_2) + b_1 \delta_2(1 - \delta_2)(1 - \beta) + b_2 \delta_1(1 - \beta)(\delta_2 - 1) = \]
\[ (\delta_2 - \delta_1)(y\delta_2 - \rho - b_2) + (1 - \delta_2)(1 - \beta)(\delta_1 \delta_2 - b_2 \delta_2). \quad (A9) \]

Equation (A9) is positive if

\[ \frac{\delta_2}{\delta_1} > \frac{b_2}{b_1}, \quad (A10) \]

which agrees with our definition of a stabilized economy.

If Equation (A10) is satisfied, then \( \frac{\partial U_{22}}{\partial \pi_2} > 0 \Rightarrow \pi_2 = 1 \). Substituting \( \pi_2 = 1 \) into Equation (A6), and simplifying, we get:

\[ C_2 = \frac{\rho(\delta_2 - \delta_1)}{(1 - \delta_2)\delta_2 - \delta_1(1 - \delta_2)\beta}. \quad (A11) \]

Next we have to check if the (IC2) is satisfied.

We directly substitute the values \( C_1 = 0, \pi_1 = \pi_2 = 1 \) into (IC2):

\[ \delta_2 y - \delta_2 R_2 - (1 - \delta_2)C_2 - b_2 \geq \delta_2 y - \delta_2 R_2 - b_2. \]
Substituting for $C_2$ from Equation (A11) and for $R_1, R_2$ from the zero-profit condition of Equation (A1), we get:

\[
\frac{1}{\delta_i} + \frac{(1-\delta_2)(\beta + \delta_2 - 1)}{\delta_2(1-\delta_i) - \delta_i(1-\delta_2)\beta} \geq 0
\]  
(A12)

The left-hand side of Equation (A12) can be simplified as

\[
\frac{\delta_i - \delta_2}{\delta_i[\delta_2(1-\delta_i) - \beta\delta_i(1-\delta_2)]} > 0,
\]

which shows that (IC2) is satisfied.

The individual rationality constraint of type-2 borrower is satisfied if

\[
U_{22} = \delta_2 y - \rho - b_2 - (1 - \beta)(1 - \delta_2)C_2^* \geq 0.
\]  
(A13)

We show that the individual rationality constraint (Equation [A13]) is satisfied under the stabilized posttransition economy assumption $\frac{\beta}{\delta_2} < \frac{\beta}{\delta_i}$.

From the binding constraint (IC1), we have:

\[
\delta_i(y - R_i^*) - (1 - \delta_i)C_i^* = \delta_i(y - R_2^*) - (1 - \delta_i)C_2^*.
\]  
(A14)

The right-hand side of Equation (A14) can be rewritten as:

\[
\delta_2(y - R_2^*) - (1 - \delta_2)C_2^* - (\delta_2 - \delta_i)(y - R_2^* + C_2^*).
\]  
(A15)

Substituting Equation (A15) for the right-hand side of Equation (A14) and rearranging, we obtain:

\[
\delta_2(y - R_2^*) - (1 - \delta_2)C_2^* - b_2 = \delta_1 y - \rho - b_2 + (\delta_2 - \delta_i)(y - R_2^* + C_2^*).
\]  
(A16)

Substituting equilibrium values of $R_i^*, R_2^*$, and $C_i^*$ into the right-hand side of Equation (A16) and subtracting $b_2$ from both sides of that equation, we get:

\[
\delta_2(y - R_2^*) - (1 - \delta_2)C_2^* - b_2 = \delta_1 y - \rho - b_2 + (\delta_2 - \delta_i)[y - \frac{P}{\delta_2} + \frac{(1 - \delta_2)\beta C_2^*}{\delta_2} + C_2^*].
\]  
(A17)

The term $b_2$ can be partitioned into

\[
b_2 = \frac{\delta_i}{\delta_2}b_2 + \frac{\delta_2 - \delta_i}{\delta_2}b_2.
\]  
(A18)

Using Equation (A18), the right-hand side of Equation (A17) can be rewritten as:

\[
\delta_1 y - \rho - \frac{\delta_1}{\delta_2}b_2 + (\delta_2 - \delta_i)\left[\frac{1}{\delta_2}(\delta_2 y - \rho - b_2) + \frac{(1 - \delta_2)\beta C_2^*}{\delta_2} + C_2^*\right] > 0.
\]  
(A19)
Because of our assumption $\frac{b_2}{b_1} > \frac{b_1}{b_2}$, it is true that $\frac{b_2}{b_1} < b_1$. Equation (A19) shows that the left-hand side of Equation (A17), which is equal to $U_{22}$, satisfies a individual rationality constraint (IR2) for any positive value of $C_2^*$.

**Case C.**

If $C_2^*$ given by Equation (A11) is bigger than $W$, we have a corner solution $C_2^* = W$. Using already obtained values $C_1^* = 0$ and $\pi_1^* = 1$, we have to find the value of $\pi_2$.

We show that $\mu > 0 \land \pi_2^* < 1$.

Assume $\mu = 0 \Rightarrow \frac{d\pi}{d \xi_2} = (1-\theta)[\delta_2 y - \rho - b_2 - (1-\beta)(1-\delta_2)W] - \xi_2 = 0$.

Because $W < C_2^*$ as given by Equation (A11), we have:

$$\xi_2 = (1-\theta)[\delta_2 y - \rho - b_2 - (1-\beta)(1-\delta_2)W] > (1-\theta)[\delta_2 y - \rho - b_2 - (1-\beta)(1-\delta_2)C_2^*] \geq 0 \text{ (because of )}$$

13)),

$$\Rightarrow \xi_2 > 0 \Rightarrow \pi_2 = 1.$$  

We check if

$$\pi_1 = \pi_2 = 1, C_1 = 0, C_2 = W$$  

(A20)

satisfies (IC1). We substitute a candidate solution (Equation [A20]) into (IC1) and simplify:

$$-\rho \geq -\frac{\delta_2 \rho}{\delta_2} \left[ \delta_2 (1-\delta_2) - \delta_1 (1-\delta_1) \beta \right] W \geq \frac{\rho (\delta_2 - \delta_1)}{\delta_2 (1-\delta_2) - \delta_1 (1-\delta_1) \beta}.$$  

But we are considering the case of

$$W < \frac{\rho (\delta_2 - \delta_1)}{\delta_2 (1-\delta_2) - \delta_1 (1-\delta_1) \beta}$$

$$\Rightarrow \text{the proposed solution violates (IC1)} \Rightarrow \pi_2^* < 1 \Rightarrow \mu > 0 \Rightarrow \text{by a complementary slackness (IC1) has to hold as an equality.}$$

We express $\pi_2^*$ from (IC1):

$$\pi_2^* = \frac{\delta_2 y - \delta_1 R_2 - b_1}{\delta_2 y - \delta_1 R_2 - b_1 - (1-\delta_2)W}$$  

(A21)
\[ W \begin{bmatrix} (\delta \gamma y - \rho - b_1) - (\delta \gamma y - \rho - b_2) \delta_1 \delta_2 (1 - \delta_1 - \delta_2) \beta \delta_2 & (\delta \gamma y - \rho - b_1)(1 - \delta_1)(1 - \beta) \delta_2 \end{bmatrix} \geq (\delta \gamma y - \rho - b_1)(\delta \gamma y - \rho - b_2). \] (A24)

Under the assumption that the expression in square brackets on the left hand side of Equation (A24) is positive, we get Equation (A3), a restriction, on \( W \), under which (IC2) is satisfied.

In order to prove that the left-hand side of Equation (A24) is positive, we show that:

\[ \delta \gamma y - \frac{\delta_2}{\delta_1} \rho - b_2 > \delta \gamma y - \rho - b_1, \] (A25)

\[ \frac{\delta_2(1 - \delta_1)(1 - \delta_2) \beta}{\delta_2} > (1 - \delta_2)(1 - \beta). \] (A26)
We rewrite Equation (A25) as:

\[ \delta_2(y - \frac{\rho}{\delta_1}) - \delta_1(y - \frac{\rho}{\delta_1} - (b_2 - b)) > 0, \]

which can be further rewritten as:

\[ \delta_1 y - \rho - \frac{\delta_1}{\delta_2 - \delta_1} (b_2 - b) > 0. \] \hspace{1cm} (A27)

Equation (A27), an inequality, is satisfied if

\[ \frac{\delta_1}{\delta_2 - \delta_1} (b_2 - b) < b, \]

which is true for \( \frac{b_2}{b} < \frac{\delta_1}{\delta_2}. \)

This proves that the inequality of Equation (A25) is true.

To prove the inequality of Equation (A26), we multiply its terms out,

\[ \delta_2 - \delta_2 \delta_1 - \delta_1 \beta + \delta_1 \delta_2 \beta > \delta_2 - \delta_2 \beta - \delta_2^2 + \delta_2^2 \beta, \]

then we collect the terms,

\[ \delta_1 \beta + \delta_2 (1 - \beta) - \delta_1 \beta - \delta_2 (1 - \beta) > 0, \]

and we get

\[ \delta_1 [\beta + \delta_1 (1 - \beta)] - \delta_2 [\beta + \delta_2 (1 - \beta)] > 0, \]

which simplifies as

\[ (\delta_2 - \delta_1) [\beta + \delta_1 (1 - \beta)] > 0, \]

which is obviously positive.

The individual rationality constraint (IR2) for the low-risk borrower is satisfied since

\[ U_{22} = \pi_2^* [\delta_2 y - \rho - b_2 - (1 - \delta_2^2 (1 - \beta) W], \]

where \( \pi_2^* \) is positive, and the term in the square brackets is bigger than the corresponding term in Equation (A13), which was proved to be nonnegative.

Q.E.D.
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