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Electives Shopping, Grading Competition, and Grading Norms

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Electives Shopping, Grading Competition, and Grading Norms

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Abstract:
This paper analyzes grading competition between instructors of elective courses when students shop for high course scores, the instructors maximize class size, and the school imposes a ceiling on mean course scores to limit grade inflation. Under this grading norm, we demonstrate that curriculum flexibility (more listed courses or less required courses) intensifies the competition: in particular, both top and mean realized scores increase. To tame incentives to provide excessively large scores, we suppose that the school additionally introduces a top-score grading norm. We consider three scenarios. First, the school caps top scores directly. Then, grading competition divides students into a concentrated group of achievers and a dispersed group of laggards. Second, the school normalizes the range of scores by changing the mean-score ceiling. Upon normalization, scores of a less flexible curriculum first-order stochastically dominate scores of a more flexible curriculum. Hence, all students will prefer rigid curricula. Third, the school requires that the mean-score ceiling is evaluated for enrolled students instead of all students. Then, the instructors stop competing for students which introduces sorting inefficiencies. Overall, we show that addressing grade inflation through grading norms may generate inequalities, rigidities, and inefficiencies.

JEL: C72, D02, D21, I21, I23
Keywords: grading competition, grades compression, grading norms, Continuous Lotto games, Captain Lotto games, higher education

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1 Introduction

While course instructors and students primarily care about content and quality of courses, it is also the case that students choose courses to shop for the best grades, and some instructors attract students through lenient grading. Indeed, there is extensive evidence of both electives shopping on the side of students (Sabot and Wakeman-Linn, 1991; Bar, Kadiyali, and Zussman, 2009) as well as grading competition between instructors, motivated by concerns about teaching evaluations (Correa, 2001; Butcher, McEwan and Weeapana, 2014) and pressure to enroll many students in a class when attendance is a performance metric (Angling and Meng, 2000). Grading competition occurs and is particularly pronounced in the context of electives that are close substitutes within a single program, but it may also exist on the level of departments and schools when a program choice or even a field choice is involved.

For the shape of grading competition, the key is the set of grading norms that bind the instructors. In this paper, we focus on grading competition under a *mean-score ceiling* (a.k.a. a mean-preserving grading norm, a grade ceiling, or a mean-grade quota). Namely, a school requires a mean course score for a representative sample of students to not exceed a given ceiling, but otherwise leaves grading in the hands of the instructors. The mean-score ceiling fully controls the incentives to increase scores across the board but otherwise leaves the shape of the score distribution unregulated. An example of the norm in practice is the well-studied decision of a small elite Wellesley College by which average grades in courses with at least 10 students should not exceed a B+ average (Butcher et al., 2014). Gorry (2017) documents a similar example in a business school of a large US public university, where in 2014 an average recommended grade was set to 2.8 in introductory courses and 3.2 in intermediate courses.

We analyze how the mean-score ceiling affects grading competition, especially when the norm is combined with additional grading norms that regulate top scores. In particular, we analyze how curriculum flexibility shapes grading competition. That curriculum flexibility may have an effect on the moments of the distributions can be observed in a variety of higher education environments, but most visibly when comparing grades across divisions. For instance, Achen and Cournot (2009) report that most departments treat students much better in upper-division courses, and also that upper-division grades are higher than those in the lower divisions.

To give an illustrative example, Table 1 lists the grade distributions in the Juris Doctor (JD) program at Pitt Law School in the year 2016.[1] At Pitt Law, JD students enjoy a large degree of latitude in designing courses of study that meet their individual goals and interests, with only a handful of graduation requirements beyond the first year. In other words, curriculum structures differ across classes, with more senior students having greater flexibility. Table 1 shows that the distribution of inferred GPAs[2] for the first-year students

[1] Source: http://law.pitt.edu/grades/distribution

[2] As individual GPAs are not observable, we assign students who have received an identical grade the mid-

who face the most rigid curriculum (Class of 2018) indeed exhibits the lowest mean and the largest variance. The median grade is B+, hence the median inferred GPA (3.31) exceeds the mean inferred GPA for all classes. For all classes, Pearson skewness is negative, with the absolute value growing upward for more senior classes that enjoy greater flexibility.

Table 1: The grade distributions for JD candidates at Pitt Law School, 2016

Grade	A	A-	B+	B	B-	C+	C	C-	D	F	Mean	Variance	Skewness
Mid GPA	3.94	3.69	3.31	3.00	2.69	2.31	2.05	2.18	1.01	0.38			
Class of 2018	6	21	36	22	28	16	1	0	0	0	3.08	0.22	-0.09
Class of 2017	4	22	56	46	21	2	1	0	0	0	3.18	0.13	-0.13
Class of 2016	4	25	84	39	17	5	0	0	0	0	3.22	0.12	-0.42

In fact, there are two channels through which flexibility may affect the grades: (i) a change in the instructors' grading scheme, and (ii) an increased self-selection into courses that more likely award better grades. To separate the two channels, we build a model of grading competition based on Continuous Lotto Games (Hart 2008, 2016). In the model, students exclusively shop for good grades, and instructors only maximize enrollment. Each student has a personal fit to an elective course, when the fit is ex ante unknown both to the student and instructor. The instructor constructs the grading rule as a mapping from the fit to the course score, where the average score for all potential students (or, for a representative sample) is fixed. In the shopping period, a student costlessly attends all electives and acquires free signals of his or her fit through *interim scores*. When the shopping period closes, each student enrolls into courses with the most promising interim scores; since interim scores are assumed to be unbiased signals of *ex post* course scores, the students simply pick up courses with the highest interim scores.

In this environment, we demonstrate that the response (enrollment) rates are always linear in interim scores and with more flexibility fall proportionally; irrespective of fit, all students become more selective if grading competition intensifies. In addition, we document that increasing curriculum flexibility (i.e., more courses offered, or less courses required) motivates course instructors to offer larger maximal scores to the best students. We can also describe the ex post scores that result from interim scores and responses. Namely, even if the interim score distributions get more positively skewed with a more flexible curriculum (i.e., students with a relatively low fit are now offered lower scores), we prove that self-selection is strong enough to imply that the mean ex post score goes up with flexibility.

The gap between excessively large scores given to the best students and very low scores given to average and below-average students are among the most unwelcome consequences of

value of the interval for the corresponding grade (Mid GPA); e.g., for $A+$, the individual GPAs lie in $[3.876, 4]$, hence we consider the inferred GPA to take the mid-interval value of 3.938. Since an observable grade is only a coarse signal of the unobservable GPA, we must interpret the skewness of mid-GPAs cautiously.

flexible curricula. We analyze how the mean-score ceiling can be augmented to eliminate the excessive top scores. For this purpose, we consider adopting one of three top-score grading norms. First, we suppose the school directly caps the exam scores. Then, in response to the norm, the instructors divide the students into two groups, attractive students (receiving the maximal score) and unattractive students (receiving low scores). Notice that this division occurs even before the students self-select into the electives. In addition, unlike in the regime with a pure mean-score ceiling, we show that this combined regulation risks matching inefficiency. The reason is that the coarse signal of the fit that an attractive student obtains through a capped top score is not informative enough.

Secondly, we let the school re-parameterize the mean-score ceiling. To give a straightforward lesson, we suppose the school targets a constant range of the course scores; hence, all curricula after normalization will award exactly identical top scores. Interestingly, we demonstrate that such normalization *excessively* counteracts strategic grading since the mean realized score decreases in curriculum flexibility. Even more, the score distributions are ranked by first-order stochastic dominance, and rigidity thus increases a student's score for *any percentile* of fit. In a word, after normalization, all types of students receive lower scores when the curriculum becomes more flexible. Thus, if the market interprets GPAs constantly irrespective of flexibility, all students will demand to abolish variety and install a rigid curriculum.

The third instrument explicitly targets self-selection. Namely, we suppose that the mean-score ceiling is evaluated ex post, after the students self-select into the courses. This regulation is excessive in a sense that it doesn't motivate instructors to attract students with a better fit. As a result, there is an equilibrium where the instructors are discouraged to compete, and rather offer identical scores. Mismatches become frequent, and positive matching effects associated with self-selection now get weaker.

Overall, the paper demonstrates that counteracting lenient grading while preserving the instructors' grading autonomy is a hard task. A number of tradeoffs is associated with grading norms that target both mean and top scores. We have analyzed three specific combinations of a mean-score and top-score grading norms, and derived unintended consequences such as inequalities, rigidities (a loss of variety), and sorting inefficiencies.

The paper proceeds as follows: Section 2 surveys the related literature. In Section 3, we construct the setup. Section 4 analyzes the equilibrium distribution of scores associated with a given curriculum structure under a mean-score ceiling. Then we analyze three alternative regimes in which a top-score grading norm enriches the mean-score grading norm. Section 5 analyzes the caps on the maximal scores. Section 6 analyzes normalization of scores to a fixed interval. Section 7 shows that imposing the mean-score ceiling on the ex post grades completely suppresses grading competition. In Section 8 we conclude. In Appendices A and B, we provide proofs and specific examples. Appendix C extends the setup by the instructor's option to invest in his or her resources.

2 Literature

There is large evidence that grades affect students' choices of courses (Sabot and Wakeman-Linn, 1991; Matos-Díaz, 2012). In the first place, grades affect a student's subjective feelings of academic accomplishment (Boatright-Horowitz and Arruda, 2013). Pursuit of easy grades is also a rational response to admissions committees that seem to not properly account for course difficulty when evaluating transcripts (Bailey, Rosenthal and Yoon, 2016). Also, in response to more restrictive dismissal policies, there is evidence that students more likely enroll in leniently-graded courses (Keng, 2016). Therefore, differences in grading mechanisms and incentives to construct lenient grading rules are of central importance to economics of education. Understanding grading is especially important as a component of the analysis of origins of widely documented grade inflation—-whether the improved grades can be attributed to lenient grading, productivity growth, or improved sorting.

How is the construction of grading rules addressed in the literature? Most papers look into centralized grading rules in the hands of school administrators. In the classic tradition, the principals maximize social welfare (or, specifically, human capital) subject to the students' best responses (Costrell, 1994; Betts, 1998). A typical problem is the optimal design of absolute or relative grading rules to elicit effort, accounting for characteristics of the environment such as the class size (Becker and Rosen 1992; Dubey and Geanakoplos, 2010; Andreoni and Brownback, 2017; Paredes, 2017; Brownback, 2018). In the construction of relative grading rules, a typical tradeoff arises between the level of overall effort and an equitable distribution of effort among students, exactly as in other tournaments with heterogenous contestants.

In more recent literature, the school uses the informative content of grades as a tool in competition for attractive placements. Typically, the pool of students is fixed and placement is in the form of assortative matching between students and firms (Ostrovsky and Schwarz, 2010; Popov and Bernhardt, 2013; Boleslavsky and Cotton, 2015; Harbaugh and Rasmusen, 2018). The idea is that a school observes a true ability, and constructs the grading rule as an optimal noise to the true ability. This literature examines the optimal signal technology employed by a school and the resulting information content of transcripts. By analyzing an optimal signal structure, the literature falls under a wide umbrella of Bayesian persuasion (Kamenica and Gentzkow, 2011; Gentzkow and Kamenica, 2017).

In this paper, we adopt a decentralized perspective on grading for which we find no predecessor in the education economics literature. Instead of having full discretion over the grading rules, we suppose that the school constrains transcripts through a few regulations on the basic properties of score distributions, while the grading rules remain in the hands of individual instructors. The assumption of large discretion on the side of instructors is particularly relevant for universities that put a large value on the autonomy of academics. In such an environment, it is natural to analyze transcript structures resulting from the grading competition of non-cooperative instructors.

Our approach belongs in the family of General Lotto games that have originated from Colonel Blotto Games (Hart, 2008). In the game, for each "battlefield" a player chooses a distribution of a nonnegative random variable with a given expected value, which corresponds to the average "allocation per battlefield". The winner in the battlefield is a player with the highest realization. The Lotto games have been solved in the symmetric case of equal expectations by Myerson (1993) and Lizzeri (1999), and in the general case of unequal expectations by Sahuguet and Persico (2006). Kovenock and Roberson (2015) generalize to heterogenous valuations and asymmetric resources. Dziubinski (2012) covers the difference between continuous and discrete General Lotto games.

In this literature, typically only two players compete with each other, and therefore only one prize per battlefield is allocated. In our setting, the battlefields are represented by a continuum of students, and we consider a more generalized case with multiple courses and multiple prizes depending on the number of courses into which each student has to enroll. We exploit the early results from Continuous General Lotto games, namely that the equilibrium scores distributions for K courses and M required courses are derived analogously to the equilibrium electoral portfolios of K candidates who compete over M votes of each voter, when an electoral portfolio is a distribution of a fixed cake (Myerson, 1993).

In addition, we borrow and extend the analysis of the caps that restrain from the above values of the random variables, known as Captain Lotto games (Hart, 2016). This also creates a link between our setup and Bayesian persuasion. In particular, we will see that our setting is equivalent to a special case of competitive Bayesian persuasion (Gentzkow and Kamenica, 2017) in which the payoff of each principal (instructor) is linear in the realization of the random variable. This is because in the equilibrium, responses of the agents (students) are also linear in the random variable. The identity between the principals' problems in Captain Lotto and in Bayesian persuasion is because the constraints on the random variable in the Captain Lotto game and Bayesian persuasion are exactly the same (Boleslavsky and Cotton, 2018).[3]

3 Setup

- Curriculum: There are $K \in \mathbb{N}$ elective courses, each with a single instructor (he). Each student (she) has to enroll into exactly $M \in \mathbb{N}$ elective courses, where $1 \leq M \leq K$. A curriculum (M, K) is called more *flexible* than curriculum (M', K') if $M \leq M'$ and $K \geq K'$. Alternatively, we speak of more intensive grading competition. In the literature, flexibility is sometimes interpreted as a low level of educational standards since the

[3]In Bayesian persuasion, the principal selects a distribution of posterior probabilities. As realization of the random variable is a probability, its maximal realization is no greater than one. By Bayes plausibility, the expected value is equal to the prior. This is equivalent to the combination of a mean-score and top-score constraint in our game that we will analyze in Section 5.

standards are measured by the number of courses needed for graduation (Lillard and DeCicca, 2001).

- Shopping period: In the shopping period, each student is allowed to costlessly visit any elective course. Through a visit, she obtains an interim score (an offer), which is a signal of her ex post score. For simplicity, the signal is perfect; hence, if a student enrolls in the course, her ex post score is identical to her interim score.
- Scores: The interim (offered) score from a course $k = 1, \ldots, K$ is a realization x_k from a random variable X_k that will be described below. For analytical convenience, we will work with continuous instead of discrete scores, $x_k \in \mathbb{R}_0^+$.
- Enrollment: Students are pure score-shoppers and maximize their ex post scores.[4] Given that the interim scores are perfect signals of the ex post scores, the students simply pick up M courses with the highest signal realizations x_k.
- Students: Ex ante, students are identical; hence, we will work with a representative student.[5] For each student and each course k, there is an individual fit to the subject matter of the course k, denoted φ_k. Ex ante, for any student and any course, the fit is uncertain, and is without loss of generality uniformly distributed on the unit interval, $\varphi_k \in [0, 1]$.
- Instructors: Each course instructor k chooses how to map the student's fit φ_k into a score. Namely, the instructor selects a random variable X_k with a probability density function denoted $f_k(x)$ and cumulative distribution function denoted $F_k(x)$. When setting X_k, each instructor non-cooperatively maximizes the number of students who enroll in his course.
- Transforming fit into score: The instructor's distribution function $F_k(x)$ transforms the individual fit to the course φ_k into the interim score x_k through a monotonic quantile function $F_k^{-1}(\varphi_k)$. As a result, since the individual fit is uniformly distributed, x_k is distributed by F_k.
- Ex ante mean-score ceiling: Each distribution F_k is required to have a constant mean, which we normalize to one, $\int_0^\infty x\, f_k(x)dx = 1$. Intuitively, the school requires that each

[4] The assumption that the student's value from the course is strictly increasing in the score allows us to disregard issues associated with mapping from continuous scores to discrete grades. Among others, with flat valuation parts we would observe an incentive for the bunching of test scores (Diamond and Persson, 2016).

[5] Recall the setup features identical (symmetric) instructors as well as students. Ex ante homogeneity rules out ex ante heterogeneity in the grading standards associated with separating types of different abilities. Thus, our paper doesn't address the ability-channel documented by evidence on heterogeneous grading standards as in Bailey et al. (2016). When both courses and students are heterogeneous, sorting primarily involves signaling on the part of instructors (Herron and Markovich, 2017). Our analysis of symmetric grading competition constitutes an initial step for a more complex analysis of asymmetric competition.

instructor distributes a *fixed* amount of points to prospective students of the course. (Later we provide a broader interpretation of the fixed resources and also analyze endogenous resources.) The instructors cannot compete for students by increasing the scores overall, but can change its distribution to encourage or discourage self-selection of particular types of students.

- Efficiency: We will call the equilibrium *efficient* if each student enrolls in those courses that give her the highest values of φ_k with probability one. For symmetric equilibria, $F_1(x, K, M) = \cdots = F_K(x, K, M)$, a necessary and sufficient condition for efficiency is that F-function is continuous in x (i.e., no mass points exist). In the absence of mass points, the inverse function F^{-1} is defined and thus there exists a one-to-one mapping between x and φ. Then, each student who picks M courses with the highest values of x_k effectively picks up M courses with the highest values of φ_k. In contrast, if F is not continuous in x (hence, mass points exist), then each mass point $\hat{x}$ is associated with an interval of φ. Hence, a student who receives an excessive number of identical offers from a particular mass point (such that some have to decline), eliminates some of the offers randomly without being able to rank the offers. By eliminating the offers randomly, efficiency is violated.

- Response rates: Let $A_k \in \{0, 1\}$ be the (random) indicator variable denoting the decision of a student to take the course k. We calculate the probability of k being taken by a student conditional on her signal x_k. This probability is a representative student's *response rate*, and hence also the payoff of the instructor k. We will denote the response rate

$$\pi_k(x_k) := \Pr(A_k = 1 \mid X_k = x_k).$$

4 Equilibrium

This section derives the score distributions in a symmetric non-cooperative equilibrium. Recall that we effectively work with two distributions, interim (offered) scores and ex post (realized) score distributions, where the latter reflects the students' responses.

4.1 Interim scores

In this section, we will look into a symmetric equilibrium, denoted as a collection of identical and independent distributions of interim scores, $F(x, K, M) := F_1(x, K, M) = \cdots = F_K(x, K, M)$. The number of offered courses K and the number of required courses $M \leq K$ are two parameters of the distribution.

To understand the structure of the equilibrium, take a course k and a representative student. Recall that an average student's *response rate*, and hence also payoff of the instructor k, is $\Pr(A_k = 1 \mid X_k = x_k)$. The event $A_k = 1 \mid X_k = x_k$ is characterized such that out of $K-1$ independent draws, at most $M-1$ courses are more attractive than the course k. Given than each other X_j is identically and independently distributed, the probability that a single course is more attractive than x_k is $1-F(x_k)$. Thus, the probability of the event $A_k = 1 \mid X_k = x_k$ is the value of cumulative distribution function of a binomial distribution with parameters $M-1$, $K-1$, and $1-F(x)$,

$$\pi_k(x_k) = B(M-1, K-1, 1-F(x)).$$

Having the conditional probability of course k being taken (i.e., the response rate), we may now proceed to the unconditional probability of course k being taken by a representative student, $\Pr(A_k = 1)$. This constitutes the expected payoff of the instructor k, denoted

$$\Pi_k(X_1, \ldots, X_K) := \Pr(A_k = 1).$$

In a symmetric equilibrium, students spread equally across courses, and the instructor's expected payoff equals $\frac{M}{K}$,

$$\Pi_k(X_1, \ldots, X_k) = \int_0^\infty B(K-1, M-1, 1-F(x))dF_k(x) = \frac{M}{K}. \tag{1}$$

To obtain the equilibrium in our problem, we exploit the observation that in the symmetric equilibrium of a General Continuous Lotto game, the student's *response rate is linear in the random variable on its support* (Myerson, 1993; Hart, 2008). With this linearity property, we easily obtain Proposition 1. (Proofs are relegated to Appendix A.)

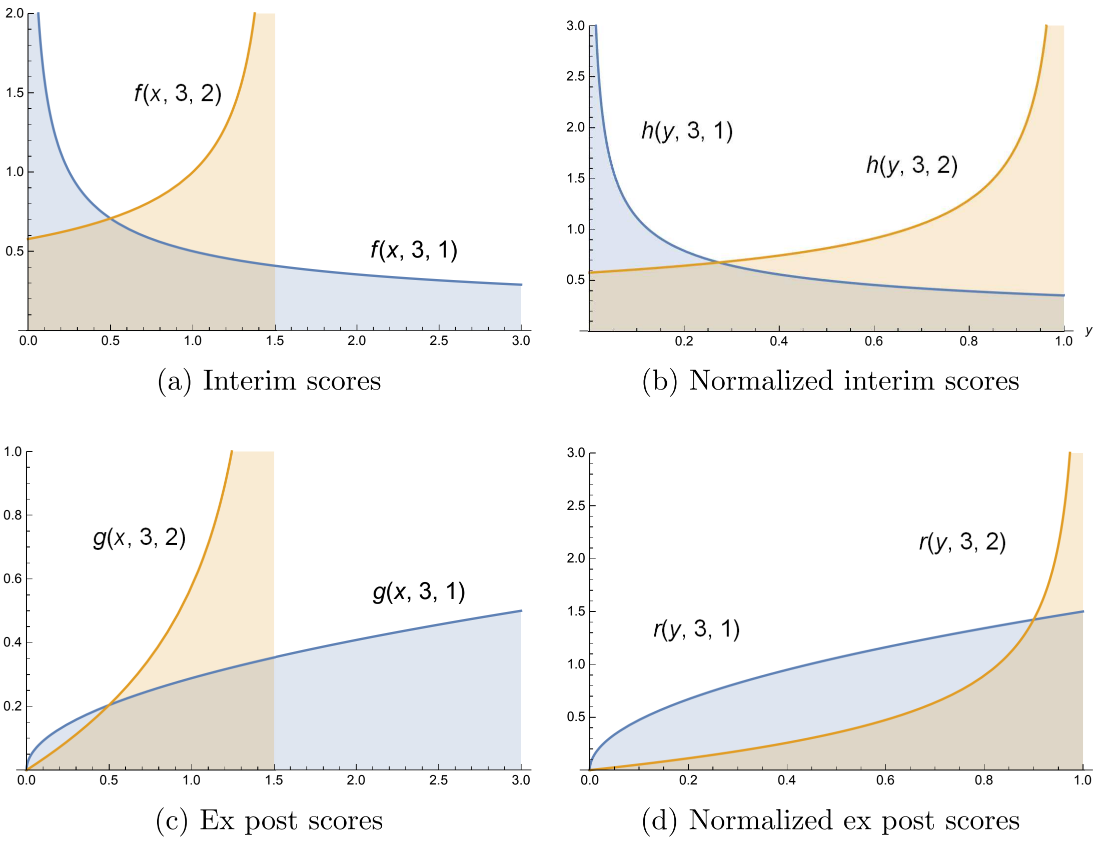
Proposition 1 (Interim scores)**.** *Under an ex ante mean-score ceiling, the symmetric equilibrium interim scores distribution $F(x, K, M)$ is characterized as a unique solution to*

$$B(K-1, M-1, 1-F(x, K, M)) = x\frac{M}{K}. \tag{2}$$

The equilibrium interim scores are derived in an implicit form. For a low K or an extremely low or high M, a closed-form solution is readily available, which is illustrated for $K = 3$ in Panel (a) of Figure 1. For a larger K and an intermediate M, the probability distribution is given by a Gauss hypergeometric function (see Appendix B.1).[6] From the continuity of the binomial function in the last argument, F-distribution doesn't contain any mass point, and hence the equilibrium is efficient.

Since the range of a binomial function is a unit interval, the support of the interim scores depends on curriculum flexibility (M, K). More specifically, if more instructors compete for

[6]The first moment of the interim scores distribution is fixed to one, but what is known about the other properties of the distribution? Above all, Myerson (1993) numerically computes standard deviations for $K \leq 8$ and shows that the standard deviation is increasing in curriculum flexibility. Not all statistics are monotonic in curriculum flexibility, though. While the maximum (100th percentile) is decreasing in the number of required courses, the median (50th percentile) may decrease but also increase in the number of required courses, as Appendix B.2 illustrates.



(a) Interim scores

(b) Normalized interim scores

(c) Ex post scores

(d) Normalized ex post scores

Figure 1: Scores densities for $K = 3$ and $M = 1, 2$

students, instructors offer larger premia to the students with the best fit for the subject. Exactly the same effect occurs when the students have to collect less credits and therefore reject more offers. Formally, the support of X is $x \in [0, \frac{K}{M}]$. Corollary 1 to Proposition 1 summarizes. (In Appendix C, we also demonstrate that the result remains unchanged if the supply is not fixed and the advisors may increase their resources at a cost.)

Corollary 1 (Top scores)**.** *Under a mean-score grading norm, curriculum flexibility increases top scores.*

How to interpret excessive scores in cases when the course scores are naturally bounded from above (e.g., by the value of 100)? One way is to interpret the offered scores more broadly than just plain test scores; we may think of extra bonuses in the form of special services that instructors provide to the best fitting students. These extra services may serve as the margin at which the most promising students are attracted to a class. Examples are promises to serve as thesis advisors, extra attention in classes and office hours, or explicit promises to write recommendation letters for graduate schools. As long as the instructors have an (ex ante) fixed supply of these extra services, the mean-score requirement is satisfied. Alternatively, interim scores exceeding natural bounds may be provided in the form of signals that effort needed for a good test score can be low, and thus effort can be saved.

4.2 Ex post scores

In this section, we derive the ex post scores distribution and examine how the ex post scores depend on curriculum flexibility. To begin, we derive the distribution using that the response rate is linear in x in the equilibrium.

Proposition 2 (Ex post scores)**.** *Under an ex ante mean-score ceiling, the density of the ex post scores, denoted $g(x, K, M)$, is in a symmetric equilibrium*

$$g(x, K, M) = xf(x, K, M). \tag{3}$$

Flexibility affects the ex post scores through two channels: offers and responses. The former is captured in the change of the interim scores distribution, $F(x, K, M)$. The latter is captured in the change of the response rate, $\pi_k(x_k)$. To analyze the latter, see that

$$\pi_k(x_k) = \begin{cases} x_k \frac{M}{K} & \text{if} \quad x_k \in [0, \frac{K}{M}], \\ 1 & \text{if} \quad x_k > \frac{K}{M}. \end{cases} \tag{4}$$

By the inspection of (4), we observe that flexibility makes students more selective across the board. Namely, higher flexibility ($K \geq K'$ and $M \leq M'$) implies that a representative student who responded with a probability π' will now respond with a lower probability $\frac{K'}{K} \frac{M}{M'} \pi' < \pi'$; the drop in the response rate is *proportional*, and the proportion is identical irrespective of which particular score is offered.

The response rate π_k is a useful measure of *self-selection*, but reflects only the side of students. A measure that captures both sides (offers and responses) is the difference between the post mean (average realized score) and the ex ante mean (average offered score, here equal to one). To get this measure of self-selection, express first variance of the interim scores,

$$\sigma^2 := \int_0^\infty (x-1)^2 dF(x,K,M) = \int_0^\infty x^2 dF(x,K,M) - 1.$$

Clearly, the variance of the interim scores σ^2 exactly measures the self-selection of average realized grades. Simply put, the ex post mean exceeds the ex ante mean by the ex ante variance σ^2,

$$\int_0^\infty x dG(x,K,M) = \int_0^\infty x^2 dF(x,K,M) = 1 + \sigma^2.$$

Since ex ante variance is increasing in flexibility (Myerson 1993, p. 861), and the interim mean is fixed by the mean-score constraint, we conclude that the ex post mean is increasing if the curriculum becomes more flexible. This also helps us to derive the student's ex ante preferences over flexibility: If the students are risk-neutral over the scores and market values of scores are exogenous to the curriculum structure, then *the students prefer as flexible curricula as possible.*

To shed light on the differences between interim and ex post scores, Figure 1 in Panel (c) illustrates the ex post distributions for $K = 3$; these ex post distributions are comparable with the corresponding interim distributions in Panel (a). In addition, Example B.3 in the Appendix demonstrates that self-selection can be strong enough such that the initially positively skewed distribution changes into a negatively skewed distribution; interestingly, the switch occurs whenever only a single course has to be taken.

5 Top-score caps

The key lesson of the previous section is that more competitive curricula motivate instructors to provide larger maximal scores. Excessive top scores can be regulated directly, namely by imposing a cap on scores. For example, Princeton University recommended in 2004 that no department give more than 35% A grades overall, only to have it reversed 10 years later (Lehr, 2016).

5.1 A Captain Lotto game

To analyze grading competition in the presence of caps that bind the values of the random variables from above, we borrow lessons from a bilateral *Cap*tain Lotto game in Hart (2016). For a special case of two players (i.e., a curriculum $(M, K) = (1, 2)$), Hart (2016, Proofs to Proposition 1 and Theorem 2) shows that the symmetric equilibrium strategy is a lottery over the (uniform) distribution from the unrestricted game truncated to an interval $[0, t]$ and a

Dirac measure on the point of the cap c, where $0 < t < c$. To construct this strategy (i.e., an i.i.d. random variable X_k) intuitively, suppose a player decides to use the distribution $F(x)$ from the unrestricted case. Then, however, the probability mass $1 - F(c)$ remains unused. This mass is optimally shifted to the atom of $x = c$; yet, the resource constraint (in our case the mean-score constraint) is now relaxed, and to make it binding, additional mass from the interval $[0, c)$ can be shifted into $x = c$. Optimally, only mass from the upper part of the interval is shifted, hence intermediate offers on an interval (t, c) are no longer offered.

In our paper, we extend this result to any feasible curriculum structure (M, K). The main difference is that the underlying distribution from the unrestricted game, in our case denoted $F(x, K, M)$, is not necessarily uniform. Proposition 3 derives a unique threshold value for the empty interval and proves that the truncation described above indeed survives in the equilibrium.[7]

Proposition 3. *In the presence of an ex ante mean-score ceiling and a binding cap on the scores,* $1 \leq c < \frac{K}{M}$*, a symmetric equilibrium cumulative distribution of the offers is as follows:*

$$\hat{F}(x, K, M) = \begin{cases} F(x, K, M) & \textit{if} \quad x \in [0, t], \\ F(t, K, M) & \textit{if} \quad x \in (t, c), \\ 1 & \textit{if} \quad x \in [c, \infty), \end{cases}$$

where the threshold $t \in [0, c)$ *is given as a unique solution to*

$$\int_0^t x \, dF(x) + (1 - F(t))c = 1. \tag{5}$$

Notice that the existence of the empty interval, $x_k \in (t, c)$, is self-fulfilling immediately. An instructor who sets his offer in an empty interval knows that all other offers are situated below or above the empty interval. Hence, increasing his offer within this interval cannot change any student's decision, and the response rate is constant. This motivates the instructor to move the offer away from the empty interval.

Additionally, notice that the existence of the cap has no effect on the responses to the offers that are given in the equilibrium. Thus, in the equilibrium, *caps change only behavior of instructors but not of students.* Formally, for any x_k on a support of X_k (i.e., $x_k \in [0, t] \cup \{c\}$), *the response rate remains linear in* x_k, $\pi_k(x_k) = B(K - 1, M - 1, 1 - F(x_k)) = x_k \frac{M}{K}$. Thus, like in the unrestricted case, the equilibrium looks such that the response rate to the given offers is linear in the offers. Differences are only for offers that are actually not given in the equilibrium; students are now less responsive to the intermediate offers and more responsive to forbidden offers. Figure 2 illustrates.

[7] If the cap is extremely binding, $c < 1$, then the resource constraint is loose for any admissible distribution. In such a case, the equilibrium distribution obviously degenerates to the atom at $x = c$, and the equilibrium threshold is in a corner, $t = 0$.

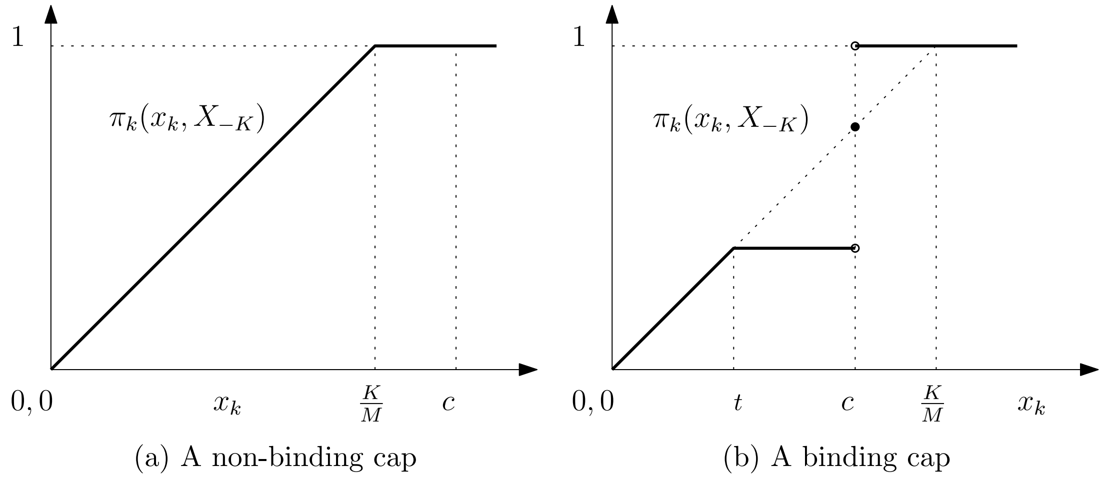


Figure 2: The students' response rate

5.2 Realized scores

How does the top-score norm change the shape of the distribution of realized offers, denoted $\hat{G}(x, K, M)$? As response rates are invariant to the top-score norm, all effects are due to a change in the offers, $\hat{F}(x, K, M)$. In other words, the cap affects the distribution of realized scores in the similar way like the distribution of offers,

$$\hat{G}(x, K, M) = \begin{cases} G(x, K, M) & \text{if} \quad x \in [0, t], \\ G(t, K, M) & \text{if} \quad x \in (t, c), \\ 1 & \text{if} \quad x \in [c, \infty), \end{cases}$$

We can now check how a cap affects (i) the interval of unused scores (empty interval), (ii) the share of top achievers, and (iii) the average realized score. First, with a more restrictive cap, the empty interval begins with a lower threshold and hence extra intermediate scores are no longer present. In a word, *a restriction on the top scores translates into a restriction on the intermediate scores.*[8] Second, a more restrictive cap (a lower c and accordingly also a lower t) increases the share of the top achievers, which is

$$[1 - F(t)]c\frac{M}{K} = \left[1 - \int_0^t x\, dF(x)\right] \frac{M}{K} = [1 - G(t)]\, \frac{M}{K}.$$

Third, in line with intuition, a more restrictive cap depresses a realized mean score. To see why, we use that the threshold t is non-decreasing in c, hence the realized mean is also non-decreasing in c. (Formally, it is increasing whenever the cap is binding and constant when the cap is non-binding.) The realized mean is, using the definition of the threshold in (5),

[8] That the threshold t is non-decreasing in c is given by the implicit-function theorem imposed on (5).

$$\int_0^\infty x\, d\hat{G}(x) = \int_0^t x^2 dF(x) + F(c)(1-F(t))c = \int_0^t (x+F(c))x\, dF(x).$$

5.3 Summary

To summarize, we may draw three lessons from the existence of the top-score caps. First, binding caps motivate instructors to submit highly compressed offers to students with the highest level of fit. The lower the cap, the larger the share of students who receive such top offers. By concentrating offers at the top, *caps resemble rigid curricula*, and caps and rigidity reinforce each other as complements. Second, caps may *reduce informative value of top scores* as the most fitting courses are indistinguishable to the students. At the top, a matching inefficiency occurs whenever a student receives at least $M+1$ top offers and consequently has to randomly eliminate some of the redundant top offers. Third, and perhaps most importantly, *caps promote inequality* in the following sense: The instructors split the students into two disjoint groups, where the share of 'top achievers' who are separated from the rest of the pack increases with a more restrictive cap. A more restrictive cap also motivates the instructors to abolish many intermediate scores and thus 'laggards' are offered lower scores on average.

6 Normalized mean-score ceiling

An indirect way to regulate the excessive top scores is to normalize the scores. This is achieved by tightening the mean-score grading norm, i.e., by lowering the ceiling for the mean. Formally, in this section we suppose that the school maintains the mean-score norm but reduces the mean-score ceiling such that all scores, including the excessive scores, exactly fall into a pre-determined range. How such *normalization* (or re-parametrization) affects the interim and ex post scores is illustrated in Panels (b) and (d) of Figure 1.

If re-parametrization occurs ex post based on the observed distributions, there is natural concern about the manipulability of the scheme by individual instructors. However, manipulability is negligible if a single instructor has a negligible effect on the inputs used for re-parametrization. For instance, the re-parametrization procedure may use the median values of the top scores of instructors instead of the maximum of top scores; then, a single instructor's deviation from a symmetric equilibrium profile has no effect on re-parametrization.

6.1 Normalized interim scores

In the first step, we focus on the normalized interim scores. The score distributions $H(y, K, M)$ are trivially obtained by normalizing the distribution $F(x, K, M)$ with a support $x \in [0, \frac{K}{M}]$ into a distribution $H(y, K, M)$ with a normalized support $y \in [0, 1]$ such that $y = \frac{M}{K}x$,

$$H(y, K, M) := F\left(y\tfrac{K}{M}, K, M\right).$$

Our key result is that the normalized H-distributions can be ranked by first-order stochastic dominance with respect to K and M. Namely, Proposition 4 shows that the score distribution for a less flexible curriculum first-order stochastically dominates the distribution with a more flexible curriculum.

Proposition 4 (Rigidity increases offers)**.** *Let* (M', K') *be a less flexible curriculum than* (M, K)*. Then, for any* $y \in (0, 1)$*,*

$$H(y, K', M') < H(y, K, M).$$

This dominance property can be alternatively stated as the property of the quantile (inverse distribution) function to H-distribution function. Namely, for any percentile $p \in [0, 1]$, let $Y(p, K, M)$ be the quantile in the normalized distribution satisfying $H(Y(p, K, M), K, M) = p$. When inserted into the implicit solution in (2), the quantile is

$$Y(p, K, M) = B(M - 1, K - 1, 1 - p).$$

Exploiting well-known properties of the binomial cumulative distribution function, we observe that for any percentile p, an increase in flexibility decreases the respective quantile $Y(p, K, M)$. We demonstrate for an increase from K to $K + 1$ and for a decrease from M to $M - 1$:

$$Y(p, K, M) - Y(p, K + 1, M) = B(K, M, p) - B(K - 1, M, p) < 0$$

$$Y(p, K, M - 1) - Y(p, K, M) = B(K, M - 1, p) - B(K, M, p) < 0$$

In other words, the H-distribution function moves to the left with an increase in flexibility. As the H-distribution function is non-decreasing y, it implies that it moves up with flexibility.

6.2 Normalized ex post scores

Importantly, the first-order dominance relation in Proposition 4 extends to the ex post (realized) scores. We know that the distribution of ex post scores $G(x)$ is such that $g(x) = xf(x)$; recall $G(x, K, M)$ denotes the distribution parameterized by K and M. Like for the interim scores, this distribution characterizes a cumulative distribution of normalized scores, denoted $R(y, K, M)$, and its probability density function, $r(y, K, M)$, again such that $y = x\frac{M}{K}$:

$$R(y, K, M) := G\left(y\tfrac{K}{M}, K, M\right).$$

Proposition 5 (Rigidity increases scores)**.** *Let* (M', K') *be a less flexible curriculum than* (M, K)*. Then, for any* $y \in (0, 1)$*,*

$$R(y, K', M') < R(y, K, M).$$

By Proposition 5, *any percentile* of the distribution $R(y, K, M)$ is decreasing in flexibility. For example, if $\tilde{y}(K, M)$ denotes the median percentile, $R(\tilde{y}(K, M), K, M) = \frac{1}{2}$, then $\tilde{y}(K, M)$ is decreasing in K and increasing in M. In contrast, Appendix B.2 shows that the median of the initial (non-normalized) distribution $G(x, K, M)$, denoted $\tilde{x}(K, M)$, may decrease but also increase in flexibility.

6.3 Students' preferences over normalized curricula

The dominance property has clear implications for students' preferences over curricula. If the students' valuations of realized scores are exogenous to the curriculum structure, then having a more flexible normalized curriculum implies lower scores. Given the dominance property, this effect holds *ex post for all types* of students, irrespective of the levels of fit. This reverts the observation from the baseline case in which students with linear valuations always benefit from more flexible curricula; here, the result generalizes beyond shape of the valuation as the preference holds both ex ante and ex post.

The assumption that the valuations of scores are exogenous is obviously simplifying as firms in the long term account for changes in score distribution, hence the market valuation at least partly offsets the normalization. Still, grading rules are often imperfectly observable (Zubrickas, 2015). Also, there is evidence that students perceive re-parameterizations such as decreases in the grade ceiling costly. For example, Gorry (2017) observes a drop in the students' satisfaction, manifested in worse teaching evaluations, after a grade ceiling dropped.

So far, we have assumed that a student who assesses various curricula maximizes the received scores (or the average of the received scores if the curricula differ in the number of course requirements). From that perspective, a high 'fit' to a course is just instrumental to receiving a high score. In addition to the market valuation of the high scores, we may consider broader benefits of a high fit; for instance, a student consumes higher intrinsic benefits, or exerts less effort if the fit is high. These extra benefits imply that the average realized fit is part of a students objective. As the average realized fit increases with a larger K (a richer menu) and a lower M (higher selection), the existence of these extra benefits sway the preferences back towards curriculum flexibility.

7 Ex post mean-score ceiling

Among potentially unwelcome consequences of flexibility is the pronounced selectivity of the students; we know that all students decrease their response rates if a curriculum becomes more flexible, and the drop (measure proportionally) is identical at each level of the offer. As a consequence, the mean ex post score even deviates more from the mean interim score. In this section we analyze a radical policy through which a school, concerned about the score-inflating effects of self-selection, eliminates the self-selection bias of the mean-score norm altogether.

To eliminate the self-selection biases altogether, the grading norm now has to apply to the self-selected groups of students, namely it must be evaluated ex post. In this section, we show that, as a consequence, the instructors no longer compete. As a result, all positive informative effect of scores resulting from benefits of sorting are wiped out. Specifically, Proposition 6 demonstrates that a symmetric equilibrium in pure strategies (i.e., an identical exam score offered to all electives shoppers) is an equilibrium with the ex post mean constraint. For completeness, we check that this profile is not an equilibrium with the ex ante mean constraint.

We will obtain the result clearly in a discrete setting, for any support of scores that is sufficiently fine-grained. Formally, an exam score is a non-negative integer, $x \in \mathbb{N}_0$. The ex ante constraint in a discrete setting writes $\sum_{x \in \mathbb{N}_0} x f(x) \leq z$; the ex post constraint writes $\sum_{x \in \mathbb{N}_0} x g(x) \leq z$. We let the required ex post mean score $z \in \mathbb{N}$ to be very large, $z \gg 0$, so that the support grid is fine enough. Next, let F^z be a degenerate cumulative distribution function that offers an identical z to all students,

$$f^z(x) = \begin{cases} 1 & \text{if} \quad x = z, \\ 0 & \text{if} \quad x \neq z. \end{cases}$$

Notice that for both constraints, only a single strategy profile is admissible, symmetric ($F_1 = \cdots = F_K$), and in pure strategies, namely $F^z := F_1 = \cdots = F_K$. Proposition 6 checks whether this profile is an equilibrium profile under the ex post constraint and the ex ante constraint.

Proposition 6 (Lack of competition)**.** *Awarding identical scores, $(F^z, \ldots, F^z)$, is an equilibrium strategy profile under the ex post mean-score ceiling but not under the ex ante mean-score ceiling.*

Intuitively, when instructors offer non-discriminative scores and thus avoid competing with each other, students spread equally and some offers are rejected. If an instructor slightly increases her offers to many students, she receives positive responses to those offers with certainty. This greatly boosts class size. The ex ante mean constraint is now slightly exceeded; to compensate, it is sufficient for the instructor to give to a few students a very low offer. For the ex post constraint, however, this compensation procedure is not feasible as low offers are never accepted, and therefore do not enter the ex post scores distribution. Therefore, the instructor is locked to the average; she cannot propose larger scores without violating the ex post mean.

8 Conclusions

High educational standards can be maintained and enforced only through appropriate grading practices. However, grading in higher education institutions is to a great extent the responsibility of individual instructors, and their objectives may differ from the maintenance of standards. For elective courses in particular, instructors are motivated to engage in grading competition whenever class size or students' satisfaction serve as the instructors' performance indicators.

To curb lenient grading associated with grading competition, schools tend to adopt various grading norms, among which the most widely analyzed is a constraint on the *average grades*. In this paper, we theoretically analyze how this grading norm combined with other grading norms may influence grading, and thus the enforcement of educational standards. Our framework cleanly identifies the effects of grading norms on the distribution of grades that abstract from the effects on effort; how the grading norms affect the effort has been studied extensively elsewhere (Becker and Rosen, 1992; Dubey and Geanakoplos, 2010; Andreoni and Brownback, 2017; Paredes, 2017; Brownback, 2018).

Technically, we apply Continuous General Lotto games into a setting with ex ante homogenous students and suppose the school regulates the mean course score. In addition, we suppose that the school doesn't punish favorable self-selection as it calculates the mean for a representative sample of students. We primarily investigate the effects of having a more intensive grading competition. With a more flexible curriculum, the gap in scores widens; the best students will receive higher scores whereas worse students will receive lower scores. In a word, inequality increases.

The very large scores awarded to the best students and the score inequality are two major concerns associated with flexibility. We analyze three scenarios of how the incentive to provide excessive scores is counteracted while the mean-score norm is preserved. All scenarios illustrate that the complex regulation of both mean and top scores comes at a cost. By imposing caps on the scores, the top course scores are less informative for both students and employers, and moreover the students get separated into two disjoint groups; a new kind of inequality emerges in the class. If the mean ceiling is lower for more flexible curricula, all types of students receive lower scores and thus will demand rigid curricula even at the cost of low variety. If favorable self-selection is eliminated by imposing the mean-score norm on the self-selected students, sorting of students into courses is distorted,

In particular, our analysis uncovers an inequality-promoting channel associated with grading standards. In the literature, grading standards are often measured by how restrictive a school is on average (Betts and Grogger 2003; Figlio and Lucas 2004). In our model, this restriction is directly captured by the level of the mean ceiling. In the absence of the cap on the scores, higher standards (a more stringent mean-score restriction) decrease received scores proportionally, hence inequality is not affected. Yet, if the scores are capped from above, the drops at the top are less pronounced, and inequality grows. This result reveals an inequality-promoting channel that works independently on the channel on effort, and corresponds to observations that exogenously measured achievement rises more for students near the top of the achievement distribution rather than for the students near the bottom (Betts and Grogger, 2003).

Our approach also suggests that admissible signal technologies of the schools may be effectively limited by the existence of decentralized grading practices; noise to ability always has to have a specific equilibrium structure, where the structure depends on the grading norms as well as curriculum structure. Thus, the literature on the optimal signal technology in the context of educational markets (Ostrovsky and Schwarz, 2010; Popov and Bernhardt, 2013; Boleslavsky and Cotton, 2015; Harbaugh and Rasmusen, 2018) may benefit from incorporating lessons from decentralized grading explicitly into the analysis.

In terms of policy consequences, we essentially illustrate that addressing grade inflation through grading norms generates nuanced effects on scores and grade distribution. In particular, we observe unintended consequences of grading norms in terms of inequality (division of the class into two separate groups, achievers and laggards), rigidity (strategic support of rigid curricula that more often give relatively high scores), and inefficiency (inefficient sorting of students into courses).

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A Proofs

A.1 Proof of Proposition 1

Proof. Any admissible $F_k(x)$ is restrained by the mean-ceilibg policy that normalizes the average signal to be exactly one. We multiply this constraint by $\frac{M}{K}$, and insert it into the objective in into (7),

$$\int_0^\infty x\frac{M}{K}\,dF_k(x)dx = \frac{M}{K} = \int_0^\infty B(M-1,K-1,1-F(x))dF_k(x). \tag{6}$$

The only solution which yields $B(M-1,K-1,1-F(x))$ linear in x (as in Myerson, 1993, or Hart, 2008) is $B(M-1,K-1,1-F(x,K,M)) = x\frac{M}{K}$. It is immediate to see that the distribution in (2) is indeed the equilibrium solution. From the linearity of the conditional probability, notice that $B(M-1,K-1,1-F(x,K,M)) = x\frac{M}{K}$ if $x \in [0,\frac{K}{M}]$, and $B(M-1,K-1,1-F(x,K,M)) = 1 < x\frac{M}{K}$ if $x > \frac{K}{M}$. Therefore, if the instructor deviates to another $F_k(x)$ that complies with the grade means restriction, $\int_0^\infty x dF_k(x) = 1$, her expected payoff cannot increase,

$$\Pi_k(X_1,\dots,X_K) = \int_0^\infty B(M-1,K-1,1-F(x))dF_k(x) \le \int_0^\infty x\frac{M}{K}\,dF_k(x) = \frac{M}{K}. \tag{7}$$

□

A.2 Proof of Proposition 2

Proof. To obtain the distribution of ex post scores, denoted $G(x,K,M)$, recall $A_k \in \{0,1\}$ is the random indicator variable denoting the decision to take the course k. Probability of taking the course conditional on $X_k = x$ is $\Pr(A_k = 1|X_k = x) = B(K-1,M-1,1-F(x))$. The joint probability is $\Pr(A_k = 1 \cap X_k = x) = B(M-1,K-1,1-F(x))f(x)$. We use that the marginal density of $A_k = 1$ is the expected payoff, $\Pr(A_k = 1) = \Pi_k(X_1,\dots,X_K) = \frac{M}{K}$. Then, the conditional probability of $X_k = x$ on $A_k = 1$ is

$$g(x) = \Pr(X_k = x|A_k = 1) = \frac{\Pr(A_k = 1 \cap X_k = x)}{\Pr(A_k = 1)} = B(M-1,K-1,1-F(x))f(x)\frac{K}{M} = xf(x).$$

□

A.3 Proof of Proposition 3

Proof. First, see that t is a unique solution, because LHS of (5) is decreasing in t for $t < c$ (i.e., its derivative is $f(t)(t-c) < 0$), while RHS is constant in t. Notice also that the distribution is defined only for $c \ge 1$; if $c < 1$, then we don't have a single-crossing as LHS evaluated at

$t = 0$ is below RHS. For $c < 1$, the equilibrium distribution is degenerate, $\hat{F}(x, K, M) = 0$ if $x < c$ and $\hat{F}(x, K, M) = 1$ if $x \geq c$.

Second, the resource constraint is satisfied, because the $\hat{F}$-distribution is a lottery over (μ', c) with probabilities $(F(t), 1 - F(t))$, where μ' is a mean of the truncated F-distribution, $\mu' = \frac{1}{F(t)} \int_0^t x f(x) dx$. Then, the mean of the $\hat{F}$-distribution is exactly equivalent to LHS in (5), and this yields that the resource constraint is just met,

$$F(t)\mu' + (1 - F(t))c = \int_0^t x f(x) dx + (1 - F(t))c = 1.$$

Third, consider any deviation of an instructor k. Notice immediately that if a deviation imposes positive probability mass on any $x_k \in (t, c)$, then there exists a more profitable deviation which (i) shifts the probability to $x_k' \in (t, x_k)$, and (ii) exploits the relaxed constraint such that it shifts the probability mass from some low x to a higher x. Simply, all shifts in (i) will be exploited as such shifts relax the constraint without affecting the expected payoff Π_k, given that π_k is flat in the interval $x \in (t, c)$. Therefore, we will consider only deviations to distributions which impose zero probability on (t, c).

We insert the resource constraint of F_k, $\int_0^t x f_k(x) dx + (1 - F_k(t))c \leq 1$, into the expected payoff upon a deviation to F_k:

$$\begin{aligned}
\Pi_k &= \int_0^t B(M-1, K-1, 1-F(x)) f_k(x) dx + [1 - F_k(t)] B(M-1, K-1, 1-F(c)) \\
&= \int_0^t x \frac{M}{K} f_k(x) dx + [1 - F_k(t)] c \frac{M}{K} \leq [1 - (1 - F_k(t))c] \frac{M}{K} + [1 - F_k(t)] c \frac{M}{K} = \frac{M}{K}
\end{aligned}$$

Hence, the deviation doesn't pay off, $\Pi_k \leq \frac{M}{K}$. □

A.4 Proof of Proposition 4

Proof. To start with, recall $F(x, K, M)$ is the distribution parameterized by K and M. This distribution characterizes a normalized cumulative distribution $H(y, K, M)$ (and its probability density function $h(y, K, M)$) such that $y = x \frac{M}{K}$, where

$$H(y, K, M) = F\left(y \tfrac{K}{M}, K, M\right).$$

Notice that $F(x, K, M)$ has a support $[0, \frac{K}{M}]$, whereas any normalized distribution $H(y, K, M)$ has a support $[0, 1]$.

In the first step, we exploit the implicit form of the equilibrium cumulative distribution. Throughout the paper, we will slightly abuse the problem by switching from discrete K to continuous K; clearly, the switch is irrelevant as long as the first derivatives are strictly monotonic. Namely, by derivating $B(m, n, p) := B(M-1, K-1, 1-H(y, K, M)) = y$ with respect to K (i.e., holding M as well as the normalized score y constant), we obtain

$$B_n(m, n, p) - B_p(m, n, p) H_K(y, K, M) = 0.$$

Notice we use subscripts for partial derivatives. We translate the monotonic property of the discrete binomial distribution $B(m, n+1, p) - B(m, n, p) < 0$ into its continuous form, $B_n(m, n, p) < 0$. Using also that $B_p(m, n, p) < 0$ holds, we obtain that the value of H-distribution is increasing in K:

$$H_K(y, K, M) > 0.$$

In exactly the same way, we obtain $H_M(y, K, M) < 0$. These properties are sufficient and necessary for a comparison of distributions in terms of the first-order stochastic dominance.

□

A.5 Proof of Proposition 5

Proof. We concentrate only on properties of the distribution of normalized scores $R(y)$. To prove that $R(y, K, M)$ is monotonic in K and M, we proceed in three steps. First, we express the density of the ex ante normalized scores $h(y)$:

$$h(y, K, M) = \frac{dF(y\frac{K}{M}, K, M)}{dy} = \frac{dF(y\frac{K}{M}, K, M)}{dx}\frac{dx}{dy} = f(y\tfrac{K}{M}, K, M)\frac{K}{M}.$$

We enter into the density $r(y, K, M)$:

$$r(y, K, M) = \frac{dG(y\frac{K}{M}, K, M)}{dy} = \frac{dG(y\frac{K}{M}, K, M)}{dx}\frac{dx}{dy} = g(y\tfrac{K}{M})\frac{K}{M} = f(y\tfrac{K}{M}, K, M)y\frac{K^2}{M^2} = h(y, K, M)y\frac{K}{M}.$$

Second, we express how the density changes in K:

$$r_K(y, K, M) = \frac{y}{M}h(y) + \frac{yK}{M}h_K(y, K, M).$$

The first component is positive. We prove that the second component is also positive. Namely, we apply Leibniz rule on the ex ante scores and our observation on the marginal effect of K on the density $h(y, K, M)$,

$$\int_0^{\hat{y}} h_K(y, \hat{K}, M)dy = \frac{\partial \int_0^{\hat{y}} h(y, \hat{K}, M)dy}{\partial K} = H_K(\hat{y}, \hat{K}, M) > 0.$$

Since $H_K(\hat{y}, \hat{K}, M) > 0$ for any $(\hat{y}, \hat{K})$, then $h_K(y, K, M) > 0$ for all $(\hat{y}, \hat{K}, M)$. As a consequence, also $r_K(y, K, M) > 0$ for any K. Finally, we impose Leibniz rule on the ex post scores to observe

$$R_K(\hat{y}, \hat{K}, M) = \frac{\partial \int_0^{\hat{y}} r(y, \hat{K}, M)dy}{\partial K} = \int_0^{\hat{y}} r_K(y, \hat{K}, M)dy > 0.$$

Obtaining $R_M(\hat{y}, \hat{K}, M) < 0$ is by analogy. □

A.6 Proof to Proposition 6

Proof. Take a course k. If this course offers a realization $X_k = x$ to a representative student and all other courses stick to F^z, then the response rate (the event is denoted $A_k \in \{0, 1\}$) is

$$\pi(x) := \Pr(A_k = 1 \mid x) = \begin{cases} 0 & \text{if} \quad x < z, \\ \frac{M}{K} & \text{if} \quad x = z, \\ 1 & \text{if} \quad x > z. \end{cases}$$

The proof now analyzes deviations under each of the two constraints separately. For each deviation, we use that the probabilities sum up to one, and also that the respective constraint must be met.

Ex ante mean constraint. We show that the instructor of course k can improve her payoff if the support is fine-grained enough. The idea is close to 'concavification' in Kamenica and Gentzkow (2010); namely, if we construct a concave closure of $\pi(x)$, denoted as $c(x)$, then the instructor can obtain the payoff $c(z)$, where $\lim_{z\to\infty} c(z) = 1$. In other words, for a sufficiently fine-grained support, the instructor can deviate such that almost all students attending the class in the shopping period will eventually enroll in the class.

The deviation will have the following form: The instructor will redistribute some probability mass to points $x = 0$ (low offer) and $x = z+1$ (high offer), $df_k(0) > 0$ and $df_k(z+1) > 0$. We suppose the ex ante mean constraint is met,

$$f_k(0)0 + (1 - f_k(0) - f_k(z+1))z + f_k(z+1)(z+1) = z,$$

which implies

$$f_k(0) = \frac{f_k(z+1)}{z}.$$

The marginal effect on the payoff is

$$\frac{d\Pi_k}{df_k(z+1)} = \frac{df_k(0)}{df_k(z+1)}\left(-\frac{M}{K}\right) + 1 - \frac{M}{K} = \frac{1}{z}\left(-\frac{M}{K}\right) + 1 - \frac{M}{K}.$$

If the support becomes fine-grained enough, i.e., for a sufficiently large z, the payoff increases with the redistribution,

$$\lim_{z\to\infty} \frac{d\Pi_k}{df_k(z+1)} = 1 - \frac{M}{K} > 0.$$

Ex post mean constraint. First, we will characterize any $F_k(x)$ by two key probabilities, $f_k(z)$ and $f_k(z^+) := \sum_{x=z+1}^{\infty} f_k(x)$. If all other courses stick to F^z, then the instructor's payoff is

$$\Pi_k := \Pr(A_k = 1) = f_k(z)\pi(z) + f_k(z^+)\pi(z+1) = f_k(z)\frac{M}{K} + f_k(z^+).$$

In the symmetric profile, $f_k(z) = 1$ and $f_k(z^+) = 0$. Clearly, a necessary condition for an increase in payoff Π_k is $df_k(z^+) > 0$. We will show that this violates the ex post constraint, and therefore the instructor's optimal distribution is $F_k(x) = F^z(x)$.

First, see that the distribution of the realized scores in the course k is

$$g_k(x) := \Pr(x \mid A_k = 1) = \frac{\Pr(A_k = 1 \mid x)\Pr(x)}{\Pr(A_k = 1)} = \frac{\pi(x) f_k(x)}{f_k(z)\frac{M}{K} + f_k(z^+)}.$$

The ex post mean satisfies

$$\sum_{x \in \mathbb{N}^0} x g_k(x) = \frac{z f_k(z)\frac{M}{K}}{f_k(z)\frac{M}{K} + f_k(z^+)} + \sum_{x=z+1}^{\infty} \frac{x f_k(x)}{f_k(z)\frac{M}{K} + f_k(z^+)} \geq \frac{z f_k(z)\frac{M}{K} + (z+1) f_k(z^+)}{f_k(z)\frac{M}{K} + f_k(z^+)}.$$

Now, if consider a deviation $df_k(z^+) > 0$, then in the new distribution, we observe $f_k(z^+) > 0$. Therefore, its ex post mean violates the ex post constraint,

$$\sum_{x \in \mathbb{N}^0} x g_k(x) \geq \frac{z f_k(z)\frac{M}{K} + (z+1) f_k(z^+)}{f_k(z)\frac{M}{K} + f_k(z^+)} > \frac{z f_k(z)\frac{M}{K} + z f_k(z^+)}{f_k(z)\frac{M}{K} + f_k(z^+)} = z.$$

□

B Examples

B.1 Closed-form solution

The existence of a binomial function affects tractability of the problem. To begin with, once students either pick a single course ($M = 1$) or avoid a single course ($M = K - 1$), closed-form solutions are straightforward:

$$F(x, K, 1) = \left(\tfrac{x}{K}\right)^{\frac{1}{K-1}}$$
$$F(x, K, K-1) = 1 - \left[1 - (K-1)\tfrac{x}{K}\right]^{\frac{1}{K-1}}$$

In the language of electoral systems (Myerson, 1993), the two cases describe campaign promises for plurality and antiplurality voting systems, where a campaign promise is a distribution of a fixed cake in a population of voters. Any other case becomes rather complicated, even for a small K. For instance, the lowest K which generates a case different from plurality and antiplurality is $K = 4$, where $M = 2$. Then,

$$B(3, 1, p) = (1-p)^3 + 3p(1-p)^2 = (1-p)^2(1+2p) = F^2(x)[3 - 2F(x)] = \frac{x}{2}.$$

Using numerical tools, we obtain an offer function

$$F(x, 4, 2) = -\frac{1}{4}(1 + i\sqrt{3})(\sqrt{x^2 - 2x} - x + 1)^{1/3} - \frac{1 - i\sqrt{3}}{4(\sqrt{x^2 - 2x} - x + 1)^{1/3}} + \frac{1}{2},$$

on $x \in [0, 2]$. Its probability distribution function has a U-shape, with many offers being very small and many offers being very large,

$$f(x, 4, 2) = \frac{(1 - i\sqrt{3})\left(\frac{x-1}{\sqrt{x^2-2x}} - 1\right)}{12(\sqrt{x^2 - 2x} - x + 1)^{\frac{4}{3}}} - \frac{(1 + i\sqrt{3})\left(\frac{x-1}{\sqrt{x^2-2x}} - 1\right)}{12(\sqrt{x^2 - 2x} - x + 1)^{\frac{2}{3}}}.$$

The case of $K = 4$ serves as a clean illustration of the property found in Proposition 4: For a small M, the distribution is skewed to the right with a higher frequency of small offers (bad scores); for an intermediate M, there are both very small offers and very large offers (separation); and, for a large M, the distribution is skewed to the left with a higher frequency of large offers (good scores).

To obtain a closed-form solution in general, we may represent the binomial function as a regularized incomplete beta function,

$$B(M - 1, K - 1, 1 - F(x, K, M)) = (M - 1)\binom{K - 1}{M - 1} \int_0^{F(x,K,M)} t^{M-2}(1 - t)^{K-M} dt.$$

Hence, its first derivative w.r.t. to x must be constant. However, when solving the integral $\int t^{M-2}(1 - t)^{K-M} dt$, we obtain a product of an exponential function evaluated at x and the Gauss hypergeometric function evaluated at $F(x, K, M)$.

B.2 Median score is not monotonic in flexibility

Suppose we increase the requirements from $M = 1$ to $M = 2$; the number of electives is fixed and satisfies $K \geq 3$. Denote the median value $\tilde{x}(K, M)$, hence in this specific case $\tilde{x}(K, 1)$ and $\tilde{x}(K, 2)$. Given than $F(x) = \frac{1}{2}$ characterizes the median, we can easily characterize the median values, and find that increasing rigidity from $M = 1$ to $M = 2$ (i.e., limiting flexibility from its maximum) in fact *increases* the median value of noise:

$$\frac{\tilde{x}(K, 2)}{\tilde{x}(K, 1)} = \frac{B(K - 1, 1, \frac{1}{2})}{2B(K - 1, 0, \frac{1}{2})} = \frac{K}{2} > 1.$$

In contrast, suppose we increase requirements from $M = K - 2$ to $M = K - 1$; again, the number of electives is fixed and satisfies $K \geq 3$. Analyze the ratio of the corresponding median values:

$$\frac{\tilde{x}(K, K - 1)}{\tilde{x}(K, K - 2)} = \frac{K - 2}{K - 1} \cdot \frac{B(K - 1, K - 2, \frac{1}{2})}{B(K - 1, K - 3, \frac{1}{2})} = \frac{K - 2}{K - 1} \cdot \frac{1 - B(K - 1, 0, \frac{1}{2})}{1 - B(K - 1, 1, \frac{1}{2})} = \frac{K - 2}{K - 1} \cdot \frac{2^{K-1} - 1}{2^{K-1} - K}.$$

We observe that $\tilde{x}(K, K - 1) > \tilde{x}(K, K - 2)$ for $3 \leq K \leq 5$, and $\tilde{x}(K, K - 2) > \tilde{x}(K, K - 1)$ for $K \geq 6$. In other words, increasing rigidity to its maximum *decreases* the median offered score if the number of the electives is sufficiently large.

B.3 For $M = 1$, self-selection turns positive skew into negative skew

We briefly show that self-selection greatly impacts scores distribution if the students pick just a single course. Consider $M = 1$. The probability density function of the ex ante grades is

$$f(x, K, 1) = \frac{1}{K-1}\left(\frac{1}{K}\right)^{\frac{1}{K-1}} x^{\frac{2-K}{K-1}}.$$

The probability density of the ex post grades is

$$g(x, K, 1) = xf(x, K, 1) = \frac{1}{K-1}\left(\frac{1}{K}\right)^{\frac{1}{K-1}} x^{\frac{1}{K-1}}.$$

Clearly, $f(x, K, 1)$ is decreasing in x (median below mean; here also positive skew), whereas $g(x, K, 1)$ is increasing in x (median above mean; here also negative skew).

C Endogenous resources

In this extension, the resource constraint (mean-score cap) of an instructor is now

$$\int_0^\infty x \, df(x) \leq r.$$

The school allocates a single unit of resources for free (i.e., total test points), while any extra resources $r - 1$ (such as the instructor's attention) are costly to the instructor. The cost function $C(r)$ is continuous, increasing, convex, twice-differentiable, and to comply with our interpretation, it is normalized at $C(1) = 0$.

The instructor k sets (r_k, F_k) simultaneously; in other words, he is flexible enough to adjust the amount of resources in the electives-shopping period if necessary. We will again look for the symmetric equilibrium characterized by (r, F). Clearly, by inserting the symmetric equilibrium value of r into (2), the implicitly characterized distribution F is now simply

$$B(M-1, K-1, 1-F(x, K, M)) = x\frac{M}{Kr}. \tag{8}$$

To derive the equilibrium value of r, consider an instructor k's payoff upon a deviation to any r_k. By inserting (8) into the payoff and combing with the resource constraint, we obtain that extra resources increases the market share $\frac{M}{K}$ proportionally,

$$\Pi_k = \int_0^\infty B(M-1, K-1, 1-F(x, K, M))f(x)dx - C(r_k) = \frac{M}{K}\frac{r_k}{r} - C(r_k).$$

The equilibrium value of r is characterized by the F.O.C., which yields the following implicit function:

$$I(\tfrac{K}{M}; r) := C'(r) - \frac{M}{Kr} = 0.$$

What is the effect of curriculum flexibility on resources? As more flexible curricula are more competitive, there is a lower 'market share' for each instructor. Since extra resources increase the market share proportionally, the marginal benefit is also lower. Hence, flexibility somewhat unexpectedly *decreases* extra attention paid to the students. Formally, we use that $\frac{\partial I}{\partial K/M} = \frac{M^2}{K^2 r} > 0$ and $\frac{\partial I}{\partial r} = C''(r) + \frac{M}{Kr^2} > 0$ to obtain

$$\frac{dr}{dK/M} = -\frac{\frac{\partial I}{\partial K/M}}{\frac{\partial I}{\partial r}} < 0.$$

In spite of this effect, we will show that the result about top scores observed in Corollary 1 is robust to the existence of the option to provide extra resources. The upper bound for the scores is $\bar{x} := \frac{rK}{M}$, and hence the F.O.C. is simply

$$\bar{x} = \frac{1}{C'(r)}.$$

By derivating, the top score is indeed increasing in curriculum flexibility,

$$\frac{d\bar{x}}{dK/M} = -\frac{C''(r)}{C'^2(r)} \frac{dr}{dK/M} > 0.$$

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