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Contagious Defaults in Inter-bank Networks

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Abstract:
This paper investigates systemic risk and contagion processes in the inter-bank network using network science methods. The inter-bank network consisting 10 banks, similar to the real world inter-bank networks, is studied to understand the contagion process in the network regarding changes in the network structure, as well as changes in the characteristics of components. Simulations support the claim that heterogeneous networks are more resilient to contagious shocks, while systemic shocks are more problematic in homogeneous networks. The study also shows that more interconnections among banks could accelerate or block contagion proces depending on the structure of the network and seniority of debts in the inter-bank network as well.

JEL Classification: G01, G21
Keywords: Complex Networks, Systemic Risk, Contagion, Default Risk, Epidemic Modeling

1 Introduction

Within the past decades, globalization and free market increase inter-dependencies among financial institutions all around the world that bring us more complex financial structure. After the global financial crisis 2007-2008 that has played a significant role in the failure of different financial institutions, many concerns arose about the stability of complex financial system and possible future systemic failures among academia, industry, and regulators. Since then, a sizeable body of literature has concentrated on measuring systemic risk and preventing a systematic failure before it passes critical thresholds.

To assess the health of a financial system and also monitoring systemic risk in a financial system, one should know: first, how much each financial institution is vulnerable to risks against its entity, and second, the structure of the financial system (financial network), which means how individual components of the system are connected to each other.

Financial distresses such as *insolvency* and *illiquidity* that occur when an institution is not able to fulfill its financial obligations, usually lead the financial institution to failure or in other words $default$. A financial institution is called insolvent when its capital is zero, while the illiquidity of a financial institution occurs when there are not enough liquid assets to repay institution's liabilities (Moussa, 2011). However, insolvency may not lead to default, if the institution can obtain financing to fulfill its payment obligations.

Individual banks failures have led to a systemic crisis in the recent years and made the concept of *systemic risk* more important to both regulators and market participants. Systemic risk could be the result of an aggregate negative shock through the system affecting individual entities. The reason is usually a macroeconomic factor such as unemployment, inflation, or even a political crisis. Another reason for systemic risk is the *contagion* of the financial distress in the system. The default of an institution may spread throughout the system and leads other connected institutions to fail. For example, if a bank fails to repay its creditor on time and the loss is larger than the creditor's capital, it would cause both institutions to default, and it might spread among other institutions in the network.

The architecture of a financial network is another crucial issue regarding systemic risk. However, the complexity of global financial structure and the interdependence of financial institutions have been the most important challenges for the assessment of systemic risk. There are different approaches to answer the problems regarding systemic problems in financial systems, especially inter-bank networks. Among approaches to deal with these challenges, *network models* are one of the most advanced methods. The general concept of a network is quite simple and intuitive. A network describes a collection of nodes and links between them. Considering a financial system as a network allows us to import and use other network models from other fields, such as biology, physics, and engineering. One may find, by help of *network theory*, common ground in the systemic risk of financial networks and other networks, which might be helpful to monitor the stability of the system, by measuring critical thresholds in the network.

1.1 Networks

A network is a set of items including *nodes* and *edges*, which depict relationship and connection of network's components. Hence it is defined by a set of N nodes(vertices), $\{N\} = \{n_1, n_2, ..., n_N\}$, and a set of E edges (links), $\{E\} = \{e_1, e_2, ..., e_E\}$. For example, a network can be illustrated as a social network, biological network, financial network or any other set of connections between individuals. A node represents a component of the network which could be a person, a firm, a country, or any kind of entity based on the type of network. The link (edge) between two nodes represents the relationship between them. In an inter-bank network, for instance, a link could be mutual exposures between ban. Network science has been evolving in the recent years to study disease spreading, social networks, and cascade failures. These networks and related processes have common characteristics with the financial network and contagion in the financial systems, which could be useful to assess and monitor the systemic risk in the financial network. Therefore, one can say the ultimate goal of network models is to understand the behavior of complex systems.

1.2 Contagion Process in Networks

1.2.1 Epidemic Approach (SI Model)

Epidemic models and approaches try to capture the dynamics of propagation of disease within a community. They face questions regarding the spread of an event (e.g., disease or default) in different kinds of networks, such as social networks, population in a country, and inter-bank networks. Of the well known epidemic methods, *SI* (Susceptible-Infected), *SIR* (Susceptible-Infected-Recovered), *SIS* (Susceptible-Infected-Susceptible) are the most used in the literature. The details of the SI method used in this paper are provided as follows.

Kermack and McKendrick (1927) introduce a simple epidemic model to study disease spreading among population networks. Although it is somehow a naive method to investigate disease spreading or in general event spreading in a network, it is still a common used method in the scientific literature. In this two-stages method, there are two kinds of nodes in the network: susceptible and infected. Once one becomes infected, susceptible nodes become infected and the disease will spread in the entire network. The model can be written as:

$$\frac{dX}{dt} = \frac{\beta SX}{n}$$
$$\frac{dX}{dt} = -\frac{\beta SX}{n},$$

where β is infection rate, which means the probability of contagion in the system, and we define parameters such that

$$s = \frac{S}{n}, \quad x = \frac{X}{n}, \quad s = 1 - x,$$

then we have reduced simple model:

$$\frac{dx}{dt} = \beta(1-x)x. \tag{1.1}$$

The solution to this *logistic growth equation* leads to

$$x(t) = \frac{x_0 e^{\beta t}}{1 - x_0 + x_0 e^{\beta t}}. \tag{1.2}$$

In the context of financial markets, one can interpret the concept in financial jargon as *Solvent-Insolvent*. At any point in time, an individual bank is either in solvent or insolvent (default) state. Suppose that a bank is susceptible with a specific default probability. To catch the contagion process, one of the connected banks must already be insolvent). Since this infected (insolvent) linked bank transmits the so called disease at the rate β, the probability that another bank becomes insolvent can be obtained using equation 1.1 with still solvent banks. Translating the SI framework into a Solvent-Insolvent one is somehow trivial. Although this simple solvent-insolvent framework is an interesting tool for the investigation of contagion in an inter-bank network, it has some drawbacks facing the real world simulations. For example, in reality, an insolvent financial institution does not suddenly declare bankruptcy and it could use extra funding to repay its obligations.

1.2.2 Balance Sheet Based Approach

To illustrate the mechanism of contagion in the inter-bank network, one assumes a network consisting banks with simplified balance sheets. The core concept behind the method is the tendency of financial institutions to borrow to each other, which could induce systemic crisis when one institution is unable to repay its liabilities.

There could be different kinds of balance sheets based on various assumptions regarding the purpose of the research. However, the balance sheet I use in the paper is based on some simple assumptions. To simplify, it is assumed that each bank has only two levels of liabilities on the balance sheet: senior borrowings b_s and junior borrowings b_j. On the assets side, there are four elements: liquid assets α, illiquid assets β, senior loans l_s, and junior loans l_j. The total assets and total liabilities of the bank i are A_i and L_i, respectively; the net worth of the bank denotes by W_i.

1.2.3 Solvency Equations

I assume a bank in the network uses its liabilities L_i to invest at interest rate R_i. Obviously, R_i can be larger or smaller than one or equal to zero with respect to the investment conditions. The profit made in this transaction can be shown as $\rho_i = (R_i - 1)L_i$ (Smerlak et al., 2014). I also assume that the borrowing and lending rates are always equal. Hence, there are 3 different solvency situations for a bank to repay its liabilities:

assets A_i	liabilities L_i
liquid assets α_i	senior borrowings b_i^s
illiquid assets β_i	junior borrowings b_i^j
senior loans l_i^s	
junior loans l_i^j	net worth W_i

Table 1.1: Balance sheet of bank i regarding its inter-bank connections. Total assets can be written as $A_i = \alpha_i + \beta_i + l_i^s + l_i^j$ and total liabilities is $L_i = b_i^s + b_i^j$. In addition, $W_i = A_i - L_i$ is the net worth of bank i.

Solvent:

If $\alpha_i + \rho_i + \sum_{j \neq i} x_{ji} - rb_i^s - rb_i^j \geq W_i$, where $x_{ji} = r(l_{ij}^s + l_{ij}^j)$. l_{ij}^s and l_{ij}^j are the senior and junior borrowings of bank j from bank i . The equation means that bank i can repay its liabilities in full.

Partial Solvent (Junior Default):

If $0 < \alpha_i + \rho_i + \sum_{j \neq i} x_{ji} - rb_i^s < rb_i^j$, bank i is in *junior default* and can repay a fraction of its liabilities. Hence for the amount repaid by bank i to bank j we have

$$x_{ij} = \frac{l_{ij}}{b_i^j}\Big(\alpha_i + \rho_i - b_i^s + \sum_{j \neq i} x_{ji}\Big).$$

Insolvent (Senior Default):

If $\alpha_i + \rho_i + \sum_{j \neq i} x_{ji} \leq rb_i^s$, bank i cannot repay any part of its liabilities and it may lead to default. Hence $x_{ij} = 0$ for each $j \neq i$.

After all payments, the net worth of bank i is

$$W_i = \alpha_i + \beta_i + \rho_i - b_i^s + \sum_{j \neq i} (x_{ij} - x_{ji}), \tag{1.3}$$

where x_{ij} and x_{ji} imply repayments of junior loans. Moreover, bank i is in the *safe* mode if $W_i > 0$, or *failed* if $W_i \leq 0$.

2 Related Literature

The current research on measuring systemic risk and contagion in the financial network, in particular inter-bank networks, can be divided into the two different categories: the network approach based on the structure and links in the system and non-network approaches, such as the econometric approach including structural and reduced form methods. In this section, I provide a short review of the related literature and findings on the network based approach.

There is a part of the literature related to systemic risk suggesting us to look at the topological structure and also the degree of connectedness of nodes (financial institutions) in a financial network. The financial network is considered a simple or complex network that its nodes are financial institutions and the links between them for an inter-bank network, for instance, are bilateral exposures and liabilities. The network modeling of financial networks has been a remarkable increasing progress since the last financial crisis 2007-2008. One of the most important issues in the network methods literature is to model financial contagion. The basic concept usually is: more linkages, more threat of contagion. Another issue in the literature is finding an optimal structure for the studied financial networks. In other words, one needs to determine how any node in the system must be connected to other nodes in order to have the minimum amount of loss throughout the network.

One of the first studies on the systemic impact of shocks and contagion in the financial network, in particular the inter-bank network, is done by Allen and Gale (2000). They study a simple inter-bank network consisting four banks to realize how a shock affects the network regarding the topology of the network. The authors find that the systemic impact crucially depends on the structure of the network. In the case of crisis, a complete network that is a network with all interconnected nodes can absorb shocks without further failures in the system. In contrast, the impact of the shock may be severe if the structure of the network is incomplete.

Similarly, Allen et al. (2010) develop a model in which institutions form linkages by the swap of projects. The authors compare systemic risk in two different network structures consisting six banks. One network is called *clustered* in which financial institutions hold identical portfolios and default together. In the other network that is named *un-clustered*, defaults are more dispersed. Allen et al. (2010) find while long term finance welfare is the same in both networks, in the case of using short term finance network structure matters. They also compare investor's rollover decisions and welfare in both networks.

Boss et al. (2004) provide an empirical analysis of the network structure of the Austrian inter-bank market. They find that the degree distribution of the network follows power laws. They also study how the network structure affects the stability of the system with respect to the elimination of a node. They suggest two general important results:

1. The inter-bank network is a small world with a very low *degree of separation* between any two nodes in the system.

2. A more realistic class of scale-free networks must be used for the future modeling of the inter-bank relations.

Moreover, Cont et al. (2012) present a quantitative methodology for analyzing contagion and systemic risk in the inter-bank network using a metric for the systemic importance of institutions: *the Contagion Index*, which is defined as the expected loss to the network triggered by the default of an institution in a macroeconomic stress scenario (Cont, 2009).

In the empirical study, Cajueiro and Tabak (2008) analyze the Brazilian network structure and find high heterogeneity of the network. They also investigate the characteristics of the nodes to understand the role of the different types of banks in the inter-bank network. The authors show that how some private domestic retail banks have a crucial role in the inter-bank network.

While some researchers use empirical data to model the networks, some researchers, such as Nier et al. (2007), Montagna and Lux (2013), Lenzu and Tedeschi (2012), Li et al. (2010), and Krause and Giansante (2012) use simulated data to capture a better understanding of the inter-bank network behavior.

Using the Erdős-Rényi network, Nier et al. (2007) study contagion in a simulated financial network. They show the effect of connectivity and concentration on contagion process in the network. The authors find that the better capitalized banks are, the less likely the banking system encounters a systemic failure. They also show that in the low levels of connectivity, an increase in connectivity would increase the chance of systemic failure, whereas high connectivity levels improve the banking system ability to absorb shocks to prevent systemic failure.

Li et al. (2010) introduce a network model for modeling inter-bank networks with features identified by empirical analysis with the real inter-bank data. Their model is based on two assumptions: first, it is easy for homogeneous banks with the same size to establish inter-bank lending, and the second is that a bank decides its credit lending relationships with other banks according to its credit degrees to them. The authors prove that their network model has common features of real inter-bank networks such as low clustering coefficient, a relatively a low value of average shortest path length (ASPL), community structures, and a two-power-law distribution of out-degree and in-degree.

Lenzu and Tedeschi (2012) develop and analyze an inter-bank market with heterogeneous financial institutions and lending agreements on different network structures by implementing an endogenous mechanism of links formation that address credit relationships between nodes. The authors show the higher vulnerability of the scale-free network compared with random networks. They also find that there are many disconnected clusters in the scale free network, which may increase the default risk. On the other hand, these clusters suggest that the scale-free network is less vulnerable to domino effects. However, the authors find that this is not the case: the scale-free network develops heterogeneous distributions through the interaction of noise and feedback effects, which means the network is more prone to systemic failure.

Krause and Giansante (2012) model a stylized banking network where nodes (banks) are characterized by their heterogeneous balance sheets, capital, cash reserves, and their exposure as a borrower or lender in the market. The authors simulate a banking crisis in the system by failing a bank in order to investigate the spread of the shock in the network. They find that the size of the initially failed bank has a crucial impact on the contagion throughout the system, while the network topology has a limited influence. On the other hand, the size of the failed bank has a very limited impact on the number of banks affected from the contagion, and the network topology has a significant influence on the spreading of the shock in the system. The authors conclude that the current regulation focusing on the balance sheet structure of banks is unable to capture the significant effects of systemic risk in the inter-bank network.

In another theoretical work, Montagna and Lux (2013) study systemic risk in scale-free inter-bank networks. The networks are produced by *fitness* algorithm, where the size of each node is used as a kind of peculiarity index for the bank itself, which means the higher the index, the higher probability that other banks will lend money to it. The networks are also combined with information of sample balance sheets of the banks, characterized by a disassortative structure. The authors show how the percentage of net worth and the percentage of inter-bank assets (both on total assets) affect spread of an idiosyncratic shock. They show that the results indicate a shell structure in the diffusion of losses in the network, i.e., creditor banks of the defaulted entity fail mostly before the others, and it is possible to classify defaults of the different shells in the cascade events. They also find that random networks or networks based on a maximum entropy principle lead to fewer contagious defaults compared with scale-free networks.

Moreover, in the epidemic modeling camp, Blume et al. (2011) study the inter-bank network as an epidemic. The study shows issues such as the trade-offs between clustered and anonymous market structures, and it exposes a fundamental sense, in which very small amounts of *over linking* in financial networks with contagious risk can have strong consequences for the possible future systemic failures.

Combining variance decompositions of vector autoregressions (VARs) and network topology theory, Diebold and Yılmaz (2014) propose different connectedess measures in inter-bank networks, particularly among major US financial institutions. Authors show that these measures are intimately related to key measures of connectedness in the network literature and can be used to measure systemic risk. Following the same approach, Baruník and Křehlík (2018) introduce a new framework based on the spectral representations of variance decompositions and connectedness measures. The authors monitor shocks in different frequencies to assess their effects on system wide connectedness or systemic risk. The results show that shocks to one asset in the system will have a long term effect when the connectedness is created at lower frequencies, while in the case of connectedness at higher frequencies, shocks will have a short term effect.

Finally, in addition to other theoretical and empirical studies regarding the network structure, there is a growing number of studies analyzing contagion and systemic risk through inter-bank networks in different countries. Most papers use balance sheets data to study exposures among banks to form connections between them in their inter-bank systems. Table 2.1 summarizes main studies on contagion in national inter-bank networks. For a comprehensive summary of contagion simulations on different inter-bank networks, one may consult Upper (2011).

3 Model and Results

The model presented in this paper is a weighted directed inter-bank network consisting 10 nodes; each node represents a bank. Assuming two banks a and b, there is a link $1 \longrightarrow 2$

Paper	Scope
Boss et al. (2004)	Austria
Degryse and Nguyen (2004)	Belgium
Guerrero-Gómez and Lopez-Gallo (2004)	Mexico
Upper and Worms (2004)	Germany
Wells (2004)	UK
Lublóy (2005)	Hungary
Müller (2006)	Switzerland
Mistrulli (2011)	Italy
Cont et al. (2012)	Brazil
Minoiu and Reyes (2013)	International
Van Lelyveld et al. (2014)	Netherlands
Martinez-Jaramillo et al. (2014)	Mexico

Table 2.1: Related literature on contagion simulation in (inter)national inter-bank networks.

implying a loan l_{12} made by bank *1* to bank *2* and the opposite exposure is shown by l_{21}. In general, the sum of all weights flowing into bank i is $b_i = \sum_{j \in banks} l_{ji}$, which is the total liabilities of bank i in the inter-bank network. On the other side, the sum of all weights flowing out of the bank i, $l_i = \sum_{j \in banks} l_{ij}$, is the total inter-bank claims of the bank i, where for both b_i and l_i, $j \neq i$. In addition, liabilities of the banks are divided into two groups of *junior* and *senior* that make the inter-bank network a duplex network. However, since inter-bank exposures are assumed senior, I focus only on one layer of this duplex network, which is the senior level. Moreover, the balance sheet used in the model is based on section 1.2.2.

To have an empirically reliable model, I try to make characteristics of the model resemble the Italian inter-bank network. It consists 10 nodes denoting biggest Italian banks and each of them represents one of the major local banks in Italy, named B1, B2, B3, B4, B5, B6, B7, B8, B9, and B10. There are two systemically important nodes, which represent *UniCredit* and *Banca Intesa Sanpaolo* and have the largest size of capital, and the largest amount of exposures to other banks in the network. There are also other 8 banks that represent *Mediobanca, Ubi Banca, Banco Popolare, Banca Nazionale del Lavoro, Banca Monte dei Paschi, Banca Popplare di Milano, Banca Carige*, and *Credito Emiliano* (ranked by market capitalization). I also adjust the size of assets and liabilities of the banks similar to their balance sheets data. It helps us to construct a more reliable network regarding the liquidity and solvency conditions of the banks. In the formation of the network, it is also assumed that larger financial institutions tend to borrow (lend) from (to) institutions of the same size of themselves. Therefore, as the network consists the largest Italian banks, this assumption holds in the model.

According to the balance sheet structure in the previous chapter, I construct networks with random data. To make it clear, each node is a bank and each edge between nodes is the bilateral exposure of those nodes including both borrowing and lending flows. One can consult Table 4.4 for more details of assets, liabilities, and other data of the banks in the networks. Therefore, there are two directed inter-bank networks of our banks; one resembles a scale-free network (s-f network 3.2a) that could be the same as other real world inter-bank networks, and the second one (d-network 3.2b) has more linkages between nodes in the network, which means a more dense and connected network compared to the first one. One should notice that, the dense network is a relative concept in this study, i.e., the dense network is only compared to the scale-free network modeled in this study. Furthermore, to form the scale-free network I use *power-law* degree distribution by using

$$p_k = k^{-\gamma},$$

where $\gamma = 2.5$. This kind of inter-bank network is consistent with previous studies on the real world inter-bank networks (Boss et al., 2004). One can check that there are repeating patterns, both triadic and 5-elements motifs in the second network, while there is not any kind of repeating patterns in the first one. Moreover, both networks could be simulations of the real inter-bank network. For example, both networks have a high disassortativity level (3.1), which is in line with previous studies on real financial networks (Soramäki et al., 2006). However, due to the lack of data for other aspects of network measures, there is not a clear evidence to confirm this claim.

3.1 Simulations and Results

3.1.1 Balance Sheet Based Simulation

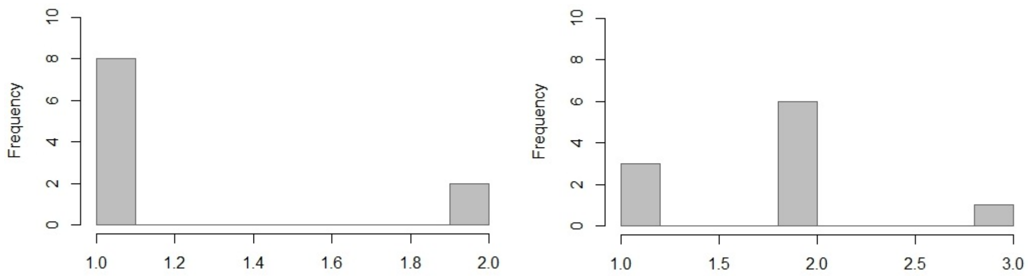
Recalling the solvent condition of solvency equations:

$$\alpha_i + \rho_i + \sum_{j \neq i} x_{ji} - rb_i^s - rb_i^j \geq W_i,$$

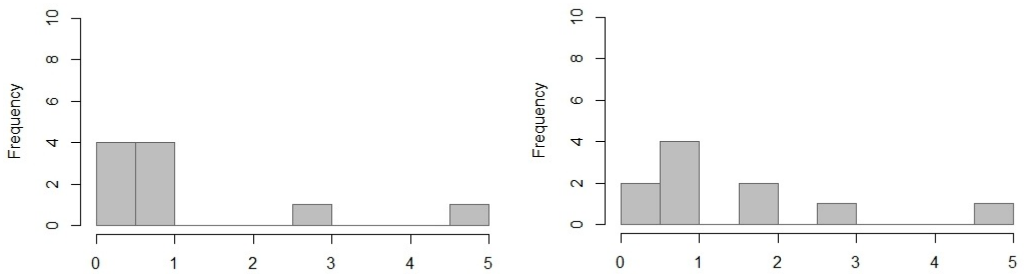
I start to apply a shock to the network by removing a node from the network. This is what we call as a *targeted attack*. I also consider the most systemically important component of the network, which is the node with the highest number of connections to other nodes, to analyze the worst case scenario in the system. Hence, to make it clear, we assume that the starting point of a shock to the system is when the largest bank (B1) investments on its liabilities do not make positive profit which lead the bank to the default (insolvent) condition. I Consequently analyze its insolvency effects, based on its size and connections to other banks in the network to understand how it affects connected neighbor nodes, its importance to affect the network measures and structure, and also check if it could lead the network to the systemic default.

Moreover, I assume a homogeneous ratio of the junior debts to the senior debts in the network which is $\frac{1}{9}$, as well as 2% and 1% for junior and senior rates respectively. I also assume a random number for the percentage of the liquid assets of the banks, which is normally

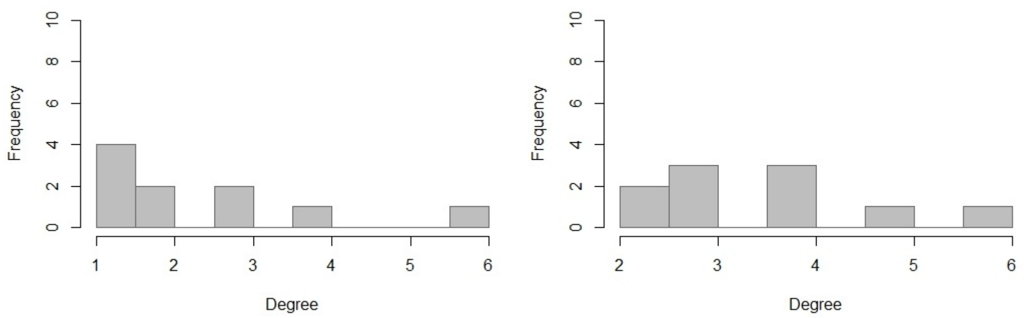
(a) Scale-free network **(b)** Dense network



In-degree



Out-degree



Total degree

Figure 3.1: Initial degree distribution of the networks including in-degree, out-degree and total degree of each network. The left column's plots belong to the scale-free network and the right column is for the dense network.

distributes as $\alpha_i \sim \mathcal{N}(0.08, 0.02)$ of L_i. Therefore, by changing the value of ρ (profit made by investment) and also inter-bank loans to the total senior loans ratio ($\frac{b^s_{interbank}}{b^s}$), one can see their effects on the contagion process in the network. The most important characteristics of the networks can be found in 3.1. In addition, the in-degree, out-degree, and total degree distribution are presented in 3.1.

After applying the shock to the system and forcing the most important bank of the system (B1) to the default position, regarding the different values of ρ and the ratio of inter-bank liabilities to total senior liabilities, there are different contagion processes in the network. For example, there is no further default in the dense network regardless of the value of ρ and inter-bank liabilities of the banks (Figure 4.3), or there is not a severe systemic contagion process in the dense network (Figure 4.4). It means in the case of the default of important banks in the dense inter-bank network with an adequate number of interconnections, the probability of contagion and systemic default is low. In other words, the idea claiming more connections in a network are against contagion and systemic default is confirmed by this simulation.

On the other hand, the scale-free network is more fragile to the shocks in comparison with the dense network. These results are consistent with Lenzu and Tedeschi (2012) and Montagna and Lux (2013). Moreover, changing the value of ρ and the ratios brings different contagion processes. By increasing the share of the inter-bank liabilities in the total senior liabilities of the banks in the system and also decreasing ρ, the contagion process would be more critical and causes severe systemic contagion to make a significant number of banks insolvent (Figures 4.2 and 4.1). In some cases contagion process makes the scale-free network transfer to a network with minimum linkages between solvent banks and also isolated banks that are not connected to each other (Tables 4.2 and 4.1), while in the dense network one cannot observe this transformation (Table 4.3). Moreover, if we assume a heterogeneous profit distribution of ρ for the banks, which is similar to the real world data, the network structure would be more robust to systemic shocks. The intuition might be that by assuming this idea we also consider higher profits for larger banks that make them able to recover their possible losses on their inter-bank exposures by using profit of their investments.

The results of simulations for both networks exhibit important factors related not only to the performance of the banks and their financial decisions, but also their connections including borrowing and lending flows from and into the other banks in the system. Although the real world inter-bank networks resemble scale-free networks, they have more complicated properties compared to the model in this paper, which make them more robust against systemic failures. Additionally, another key factor that is worth mentioning is the ratio of the inter-bank loans to the total liabilities ($\frac{b^s_{interbank}}{b^s}$) of the banks. The assumptions in the presented model are homogeneous for all components in the network, however these assumptions could not be realistic in the real world network, because these ratios are different from bank to bank.

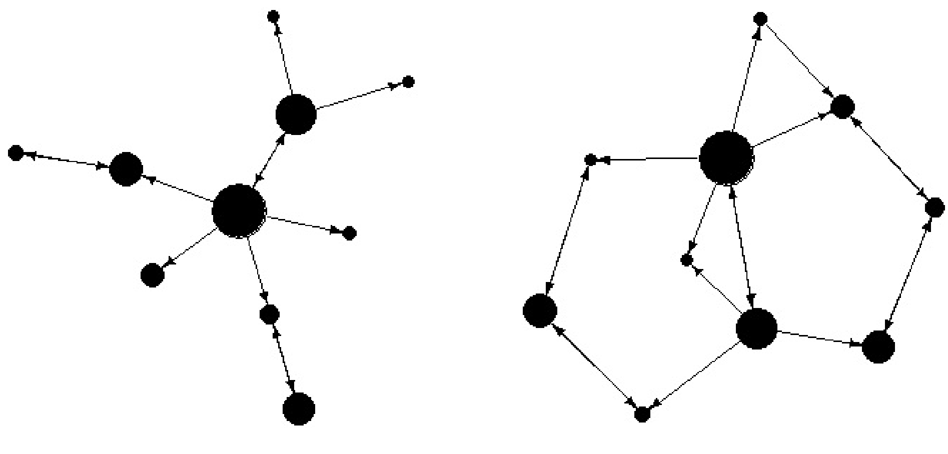
3.1.2 SI Simulation

Although this approach is somehow naive for the real inter-bank network, by assuming a homogeneous probability default (infection ratio) for all banks in the network, we run a simulation for both networks to investigate how contagion spreads over the network with homogeneous characteristics for all components. Therefore, there is no assumption about ρ, and I just assume the same ratio of junior loans to senior loans and also inter-bank loans to the total senior loans. It could help us to understand how the structure of a network could react to the contagion process, with the same conditions for both networks. Because of the simplicity of the model I just show how the contagion spreads through the networks graphically with respect to different values for the default probability (DP) of the banks.

I start the simulation when the shock is initiated by the insolvency of the biggest component (B1) to investigate the contagion process in the system. Therefore, I use two different default probabilities or *infection rate* (in epidemic modeling jargon), 0.4 and 0.6. Unlike the balance sheet based model, the dense network seems to be more vulnerable to the systemic shock in comparison with the scale-free network. In both cases the contagion process takes shorter time to cause a systemic failure in the dense network. Figures 4.5, and 4.6 show the contagion process where $DP = 0.4$. The results support the idea that more interconnections in a network makes it vulnerable to the systemic shocks. Although the results look interesting, they are not reliable for the real inter-bank networks. In the case of default in the real network, a bank might recover its debts and transforms from insolvent position to solvent positions, which happens frequently.

Measures	Scale-free network	Dense network
Density	$0.1\bar{3}$	0.2
Average Degree	2.4	3.6
ASPL	1.54	$1.4\bar{8}$
Reciprocity	0.5	0.56
Assortativity	-0.069	-0.35
Transitivity	0	0.23

Table 3.1: Network measures before applying shocks and individual defaults.



(a) Scale-free network **(b)** Dense network

Figure 3.2: Two sample directed networks resembling 10 major Italian banks, which are formed in two different method. The left one is a scale-free network, while the right one is a more dense network with more connections between the nodes. The size of a node is related to its net worth (W_i).

4 Conclusion and Discussion

This paper is set up to study default and contagion processes in inter-bank networks considering the seniority level of inter-bank loans among them. The contagious defaults in the financial networks and methods to measure them in order to prevent further possible financial crisis have become one of the major topics of financial literature in recent years. Hence to answer this problem, this study considers not only the seniority level of debts within inter-bank networks, but also examines the structure of inter-bank networks. Although it is difficult to access non-public inter-bank data including the inter-bank liabilities, the networks constructed in this paper are based on Italian banks' public balance sheets to have them similar to the real network as much as possible.

The first finding of the study suggests that networks with the higher ratio of senior liabilities to junior liabilities in the whole network are less vulnerable to the targeted shocks (which are defined as failures of the biggest banks of the network), while the higher ratio of inter-bank liabilities to total senior liabilities makes a network vulnerable to the contagious defaults. Therefore, there must be an optimal structure for the inter-bank network regarding the amount of junior loans, senior loans, and inter-bank loans which belong to senior loans.

The second main finding from the simulations of both *scale-free network* and *dense network* of the Italian banks is that regardless of the network structure, in the same seniority level conditions of the debts, an inter-bank network consisting nodes with heterogeneous characteristics is more resilient against contagion and systemic defaults.

Lastly, the results of the balance sheet based approach show that a heterogeneous network with components that have different financial characteristics, is resilient to the contagion processes when there are more linkages between its nodes (dense network). In contrast, a dense network consisting homogeneous banks with the same financial metrics is more vulnerable against systemic shocks and failures. Moreover, simulating shocks with *SI* (Susceptible-Infected) method supports this claim that in the same seniority level of debts in the network, a homogeneous inter-bank network with more linkages between its components (dense network) is more vulnerable to systemic risk compared to an inter-bank network with fewer linkages (scale-free network).

Further studies can be conducted in a more efficient way by having the data of inter-bank liabilities to find an optimal network structure and liabilities distribution with the highest degree of resilience against systemic defaults. Moreover, future research could also be related to the other aspects of inter-bank liabilities, such as the maturity of inter-bank loans and information about collateralized loans. Finally, network measures fluctuations can be interpreted as indicators for the inter-bank network in order to react against shocks. Therefore, as a recommendation, policy makers may use network measures as navigators of the possible upcoming crisis in the future by studying them in the past pre-crisis and post-crisis periods.

Scale-free Network $\rho = 1\%$ of L $\frac{b^s_{interbank}}{b^s} = 0.5$	Initial	Shock	After shock
Insolvent nodes	0	1 (B1)	5 (B1, B6, ,B7, B9, B10)
Density	$0.1\bar{3}$	$0.08\bar{3}$	0.1
Average Degree	2.4	$1.\bar{3}$	0.8
ASPL	1.54	1	1
Reciprocity	0.5	0.66	1
Assortativity	-0.069	None	None
Transitivity	0	0	0

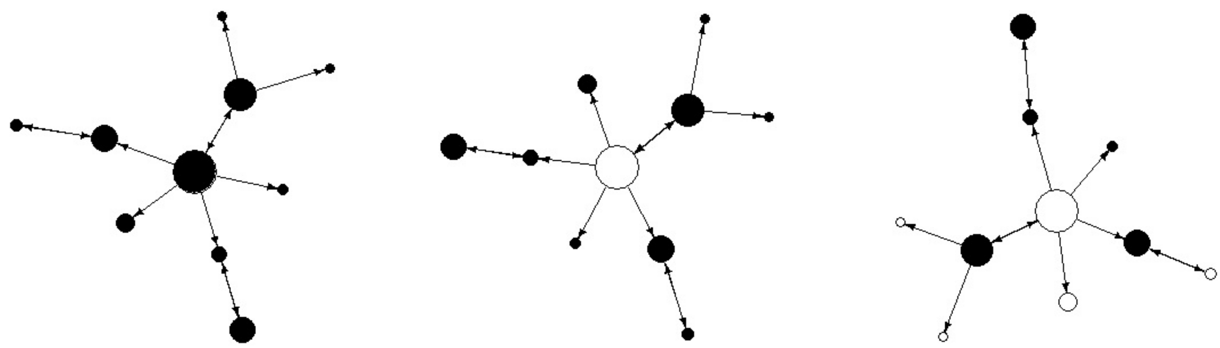
Table 4.1: Changes in network measures before, during, and after shock with respect to $\rho = 1\%$ of L, $\frac{b^s_{interbank}}{b^s} = 0.5$.

Scale-free Network $\rho = 1\%$ of L $\frac{b^s_{interbank}}{b^s} = 1$	Initial	Shock	After shock
Insolvent nodes	0	1 (B1)	6 (B1, B4, B6, B7, B9, B10)
Density	$0.1\bar{3}$	$0.08\bar{3}$	0
Average Degree	2.4	$1.\bar{3}$	0
ASPL	1.54	1	0
Reciprocity	0.5	0.66	0
Assortativity	-0.069	None	None
Transitivity	0	0	0

Table 4.2: Changes in network measures before, during, and after shock with respect to $\rho = 1\%$ of L, $\frac{b^s_{interbank}}{b^s} = 1$.

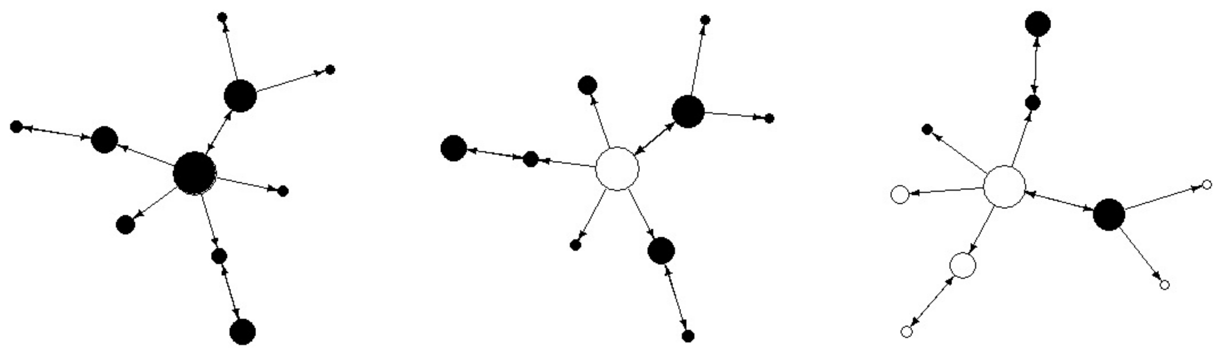
Dense Network $\rho = 1\%$ of L $\frac{b^s_{interbank}}{b^s} = 1$	Initial	Shock	After shock
Insolvent nodes	0	1 (B1)	1 (B1)
Density	0.2	0.16	0.16
Average Degree	3.6	2.66	2.66
ASPL	1.48	1.59	1.59
Reciprocity	0.56	0.66	0.66
Assortativity	-0.35	-0.37	-0.37
Transitivity	0.23	0	0

Table 4.3: Changes in network measures before, during, and after shock with respect to $\rho = 1\%$ of L, $\frac{b^s_{interbank}}{b^s} = 1$.



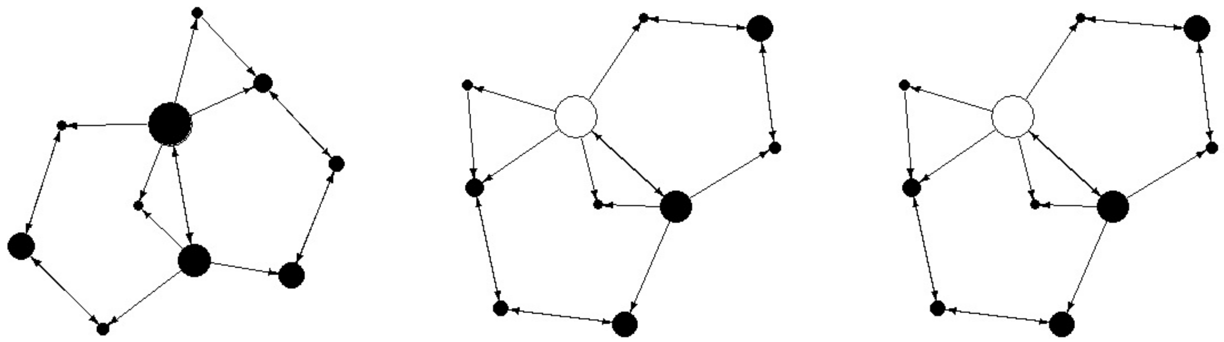
(a) Initial. **(b)** Shock. **(c)** After shock.

Figure 4.1: The contagion process after the default of the largest component (B1) and its shock to the network where $\rho = 1\%$ of L and $\frac{b^s_{interbank}}{b^s} = 0.5$ for all components of the scale-free network (The black nodes are solvent banks and the white ones are insolvent banks in each level).



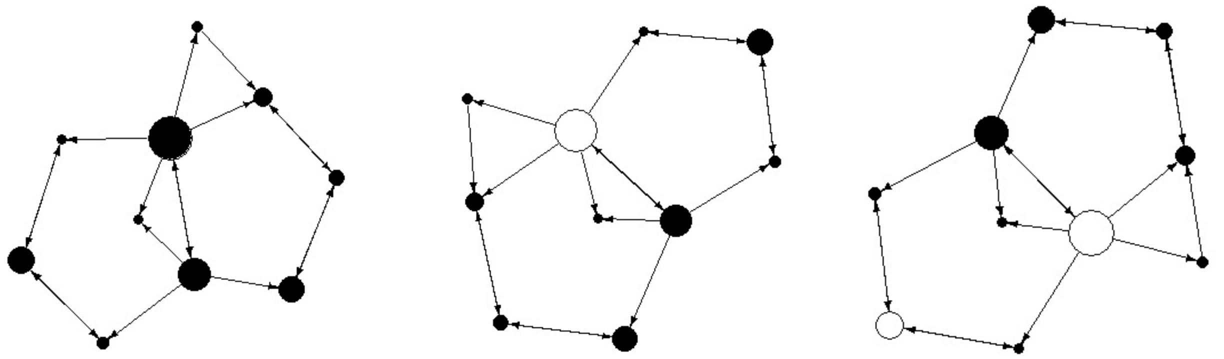
(a) Initial. **(b)** Shock. **(c)** After shock.

Figure 4.2: The contagion process after the default of the largest component (B1) and its shock to the network where $\rho = 1\%$ of L and $\frac{b^s_{interbank}}{b^s} = 1$ for all components of the scale-free network (The black nodes are solvent banks and the white ones are insolvent banks in each level).



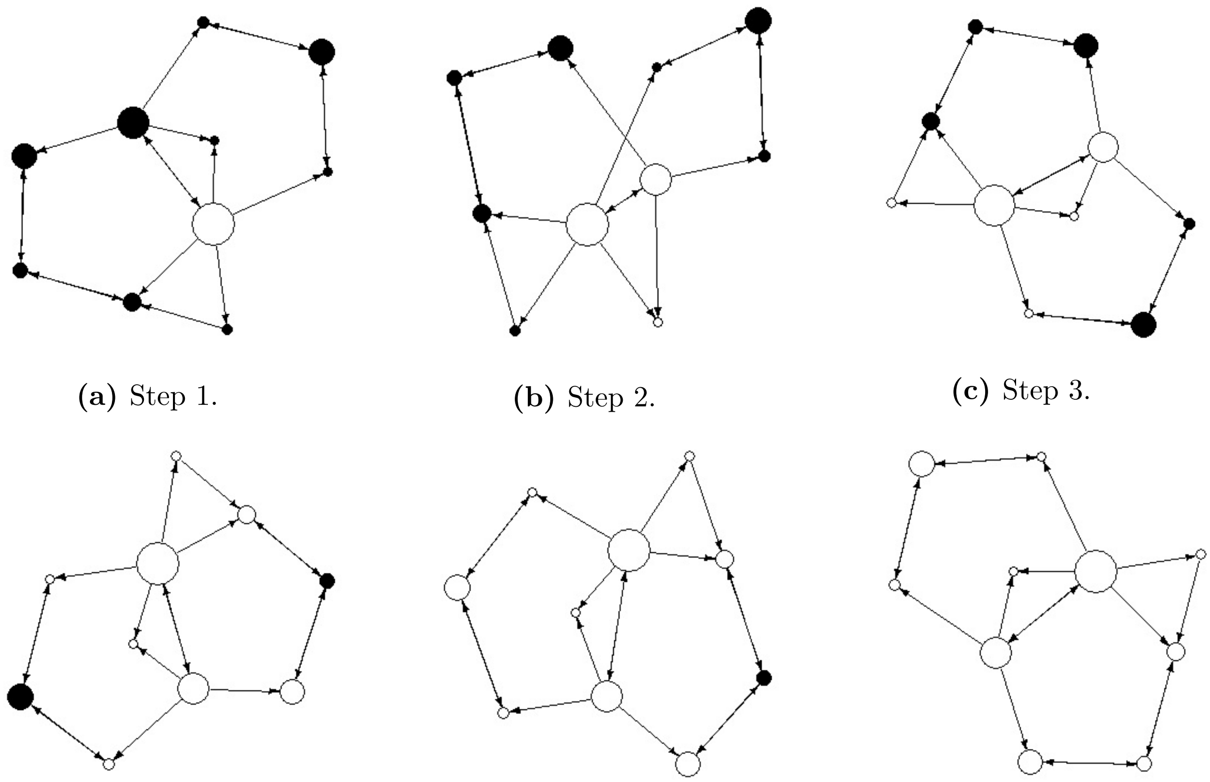
(a) Initial. **(b)** Shock. **(c)** After shock.

Figure 4.3: The contagion process after the default of the largest component (B1) and its shock to the network where $\rho = 1.2\%$, $\rho = 1\%$ of L, and also $\frac{b^s_{interbank}}{b^s} = 1$ and $\frac{b^s_{interbank}}{b^s} = 0.5$ for all components of the dense network (The black nodes are solvent banks and the white ones are insolvent banks in each level).



(a) Initial. **(b)** Shock. **(c)** After shock.

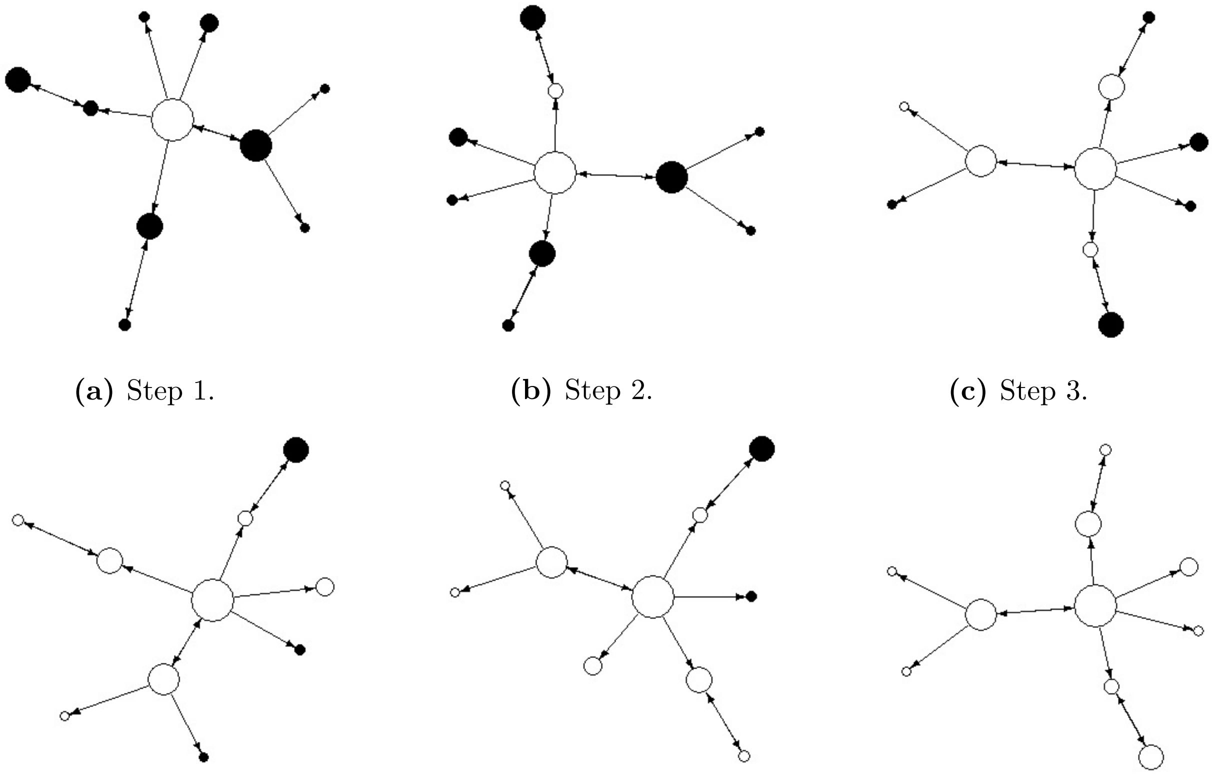
Figure 4.4: The contagion process after the default of the largest component (B1) and its shock to the network where $\rho \sim \mathcal{N}(1.5\%, 0.5\%)$ of L and $\frac{b^s_{interbank}}{b^s} = 1$ for all components of the dense network (The black nodes are solvent banks and the white ones are insolvent banks in each level).



(a) Step 1. **(b)** Step 2. **(c)** Step 3.

(d) Step 4. **(e)** Step 5. **(f)** Step 6.

Figure 4.5: SI model with $DP = 0.4$, for the contagion process after the default of the largest component (B1) and its shock to the dense network (The black nodes are solvent banks and the white ones are insolvent banks in each level).



(a) Step 1. **(b)** Step 2. **(c)** Step 3.

(d) Step 4. **(e)** Step 5. **(f)** Step 6.

Figure 4.6: SI model with $DP = 0.4$, for the contagion process after the default of the largest component (B1) and its shock to the scale-free network (The black nodes are solvent banks and the white ones are insolvent banks in each level).

Banks	Assets	Junior borrowing	Senior borrowing	Inter-bank borrowing	Net worth
B1	950	88.87	799.83	399.915	61.3
B2	646	60.13	541.17	270.585	44.7
B3	200	19.4	174.6	87.3	6
B4	135	12.35	111.15	55.575	11.5
B5	120	11	99	49.5	10
B6	80	7.65	68.85	34.425	3.5
B7	75	6.84	61.56	30.78	6.6
B8	54	5.03	45.27	22.635	3.7
B9	40	3.82	34.38	17.19	1.8
B10	30	2.82	25.38	12.69	1.8

Table 4.4: Approximated balance sheet data of the banks, assuming ratio of junior liabilities to total liabilities is equal to 0.1 and also $\frac{b^s_{interbank}}{b^s} = 0.5$ (all numbers in Euro).

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