

# Optimal Renegotiation Proof Financial Contracts with Multiple Lenders

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# OPTIMAL DEBT CONTRACTS

Costly state verification: Townsend (1979), Gale and Hellwig (1985), Williamson (1987).

Unobservable outcome  $\Rightarrow$  ex post moral hazard.

Standard (simple) debt contract minimizes verification cost.

Assumptions: lender fully committed, strategies deterministic.

# MOTIVATION

Emerging market economy.

Weak business culture and economic environment, significant exemptions.

Insufficient internal resources, no collateral, needs outside investors in addition to internal resources.

Strong and strategic versus weak investors.

Franks and Sussman (2005): low bargaining power of many small lenders, who do not collude.

Brunner and Krahnert (2006): big lenders collude.

Weak contracting system, high uncertainty  $\Rightarrow$  no commitment, allowing renegotiation.

Weak legal system  $\Rightarrow$  APR violation.

# MODEL

Risk neutral borrower (firm) and investors (lenders) in 4 periods model.

Project's return  $0 < x_1 < \dots < x_n$ .

Firm borrows:

$\beta\%$  ... from strategic investor

$\frac{(1-\beta)}{m}\%$  ... from each of  $m$  small investors.

**t=0** Common prior  $\mu$  over  $X$ .

Enforceable payments  $\frac{\eta_{xx}}{m}, \eta_f F(x, v)$  stated.

**t=1** Firm observes  $x$ , pays  $v$ .

**t=2** Renegotiation: firm pays more to strategic investor ( $v' > v$ ) in exchange for no verification. Non-strategic investors obtain  $v$ .

**t=3** Verification? Yes  $\Rightarrow$  firm pays  $\eta_f F(x, v), \frac{\eta_{xx}}{m}$ , cost  $c(x)$  paid by strategic investor.

Absolute priority violation: firm able to keep part of outcome.

# DEFINITION OF RENEGOTIATION PROOF CONTRACT

Firm's and investors' payoffs at  $t = 3$ :

$$\pi_F(x, v, e) = x - v - e[\eta_f F(x, v) + \eta_x(x - v)] \quad (1)$$

$$\pi_I(x, v, e) = \beta v + e[\eta_f F(x, v) - c] \quad (2)$$

$$\pi_i(x, v, e) = \frac{1 - \beta}{m}v + e\frac{\eta_x x}{m}, \forall i \in \{1, \dots, m\}. \quad (3)$$

**Definition 1** *A contract  $F$  with associated PBNE strategies  $\sigma_F, \sigma_I$  is renegotiation proof if and only if there does not exist  $v'$  that makes the strategic investor strictly better off and the firm weakly better off in all states, i.e.,*

$$\beta v' > \sum_{x \in X} \sum_{e=0}^1 \pi_I(x, v, e) \sigma_I(e|v) \mu(x|v) \quad (4)$$

$$x - (1 - \beta)v - \beta v' \geq \sum_{e=0}^1 \pi_F(x, v, e) \sigma_I(e|v), \forall x | \mu(x|v) > 0.$$

# DETERMINISTIC RENEGOTIATION PROOF CONTRACT

Intuition:

Assume the strategic investor's expected continuation payoff from verification  $<$  the lowest payment which firm has to give to all investors if verified.

Then firm able to bribe strategic investor not to verify.

**Proposition 1** *For given  $V$  and  $F$ , let  $\sigma_F, \sigma_I$  be a PBNE strategies. Then  $V, F, \sigma_I, \sigma_F$  is renegotiation proof for all  $v \in V$  with  $\sigma_I(e = 1|v) > 0$  if and only if*

$$\sum_{x \in X} \eta_f F(x, v) \mu(x|v) - c \geq \min_{x \in X, \mu(x|v) > 0} \eta_f F(x, v) + \eta_x x.$$

Proposition (1)  $\Rightarrow$  verification deterministic (i.e.  $\sigma_I(e = 1|v) \in \{0, 1\}$  for all  $v$  such that  $\sigma_F(v|x) > 0$  for some  $x \in X$ ).

# OPTIMIZATION PROBLEM

In stage  $t = 0$  we will assume that the strategic investor chooses  $\{F, \sigma_I, \sigma_F\}$  to maximize the firm's payoff subject to a set of restrictions. This means that the strategic investor solves

**Problem 1** *At  $t = 0$  choose  $F, \sigma_I, \sigma_F$  to maximize*

$$E_0[u_F(x)] = \sum_{x \in X} \sum_{v \in V} \sum_{e=0}^1 \pi_F(x, v, e) \sigma_I(e|v) \sigma_F(v|x) \mu(x)$$

*subject to*

$$E_0[u_I(x)] = \sum_{x \in X} \sum_{v \in V} \sum_{e=0}^1 \pi_I(x, v, e) \sigma_I(e|v) \sigma_F(v|x) \mu(x) \geq \bar{u}_I$$

$$E_0[u_i(x)] = \sum_{x \in X} \sum_{v \in V} \sum_{e=0}^1 \pi_i(x, v, e) \sigma_I(e|v) \sigma_F(v|x) \mu(x) \geq \bar{u}_i, \forall i$$

$\sigma_F, \sigma_I, \mu, \mu(\cdot|v)$  is a PBNE at  $t = 1$ ;

$v, F, \sigma_I$  is renegotiation proof  $\forall x_i \in X$  and  $\forall v | \sigma_F(v|x_i) > 0$ ;

$$0 \leq F(x, v) \leq x - v - \frac{\eta_x}{\eta_f} v, \forall v \leq x, x \in X.$$

# STANDARD DEBT CONTRACT

2 features:

1. Firm pays fixed face value  $\bar{v}$  or it does not pay anything and defaults.
2. Default triggers the verification process.

**Proposition 2** *Let  $\{F, \sigma_I, \sigma_F\}$  solve Problem 1. Then there exists  $\{\tilde{F}, \tilde{\sigma}_I, \tilde{\sigma}_F\}$  which solves Problem 1 with the following properties*

1. *At most the two payments 0 and  $\bar{v}$  occur with positive probability, i.e.,  $\mu(v|x) = 0$  for all  $x \in X, v \notin \{0, \bar{v}\}$ .*
2. *Verification takes places if and only if  $v < \bar{v}$ , i.e.,  $\sigma_I(e = 1|v) = 1 \Leftrightarrow v < \bar{v}$  and  $\sigma_I(e = 1|v) = 0 \Leftrightarrow v \geq \bar{v}$ .*

Therefore  $F(x, \bar{v} = 0$

**Proposition 3** *As long as renegotiation proofness condition is satisfied as a strict inequality, the optimal payment schedule solving Problem 1 is  $F(x, 0) = x$ .*