Arithmetics of Property Rights:
A Leontief-type Model of Ownership Structures

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Abstract A simple algebraic model of a property structure leading to the Leontief-type input-output scheme is developed and used to eliminate indirect ownership relations and evaluate the final distribution of national property among individual owners. Concepts of 'family capitalism' and 'capitalism of agents' type of corporate governance are defined and compared. Implications of different designs of corporate governance for general equilibrium theory, profit distribution and decision making are discussed.

JEL Classifications C60, L33, K11
Keywords corporate governance, Leontief input-output model, ownership structure, primary owners, principal-agent problem, secondary owners

1. Introduction

One of the basic paradigms of neo-classical economics reflected in general equilibrium theory and welfare economics is an assumption about economic organization of the society based on private ownership of production factors and services and their use to maximize 'selfish' benefits of owners. Individuals as consumers are maximizing utility subject to budget constraint having on the right hand side incomes from selling production factors and services owned by them and the revenues from profits of firms they are co-owning, firms are maximizing profits and invisible hand of competition leads to Pareto optimal equilibrium states (e.g. Arrow, 1951; Debreu, 1959; Feldman, 1989). One can call such an ideal picture a 'family capitalism'; everything is owned by households and there are no indirect ownership relations.

Facing reality one can observe a significantly different picture: a universe

of corporations and non-transparent networks of ownership relations. Citizens are owners of a fraction of shares, but the ownership is dominated by big anonymous companies, banks and funds, co-owning a significant part of national property on institutional basis. A citizen A has a share in corporation B, corporation B has a share in corporation C, corporation C has a share in corporation D, and corporation D has a share in corporation B. Is there some relation between citizen A and corporation D? One can call such a structure a 'capitalism of agents'.

The legitimate question is: can an anonymous institution as an institution own anything? Because of transaction costs modern economy cannot be governed by individual owners directly. Thus the system of agents had been developed consisting of intermediary institutions and their professional management, mostly distinct from owners. But, in principle, intermediary institutions are only authorised to execute some of the property rights as agents and on behalf and for benefit of individual owners. The final owners of national property can be only individuals or their non-profit associations.[1]

Accepting this point of view one can ask a rather technical question: In non-transparent networks of ownership relations is there a possibility to disclose a final assignment of the whole national property to individual owners only? Can we decompose the ownership structure of 'capitalism of agents' to a 'family capitalism' structure? In the paper we are trying to answer this question, amending an earlier model of direct and indirect property rights which worked under the assumption of 'transparency' defined as absence of cross-ownership relations (Turnovec, 1999).

A simple algebraic model of ownership structures is formulated reflecting direct and indirect ownership relations[2] independently on cross-ownership and leading to a well known Leontief input-output scheme (Leontief, 1996).

2. Leontief-type model of ownership structures

Let us consider two types of economic agents: the primary owners, who can own, but cannot be owned (citizens, citizens' non-profit associations, state,

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[1] 'Property rights are of course human rights, i.e. rights which are possessed by human beings. The introduction of the wholly false distinction between property rights and human rights in many policy discussions is surely one of the all-time great semantic flimflams'. (Jensen and Meckling, 1976, 307).

[2] Speaking about direct relation we have in mind relation between individual A and company B providing that individual A owns a share in company B, while indirect relation means that individual A, having a share in company B and not having a share in company C, has through company B a relation to company C that is co-owned by company B.

municipalities, etc.), and the secondary owners, who can be owned, and at the same time, can own (companies, corporations). Let

m be the number of primary owners, $i = 1, 2, ..., m$,

n be the number of secondary owners (companies), $j = 1, 2, ..., n$,

s_{ji} be the direct share of the primary owner i in the secondary owner j (as a proportion of total number of shares),

t_{jk} be the direct share of the secondary owner (company) k in the secondary owner (company) k,

x_{ji} be the full (direct and indirect) share of the primary owner i in the secondary owner j,

y_{ji} be the indirect share of the primary owner i in the secondary owner j.

Then the $n \times m$ matrix

$$S = (s_{ji})$$

where the row j expresses shares of the primary owners $i = 1, 2, ..., m$ in the secondary owner j, and the column i expresses the shares of the primary owner i in the secondary owners $j = 1, 2, ..., n$, provides a primary property distribution, and the $n \times n$ matrix

$$T = (t_{jk})$$

where the row j expresses shares of the secondary owners $k = 1, 2, ..., n$ in the secondary owner j, and the column k expresses shares of secondary owner k in the secondary owners $j = 1, 2, ..., n$ provides a secondary property distribution. The couple (S, T) characterises an initial property distribution in an economy.

Clearly

$$\sum_{i=1}^{m} s_{ji} + \sum_{k=1}^{n} t_{jk} = 1$$
for any $j = 1, 2, ..., n$

Matrices S and T are observable. The question is: Is it possible to derive unobservable matrices

$$Y = \left(y_{ji} \right) \text{ and } X = \left(x_{ji} \right)$$

of indirect and full shares of primary owners in secondary owners?
   It must hold

$$X = S + Y \text{ and } y_{ji} = \sum_{k=1}^{n} t_{jk} x_{ki}$$

(where $t_{jk} x_{ki}$ is the part of the full property share x_{ki} of a primary owner i in a company k which follows from the share t_{jk} of company k in company j). In matrix form we have

$$Y = TX \text{ and } X = S + TX$$

Assuming, that the matrix $(I - T)$, where I is an identity matrix, is a non-singular Leontief matrix (i.e. all elements of T are non-negative and some of its norms is less than 1), then there exists a non-negative inverse matrix $(I - T)^{-1}$ and we can express X as

$$X = (I\text{-}T)^{-1} S$$

The last expression is nothing else but Leontief input-output scheme that gives X as a function of T and S. We are able to eliminate indirect relations and to find a final assignment of the total property to the primary owners only.
   If $T = 0_{n \times n}$, where $0_{n \times n}$ is $n \times n$ zero matrix, we can speak about the 'family capitalism' structure, if T is a non-zero matrix, we can speak about 'capitalism of agents' structure. We are living in the world of corporate stake-holders (the capitalism of agents). In the latter case the corporate governance (decision making rights and profit shares of stakeholders) can be based on matrices S and T, but it is theoretically possible (while, perhaps, not very practical) to simulate the 'family capitalism' governance based on the matrix X.

3. An illustrative example

Let us consider a hypothetical initial ownership structure with the four primary owners P_1, P_2, P_3, P_4 and the three companies C_1, C_2, C_3 (secondary owners), described in Table 1.

Table 1

	Matrix S				Matrix T			
	P_1	P_2	P_3	P_4	C_1	C_2	C_3	total
C_1	0.3	0.2	0	0	0	0.5	0	1
C_2	0.3	0	0.3	0.2	0	0	0.2	1
C_3	0	0	0	0	0.6	0.4	0	1

In this case

$$(S) = \begin{pmatrix} 0.3 & 0.2 & 0 & 0 \\ 0.3 & 0 & 0.3 & 0.2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{ and } (T) = \begin{pmatrix} 0 & 0.5 & 0 \\ 0 & 0 & 0.2 \\ 0.6 & 0.4 & 0 \end{pmatrix}$$

Matrices S and T provide an observable property distribution.
   Let us use our model. In our particular case

$$(I - T) = \begin{pmatrix} 1 & -0.5 & 0 \\ 0 & 1 & -0.2 \\ -0.6 & -0.4 & 1 \end{pmatrix} \text{ and,}$$

$$(I - T)^{-1} = \begin{pmatrix} 1.069767 & 0.581395 & 0.116279 \\ 0.139535 & 1.162791 & 0.232558 \\ 0.697674 & 0.813953 & 1.162791 \end{pmatrix}$$

Then

$$X = (I - T)^{-1} S =$$

$$\begin{pmatrix} 1.069767 & 0.581395 & 0.116279 \\ 0.139535 & 1.162791 & 0.232558 \\ 0.697674 & 0.813953 & 1.162791 \end{pmatrix} \begin{pmatrix} 0.3 & 0.2 & 0 & 0 \\ 0.3 & 0 & 0.3 & 0.2 \\ 0 & 0 & 0 & 0 \end{pmatrix} =$$

$$\begin{pmatrix} 0.495349 & 0.213953 & 0.174419 & 0.116297 \\ 0.390698 & 0.027907 & 0.348837 & 0.232558 \\ 0.453488 & 0.139535 & 0.244186 & 0.162791 \end{pmatrix}$$

which gives the final distribution of shares and voting rights of primary own-

ers in corporations (secondary owners) after elimination of indirect links (i.e. 'family capitalism' type of corporate governance in 'capitalism of agents' ownership structure from Table 1). For example, this arrangement shows the decision making rights of the primary owners in corporation C_3 which is fully owned by corporations C_1 and C_2.

To illustrate the difference in the distribution of profits in 'capitalism of agents' and 'family capitalism' type of corporate governance, let us assume that profits of corporations C_1, C_2 and C_3 are 100 in all three cases:

$$\pi = \begin{pmatrix} 100 & 100 & 100 \end{pmatrix}$$

Then in 'capitalism of agents' the profits will be distributed among all seven actors:
i.e. 60 for P_1, 20 for P_2, 30 for P_3, 20 for P_4, 60 for C_1, 90 for C_2 and 20 for C_3. In 'family capitalism' type of governance the profits will be distributed only among the primary owners:

$$\pi X = \begin{pmatrix} 100 & 100 & 100 \end{pmatrix} \begin{pmatrix} 0.495349 & 0.213953 & 0.174419 & 0.116297 \\ 0.390698 & 0.027907 & 0.348837 & 0.232558 \\ 0.453488 & 0.139535 & 0.244186 & 0.162791 \end{pmatrix} =$$

$$\begin{pmatrix} 133.9535 & 38.13953 & 76.74419 & 51.16279 \end{pmatrix}$$

i.e. 133.9535 for P_1, 38.13953 for P_2, 76.74419 for P_3 and 51.16279 for P_4.

4. Some implications

There can be a significant difference between a primary 'face' image of the ownership structure and a 'true' position of the subjects of property rights. This difference as a difference between 'family capitalism' and 'capitalism of agents' types of corporate governance has serious theoretical implications.

Just a few questions:

(a) How are and should the profits be distributed? We established that the final allocation of property to the individual property owners, after elimination of indirect relations, is $X = (I - T)^{-1} S$ while, in reality, only listed direct initial distribution S is taken into account.

(b) What are the implications for voting power in the corporate governance?

(See, e.g. Maeland, 1991; Gambarelli, 1994; Gambarelli and Owen, 1994). How should be allocated the voting weights of different actors in corporate decision making: directly according to *X* or indirectly according to *S*?[3]

(c) Another issue for theoretical research is an implication of corporate governance design for general equilibrium and welfare theory. Indirect ownership relations reflected in 'capitalism of agents' corporate governance clearly generate externalities in the profit maximisation doctrine of general equilibrium theory: total profit of one company might depend (through redistribution of profits among institutional owners) on profits of other companies.

Many problems associated with the inadequacy of the current general equilibrium theory and welfare economics can be related to the theory of agency relationships (principal-agent problem). An agency relationship is a contract under which one or more persons (the principal(s)) engage another person (the agent) to perform some service on their behalf. This involves delegation of some decision making authority to the agent and presupposes some incentive scheme for the agent to maximise the welfare of the principal. Agency relations have been intensively investigated on the level of the firm (e.g. Jensen and Meckling, 1976). But here we face the economy level principal-agent problem. Indirect ownership relations, generally viewed as full ownership relations, are frequently agency relations. We are living in economy of agents behaving as owners. There is a hierarchical structure of agents in economy. Primary owners are principals and secondary institutional owners are in many cases just labels for agents. But in the network of indirect ownership relations an agent A becomes a principal with respect to some other agent B, the agent B becomes a principal with respect to some agent C, and C can become a principal with respect to A, principal of his principals. The principal-agent ownership relation is not transitive and it is not always clear who is an agent and who is his principal. Such situation can be considered a market imperfection and can lead to market failures.[4]

It is interesting that one of the major differences between the USA, on the

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[3] An agenda for future research is to apply the methodology developed here to the control structures that are given not only by direct shares, but by hierarchical relations in networks of principals and agents. Extension of voting models and power indices methodology for such structures could bring new ideas also into studies of political behaviour.

[4] One of the reasons of problematic results of the Czech transitional privatization can be found in not understanding clearly the distinction between principals and agents and in the absence of agency relation regulation (e.g. Bohatá, 1998; Schwartz, 1997).

one hand, and Germany and Japan, on the other, is in the role of corporations as each other's shareholders. In the USA it is rare that one corporation owns large blocks of shares in other companies; in some situations this is even forbidden by law. This is not so in Germany and Japan where high proportions of company shares are held by other corporations (Marer, 2000).

A hierarchical principal-agent problem and corporate governance design within the framework of general equilibrium theory and welfare economics is a challenge for economic theory.

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