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BAKALÁŘSKÁ PRÁCE

Impact of HIV/AIDS on human capital accumulation in Sub-Saharan Africa

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Prohlášení

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Poděkování

Rád bych za věcnou kritiku a postřehy poděkoval vedoucímu této bakalářské práce PhDr. Pavlu Streblovovi MSc a Ing. Ivo Koubkovi za rady stran použité mikroekonomie.

Abstrakt

Tato práce se zabývá vlivem HIV/AIDS epidemie na růst lidského kapitálu v Sub-Saharské Africe. V rámci převzatého teoretického modelu se snažíme porovnat vliv HIV/AIDS epidemi na akumulaci lidského kapitálu při různých rodinných uspořádáních. Tj. nukleární rodina na jedné straně a pro Sub-Saharskou Afriku typická "Široká rodina" na druhé straně. Na konci Bakalářské práce se zabýváme identifikací optimálního rodinného uspořádání a možnou vládní politikou minimalizující dopady HIV/AIDS.

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Abstract

This study examines impact of HIV/AIDS epidemics on human capital accumulation in Sub-Saharan Africa. In framework of taken over theoretical model impact of HIV/AIDS on human capital accumulation under different family settings is examined. We are comparing for Sub-Saharan Africa typical Extended Family with for developed countries typical Nuclear Family. At the end optimal policy for coping with HIV/AIDS epidemics is identified.

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Introduction

With the outbreak of HIV/AIDS epidemics the disease has become concern not only from the medical point of view, but also from the economical one. The negative impacts of the disease on the micro and macro level are undisputable, and prove by a number of studies (Bell *et al.*, 2004), (McDonald & Roberts, 2004), (Canning *et al.*, 2006). The estimated impact on African GDP growth ranges from 0.3 to 1.5 per cent lower annual growth in comparison to the no HIV/AIDS scenario, which is significant, but not as drastic as one could expect. The effects on GDP per capita are much more blurred, as the numerator as well as denominator in the formula is reduced. Estimates about the effect on GDP per capita done are highly sensitive to assumptions of the employed model, to name one example (Quattek, 2000) estimates, that by 2010 South African GDP per capita will be about 9 per cent higher than in the no HIV/AIDS scenario. One year later these findings were challenged in (Arndt & Lewis, 2001), where 8 per cent lower GDP per capita in comparison to no HIV/AIDS scenario is estimated. For now no consensus on effect of HIV/AIDS on GDP per capita growth has been reached, and "finding the truth" is well behind the scope of this thesis.

Since GDP per capita does not include many by HIV/AIDS affected proxies as shortened life span, suffering, trauma of the relatives and other, we can not take GDP per capita as comprehensive welfare measure, thus fact that the effects of HIV/AIDS on GDP per capita are unclear, is no drawback in assessing the significantly negative impact of HIV/AIDS on welfare (Crafts & Haacker, 2003).

Aside of impact on the macro level AIDS strikes with great force some key elements of the functioning society and so threatens its stability. During the outbreak of the epidemics well educated, most productive members of the society were at greatest risk of getting infected by HIV/AIDS (Over, 1992). This definition includes teachers and doctors, both of them crucial elements in fighting the disease (Stover J., 1999). The importance of doctors in fighting the disease, prolonging life of hers patients and thus minimizing the impacts of the epidemics is clear. Teachers on the other hand act as a source of information-

prevention, which has prooven to be by far the most effective measure in fighting the HIV/AIDS epidemics (Canning, 2006).

All of the above mentioned economic impacts emphasize the present impacts or those that are to come in the short term. We believe, that the real HIV/AIDS threat lies in the log run impacts, which is often unjustly overlooked. We identify with (Bell *et al.*, 2004) and (Bell *et al.*, 2003) and believe, that the real threat of HIV/AIDS lies in damaging the transmission mechanism of knowledge from parent to child and in lowered incentive to invest in to schooling. Lowered incentive to provide children with education is consequence of increased pre mature mortality, as in case of children's death the investment in to education is wholly wasted, above that if one or both of her parents die, the transmission of knowledge from parent to child is significantly weakened or even destroyed. The rise in human capital is in comparison to no HIV/AIDS scenario significantly slowed and even reversed (Bell *et al.*, 2003) . Given that human capital is main driving force behind economic growth, HIV/AIDS can can prolong suffering and poverty in Sub-Saharan Africa by many years.

Chapter 1

HIV/AIDS

1.1 Historical perspective

AIDS - Acquired Immune deficiency syndrome, fatal disease first recognized in 1981 in the USA, is the final stage of HIV Human Immunodeficiency virus, discovered and studied independently in the USA and France in 1983. There are two competing theories on the origin of HIV/AIDS, according to the most widely accepted and less controversial theory, so called "Natural theory", the disease originated in central Africa in the 1950's probably through direct contact with primates during hunting (Cohen, 2000), when the domestic hunters got infected with SIVcpz virus, that mutated in to HIV virus. The second theory, so called "OPV" theory claims, that HIV originated during experiments with oral polio vaccine called CHAT, which was prepared in the chimpanzee kidney cells infected with SIVcpz, later injected to humans in whose bodies the virus mutated in to HIV. Either way only very narrow group was inffected for the first one or two decades, but the effects of urbanization, population mobility and social changes allowed the epidemics to spread rapidly in Sub-Saharan Africa and consequently in to the rest of the World. There are three possible ways in which was HIV spread from Africa to the rest of the World(Grmek, 1993). There is no strong evidence supporting one or the other, thus all three ought to be mentioned and briefly explained, first of them "Haiti, Cuba and the Peace corps". Above 10 000 Haitans did work in Africa in the 1960's, picked the virus and emigrated to USA after return home, the virus did spread through USA and via numerous channels to other countries. Second options are Cuban soldiers fighting in the Angola and Zaire civil wars, bringing the virus back to Cuba, some of them later emigrating and spreading the virus in to the USA. Third possible route are the Peace corps volunteers in Africa, which could

have picked up the virus and brought it back to their home countries. We believe the virus did spread through all of these three channels, some of them more, some of them less important, the evaluation is behind the scope of this thesis though. May we support one or the other view, we must agree, that AIDS proved to be extremely sucesfull virus, that in the end threatens stability of countries and perhaps entire Sub Saharan region.

1.2 On the medical side of the problem

AIDS is a group of symptoms that appear in body with by HIV severally damaged immune system. HIV is a retrovirus that primarily infects vital parts of the human immune system, CD4+ cells. The role of CD4+ cells is activation and direction of white cells responsible for fighting the infection. HIV significantly lowers the CD4+ cells count and thus consequently lowers ability of the organism to fight off infections. When the CD4+ cells count reaches 200[1] per microleter the patient enters the AIDS stadium, at this point the cellular immunity is nearly lost and is further declining as the HIV progresses.
AIDS symptoms result from conditions that would never appear or extensively develop in healthy immune system, these conditions such as bacteria infections, parasites, certain types of cancer etc. would be normally controlled by those parts of immune system, that HIV attacks and damages. Above that most patients experience systematic symptoms such as fewer, swelling, tiredness, weight loss etc. When examining AIDS as a group of symptoms caused by opportunistic infections, it is fit to mention, that in some very rare cases low CD4+ cells count and complex damage of the immune system very similar to the one done by HIV could result from different conditions than HIV, patient could thus theoretically develop AIDS without being HIV+. Yet, those cases are extremely rare, still discussed by the scientific community and well behind the scoup of this bachelor thesis. We will focus purely on AIDS as a result of HIV infection.
The net median survival time for those infected with HIV with no access to ARV treatment [2] is 9-10 years, it is worth mentioning, that the course of the HIV sickness may vary greatly, in some case the AIDS stadium and death occurs within months after infection, on contrary in same cases there are no signs of HIV progress after more than 12 years,

[1] So is stated by the USA law, the situation is a bit different in each country, there is no exact point when we start talking about AIDS

[2](UNAIDS, 2007) 85% of infected in Subsaharan Africa

either way after the outbreak of the final stage of the disease, the AIDS stadium, the estimated survival time ranges from 6 to 19 months. At this point the doctors can postpone the final stadium of HIV (AIDS) by many years and with ARV significantly prolong to on average 5 years the AIDS stadium, again the survival lenght of person with AIDS varies greatly and is aside of ARV determined by many factors, that do influence the course of the opportunistic infections which are in the end direct cause of death. The key ifluential factors are host susceptability, general health care quality, diet, specific virae strain in the geographic etc. Carefull mix of drugs and health care available in developed countries ensures thus quality of life, that can be compared with the quality of life of a completely healthy individual. Yet, it cannot be forget, that at this point there is no definite cure for HIV, the final stage of the virus - AIDS can thus be postponed and prolonged,but in all cases HIV infection in the end results in colaps of the immunity system and death of the patient.

1.3 On the development

There are 33,2 million people living with AIDS in the world by now(UNAIDS, 2007), the epidemic took estimated 28 million lives since its outbreak. Last year 2,5 million people were newly infected with HIV, while 2,1 died of AIDS, this means 6800 new infections and 5700 deaths of AIDS daily. Even though the number of infected has risen, there are encouraging elements. We must note that percentage of infected on the population is remaining constant. Due to increased availability of ARV the life expectancy of HIV infected has prolonged, less of them entering the AIDS stadium and dieing, thus increasing the number of HIV infected. Last, but not least, the number of newly infected with HIV has globally declined, this reflects both natural trend in the epidemic and result of prevention programs. By now, the pandemic has formed two clear patterns, the Sub-Saharan Africa and "the Rest of the World". While in the main portion of the World most of the HIV infected belong to three narrow groups - Homosexual man, sex workers and injection drug users, in Sub Saharan Africa the pandemic is widely spread throughout the population, threatening thus every part of the socio-economic establishment, considering that 68% of adults 90% of children infected with HIV live in this region and 76% deaths due to AIDS occures here, it is clear how serious the impact on the region are and in the future will be.

1.3.1 Sub-Saharan Africa

Sub-Sahran Africa is the by HIV/AIDS most affected region in the world. Given its limited resources and socio-political instibility it is also the most vulnerable one. About twenty per cent of all deaths in 2000 in the region were caused by AIDS (Bank, 2008) although the share is slowly declining as the disease has in most of the region already peaked, still in 2006 about 1,7 million people died of the disease in the region (UNAIDS, 2007). Above that, the burden of the epidemics is distributed unevenly across the Sub-Saharan region. The least affected Madagascar has prevalence rates of only about 0,5 per cent, which is comparable to most of the developed countrie. Whereas in South African countries, the worst affected part of Sub-Saharan Africa the prevalence rates range from 16,1 per cent in Mozambique to 33,5 per cent is Swaziland (UNAIDS, 2006) [3].

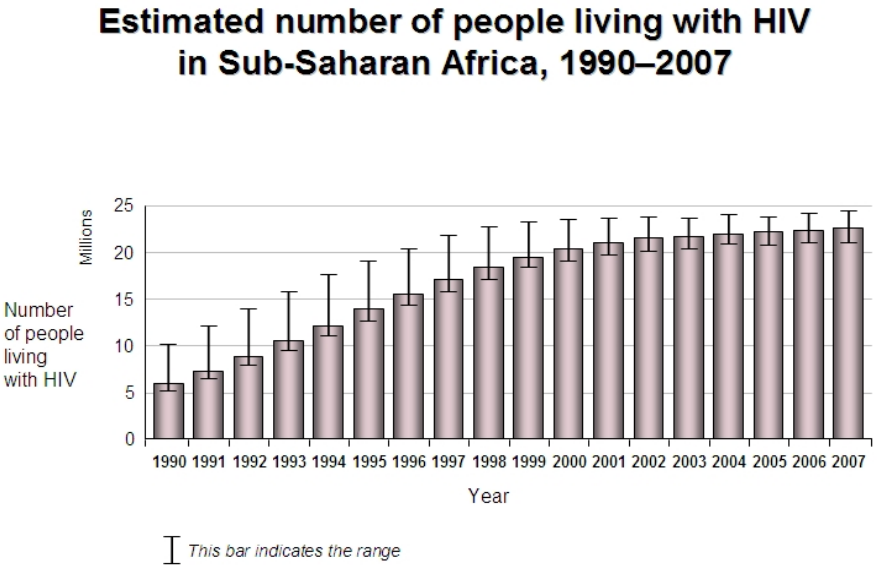

Figure 1.1: source:(UNAIDS, 2006)

On (Figure:1.1) could be seen, that despite decreasing prevalence rate the absolute number of HIV/AIDS infected in Sub-Saharan Africa is still slowly increasing[4]. Eventhough the epidemics has in most of the countries already peaked and prevalence rates are slowly falling, it will take many generations till the burden of HIV/AIDS in Sub-Saharan Africa disappers copletely.

[3]Botswana (24,1 %), Lesotho(23,2 %), Mozambique(16,1 %), Namibia(19,6 %), South Africa(18,8 %), Swaziland(33,4 %), Zambia(16,9 %) and Zimbabwe(20,1 %)

[4]South Africa alone accounts for 32 % of new infections globaly in 2007

Chapter 2

Impact of AIDS on human capital formation

HIV/AIDS much like any other disease negatively influences socio-economic situation. Unlike other major diseases we have seen in Sub Saharan Africa in past century, like ebola, sleeping sickness , cholera etc. HIV/AIDS seems to have enough force to threaten social establishment itself. The reasons are high prevalence rates and unfavorable properties of the epidemics. Let us briefly go over impacts and characteristics of the disease.

HIV/AIDS unlike most of other diseases does not strike groups with lowest immunity such as the youth and the elderly, but strikes people in the most productive age, thus on one side lowering labor supply on the other leaving millions of orphans and so threatening the social fabric itself. In the by state co funded health care system high prevalence rates create substantial burden on the fiscal stability. Also households are effected as some of the medical treatment costs are funded from their savings. Due to HIV/AIDS lowered public and private savings lead to decreased level of investment in the economy.
Time spent while taking care of the sick person by family members may make them miss out school or work, directly influencing creation of GDP and formation of human capital. The formation of human capital suffers from other reasons as well, as proved by number of economic models, lowered life expectancy in the Sub-Saharan region lowers overall incentive to invest in to human capital, so damaging one of the main driving forces behind economic growth. HIV/AIDS also creates significant pressure on the health care system and its quality. Increased number of HIV/AIDS patients takes up large number of beds in hospitals, leaving patients with other serious diseases with no place to stay, and so further negatively influencing life expectancy, number of sick days, life quality in the

countries. It is also worth mentioning, that amongst the health care personal is relatively high prevalence of HIV/AIDS infections creating direct recruitment costs, training costs and demand for risk environment benefits. All above described negative influences are strengthened by, relatively low level of economic development of the region, and thus insufficient funds to fight the epidemics, its consequences and prevent further spread.

The direct costs of AIDS are substantial, direct impact on GDP indisputable, yet there are impacts of HIV/AIDS that are going to be felt in the economy only in the generations to come and that might turn out to be more disastrous than the direct impact. We will discuss influence of HIV/AIDS on building and accumulation of human capital in the rest of the thesis. The influence of the epidemic on human capital accumulation is very complex and in our eyes not yet fully understood, we will identify the main channels through which human capital accumulation gets influenced.

As was indicated above, main portion of AIDS victims are young people in the prime of their lives, majority of them with family to take care of. Their sickness and consequent death creates enormous burden on their families. In the early stages of HIV infection, the burden is not large as there are very few symptoms, however as the disease progresses both the direct and indirect costs are becoming substantial, not only the family looses income of the sick person, but has to pay for the health care and medications during the sickness, often the children are kept out of school to take care of the sick family member, or have to leave the school and go to work as the family can not afford paying for the school or going without children's wage. With the death of the parent while the children are young the possibility to raise them properly and educate them are greatly reduced. Not only that the child will most likely due to financial troubles be kept out of school, but the knowledge, love, guidance provided by the parents as a complement to formal education will not be passed to him. When loosing one parent, the children still has a chance to become a productive, capable citizen, however loosing both parents in the Nuclear family establishment, might well doom the child to live "on the edge" of the society, with no education and social skills whatsoever. The formation of human capital in such establishment is significantly weakened by HIV/AIDS. In the traditional family establishment, the orphaned children are adopted by their close relatives, but even so, the effect of HIV/AIDS are significant, as it is harder to raise, educate and take care of more children.

Moreover increased premature mortality leads to lower incentive to invest in to children's education, as in the presence of the epidemics child might well die soon after finishing

school. We believe HIV/AIDS impact on human capital formation might in the long run prove disastrous and might threaten the economic growth and stability for many generations after the peak of the epidemics. Researching such topic is crucial as the relationship is not yet fully understood and less attention than it deserves is given to it. Hopefully in the future we will have enough knowledge to correctly asses the Long Run impact of HIV/AIDS and employ reasonable policies.

2.1 Human capital accumulation model - Description

In this section theoretical model introduced in (Bell *et al.*, 2003) will be described. The model argues that the main impact of HIV/AIDS on human capital formation is through two channels. Due to one side or both orphaning of children, which weakens or in the case of both side orphaning wholly destroys the transmission mechanism of knowledge from parents to children. And due to decreased incentive to invest in to children's education as the investment will be wholly wasted as a consequence of death. The model, assumptions used and conclusions will be described in to detail, because perfect understanding is necessary for the next section, in which based on the model ability of different family establishments to cope with HIV/AIDS epidemics will be analyzed.
The description of the model will start with its assumptions:

- Family structure is nuclear.
- Every adult chooses partner with the same level of human capital - assortative mating.
- Couples have children until they reach some exogenously fixed number of children.
- All marriages take place at the beginning of period t .
- There is equal number of man and women for each generation - each finds a partner.
- The death of one or both parents is assumed to occur at the beginning of period t, that is shortly after the marriage.

Every person gets married and has children at the beginning of period "t", for first years family spends whole income on consumption, as children are too young to attend school. After this period, at the time child is supposed to start schooling deaths due to HIV/AIDS occurs. By the time child starts attending school, the family is in one of the four states: Both parents alive, father deceased, mother deceased, both parents deceased. Such assumption is more plausible than might seem at first. Transmission via sexual contact is by far the main mean of HIV/AIDS spread in Sub-Saharan Africa, assuming that adults are faithful after getting married, the risk of being infected other than by husband/wife is minimal. the death due to HIV/AIDS with no access to ARV treatment occurs on average within 9 years, which given that he/she got infected before marriage is about the time or shortly after the time, the children start attending school. The family thus ends up in on of the four states denoted by the variable s_t:

- both parents survive into old age ($s_t = 1$)
- mother died ($s_t = 2$)
- father died ($s_t = 3$)
- both parents died ($s_t = 4$)

The total parents human capital is given as $\Lambda_t(s_t)$, since both parents are assumed to be endowed by same amount of capital we have:

$$\Lambda_t(1) = 2\lambda_t, \Lambda_t(2) = \Lambda_t(3) = \lambda_t$$

As was indicated above, there are two main influences on human capital formation, first of them the knowledge passed on child from parents second of them formal education. Formal education and knowledge passed from parents are assumed to be complements in human capital formation. Child with full education, but no social intelligence gained from the parents will hardly be an asset to the society and vice versa. Formal education denoted by e_t, will represent fraction of childhood spent in school $e_t \in < 0, 1 >$. The level of persons human capital Λ_{t+1} in period $t + 1$ based on schooling and surviving parents human capital in period t is than given by:

$$\lambda_{t+1} = \begin{cases} z(s_t)g(e_t)\Lambda_t(s_t) + 1 & s_t = 1, 2, 3 \\ \xi & s_t = 4 \end{cases} \tag{2.1}$$

z(s_t) elasticity given by number of surviving parents, denoting influence of parents human capital on childerns human capital formation. In other words it denotes strength

with which human capital is transmitted from one generation on the other. We will assume, that having both parents is better than having just one. Than child with both parents will attain under same level of education same or higher level of human capital, than if it lost one parent to HIV/AIDS. Formally:

$$z(1)g(e_t)2\lambda \geq z(s_t)g(e_t)\lambda \quad s_t = 2,3 \tag{2.2}$$

$$2z(1) \geq z(s_t) \quad s_t = 2,3 \tag{2.3}$$

$2z(1)$ equals to $z(s_t)$ if the parents are perfect substitutes[1]. Other extreme is allowed, if parents perfect complements[2] $z(1)$ equals $z(s_t) \quad s_t = 2,3$. Since $z(2) = z(3)$, the transmission factor for both parental house will than lie in following range $z(1) \in < z(2)/2, z(2) >$

g(e_t) is function, denoting effect of schooling on human capital formation, the variable is fraction of childhood spent at school, g is strictly increasing differentiable function on $< 0, 1 >$

Let us examine the function (2.1) for s_t=1,2,3. If at least one parent survives, the first part of the equation is equal to zero only if the child does not attend any school at all. Which implies, that child has to attend at least some school to get the benefits from childrearing $z(s_t)\Lambda_t(s_t)$ and so child deprived of all formal schooling will not benefit from level of human capital of his parents. Since person with no school attendance will still have some level of human capital there is added "+1" to the equation. $\Lambda_t = 1$ is thus normalized lowest level of human capital attained by child with both parents or one side orphan. If $s_t = 4$, which means, that the child is both side orphan, the equation equals ξ. It is assumed, that in such scenario the child does not have access to education and is deprived of parental care, thus $\xi \prec 1$. Both side orphan will according to such model, no matter how hard he tries obtain lower level of human capital than one side orphan or

[1]We are somewhat puzzled by the term "perfect substitutes" in this context. Even if the parents were perfect substitutes, in other words both of them would be passing exactly same knowledge on child. Having both of them seems still more beneficial than just one. (2.3) would than hold as equality, only if parents were perfect substitutes and would manage to teach child all of their knowledge in all of the following states $s_t = 1, 2, 3$.

[2]The term complements is in our eyes again slightly misused. Theoretically some knowledge passed from mother on child could need some knowledge passed from father on child. With the father being deceased, mother can not teach child the desired skill and the transmission factor $z(3)$ is strictly smaller than $z(1)$

child with both parents.

In the next part the model is determining the level the level of school attendance e_t. In order to develop the model we will be working with following assumptions:

- Family is choosing only between aggregate consumption and children's education
- Constant returns to scale on production
- Only input is labor, efficiency is given by human capital endownmend λ
- Labor supplied completely inelastically

Even though not explicitly stated, the paper assumes that in the less developed countries scenario, the state does not enforce school attendance and that school attendance is purely decision of the parents. Family than has full income measured in units of the consumption good equal to:

$$\Omega_t(\Lambda_t, N_t, s_t) = \alpha(\Lambda_t(s_t) + n_t\gamma) \tag{2.4}$$

$\alpha \succ 0$ Denotes productivity of human capital
n_t Is the number of children in the family
$\gamma \in (0, 1)$ Denotes lower number of efficiency units of labor supplied by children
$e_t(s_t)$ denotes fraction of childhood spent at school.
Now we will assume that full income is spent on consumption and on education, both partners consume same amount of goods and each child receives the fraction $\beta \in (0, 1)$ of adults consumption. We will further suppose, that the only cost of schooling are opportunity costs (fees could be easily included in to the equation though). The budget line can than be written in following form:

$$[(3 - s_t) + \beta N_t]c_t(s_t) + \alpha\gamma n_t e_t(s_t) = \alpha(\Lambda_t(s_t) + n_t\gamma) \qquad s_t = 1, 2 \tag{2.5}$$

$c_t(s_t)$ denotes the level of adults consumption
We do not differentiate between mother's and father's consumption, constraint for states $s_t = 2, 3$ is thus identical. It could be directly seen, that families with one of the parents deceased face higher relative costs of education. Based on (2.1) both side orphans (family in state $s_t = 4$) have no chance to attend school, all money they earn as child laborers goes toward consumption. They will attain human capital $\xi \prec 1$

Based on above presented budget constraint the household will maximize its utility subject to consumption $c_t(s_t)$ and the education of their children $e_t(s_t)$. Let mothers and fathers have same preferences. Male with same level of human capital as female will choose same pair ($c_t(s_t)$,$e_t(s_t)$). In the case of two parental household the consumption will be treated as private good, while education of children will be public good within the marriage. The utilities in the marriage are assumed to add up. The only form of uncertainty the parents have to face is the number of children who will die prematurely, as the death of the child wastes completely the investment in to the education. The parents are assumed to be risk neutral, their expectations about children's premature death are going to be based on publicly available life expectations data. The Utility function is given as follows:

$$U_t(s_t) = (3 - s_t)\big(u(c_t(s_t)) + n_t a_{t+1} v(z(s_t)g(e_t)\Lambda_t(s_t) + 1)\big) \quad s_t = 1, 2 \tag{2.6}$$

a_{t+1} is the probability, that children will survive in to old age.
The utility functions u(.) and v(.) are assumed to be increasing, continuous concave, twice differentiable.
We do not differentiate between mother's and father's preferences, budget constraint for states $s_t = 2, 3$ is identical. Since state $s_t = 2$ cover of of the states $s_t = 2, 3$, we will use only $s_t = 2$ from now on. Family in time t, in one of the three states ($s_t = 1, 2$) solves following maximization problem:

$$\arg\max_{c_t, e_t} U_t(s_t) \tag{2.7}$$

- The budget line $\Omega_t(\Lambda_t, N_t, s_t)$ is constraint
- $c_t \geq 0$
- $e_t \in < 0, 1 >$

It is further assumed, that both goods are non-inferior[3]. Let us briefly examine impact of family's human capital on consumption and education of their children. Difference in total human capital of the family is given by the number of parents and/or by their education. We will examine these two effects separately.

[3]The goods are assumed not to be non-inferiorin the traditional ceteris paribus sense, as increase in Λ_t changes aside of budget also the marginal utility of education.

The **larger human capital** of the parents implies higher productivity and thus earnings given by equation (2.4). Given the assumption, that both goods are non-inferiorwe get:

$$\frac{\partial e_t(s_t)}{\partial \lambda_t(s_t)} \geq 0 \qquad \frac{\partial c_t(s_t)}{\partial \lambda_t(s_t)} \geq 0 \tag{2.8}$$

The **larger number of parents** (both parents instead of just one) will indicate larger family's earning, and higher family's consumption, it is however not immediately seen, that it will indicate same or higher level of consumption per head and higher or same level of education for their children. From equations ((2.4)) and (2.5) could be seen, that the consumption per head and education will increase or remain same, only if the net contribution[4] of the parent to the family's budget is equal or larger than zero. Formally:

$$(3 - s_t)c_t \leq (3 - s_t)\alpha\lambda \tag{2.9}$$

Which for both $s_t = 1, 2$ results in :

$$c_t \leq \alpha\lambda \tag{2.10}$$

Equation (2.10) is assumed (even though not explicitelly stated) to always hold, which gives us non negative impact of increased number of parents on consumption and education . Putting equation (2.8) together with above drawn conclusions, we obtain:

$$\frac{\partial e_t(s_t)}{\partial \Lambda_t(s_t)} \geq 0 \qquad \frac{\partial c_t(s_t)}{\partial \Lambda_t(s_t)} \geq 0 \tag{2.11}$$

Given that consumption and education are by (2.11) non decreasing in parent's human capital. We can start evaluating impact of increased human capital on utility. In order to do so, the chain rule will be employed:

$$\frac{\partial U_{t_{\lambda_t}}(s_t)}{\partial \Lambda_t} = \left(u'_{\lambda_t} c'_{t_{\lambda_t}} + n_t a_{t+1} v'_{\lambda_t} z(s_t) g'_{\lambda_t} e'_{t_{\lambda_t}}\right)(3 - s_t) \tag{2.12}$$

u'_{λ_t} is derivation of $u(.)$ with respect to λ_t
c'_{λ_t} is derivation of $c(.)$ with respect to λ_t
v'_{λ_t} is derivation of $v(.)$ with respect to λ_t
g'_{λ_t} is derivation of $g(.)$ with respect to λ_t

[4]By net contribution is meant his/hers earnings minus consumption

e'_{λ_t} is derivation of $e(.)$ with respect to λ_t
Based on our knowledge about the functions for (2.12) holds:

$$\Big(\overbrace{u'_{\lambda_t}}^{\geq 0} \times \underbrace{c'_{t_{\lambda_t}}}_{\geq 0} + \overbrace{n_t}^{\succ 0} \times \underbrace{a_{t+1}}_{\succ 0} \times \overbrace{v'_{\lambda_t}}^{\geq 0} \times \underbrace{z(s_t)}_{\succ 0} \times \overbrace{g'_{\lambda_t}(e_t)}^{\geq 0} \times \underbrace{e'_{t_{\lambda_t}}}_{\geq 0} \times \Big) \overbrace{(3 - s_t)}^{\succ 0} \tag{2.13}$$

$n_t \succ 0$ is the number of children in the family.
a_{t+1} is the probability, that children will survive in to old age, it has to be larger than zero, otherwise the whole society would die out.
v' is assumed by (2.11) to be larger than zero.
$z(s_t)$ denotes influence of parents human capital on children human capital formation, as the influence is assumed to be positive, it is strictly larger than zero.
$g(e_t)$ represents the education technology, the impact of school on capital formation has to be positive, otherwise there would be no incentive to attend school.
$(3 - s_t)$ is always larger than zero, as $s_t = 1, 2$
Since both of the goods have positive marginal utility, the consumer will maximize his utility by spending the whole budget. Together with (2.11) we obtain, that either consumption, education or both will have to increase. That gives us that either the term $u'_{\lambda_t} c'_{t_{\lambda_t}}$ or $n_t a_{t+1} v'_{\lambda_t} z(s_t) g'(e_t)_{\lambda_t} e'_{t_{\lambda_t}}$ or both of them are strictly larger than zero. Together we obtain:

$$\frac{\partial U_{t_{\lambda_t}}(s_t)}{\partial \Lambda_t} \succ 0 \tag{2.14}$$

Assessing the impact of decreased premature adult mortality in time $t + 1$ is going to be much simpler. Observe, that given the properties of (2.6). For given optimal bundle (c_t, e_t) increase in a_{t+1} increases utility. The feasibility of (c_t, e_t) is not effected by a_{t+1} . If the consumers changes the optimal bundle he does it only to further increase the utility. We have obtained, that utility is increasing in decreasing premature adult mortality in the next period. Together we have obtained:

$$\frac{\partial U_t(\mathbf{a})}{\partial \Lambda_t} \succ 0 \qquad \frac{\partial U_t(\mathbf{a})}{\partial a_{t+1}} \succ 0 \tag{2.15}$$

We can draw following conclusions:
(a) One side orphans receive less schooling than children from two parent families and strictly less if the former chooses some but not full schooling. This is given by shift in preferences as schooling becomes less efficient in one parent households and because both

education and consumption are non-inferior goods.
(b) Children with both parents accumulate larger human capital than one side orphans and strictly larger if the parents are not perfect substitutes. If parents were perfect substitutes (2z(1)=2(2)) and the children received full schooling in case of survival of both parents as well as in case of death of one of the parents, than it would accumulate same human capital in both scenarios.
(c)Increase in expected child mortality puts lower weight on utility of education relative to utility of consumption. For $e_t \in (0,1)$ will thus increase in premature adult mortality in period $t+1$ reduce schooling,.
To asses the dynamics of the model assumption about non functioning altruism is going to be introduced. For uneducated couples, as they do not have enough money to provide children with education or they do not realize the importance of it, the altruism toward their children is not functioning. Formally:

$$\Lambda_t(1) \leq 2 : e_t^*(1) = 0 \tag{2.16}$$

Conclusion (a) immediately gives us that $e_t^*(2) = e_t^*(3) = 0$

2.1.1 Dynamics of the model

The level of education is chosen as to maximize (2.7). We can rewrite equation (2.1) in following manner:

$$\lambda_{t+1} = \begin{cases} z(s_t)g(e_t(\Lambda_t, s_t, a_{t+1}))\Lambda_t(s_t) + 1 & s_t = 1,2,3 \\ \xi & s_t = 4 \end{cases} \tag{2.17}$$

Equation (2.17) describes level of human capital person will be endowed with in time t+1, provided he/she will not die before reaching adulthood. The equation however does not tell anything about the dynamics of the human capital accumulation across generations, as he/she can end up in any of the four states describing family status in t+1 time $s_{t+1} \in 1,2,3,4$, which dramatically influences level human of capital passed to the next generation (generation reaching adulthood in t+2 time).
For a while we will assume, that there is no premature mortality, all adults manage to raise their children and die of old age. In such society only state $s_t = 1$ exists. According to (Bell & Gersbach, 2002) the system has at least two steady states if completely uneducated people choose not to educate their children and if for lowest level of parent's human

capital such, that family in state $s_t = 1$ chooses full education for their children holds:

$$z(1)g(1)2\lambda^a + 1 \geq \lambda^a \tag{2.18}$$

Equation (2.18) tells us, that children with full education, whose parent's are endowed with human capital larger than λ^a will acquire same or higher level of human capital than their parents. Note, that $\lambda^a \succ 1$, in such scenario the difference between uneducated - poor and educate - wealthy, would be increasing.
$\Lambda^d \succ 2$ stands for smallest capital endowment of the parents such that they provide their children with some formal education. Formally:

$$\forall \Lambda, \Lambda \succ \Lambda^d : e(\Lambda(1), 1) \succ 0 \tag{2.19}$$

Families endowed with human capital $\Lambda \prec \Lambda^d$ will find themselves in poverty trap for all next generations as this steady state is stable. They will not be able or willing to provide their children with education.
Provided equation (2.18) holds as strict inequality, we will have third steady state Λ^*, such that $\Lambda_t = \Lambda^*, \forall t$. This relationship tells us, that all generations will be endowed with exactly same level of human capital, as this state is unstable, note, that any shock that will favor or disfavor education will dramatically influence families that find themselves in this unstable state. For the typical dynamics is assumed,that $z(1)g(e_0^t(\Lambda_t, 1))\Lambda_t(1) + 1)$ is strictly convex on $< \Lambda^d, \Lambda^a >$. The human capital accumulation is than depicted on graph (2.1). In some finite time t+x all families will converge in to one of three steady

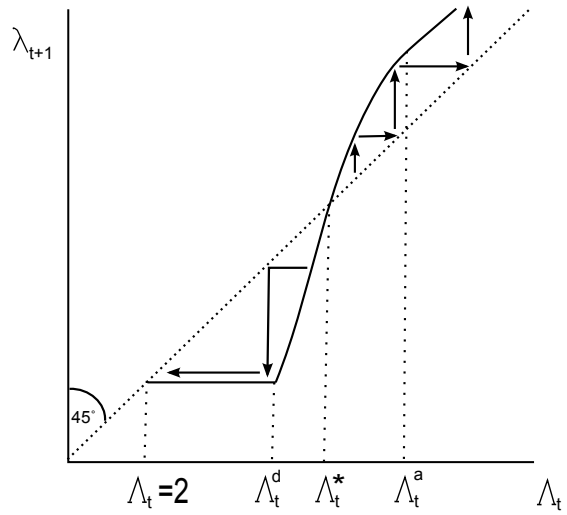

Figure 2.1: Human capital accumulation source:(Bell *et al.*, 2003)

states $e = 1, 0$ or $e(\Lambda^*) \in (0, 1)$. Observe, that both side orphan will marry spouse with same level of human capital ξ, the human capital of such family will than be equal to $\Lambda_t = 2\xi \prec 2 \prec \Lambda_d$. They will thus choose no education for their children, which will by (2.17) attain human capital $\lambda_{t+1} = 1$. When they get married, their family will have human capital $\Lambda_{t+1} = 2 \prec \Lambda_d$ they will thus choose again no education for their children. All descendants of both side orphan are thus trapped in the poverty trap.
From this perspective increased premature mortality is particularly dangerous as the appearance of both side orphans affects all levels of society without any exception. Without certain safety net for orphans the whole society will find itself in the poverty trap in finite number of periods.

2.1.2 Family Setting a Key Variable

The above described model with its disastrous outcomes is describes the consequences of HIV/AIDS in the Nuclear family setting. In such setting only parents are responsible for their children and if death of both of the parents occurs, the children are left to care for themselves, with no help from Extended family or state whatsoever.

Second arrangement is partial pooling, that occurs when subset of society pools it's resources and takes responsibility for the orphaned children. Such subset could be represented by Extended family, tribe, community etc. The Extended family setting is specially relevant in case of Africa and it's positive sides will be discussed in the next section. Note that in order to get rid of the risk aversity, the subset has to be large enough, the number of children raised in such subset per person is than given as $\frac{n_t}{2a_t}$.

Third arrangement is complete pooling that is to say special case of partial pooling. The subset is the whole society, in other words all surviving parents take on responsibility for all orphaned children. It is also assumed, that after deaths occurs, surviving parents form new pairs.

In the following section of the paper (Bell *et al.*, 2003) complete pooling is described. As Extended family setting is more realistic setting for Sub-Saharan Africa, we will slightly modify the complete pooling case described in paper (Bell *et al.*, 2003).
We will start with few plausible assumptions. The Extended family is large enough to diversify premature mortality idiosyncratic risk. We will assume, that after the deaths occur,

surviving adults form new pairs, in such manner, that in comparison to pre HIV/AIDS scenario, the number of adult pairs per family decreased by $a \times 100$ per cent. In other words, when forming new pairs equal number of adults moves to live with different families as joins the original one. In such arrangement the pair raises increased number of children given as:

$$n_t(0) = \frac{n_t}{a_t} \tag{2.20}$$

$s_t = 0$ Denotes the Extended family pooling setting.
a_t Denotes premature adult mortality in time t.
n_t Stands for original number of children per pair.
The modified budget line looks after inclusion of extra children as follows:

$$(2 + \frac{n_t}{a_t}\beta)c_t(0) + \alpha\gamma\frac{n_t}{a_t}e_t(0) = \alpha(2\lambda_t + \frac{n_t}{a_t}\gamma) \tag{2.21}$$

Note, that in case of pooling $(a_t) \times 100$ percent of original families take care of n_t/a_t children each. It is plausible to assume, that increased number of children will negatively influence the transmission factor $z(0)$, the influence of premature adult mortality on the transmission factor is thus introduced, $z(0, a_t)$. It is further assumed, that newly formed family can pass higher potential on children, that one parental household, if $a_t \geq 1/2$. Formally:
$z(0, a_t)$ is non decreasing, continuous and differentiable function in a_t, and holds: $z(0, 1/2) \geq z(2)/2$.

Putting term $z(0, a_t)$ in to equation (2.1) we obtain:

$$\lambda_{t+1} = 2z(0, a_t)g(e_t)\lambda_t + 1 \tag{2.22}$$

Note that, both side orphans are "eliminated" from the equation, as their relatives take care of them. The utility function is given as.

$$U_t(0) = (2)\big(u(c_t(0)) + (n_t/a_t)a_{t+1}v(2z(0, a_t)g(e_t)\lambda_t + 1)\big) \tag{2.23}$$

Note that in utility function (2.23) is assumed, that "adoptive" parents feel same altruism toward their biological as well as adopted children. If it was not so, and the parents were obliged by some social norm to provide their adoptive children with same education as biological children, while not feeling the altruism toward them, the marginal utility of education would decrease, while price remain same. The optimization would lead to

higher level of consumption and lower level of education. Considering that adoptions are taking place within family, the assumption, that parents feel altruism even toward adopted children is kept.

Provided, that sub utility functions u(.) and v(.) are concave (not strictly concave), we can claim, that the new steady state $\Lambda^*(0)$ requires same or higher level of adults human capital than the steady state for the two parent household $\Lambda^*(1)$. On the other hand parents human capital required to attain the steady state of one parental household $\Lambda^*(2)$, is higher or same as in the case of pooling. Formally we have:

$$\Lambda^*(1) \leq \Lambda^*(0) \leq \Lambda^*(2) \tag{2.24}$$

That means, that some fraction of the society with both parents alive, which could have been enjoying sustainable growth and in few generations provide their children with full education, will under pooling descent in to poverty. The logic is reverse in case of one parental household. In the short run pooling act as any kind of insurance, is paid by the lucky ones, while benefiting the unlucky ones. The important benefit is seen it the lung run, where thanks to absence of both side orphans unlike in the case of Nuclear family establishment, the whole society does not descent in to poverty, where each member attains the lowest level of human capital ξ.

Chapter 3

Inclusion of family establishment

HIV/AIDS is social, economic and political issue that can lead to collapse communities, social establishments and even states. We have identified worsened conditions for human capital accumulation as major long run threat of HIV/AIDS. If the whole society starts falling to general state of backwardness resources to fight the epidemics and avert the inevitable collapse will be reduced with every generation. On the other hand if the society manages to increase its' human capital stock with every coming generation, its' tax base and thus resources to fight HIV/AIDS will raise. We believe family, the building stone of each society, is the key to determine impact of HIV/AIDS on human capital accumulation in the country. We will examine impact of HIV/AIDS in resource limited setting on two completely opposite family establishments. On one side stands Extended Family, setting typical for Sub-Saharan Africa. In such setting the broad family act in a sense as an insurance. Relatives help each other in need, while expecting same favor sometimes in the future. Such family establishment act as safety net for orphans, seniors and disabled. We will discuss vulnerability of such setting to HIV/AIDS and its' ability to educate children and orphans after HIV/AIDS outbreak.
On the opposite side stands Nuclear Family. Setting typical for developed societies. In such setting "Every man stands for himself". Obviously family can lend help to its' unlucky member, however the insurance incentive diminishes, only altruism remains. Seniors have to save for their pension during their productive years, children get still adopted by relatives, the incentive is ceteris paribus[1] lower than in Extended Family.

Given the complexity of the problem and our limited knowledge about the utility

[1]The "ceteris paribus" assumption is crucial to make such claim

functions, we were not able to employ the "method of Lagrangean Multipliers" to solve the utility maximization problem. We will thus have to settle with conclusions based on changes in optimality conditions.

3.1 Nuclear Family under pension system

As first impact of HIV/AIDS on human capital accumulation in the case od nuclear family will be described. We will slightly modify model developed by (Bell *et al.*, 2003). In such setting seniors will not be supported by their offspring. Every adult will have to save enough money during his productive years to go through senior hood.

One of the key features of Extended Family is its' role as safety net for orphans. We believe that even in Nuclear Family setting orphans can be taken in and fostered by their relatives same as in the case of Extended Family. It is however clear that the incentive to adopt orphans will be smaller than in the case of Extended Family. The help in the case of nuclear Family is based purely on altruism, whereas the help in the case of Extended Family is based on altruism and knowledge, that the rest of the family will help me in need. We will introduce altruism utility function in to the equation, that will reflect satisfaction from orphans adoption.
We will further assume, that the model does not change over time, terms and preferences are same across generations [2]. The mandatory pension P will be appointed by some central authority. The level of pension savings P will be paid at the beginning of seniorhood in one installment. The productive adult will than be choosing between three goods. Children's education, consumption and child adoption. The budget constraint will look as follows:

$$[(3-s_t)+\beta n_t\kappa]c_t(s_t)+\alpha\gamma\kappa n_t e_t(s_t)+(3-s_t)(P)=\alpha(\Lambda_t(s_t)+n_t\gamma\kappa) \qquad s_t=1,2 \quad (3.1)$$

$n_t\kappa$ will be the number of children per family after adoptions. $n_t(\kappa-1)$ is than the number of adopted children.
As the pension system is mandatory, everyone has to pay contribution P appointed by

[2] Observe that such assumption was adopted, even though not explicitly stated by (Bell *et al.*, 2003), as without such assumption the human capital accumulation stable states would shift over time.

the government.For now we will make no further assumptions on the magnitude and properties of the social security contribution. The corresponding utility function looks as follows:

$$U_P(s_t) = (3 - s_t)\big(u(c_t(s_t)) + n_t a_{t+1}\kappa v(z(s_t)g(e_t)\Lambda_t(s_t) + 1) + \frac{xu(P/x)}{(1+\rho)} + w(\kappa)\big) \quad (3.2)$$

ρ stands for discount factor.
x stands for the fraction of the period senior will live. (If x equals 1/3, his consumption will be tripled in comparison if he lived for whole period, however his utility will be divided by three, as he lives only one third of the whole period.) x is assumed to be known (based on life expectancy).
Where sub-utility functions $u(.)$ for present and future consumption are assumed to be same.
w is the altruism utility function, denoting the utility from adopting another child. We will assume, that w(.) is concave, twice differentiable, and that the children are treated identically and parents enjoy their education as in the case of biological children (Bell *et al.*, 2003).
The optimization problem, subject to (3.1) will look as follows:

$$\arg\max_{c_t, e_t, \kappa} U_P(s_t) \quad (3.3)$$

For the sake of simplicity we have not allowed each subject to choose hers desired level of pension savings P. Only costs of schooling are opportunity costs. For consumption and education holds$c_t \succ 0$ and $e_t \geq 0$, much like in the original model (Bell *et al.*, 2003). Number of children families are willing to adopt depends on all three sub utility functions $u(.), v(.), w(.)$. Observe, that there might not exist such level of human capital, at which children get adopted. fig:1 illustrates such situation. As the human capital endowment of family increases, consumption and education rises, consequently increasing price of adoption. The marginal utility of child adoption divided by its price (3.7) is thus falling. The incentive to adopt children might thus never be strong enough.
Whether children will be adopted for all levels of human capital λ_t, for certain interval (λ_1, λ_2), from certain level or up to certain level of human capital or not at all. Depends on every single term included in the utility equation. Assessing the impact would "cost" us enormous number of assumptions, and is not crucial for our comparison. Knowing that adoption might not always take place is sufficient for now.

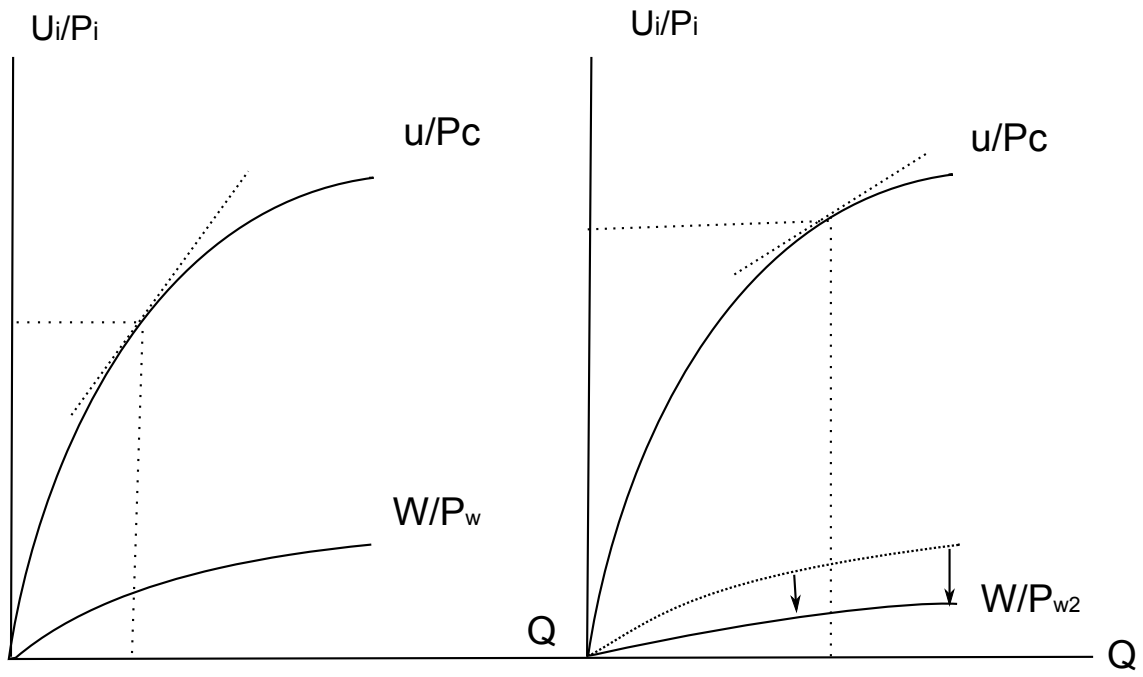

Figure 3.1: Utility of orphan adoption

Notice that for pension holds following inequality : $P \succ 0$. As consumption and education are non-inferior goods, we can safely claim, that an introduction of social security in to the model changes level of consumption c_t and level of education e_t in following manner:

$$(c_{t,P} \leq c_{t*} \quad e_{t,P} \leq e_{t*}) \quad \wedge \quad (c_{t,P} \prec c_{t*} \vee e_{t,P} \prec e_{t*}) \tag{3.4}$$

Where "$c_{t,P}, e_{t,P}$", stand for optimal consumption and education if social security is available."c_{t*}, e_{t*}" Stand for optimal consumption and education in hypothetical state, in which people do not retire, thus do not save for retirement.

We can conclude, that introduction of mandatory social security will have in comparison to no retirement scenario no or negative effect on education and so on human capital accumulation.The human capital accumulation steady states and the state from which full education is provided will remain or be shifted toward higher level of parents human capital $\Lambda^*_P \geq \Lambda^*, \Lambda^d_P \geq \Lambda^d, \Lambda^a_P \geq \Lambda^a$. Such conclusions are rather intuitive.

3.1.1 Effect of HIV/AIDS epidemics outbreak

We will examine the impact of HIV/AIDS epidemics outbreak on Nuclear Family setting. In order to keep the analysis simple, we will assume, that same as in the case of Extended family, those who loose partner will immediately find a new one. They will rise κn_t children, where $\kappa n_t \in < n_t/a_t, 2n_t >$. Where the pair raises $2n_t$ children when the relatives refuse to help with child rearing, whereas n_t/a_t is reached when burden of HIV/AIDS is distributed evenly. Observe, that for z(0) for $2n_t$ children holds $z(0,2) \succ z(2)/2 = z(3)/2$. The relationship is given in (Bell *et al.*, 2003), where is assumed that for n_t/a_t raised children holds: $\forall n_t/a_t \quad a_t \in < 1/2, 1 >$: $z(0) \succ z(2)/2 = z(3)/2$. Given that, z(.) is non-decreasing in a_t, we obtain, that $z(0,\kappa)$ is non increasing in κ and holds: $z(0,\kappa) \succ z(2)/2 = z(3)/2$. In this section the impact of HIV/AIDS outbreak will be assessed, whereas when describing impact of HIV/AIDS outbreak on the Extended Family, comparison of both settings will be included. We have furthermore assumed, that families will act altruistically, taking care of their orphaned relatives. We will make no assumptions on the number of adopted children they rise, it might well be zero. Let us than divide the population in to following subgroups: Family with both parents, rising n_t children (not affected by HIV/AIDS, not raising an orphan). Family rising $n_t\kappa_i$ children, where index $i = 0$ stands for remarried couple, whereas $i = 1$ stands for family that is taking care of their orphaned relatives. Finally both side orphan, that is left to take care of herself. Please notice, that we make absolutely no claim that all three of those groups will exist. The reality might be such, that all orphans are adopted and nobody falls in to the "Both side orphan" group.

3.1.1.1 Both side orphan

Both side orphan that is left to take care of herself will attain human capital equal to $\xi \prec 1$. Provided assumption about non-functioning altruism toward children of parents with human capital $\lambda_t \leq 1$ from (Bell *et al.*, 2003) holds. All generations after the orphan will end up in poverty with human capital $\lambda \leq 1$.

3.1.2 Two parental household with n_t children

Such household will be intuitively the least affected by HIV/AIDS epidemics. The effect of HIV/AIDS outbreak on the steady states of human capital accumulation under Pension system $\Lambda_P^d, \Lambda_P^*, \Lambda_P^a$ will be examined. This situation takes place, when the altruism toward

orphaned children is not sufficiently strong to adopt at least one. In all of the steady states and in all non-corner solutions, for consumption and education optimality condition $M_i/P_i = MU_j/P_j$ holds, whereas the marginal utility of extra child, divided by its price is smaller $M_i/P_i = MU_j/P_j \geq M_w/P_w$. The shape of marginal utility functions follows:

$$MU_{c_t,P}/P_{c_t} = \frac{2u^{'}(c_t)}{2+\beta n_t \kappa} \tag{3.5}$$

$$MU_{e_t,P}/P_{e_t} = \frac{2a_{t+1}z(1)g^{'}(e_t)\lambda_t v^{'}(\lambda_{t+1})}{\alpha\gamma} \tag{3.6}$$

Notice that since the term n_t is both in denominator and numerator it cancels out.

$$MU_{\kappa,P} = \frac{2w^{'}(\kappa)}{\beta n_t c_t + \alpha\gamma n_t(e_t-1)} \tag{3.7}$$

Notice, in this case (3.5) and (3.6) are larger than (3.7), no child is thus adopted.
The HIV/AIDS epidemics will directly influence only (3.6). Increase in premature adult mortality, thus decrease in survival probability a_{t+1} lowers the marginal utility of education, lowering the whole term (3.6). Family consequently lowers education for their children, while increasing consumption. The denominator in (3.7) decreases, as $\Delta\beta n_t c_t + \Delta\alpha\gamma n_t e_t \prec 0$, where Δ stands for difference between AIDS and no AIDS scenario. The whole term (3.7) consequently increases. Intuitively, as children attend less school thus work more, while some of their wage is consumed by parents, their price decreases, if term a_{t+1} decreases sufficiently, families might start adopting children as consequence of worsening HIV/AIDS epidemics and move in to situation that is described in following paragraph. Such finding is however not positive, as the cost of adopting children is reduction in schooling. The steady states of human capital $\Lambda_P^d, \Lambda_P^*, \Lambda_P^a$ shift after HIV/AIDS epidemics outbreak toward higher level of human capital. The effect of HIV/AIDS from (Bell *et al.*, 2003) is not changed by inclusion of pension and orphan adoption.

3.1.2.1 Remarried pair or family taking care of κn_t children

This case is somewhat tricky to describe. In ideal situation, when pair gets remarried, the family helps them raise their children. Than each pair would be taking care of n_t/a_t children, HIV/AIDS burden is distributed evenly. The altruism alone might not be sufficiently strong to assure complete pooling of resources. Two pairs with same level of human capital Λ_t, one of them remarried after death of spouse the other not affected by HIV/AIDS, might thus end up with different number of children (We will assume, that

family will not get rid of their own child even if it could increase utility). The burden of HIV/AIDS will thus be distributed unevenly. Observe, that increased number of children means higher income, relatively expensive consumption, if the consumption has sufficiently large price elasticity on κ, we can claim, that overall expenditures on education given as $\alpha\gamma n_t\kappa_i$ rise, and theoretically e_t rises.

The equilibria will be affected in comparison to previously described situation thrice. At first the death of one or both parents[3], that leads to higher number of children per family, than change in the transmission factor z(.) and finally lower marginal utility of education due to decrease in survival probability a_{t+1}. In order to study the impact of HIV/AIDS the marginal utilities will be defined:

$$MU_{c_t,P}/P_{c_t} = \frac{u^{'}(c_t)}{2+\beta\kappa_i n_t} \quad i = 0, 2 \tag{3.8}$$

$$MU_{e_t,P}/P_{e_t} = \frac{2a_{t+1}z(0,\kappa_i)g^{'}(e_t)\lambda_t v^{'}(\lambda_{t+1})}{\alpha\gamma} \quad i = 0, 1 \tag{3.9}$$

$$MU_{\kappa,P} = \frac{2w^{'}(\kappa_i)}{\beta n_t c_t + \alpha\gamma n_t(e_t-1)} \quad i = 0, 2 \tag{3.10}$$

$i = 0$ stands for the remarried pair $i = 1$ for the original(not directly affected by HIV/AIDS). From previous discussion should be clear, that $\kappa_0 \geq \kappa_1$. Observe, that if $\kappa_0 \succ \kappa_1$, in other words there is not sufficiently strong incentive to adopt orphans. Than one side orphans will be raised by remarried pair with large number of sybillings, where both side orphans might be not adopted at all. If so, than the society will head toward state of general backwardness, though much slower, than was described in (Bell *et al.*, 2003).

In the example of two parental household taking care of n_t children the only impact of HIV/AIDS was increased probability of children s' death in period $t+1$ (productive adult at that time). Aside of increased premature adult mortality in $t+1$ and its' impact on equilibria HIV/AIDS influences remarried pairs and families that adopt children by the increase in raised children. The impact is than given as follows.

We clearly observe change in the transmission factor $z(0)$, given the properties of $z(.)$, we obtain, that $z(0,\kappa_i) \leq z(1,1)$. The impact of the change in transmission factor on education is somewhat tricky to describe. We will be looking at the change in the term

[3]Obviously there is much suffering and pain associated with death of their parents, the model does not take in to account. We should thus keep in mind, that behind HIV/AIDS is usually more than theoretical model or regression tells.

MU_i/P_i, since price of education given as $\alpha\gamma$ does not change, we will be looking only at the numerator of (3.9) which is geiven as:

$$MU_{e_t,P}/n_t\kappa_i = 2a_{t+1}z(0,\kappa_i)g^{'}(e_t)\lambda_t v^{'}(2\lambda_t z(0,\kappa_i)e_t(0)+1)/(\alpha\gamma)$$

We will derive the term partially with respect to κ_i and obtain:

$$2z'(0,\kappa_i)a_{t+1}g^{'}(e_t)\lambda_t\Big(v^{'}(\lambda_{t+1}) + v^{''}(\lambda_{t+1})2z(0,\kappa_i)\lambda_t e_t(0)\Big) \tag{3.11}$$

We know, that increased number of children negatively influences the transmission factor $2z'(0,\kappa_i) \leq 0$. The sign of the whole derivation depends on the term in the brackets. Observe, that $v^{''} \leq 0$ and $v^{'} \geq 0$. If the term is negative, and the change is strong enough to offset decline in term a_{t+1} than marginal utility of education increases and holds : (3.8) $\leq$ (3.10) $\prec$ (3.9), provided that price elasticity of consumption on child adoption[4] is larger than one, than education will rise. In theoretical case, was the elasticity sufficiently large, and the positive effect of increased κ_i on (3.9) sufficiently strong, than even the attained level of human capital could rise as consequence of child adoption. If equation (3.11) was negative or the change not strong enough to offset decline in a_{t+1}, and consumption elastic, the education will rise or decline depending on the shape of utility functions. If consumption is not elastic, than education will decline.
As many contradictory affects are at work, impact of HIV/AIDS can not be with the given information precisely described. Adoption increases budget, makes consumption relatively expensive, on the other hand lowers the education transmission factor, and due to HIV/AIDS increased premature mortality a_{t+1} disfavors education. The overall effect on education and on human accumulation steady states can be evaluated only with the precise knowledge of utility functions and prices.

3.2 Extended family setting

At this point virtually no social security is present in Sub-Saharan Africa, only estimated 5%- 10% do have at least some coverage. The money to satisfy basic human needs in seniorhood have to come from different source. The source in following setting will be relatives, involved in so called Extended Family. Extended Family is setting in which

[4]Observe, that such elasticity is not price elasticity in the "ceteris paribus" sense, as increased number of children does not affect only price of consumption

individual families are united in to social net, in which resources are in need pooled, help lended and seniors supported. In such setting the source of seniors income are their grown offsprings. Adults do take care of their retired parents, while expecting the same from their children once they retire. In such setting is plausible to assume, that grandparents will consume fraction η of their offsprings (productive adults at that time) consumption. The logic is very much same as behind the childrens consumption. Note, that as the only costs of schooling are opportunity costs, all earned money goes towards consumption, seniors are thus given fraction of parent's and children's wage. It is also worh noticing, that depending on the number of their offsprings, more than one pair of productive adults [5] might be contributing towards seniors.

The Extended family setting can be seen as a form of insurance based on altruism toward relatives and on the knowledge, that same altruism and sense of responsibility is felt by the other members of the wide family. The incentive to adopt child is in such setting aside of altruism fueled by the incentive to buy insurance, as by child adoption the pair lends help to Extended Family, where same kind of help will be given back in need. In reality such behavior is widely observed in Africa, where help is lended and orphans adopted by their close relatives, or in some case rotate throughout relatives' families (Kakwani & Subbarao, 2005)(Foster & Williamson, 2000). With the outbreak of the HIV/AIDS epidemics the number of breadwinners declines while the number of orphans dramatically rises, increasing the burden carried by survivals, adoptive parents at that time. In the case of severe HIV/AIDS epidemics even the "insurance-like" Extended Family might not be able, or the incentive might be insufficient to support all orphaned children (Foster, 2000). In such disastrous scenario, when productive adults are unable to carry the burden of the disease anymore, orphans are sometimes adopted by their grandparents, all of them often consequently suffering from severe poverty with no, or very limited access to education (Kakwani & Subbarao, 2005). In the worse case, even the grandparents are not able to lend help anymore and children end up on their own. The refusal to adopt child comes however with some penalisation, the family will be less willing to help someone who refused to help them previously. We will for simplicity assume, that refusal means excommunication from the family circle. By refusing child adoption, the pair looses chance to get help from their relatives in the future. The Extended Family as such is than under threat as relatives loose confidence in each other and the role of

[5] Given the properties of the model in period $t+1$ $a_{t+1}(n_t/2a_t)$ adults contribute towards senior in E.F. setting. Observe, that if PAYG pension system was introduced, the rate of contributors to seniors would be same.

Extended Family in the society might be significantly weakened or even diminish. As for human capital accumulation Extended Family acts very often as the best setting, on following pages need for supportive intervention will be stressed out.
We will assume, that after death of partner, person finds adequate new partner, as was described in previous section. The extended family will then help them raising their extra children. The burden of HIV/AIDS will than be unlike in previous case distributed evenly. The budget of the pair and their offspring, that are part of the extended family than looks as follows:

$$[2 + \beta n_t \kappa_i + 2\eta]c_t(0) + \alpha\gamma n_t e_t(0)\kappa_i = \alpha(\Lambda_t(s_t) + n_t\gamma\kappa_i) \tag{3.12}$$

η stands for the level of contribution toward senior, we will assume that η is fixed across generations, as children will feel obliged to contribute same fraction of income as their parents did.
κ_i is multiplier, that denotes the number of adopted children, which is given as $(\kappa_i - 1)n_t$. $i = 0, 1$ stands for remarried pair (one of the original pair was deceased), $i = 1$ for pair not directly affected by HIV/AIDS. Broader explanation will be given in the section dealing with HIV/AIDS epidemics outbreak. The following analzyssis will be conducted for non-AIDS scenario, $a_t = 1$, we will however always write a_t in the equations, as we will use them when describing impact of HIV/AIDS outbreak in which case $a_t \prec 1$
s_t Denotes the family state. Since pooling is assumed to take place in the case of Extended Family, s_t will equal to zero.
The utility function of the adult pair in the extended family setting U_E, where the subscript E will from now on stand for Extended family stetting, will look as follows:

$$U_E(0) = 2\Big(u(c_t(0)) + n_t\kappa_i a_{t+1} v(\lambda_{t+1}) + \frac{xu\big(n_t\kappa_i a_{t+1}\eta c_{t+1}/xa_t\big)}{(1+\rho)} + \tilde{w}(\kappa_i)\Big) \tag{3.13}$$

x stands for the fraction of period seniors will live. She will thus consume $1/x$ times more than if she lived throughout whole period. However hers utility of consumption will be fraction x of the utility she would have if she lived throughout the whole period, as she will enjoy her consumption only for the fraction of period. We will assume that $x \in (0, 1)$, is known, as the seniors base their expectations on publicly available life expectancy.
nc_{t+1} is seniors consumption based on hers child consumption.
$\tilde{w}(\kappa_i)$ is the function denoting utility from child adoption. In comparison to Nuclear family under pension system, utility from child adoption is changed. Adoptive parents

do feel same altruism as in previous example given by function $w(\kappa)$. Above that the the incentive to invest in to the "insurance" provided by the Extended family favors child adoption. As was described above, if they adopt child, they can expect help from the family in the future, if they do not adopt child, they loose favor and the family will not help them in need. The utility function of insurance $O(j)$ is given as follows:

$$O(j) = \begin{cases} O(0) = 0 \\ 0(1) = k \quad k \in R^{+} \end{cases} \tag{3.14}$$

Where state $j = 0$ denotes refusal to adopt child, whereas state $j = 1$ denotes child adoption. The marginal utility of child adoption is than severely modified. Assuming altruism is same in both of the scenarios, and that refusal to adopt child equals complete loss of Extended Families' support, we can claim, that:

$$MU_{\kappa,E} = MU_{\kappa,P} + O(1) \tag{3.15}$$

The situation is illustrated on graph (3.2).

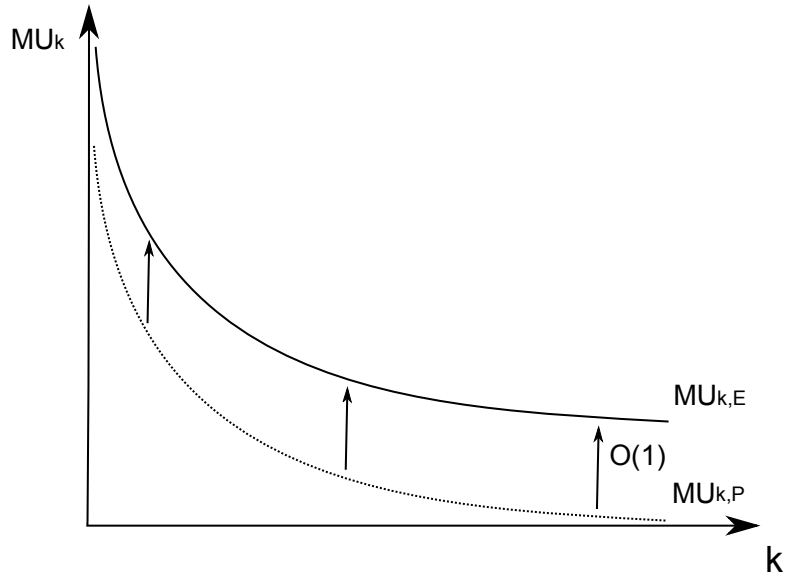

Figure 3.2: Marginal utility of child adoption under Extended Family setting

The incentive to adopt children is in Extended Family setting larger than in the Pension system setting. We will make no assumptions about the magnitude of the difference, which will depend on the utility of insurance.

Now the effect of increased education on future consumption will be assessed.The chil-

dren's (In time $t+1$ productive adults) budget line is written as:

$$\Omega_{t+1}(\Lambda_{t+1}, s_{t+1}, n_{t+1}, a_{t+1}) = \alpha(\Lambda_{t+1}(s_{t+1}) + \gamma n_{t+1}/a_{t+1}) \tag{3.16}$$

If we derive (3.16) with respect to education e_t we obtain:

$$\frac{\partial \Omega_{t+1}}{\partial e_t} = \underbrace{\alpha}_{\succ 0} \overbrace{(3 - s_{t+1})}^{\succ 0} \underbrace{z(0)}_{\succ 0} \overbrace{\Lambda_t(0)}^{\succ 0} \underbrace{\frac{\partial g(e_t)}{\partial e_t}}_{\succ 0} \Rightarrow \frac{\partial \Omega_{t+1}}{\partial e_t} \succ 0 \tag{3.17}$$

Given that consumption is non-inferior good and (3.17) epartially deriving, we obtain at once that:

$$\frac{\partial n_t a_{t+1} \eta c_{t+1}(\Omega, 0)/x a_t}{\partial e_t} \geq 0 \tag{3.18}$$

The effect of increased education of children has positive or no effect on parent's consumption during their seniorhood, such conclusion accords with basic intuition. We will employ partial derivation once more, and given that u(.) is increasing we immediately obtain, that:

$$\frac{\partial \frac{xu\left(n_t a_{t+1} \eta c_{t+1}(s_t, s_{t+1})/x a_t\right)}{(1+\rho)}}{\partial e_t} \geq 0 \tag{3.19}$$

Parents' utility of consumption in hers retirement is non decreasing in childrens' education.Such conclusion again accords with basic intuition, as increased level of education brings increase or same level of consumption, which in turn brings increased or same utility.

Parents will maximize their utility (3.13) subject to (3.12) in following manner:

$$\arg\max_{c_t, e_t, \kappa} U_E(0) \tag{3.20}$$

As we have no assumptions about the price elasticity of consumption, we can not estimate, how introduction of seniors to households' budget effects education. It is a slight drawback, as the model gets less straightforward. Since we will be able to produce desired results without any assumption about price elasticity, we will sacrifice straightforwardness of the model. For now, we would like to have at least some rough comparison between Extended Family Setting and Mandatory Pension System. We can claim, that if pension contribution P, is as high as contribution toward seniors in the extended family setting, than the level of education in Extended Family Setting will be strictly higher. Detailed

proof is given in the appendix.

For comparable scenario both of the steady states have in comparison to Mandatory Pension System shifted toward lower level of human capital. We can conclude, that in the case of social security, provided levels of contribution are same, the Extended family is more convenient. Provided only 5 to 10 percent of the population in Sub-Saharan Africa do have at least some social security, and based on recent studies (Kakwani & Subbarao, 2005)(Foster & Williamson, 2000) we believe that Extended Family is still the prevailing reality in Sub Saharan Africa. On following pages its' ability to cope with HIV/AIDS epidemics will be assessed.

3.2.1 HIV/AIDS epidemics outbreak

In this section impact of HIV/AIDS epidemics outbreak on Extended Family will be analyzed. Let us remind equation (2.1), in which both side orphan end up with low level of human capital in general state of backwardness, from which hers descendants cannot for lack of money or knowledge escape. One of the main threats of HIV/AIDS is than appearance of both side orphans throughout all levels of society, to which no help is given. Since everyone has some probability of becoming both side orphan, if no help was given to them the whole society would "family by family" descent in to poverty. Since orphans are adopted and resources pooled the Extended family setting is very durable to HIV/AIDS threat. However if the epidemics spreads too much not even such durable social establishment can with its' limited resources withstand it.

We will briefly compare child adoption under Extended Family and Nuclear Family under Pension system in following paragraph. We have prooven, that in the Extended Family setting children are better educated than in Nuclear Family setting. Observe that in comparison to Nuclear Family, increased level of education e_t and decreased level of consumption c_t in the Extended Family Setting turns equation (A.3) in to an equality. Marginal utility of education decreases in comparison to Pension system scenario. As price of education remains unchanged, marginal utility of education divided by its' price declines. In optimum, for all bought goods - "consumption, education, adoption" holds : $MU_E i/P_E i \prec MU_P i/P_P i$. The index "E" denotes Extended Family setting, index "P" denotes Pension system. Let us look more closely on the term denoting marginal utility

of child adoption:

$$MU_{\kappa,E} = \frac{2\tilde{w}'(\kappa)}{\beta n_t c_t + \alpha\gamma n_t(e_t - 1)} \tag{3.21}$$

Observe, that in comparison to Pension system the marginal utility of child adoption is higher, however the price increases. The increase in price of child adoption is given as decrease in consumption affects both parents and children, whereas increase in education, to which the saved money from decrease consumption are invested affects only children. The denominator will thus be larger than in the case of Pension system. The overall affect can not be established. In the absence of policy interventions Extended Family always leads to higher level of education, however if our goal was adoption of both side orphans, the Nuclear Family might theoretically win. In other words being member of Extended Family might not be enough beneficial to keep on adopting children, adults might increase their utility by not adopting children and facing excommunication from the family. The adoption would theoretically stop sooner than in the case of Nuclear Family under Pension system. Notice, that whether the Extended Family will or will not be better both in providing education and adopting children depends crucially on the function denoting the insurance effect of the family O(j) and on both of the utility functions u(.),v(.) as based on these consumption decreases and education increases in the case of Extended Family. Such findings are somewhat unexpected, however if we think about them for a moment logic ones.

Conclusion 1. In the absence of policy intervention, Nuclear Family setting could act as better safety net for orphans than Extended Family setting.

Let us now look at the impact of HIV/AIDS on child adoption in the case of Extended Family setting. For non-corner solution and both of the steady states Λ_F^a and Λ_F^d the optimality condition $MU_i/P_i = MU_j/P_j$ has to hold. In our case the condition will look as follows:

$$MU_{e_t,E}/P_{e_t} = MU_{c_t,E}/P_{c_t} \leq MU_{\kappa,E}/P_\kappa \tag{3.22}$$

Where $P_{e_t}, P_{c_t}, P_\kappa$ are the prices of education, consumption and child adoption.
$MU_{e_t,E}MU_{c_t,E}MU_{\kappa,E}$, are marginal utilities of education, consumption and child adoption under Extended Family setting. Observe, that if (3.22) holds for all levels of human capital λ_t as an inequality:

$$MU_{e_t,E}/P_{e_t} = MU_{c_t,E}/P_{c_t} \prec MU_{\kappa,E}/P_\kappa \tag{3.23}$$

than families are willing to adopt more children than are available, all orphans due to HIV/AIDS epidemics outbreak or progress will be taken in and fostered. Let a_t denote level of adults survival rate, at which (3.22) holds as an equality. For all levels of premature adult mortality a_t for which $a_t \in < a_{t,1}, 1 >$, all both side orphans are adopted by their relatives, if newly formed pair has excessive number of children, the extended family helps with their rearing. In such situation $\kappa_i = 1/a_t$, one side orphans as well as both side orphans are distributed evenly amongst relatives. Resources of the Extended Family are perfectly pooled.

If a_t falls bellow $a_{t,1}$ (3.22) does not hold for certain level of human capital$\lambda_{t,a_{t,1}}$, marginal utility of child adoption, divided by its' price is too low. Certain group of people endowed with human capital $\lambda_{t,a_{t,1}}$ will stop adopting children and leave the Extended Family setting. In such group both side orphans appear and consequently descent to general state of backwardness begins. If the mortality further rises, increasing portion of the society leaves the Extended Family setting and stops with adoptions, finally when survival rate falls bellow point $a_{t,2}$ the whole society under Extended Family setting stops adopting children. The system breaks down, insurance role of the Extended Family disappears, society moves to Nuclear Family setting. Only incentive to adopt children remain altruism. If the adults continue to support their parents (the role of children as source of pension remains), than for all levels of λ_t holds that remarried pairs will rise $\kappa_0 \in (n_t/a_t, 2n_t >$ children[6], whereas not by AIDS directly affected families will raise $\kappa_1 n_t \prec n_t/a_t$ children, the burden of the epidemics is distributed unevenly. Both side orphans who have to take care of themselves appear and the society heads toward general state of backwardness. Observe, that in comparison to Nuclear Family under Pension system children get better education, but the incentive to adopt them is lower. If the Extended Family was to break down (provided offspring will still take care of their parents) and we were to minimize the number of both side orphans, than Nuclear Family would be better or same option. The Nuclear Family would be strictly better option, if:

$$\exists \Lambda_t, \Lambda_t \geq \Lambda_t^*(a_t, a_{t+1}) : \kappa_0 = \kappa_1 = n_t/a_t \tag{3.24}$$

(3.24) is telling us, that Nuclear Family will be strictly better, if children are adopted for certain level of human capital that is not heading toward general state of backwardness due to decreased marginal utility of education. In such case government would have to introduce Pay as You Go pension system, to take care of the one generation of seniors, that did not contribute toward pension system. The situation will be identical even if

[6]we will assume, biological parent will not get rid of their own child even if it was to rise utility

only some fraction of the society abandons the Extended Family establishment. For those groups pension should be introduced. Fraction of the society, that manages to pool its' resources under Extended Family setting, should remain in the Extended Family setting.

We have illustrated, that in the absence of supportive policy, the Extended Family setting can break down as a consequence of HIV/AIDS epidemics. Moreover, if that was to happen, the Nuclear Family under Pension system would be better option.
Conclusion 2. If Extended Family setting breaks down and children continue to support seniors. Nuclear Family setting will be better option to fight HIV/AIDS epidemics.

Let us for a while leave the most pessimistic scenario and focus on HIV/AIDS epidemics outbreak, provided that there are enough resources in the society to adopt all orphans and thus cope with the epidemics. We will discuss impacts on the optimal education. To analyze the situation the marginal utilities divided by its price have to be defined more precisely:

$$MU_{e_t,E}/P_{e_t} = \frac{4a_{t+1}z(0,\kappa_i)g^{'}(e_t)\lambda_t\big(v^{'}(\lambda_{t+1}) + \frac{u^{'}(C)\eta c^{'}_{t+1}(\lambda_{t+1})}{1+\rho}\big)}{\alpha\gamma} \tag{3.25}$$

$$MU_{c_t,E}/P_{c_t} = \frac{2u^{'}(c_t)}{2+\beta\kappa n_t + 2\eta} \tag{3.26}$$

$$MU_{\kappa,E} = \frac{2Q^{'}(\kappa)}{\beta n_t c_t - \alpha\gamma n_t(1-e_t)} \tag{3.27}$$

We can see, that outbreak of HIV/AIDS epidemics will have impact on all three terms represented by equations (3.25)(3.26)(3.27).
At first impact on $MU_{e_t,F}/P_{e_t}$ will be assessed. Outbreak of HIV/AIDS will increase pre-mature adult mortality, and so decrease probability of survivor given by a_{t+1}. We observe two contradictory impacts on (3.25). The multiplier a_{t+1} decreases. On the other hand the effect of HIV/AIDS outbreak on term $v^{'}(\lambda_{t+1}) + \frac{2u^{'}(C)\eta c^{'}(\lambda_{t+1})\alpha\kappa}{1+\rho}$ can not be established. Let C denote the seniors expected consumption κ $C = n_t a_{t+1}\kappa\eta c_{t+1}(\lambda_{t+1})/x$. Than decrease in a_{t+1} influences expected consumption C negatively, thus increasing the derivation $u'(C)$, whereas number of adopted children κ influences the term positively, thus lowering $u'(C)$ Not knowing the progress of the disease in the next period we can not make any claims about change in $u^{'}(C)$. Given our knowledge concerning shape of

the functions, the overall impact can not be estimated.

$$\frac{\partial MU_{e_t,F}/P_{e_t}}{\partial a_{t+1}\partial a_t} \quad ? \quad 0 \tag{3.28}$$

Observe, that if the disease did already peak, which many argue is the case of sub Saharan Africa. Than the premature adult mortality would decline or remain the same for all subsequent periods, than we could rewrite (3.28) as follows:

$$\frac{\partial MU_{e_t,F}/P_{e_t}}{\partial a_{t+1}\partial a_t} \prec 0 \tag{3.29}$$

The HIV/AIDS epidemics would have negative effect on the level of schooling in comparison to no AIDS scenario.

Same as in the case of pension system, two more influences are at work. First of them is increased price of consumption given as: $2 + \beta\kappa_i n_t$. As we have no assumption on the elasticity of consumption on the number of children per family, we can make no claim about the change of expenditures on consumption.

Second effect is change in the transmission factor $z(0)$, given the properties of $z(.)$, we obtain, that $z(0,\kappa_i) \leq z(1,1)$. The effect of such change is same as in the case of Pension system hard to identify. We will discuss the impact of HIV/AIDS on the marginal utility of education per child, which is given as:

$$MU_{e_t,E}/n_t\kappa_i = 4a_{t+1}z(0,\kappa_i)g^{'}(e_t)\lambda_t\big(v^{'}(\lambda_{t+1}) + \frac{u^{'}(C)\eta c^{'}_{t+1}(\lambda_{t+1})}{1+\rho}\big)$$

We will derive the term partially with respect to κ_i and obtain:

$$4a_{t+1}z^{\prime}(0,\kappa_i)g^{'}(e_t)\lambda_t\Big(v^{'}(\lambda_{t+1})+\frac{u^{'}(C)\eta c^{'}_{t+1}(\lambda_{t+1})}{1+\rho} + z(0,\kappa_i)g(e_t)\lambda_t\big(v^{''}(\lambda_{t+1})+\frac{u^{'}(C)\eta c^{''}_{t+1}(\lambda_{t+1})}{1+\rho}\big)\Big) \tag{3.30}$$

As in previous case, increased number of children negatively influences the transmission factor $2z^{\prime}(0,\kappa_i) \leq 0$. The sign of the whole derivation depends on the term in the brackets. Observe, that the first two terms are positive, term $v^{''}(\lambda_{t+1}) \leq 0$ where the last term $c^{''}_{t+1}$ can have for certain intervals of human capital different sign. The overall impact can not be evaluated. If the term is negative, and the change is strong enough to offset decline in term a_{t+1} than marginal utility of education increases and holds : (3.26) $\prec$ (3.25), provided that elasticity of consumption on number of children is larger than one, than education will rise. In theoretical case, were the elasticity sufficiently large, and the positive effect of increased κ_i sufficiently strong, than even the attained level of human capital could rise. If equation (3.11) was negative or the change not strong enough to offset decline

in a_{t+1}, and consumption elastic, the education will rise or decline depending on the shape of utility functions. If consumption is not elastic, than education will decline. The discussion is very much same as in the case of Nuclear Family. However the term (3.11) is different to (3.30). It may well be that under one setting increased number of children per family leads to increased education and even human capital, whereas under the other completely opposite effect takes place.
As many contradictory affects are at work, impact of HIV/AIDS can not be with the given information precisely described.

If the overall impact was positive, given that Marginal utility function if continuous, the steady states would shift toward lower level of human capital. Were the impact negative, the steady states would have shifted toward higher level of human capital. We are unable to conclude preciously what impact will the HIV/AIDS epidemics have on the human capital accumulation.

3.3 Policy recommendation

Governments do play a crucial role in todays economy. Their interventions are sometimes for better sometimes for worse. We believe, that sound, well coordinated policy can significantly ease the impact of HIV/AIDS pandemic. In this section we will identify the most fit setting and policy to fight HIV/AIDS epidemics long run consequences. It should be clear from the preceding text, that in our eyes the real threat imposed by HIV/AIDS is its impact on economy and society via human capital accumulation distortion. We will assume, that the government already has some source of money to cover its' expenditures. The money raised through on following pages proposed taxation can thus go exclusively toward fighting AIDS. Original taxation could be for example included in the parameter α, where employee would get only part of his production, while the other part would be collected by the state in the form of taxes. From such taxes reasonable health care system, schooling system, infrastructure etc. is financed.
With the breakout of HIV/AIDS epidemics government will decide on the strategy to minimize impacts of HIV/AIDS. Increased spendings on health care to prolong life of HIV/AIDS positive and preventive measures on one hand, increased investment in to education that will prevent the human capital collapse in the long run on the other. The

authors believe, that in Sub-Saharan Africa the resources available for medical treatment and HIV/AIDS spread prevention are sufficient (WHO, 2003)(PEPFAR, 2008). The resources come from the taxation, but more importantly from big foreign donors[7], that are focusing on short run impacts of AIDS and provide money for ARV, hospital equipment etc. The problem with resources contributed toward medical sector is its inefficient use on expensive ARV treatment rather than cheap and effective prevention (Canning, 2006), close to 80 percent of PFPFARs' contributions goes toward direct tretnment, whereas only slightly above 20 percent goes toward prevention. The main threat of the epidemics as we have identified it, worsened conditions for human accumulation and emergence of both side orphans are unfortunately not a main issue for the key foreign donors . The Government should thus focus on increasing incentive to send children to school and supporting orphans' adoptions. So preventing dramatic decrease in country's human capital, growth, and increasing poverty and income inequality.

Our goal will be identification of the most fit setting to cope with the HIV/AIDS pandemic provided government actively supports education and child adoption. We will start with brief summary of results from previous pages.
We have previously concluded, that the choice of optimal scenario will depend on the variable we want to maximize. Were we to maximize education, the Extended Family would always win [8]. However were we to minimize number of unadopted both side orphans in the society, Nuclear Family could theoretically prove to be more fitting. Which scenario is better will crucially depend on the utility of "being member of the Extended Family". If people value their membership very high, they will not risk excommunication from the family as a consequence of child adoption refusal. Since children get better education under Extended Family than in Nuclear Family under Pension system, they cost more. Than if the person values membership in the family very little, she will stop adopting children earlier than in the case of Nuclear Family, under which "every man for himself" holds. We will examine, how the merit of each of these settings changes under sound policy. Considering the quality of many Sub-Saharan governments we might get a little skeptical about implementation of "sound policy". The reality might be such, that in to some extend autonomous Extended Family, which is able to solve its' own problems might be the best option which is only distorted by interventions from outside. For now

[7]PFPFAR, UNAIDS,WHO etc.

[8]provided contribution toward seniors in Extended Family setting is same as toward pension fund in Pension system

we will assume (and hope) that African governments are able to identify and willing to introduce the best policy to fight HIV/AIDS epidemics outbreak.

3.3.1 Support to education and family structure

From previous discussions should be clear, that providing full education to whole society[9], when both side orphans exists is as pointless as ensuring, that all orphans are adopted, when the society is bellow steady state of human capital accumulation Λ^*. In both of the scenarios the society will over some finite number of periods end up in state of general backwardness. These two extreme cases are depicted on following graph (3.3).

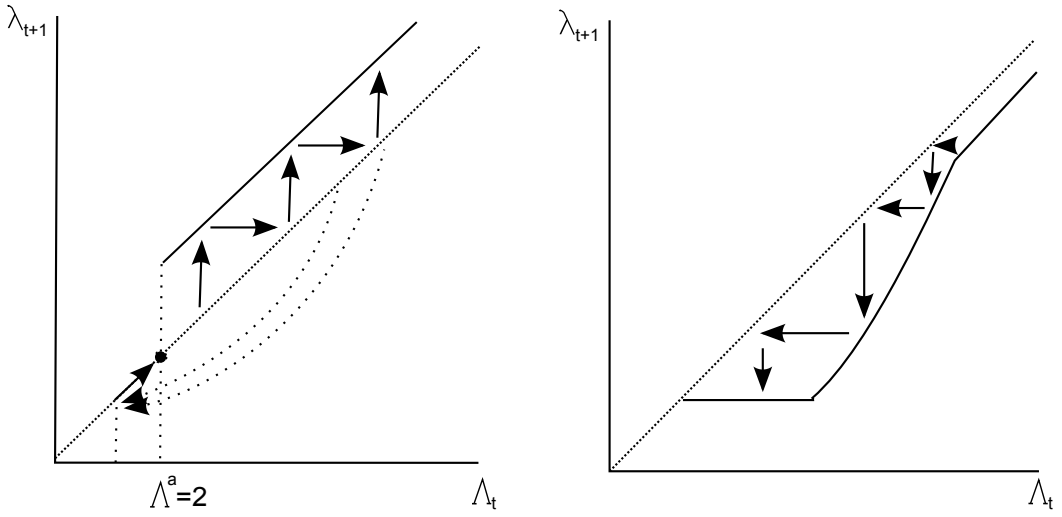

Figure 3.3: Human Capital Accumulation - Extreme Cases

We could loose the assumption, that parents with low level of human capital do not feel altruism toward their children s' education. Than system with following properties will prove to be unfit to withstand HIV/AIDS:

1. Altruism toward children is functioning for all levels of human capital.
2. Not all orphans get adopted for all level of human capital.
3. Such subsidy is introduced, that all non-orphaned children attain higher level of human capital than their parents

[9]Provided that holds assumption, that parents endowed with human capital $2 \geq \Lambda_t$ will not feel satisfaction from childrens' education and will not educate them

Such situation is depicted on the first graph of figure (3.4). We can see, that very few people do attain high level of human capital, as in each period in each state the probability of falling back to state ξ attained by both side orphans equals to $(1 - a_t)^2$. In such situation the unbounded growth of human capital would not be possible, as the probability of not occurring death of both parents in the family line until period "n" given as $(1 - (1 - a_t)^2)^n$ would be for increasing number of periods approaching zero. However, depending on the size of the population it could take enormous amounts of periods, till each family returns with high probability at least once to state od general backwardness. Provided that we will keep on subsidizing only education, the amount of human capital attained by the whole society would than stabilize at some level and would stagnate. If from certain level of human capital orphans would get adopted, subsidy toward education would be sufficient to educate whole society over some finite number of periods. The costs would be in the absence of orphan adoption subsidy immense. Such option is thus again not optimal one.
For comparison second graph was included in to figure (3.4). On the second graph the policy intervention was such, that only parents with some level of human capital Λ_t^T started providing children with such education, that each child attains higher level of human capital than than the parent. Adoption of both side orphans amongst the families with level of human capital above or equal to Λ_t^T was promoted and subsidized in such manner, that all both side orphans were adopted. On the second graph part of the population starting with human capital above or equal to Λ_t^T enjoys unbounded growth. Over some time, all of them will reach such level of human capital, that no intervention toward education is necessary and more families will be becoming net contributors toward adoptions[10]. With increasing level of human capital more money is raised from taxation and saved on abandoned subsidies. The money can than be used to subsidize education and adoptions in the part of population which did not qualify for the first round of interventions. Over some time the whole population could thanks to right policy mix get on track toward ever increasing stock of human capital.

It is thus clear, that policy intervention should go both toward education and orphan adoption. Observe, that depending on the type of subsidy toward education, certain instruments might decrease price of adoption as by-product of the policy. Were we to introduce some measure to offset the opportunity costs of schooling, for example paying to children for school attendance, the cost of orphan might, depending on price elasticity

[10]More on that in following section

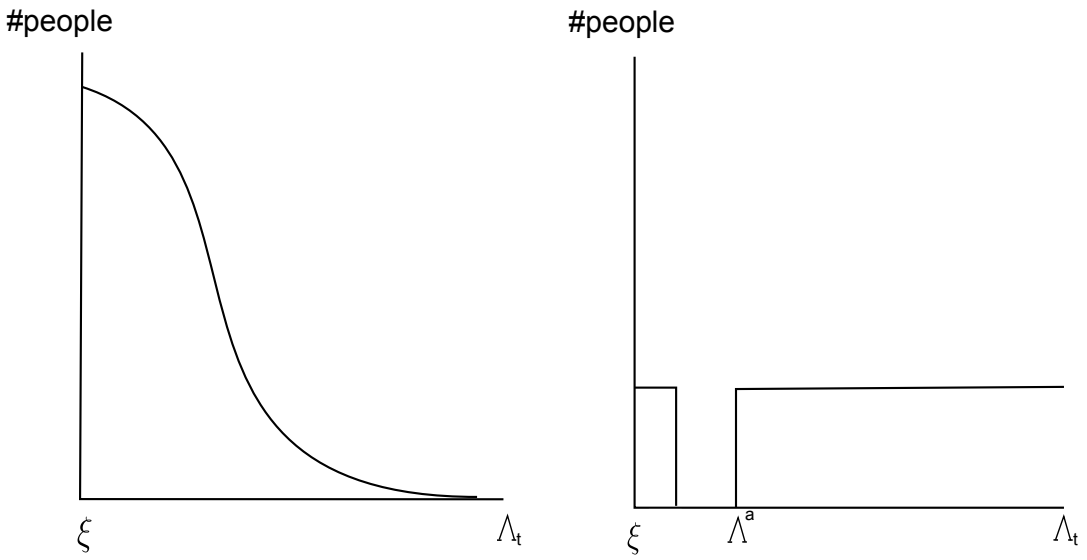

Figure 3.4: # number of people attaining certain level of human capital

of education decrease.

3.3.2 Taxation

From previous paragraph is clear, that society might not be wealthy enough to support all its' members in need. Heavy taxation should be imposed on those, who are not willing or even under subsidies able to educate their children and on the wealthiest members of the society, which can afford paying. For each level of taxation, optimal redistribution toward education and child adoption happens. Those who get subsidized education should also be adopted in case they get orphaned. In other words, if we provide education to certain group we must support adoptions as well, as without adoptions the group will slowly fall in to state of general backwardness as a consequence of HIV/AIDS. Let us for each level of taxation and resulting optimal subsidies find lowest state of human capital Λ_t^S, under the system of subsidies from this level of human capital on hold:

1. Children attain higher level of human capital than their parents.
2. All orphans get adopted.
3. For all following periods enough taxes to cover the subsidies are going to be raised.

Since our model did not differentiate between both side orphan and one side orphan (raised in remarried family with $2n_t$ children), the remarried pair will raise their utility

by letting relatives adopt some of their children. The burden of HIV/AIDS will be distributed evenly.

3.3.2.1 Tax with respect to education

Observe that provided:

1. $(\Lambda_t z(\kappa_i) g(e_t) + 1) \geq 1$[11]
2. $\Lambda_t^*(a_t, a_{t+1}, \kappa_i) \prec \Lambda_t^a(a_t, a_{t+1}, \kappa_i)$

Where $\Lambda_t^*(a_t, a_{t+1}, \kappa_i)$ is the steady state of human capital accumulation after HIV/AIDS outbreak without the presence of taxation, provided each family raises n_t/a_t children (adoption given by law, in next section subsidy toward adoption will be discussed). Than, for each number of orphans κ_i exists level of human capital $\Lambda_t^A \in < 2, \infty)$, from which imposing tax $T\Omega(\Lambda_t, \kappa_i, e_t)$ changes education e_t in such fashion, that even after adoption $(\Lambda_t^A z(\kappa_i) g(e_t) + 1) \succ \lambda_t^A$. Introduction of taxes thus does not affect unbounded growth of human capital and provides us with resources to promote education amongst families with lower level of human capital. Observe, that T could be progressive $T = T(\Omega(\Lambda_t, \kappa_i, e_t)$. The progression ought to be modest though, increased level of human capital should always lead to higher net income. If it was not this way, shift of preferences away from education could occur.

3.3.2.2 Tax with respect to adoption

Observe, that provided:

1. $(\Lambda_t z(\kappa_i) g(e_t) + 1) \geq 1$[12]
2. $\Lambda_t^*(a_t, a_{t+1}, \kappa_i, T) \prec \Lambda_t^a(a_t, a_{t+1}, \kappa_i)$

Where $\Lambda_t^*(a_t, a_{t+1}, \kappa_i)$ is the steady state of human capital accumulation after HIV/AIDS outbreak in the presence of taxation described in previous paragraph, provided each family raises n_t/a_t children (adoption given by law). Than, for each number of orphans κ_i exists level of human capital $\Lambda_t^B \in < 2, \infty)$, from which imposing tax $T = \beta c_t + \alpha\gamma(e_t - 1)$ changes education e_t in such fashion, that even after adoption holds: $(\Lambda_t^B z(\kappa_i) g(e_t) + 1) \succ$

[11] Taken from the original model (Bell *et al.*, 2003)

[12] Taken from the original model (Bell *et al.*, 2003)

λ_t^B. If we thus take the money needed to raise an orphan from the family and than give it back as contribution toward orphan, the price of orphan adoption will be zero. The children raised by such family would still attain higher level of human capital than their parents. All families with level of human capital above Λ_t^B, $\Lambda_t \succ \Lambda_t^B$ could be taxed more, whereas the money ought to be used to promote education and adoptions amongst net recipients of subsidies. We have prooven, that after HIV/AIDS outbreak some fraction of the san under reasonable taxes and subsidies enjoy unbounded growth. In reality, will the tax and contribution toward orphans adoption be lower, than $\beta c_t + \alpha\gamma(e_t - 1)$ as marginal utility of child adoption is both in the case of Nuclear Family under pension system and Extended Family larger than zero $w'(\kappa_i) \succ 0, \tilde{w}'(\kappa_i) \succ 0$.

3.3.2.3 Taxing the poor

Aside of wealthy people, those with lower human capital that are heading toward general state of backwardness could be taxed. Let Λ_t^S denote the lowest level of human capital that enjoys subsidies, fulfilling condiditions specified at the beginning of section "Taxation". Than on families in interval (ξ, Λ_t^S) could be theoretically imposed relatively high tax, as for now the society does not have enough wealth to help them. Since we want to help them eventually, when the society gets wealthier, we should keep in mind, that heavy taxation will speed up their descent in to poverty. Broader discussion is provided in section "Summary".

3.3.2.4 summary

We have shown, that for each level of HIV/AIDS epidemics outbreak, and each number of orphans, there exists such level of human capital Λ_t^B possessed by family, from which net taxes are paid, orphans are adopted and family enjoys unbounded growth of human capital. Let Tn denote the net level of contribution - Tax minus subsidy. For the tax system, under the condition that net income increases with increased level of human capital holds:

$$\int_{\Lambda_t^S}^{\infty} (Tn(\Lambda_t))Sn(\Lambda_t)d\Lambda_t \prec \int_{\Lambda_{t+1}^S}^{\infty} (Tn(\Lambda_{t+1}))Sn(\Lambda_{t+1})d\Lambda_{t+1} \tag{3.31}$$

Where $Sn(\Lambda_t)$ stands for number of pairs endowed with human capital Λ_t in time t.
As people endowed with human capital $\Lambda_t \prec \Lambda_t^S$ will get closer to state of general backwardness their tax contribution will decrease. Over finite number of periods all families

originally endowed with human capital $\Lambda_t \prec \Lambda_t^S$ will end up with general state of backwardness with human capital $\Lambda_{t+s} = 2$. Since (3.31) holds and net taxes paid by people in state of general backwardness do not change anymore, we can claim, that:

$$\int_{2\xi}^{\infty} (Tn(\Lambda_{t+s}))d\Lambda_{t+s} \prec \int_{2\xi}^{\infty} (Tn(\Lambda_{t+s+1}))d\Lambda_{t+s+1} \tag{3.32}$$

By (3.32) we obtain, that in some point $t + s + x$ in the future, enough money to subsidize education thus human capital growth and adoptions of families that are in time $t + s$ in general state of backwardness is going to be raised.

Certain portion of the subsidized population is subsidized only from contributions of the uneducated poor families with level of human capital bellow Λ_t^S, in the absence of taxation of poor class of the society and following redistribution families in certain interval $(\Lambda_t^S, \Lambda_t^1)$ would too head toward general state of backwardness[13]. The taxation of the population in interval (ξ, Λ_t^1) will be thus, as to minimize time, in which all families from the interval attain sustainable level of human capital, from which no net subsidies toward education and adoption are necessary in order to increase human capital of following generations.

3.3.3 Comparison

Comparison of Extended Family with Nuclear Family under pension system follows. We will examine situation when income tax is introduced and raised money are contributed toward education and orphan adoption promotion. Let us introduce level of taxation t, the tax will be paid as portion of income. We will keep the simplifying assumption, that all widows/widowers get immediately remarried. The budget lines for Nuclear Family under pension system and Extended Family will than have following shape:

$$(2 + \beta n_t \kappa_i)c_t(\kappa_i) + (1 - t(\Lambda_t))\alpha\gamma\kappa_i n_t e_t(\kappa_i) + 2(P) = (1 - t(\Lambda_t))\alpha(\Lambda_t + n_t\gamma\kappa_i) \tag{3.33}$$

$$(2 + \beta n_t \kappa + 2\eta)c_t(\kappa_i) + (1 - t(\Lambda_t))\alpha\gamma\kappa_i n_t e_t(\kappa_i) = (1 - t(\Lambda_t))\alpha(\Lambda_t + n_t\gamma\kappa_i) \tag{3.34}$$

[13]either because each following generation attains lower human capital or because both side orphans appear in the interval

Consumer will still maximize utility functions (3.2) and (3.13), only this time subject to (3.35), and (3.34) respectively. We can clearly see, that if pension savings P equals to contribution per senior $c_t\eta$, than proof given in appendix still holds and Extended Family provides children with higher level of education than Nuclear Family under pension system. Before the redistribution following inequalities hold:

$$e_t^E \succ e_t^P \wedge c_t^E \prec c_t^P \tag{3.35}$$

Let us find state Λ_t^S from which Nuclear Families under pension system get subsidized, as was described in section "Summary". Just to remind, for parents whose human capital lies in $\lambda_t \in <\Lambda_t^S, \infty)$, each generation of offspring will attain higher level of human capital than their parents. All both side orphans are adopted, burden of HIV/AIDS is distributed evenly.

The marginal utilities will than look as follows:

$$\frac{MU_{c_t,P}(c_{t,P})}{2+\beta\kappa n_t} = \frac{MU_{e_t,P}(e_{t,P})}{(1-t(\Lambda_t))\alpha\kappa\gamma n_t - S_{e_t,P}(\Lambda_t)} = \frac{MU_{\kappa,P}(\kappa_t)}{\beta n_t c_{t,P} + \alpha\gamma n_t(e(t)-1) - S_{\kappa,P}(\Lambda_t)} \tag{3.36}$$

$S_{e_t,P}(\Lambda_t)$ stands for the subsidy toward education. The subsidy is function of human capital.

$S_{\kappa,P}(\Lambda_t)$ stands for the subsidy toward child adoption. The subsidy is again function of human capital.

κ denotes the number of adopted children, which is given as $(\kappa-1)n_t$. Observe, that since the system of subsidies is constructed in such fashion, that remarried pairs leave their children to adoption and adoptive parents do not differentiate between both side orphan and one side orphan (coming from remarried family) $\kappa_1 = \kappa_0$, we have thus abandoned the indexation.

Now the required level of subsidies in both of the family settings will be discussed. Both of the settings will be compared if same level of education is to be provided and same number of orphans to be adopted for identical fraction of society. If we introduce exactly same level of subsidies $S(\Lambda_t) = S_{e_t}(\Lambda_t) + S_\kappa(\Lambda_t)$ in to both of the settings. The proof given in Appendix A will hold. The level of education will be higher in the case of Extended Family setting. Thus if we want to provide exactly same education to exactly same population in the Nuclear Family with pension and Extended Family setting, the subsidy in the case of Extended Family setting $S_{e_t,E}(\Lambda_t)$ will be lower.

$$S_{e_t,P}(\Lambda_t) \succ S_{e_t,E}(\Lambda_t) \tag{3.37}$$

Observe, that the subsidy toward education was as to lower opportunity costs of education. It could take form of wage given to children as a reward for school attendance. The budget constraints (3.35) and (3.34) would be modified in following manner:

$$(2+\beta n_t\kappa)c_t(\kappa)+(1-t(\Lambda_t))\alpha\gamma\kappa n_t e_t(\kappa)+2(P)=(1-t(\Lambda_t))\alpha(\Lambda_t+n_t\gamma\kappa)+S \quad (3.38)$$

$$(2+\beta n_t\kappa+2\eta)c_t(\kappa)+(1-t(\Lambda_t))\alpha\gamma\kappa n_t e_t(\kappa)=(1-t(\Lambda_t))\alpha(\Lambda_t+n_t\gamma\kappa)+S \quad (3.39)$$

We will subtract (3.39) from (3.38) and obtain:

$$(2+\beta n_t\kappa)\big(c_{t,P}(\kappa)-c_{t,E}(\kappa)\big)=0 \quad (3.40)$$

Where index P denotes Nuclear Family with pension, whereas index E denotes Extended Family. By the virtue of (3.40) (3.15) and (3.14) we obtain, that lower subsidy toward child adoption is needed in the case of Extended Family, than in the case of Nuclear Family. Let us set the aggregate level of subsidies S to such level, that holds:

$$S(\Lambda_t)=S_{e_t,P}(\Lambda_t)+S_{\kappa,P}(\Lambda_t) \quad (3.41)$$

Than holds:

$$S(\Lambda_t)-S_{e_t,E}(\Lambda_t)-S_{\kappa,E}(\Lambda_t)=\Delta S_E\succ 0 \quad (3.42)$$

The term $\Delta S_E(\Lambda_t)$ than takes form of some social security. By (3.40) we obtain, that whole term $\Delta S_E(\Lambda_t)$ is invested in to consumption, however if we take the term ΔS away from the family, education, consumption and number of adopted children, being all non inferior will decline or remain the same. Since social security transfer will never be more efficient in education and adoption promotion, than direct subsidies. We can redistribute the term $\Delta S_E(\Lambda_t)$. $\Delta S_E(\Lambda_t)$ will be split in to four groups: extra education subsidy $\Delta S_{e_t,E}(\Lambda_t)$, extra adoption subsidy $\Delta S_{\kappa,E}(\Lambda_t)$,social security transfer to the family $\Delta_2 S_E(\Lambda_t)$ and save money $S(\Lambda_t)$. Now our goal will be find minimal transfers needed to promote education and child adoption equivalent to Nuclear Family:

$$\arg\min_{\Delta S_{e_t,E},\Delta S_{\kappa,E},\Delta_2 S_E}\Delta S_{e_t,E}(\Lambda_t)+\Delta S_{\kappa,E}(\Lambda_t)+\Delta_2 S_E(\Lambda_t)\Leftrightarrow(e_t^E=e_t^P\wedge c_t^E=c_t^P) \quad (3.43)$$

We can always set $\Delta_2 S_E(\Lambda_t)=\Delta S_E(\Lambda_t)$, than Extended Family and Nuclear Family will be equal. However if there exists such combination of $\Delta S_{e_t,E},\Delta S_{\kappa,E},\Delta_2 S_E$, which

summed is lower than $\Delta S_E(\Lambda_t)$, than Extended Family is strictly better than Nuclear Family. Saved money will be used to subsidize extra portion of the population, that could not be supported in the case of Nuclear Family under pension system. The optimality condition for all levels of family s human capital $\Lambda_t \in < \Lambda^S_{t,E}, \infty)$ looks as follows:

$$\frac{MU_{c_t,E}(c_{t,E})}{2+\beta\kappa n_t} = \frac{MU_{e_t,E}(e_{t,E})}{(1-t(\Lambda_t))\alpha\kappa\gamma n_t - S_{e_t,E}(\Lambda_t)} = \frac{MU_{\kappa,E}(\kappa_t)}{\beta n_t c_{t,E} + \alpha\gamma n_t(e(t)-1) - S_{\kappa,P}(\Lambda_t)} \tag{3.44}$$

$\Lambda^S_{t,E}$ is the subsidized state of human capital, from which human capital starts growing from generation to generation. For the relationship between Λ^S_t in the case of Extended Family and Nuclear Family under pension system holds:

$$\Lambda^S_{t,E} \leq \Lambda^S_{t,P} \tag{3.45}$$

3.3.4 Comparison under adjustable pension savings

In this section assumption, that contributions toward pension system are equal to contribution toward seniors in the extended Family setting is going to be loosed. Note, that level of taxation plus pension savings is highest bearable level of levies. If we could tax more, without changing the preference map, we would have done it and set more people on track from poverty. We could be in optimal state without attaining the highest level of possible levies only if the subsidies in such state would be sufficient to subsidize human capital growth for the whole population. For the highest level of levies holds:

$$2P(\Lambda_t) + T(\Lambda_t) = \varphi(\Lambda_t) \tag{3.46}$$

Were $T(\Lambda_t) = t\alpha(\Lambda_t\gamma n_t\kappa(1-e_t))$. Than $\varphi(\Lambda_t)$ will be the highest level of contributions toward state family with human capital Λ_t can afford to pay. Term $T(\Lambda_t)$ can be however increased on the expense of pension savings. Let P_m denote the minimal level of pension senior can live off and $P(\Lambda_t)$ pension equal to contribution toward senior in the Extended Family setting (The level we have been working with so far.). The maximal level of taxation will than be equal to:

$$T(\Lambda_t) = \varphi(\Lambda_t) - 2P_m \tag{3.47}$$

We can claim, that:

$$P(\Lambda_t) \geq P_m \quad \forall\Lambda_t \tag{3.48}$$

If the equation did not hold, seniors under Extended Family setting would be suffering, which we assume is not happening. From some level of human capital the equation (3.48) holds as strict inequality. We immediately see, that setting the pension to minimal level P_m allows us to raise more money in the form of taxes $T(\Lambda_t)$. This allows us to provide education and orphan adoption support for some fraction of population, that was not subsidized previously. We immediately obtain, that:

$$\Lambda^S_{t,Pm} \succ \Lambda^S_{t,P} \tag{3.49}$$

Where index P_m denotes Nuclear Family under pension system with minimal level of contribution. The comparison with Extended Family setting follows. Nuclear Family under pension system with minimal level of contributions generates in comparison to two previous settings higher tax revenues. The system will be same or better than Extended Family, only if the extra revenues are enough to provide families with human capital in interval $(\Lambda^S_{t,P}, \Lambda^S_{t,E})$ with such subsidies $S_{e_t,Pm}(\Lambda_t), S_{\kappa,Pm}(\Lambda_t)$ that (3.44) holds in interval $(\Lambda^S_{t,P}, \Lambda^S_{t,E})$ as an equality. The condition can be slightly reformulated to obtain:

$$\int_{\Lambda^S_{t,E}}^{\Lambda^S_{t,P}} (S_{e_t,Pm}(\Lambda_t) + S_{\kappa,Pm}(\Lambda_t))d\Lambda_t \leq \int_{\xi}^{\infty} T(\Lambda_t)d\Lambda_t - \int_{\Lambda^S_{t,P}}^{\infty} (S_{e_t,P}(\Lambda_t) + S_{\kappa,P}(\Lambda_t)) \tag{3.50}$$

Equation (3.50) seems complicated, however its' message is rather simple. It basically tells us that rise in tax revenues has to be sufficiently large to cover costs due to increased number of families under governments policy to promote education and adoptions. Provided (3.50) holds, Nuclear Family setting under minimal pension will be as good as or better than Extended Family setting.

Provided contribution toward seniors equals to pension contribution, the Extended Family setting will be better option to fight the HIV/AIDS epidemics. However if the government in the Nuclear Family setting increases subsidies toward education and child adoption on the expense of seniors, Nuclear Family might cope better with the HIV/AIDS epidemics than the Extended Family. Irespectably of chosen setting, key condition to fight the epidemics is sound, coordinated and well planned policy intervention. The authors are afraid, that expecting such policy intervention in the Sub-Saharan Africa might not be realistic.

Conclusion

HIV/AIDS affects economic growth and social stability via numerous channels. Most of them are going to become less important as the prevalence rates of HIV/AIDS are beginning to fall in most of the Sub-Saharan Africa. Unlike the majority of negative imapcts of the HIV/AIDS epidemics the human capital accumulation distortion is going to be felt significantly over large number of periods and in the absence of sound government intervention for ever. In the framework of theoretical model developed in (Bell *et al.*, 2003), vulnerability of human capital accumulation under two different family settings was examined in this bachelor thesis. We have concluded, that against widely estabilished believe, the traditional Extended Family is not always in the absence of intervention best setting to cope with the HIV/AIDS epidemics. Nuclear Family might be better option, if people value relativelly little their membership in the Extended Family.

In the presence of sound government intervention could Nuclear Family again end up being better setting, as the government can on the expense of seniors subsidize education and child adoption.

At the end must be mentioned, that to cope with the epidemics' impact on human capital accumulation, well coordinated government intervention is crucial. The authors are affraid that this viable condition might not be fulfilled in many of the Sub-Saharan countries.

Appendix A

Extended Family Setting - Proof

Proposition 1. Let us have the extended family setting, with contribution toward seniors[1] $2\eta c_t$. Let us have exactly same situation only with mandatory pension system, the government will try to distort the market as little as possible, they will thus impose tax which is equal to hypothetical contribution ηc_t contribution toward seniors $\eta c_t = P$, the extended family setting always leads to higher level of education than mandatory social security.
Such proposition is intuitive one, however we were not able to employ traditional means of microeconomic analysis (Slutski equations etc.) to get results in a simpler way. Easy prove of the proposition thus follows.

*Proof 1.*Let $MU_{e_t,E}(s_t), MU_{c_t,E}(s_t)$ denote the marginal utility of education and consumption in the extended family setting and $MU_{e_t,P}(s_t), MU_{c_t,P}(s_t)$ denote marginal utility of education and consumption in the case of mandatory Pension system. Let us have tax P=$t\alpha\lambda_t$ such, that contribution toward seniors in the equlibria $c_{E,t}, e_{E,t}$ of (3.13) subject to (3.12) equal to tax t. formally:

$$P = \eta c_{E,t} \tag{A.1}$$

Let us find such $e_{t,P}, c_{t,P}$, that maximizes utility equation (3.2) subject to (3.1). We will put the pair $e_{t,P}, c_{t,P}$ in to equation (3.13). Note that, for marginal utilities in the non corner and consumptions in the steady states of human capital accumulation Λ^a_P, Λ^d_P

[1] Observe, that with the population growth in Sub-Saharan Africa, more than one family is likely to contribute toward senior, the system should thus be compared with Pay as You Go pension system, as the logic behind is similar.

holds:

$$\frac{MU_{c_t,P}(c_{t,P})}{3 - s_t + \beta n_t} = \frac{MU_{e_t,P}(e_{t,P})}{\alpha\gamma n_t} \tag{A.2}$$

That implies, that for the marginal utilities derived from (3.20) holds:

$$\frac{MU_{c_t,E}(c_{t,P})}{3 - s_t + \beta n_t + \eta(3 - s_{t-1})} \prec \frac{MU_{e_t,E}(e_{t,P})}{\alpha\gamma n_t} \tag{A.3}$$

The inequalities in (A.3) are given by following characteristics of following term: $\eta \in (0, 1)$

Note, that contribution toward seniors $2\eta c_{P,t}$ creates adittional pressure on the budget, but tax $T = 2\eta c_{E,t}$ is not paid, as the state is not optimal, too much c_t is bought $2\eta c_{P,t} \succ T$ such state is not feasible for Extended family setting. Now the subject will move directly to the optimal level of consumption $c_{t,E}$ of c_t given by (3.20) subject to (3.12), as (A.3) must hold, we claim, that $c_{t,E} \prec c_{t,P}$, such inequality implies, that: $c_{t,E}(3 - s_t + \beta + \eta) - t \prec c_{t,P}(3 - s_t + \beta)$. That implies, that for $(c_{t,E}, e_{t,P})$ not whole budget has been used up. As $c_{t,E}$ is the optimal quantity, whole "left over" budget will be contributed towards e_t, with increasing e_t the budget shrinks, as children attend school instead of work. e_t rises until it reaches optimal state $e_{t,E} \succ e_{t,P}$.

end of proof.

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