Charles University in Prague
Faculty of Social Sciences

Institute of Economic Studies

DIPLOMA THESIS

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Implied Market Loss Given Default

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Prohlášení

Prohlašuji, že jsem diplomovou práci vypracoval samostatně a použil pouze uvedené prameny a literaturu.

Declaration

Hereby I declare that I compiled this master thesis independently, using only the listed literature and resources.

Prague, 30 June 2008

Jakub Seidler

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Bibliographic Evidence Card

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Abstract

This thesis focuses on the key credit risk parameter – Loss Given Default (LGD). We describe its general properties and determinants with respect to seniority of debt, characteristics of debtors or macroeconomic conditions, and discuss its role in Basel II framework. Further, we illustrate how the LGD can be extracted from market observable information with help of both the structural and reduced-form models. Finally, by using the adjusted Mertonian approach, we estimate the 5-year expected LGDs for companies listed on Prague Stock Exchange and find out, that the average LGD for this analyzed sample is around 20%. To the author's best knowledge, those are the first implied market estimates of LGD in the Czech Republic.

Keywords: loss given default, credit risk, structural models, reduced-form models

JEL class: C02, G13, G33

Abstrakt

Tato práce se zabývá klíčovým parametrem kreditního rizika – ztrátou při defaultu (loss given default – LGD). V první části práce jsou popsány hlavní determinanty ztráty následkem defaultu odvíjející se od seniority dluhu, charakteristik dlužníka, nebo makroekonomických podmínek, a je probírána role LGD v rámci Nové basilejské dohody (Basel II). Dále jsou podrobně rozvedeny metody, pomocí nichž lze extrahovat LGD z tržních dat jak s využitím strukturálních, tak redukovaných modelů. Na závěr jsou pomocí upraveného Mertonova modelu odhadnuty pětileté LGD pro společnosti kotované na Pražské burze. Výpočty ukazují, že průměrné LGD analyzovaného vzorku firem se pohybuje kolem 20 %. Uvedené hodnoty tržně odhadnuté ztráty z defaultu patří mezi první v České republice.

Klíčová slova: ztráta z úpadku, kreditní riziko, strukturální modely, redukované modely

JEL klas.: C02, G13, G33

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List of abbreviations

ALGD	Aggregated Loss Given Default
APR	Absolute Priority Rule
ARS	Adjusted Relative Spread
BCBS	Basel Committee on Banking Supervision
CAR	Capital Adequacy Ratio
CDS	Credit Default Swap
CNB	Czech National Bank
CZSO	Czech Statistical Office
DB	Default Barrier
DD	Distance to Default
EAD	Exposure to Default
ECAI	External Credit Assessment Institution
EL	Expected loss
ELGD	Expected Loss Given Default
IAS	International Accounting Standard
IFRS	International Financial Reporting Standard
IRB	Internal Rating-Based
IRS	Internal Rating System
LGD	Loss Given Default
OECD	Organization for Economic Co-operation and Development
PD	Probability of Default
PSE	Prague Stock Exchange
RFV	Recovery of Face Value
RMV	Recovery of Market Value
RR	Recovery Rate
RS	Relative Spread
RT	Recovery of Treasury
RWA	Risk-Weighted Assets
UL	Unexpected loss

♦ Introduction

The awareness of the credit risk has largely enlarged in last decades due to an increase in the volatility in the underlying real economy, integration of financial markets and development of new financial instruments. The increased uncertainty has lead to development of new procedures and mechanisms how to determine the causality between the attributes of the borrowing entity and its potential bankruptcy. The credit risk techniques have therefore experienced a significant development of new refined methods concerning the estimation of risks and other parameters specifying possible losses.

One of those parameters is also Loss Given Default (LGD), which has obtain a greater acceptance only in recent years as the New Basel Accord identified LGD as one of the key risk parameters.[1] While estimation of probability of default (PD) has received considerable attention over the past 20 years, loss given default modeling is still a quite new open problem in the credit risk management. The estimation of LGD is not so straightforward, because it depends on many driving factors, such as the seniority of the claim, quality of collateral or state of the economy. Moreover, the insufficient database with experienced LGDs makes it more difficult to develop accurate LGD estimates based on the historical data. Hence, the extraction of LGD for credit-sensitive securities based on the market observable information is an important issue in the current credit risk area and may bring other improvements into present credit risk management.

This diploma thesis therefore discusses this key risk parameter for single corporate exposures and deals with the possibility of LGD's extractions from market information. This type of LGD modeling is denoted as *implied market LGD* and is also the main object of this work. Since the idea of LGD is relatively new and not fully understood, the thesis contributes to the efforts to explain overall concept of LGD. However, the methods of estimates and empirical applicability deal solely with implied market type of LGD. The structure of the thesis is following.

The first chapter brings an overview of the main characteristics of Loss Given Default in the credit risk framework and provides a general survey of LGD properties. It characterizes different types of LGD and the current practice in their measurement. Further, it brings the main determinants of LGD with respect to the seniority of debt, presence of the collateral, or properties of the debtor. It is also demonstrated, why debtors' industry specificity might have significant importance for value of recovered debt after default and how macroeconomic conditions implicate positive relationship between LGD and PD. The final subchapter is denoted to the role of LGD in Basel II framework. It shows how LGD enters capital calculation both in the standardized and in the internal rating based (IRB) approach.

[1] Before Basel II formalized the use of LGD, this concept was also called *Severity* (see Stephanou and Mendoza 2005).

Chapter 2 focuses on methods for implied market LGD estimation. We therefore utilize models, which use the market information for extracting credit risk parameters from specific facility or borrower and thereby evaluate its credit quality. The value of firm's assets is the primary source of information for so called structural models, which are based on the initial Merton's framework founded on contingent claim analysis. Those models of credit risk describe the default process by explicit modeling of the assets and liability structure of the company. Default occurs, if the value of firm's assets hits the particular default barrier.

We show closed-form formula for LGD in the basic Merton's approach and present the sensitivity analysis of LGD with respect to other structural parameters of the firm. Further, we illustrate how the option approach can be utilized for LGD estimation in the cases when the collateral is present, which serves as the back for the debt. After discussion of main criticism of initial Mertonian approach we describe a more complex structural model, which solves some of the Merton's simplifications and incorporates stochastic interest rate and possible default before maturity time. Within this framework we provide heuristic analysis of consequences of interest rate's and default barrier's development on LGD.

Other part focusing on LGD modeling deals with reduced-form models, which do not condition default and recovery like the structural model on the fundamental values of borrowers and use for specification of default an exogenous intensity process instead. The main source of information for LGD extraction within reduced framework is the price of risky debt. After discussion of main building blocks of reduced-form models and their assumptions about recovery rates parameterization we present a method of extraction LGD information based on the prices of corporate bonds of different seniority.

Last chapter empirically implements the structural approach and so illustrates the potential of structural models for LGD estimation. Since the application of structural models requires a value of firm's asset and its volatility as input parameters, which are non-observable variables, we also present the methods for their estimation using equity prices and balance sheet data. We estimate 5-year expected LGD for almost thirty companies listed on Prague Stock Exchange in period 2000–2008. Those are to the author's knowledge the first estimates of LGD from market information in Czech Republic.

To summarize it, the main goal of this thesis is to approach the concept of implied market LGD with its basic characteristics, estimation techniques and their possible empirical implementations.

1. Loss given default in the Credit risk

"Any scientific truth is an awaiting fallacy."

Karl R. Popper *

Credit risk is hardly a new concept. The chance that owed money may not be repaid has always been a daily fact of economic life. There is no doubt that awareness of loss has continued to grow. This has been accompanied by an increasing perception that credit risk exposures need to be more actively and effectively controlled. Not only for banks, but for all types of organizations, the necessity to use their capital as efficiently as possible is a key driver for a focus on credit risk management.

Credit risk is usually defined as the risk that "*...unexpected change in a counterparty's creditworthiness may generate a corresponding unexpected change in the market value of the associated credit exposure"* (Resti, Sironi 2007, p. 277). Credit risk is not so limited only to counterparty's default and loss resulting from its insolvency, but also to losses arising from the deterioration in credit quality, which is expressed by downgrading of its credit rating.[1]

While credit risk has been traditionally mitigated through collaterals, covenants and selections of obligors, the development of the market for credit derivatives and securitization increased the quest for advanced methods to price credit risk correctly as well as to develop better tools and techniques. That is why the quantification of credit risk has become an important topic in research in recent years and it has been further accelerated by the introduction of the Basel II Capital Adequacy Accord. With this increased vigilance on credit risk, comes a growing need to better understand its elements.

Credit risk is usually divided into several key risk parameters. The probability of default (PD), representing the likelihood of a borrower's defaulting within a certain period in the future. Exposure to default (EAD), estimating outstanding exposure at the time of default and finally loss given default (LGD), expressing percentage of exposure, which will be not recovered after counterparty's default. While in the past a lot of effort was put into the estimation and understanding of PD, the LGD received less attention and at the present leaves the most unanswered questions, mainly because LGD is difficult to quantify and it varies with the country, industry, product, or seniority of the claim.

Accurate LGD estimation is important for lending, investing or pricing of loans, bonds or credit risky instruments. It is also essential for provisioning reserves for credit losses,

* Popper, K. (1934, 1959): *The Logic of Scientific Discovery*, 1934 (as *Logik der Forschung,* English translation 1959), ISBN 0415278449.

[1] Basel II classifies changes in bond prices as "specific risk" within market risk. However, if price movements arise from factors specific to individual issuers (idiosyncratic factors) then the associated risk should rather be classified as a credit risk (Resti and Sironi 2007, p. 278).

calculating risk capital and determining fair value for any credit risky obligation. In its advanced internal rating based approach, Basel II underlines the importance of the LGD parameter by allowing financial institutions to apply own estimates of LGD in the computation of regulatory capital. Appropriate LGD estimation therefore improves the precision of both regulatory and economic capital allocation.

LGD numbers, however, will not only play a significant role in the credit risk management and future regulatory reporting, but may also be used in accounting. Considering IAS/IFRS, LGD may help with fair value[1] computations and impairment tests that provide further potential for connecting accounting and credit risk management processes, e.g. IAS asks banks to disclose fair values for financial assets and liabilities at least in the notes of the annual statement (see IAS 39.8). Furthermore, *incurred loss* as defined by IAS/IFRS and *expected loss* used for credit risk management, are not so different, or the best LGD estimate required for regulatory use and specific provisions according to IAS/IFRS are both based on expected future cash flows from a defaulted facility (see Christian 2006).[2]

As we can see, complexity of LGD parameter can be really high and despite all these fields of application, LGD has just recently began getting more attention. However, the general term LGD may express various concepts, which may differ in many features.

1.1. Definition of Loss Given Default

LGD is usually defined as the loss rate experienced by a lender on a credit exposure if the counterparty defaults.[3] Thus, despite default the lender still recovered $1 - \text{LGD}$ percent of the exposure. One minus LGD is therefore called recovery rate (RR). In principle, LGD comprises also other costs related to default of the debtor, and the correct formula should rather be

$$LGD = 1 - RR + Costs \tag{1.1}$$

Nevertheless, costs are relevant only in a specific type of LGD and are not usually so high to influence losses markedly in comparison with recovery rate. Therefore we use recovery rate as the complement of LGD in the following text and take these two parameters as conceptually the same.[4]

The stress should be admittedly put on distinguishing between measuring LGD ex-post and estimating it ex-ante; however, to gain the simple ex-post LGD, as we will see, is not so straightforward and unambiguous and that is why one can also speak about its "estimation".

[1] For definition of fair value see IASB (2005).

[2] Of course there have to be differences due to diverging intentions of Basel and IAS/IFRS – stability of the bank versus objective reporting of the bank's assets.

[3] In principle we should mark the loss rate given default as LGDR and LGD use for the absolute amount of loss. However, LGD is used to indicate the loss rate by many practitioners including the Basel II, while the absolute loss is indicating as LGD.EAD (see BCBS 2005).

[4] According to Bhatia (2006, p. 281), LGD and RR may be used in different meaning. LGD could be the term generally used in the context of tradable assets and represents loss in the market value of the bond immediately after default, while the RR term is more used for the amount recovered after default for non-traded assets such as bank loans.

The quality of ex-post LGDs' database is influencing development of models for ex-ante LGD predictions, therefore both concepts of LGD have the indispensable importance in a credit risk area.

However, any study of LGD has to exactly specify, which criterion should be used to define default event and which method should be used to measure recovery rate on a defaulted transaction because a different classification will lead to diverse results in LGD's both ex-ante and ex-post estimates.

1.1.1. Definition of default

The criterion used for classification of credit exposure in the default category is critical for a study of a recovery rates. A widely chosen definition of default leads to a lower estimate of PD but higher estimate of LGD because fewer exposures will be classified as "in default", but those will have relatively lower quality with a low recovery outlook. Conversely, from a narrower and more severe definition stems higher default rates and also higher recoveries.

Although Basel provides a standard default definition, it varies from bank to bank and country to country. In the literature are usually used following classifications (see Dermine, Neto 2005).

- *doubtful* – as soon as full payment appears to be questionable on the basis of the available information
- *in distress* – as soon as a payment of interest or principal has been missed
- *in default* – when a formal restructuring process or bankruptcy procedure has started. The legal definition is linked with the bankruptcy of the firm and will typically depend on legislation in different countries

Standard & Poor's defines default as the first occurrence of a payment default on any financial obligation. An exception occurs when an interest payment missed on the due date is made within the grace period, which typically ranges from 10 to 30 days. Distressed exchanges are considered defaults when the debt holders are forced to accept substitute instruments with lower coupons, longer maturities, or any other diminished financial terms (see S&P's 2005).

Moody's (2005) definition of default is any missed or delayed disbursement of interest or principal, including delayed payments made within a grace period, if issuer files for bankruptcy [1] or legal receivership occurs. Distressed exchange arises when (i) the issuer offered bondholders a new security that amounts to a diminished financial obligation or (ii) the exchange had the apparent purpose of helping the borrower avoid default.[2]

[1] Chapter 11, or less frequently Chapter 7, in the USA (see Moody's 2004).

[2] The default definition should capture events that change the relationship between the bondholder and bond issuer from the relationship which was originally contracted, and which subjects the bondholder to an economic

The reference Basel II default definition must be used by banks while estimating internal rating-based models. Even before Basel common habit was a one year time horizon for PD estimate. According to BCBS (2005, p. 96, §452) is default defined as

"A default is considered to have occurred with regard to a particular obligor when either one or both of the following events have taken place.

- *The bank considers that the obligor is unlikely*[1] *to pay its credit obligations to the banking group in full, without recourse by the bank to actions such as realising security (if held).*
- *The obligor is past due more than 90 days on any material credit obligation to the banking group.*[2] *Overdrafts will be considered as being past due once the customer has breached an advised limit or been advised of a limit smaller than current outstandings*."

Obviously, different definitions of default event will lead to inconsistent PD and LGD estimations. There can be differences in qualitative default criteria (as applying "unlikely to pay" from Basel II definition to companies that are in the lowest external non-default rating grades), number of days of delayed payments that will be considered as default, or other disparities mentioned by Erlenmaier (2006) e.g. while external agencies measure defaults without respect to the size of amount due, in Basel II delayed payments that are small with respect to the company's overall exposure are not counted as default.

Above mentioned problems raise a question of using agencies' ratings in Basel II framework, because Basel's definition is weaker and leads to lower PDs but higher LGDs, than would be observed under rating agencies' definition of default. However, S&P's (2006, p. 39) state, that it is relatively straightforward for agencies to produce a Basel II aligned results. Anyway, it is advisable that banks and rating agencies adopt a homogeneous definition of default in order to be able to pool and compare their estimates.[3] Especially in the light of Basel's II standardized approach, where banks rely on agencies' rating when computing capital adequacy, it is important to have unified definitions across agencies (see Chapter 1.3).

Default is not a logical consequence of a unique, well-defined process and can occur from many reasons.[4] Occasionally the approaching default can be observed ahead, e.g. by a breach of overdraft limits or deteriorating balance sheet ratios; however, the bank may not

loss. Technical defaults (covenant violations, etc.) are not included in Moody's definition of default (see Moody's 2005, p. 39).

[1] Definition of „unlikely“ is presented in paragraph 453 (see BCBS 2005, p. 96).

[2] In the case of the retail and public sector entities obligations, a supervisor may modify a figure up to 180 days for different products, if it seems appropriate for local conditions. This applies to a transitional period of 5 years (see BCBS 2005, p. 96).

[3] Non-ambiguous definition of default event is especially important for credit derivatives, where a payoff is conditional on default event. If it is not accepted by both counterparties, it is a source of conflicts and legal suits and reduces thus the usefulness of credit derivatives as an insurance product.

[4] We could divide the reasons into (i) firm specific reasons (bad management, fraud, project failure, etc.), (ii) industry specific reasons (sector shocks such as overcapacity or a rise in the price of materials), (iii) general macroeconomic conditions such as interest rate changes, recession, etc. (see Servigny, Renault 2004, p. 168).

always be in position to observe a state of distress and default can occur directly without any intermediate phase.

A firm can default on the debt obligations and still not declare bankruptcy. It depends on the negotiations with its creditors. Thus, not all losses are the result of bankruptcy after the default event. Therefore we should examine after-default performance more closely because the state of default may not lead to a simple and straightforward recovery process.

1.1.2. After default scenarios

Default event itself is a significant point to the possible recovery estimation, e.g. from the amount of the exposure at the time of default, but more crucial is a facility's after-default performance, which substantially determines a future recovery rate. One can observe a certain pattern of typical developments, which we call after default scenarios. Their numbers and exact definitions can slightly differ, depending on the bank's portfolio and also its workout strategy; however, Christian (2006) states three typical after-default developments

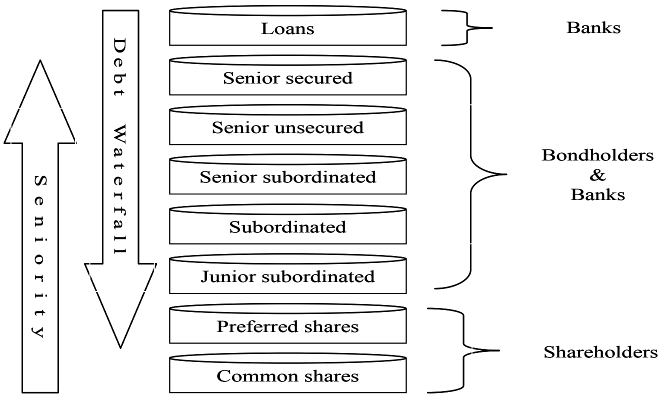
- *Cure* – the defaulted subject recovers itself of its financial difficulties after a short time and continues to fulfill its contractual obligations. This scenario means minimal losses and usually does not cause any changes in the initial structure or conditions.
- *Restructuring* – the defaulted subject recovers after a restructuring of its facilities. Restructuring [1] can be implemented through: (i) reduction in the principal amount of the facility or the amount payable at the maturity, (ii) lower interest rate than it is usual for other similar facilities, (iii) reduction of accrued interest, including full forgiveness of interest, and (iv) extension of the maturity date. Loss amount may vary but customer relationship is maintained.
- *Liquidation* – all facilities of the defaulted subject are liquidated. Loss amount is generally higher than the one by the restructuring process and customer relationship is ended.

When firm goes to the bankruptcy [2] and there is no other possibility than liquidation, the capital structure of the firm and absolute priority rule (APR)[3] is an important determinant of recovery rate. This states that the value of the bankrupted firm must be distributed to suppliers of capital so that "*...senior creditors are fully satisfied before any distributions are made to more junior creditors, and paid in full before common shareholders"* (Schuermann 2004, p. 11). The resulting cascade of payments is often referred to as the debt waterfall. The capital structure can be generally divided as it is shown in Figure 1.

[1] Restructuring is done only for some facilities that fulfill conditions of minimal age and a reassessment of the borrower's capacity to repay (see Bhatia 2006, p. 321).
[2] The bankruptcy has a form of reorganization or liquidation.
[3] Eberhart and Weiss (1998) are confirming that APR is routinely violated because of speed of resolution. Creditors agree to violate APR to resolve bankruptcies faster.

Figure 1

The capital structure of a firm



Source: adopted and changed from Schuermann (2004)

The rate of recovery of the defaulted bond depends on where the claims in the firm's capital structure are. Bonds are frequently classified in terms of seniority and allocated collateral. Seniority is capturing the order mentioned above of the claimants‘priority over the assets of the defaulted company and collateralization (called secured versus unsecured) measures the allocation of specific assets as guaranties for the facility in the case of default. The bank loans are on the top of the debt waterfall and are often highly collateralized. What are the RRs of the corresponding debts class in capital structure is analyzed in chapter 1.2.

Having outlined main definitions influencing LGD measures, we can now take a closer look at its different concepts.

1.1.3. Types of LGD

Usually two basic measures of recovery for defaulted facilities are used. The first one is the *ultimate recovery* which is the amount that the debt holder recovers after default. The second measure is represented by *post-default market prices of defaulted security* (see Servigny, Renault 2004, p. 123).[1] Both methodologies have their drawbacks. For example, ultimate recoveries are gradually traced in long-running work-out process, while post-default prices for defaulted facilities are observable soon after default. However, post-default prices are available only for debt, for which distressed market exists. Ultimate recoveries are therefore the only way to measure RR for illiquid bank loans.

From ultimate recoveries could be measured so called *accounting* and *workout LGD*, post-default prices category comes under *market LGD*. A different approach is already mentioned *implied market LGD* which is estimated from market information of non-defaulted securities. All types of LGD will be examined in a little more detailed way.

[1] Ultimate recovery belongs to the internal data-based approach whereas post-default prices fall into market-based approach of RR measure.

- **Market LGD**

As it follows from the title, this methodology considers the price of bond after default as a proxy of the recovered amount. "*The market value is the best estimate for the expected recovery and the market price reflects the expected recovery suitably discounted*" (Bhatia 2006, p. 285). Nevertheless, market prices are impacted by a supply and demand and therefore a question about market's ability to price defaulted debt efficiently comes up. On the basis of several studies it has been confirmed that market prices are efficiently stated and are predictive of future recoveries (see Gupton, Stein 2005).[1]

A variation of this approach could be the estimation of the RR based on the market value of a new financial instrument (price of a new bond) that issues a defaulted company after reorganization and restructuring the initial debt which the company emerges from default. New bond's price must then be discounted back to the moment of default using adequate discount rate. This approach is called *emergence LGD*.

Market data are an objective and updated source of information for LGD observations. However, as it was mentioned, post-default price is available only for the fraction of the debt that is traded and for which after-default market exists – very often they are available only for corporate bonds issued by large companies.[2] Market LGD is therefore highly limited for defaulted bank loans that are traditionally not traded. For them one must turn to the "ultimate" approach.

- **Accounting LGD**

Accounting LGD is based on charge-off amounts – the amount of non-performing facilities that an institution writes off its books. The charge-offs are determined by product types, past due days, collateral and by accounting standards which focus on prudence what may not be consistent with risk management policies. Also, the problem of charge-offs is that they can occur before the final resolution. All mentioned above indicates a limited use of accounting charge-off for LGD measurement and accounting data are just a starting point for collecting "true losses".

- **Workout LGD**

This method is rather analyzed from an "economic" perspective, than based on mere accounting data.[3] When measuring it, all relevant facts that may reduce the final economic value of the recovered part of the exposure must be considered. LGD is then determined by (i) loss of principal, (ii) carrying costs of non-performing assets, e.g. interest income lost or foregone, and (iii) recovery and workout expenses for example direct and indirect administrative costs.

[1] Mentioned efficiency definitely does not mean that supply and demand would not influence the price of defaulted bonds. The higher volume of defaulted bonds leads to a higher supply and to lower price, i.e. to lower RR (see Altman et al. 2005a for closer relationship between the LGD and the volume of defaulted bonds).
[2] What's more, outside the USA the market for defaulted bonds either non-exist or does not have the required depth and liquidity.
[3] Because of this economic perspective Workout LGD is sometimes called Economic LGD.

The simple definition of Workout LGD for facility *j* at time of default could be

$$LGD_j(t_{DF}) = \frac{EAD(t_{DF}) - NPV\sum_{t=t_{DF}}^{t_E} R_j(t) + NPV\sum_{t=t_{DF}}^{t_E} C_j(t)}{EAD(t_{DF})} \qquad (1.2)$$

Where *NPV* is the net present value and $R_j(t)$, $C_j(t)$ represents all recoveries and costs observed from the time of default t_{DF} to the end of workout process t_E. However, bankruptcy claims are often not settled in cash but with securities (equity, options, warrants, etc.) with no secondary market, which means that their value will be unclear for years. Another problem is that appropriate discount rate is not known, but should reflect the risk of holding defaulted asset. "*Worse still, as default is being resolved, risk changes at time passes and expectations become more definite"* (Frye 2005, p. 2) and therefore these "ultimate cash-flow" methods depend on an unknown and variable discount rate which is difficult to estimate for particular situation.[1] Therefore we must speak only about estimate of LGD even if we are trying to measure it from ex-post data.

Despite the ultimate recovery is hard to measure, the main interest of a bank is to estimate just workout LGDs, because these best reflect its losses. Market LGDs include components as a risk premium for unexpected loses, which may not be considered in workout LGDs. Moreover, workout specific costs of institution are not part of these market loss quotas. Ex-post observations of market LGDs can be used for model development, but it is necessary to make some adjustments, which take into account those differences.

- **Implied market LGD**

The basis of the implied market LGD is to derive LGD estimates from market prices of non-defaulted loans, bonds, or credit default instruments by structural or reduced-form models.[2] The idea is that prices of risky instruments reflect market's expectation of the loss and may be broken down into PD and LGD. Implied market LGD estimation does not rely on historical data and can be especially used for low default facilities, however, "*if default experience is rare for all market participants, one should not expect implied market LGD to provide more than an expert judgment of the market"*(Christian 2006, p. 150). Calculation of implied market LGD is based on asset pricing models that are usually working with risk-neutral measure. Therefore probabilities of default as well as recovery rates extracted from these models are usually also in risk-neutral measure.[3] What is the coherence between risk-neutral and observed (physical) measures will be further dealt throughout the following chapters.

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[1] Sometimes the discount rate based on historical values is used. For example, on average market rates observed between the default and the end of workout process, this would lead according to Resti and Sironi (2007, p. 349) to backward-looking measure, because in estimation of LGD on future bad loans we are interested in interest rate that might be on the market after a new default. The use of past interest rate is not appropriate for the present and future market conditions. What discount factor should be used is dealt in e.g. Maclachlan (2005).

[2] See Chapter 2.

[3] Risk-neutral measures are adjusted for the risk premium. Risk-neutral means that investors are indifferent between the same expected value of the risky cash flow and risk-free cash flow.

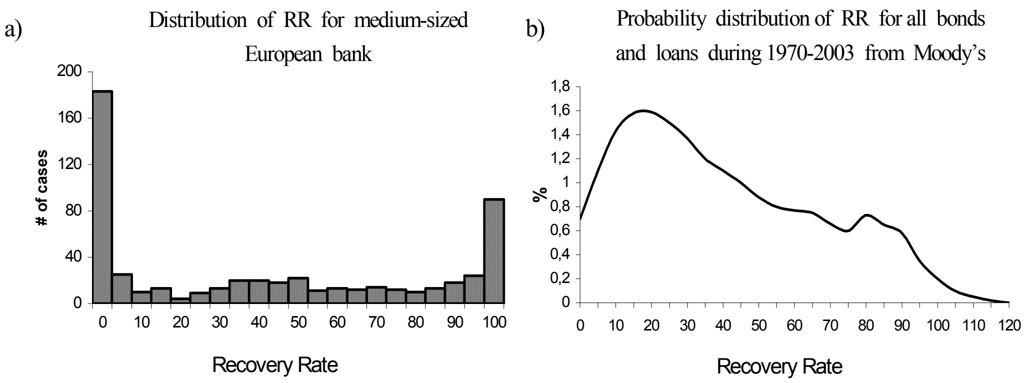
Apparently, when classifying LGD we can use different approaches that are significantly influencing its estimate. Nevertheless, these LGDs have some characteristics in common regardless what methodology was used.

1.2. Properties of Loss Given Default

In most empirical analyses LGD is implied to be a constant. However, LGD is better represented by a distribution than by a single figure. If we take a look at the distribution of recoveries regardless of any factors or characteristics, it is with two distinct humps (Figure 2). Recovery is either quite high or very low. This happens because some exposures (e.g. leasing) tend to have high RR close to 100 %, while others (e.g. unsecured overdrafts) have RR close to zero. This bimodality makes parametric modeling of recoveries arduous and therefore it is convenient to use beta kernel method (see Servigny, Renault 2004). Evidently, observed mean as appraisal of future LGDs is really poor indicator, as most values tend to cluster near 0 or 1. In such U-shaped distribution, the probability of observing values which are close to the mean is dramatically low.

Figure 2

U-shaped (bimodal) distribution of recovery rates



Source: adopted from Resti, Sironi (2007) and Schuermann (2004)

A better information capability than a simple overall average value would have the conditional means. It is therefore convenient to break down dataset of ex-post LGDs to some clusters with similar characteristics for which is the “within” variance of RR relatively low. The division into groups should be based on the factors that are significant in explaining empirical differences among recovery rates such as the level of seniority of underlying instrument, availability of collateral or presence of any guarantees. These cluster-conditional means would then offer a more reliable approximation of the expected loss rates for different types of debts.

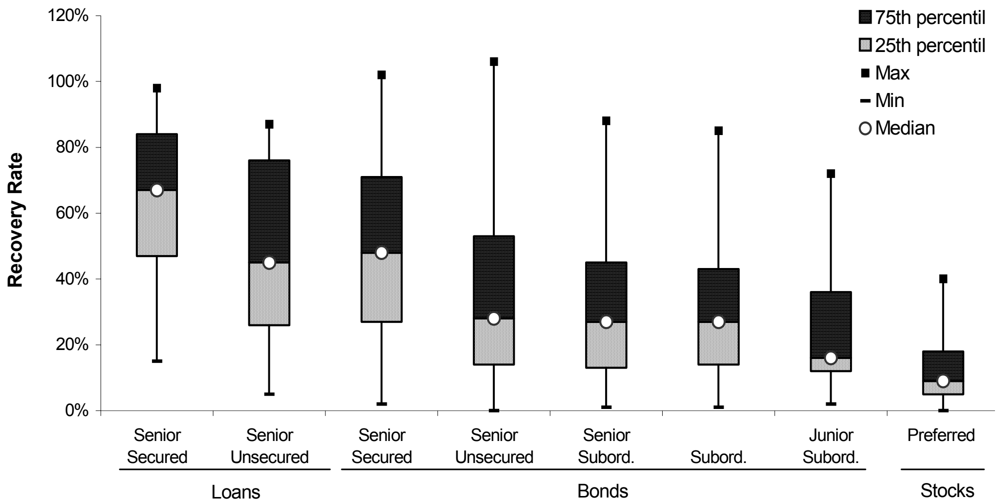
The objective of this part is therefore to review some empirical findings that have been reported on various datasets and to highlight the impact of the different factors on recovery rates. One of the most persistent results is that seniority and presence of collateral are the most significant determinants of debt recovery (see e.g. Izvorski 1997).

1.2.1. Seniority and Collateral

Empirical evidence on recovery rates is usually based on defaulted bonds because the LGDs data is simply available through after-default market information. Loans are usually expected (ceteris paribus) to perform better because they are typically more senior in capital structure, have tighter covenants and banks can more actively monitor the evolving financial health of the obligor (see Amihud et al. 2000, p. 6).[1] This fact also confirms following figure that presents the dispersion of recoveries for facilities based on debt type and seniority for 1981–2004, global.[2]

Figure 3

Recovery rates by debt type and seniority class



Source: Moody's (2005), Gupton, Stein (2005), adjustments

The shaded boxes cover the quartile range with grey extending 25th percentile to median and dark from median to the 75th percentile and vertical lines show the maximum and minimum recovered rates. This box-and-whisker illustrates the variability in each category. For example, even thought the median of recovery rate for "senior secured loans" is about 67%, in half of the cases the experienced RR was in the range of 47–84%. Evidently from other bars in different classes, grouping instruments by debt class and seniority show a pattern, but still leaves a wide variability in recovery values.

The results of several empirical studies have confirmed that the RR increases with the seniority and security of the defaulted bonds and decrease with its degree of subordina-

[1] Banks are sometimes able to change their lending relationship to better position in capital structure of the firm with anticipation of forthcoming debtor's bankruptcy and thereby raise expected recovery. The dispersed nature of bond ownership makes it impractical for bondholders to renegotiate the terms of contract as debtor's conditions changes (see Schuermann 2004).

[2] Moody's (2005) estimates defaulted debt recovery rates using market bid prices observed roughly 30 days after the date of default. Recovery rates are measured as the ratio of price to face value.

tion. Results tend to be also rather similar in term of average recovery rates – for bank loans (70–84%), for bonds: senior secured (53–66%), senior unsecured (48–50%), senior subordinated (34–38%), and subordinated (26–33%). All studies also reported high standard deviation that characterizes recovery rate across all bond debt-classes, regularly overrunning 20%.[1] This implies a high degree of uncertainty concerning the expected RR and observed ex-post results may significantly differ from ex-ante estimates (see Altman, Kishore 1996, Castle, Keisman 1999, Keenan et al. 2000 or Hu, Perraudin 2002).

Except the seniority of the debt also the presence of the collateral determines significantly the recovery. Collateral consists of assets serving as a guarantee in case of debtor's default. There is a general understanding that collateral can help to reduce LGD radically, which is also empirically confirmed, but usage of collateral should not lead to non-vigilance. Firstly, value of collateral can fluctuate and falls in recession (see following chapter), secondly, it may have an adverse impact on bank monitoring, as bank does not feel a need to monitor heavily collateralized loans. Collateralized facilities sometimes experience higher default rate. According to Dermine and Neto (2005, p. 110) it is caused by a fact that guarantee or collateral may not be requested from reliable clients, so that the existence of guarantee could be an indicator of greater risk and higher probability of default.

From above mentioned follows, that average RR's values confirm the intuitive relationship between RR and debt-types showed in Figure 3. Together with loans, senior secured bonds have the highest RR whereas subordinated and junior subordinated ones have with preferred stocks the highest LGD. However, it is necessary to take into account not only the "absolute" but also the "relative" seniority.[2] Preferred stock in the lowest seniority class might hold the higher seniority rank in a firm that has no funding from loans or bonds. In addition, if firm issues debt sequentially in order of seniority, senior debt might mature earlier leaving junior debt outstanding. Castle and Keisman (2000) affirmed that the greater the proportion of junior securities in firm's liabilities is, the greater is the RR on the senior securities, because there is bigger "equity cushion" for them in liquidation process. Nevertheless, relative seniority tested in Hamerle (2006) did not show a significant influence on LGD.

By sorting the debt by the seniority and presence of the collateral we get better notion about recoveries across particular types of debts, however, their dispersion was still high because RR is still influenced by other factors, such as firm and industry specific ones.

[1] It is interesting to note that if the recovery rate probability distribution were uniform, which means that the probability of occurrence of values from 0 to 100% is the same, then its standard deviation would be approximately equal 29%. This clearly shows the big variance in RRs.

[2] If issuer A has two bonds, one in the category *subordinated* and the other *junior subordinated* and issuer B has three bonds with seniority *senior secured*, *senior subordinated* and *subordinated*, then the subordinated bond from issuer A will be served first and therefore has possibly lower LGD than the subordinated bond from B, which is served after the two other bonds (see Hamerle 2006, p. 7).

1.2.2. Firm and Industry Characteristics

Recovery rates are ultimately determined by the value of assets that can be seized in case of default. Because many asset types differ between industries,[1] it is therefore intuitive to assume that the debtor's industry characteristics can influence LGD. Also firm-specific characteristics, mostly financial, which contribute towards reducing leverage help to improve RR. Leverage indicates the extent of claimants for assets in case of default; therefore its lower value improves the enforcement of claim.[2] Empirically it was found that the leverage impacts recovery to the size of 5–15 % (see Bhatia 2006, p. 299). The firm-specific quality of assets has also its importance, as their values are the source of repayment after default. Assets, whose quality have lower likelihood to deteriorate over time and are less likely to "disappear", provide better guarantee.

Although industry-type seems like a straightforward determinant of RR, the literature does not give wholly unified answers. Altman and Kishore (1996) have broken out LGD of corporate bonds by industry and have found evidence that some industries such as *public utilities* and *chemicals*[3] do evidently better than the others. Nonetheless, they also showed that the standard deviation of RR per industry and within a given industry is still very large (see following figure).

Figure 4
Average recoveries per industries

Altman and Kishore			Acharya et al.			Moody's	
1971-1996			1982-1999			1982-2003	
Industry Description	Mean (%)	Std. Dev. (%)	Industry Description	Mean (%)	Std. Dev. (%)	Industry Description	Mean (%)
Public Utilities	70,5	19,5	Utilities	74	18,8	Utility-Gas	51,5
Chemicals*	62,7	27,1	Energy, Resources*	60	31,0	Oil and Oil Service	44,5
Machinery*	48,7	20,1	Financial Institutions	59	44,3	Hospitality	42,5
Services*	46,2	25,0	Healthcare, Chemicals	56	40,8	Utility-Electric	41,4
Food*	45,3	21,7	Building Products	54	42,1	Media and Broadc.*	38,2
Wholesale and retail	44,0	22,1	Telecommunications	53	38,1	Finance and Banking	36,3
Divers. manufacturing	42,3	25,0	Aerospace, Auto*	52	38,1	Industrial	35,4
Casino, hotel*	40,2	25,7	Leisure Time, Media	52	37,2	Retail	34,4
Building material*	38,8	22,9	High Technology*	47	32,4	Automotive	33,4
Transportation*	38,4	27,9	Consumer, Service	47	35,6	Healthcare	32,7
Communication*	37,1	20,8	Transportation	39	36,1	Consumer Goods	32,5
Financial institutions	35,7	25,7	Insurance and Real Es.	37	35,4	Construction	31,9

* Industry description is reduced

Source: Altman, Kishore (1996), Acharya et al. (2003), Moody's (2004)

Likewise Izvorski (1997) tabulated average RR for seemingly similar groups and confirmed their results. He found out that industries with higher growth tend to have signifi-

[1] Firm in some sectors have a large amount of asset that can be easily sold on the market in case of default, while other sectors can be more e.g. labor-intensive.
[2] Even for the secured debt holders can higher leverage influences the collateral enforcement and recovery "*...since concessions are often extended to the junior and unsecured debt holders for obtaining their consent to various settlement schemes"* (Bhatia 2006, p. 299).
[3] The exact name of the group is "Chemicals, petroleum, rubber, and plastic products" (see Altman, Kishore (1996).

cantly higher recoveries and more competitive industries (measured by Herfindahl index) are also associated with higher RR, as assets can be readily reused by another party which increase the liquidation value and thereby RR. A more recent study, Grossman et al. (2001) corroborated industry's influence on RR as well as Acharya et al. (2003). Their results of variations of RR across industries are even higher than in the older Altman, Kishore study, which is also reported in Figure 4 together with average RR per industry from Moody's database. However, across the board comparisons are not possible since the industry classifications are not identical, but utilities remain the industries with highest RR.[1]

An opposite view of the industry influence is presented by a study on bank loans by Gupton et al. (2000) which has on the contrary found no evidence of different LGDs across industries. Although industry's means of LGD were different, they were not statistically distinguishable. Also Araten et al. (2004) could not find significant impact of industry on LGDs observed for loans. Gupton and Stein (2005) state, that the use of recovery averages broken out by industry does not capture the industry variability in recovery rates across time. Some sectors may enjoy periods of high recoveries, but can fall later below average recoveries at other times, it means that industry recovery distributions change over time and therefore cannot be expected to hold in the future. "*Thus, recoveries in one industry may be higher than those in another during one phase of the economy, but lower than the other industry in a different economic environment"* (Gupton and Stein 2005, p. 20). As a result they concluded that industry type is not so appropriate factor for future RR predictions.

These unambiguous results of different studies might be attributed to a number of reasons. Firstly, studies focus on different facilities and use diverse sample sizes. Secondly, there are differences in industry grouping definitions among studies and finally, the authors are focusing on different periods what can puzzle the comparability because of above mentioned LGD cyclicality in relation with economic environment. The result is that the use of industry factors can bring some new outlook on LGD variations, but it has to be taken into account with macroeconomic conditions.

1.2.3. Macroeconomic conditions

Every industry has specific traits and can be in different stage of economic cycle, which can more influence LGD than the industry-type itself. In the following figure it can be clearly seen that LGD is not stable in time and obviously is underlying cyclical variability, which can be taken in relation with macroeconomic conditions. Acharya et al. (2003) showed that when the industry is in distress, mean LGD is on average 10–20% higher than otherwise.

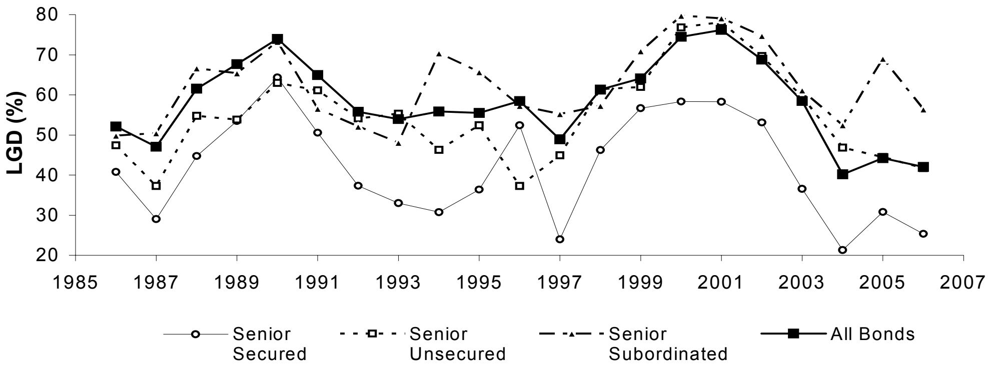
Behind the cyclical variation is the fact that as the economy enters into recession, default rates increase. Recoveries from collateral will depend on the possibility of selling the respective assets. We can generally suppose that greater supply of collateral-assets will lead

[1] In Acharya et al. (2003) study recovery numbers seem consistently higher than in Altman, Kishore (1996), this can be resulted by differences in discounting methodology.

to their lower prices, of course, depending on the market size and structure observed for a certain asset.[1] Also banks have to accept discounts for distressed sale. Moreover, the demand for these assets declines because non-defaulted companies are not able to invest the same amount of money in recession as during an expansion. The result is that macroeconomic situation can significantly influence the recovery rate,[2] which was as well demonstrated by several authors (see Araten et al. 2004 or Altman et al. 2005a).

Figure 5

The development of LGD in 1986–2006



Source: computed from Moody's (2007a)

From aforementioned fact that RRs tend to go down when default rates increase in economic downturn follows the relationship between LGD and PD. This joint dependence on macroeconomic conditions indicates positive correlation between them and implies that LGD and PD can not be treated as independent. On the contrary, LGD was traditionally assumed to be dependent on individual features that do not respond to systematic factors and hence as independent on default rates.[3] Nonetheless, as it was said, the firm's asset value after default influencing RR might be dependent on macroeconomic factors.

Frye (2000) used for modeling of the recovery process the assumption that the same economic conditions causing increase in default rates might also increase LGDs. In this model both PD and LGD are dependent on the systematic factor. Empirical tests on US corporate bond data show a strong positive correlation between default rates and LGD. He also found out that bond RR in economic downturn might decline 20–25% from normal year average. The same conclusion with inverse relationship between PD and RR presents Madan et al. (2006), Hu, Perraudin (2002) or Jokivuolle, Peura (2003). Frye (2005) also distinguishes

[1] For instance, a substantial number of defaulted firms in the telecom industry in 2001 in US. The very large inflow of specific telecom assets being liquidated increased the imbalance between supply and demand and depressed the value of these assets in the market. Similarly, the subprime-mortgage crises in the US in 2007 caused deterioration in values of real estate, which served as collateral for mortgages.

[2] Also interest rate should impact recovery rates. Higher interest rate leads ceteris paribus to lower discounted values of future recovered values and should therefore lead to higher LGD.

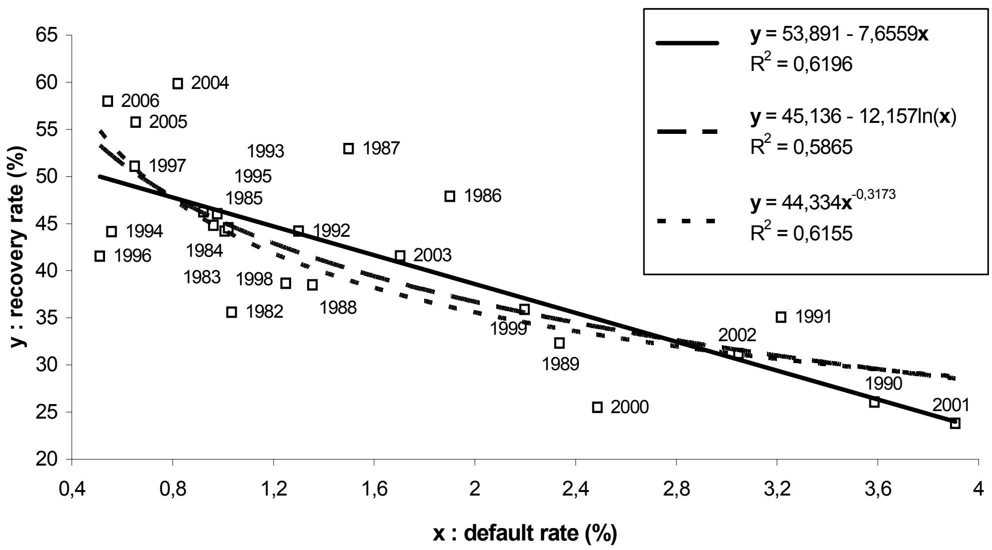
[3] The recovery rates are treated as independent on default rates for instance in Gupton, Finger, Bhatia (1997) or Jarrow, Lando, Turnbull (1997).

sensitivity of LGD's change in economic downturn for different types of debt instruments. His conclusion is, that debt type with average low LGDs in "good" years (e.g. senior secured bonds) are more sensitive to the deterioration of macroeconomic conditions than "high-LGD" instruments, whose LGD is not increasing so substantially in economic downturn. As a result, security on debt instrument does not protect lender against systematic risk.

The Figure 6 presents relationship between bond recovery and default rates and also displays the results of linear, logarithmic and power univariate regression carried out using these fundamental variables.

Figure 6

The link between default and recovery rates, 1982–2006



Source: computed from Moody's (2007a)

We can find that simple linear regression can explain about 62% of the variation in the annual RR with the level of default rate. The dependence is significantly negative, which is consistent with previous discussion about the inverse PD, LGD link. Although linear regression measures only partial connection between variables and is therefore a weaker concept than dependency that includes more complex affects such as the co-movement with time lag or causality effects, it results in a signal that recovery rates show a considerable share of systematic risk. The correlation between bond's recovery and default rates falls with decrease of seniority and security of bonds which is coincident with Frye's result of different LGD sensitivity with respect to economic downturn.[1]

Merton (1974) suggests that PD and RR may be also correlated because the borrower's leverage affects both the default probability and the amount of company assets per unit of debts (see Chapter 2). These firm specific causes should be also considered in PD,

[1] Sen. Secured = –0,79; Sen. Unsecured = –0,76; Sen. Subordinated = –0,5.

LGD correlation; however, asset value, the main determinant of PD and LGD in structural Merton type models is still dependent on the macroeconomic conditions. Therefore we can conclude that important property of LGD is its dependence on systematic factors.

Another question rises with the size of the debt and its influence on LGD. Study by Carty and Lieberman (1996), who used Moody's data on syndicated lending, did not confirm the size of exposure to have any significant impact on LGD. In contrast with this finding is a work by Eales and Bosworth (1998) that presents increase in RRs with the size of exposure, but for exposures exceeding $2 mill. are RRs again falling. Similarly Dermine and Neto (2005) state, that the size of loans has a significant negative effect on RRs.

Possible impact on LGD could be given also by the maturity of the debt. While Hamerle (2006) admits that a longer maturity leads to higher LGDs because more future payments are insecure and more uncertain, Gupton and Stein (2005) negate this influence and in their opinion, the maturity does not play a role for defaulted bonds.

Also the geographic variance in LGD should be noted. Default and recovery rates will naturally differ over time from country to country because of possible diverse stages of the economic cycle. Additionally, different legal insolvency procedures exist among countries and specific legal procedure surely influences the level of recovery rates too.[1] For instance, average bond's recovery rates in Europe between years 1985–2006 are lower than in North America (see Moody's 2007b, p. 16).[2]

As it was shown, LGD is influenced by many factors as facility's seniority and presence of collateral, borrower's industry characteristics or more general factors as macroeconomic conditions. However, previous research gives ambiguous results concerning some LGD's properties. The relatively rare occurrence of default events for some facilities could cause that the research was based on relatively small empirical samples. Also a non-homogenous methodology was used (e.g. for extracting LGD in workout process), which could also influence some conclusions. It is clear that further research is needed and hopefully with the acceptance of Basel II accord, setting rules for LGD's data gathering and its estimates, this research will be based on better data sample offering more exact outcomes.

1.3. Loss Given Default in Basel II

The Basel I Capital Accord represented a major breakthrough in the bank convergence of supervisory regulations regarding capital adequacy. The imposition of minimum capital requirements for credit risk by setting minimum ratio of regulatory capital to total risk-weighted assets helped to stabilize the international banking system and promoted its soundness (see

[1] Differences in bonds and loans insolvency regimes across USA, UK, Germany and France are described in Servigny, Renault (2004), Appendix 4A.

[2] Bonds type (RR (%) in Europe; RR (%) in North America): Sr. Secured (44,5; 53,8), Sr. Unsecured (27; 37,96), Sr. Subordinated (36,7; 32,5), Subordinated (30,8; 31,2).

BCBS 1988). However, this framework showed several problems that became more and more evident over time, e.g.[1]

- *Insufficient risk differentiation* – the weights did not differentiate enough the credit risk by counterparty characteristics as collateral, covenants, maturity or actual borrower's rating. The definition of risk buckets did not reflect the true level of risk of obligors and consequently the banks with the same capital adequacy ratio (CAR) could have very different risk profiles.
- *No recognition of benefits from diversification* – capital treatment did not take into account risk reduction attained by diversification that is why there was no distinction between a well diversified and less risky portfolio and one that was correlated and hence riskier.
- *Unsuitable treatment of sovereign risk* – for instance, lending to all OECD governments with substantially different credit ratings (Mexico, Turkey) incurred no regulatory capital charge.

Furthermore, by the time a variety of products in banks was developed primarily to overcome regulatory capital rules. Arbitrage between the banking and trading books has increased significantly especially with development of credit derivatives. All above-mentioned facts reduced meaning of regulatory capital ratios as measures of true capital adequacy. Also, the risk measurement extensively evolved in the last decade and many banks developed their own internal economic capital models to guide their decisions and bank practice became more distant from Basel I required rules. The main motivation for a revised capital adequacy framework known as Basel II was therefore to bring regulatory capital requirements more in line with good bank practice, escape from "one-size-fits-all" setting of Basel I and further strengthen the stability and security of the banking system via better risk management.[2] The New Accord started to be implemented by banks at the end of 2006.[3]

Basel II identifies three types of approaches dealing with credit risk: (i) standardized, (ii) an internal rating based (IRB) foundation, and (iii) IRB advanced approach. The main break with the previous Basel is the fact, that facilities will require different capital coverage depending on their intrinsic riskiness, as evaluated by some external rating agency – standardized approach, or by the bank itself – IRB approach.

[1] For other problems in Basel I framework see Stephanou and Mendoza (2005).
History of banking regulation can be found in Servigny and Renault (2004), Chapter 10.

[2] Basel II was agreed by Basel Committee members in mid-2004 after round of proposals between 1999–2003 and consists of a set of supervisory standards which are structured along three pillars:
Pillar 1 – Capital Requirements; concerns minimum requirements for credit and operational risk.
Pillar 2 – Supervisory Review; provides guidance on the supervisory oversight.
Pillar 3 – Disclosure; requires banks to publicly disclose information on their risk profile.
(see BCBS 2005).

[3] Basel II applies at a consolidated level to internationally operating banks and banking groups. Some national supervisors choose to apply it also to domestic banks – this is the approach followed by the EU Directives 2006/48 and 2006/49, whereby Basel II was introduced in the EU legislation.

1.3.1. The Standardized Approach

The attitude of this "new standardized approach" to credit risk is similar to Basel I, but has higher risk sensitivity due to employing the credit rating of external credit assessment institutions (ECAI) to define the weights for calculating RWA.[1] Yet, ratings from various agencies do not carry the same information. Whereas S&P's perceive their rating primarily from the likelihood of default of an issuer, Moody's rating reflects its opinion on the expected loss (PD.LGD) on a facility.[2] It is therefore important that ECAIs provide roughly similar ratings because wide discrepancies across institutions would incite banks to use the most moderate and favorable rating provider to deliberately reduce total capital requirements. In order to prevent such "agency arbitrage", the national supervisors have to ensure that there is no obvious risk underestimation by using certain institutions ratings.

LGD is not explicitly quoted in the standardized approach, but we can observe its presence indirectly, because the portfolios comprise exposures secured by residential property and by commercial real estate have lower risk weights. For instance, loans collateralized by mortgages on residential property that will be occupied or rented by borrower, is risk-weighted at 35% compared with 75% weight for other exposures to individual in the retail portfolio.[3] Basel II also states that loans secured by commercial real estate may receive a lower capital requirement than unsecured corporate exposure; risk weights may be reduced from 100% to 50%. Because no other specific limitations are imposed on the exposure type or the borrower's PD, "*...the presence of a lower risk weight seem to be motivated mainly by LGD consideration"* (Resti, Sironi 2005, p. 27).

Banks which follow the standardized approach can adjust the exposure of each asset by taking into account the positive impact of guarantees, collateral and other hedging tools such as credit derivatives. This can be done in two different ways, the so-called simple and comprehensive. Under the simple approach there are defined rules for changes in the risk weights considering the quality of the collateral, while leaving the exposure unchanged. The exposure portion covered with valid collateral receives the risk weight applicable to the collateral itself instead of using debtor's coefficient,[4] subject to a floor usually of 20%.

[1] Such institutions may be rating agencies or other institutions recognized by the national supervisors. In order to be admitted as ECAI agencies, they must meet requirements in terms of independence, transparency and consistency of rating criteria. For more details see Resti and Sironi (2007).

[2] Beattie and Searle (1992) provided a comprehensive study of the assessment of eight rating agencies and they find that large players (Moody's, S&P) despite different methodology exhibit similar assessments. Some large differences were found by more specialized and regional agencies (MCM, JBRI). This study is nevertheless quite old and agencies have recently put a lot of emphasis on ratings homogeneity.

[3] Such loans are usually highly fractioned with good risk diversification; hence weights are reduced. Also, as in 1988 Accord, loans backed by a mortgage on the borrower's home were weigh-reduced to 50%.

[4] It works similarly with the guarantees. The debtor's risk weight is replaced by the guarantor's one, which usually means a lower capital requirement due to better guarantor's rating. Furthermore, the losses can occur only with simultaneous default of debtor and guarantor – an event that is more rare than a default of the guarantor only. Nonetheless, this low probability of double default must not be considered. Neither in the IRB approach can be taken into account in the case of risk mitigation by guarantees or credit derivatives (see BCBS 2005,

In the comprehensive method, no capital requirement is applied on the exposure portion backed by valid collateral. However, its current value C must be reduced by a haircut H_C, reflecting the risk which the market value of the financial instrument may decrease. Banks can use standard supervisory haircuts or their own estimates. Once the appropriate haircut is chosen, the current value of the collateral is trimmed down by multiplying it by a haircut factor, defined as $1-H_C$, obtaining the adjusted collateral value. The value of the credit exposure covered by the adjusted collateral E_C is not subjected to any capital requirement. The remaining part of exposure after risk mitigation E^* is subjected to a "full-risk weight" and can be easily calculated as follows

(1.3) $$E^* = \max\left\{0, \left[E - C\left(1 - H_C\right)\right]\right\}$$

where E is the original exposure and holds $E = E^* + E_C$.

When the loan is not issued in cash, e.g. in the case of securities lending, its value can increase over time and cause that the collateral becomes insufficient for the exposure to be fully secured. For this case the current value of the exposure E must be adjusted by haircut H_E. Further, when currency mismatch is present between collateral and exposure, another haircut H_{FX} has to be used to account for foreign exchange risk. The remaining exposure may be expressed as shown in BCBS (2005, §147)

(1.4) $$E^* = \max\left\{0, \left[E\left(1 + H_E\right) - C\left(1 - H_C - H_{FX}\right)\right]\right\}$$

The exposure amount after risk mitigation E^* will be multiplied by the risk weight of the counterparty to obtain the risk-weight asset amount.

The appeal for banks to leave standardized approach and pass to IRB is that it may allow them to obtain a lower level of capital requirements.

1.3.2. The Foundation IRB Approach

The IRB approach is more sophisticated, because it relies on banks' own internal estimates of certain risk parameters for determining credit capital requirements. Nonetheless, both in the foundation and in advanced IBR approach there must be used a risk-weight function provided by Basel Committee for deriving regulatory capital to ensure that overall capital levels across countries remain homogenous.

The IRB is based on measures of expected and unexpected losses (EL, UL).[1] In both regulatory and economic capital determination plays those factors an important role because the risk-weight functions produce capital requirements for the UL portion (see BCBS 2005,

§301). However, for some hedged exposures is in the IRB double default effect deliberated (see BCBS 2005, §284).

[1] The distinction between EL and UL was introduced only in final version Basel II in 2004. Before that, banks were simply required to cover both types of losses.

§212). EL can be generally expressed as product of PD, EAD, LGD and *f*(M), where *f*(M) presents the effective maturity.[1]

Expected loss stands for the anticipated average loss rate that a bank should expect over time for particular facility or portfolio. It is the mean value of the probability distribution of future losses and because it is estimated by lender *ex-ante***,** it should be reflected in the price of risky product. From this point of view, as Bhatia (2006, p. 10) refers, EL does not represent risk but a cost of providing credit, which must be recovered as a part of pricing.

The real credit risk is associated with unexpected loss that is usually defined as the volatility of the EL. In order to compute it, it is necessary to know standard deviations of all above mentioned variables, which occur in the formula for EL. The accuracy of EL and UL computation therefore depends on estimates of those risk parameters. However, some of them may be stochastic and considering them as deterministic leads to underestimating UL's values. This is also the case of LGD.

It is good to note that in Basel II framework, UL corresponds to value at risk, rather than to standard deviation of losses. UL is defined as a difference between LGD (multiplied by the factor taking into account correlation[2] between assets in portfolio and maximal loss on given confidence level) and the bank's best estimate of EL, which are all adjusted by maturity factor (see BCBS 2005, §272). UL then represents the capital requirements, the amount, that must be covered by capital, while EL may be covered by provisions.[3]

We can see that LGD enters capital calculation in IRB approach in a direct and explicit way. However, a foundation approach allows banks to use own estimate only of the debtor's PD, while all other variables are set by regulators, LGD makes no exception.[4] Accordingly to BCBS (2005, §287), LGD is fixed 45% for all senior, unsecured exposures. This value must be increased to 75% for subordinated exposures, but can be reduced again when some adequate collateral is pledged. Such a reduction cannot be based on banks' internal models or past experience but has to come out from rules that quantify the effect of financial and non-financial collaterals. The similar haircut system as in standardized approach is used; however, in this case the haircuts are applied directly to LGDs and not to the value of exposure. LGD applicable to a collateralized transaction is called *effective loss given default,* LGD^*.

Concerning the financial instruments as collateral,[5] the formula for the adjusted LGD is the following

---

[1] The effective maturity *f*(M) must be computed for a given exposure only in some cases. In BCBS (2005) in paragraphs 318 to 324 the circumstances in which the maturity adjustment applies are discussed.

[2] Unlike EL, UL for portfolio is not equal to the sum of individual ULs, because variance is not additive parameter and depends on the loss correlation among portfolio's assets.

[3] This rule is not absolutely rigid; a limited percentage of UL can also be covered with provisions (see Resti, Sironi 2007, p. 610).

[4] For retail exposures, banks are allowed to provide own estimates not only for PD, but also for LGD and EAD (see BCBS, 2005, §252).

[5] Eligible financial collaterals are the same as in the standardized approach and include cash and deposits issued by the lending bank, gold, bonds with a rating of at least BB, and others (see more in BCBS 2005, p. 31).

(1.5) $$LGD^{*} = LGD\left(E^{*}/E\right)$$

taking the "basis" 45% LGD and substituting into equation (3) we get

(1.6) $$LGD^{*} = 45\% \max\left\{0, \left[1 + H_E - C/E\left(1 - H_C - H_{FX}\right)\right]\right\}$$

where all of the symbols have the same meaning as above. LGD is increasing (i) when the loan is not issued in cash, (ii) when H_E is greater than zero, and (iii) the coverage ratio C/E (current value of collateral to original value of exposure) is reduced according to the size of collateral and currency mismatch haircut.

As far as non-financial assets are concerned, three different categories of collateral are defined. Receivables, real estate and other collateral,[1] for which haircuts are replaced by a system of minimum and maximum thresholds that are used for adjusted LGD's computation as follows

(1.7) $$\text{LGD*} = \begin{cases} 45\% - \left(\min\left[C/E, T_{max}\right]/T_{max}\right)\left(45\% - LGD_{min}\right) & \text{if } C/E \geq T_{min} \\ 45\% & \text{if } C/E < T_{min} \end{cases}$$

where LGD_{min} is the minimum value that can be attained by the adjusted LGD, when $C/E \geq T_{max}$. The threshold values together with LGD_{min} are for particular collateral types reported in footnote.[2]

As we presented, in the foundation IRB approach LGD levels are predetermined and adjusted in case, that collateral is present according to given rules. Own estimate of LGD is required only for retail exposure and LGD is estimated at a pool level. The use of banks' own estimate of LGD for corporate, sovereign and bank exposure is allowed in the advanced IRB methodology (see BCBS 2005, §297).

1.3.3. The Advanced IRB Approach

When using the advanced IRB approach, banks are authorized[3] to provide more of their own estimate of risk parameters (PD, LGD and EAD) and their own calculation of effective maturity. BCBS (2005, §468–§505) sets several requirements on LGD estimates concerning data collection, types of LGD's estimates, adjustment reflecting effect of guarantee or validation process. Those rules are shortly summarized below.

Banks must collect and store a complete history of data on the LGD associated with each facility and also, the data on the estimated and realized LGDs that should be measured in an economic and not merely in an accounting manner.[4] Data set, on which estimates must

[1] Other collateral, including physical capital but excluding any assets which are acquired by the bank as a result of a loan default (see Resti, Sironi 2005, p. 30).
[2] Collateral type (T_{min} (%); T_{max} (%); LGD_{min} (%)): Receivables (0, 125, 35), Commercial and residential real estate (30; 140; 35) and other collateral (30; 140; 40).
[3] Banks' internal models for credit risk parameters estimation has to be approved by Regulator.
[4] See Chapter 1.1.3 for difficulties with ex-post LGD measurement.

be consequently based, should ideally cover an entire economic cycle, but no shorter than seven years or five for retail exposure.[1]

"*A bank must estimate an LGD for each facility that aims to reflect economic downturn conditions where necessary to capture the relevant risks*".[2] This estimated LGD cannot be less than *"the long-run default-weighted average loss rate given default calculated based on the average economic loss of all observed defaults"* for that type of facility. In addition, a bank must take into account the LGD's cyclical variation of some facility and possibility of higher LGDs during a period with higher credit losses than the default-weighted average (§468). The extent of relation between the risk of the borrower and risk of the collateral or its provider must be considered and in case of significant degree of dependence it must be by loss estimate treated conservatively. Similarly, any currency mismatch between obligation and the collateral must be addressed in a conservative manner (§469).

"LGD estimates must be grounded in historical recovery rates and when applicable, must not solely be based on the collateral's estimated market value". This requirement is set, because banks are usually unable to expeditiously gain control of their collateral and liquidate it (§470). Bank must determine the best estimate of LGD for each defaulted exposure, based *"...on the current economic circumstances and facility status"*. Since realized losses can exceed expected values, banks must also estimate conservative projection reflecting"*...the possibility that the bank would have to recognize additional, unexpected losses during the recovery period"* (§471).

Banks may reflect the risk mitigation effect due to presence of guarantees by an adjustment of PD or LGD.[3] The adjustment criteria must be plausible, intuitive and consistent over time and across types of guarantees. *"In no case can the bank assign the guaranteed exposure an adjusted PD or LGD such that the adjusted risk weight would be lower than that of a comparable, direct exposure to the guarantor"*. Advantageous effects of borrower's and guarantor's imperfect expected correlation of default is permitted to influence capital requirements. *"As such, the adjusted risk weight must not reflect the risk mitigation of double default"* (§482). However, for some hedged exposures is the double default effect deliberated (§284). No restrictions are put on the types of eligible guarantors.

The validation of internal estimates is also considered. "*Banks must have a robust system in place to validate the accuracy and consistency of rating systems, processes, and the estimation of all relevant risk components"* (§500). Comparisons between estimated and realized LGDs should be at lease annually performed to demonstrate that realized recoveries

[1] According to EU Directive 2006/49, EU banks may be granted up to three years' discount when the New Accord is implemented for the first time.
[2] For estimation of downturn LGD demanded by Basel II, "point in time" approach presented by Hamerle (2006) can be used.
[3] Also credit derivatives can be used for the credit risk mitigation. *"The criteria used for assigning adjusted borrower grades or LGD estimates (or pools) for exposures hedged with credit derivatives must require that the asset, on which the protection is based (the reference asset), cannot be different from the underlying asset, unless the conditions outlined in the foundation approach are met"* (§488).

are within the expected range. *"Banks must also use other quantitative validation tools and comparisons with relevant external data sources"* (§503) and must also demonstrate that used methods do not systematically vary with the economic cycle (§504). For the case when significant deviations between estimated and realized LGDs are found and validity of the bank's estimates is queried, banks must have prepared well-articulated internal standards dealing with such a situation (§504).

The Basel II framework represents a significant improvement in the risk sensitivity of capital regulation and corresponds to a clear progress in credit risk management compared to Basel I. However, there are still a lot of gaps and limitations that should be considered.[1] In the credit risk area, treatment of correlations is still inadequate. Advantage of diversification of retail portfolios with exposure to individuals and small enterprises is recognized, but there is unrealistic way in which IRB system measures the concentration and correlation among borrowers. Furthermore, a correlation of losses is modeled only as a function of PD and thereby ignores possible portfolio characteristics such as geographic or industry diversification.

Another possible problematic issue is the impact of Basel II regulation on the macroeconomic level. Indeed, if capital adequacy depends on internal or external ratings of the counterparts, recession will lead to higher default rates and to following increase in the minimum required capital. The consequence is a procyclical effect leading to overlending in the time of strong economic growth and credit crunch in the recession, which exacerbates economic downturn. Furthermore, Altman et al. (2005b) used the Monte Carlo simulation to assess the impact of not considering LGD and PD as correlated in the IRB credit risk models. They found that EL and UL are in this case importantly understated.[2] Taking into account the link between PD and LGD in IRB estimation, it is likely to even increase the procyclicality effects of Basel II. However, neglecting this correlation might lead to insufficient bank reserves.

The first chapter provided the range of information about LGDs with respect to the types of borrowers, seniority of debt, or development of macroeconomic conditions. Although we could notice different relations among those variables, a major difficulty of such information is their complete dependence on historical data. The LGD predictions based on their past data are not thus necessarily coherent with the evolution of fundamentals across time and can result in inaccurate estimates being not able to capture the real trend in economy.

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[1] As the general shortcomings of Basel II is usually mentioned gap between industry best practices and 1st Pillar, non-capability to offer equal treatment to banks operating in different environment, bad coherence with new accounting rules, high implementation costs, etc. For more criticism see Servigny and Renault (2004), Stephanou and Mendoza (2005) or Das (2007).

[2] Taking LGD as stochastic and correlated with PD increases EL and UL approximately about 30% (see Altman et al. 2005b).

2. Loss Given Default modeling

"All models are wrong, but some are useful."

George E. Box [*]

This chapter focuses on analytical tools which enable forward-looking estimates of LGD from market observable information. Those implied market LGDs incorporate specific conditions in economy and therefore should bring additional piece of knowledge to already presented properties of ex-post historical LGDs.

Modern credit analysis is more and more in line with integration of uncertainty into new theories with inclusion of sophisticated mathematical tools such as stochastic calculus. Its contribution to modern finance is the ability to give a deterministic solution out of an uncertainty which is modeled as a random process. Furthermore, stochastic calculus allows the fining of the time into infinitesimal points as a limit of the discrete time approach. Despite the discrete approach it is still practical to visualize the development through time (e.g. binominal trees); the continuous-time framework is very useful, since it enables more easily to obtain closed-form solution.

This also enabled the dynamics approach to asset pricing models which are aiming at determining the equilibrium arbitrage-free price of risky assets. Each risky asset should offer an expected return corresponding to its degree of risk; therefore all risky parameters must be evaluated by market in order to get the equilibrium price. This assumption, that prices include all information is then used by credit risk pricing models which utilize market information (e.g. share or bond price) to measure credit risk and with help of asset pricing models are trying to extract the key risk parameters such as PD or LGD from the prices. Those models are forward-looking, estimating the risk parameters which are expected by the market in the future and not those that occurred in the past. From the nature of this method such estimate of LGD is called *implied market LGD*.

The market information based models can be further classified as *structural* and *reduced-form* models. Many theoretical developments have appeared in this field only during the last few years. The goal of this chapter is to give a summary of different approaches to credit risk pricing and how it can be further used for extracting credit risk parameters, especially LGD. The analysis begins with older structural models, whose imperfections gave rise two decades later to new reduced-form approach. However, both types of models have their limitations and stand on many assumptions, which hinder their practical usage.

[*] Box, G. (1979): *Robustness in the strategy of scientific model building*, In Launer, R., Wilkinson, G. (eds): Robustness in Statistics, 1979, p. 202.

2.1. Structural Models

The category of structural-form models of credit risk are based on the framework developed by Merton in 1974 using the theory of option pricing presented by Black and Scholes (1973).[1] The intuition behind is quite straightforward, a company defaults, when the value of its assets becomes lower at the time of debt's maturity than that of its liabilities. For that reason, the default process is driven by the value of the company's assets and the risk of default is explicitly related to the assets variability.

The term *structural* comes from the fact, that these models focus on the structural characteristics of the company such as asset volatility or leverage that determine relevant credit risk elements. Default and RR are a function of those variables. In contrast, *reduced-form* models generally ignore the structural parameters as the cause of the default and simply assume, that default is possible and is driven by some exogenous random variable. The result is, that default and recovery is modeled independently from the firm's structural features, which lacks the clear economic intuition behind the default event.

Although the original Merton's model introducing contingent claim approach brought a whole new perspective for credit pricing analysis, it was based on some simplifying assumptions, for example, that default can occur only at maturity of the debt, company's liabilities are represented only by one zero-coupon bond,[2] or interest rate is taken as constant. In response to such problems there have been developed alternative approaches, which try to remove one or more of those problematic drawbacks of the model. Black and Cox (1976) introduced the possibility of more complex capital structure of the company's liabilities, Geske (1977) presented the interest paying debt, or Vasicek (1984) established the distinction between the short and long-term debt. All previous authors also enhanced the model by treating default as an event that can occur any time before debt's maturity. More recent improvements such as works by Longstaff and Schwartz (1995), Hull and White (1995), or Collin and Goldstein (2001), reject the constant risk-free interest rate and considered interest rate as stochastic variable instead of that.[3]

All of mentioned structural models deal primarily with PD of specific company. What were the initial assumptions, how they developed with models' improvements, and foremost, how LGD and RR can be modeled are the main objects of the following text. Since all later structural models are more or less based on this original Merton's framework, its derivation is described in this chapter in more details and further there will be presented its expansions in form of presence for collateral, which pledges the debt, or stochastic interest rate.

[1] Therefore are structural models also called Merton type models. In the original option pricing paper, Black and Scholes (1973) suggested that their technique could be used to price corporate securities.
[2] In reality companies have complex financial structure with claims of different maturities, interest payments, or levels of security and seniority.
[3] For detailed development of later structural models see e.g. Altman et al. (2005a) or Jarrow and Protter (2004) and the references therein.

2.1.1. PD and Recovery Rate in Merton's model

The seminal structural Merton's (1974) model relies on many hypotheses which are mostly coming from the Black–Scholes option pricing theory. Some of them became source of criticism and have been later relaxed. The original framework in which is the valuing process of firm's assets embedded requires for the application of standard corporate credit risk pricing many assumptions, which are following

(A. 1) Markets are frictionless. There are no transactions costs, taxes, short-selling restrictions, bid-ask spreads and assets are perfectly divisible.

(A. 2) There is a sufficient number of investors with comparable level of wealth such that they can buy or sell as much of an asset as they want.

(A. 3) The term structure of risk-free interest rate is flat and known with certainty. The price of riskless bond paying \$1 at time T is hence $B_0(T) = \exp[-rT]$, where r is the instantaneous riskless interest rate.

(A. 4) Total value of firm V is financed by equity E and one zero-coupon non-callable debt contract D, maturing at time T with face value F. Also holds $V_t = D_t + E_t$, with (A.1) this implies that the value of the firm and the values of assets are identical and do not depend on the capital structure itself. This corresponds with Modigliani–Miller theorem.

(A. 5) There is neither cash flow payout, nor issues of any type of security during the life of the debt, nor bankruptcy costs. Default can only happen at the maturity.

(A. 6) There is no possibility of the absolute priority rule violation. Shareholders are paid off after firm's default only after full compensation of the debtholders.

(A. 7) The dynamics of the firm's value through time can be described by the stochastic differential equation called geometric Brownian motion

$$dV_t = \mu_V V_t dt + \sigma_V V_t dW_t^V, \tag{2.1}$$

where μ_V is the assets drift (i.e. the instantaneous expected rate of return on the firm's value V per unit time), σ_V is the standard deviation of its return, and dW_t^V is a standard Gauss–Wiener process.[1] In case of cash outflow per unit time in form of dividends or coupons (δ_V), the equation is adapted to[2]

$$dV_t = (\mu_V - \delta_V) V_t dt + \sigma_V V_t dW_t^V \tag{2.2}$$

In such framework based on those assumptions, credit risk concerns the possibility that the value of the company evolves stochastically, will be on the maturity day T less than the repayment value of the loan F. The debtholders receive at T neither the value F (if $V_T > F$), or they receive the entire value of the firm and the owners of the firm receive nothing (if $V_T < F$). The risk of default is therefore explicitly linked to the volatility on the firm's asset value. The Merton's contingent claim analysis shows, how this risk should be priced.

[1] Definition of Wiener process and other stochastic concepts is given in Appendix A.

[2] In the seminal Merton's model is the drift in the form of $\mu_v V - \delta$, not like in (2.2) in form $(\mu_v - \delta)V$. The later is more often used in newer models; however, there is no difference for model derivation.

Dynamics of debt security, whose market value is at any time t a function of the value of the firm and time, i.e. $D_t = f(V_t, t)$, can be expressed by stochastic equation as

(2.3) $$dD_t = \left(\mu_D D_t - \delta_D\right)dt + \sigma_D D_t dW_t^D$$

where the symbols are the same as in (2.2). By using Itô's Lemma,[1] it is possible to derive (see Appendix B) the fundamental differential equation, which determines the value of the debt and which is the basis cornerstone of Merton's model. The equation is in the form

(2.10) $$\frac{\partial D_t}{\partial t} + rV_t \frac{\partial D_t}{\partial V_t} + \frac{1}{2}\sigma_V^2 V_t^2 \frac{\partial^2 D_t}{\partial V_t^2} - rD_t = 0$$

and it must be satisfied by each security, whose value is a function of the value of the firm and time. In addition to those variables, D_t depends also on the volatility of the firm's value and on the interest rate.[2] As we can see, the value is independent of the expected rate of return of the firm and therefore independent of the risk preference of investors. From this result comes out a principle of risk-neutral valuation, which allows assuming the risk-neutral world when pricing a corporate debt. This concept will be used also later to get risk-neutral probabilities of default and recovery rates.

To solve this equation for the value of the debt D, there is necessary to get two boundary and initial conditions. By (A.4), $V = D(V,\tau) + E(V,\tau)$, where $\tau = T - t$ is the length of time until maturity. The boundary conditions are then given by

(2.11a) $$D(0,\tau) = E(0,\tau) = 0 \text{ and } D(V,\tau) \le V$$

The initial condition is given by fact that debtholders at maturity day receive the face value of debt F or remaining value of the firm, what can be expressed as

(2.11b) $$D(V,0) = \min\left[V; F\right]$$

The equation (2.10) can be using (2.11) solved by standard methods of *separation of variables* or *Fourier transforms*. However, as noticed Merton, it is possible to easily determine the value of equity $E(V,\tau)$ if we substitute for D in (2.10) and (2.11) the expression $D(V,\tau) = V - E(V,\tau)$ and deduce the partial differential equation for E. This is

(2.12) $$\frac{\partial E_t}{\partial t} + rV_t \frac{\partial E_t}{\partial V_t} + \frac{1}{2}\sigma_V^2 V_t^2 \frac{\partial^2 E_t}{\partial V_t^2} - rE_t = 0$$

subjected to the boundary conditions (2.11a) and the initial condition

(2.13) $$E(V,0) = \max\left[0; V - F\right]$$

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[1] See Appendix A

[2] The value of D_t depends also on the payouts of the firm and the security, however, the initial Merton's model assumes no payout policy (Merton 1974). The incorporation of dividends' payouts will be developed in Chapter 3, which utilizes Merton's model for estimation of LGD for the sample of companies in the Czech Republic.

This is identical to Black and Scholes (1973) formula for pricing "*...an European call option on a non-dividend paying common stock where firm value corresponds to a stock price and F corresponds to the exercise price*" (Merton 1974, p. 10). Indeed, at maturity time T, the equity holders will exercise option and pay to debtholders face value of liabilities if $V_T \geq F$, otherwise they let this option expire. By applying the Black–Scholes option pricing formula it is straightforward to get solution for equations (2.12), (2.13) as

(2.14) $$E(V,\tau) = V\Phi\left(d_1\right) - Fe^{-r\tau}\Phi\left(d_2\right)$$

where $d_1 = \dfrac{\ln\dfrac{V}{F} + \left(r + \dfrac{1}{2}\sigma_V^2\right)\tau}{\sigma_V\sqrt{\tau}}$, $d_2 = d_1 - \sigma_V\sqrt{\tau} = \dfrac{\ln\dfrac{V}{F} + \left(r - \dfrac{1}{2}\sigma_V^2\right)\tau}{\sigma_V\sqrt{\tau}}$ and Φ (.) is cumulative standard normal distribution. If we use again $D(V,\tau) = V - E(V,\tau)$ in which we appoint the expression for E from equation (2.14), we can express the value of debt at time τ as

(2.15) $$D(V,\tau) = V\Phi\left(-d_1\right) + Fe^{-r\tau}\Phi\left(d_2\right)$$

where d_1 and d_2 are defined above. This equation could be later adjusted to express risk premium of corporate debt or its yield to maturity.

Armed with the aforementioned logic of the model we can already look how credit risk parameters as PD and RR can be extracted. The default occurs, when firm's value drops below some *default barrier* (DB), that is in the seminal Merton's model represented by face value of debt F at its maturity. The probability of default is therefore the likelihood that the value of the firm will be at the maturity day T lower than the value of debt F. Simply expressed as

(2.16) $$PD = \Pr(V_T \leq F)$$

To get this probability, the more information about probability distribution of V has to be known. However, we can use the assumption that the value of the firm V is log-normally distributed, what is according to Crouhy et al. (2000) quite robust hypothesis confirmed by actual data, and we can get information about probability distribution of $\ln V_T$,[1] what is

(2.17) $$\ln V_T \sim \Phi\left[\ln V_0 + \left(\mu_V - 0{,}5\sigma_V^2\right)T, \sigma_V^2 T\right]$$

from properties of natural logarithm can be the probability (2.16) expressed as

(2.18) $$PD = \Pr(\ln V_T \leq \ln F)$$

and from that by using (2.17) we can get

[1] The Itô's Lemma can be again used to get dynamics for $d\ln V_t$ and from that can be determined parameters of normal distribution for $\ln V_t$ (see Hull 2002 p. 227).

(2.19) $$PD = \Phi\left(-\frac{\ln\frac{V_0}{F} + \left(\mu_V - \frac{1}{2}\sigma_V^2\right)T}{\sigma_V\sqrt{T}} \right) = \Phi(-d_2^*)$$

which is the PD of a company at the time of maturity T expected at time $t = 0$, $(\tau = T)$, when the value of the firm V_0 is known with certainty.[1] Nearer look at values d_2^* and d_2 discloses that probability of default occurs also in final equation for pricing risky debt (2.15). This comes from the fact that $\Phi(d_2)$ is the probability that the European call option will be exercised by equity holders (see Hull 2002, p. 247), and company will not default. The term $1-\Phi(d_2)=\Phi(-d_2)$ then characterize default probability. However, while $\Phi(-d_2^*)$ in (2.19) gives the real-world (physical) probability of default, $\Phi(-d_2)$ presents the default probability in the risk-neutral world. This is caused by using riskless interest rate r instead of expected rate of return μ_V in the formula for d_2. In the real world, investors are demanding more than risk-free rate of return and therefore $d_2^* > d_2$ what implies $\Phi(-d_2^*) < \Phi(-d_2)$ and the fact that risk-neutral PD overstates its physical measure. Similarly it has to be distinguished between physical and risk-neutral RR.[2]

The recovery rate, when assuming no liquidation costs after default, will by given by the ratio of firm's value at T to the debt F, (V_T/F). More formally expressed as

(2.20) $$RR = E\left(\frac{V_T}{F}\middle| V_T < F\right) = \frac{1}{F}E\left(V_T \middle| V_T < F\right)$$

as was already mentioned, V is log-normal variable, therefore to get an explicit formula for RR we can use for the method presented in Liu et al. (1997), that derives conditional mean for log-normal distributed variable, what is exactly the case of equation (2.20).

Let's suppose that variable Y is log-normal and $\ln Y$ is normally distributed with mean μ and variance σ^2. Then variable $Z = (\ln Y - \mu)/\sigma$ has a standard normal distribution. The conditional mean of Y, giving $Y < c$, can be then expressed as follows

$$E\left(Y \middle| Y < c\right) = E\left(\exp[\sigma Z + \mu] \middle| \exp[\sigma Z + \mu] < c\right)$$

(2.21) $$= E\left(\exp[\sigma Z + \mu] \middle| Z < (\ln c - \mu)/\sigma\right)$$

to simplify following expression, let's define

(2.22) $$g = (\ln c - \mu)/\sigma \text{ and } h = \Phi(g)$$

[1] From (2.19) it can be seen that PD is the function of the distance between current V_0 and the face value of debt F, adjusted by the expected growth of asset μ_V relative to its volatility σ_V^2. The d_2^* is hence called distance-to-default (DD) and the higher its value is, the lower is PD.

[2] As e.g. Deliandes and Geske (2003) state, risk-neutral default probabilities can serve as an upper bound to physical default probabilities. For recoveries hold reverse relation – the risk-neutral expected recovery rate is less than its physical (real-world) counterpart (see Madan et al. 2006, p. 5).

where Φ (.) is again normal c.d.f. with these notations, the equation (2.21) becomes

$$E\left(Y\middle|Y<c\right)=h^{-1}\int_{-\infty}^{g}\exp[\sigma Z+\mu]\left(2\pi\right)^{-1/2}\exp[-z^2/2]dz$$

$$=\exp[\mu+\sigma^2/2]h^{-1}\int_{-\infty}^{g}\left(2\pi\right)^{-1/2}\exp[-(z-\sigma)^2/2]dz$$

(2.23) $$=\exp[\mu+\sigma^2/2]\frac{\Phi\left(\left(\ln c-\mu\right)/\sigma-\sigma\right)}{\Phi\left(\left(\ln c-\mu\right)/\sigma\right)}$$

considering the parameters of normal distribution of ln$V$ stated in (2.17), we can write conditional mean of V_T, giving $V_T<F$ as

(2.24) $$E\left(V_T\middle|V_T<F\right)=\exp[\mu_v^*+\sigma_V^{*2}/2]\frac{\Phi\left(\left(\ln F-\mu_v^*\right)/\sigma_V^*-\sigma_V^*\right)}{\Phi\left(\left(\ln F-\mu_v^*\right)/\sigma_V^*\right)}$$

where $\mu_v^*=\ln V_0+\left(\mu_V-0{,}5\sigma_V^2\right)T$ and $\sigma_V^{*2}=\sigma_V^2T$, after substituting and rearranging we get

$$E\left(V_T\middle|V_T<F\right)=\exp\left[\ln V_0+\mu_V T\right]\frac{\Phi\left(-\frac{\ln\left(V_0/F\right)+\left(\mu_V+0{,}5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}\right)}{\Phi\left(-\frac{\ln\left(V_0/F\right)+\left(\mu_V-0{,}5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}\right)}$$

(2.25) $$=V_0\exp[\mu_V T]\frac{\Phi(-d_1^*)}{\Phi(-d_2^*)}$$

using this term in equation (2.20) we get final expression for the expected recovery rate at time $t = 0$ in the form

(2.26) $$RR=\frac{1}{F}E\left(V_T\middle|V_T<F\right)=\frac{V_0}{F}\exp[\mu_V T]\frac{\Phi(-d_1^*)}{\Phi(-d_2^*)}$$

which is the physical recovery rate and the risk-neutral RR would be obtained by replacement μ_V with r. RR function is homogenous of degree zero in V_0 and F, which means that proportional change in those variables does not influence its value (ceteris paribus). Moreover, RR is dependent, as the PD, on the uncertain development of firm's value and therefore is not constant through the time but stochastic.

As it was shown above, risk-neutral PD is direct component in the formula for pricing risky debt. Similarly we can find out that RR is there embedded. To see it more clearly, we can rewrite eq. (2.15) at $t=0$ as follows

$$D_0(V,T)=V_0\Phi\left(-d_1\right)+Fe^{-r\tau}\Phi\left(d_2\right)$$

$$= Fe^{-rT} - \left(-\Phi\left(-d_1\right)V_0 + Fe^{-rT}\Phi\left(-d_2\right)\right)$$ [1]

(2.27) $$= Fe^{-rT} - \Phi\left(-d_2\right)\left[Fe^{-rT} - V_0\,\Phi\left(-d_1\right)/\Phi\left(-d_2\right)\right]$$

F^{-rT} represents the current value of the riskless debt, using (2.26), we can call the second term in bracket the risk-neutral expected discounted recovery value, conditional on $V_T<F$ (see Crouhy and Galai 1997). The bracket is hence the present value of expected loss, if the firm goes bankrupt at time T. With result of (2.19) we easily get formula

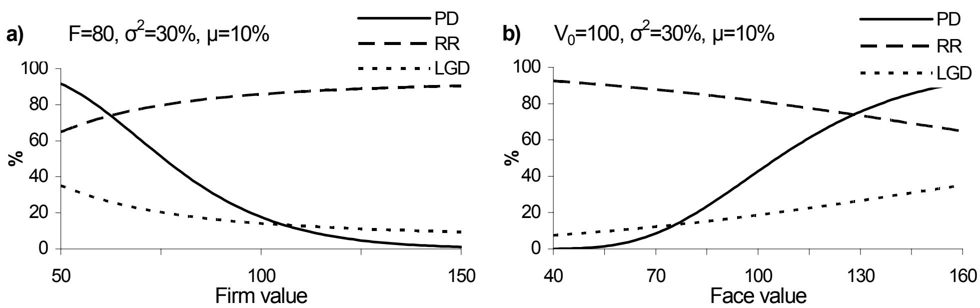
(2.28) $$D_0(V,T) = Fe^{-rT} - PD.ELGD_D$$

where $ELGD_D$ is the expected discounted LGD. Thus, $PD.ELGD_D$ can be extracted from the prices of particular risky and riskless bond with the same maturity. By utilizing the knowledge about borrower's PD (e.g. external estimate from rating agencies) is then possible to get separate value of expected discounted LGD.

As we could see from the above derived model's dynamics, both PD and RR are simultaneously given from arbitrage-free equilibrium conditions. Using the presented expression for PD and RR, the sensitivity analyses with respect to other company's structural parameters can be made. Consider the firm with given $F=80$, $V_0=100$, $\sigma^2=30\%$, $\mu=10\%$ and T=1. The variables will be shocked to see how PD and RR change.

Figure 7

The sensitivity analysis for PD and RR (LGD) – part 1



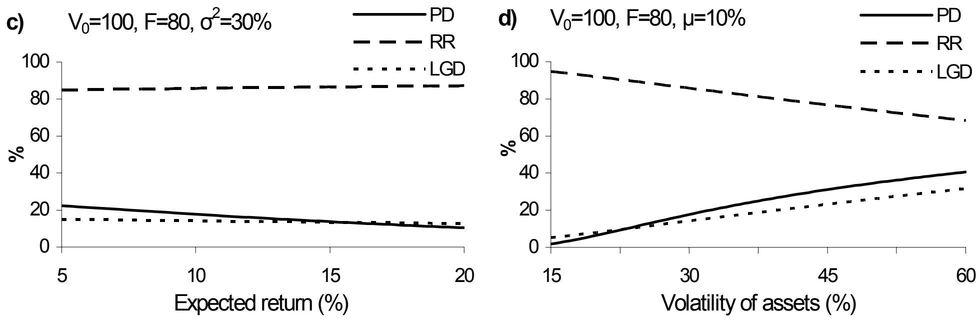
Source: computed from eq. (2.19) and (2.26)

The figure presents results for RR and PD in physical measure. It could be supposed, that the higher is the firm's value at the time of risk parameters prediction, the lower is the expected LGD and lower PD – part a), the linkage is reverse with the value of debt F. An increase in firm's leverage brings about higher both PD and LGD. The similar impact also has an increase in assets' volatility (leaving leverage unchanged) which causes higher uncertainty of future firm's value at the maturity T and therefore fall in RR.

[1] The derivation of eq. (2.15) came from the relation $D=V-$European call hold by shareholders. Equivalently we could consider that pay-off to debt holders is analogous to the pay-off from writing European put and $D=F^{-rT}-$European put. This is also the form of rewriting in eq. (2.27).

Figure 8

The sensitivity analysis for PD and RR (LGD) – part 2



Source: computed from eq. (2.19) and (2.26)

In summary, Merton's approach evidently generates the negative correlation between PD and RR because both variables depend on the same firm's structural characteristics. The RR is significantly determined by the value of firm's assets at the maturity time T. However, possible RR gained by debt holder must not always depend only on firm's value V_T because the debt might be pledged by collateral.

2.1.2. Impact of Collateral on LGD in Merton's model

From the previous part it is obvious, that the „collateral" for debt holders in the seminal Merton's model is the value of the firm at the maturity time.[1] Firm's value therefore determines not only the PD but also the RR. However, as a matter of fact, the debt is usually backed by collateral, and its value at the time of default determines RR. Furthermore, the value of collateral is not perfectly correlated with firm's value. As a consequence, the basic Merton's model is not sufficient for extracting RR from arrangement with collateral and hence it is necessary to make a few adjustments for more flexible LGD predictions.

The extension, in which the asset value and the collateral are less than perfectly correlated, was made by Jokivuolle and Peura (2003, 2005) by adding a separate process for the collateral's value. The collateral is used to back the debt with face value F and maturity T. However, the firm can have more than one debt F, and the default occurs if the firm's value V at the maturity time is less than the overall debt value D, $D \geq F$. As in Merton's model, V satisfies the stochastic dynamics $dV_t = \mu_V V_t dt + \sigma_V V_t dW_t^V$ and the recovery rate for the debt F is determined by the value of the collateral, which follows the same geometric Brownian motion as follows

$$dC_t = \mu_C C_t dt + \sigma_C C_t dW_t^C \qquad (2.29)$$

[1] Since in the initial Merton's model holds the assumption of Modigliani–Miller theorem, value of firm and value of firm's assets are identical.

with instantaneous correlation between W^V and W^C denoted as ρ. If the debtor owns the collateral, it can be considered that $V = V^* + C$, where V^* and C would be correlated log-normally distributed diffusion processes. The correlation parameter than would be an endogenous variable derived from the correlation between V^* and C. However, this is not very tractable, since the sum of log-normal variables is not log-normal and the model would lose much of its applicability. Therefore is ρ assumed to be exogenous, which corresponds to the case when the collateral is provided by a third party.[1]

Loss in the event of firm's default can be expressed as max $[0;F-C_T]$. The expected LGD, conditional on the default is then

(2.30) $$ELGD = \frac{1}{F} E\left(C_T \middle| V_T < D\right)$$

considering the relation (2.17) that also holds for the collateral value, we can write V_T and C_T in terms of standard normal variables x and y as $\ln V_T = \ln V_0 + \left(\mu_V - 0{,}5\sigma_V^2\right)T + \sigma_V \sqrt{T} y$ and $\ln C_T = \ln C_0 + \left(\mu_C - 0{,}5\sigma_C^2\right)T + \sigma_C \sqrt{T} x$. By applying the property of bivariate normal distribution,[2] we get the distribution for conditioned

$$\ln C_T \left| y \right. \sim \Phi\left[\left(\ln C_0 - 0{,}5\sigma_C^2 \rho^2 T + \sigma_C \rho y \sqrt{T}\right) + \left(\mu_C - 0{,}5\sigma_C^2\left(1-\rho^2\right)\right)T, \sigma_C^2\left(1-\rho^2\right)T\right]$$

where the event $V_T = D$ corresponds to $y = h = -\dfrac{\ln V_0 - \ln D + \mu_V T - 0{,}5\sigma_V^2 T}{\sigma_V \sqrt{T}}$.

The equation (2.30) for expected LGD can be then evaluated as

$$ELGD = \frac{1}{F} E\left[\max\left[0; F - C_T\right] \middle| V_T < D\right]$$

$$= \frac{exp\left[\mu_C T\right]}{F} E\left[E\left[exp\left[-\mu_C T\right]\max\left[0; F - C_T\right] \middle| y\right] \middle| y < h\right]$$

(2.31) $$= \frac{exp\left[\mu_C T\right]}{\Phi(h) F} E\left[BS^{Put}\left\{C_T exp\left[-0{,}5\sigma_C^2 \rho^2 T + \sigma_C \rho y \sqrt{T}\right], F, \mu_C, \sigma_V^2\left(1-\rho^2\right), T\right\} \mathbf{1}_{\{y<h\}}\right]$$

where $\mathbf{1}_{\{y<h\}}$ is indicator function giving 1 if $y < h$ and 0 otherwise, BS^{Put} is the Black–Scholes formula for the value of put option with arguments corresponding to value of the underlying asset, exercise price, risk-free interest rate, variance and maturity, respectively. The $\Phi(h)$ is the probability of default as was defined in (2.19). After a few rearrangements it is possible for ELGD to get following expression

(2.32) $$ELGD = \frac{e^{\mu_C T}}{F} \frac{1}{PD} \int_{-\infty}^{\Phi^{-1}(PD)} BS^{Put}\left\{C_0 e^{-0{,}5\sigma_C^2 \rho^2 T + \sigma_C \rho \sqrt{T} y}, F, \mu_C, \sigma_V^2\left(1-\rho^2\right), T\right\} \varphi_{0,1}(y) dy$$

[1] A common example is an entrepreneur whose company is the debtor but who provides the collateral as a private person.

[2] See Appendix A for detailed properties of the bivariate normal distribution.

where $\varphi_{0,1}$ is the density function of the standardized normal distribution. Equation is integral over Black–Scholes put option formula, where the upper integration boundary is a function of the firm's probability of default, which does not depend on the collateral value parameters. ELGD is a function of the dynamics of firm's value only through PD. However, if the C and V are uncorrelated (ρ =0) then ELGD does not depend on firm's PD at all.

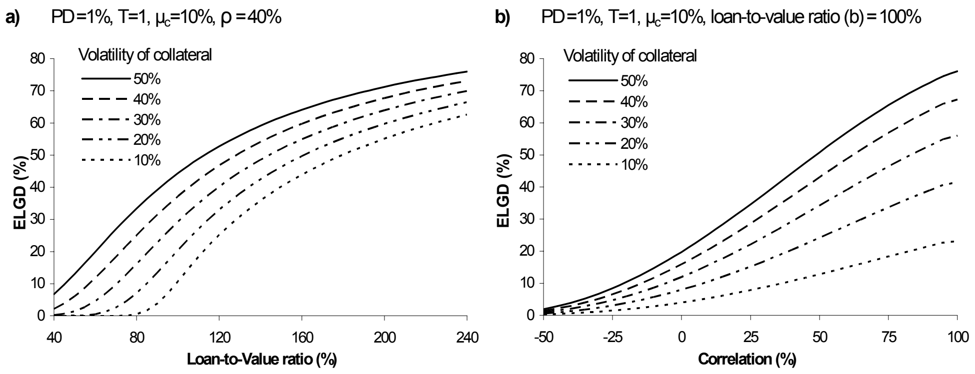
To present ELGD sensitivity to model parameters, it is convenient to define loan-to-collateral value ratio (F/C_T) because ELGD is homogenous of degree zero in C and F, this can be seen after rewriting (2.32) as

$$(2.33)\ ELGD = \frac{1}{PD}\int_{-\infty}^{\Phi^{-1}(PD)} BS^{Put}\left\{\frac{C_0}{F}e^{\mu_C T-0,5\sigma_C^2\rho^2 T+\sigma_C\rho\sqrt{T}y}, e^{\mu_C T}, \mu_C, \sigma_V^2\left(1-\rho^2\right), T\right\}\varphi_{0,1}(y)dy$$

and therefore it is more useful to analyze the changes of F/C_T ratio than value of F and C itself. Following figure shows that ELGD is always increasing function of F/C_T which is the expected result, since with lower value of collateral with respect to the loan, the possible loss given by default rises. Also ELGD gets larger with collateral's value volatility σ_V as the uncertainty about future value of collateral increases. The highest ELGD's sensitivity to volatility measured by its partial derivative is for loan-to-value ratio around 100%.

Figure 9

ELGD for different levels of volatility as a function of F/C_T ratio and correlation



Source: computed from eq. (2.32)

Part b) of the figure illustrates the relationship between ELGD and correlation between dynamics of collateral's and firm's value. ELGD rises with higher correlation because the default event occurs, if the firm's value drops under default barrier D; higher mutual dynamics implies that in the case of default it also declines the collateral value.

The stochastic variables C and V are treated separately, which can be useful in the application of the model because external estimates for borrower's PD can be used for getting ELGD for specific debt contract F. The most difficult parameter to estimate is then the correlation between firm's and collateral's value. However, this can be estimated from historical implied asset values, if they are available.[1] The model can be used in the Advanced IRB since the banks estimating internal LGD has to consider the degree of dependency between borrower's risk and collateral, as it was mentioned in previous chapter. Despite that the Basel requires estimate of *economic loss* including all costs arising in workout process, the ELGD in this model consists only of interest and principal loss, however; the calibration on historical data can be used to incorporate other collection costs to ELGD. The result is that the model may be useful as a quantitative tool for estimation of expected LGD in advanced IRB approach, although it comes out from the basic Mertonian approach that is assumed to be oversimplifying the reality. The subsequent chapter therefore deals with later adaptations, which release some of the seminal assumptions.

2.1.3. First-Passage-time approach

In the former approach, the default event could occur only at maturity time. This assumption was firstly relaxed by Black and Cox (1976) who come with the idea of default barrier (DB) that causes the default anytime during life of contract, if borrower's assets touch barrier level. The economic interpretation of DB is a safety covenant, which is "*...contractual provision which gives the bondholder the right to bankrupt or force a reorganization of the firm if it is doing poorly according to some standard"* (Black, Cox 1976, p. 5). Safety covenants hedge debt holders against the big losses in case that the borrower's asset would deeply drop under the value of debt.

Default barrier can be treated in models as a constant as e.g. in Leland (1994), Longstaff and Schwartz (1995), deterministically as in Black and Cox (1976), Leland and Toft (1996), or stochastically as in Briys and de Varenne (1997) or Collin and Goldstein (2001). Also it can be distinguished whether the barrier is set exogenously as a percentage share of debt value or it is endogenously determined in the model as the output from an optimization task. If the DB is touched and default occurs, the recovered amount is simply equal to the DB, however, it must be considered, that default time is an uncertain event, which makes it difficult to determine time value of RR.[2]

Briys and de Varenne (1997) also relaxed the fixed default threshold and allowed it to be itself dependent on the term structure of interest rate. The integration of interest rate dynamics in the credit pricing framework is an appropriate step, partly it is hard to analyze bond's value assuming constant interest rate, and also dynamics of interest rate and its spread is indirectly linked to the incorporation of business cycle effects, since the RR is related to macroeconomic variables, as it was shown in first chapter.

[1] The second option according to Jokivuolle and Peura (2005) would be to use stock market industry index returns to proxy for firm's value returns.

[2] One could use for determining the time of hitting the barrier a cumulative distribution function for the first passage time of the Wiener process (see Karatzas, Shreve 1991). The value of equity is in the first-passage time models determined by using the formula for the barrier (down-and-out) call option instead of standard European call option, which was used in the seminal Merton's model (see Hull 2002).

In the following part the pricing approach for zero-coupon bond developed in Pirotte (1999) is presented, where there is integrated the stochasticity of interest rate and possibility of before-maturity default. This framework is coherently rooted in Merton's model from the previous sections and therefore it allows better comparison of changes in estimated LGDs coming from above mentioned modifications.

The asset value follows again the log-normal diffusion process with constant instantaneous drift and volatility. Equity holders receive a continuous dividend rate δ_V that are implicitly financed by asset sales. This means

(3.1) $$dV_t = \left(\mu_V - \delta_V\right)V_t dt + \sigma_V V_t dW_t^V$$

The assumption that all assets can be traded continuously without any restriction in a liquid and frictionless market still holds as well as that the firm is financed from equity and single non-callable zero coupon bond of finite maturity.[1]

The term structure of interest rate is assumed to be given by a two-factor model following a bi-dimensional Ornstein–Uhlenbeck process, a mean-reverting process with a constant variance.[2] This allows decomposing the interest rate into two components linked to the business cycle of the economy

$$ds = a_s(b_s - s)dt + \sigma_s dW_t^s$$

(3.2) $$dl = a_l(b_l - l)dt + \sigma_l dW_t^l$$

where s is the spread between the short-term and long-term[3] interest rate (described by process dl), $r = s + l$, b_s, b_l are the long-run averages to which s and l revert with speeds a_S, a_l, furthermore holds $dW_t^s.dW_t^l = \rho_{s,l}$. The asset value of the firm is assumed to be correlated with the spot interest rates. Also is assumed that a risk-free asset $P_{0,T}$ exists with maturity T whose value depends on a term structure of interest rate $R_t(T)$ and whose dynamics under the risk-neutral probability is given by

(3.3) $$dP_t = r_t P_t dt + \sigma_{P_{t,T}} P_t dW_t^P$$

where W_t^P is linear combination of $W_t^s$and dW_t^l, and also holds relation

(3.4) $$dW_t^P dW_t^V = \rho_{P,V}\sigma_{P_{t,T}}\sigma_V$$

with $\sigma_{P_{t,T}} = \sqrt{\eta_s^2 + \eta_l^2 + 2\rho_{s,l}\eta_s\eta_l}$, $\eta_s = \sigma_s \frac{1}{P_{t,T}}\frac{\partial P_{t,T}}{\partial s}$ and $\eta_l = \sigma_l \frac{1}{P_{t,T}}\frac{\partial P_{t,T}}{\partial l}$ for formal proof see Cossin and Pirotte (2001).

[1] Both of them are assumed to be continuously traded.
[2] For additional information involving two-variable model of the interest rates' term structure see e.g. Schaefer and Schwartz (1984).
[3] The long-term rate captures mostly the business cycle effects.

The default event is triggered any time between the origin of the contract and maturity T if the asset value reach default barrier level, which is considered to be defined exogenously as constant fraction of the face value of debt F times a risk-free discount bond $P_{t,T}$ with maturity T. DB is therefore stochastic given as

$$DB_t = HP_{t,T} = \phi FP_{t,T} \qquad 0 \le \phi \le 1 \tag{3.5}$$

and can be viewed as the Black and Cox's safety covenant, however extended with stochastic interest rate. It results in the fact, that higher discounting rate means a lower DB but also a lower discounted value to recover, if the barrier is reached.

The default time occurs when $V_t \le DB_t$ and can be simply expressed as follows

$$\tau_D = \inf\left\{t \in [0,T] : V_t \le DB_t\right\}. \tag{3.6}$$

The assumption of an absolute priority rule still holds under this framework, however, the liquidation cost are not assumed to be zero as in Merton's model and when default occurs, the exogenous cost $(1-\varphi)$ is taken from the value of assets at that time.[1] The recovery value at default time is then given as

$$RV_{\tau_D} = \begin{cases} \varphi V_T & \text{if } \tau_D = T \\ \varphi V_{\tau_D} = \varphi DB_{\tau_D} & \text{if } \tau_D < T \end{cases} \tag{3.7}$$

where φ is exogenously given RR on the assets' value available at τ_D .The expected value of RR will keep varying as it is a function of V_t and interest rate dynamics. Also the value of the assets of the firm V is different from the total value of the firm, which depends on the leverage and bankruptcy parameters. This holds because the model enables to treat the value of an asset V as an exogenous variable

$$V_t = E_t + D_t + L_t(V) - C_t(V) \tag{3.8}$$

where E is the equity value, D is debt value, $L(V)$ is the present expected value of bankruptcy costs and $C(V)$ is the present expected value of dividends' payout.[2]

The formula for the value of corporate bond at maturity T is given by payoffs in the different states and by considering (3.5) it can be expressed as

$$D_T(V,s,l,T) = F\mathrm{I}_{\{\tau_D > T, V_T > F\}} + \varphi V_T \mathrm{I}_{\{\tau_D = T, V_T < F\}} + \varphi\phi F\mathrm{I}_{\{\tau_D < T\}}. \tag{3.9}$$

Further there is assumed that all traded securities can be priced in terms of risk-neutral (martingale) probability measure Q. Since the debt and equity are traded assets and if liquidation costs and the dividend payout rate are known, V can be observed by using (3.8) and the debt at time $t=0$ is given

$$D_0(V,s,l,T) = E^Q\left[e^{\int_{t_0}^{T} r_t dt}\left(F\mathrm{I}_{\{\tau_D > T, V_T > F\}} + \varphi V_T \mathrm{I}_{\{\tau_D = T, V_T < F\}} + \varphi\phi F\mathrm{I}_{\{\tau_D < T\}}\right)\right] \tag{3.10}$$

[1] For the simplicity is assumed that the liquidation costs are the same and does not change with default time.
[2] Total value of the firm is in model expressed as $\upsilon_t(V) = V_t - L_t(V) + C_t(V)$.

which can be after some computation (see Pirotte 1999) further written as

$$D_0(V,s,l,T) = FP_{0,T} - \text{Default put}$$

$$= FP_{0,T} - FP_{0,T}\left(\Phi(-d_2) + (DB_0/V_0)^{2\gamma-2}\,\Phi(l_2)\right) \qquad \Big\}\, FP_{0,T} - FP_{0,T}X$$

$$+\varphi V_0 e^{-\delta T}\left[\Phi(-d_1) - \Phi(-k_1) - (DB_0/V_0)^{2\gamma}\left(\Phi(h_1) - \Phi(l_1)\right)\right] \qquad \Big\}\, \varphi V_0 e^{-\delta T} Y$$

$$+\varphi HP_{0,T}\left[\Phi(-k_2) + (DB_0/V_0)^{2\gamma-2}\,\Phi(h_2)\right] \qquad \Big\}\, \varphi HP_{0,T} Z \tag{3.11}$$

where $FP_{0,T}$ is the value of risk-free bond with face value F and maturity T and

$$DB_0 = HP_{0,T} = \phi FP_{0,T}$$
$$\gamma = 0,5 - \delta T/T^*$$
$$T^* = \int_0^T \left[\sigma_V^2 + \sigma_P^2(u,T) + 2\rho\sigma_P(u,T)\sigma_V\right] du$$
$$d_1 = \ln(V_0 e^{-\delta T}/FP_{0,T})\Big/\sqrt{T^*} + 0,5\sqrt{T^*} = d_2 + \sqrt{T^*}$$
$$h_1 = \ln(DB_0^2\, e^{-\delta T}/V_0 HP_{0,T})\Big/\sqrt{T^*} + 0,5\sqrt{T^*} = h_2 + \sqrt{T^*}$$
$$k_1 = \ln(V_0 e^{-\delta T}/HP_{0,T})\Big/\sqrt{T^*} + 0,5\sqrt{T^*} = k_2 + \sqrt{T^*}$$
$$l_1 = \ln(DB_0^2\, e^{-\delta T}/V_0 FP_{0,T})\Big/\sqrt{T^*} + 0,5\sqrt{T^*} = l_2 + \sqrt{T^*}$$

The better look at equation (3.11) can show, that the first parenthesis denoted as X is the overall probability of default and expression Y represents the probability of default at maturity $PD_{\{\tau_D = T\}}$ and Z stands for the probability of default before maturity $PD_{\{\tau_D < T\}}$. Using the conditional probability, the term $\varphi V_0 e^{-\delta T} Y/X$ then expresses the expected discounted recovered amount in the case of default at the maturity T and $\varphi HP_{0,T} Z/X$ in the case of default at $\tau_D < T$. Dividing them by $FP_{0,T}$ gives us the recovery rates for different scenarios and (3.11) can be rewritten as

$$D_0(V,s,l,T) = FP_{0,T}\left[1 - PD\left(1 - RR_{\{\tau_D = T\}} - RR_{\{\tau_D < T\}}\right)\right] \tag{3.12}$$

The sum of expected discounted RRs for different scenarios of default can be denoted as aggregated discounted expected recovery rate (ERR_D). Similarly we can get aggregated discounted LGD as the $1-ERR_D$ and equation (3.12) express in terms of individual LGDs which gives

$$D_0(V,s,l,T) = FP_{0,T}\left[1 - PD\left(LGD_{\{\tau_D = T\}} + LGD_{\{\tau_D < T\}} - 1\right)\right] \tag{3.13}$$

Finally, the aggregated discounted LGD we can therefore express as

$$ELGD_{A,D} = LGD_{\{\tau_D = T\}} + LGD_{\{\tau_D < T\}} - 1$$

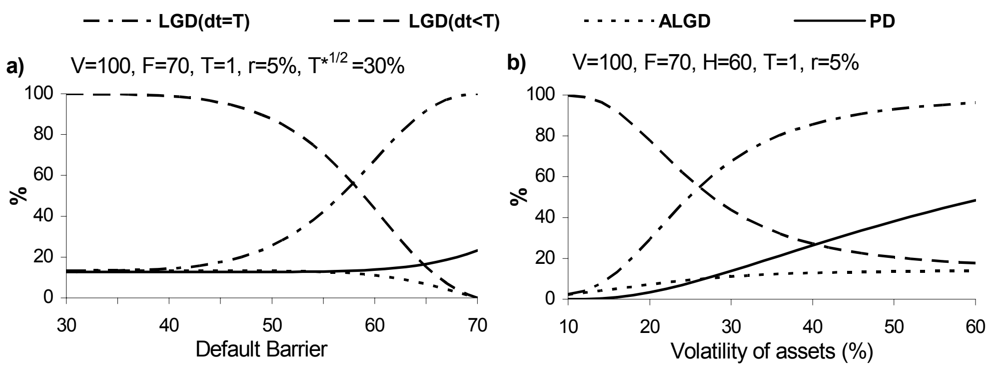
$$
\begin{aligned}
&= 1 - \frac{\varphi V_0 e^{-\delta T}}{FP_{0,T}} \frac{\Phi(-d_1) - \Phi(-k_1) - (DB_0/V_0)^{2\gamma}\left(\Phi(h_1) - \Phi(l_1)\right)}{\Phi(-d_2) + (DB_0/V_0)^{2\gamma-2}\,\Phi(l_2)} && \Big\} LGD_{\{\tau_D = T\}} \\
\textbf{(3.14)} \quad & \\
&+ 1 - \frac{\varphi HP_{0,T}}{FP_{0,T}} \frac{\Phi(-k_2) + (DB_0/V_0)^{2\gamma-2}\,\Phi(h_2)}{\Phi(-d_2) + (DB_0/V_0)^{2\gamma-2}\,\Phi(l_2)} && \Big\} LGD_{\{\tau_D < T\}} \\
&-1
\end{aligned}
$$

On the basis of this formula is presented comparative statistics of development of PD, and both aggregated and individual LGDs ($LGD_{\{\tau_D=T\}}, LGD_{\{\tau_D<T\}}$). This is presented for some sets of the most relevant parameters in the Figures 10 and 11. The costs of default are not taken into account and therefore $\varphi = 1$.

The part a) shows the impact of default barrier's level on PD and expected LGDs. As it was already mentioned, the higher the barrier is, the higher the level of assets is at the time of default which raises the recovery rate. However, the higher default barrier increases the probability of default, because it is more likely for assets to drop to the DB level and hence PD starts from some level of DB rise.

Figure 10

The sensitivity analysis for PD and ELGD to DB and volatility of assets

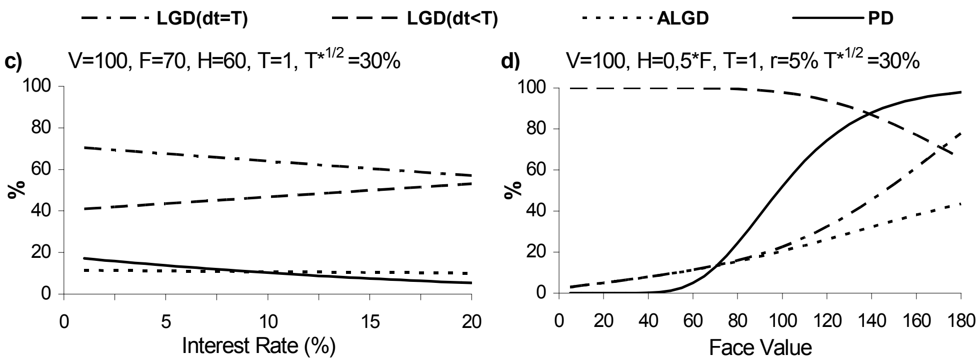


Source: computed from eq. (3.14)

For the low level of barrier (with respect to assets and debt level) is the likelihood of default before maturity very small, but if it occurs, the loss would be large as the value of assets will be low (equal to DB). Therefore is the expected LGD in case of default before maturity for the low level of DB high and stepwise falls with rise of barrier. Contrariwise it holds for expected LGD at maturity time conditioned by no default till maturity. Note, that the low level of DB represents the case when default can occur only at the maturity day which is the Mertonian case. The overall effect of DB on LGD is represented by aggregated LGD curve, which with higher level of DB goes down. Part b) illustrates the fact that the increase in volatility of assets leads to higher PD and expected LGD, since it raises the probability of reaching the DB.[1] However, if we want to analyze individual LGDs, it is necessary to look closer at the ratio between individual and overall PD. For low level of volatility $PD_{\{\tau_D<T\}}$ is very low compared to overall PD and hence the expected $RR_{\{\tau_D<T\}}$ is low and expected $LGD_{\{\tau_D<T\}}$ is high. Oppositely it holds for $LGD_{\{\tau_D=T\}}$ that increases with volatility as $PD_{\{\tau_D=T\}}$ is falling compared to overall PD. This can be deduced from eq. (3.14).

Figure 11

The sensitivity analysis for PD and ELGD to *r* and face value of debt



Source: computed from eq. (3.14)

Another sensitivity analysis displays the effects of different interest rates. We know that DB is function of interest rate through the value of riskless bond, see eq. (3.5). With rise of interest rate the value of riskless bond falls and hence also DB drops. This causes a lower RR in case of default before maturity. Also, higher discounting rate means a lower discounted value of recovered amount, if the barrier is touched. The $LGD_{\{\tau_D<T\}}$ is therefore increasing in the interest rate. However, in the martingale measure the riskless interest rate is also the drift of borrower's assets, which induces decrease of $LGD_{\{\tau_D=T\}}$. Those two effects are against each other, which leads to almost constant aggregated LGD.

In the d) part the DB is determined as a proportion of face value of the debt. The figure presents behavior of LGD depending on leverage of the borrower. As it can be expected, the PD and aggregated LGD rise with leverage.[2] The highest increase in PD measured by marginal changes is about 1 for values of leverage.

The presented expansion of seminal Merton's model by two factor term structure of interest rate causing the stochasticity of DB is more in line with presented evidence of the business cycle effects on recovery rates. Thus, negative correlation between PD and RRs is not only caused by dependency on the same fundamentals of the borrower, but also because of decrease of DB's value in the time of recession. The impact of volatility of assets or lever-

[1] The volatility of assets has the same effect on LGD as in the seminal Merton's model (Figure 8).
[2] We assume all other things being equal, also the volatility of assets.

age on LGDs is consistent with previous simpler models; nevertheless, the presence of DB causes (ceteris paribus) overall decrease of LGD.

The benefit of structural models is their economic intuition, since they explicitly state, which parameters trigger off the default event and how is their influence on LGD. However, this has also the drawbacks, because many of those parameters are not directly observable and have to be estimated (e.g. the volatility and value of assets). This represents the main difficulty in terms of models' application, although the wide range of methods was developed to extract those input parameters from other market data.[1] The attempt to overcome those shortcomings gave rise to reduced-form models, which do not rely on the structural parameters of borrowers and hence they do not have to be estimated.

2.2. Reduced-form Models

As should become apparent from the previous part, the probability of default and recovery rates in structural models are mainly determined by the value of assets of the borrower, which also determines the fair value of borrower's debt. Contrary to this fact, reduced form models do not condition default and recovery on the fundamental values of borrowers and lack the link and intuition between credit risk and firm's financial situation.[2] These models take a purely probabilistic approach and simply assume that the default event is possible, modeled by exogenous default process, without any attempt to explain it economically. Assuming no arbitrage opportunity, defaultable instruments' prices are driven by the market's expectation about default and recovery rate. Our effort will be therefore to use pricing mechanism of reduced model for extracting the credit risk parameters from market prices.[3] Therefore we need to look closer, what assumptions the models have about PD and LGD.

2.2.1. Assumptions about RR and PD in reduced models

It was already said that in the reduced-form model's framework is default unpredictably driven by a default process. This process is usually modeled by a different type of Poisson jump processes and if the jump occurs, default is triggered. PD of this event in reduced models is described in terms of intensity parameter. This means, the conditional probability at time t that default occurs between t and $t+\Delta$ (if survived till t), is given approximately as $\lambda_t\Delta$. Parameter λ is called the *intensity* or *hazard rate*.[4] At the basic level, the intensity parameter is constant, what correspond to homogenous Poisson process. However, λ can

[1] Those methods and their limitations are described closer in Chapter 3, which is dealing with empirical application of structural approach in the Czech Republic.

[2] Therefore they are termed as „reduced-form“ models, because they rely on reduced information and do not consider firm's fundamental parameters.

[3] Examples of reduced-form models include Jarrow and Turnbull (1995) Jarrow, Lando and Turnbull (1997), Duffie (1998), Madan and Unal (1998), Duffie and Singleton (1999) or Madan et al. (2006).

[4] For example, a constant hazard rate of 0,04 for time interval of 1 year represents a mean arrival rate of 4 times per a 100 years, what means that probability of default during one year is $1-\exp(-0{,}04)=3{,}92\%$ (for details see Appendix A).

be also deterministic and can change over time, in this case the default time is modeled by so called homogeneous Poisson process. Furthermore, if the intensity rate itself is stochastic, we speak about Cox process, where not only time of jump is stochastic, but also is the conditional probability of the jump over a given time interval (see Appendix A).

Using the hazard rate setting presented above, we can now specify the pricing of defaultable bond. Let's assume constant λ and possibility of multiple defaults. This means that every default causes fractional constant percentage loss L of the bond's price. If we denote D_t as the price at time t of a defaultable zero coupon bond with maturity T, then using the Itô's lemma, we can express the dynamics of the risky bond's price according to Servigny and Renault (2004) as follows

(4.1) $$dD = \frac{\partial D}{\partial t}dt + \frac{\partial D}{\partial r}dr + \frac{1}{2}\frac{\partial^2 D}{\partial r^2}(dr)^2 - LDdN$$

where the first three terms represent the dependence of the bond price on time and riskless interest rate r. The last term shows that when the jump occurs ($dN=1$), the price drops by a fraction L. If we assume that risk-free interest rate follows dynamics described as

(4.2) $$dr_t = \mu_r dt + \sigma_r dW_t$$

then under a risk-neutral measure Q, it must hold $E^Q[dP]=rPdt$ and since expected default probability is represented by λdt and $E^Q[dr]=\mu_r dt$, equation (4.1) can be rewritten as

(4.3) $$0 = \frac{\partial D}{\partial t} + \frac{\partial D}{\partial r}\mu_r + \frac{1}{2}\sigma_r^2\frac{\partial^2 D}{\partial r^2} - (r + \lambda L)D$$

and comparing this equation with the one that holds for default free bond B with same maturity

(4.4) $$0 = \frac{\partial B}{\partial t} + \frac{\partial B}{\partial r}\mu_r + \frac{1}{2}\sigma_r^2\frac{\partial^2 B}{\partial r^2} - rB$$

we see that only one difference is in the last term λL. If we solve those equations we can then express the price of risky bond at time t as $D_t(T)=B_t(T)exp[-\lambda L(T-t)]$. The term λL therefore represents the spread and is the only difference between price of risk-free and risky bond.

Even if this example with zero coupon bonds and constant intensity parameter is simplified, it shows us the basic intuition behind reduced-form models. This is, that on the arbitrage-free setting where all securities may be priced in risk-neutral measure Q, it is possible to price defaultable bond as if it was default-free by using default adjusted short-rate $R_t=r_t+\lambda_t L_t$, where r is a riskless rate. In this case, as Duffie and Singleton (1999) state, the price of risky bond D_t promising payment F at maturity T can be expressed as

(4.5) $$D_t = E_t^Q\left[\exp\left(-\int_t^T R_t dt\right)F\right]$$

Similar formula can be easily derived for defaultable securities with more general payoffs (see Servigny and Renault 2004). We can see that the adjusted R is composed from riskless r and λL what is actually a product of PD and LGD representing the risk-neutral expected loss. We see that the risk-neutral spread therefore reflects information about both PD and LGD. An important feature for valuation of equation (4.5) is taking λ_t, L_t as exogenous. The assumption, that default intensity rate and fractional loss L does not depend on the value of D_t, is typical for reduced-form models.[1]

Further using of eq. (4.5) depends on parameterization of default adjusted rate. One can either parameterize by some single or multi factor model directly R_t or each of its component r_t, λ_t and L_t. The later approach enables to consider better the possible dependencies among λ_t, L_t and r_t, or model those variables as the function of the state of the economy, what better fits the evidence that λ_t, L_t vary with the business cycle. Thus, allowing for correlation between λ_t and riskless r_t seems desirable. Intensity rate can be also modeled on the base of rating migration matrices as Jarrow, Lando and Turnbull (1997) did, who used for migration process Markov chain dynamics with default as an absorbing state. All methods are trying to obtain reliable estimate for risk-adjusted R_t and for credit parameters λ_t, L_t.

The spread of a corporate bond reflects the risk of default, however, its magnitude for given borrower depends also on the bond's maturity, coupon degree of subordination or expected future interest rate (see Litterman and Iben 1991). Nevertheless, market observed spreads are not containing information about credit quality of the defaultable security only, but also the market risk, the liquidity premium and the tax impact (see Fisher 1959 or Elton et al. 2001). Moreover, the influence of each of these segments keeps changing over time what makes difficult to segregate them. Therefore, it is not so straightforward to analyze, to what extent the change in price is linked to the change in PD or LGD expectations. To capture other effects influencing spread, Duffie and Singleton (1999) introduced a stochastic process l that adjusts short rate as $R_t = r_t + \lambda_t L_t + l_t$. Thus, reduced-form models can be distinguished by the manner in which λ, L and dependencies between r and value of debt D are parameterized.

Presented part considered LGD or RR as an exogenous fraction of debt's value just before default time (see variable L in eq. 4.1). This approach to RR parameterization is in the literature denoted as RMV – Recovery of Market Value; it represents the loss given by default, which is measured as the difference between price before and after default date (see Madan et al. 2006 or Bhatia 2006). Another assumption about recovery used Jarrow and Turnbull (1995) who took RR at default as an exogenous fraction of the market value of default-free bond with the same maturity and face value as the defaulted bond have. This stipulation of RR is therefore referred as Recovery of Treasury (RT) and it tries to consider the fact that "*...amounts of principal with long maturity should be discounted more than principal payments with short maturity"* (Lando 2004, p. 121). However, sometimes it would be more suitable if the bond of the same issuer, seniority, and face value has the same RR, re-

[1] However, this assumption is for some cases (e.g. swap contract with asymmetric credit quality of counterparty) counterfactual and hence is later in Duffie and Singleton's (1999) work released.

gardless of remaining time to maturity or coupon rate. This can be measured as the fraction of face value (RFV model). This concept of RR can have legal interpretation based, for example, on the assumption of absolute priority or liquidation at default.[1] This is also the measure typically used by rating agencies. RR modeling based on exogenous recovery of face value was used e.g. by Duffie (1998) who parameterized RR on the base of those statistics.

Which of above mentioned assumptions is more appropriate is still investigated. For example, study by Madan et al. (2006) tested on a sample of tripple B-rated corporate bonds, what assumption has better empirical support and they found out that recovery as a fraction of discounted face value (recovery of Treasury) demonstrates lower average error and therefore is more empirically supported. Contrary to that, Guha (2002) claims that defaulted bonds of equal seniority are traded at identical price, independent of their maturity time or coupon rate. This corresponds to framework of RFV. Also, Duffie and Singleton (1999) suggest that calculation with assumption of RFV or RMV generates rather similar results and therefore, even in the cases when "recovery-of-face" is the more appropriate assumption, one can exploit the more analytically tractable RMV framework. Indeed, recovery concept is often driven by practical requirements rather than by the quest for accuracy as the appropriate choice of recovery function enables to obtain closed-form solution for debt's price. Thus, RMV is easier to use, since standard default free term structure modeling techniques can be used, as we presented above.

Nevertheless, if we want to separately identify RR from market prices, the assumption of RMV is not appropriate, since it allows the estimation of the expected loss only. It means that knowledge of defaultable bond prices alone is not sufficient to separately identify hazard and loss rate, because they enter the pricing relation (4.5) only through the mean-loss rate λL (see Duffie and Singleton 1999). Contrariwise, as it can be seen in Karoui (2005), the RT and RFV approach allows for identification of the separate impact of the hazard rate and the recovery rate on bond prices. This results in the fact that although the RMV has become a standard assumption in credit risk modeling due to its mathematical tractability, however, its limitation has lead also to application of RT and RFV assumptions for RR modeling.

For example, Madan et al. (2006) developed a reduced-form model under RT and RFV assumption in which it is possible to extract a term structure of recovery rates. They derived the general pricing solution when the RR is stochastic. Unfortunately, closed-form expression for price of risky bonds is still difficult to obtain and computational costs of numerical techniques are high even for one single factor model, by which RR is parameterized. Therefore this model has only one factor – interest rate, explaining both default and recovery risk. This is according to Christensen (2007) the model's main weakness.

To learn more about the hazard and recovery rates implicit in market prices within RMV framework, it is necessary to utilize other additional information relating to particular

[1] It is understood in the sense that debt with the same priority is assigned to a recovery rate depending only on the outstanding amount of debt but not on maturity or coupon.

security. Das and Hanouna (2007) developed a technique for bootstrapping implied risk-neutral forward recovery rates exploiting additional information from credit default swap (CDS) spread curve. Also Christensen (2007) provided evidence that separation of default and recovery risk is in reduced-form credit setting possible using actual data about CDS rates for particular issuer.[1] Further, Jarrow (2001) proposed a methodology for extracting RRs and PDs by using not only the debt prices but also extra information from equity prices. Another possibility is to examine bonds prices that share some but not all of the same default characteristics. This quite intuitive and not so technically demanded approach we will examine in detail in the following part.

2.2.2. Extracting expected RR from different bonds' seniority

It was already stated that the recovery rate differs by seniority of the debt. This is used in the following approach that exploits the prices of bonds of different seniority of the same issuer to extract information about possible recovery rate and to get its risk-neutral density. The following framework comes from Unal et al. (2003) who also state that "*...relative prices of securities facing identical arrival risks but differing in their default conditional recovery rates are an important source of information"*. Within this approach there is a new general statistics termed as *adjusted relative spread* (ARS) presented, that may be derived from market prices and yields other pieces of knowledge for recovery rate expectation.

Let's consider a frictionless economy where two classes of zero coupon bonds are traded: (i) default-free bond with price $P(\tau)$, time until maturity $\tau = T - t$, and unit face value and (ii) defaultable bond with price $D_i(\tau)$, i = {S: senior bond, J: junior bond}, where its holders receive the promised unit face value if the firm survives till maturity or reduced value of face, if default occurs. As it is shown in Madan and Unal (1998), the price of defaultable bond then can be expressed in terms of default-free bond as

(5.1) $$D_i(\tau) = P(\tau).(1 - PD(\tau)) + P(\tau)PD(\tau)E\left[RR_i\right] \qquad i = \{S,J\}$$

where $PD(\tau)$ is the probability of default of the issuer and $E[RR_i]$ denotes expected recovery for the bond of given seniority in the case of default. The both of those bonds have identical risk of default but they have different recovery arising from their seniority.

If we denote p_s as the ratio of the promised face value of the senior debt to the value of all promised face values (senior plus junior), then the aggregated recovery rate of the issuer after default can be expressed as

(5.2) $$RR = p_s RR_S + (1 - p_s) RR_J$$

assuming that RR for senior and junior debt is the same regardless of time to maturity.

[1] Houweling and Vorst (2005) state that default swaps are relatively insensitive to changes in the recovery rate as long as the hazard function is scaled accordingly. This would mean that as long as the recovery rates take a reasonable value, there is no need to determine the recovery rate for CDS very accurately.

By using the equation (5.1), we can get the relative spread of the prices of senior and junior debt over the spread of default-free bond and junior debt as

(5.3) $$RS = \frac{D_S(\tau) - D_J(\tau)}{P(\tau) - D_J(\tau)}$$

what is after substitution of (5.1) possible to express without probability of default simply as

(5.4) $$RS = \frac{E\left[RR_S\right] - E\left[RR_J\right]}{1 - E\left[RR_J\right]}$$

We can see that RS gives the information independently on the PD and therefore this approach based on the RS is the pure recovery framework as the equation for RS is free of default timing risk. The attractiveness of RS is hence in the fact that it gives the information about expected recoveries from observed market prices. Using the relation (5.2), it is possible to get relative spread in terms of expected overall *RR*. This is

(5.5a) $$ARS = p_s.RS = \frac{E\left[RR\right] - E\left[RR_J\right]}{1 - E\left[RR_J\right]}$$

denoted as *adjusted relative spread* that may also be computed from market prices and information about senior debt's proportion. The ARS, however, can be also rearranged in terms of expected *LGD*.

(5.5b) $$ARS = 1 - \frac{E\left[LGD\right]}{E\left[LGD_J\right]}$$

As Unal et al. (2001) state, to be able to better analyze the dynamics of RS or ARS, it is necessary to obtain the risk-neutral recovery density for variables in equation (5.5a) or (5.4). Let's suppose that RR_J is the function of aggregated *RR*. It means that $RR_J = J(RR)$. If we have a density function for default conditional aggregate recovery $f(RR)$, we may write $E[RR_J]$ as

(5.6) $$E\left[RR_J\right] = \int_0^1 J(RR) f(RR) dRR$$

and yield the expected value of recovery for junior debt.

The form of function $J(RR)$ depends on the fact, whether the absolute priority rule is violated or not. Under strict APR, holders of junior debt are paid only after senior debt's holders receive full satisfaction of their claims, $RR = p_s$[1] If the APR is violated, the proportion of the junior debt is paid after default even if senior debt claims are not wholly settled. Breaking the APR is captured by parameter λ ($0 \le \lambda \le 1$) which reflects the amount of overall *RR*, from which APR is violated. It means, as long $RR \le \lambda p_s$, the junior debt holders receive nothing. They start receiving payoff and sharing the aggregated *RR* with senior debt when

[1] This amount of *RR* satisfies fully all claims of senior bonds. For $RR > p_s$ the recovery payments to junior bond holders begin.

$RR \geq \lambda p_s$. No APR violation is captured by $\lambda = 1$. If APR is broken, it still has to be determined which proportion of recovered amount is paid to junior and which to senior debt. This is described by parameter θ ($0 \leq \theta < 1$). The lower its value is, the more the recovery rate of senior claimant in favour of the junior ones is reduced. Armed with all described parameters, we can now closely specify the function $J(RR)$ for different scenarios as

(5.7) $$RR_J = J(RR) = \begin{cases} 0 & RR \leq \lambda p_s \\ \dfrac{(1-\theta)(RR - \lambda p_s)}{1 - p_s} & \lambda p_s < RR \leq RR^* \\ \dfrac{(RR - p_s)}{1 - p_s} & RR^* < RR \leq 1 \end{cases}$$

where RR^* is the rate of recovery that fully satisfied senior claims (and other increase in RR increases directly junior debt's RR_J) what is given as $RR^* = \lambda p_s + (1-\lambda) p_s / \theta$

As it was shown already by Black and Cox (1976), the function $J(RR)$ under APR represents the payoff to a long position on a call option written on the default conditional recovery rate with exercise price equal to the proportion of senior debt p_s.[1] The violation of APR increases the value of the call option by reducing its exercise price (this makes the junior debt holders better off and oppositely worse off to senior ones). The equation (5.7) can be rewritten alternatively as

(5.8) $$RR_J = J(RR) = \frac{1-\theta}{1-p_s} \max\left[RR - \lambda p_s ; 0\right] + \frac{\theta}{1-p_s} \max\left[RR - RR^* ; 0\right]$$

what can be interpreted, that junior debt holders' function $J(RR)$ is given as a sum of two call options. More accurately, $(1-\theta)/(1-p_s)$ units of call written on the debtor's expected default conditional aggregate RR with exercise prices λp_s, and $\theta/(1-p_s)$ units of call written also on RR with exercise price RR^*.

To evaluate eq. (5.6), a second component, $f(RR)$, has to be known. It is assumed that RR is normally distributed.[2] Nevertheless, it is necessary to make some adjustments in order to have RRs between 0 and 1. Therefore there is made a suggestion that RR is related to a normal random variable x by the logit transformation $RR = e^x/(1+e^x)$ and x is normally distributed $x \sim N(\mu, \sigma^2)$. From the property of normal distribution, using $x = \ln(RR/1-RR)$, it is straightforward to get the conditional density for overall recovery rate as

(5.9) $$f(y) = \frac{1}{(1-y)\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}\left(\ln\left(\frac{y}{1-y}\right) - \mu\right)^2\right] \qquad 0 < y < 1$$

[1] The function of recovery payoff for senior debt $S(RR)$ is represented in the same manner by payoff of default-free bond and a short position on a put option written on the defaulter's conditional payout.

[2] In this case we speak about RR distribution of one specific company. Assumption of normality therefore does not contradict aforementioned fact about bimodal distribution of RRs observed across economy.

and by using this density function it is possible to express expected value and variance of *RR* for given μ, σ^2 as follows [1]

(5.10) $$E(y) = 1 - \int_0^1 N\left(\frac{\ln(y/1-y)-\mu}{\sigma}\right)dy$$

(5.11) $$Var(y) = \int_0^1 2(1-y)N\left(\frac{\ln(y/1-y)-\mu}{\sigma}\right)dy - \left(\int_0^1 N\left(\frac{\ln(y/1-y)-\mu}{\sigma}\right)dy\right)^2$$

where *y* denotes the overall *RR*. With the help of density function one can also determine the probability that call options, given in eq. (5.8), are in the money once default occurs. And because the price of a call option written on the underlying asset *z* with strike price k is

(5.12) $$C(k) = \int_k^1 (z-k)f(z)dz$$

we can express after rearrangements the price of call written on the overall defaulter's *RR* with strike k as follows

(5.13) $$C(k) = 1 - k - \int_k^1 N\left(\frac{\ln(y/1-y)-\mu}{\sigma}\right)dy$$

where *z* is again substitution for *RR*.

Using this formula for pricing conditional *RR* call option, one can get the expression for $E[RR_J]$ from equation (5.8) as

(5.14) $$E[RR_J] = \frac{1-\theta}{1-p_s}C(\lambda p_s) + \frac{\theta}{1-p_s}C(RR^*)$$

and considering the fact resulting from (5.10) and (5.13) that $E[RR]$ is actually equal to $C(0)$, we have all necessary expressions to get formula for ARS from eq. (5.5) which is

(5.15) $$ARS = \frac{E[RR]-E[RR_J]}{1-E[RR_J]} = \frac{C(0) - \frac{1-\theta}{1-p_s}C(\lambda p_s) - \frac{\theta}{1-p_s}C(RR^*)}{1 - \frac{1-\theta}{1-p_s}C(\lambda p_s) - \frac{\theta}{1-p_s}C(RR^*)}$$

where value of call option with corresponding exercise price is given in (5.13).

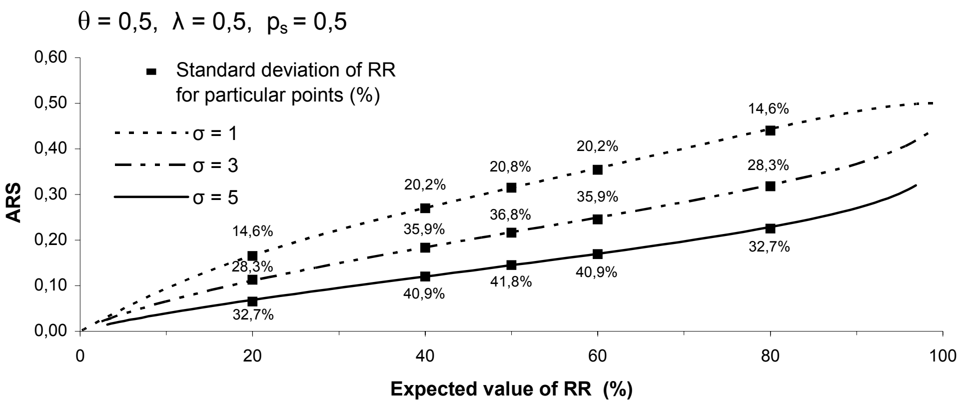
To be able to describe the dynamics of ARS with respect to parameters on which it depends, following figures present sensitivity analysis for some set of parameters' values. Figure 12 displays relation between ARS and overall *RR*'s expected value and standard deviation. As it can be seen, ARS increase with expectation about future RR. This is obvious from definition of ARS. The RR_J of junior debt is an increasing function of overall *RR*, denominator in eq. (5.5a) has to therefore drop with increase of $E[RR]$. Conversely, the numerator will rise, because *RR* of the junior debt is increasing "more slowly" than aggregated

[1] The derivation is based on integration of eq. (5.6) by parts and then using standard properties of normal distribution. For formal proof see appendix in Unal et al. (2003).

RR, since senior claimants are still favored in repayments.[1] Alternatively we can look at ARS as the development of LGD_J/LGD ratio which comes from eq. (5.5b). Higher expected *RR* lowers expected *LGD*, but junior debt's LGD_J will fall to a lesser extent as the recovery payments receive more senior debt holders.

Figure 12

The sensitivity analysis of ARS to expected value and volatility of RR



Source: computed from eq. (5.10), (5.11) and (5.15)

Higher volatility brings more uncertainty into recovery process. This concerns mainly senior debt holders, because their RR_S is more sensitive to *RR* changes. ARS is therefore decreasing with volatility of *RR*. Note that the parameter σ does not represent volatility of *RR* but volatility of normally distributed variable x used in logit transformation. Volatility of *RR* is dependent due to property of transformation on mean of *RR*, see equation (5.11). With $E[RR]$ approaching to 0 and 100%, volatility of *RR* becomes zero. Mean and volatility are therefore related and it is obvious from the figure, that the highest values have for mean equal to 50% and from that point the volatility is decreasing in both directions.

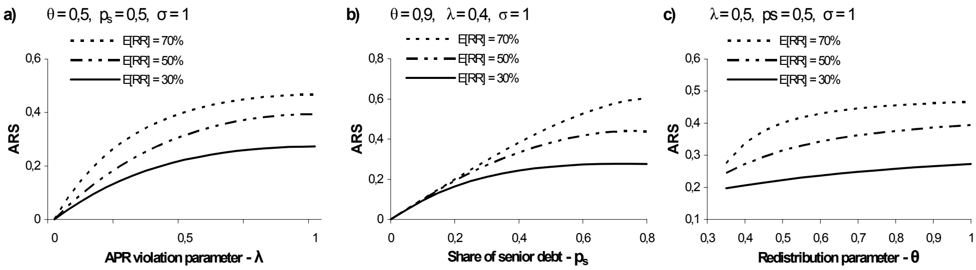
The dependence of ARS on other parameters is presented in Figure 13. Part a) describes impact of λ (APR violation parameter). Higher values of λ mean that APR becomes violated after a higher portion of senior debt is repaid. Hence, expected RR for senior debt increases as well as *LGD* for junior debt. Therefore ARS is increasing with λ. The similar impact on ARS has the parameter p_s describing proportion of senior debt among borrower's claims. From eq. (5.15) one can see that higher p_s increases the exercise price of call option written on the overall defaulter's *RR* what decreases its value. Thus, $E[RR_J]$ falls and since $C(0)=E[RR]$ is assumed to be lower than 100%, the numerator in (5.5a) grows larger than denominator and ARS hence rises.

[1] APR violation is taking into account, however, not such a extreme case where would be junior debt holders paid before senior ones.

In the last part of the figure the relationship between ARS and parameter θ can be seen. The higher values of θ indicate that less of recovered value is shared with junior debt holders; the junior debt's expected LGD_J rises. It also causes rise in ARS, see eq. (5.5b).

Figure 13

The sensitivity analysis of ARS to parameter λ, p_s and θ



Source: computed from eq. (5.10) and (5.15)

As it follows from the presented results, the ARS statistic is related to the expected overall *LGD* of borrower and other parameters describing APR violation. However, these parameters should be stable for specific debt as they depend on the particular borrower and bankruptcy procedures in given country. The dynamics of ARS in the time hence should bring an insight also on the dynamics of market expectation about borrowers LGD. Furthermore, this can be acquired on the base of observable information like the prices of debt for different seniority and their mutual share and can serve as early warning model for given borrower.

As it was demonstrated, both structural and reduced models are commonly used to price credit-sensitive securities and might be utilized for extracting LGD parameter from market observable information. While structural approach focuses on company's development of asset value using option-pricing techniques, reduced-form models extend to take into account those inter-firm dependencies and default event specify poorly in probabilistic view by some type of jump process. Both types of models have evolved over last decade and are still developing. The initial restrictive assumptions were in many cases relaxed and the models became more sophisticated and complex.

Thus, this part attempted to identify main building blocks, assumptions and restrictions of structural and reduced-form models and to present how it can by utilized for extracting LGD. This knowledge is later used in the following chapter, where the structural approach to identify LGD for particular sample of companies in the Czech Republic is used.

3. Estimate of LGD in the Czech Republic

"Prediction is very difficult, especially if it is about the future."

Nils D. Bohr [*]

The previous chapter presented different ways of extracting LGD parameter from market-observable information. The added value of any model comes also from its implementability, therefore this section is devoted to empirical computation of LGD in the Czech Republic based on the market data.

The basic input parameters for extracting LGD in reduced-form approach are the prices of risky corporate bonds. However, the companies in the Czech Republic are still using more traditional bank loans as the source of finance than issuing bonds (see Dvořáková 2003). It results in the fact that the domestic market with corporate debt is rather illiquid and incomplete and can hence barely reflect market expectations about default and recovery risk of particular company or its security. Also reduced-form models are based on the risk-neutral measure that is defined as a unique equivalent martingale measure only when the markets are complete. The result is that the reduced-form models are nowadays hardly applicable for LGD estimation in the Czech Republic.

The stock market provides an alternative source of information assuming that the share prices incorporate all available information including future prospects of the company as well as its creditworthiness.[1] Structural models for extracting company's default risk typically utilize the observed stock prices, stock volatility and specifics about the company's capital structure. Even if the number of quoted companies in the Czech Republic is also limited, for some of them seems to be sufficiently liquid to apply structural models and estimate demanded credit risk parameters.

As the result, we will implement the Merton's structural approach on a sample of firms, which are listed on Prague Stock Exchange (PSE) and present dynamics of expected LGD for each company between 2000 and 2008. We restrict our sample to non-financial firms, so that the leverage ratios could be comparable across them. In addition, we exclude enterprises that become listed after 2007 to obtain at least one year time series of share prices necessary to estimate asset volatility. The list of 27 analyzed companies can be found in Appendix C, Table1.

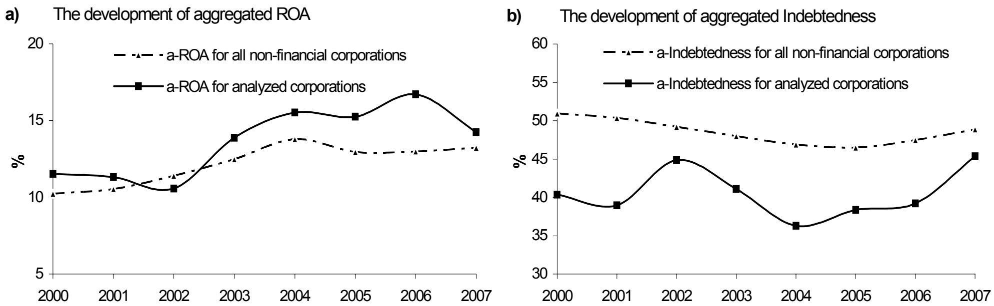
[*] Danish physicist, the Nobel Prize in Physics in 1922.

[1] This is true only if an efficiency hypothesis holds, which was doubted by some studies (see e.g. Sloan 1996). There is also a question, whether the volatility of stock price is caused solely by incorporation of new information about future stocks' returns, or if it is caused largely by trading itself (see French 1980 or French and Roll 1986).

On the basis of aggregated indicators (Figure 14) we can assume that the analyzed sample is on average less risky than other non-financial corporations in the Czech Republic.[1] The performance expressed by aggregated ROA is for the sample of companies listed on PSE higher in the recent years and the ratio between total liabilities and total assets (Indebtedness) is contrariwise lower during the whole period. This could indicate that the credit riskiness of our sample is lower what consequently means that the average expected LGD will be for those companies lower than it would be for non-financial corporate sector overall. However, analyzing companies with no traded equity or debt is constrained since one has to utilize only accounting information, which is not designed to measure directly the market value of the company and therefore can not provide reliable estimates of LGD.[2]

Figure 14

The comparison of analyzed sample with other non-financial corporations



Source: CNB (2008), CZSO (2008), Magnus (2008), author's computations

Income statements and the balance-sheet items for our set of PSE corporations were obtained from Magnus (2008) database and for some of them were completed from company's annual reports. Share prices, dividend yields and the number of shares outstanding are available on the web of Prague Stock Exchange.[3] We used time series of share prices from the beginning of 1999 to the end of 2007 and accounting information reported at the end of fiscal year. The series of five year risk-free interest rates comes from ARAD database of Czech National Bank (CNB).

One of the difficulties with structural-models approach lies in the estimation of the borrowers' fundamental parameters like the asset value or default boundary. Therefore, the following section describes an estimation procedure that is inevitable for empirical application of structural model. Next, other additional extensions of initial Merton's model are introduced to better capture dynamics of company's credit risk and provide more reliable estimates of expected LGDs.

[1] The comparison is based on the economic results of non-financial enterprises with more than 100 employees, which provides Czech Statistical Office.

[2] Especially for companies with growth opportunities book value-based valuations will yield significantly lower values than is the true market value (see Lockridge and Sridharan 2005).

[3] The information is also available for the Czech companies in Magnus (2008) database.

3.1. Model's implementation

The computation of expected LGD is based on derivations presented in the previous chapter on the basis of the seminal Merton contingent claim approach. It was shown that expected LGD in the physical measure can be expressed as

(6.1) $$LGD = 1 - \frac{V_0}{F}\exp[\mu_V T]\frac{\Phi(-d_1^*)}{\Phi(-d_2^*)}$$

However, the original Merton's model does not include payouts to security holders. Those are usually difficult to incorporate and still to maintain the closed-form solution (see Dalianedis and Geske 2001).[1] The payout structure during the life of the security is therefore alternatively included into models as the interest and dividend payout at the debt's maturity (see e.g. Vasicek 1984).

Since the interest payouts occur over the life of the debt and they are considerably lower than the principal amount, they represent lower default risk. Their neglecting should not hence bring important bias into our analysis. However, to disregard dividend stream, as Hillegeist et al. (2004) state, could introduce significant errors in estimation of current market value of the firm and its volatility and influence resulting LGD estimate. Therefore we modify the seminal Merton approach and incorporate into model payout of dividends.

3.1.1. Payout of dividends

The original model considers company's equity as a call option on the value of the company's asset. When the value of the assets is below the face value of the debt at its maturity, the option is left unexercised and bankrupt firm is turned over to its debtholders. The equation for value of equity at time $t=0$ is then given in (2.14) as follows

(6.2) $$E(V,T) = V\Phi\left(d_1\right) - Fe^{-rT}\Phi\left(d_2\right)$$

If we denote dividend rate δ as the ratio between the sum of the prior year's common and preferred dividends and the market value of the firm's asset, then the equation for the equity value reflecting the dividend stream paid by the firm accrues to equity holders would change as proposed by Hillegeist et al. (2004) in

(6.3) $$E(V,T) = V\exp[-\delta T]\Phi\left(d_1\right) - Fe^{-rT}\Phi\left(d_2\right) + (1-\exp[-\delta T])V$$

where the additional $\exp[-\delta T]$ in the first term accounts for the reduction in the assets' value due to dividends distributed before maturity T. The last expression $(1-\exp[-\delta T])V$ does not appear in the traditional equation for call option on a dividend paying stock since dividends do not accrue to option holders. This term therefore represent the fact that equity holders receives the dividend and for $\delta=0$ is this term equal to zero. Equation (6.3) is derived under

[1] Black and Cox (1976) presented a closed-form solution for debt with coupon payment assuming that the debt is perpetuity.

risk-neutral measure, therefore risk-free rate is taken as the expected rate of return on the firm's value. This rate is however lowered by the dividend rate and hence the terms d_1, d_2 have to be modified as

$$d_1 = \frac{\ln(V_0 / F) + \left(r - \delta + 0,5\sigma_V^2\right)T}{\sigma_V \sqrt{T}}, \; d_2 = d_1 - \sigma_V \sqrt{T} \; .$$

The empirical use of any structural model is based on variables, which are not directly observable. Similarly in our case, the market value of assets V and also asset volatility σ_V, must be estimated in order to compute expected LGD.[1] The procedure for estimation of those variables was firstly proposed by Jones et al. (1984) for publicly listed companies exploiting the prices of their shares. Their approach is based on simultaneous solving two equations, which are matching the value of equity E and its volatility σ_E with two unknown variables V and σ_V. The equity data is generally used since actual daily prices are observable and equity is the firm's most liquid security.[2] Jones et al. (1984) used as the first equation the relation (6.2). Nonetheless, this equation does not consider dividends' payout and we will hence utilize modified equation (6.3). The second equation linking the observable and unknown values is in the form

$$\sigma_E E = \sigma_V \exp[-\delta T] V \Phi(d_1) \tag{6.4}$$

and its derivation uses the Itô's lemma and is presented in Appendix C. This system of two equations has to be solved to arrive at unobservable market value of firm's asset and its volatility. Due to the non-linearity of those equations it is necessary to solve the system iteratively.[3]

The accuracy of the expected LGD estimate is therefore dependent on the estimates of parameters in equation (6.1). Although some of them as the debt's face value[4] or its maturity are observable, some assumptions about them must be made to be able to implement Merton's simplifying approach.

3.1.2. Estimation of parameters

The implementation of the original Merton's model requires reducing firm's capital structure into one single liability. Since the large share of the firm's debt is not very often traded, we have to use the book values as a proxy. As a result, the book value of total liabilities reported in firms' balance sheets is used as the notional face value of the zero coupon bond. This approach is often used because equity holders earn the residual value of the firm

[1] The market value of the firm is the sum of the equity and debt's market value. However, the market value of the debt is not usually available since companies are not financed entirely by traded debt.

[2] It could be also possible to match firm's bond price and its volatility with unknown V and σ_V (see Delianedis, Geske 2001). However, as it was already mentioned, the bond market usually suffers with higher illiquidity than the stock market what could be reason of higher inaccuracies in calculation of unknown parameters.

[3] To solve two non-linear equations of the form $F(x,y) = 0$ and $G(x,y) = 0$ can be minimized the function $[F(x,y)]^2 + [G(x,y)]^2$ (see Kulkarni et al. 2005).

[4] This holds only if the debt is traded.

once all debt is paid off (see e.g. Helwege et al. 2004 or Hillegeist 2004).[1] Further, for the LGD's estimation this approach seems convenient as we are interested in the ratio between the value of the firm after possible default and the value of all claims that will be demanded by creditors.

To determine the maturity time of zero coupon bond representing all firm's liabilities, we could compute the weighted maturity of the individual claims' maturities. Another method widely used among academics is to group the short-term and long-term obligations and find out the maturity by weighting the maturities of those two groups. For example Dalianedis and Geske (2001) made assumption of 1 year maturity for short-term and 10 years for long-term debt. The weights would be the book values of claims. However, our intention is to provide LGD comparable across the sample of analyzed companies, which would be hardly practicable in case of different maturities. Therefore we will assume five years debt's maturity for all companies, which should be an assumption considering the length of both short-term and long-term debt's maturity. By setting the longer time horizon we should also avoid inaccuracies coming from the fact that we use for firm's asset value dynamics poor diffusion process without possible jumps.[2]

From our previous discussion is obvious that V and σ_V estimates are highly dependent through the system of two equations on the value of equity and its volatility. While the market value of equity E is simply obtained as the shares' closing price at the end of the fiscal year multiplied by outstanding number of stocks, the value of equity's volatility depends on chosen method of estimation. For that reason it is desirable to use different types of estimation techniques for mutual comparison.

The standard approaches of estimating σ_E can be based on the historical data of stock prices or can exploit bond prices for getting so called implied volatility. Bond implied volatility is acquired when one chooses the asset volatility such that the price generated by our model fits to actual bond's market value.[3] Nevertheless, since this volatility's estimates incorporates all possible errors of used model and also considering our discussion about illiquid and insufficient bond market, we will use only historical approach using the development of stocks' returns.

Let P_i denotes the day i closing price of the stock. Then the continuously compounded one day return r_i is defined as

(6.5) $$r_i = lnP_i - lnP_{i-1}$$

[1] Moody's KMV model specifies the notional default point as the book value of short-term liabilities plus half of the value of long-term liabilities (see Crosbie and Bohn 2003). They put a greater weight on short-term obligations because debts due in the near term are more likely to cause a default. However, this approach is probably more convenient in the first-passage time models than in seminal Merton, where the default may occur only at debt's maturity.

[2] See the discussion in subchapter 3.2.2.

[3] Similarly, one could get the option-implied volatility for the companies with options written on their stock by using standard Black–Scholes formula for pricing option (see Hull 2002).

and the unbiased estimate of the one day volatility using the m observations of the r_i is

(6.6) $$\sigma_E = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}\left(r_i - \overline{r}\right)^2}$$

where $\overline{r}$ denotes the mean of r_i's (see Hull 2003). The appropriate observation interval depends on the time horizon, which we are dealing with. Since we set the maturity time to 5 years, we should also use the long-term volatility for our predictions. From that reason we used volatility of 5-trading years.[1] In addition, to take into account the possible changes in volatility in the shorter time period, we also estimate last 250 trading days' volatility similarly as did e.g. Kulkarni et al. (2005).

The improvement over those traditional methods of volatility estimate that give equal weigh to each observation, is the estimate using the exponentially weighted moving average (EWMA), where more recent observations carry higher weights. This method, capturing better the volatility dynamics, is recommended in RiskMetrics™ (1996) and for a given set of m observations can be exponentially weighted volatility computed as

(6.7) $$\sigma_E = \sqrt{(1-\lambda)\sum_{i=1}^{m}\lambda^{i-1}\left(r_i - \overline{r}\right)^2} \qquad 0 < \lambda < 1$$

where λ is referred as the *decay factor* that determines the relative weights for particular observation. For our sample of companies we used monthly observations over the five years with decay factor equal to 0,97. This value is based on the analysis relating to optimal λ that was provided in RiskMetrics™ (1996).

The fourth and the last method that we used is GARCH(1,1) from class of ARCH models that consider the fact that variance of time series returns tends over time to revert to its long-run average (see Bollerslev 1986). We estimate GARCH(1,1) model for daily data over the five year interval in the form

(6.8) $$\sigma_t^2 = b + \alpha_1 r_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 \qquad \alpha_0 > 0, \alpha_1 \geq 0, \alpha_2 \geq 0$$

where $b = \alpha_0 \sigma_{LR}^2$, σ_{LR}^2 represents the long-run unconditional variance of daily returns r and $\alpha_0, \alpha_1, \alpha_2$ are the weights whose sum is equal to 1. Since we are concentrating on the long-run volatility, we use only the long-run average variance σ^2_{LR} to which the process will convert in the future. The long-run volatility is therefore computed from estimated parameters as

(6.9) $$\sigma_E = \sqrt{\frac{b}{1-\alpha_0-\alpha_1}}$$

However, for some companies was not the GARCH's long run volatility estimated as their return's time series was not weakly stationary. The GARCH is also unstable, when fitted parameters $\hat{\alpha}_1 + \hat{\alpha}_2$ are close to 1. This leads to integrated IGARCH(1,1) model with

[1] In the case of insufficient long time series, we use the longest available one. This holds also for other 5-year estimates computed later in this section.

additional constraint $\alpha_1 + \alpha_2 = 1$. However, the unconditional variance σ^2_{LR} is not in this case defined. Nonetheless, as it can be found in Tsay (2005), this special IGARCH(1,1) model can be rewritten as EWMA formula, with that we have already σ_E estimated.

For the most of the companies in our sample we estimated by aforementioned methods four types of daily equity's volatility. Those must be still scaled to obtain the annualized volatility used in later computations. This is simply done by using $\sigma_{annual} = \sigma_{daily} \cdot \sqrt{\#\text{days}}$, where #days is the number of trading days that we assume to be 250.

All estimates are enclosed in Appendix C, Table 2. Since the higher volatility of equity results in higher volatility of firm's value and higher default risk, the choice of estimated σ_E can significantly influence further results. However, we consider it more desirable to provide as the rule of prudence rather overstated values of LGD than vice versa. Therefore we use the average of the two highest σ_E estimates, σ_E^* as a parameter entering the system of two equations.

The derived system for obtaining unobservable values of V and σ_V exploits as the firm's expected rate of return the risk-free rate r_f, for which we used the yield of 5-years government bond. Therefore, the last parameter that must be estimated, in order to solve the equations, is dividend rate δ. Nonetheless, for acquiring δ, one needs to get the market value of the firm V. Hence we use the approximate market value V' as the sum of equity's market value E and book value of debt.[1] Since we are estimating 5-year horizon, we will use in computations the adjusted rate δ^* capturing dividend stream in the last five years, instead of one year dividend rate δ.[2]

We solved the two equations simultaneously by the iterative Newton search algorithm. As the starting values for V and σ_V the approximate value V' and volatility of equity were used, respectively. In almost all of the cases, the process converges within ten iterations. Note that the equation linking equity and asset volatility given by (6.4) holds only instantaneously, what causes the bias in V and σ_V estimates when the leverage changes. Crosbie and Bohn (2003) assert that a quick decrease in the leverage would lead to overestimation of asset volatility and vice versa, if the leverage increases. The impact of the change in firm's leverage on ELGD is presented later in the sensitivity analysis section.

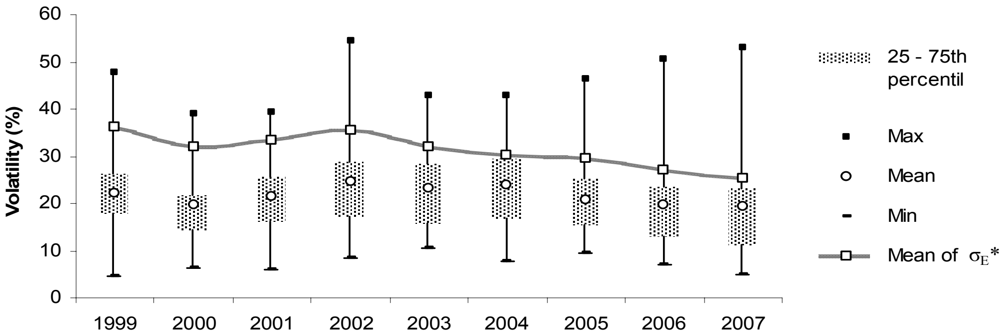
The Figure 15 presents the dynamics of the average equity's volatility and estimated volatility of firms' assets. The highest average σ_V reached almost 25% in 2004 and since then it is decreasing to 19% in 2007. The average spread represented by 25th and 75th percentile is slightly increasing in the last three years, however, across time it is quite stable in the range between 15 to 25%. Note that dynamics of estimated σ_V follows the equity's volatility σ_E^*, nevertheless, σ_V is always lower than σ_E^*. This is caused by presence of leverage, since the debt is considered as non-traded. With the increase of leverage, the equity occupies a lower

[1] This approach, as Wong and Li (2004) show, overestimates the true market value of the firm.
[2] We used exponentially weighted average with decay factor $\lambda = 0{,}9$.

share in the overall value of the firm and therefore V is less volatile than E. The difference between the average σ_E^* and σ_V in particular years is hence given by the size of average leverage.[1]

Figure 15

Development of estimated volatility σ_V and σ^*_E



Source: computed from system of eq. (6.3) and (6.4)

For estimate of expected LGD in risk-neutral measure we already know all necessary parameters, however, as the risk-free rate can significantly differ from the real firm's value rate of return, we estimate also the expected market return on assets, μ_V, as the return on assets during previous year. We can easily utilize estimated values of firm's market value V and one-year return μ_V get as

(6.10) $$\mu_V(t) = \frac{V(t) + Div(t) - V(t-1)}{V(t-1)}$$

where $V(t)$ is the firm's market value at the end of year t and $Div(t)$ denotes the sum of the common and preferred dividends declared during this year. Since the 5-year expected return will not be solely based on a one year observation only, we use in our calculations adjusted μ_V^* again as the five-year weighted average, in which recent years carry more weight to react faster to current information.

[1] It is assumed that debt is not traded. Therefore the volatility of firm's asset depends only on the volatility of equity.

3.2. Final results

The initial Merton's model is based on the framework of simplifying assumptions like the absence of transaction costs, dividends, bankruptcy costs, taxes or problems with continuous time trading. The non-existence of dividends' payouts was modified in the last section. Still, one should also incorporate the costs of bankruptcy which result that debt holders in the case of default receive less than the total firm value. Additional default costs also arise from deviations in APR where equity holders gain at the expense of bondholders. While Betker (1997) estimated the direct administration costs relating to bankruptcy around 5% of firm value, study by Andrade and Kaplan (1998) indicates higher costs of financial distress in the range of 15–20%. Based on those empirical studies we consider exogenous common bankruptcy costs $(1-\varphi)$ equal to 10%.

The final formula for 5-year expected LGD at the beginning of year t in physical measure, including both dividends payout and bankruptcy cost, is then

(6.11) $$ELGD_t = 1 - \varphi \frac{V_t}{F_t} \exp[(\mu^*_{V,t} - \delta^*_t)T] \frac{\Phi(-d^*_1)}{\Phi(-d^*_2)}$$

$$d^*_1 = \frac{\ln(V_t / F_t) + \left((\mu^*_{V,t} - \delta^*_t) + 0,5\sigma^2_{V,t}\right)T}{\sigma_{V,t}\sqrt{T}}, \text{ and } d^*_2 = d^*_1 - \sigma_{V,t}\sqrt{T}$$

where time indexes represent particular values at the beginning of year t (end of the previous year), and $\mu_{V,t}^*$, δ_t^* denotes adjusted rates considering 5-year historical observations. One can get the expected LGD in risk-neutral measure by replacing $\mu_{V,t}^*$ by r_f.

The results are given in the Figure 16 which presents the expected LGD for each company estimated at the beginning of every year during the period 2000–2008 in both risk-neutral and physical measure. The estimates in physical measure begin from year 2001 since we lost one observation for acquiring firm's growth rate. All parameter used for computations are given in Appendix C, Table 2.

In the theoretical framework of second chapter the risk-neutral LGD was always an upper bound to its physical counterpart. Nevertheless, this holds only if assets drift μ_V is greater than the risk-free rate. In the conventional analysis the r_f rate is supposed to be always lesser than drift μ_V. For example, Hillegeist et al. (2004) compute μ_V for PD estimates and use r_f as a minimal bound for μ_V, since their claim that lower expected growth rates than r_f are inconsistent with asset pricing theory. However, this approach can result in highly underestimated values of LGD if the real growth rate is lower than r_f. This can be demonstrated from given results.

Company Paramo ended year 2000 with a loss counting more than 430 mil. CZK and almost 24% drop in the market firm's value. This negative result has no impact on expected risk-neutral LGD at the beginning of 2001 and its value is even below-average for given year. However, the physical estimate captures the huge deterioration in firm's asset value which leads to more than four times higher expected LGD. Also Spolana recorded as a result of negative development in the market with plastics in 2001 looses about 700 mil. CZK. Subsequent year was negatively affected by floods which lead to other losses. While risk-neutral LGDs in these years do not incorporate any problem comparing to other years estimates, the physical measure counterparts indicate company's poor performance quite well. The same situation can be also found in the case of Lázně Jáchymov in 2001, Slezan FM in 2001, 2002 or e.g. Papírny Větřní in 2002 and 2004. Contrary to that, when the growth rate of firm's assets μ_V is higher than r_f, risk-neutral estimates overstate ELGD. Considerable overestimation emerged for example in the case of Energoaqua in 2002 and 2003, where $ELGD^Q$ is almost twice higher than physical ELGD because in those years there was a high dividend rate and low r_f in the comparison with μ_V.

Figure 16

The 5-year expected LGD in the period 2000–2008.

	Expected LGD (%) - risk neutral measure									Expected LGD (%) - physical measure							
Company	2000	2001	2002	2003	2004	2005	2006	2007	2008	2001	2002	2003	2004	2005	2006	2007	2008
CETV	–	–	–	–	–	–	18,0	22,5	21,4	-	-	-	-	-	-	23,1	19,1
Č. NÁM. PLAVBA	28,7	26,1	23,1	24,0	22,2	34,8	16,2	13,7	12,5	23,5	28,6	24,6	22,8	34,7	16,0	12,9	12,6
ČEZ	24,1	27,7	34,4	35,7	35,3	30,7	29,3	29,2	24,1	32,7	47,1	39,3	29,6	21,2	18,1	18,7	16,7
ECM	–	–	–	–	–	–	–	13,8	18,4	-	-	-	-	-	-	-	13,3
ENERGOAQUA	13,0	24,4	37,7	35,7	33,4	22,4	17,8	14,0	13,6	17,2	22,7	20,8	19,6	17,4	14,6	12,4	12,2
JČ PAPÍRNY VĚTŘNÍ	29,2	23,7	26,3	26,5	21,3	32,4	23,1	23,0	33,6	30,3	52,6	33,2	57,9	33,2	13,0	14,1	36,2
JM PLYNÁRENSKÁ	44,7	38,3	34,4	45,6	32,2	23,8	19,4	14,9	11,6	48,2	21,5	27,7	19,5	14,8	17,4	13,4	11,2
LÁZNĚ TEPLICE	17,7	16,4	16,9	15,5	17,0	17,2	16,9	15,9	14,7	49,5	12,6	11,5	12,9	13,7	14,1	14,7	14,9
LEČ. L. JÁCHYMOV	30,1	18,0	25,9	20,4	19,1	16,8	14,0	15,6	13,3	85,8	49,5	19,7	19,0	17,1	13,7	14,9	13,4
ORCO	–	–	–	–	–	–	21,3	22,5	29,5	-	-	-	-	-	-	13,2	16,7
PARAMO	30,4	17,6	16,2	20,5	19,5	23,8	25,0	21,4	22,5	78,4	65,4	44,3	16,5	20,6	19,1	18,7	19,6
PEGAS	–	–	–	–	–	–	–	28,4	19,0	-	-	-	-	-	-	-	20,4
PHILIP MORRIS	–	17,0	25,4	36,9	32,1	31,1	28,9	32,5	41,2	-	15,8	21,7	18,8	20,8	21,0	29,5	41,2
PR. ENERGETIKA	51,5	40,8	42,5	44,0	35,9	28,8	25,1	22,9	20,6	52,7	53,5	40,4	28,5	22,0	18,5	17,4	16,2
PR. PLYNÁREN.	30,2	33,0	34,6	40,3	38,5	36,8	30,9	29,1	26,4	66,7	33,2	36,3	36,4	26,6	19,4	19,2	18,9
PR. SLUŽBY	18,3	25,6	22,2	22,0	20,1	14,4	12,1	11,2	10,7	17,0	22,1	21,9	17,1	13,3	11,7	11,0	10,6
RM-S HOLDING	29,2	34,6	32,7	27,1	34,5	35,4	24,6	12,5	11,4	58,8	50,4	33,4	29,1	31,2	24,5	12,6	11,5
SETUZA	30,0	30,6	28,2	28,0	28,7	29,8	29,8	28,4	27,4	13,3	14,3	17,4	21,3	23,5	31,0	30,7	22,7
SLEZAN FM	26,4	34,4	34,4	32,4	29,4	30,2	25,9	23,1	18,5	88,3	70,4	27,8	23,1	27,9	25,3	23,2	19,8
SM PLYNÁREN.	31,5	25,1	40,6	29,9	33,7	33,7	23,9	21,7	19,2	25,2	36,5	21,1	29,6	33,5	21,5	19,6	17,4
SPOL. CH.H..VÝR.	20,0	16,2	23,0	23,4	24,9	22,4	25,5	22,0	20,0	70,1	37,8	28,1	23,9	15,8	14,5	13,7	13,2
SPOLANA	33,3	33,5	36,1	34,2	35,0	34,9	27,8	27,5	26,6	42,9	76,6	58,5	44,3	45,0	28,9	27,1	30,0
TELEFÓNICA	23,9	32,5	36,7	36,0	33,4	33,3	26,3	22,9	43,4	40,2	49,5	51,7	35,4	32,7	23,0	20,9	37,1
TOMA	29,9	29,1	23,0	23,5	21,0	19,7	23,5	21,4	18,7	67,5	24,2	29,6	18,4	15,6	16,5	15,8	13,4
UNIPETROL	36,1	30,1	26,5	24,8	26,4	29,8	35,0	36,3	31,3	24,0	25,3	23,4	22,1	27,0	18,8	22,3	21,6
VČ PLYNÁRENSKÁ	42,8	34,9	48,6	63,5	56,9	55,3	48,4	49,1	30,7	33,6	33,1	41,1	39,7	42,5	39,0	41,0	28,9
ZENTIVA	–	–	–	–	–	18,6	22,6	22,9	24,0	-	-	-	-	-	15,3	18,7	19,2
Mean (%)	29,6	27,7	30,4	31,4	29,6	28,5	24,5	22,9	22,4	46,0	38,3	30,6	26,6	25,0	19,8	19,2	19,5
Std. Dev. (%)	9,1	7,4	8,4	10,8	9,0	8,9	7,4	8,3	8,7	23,4	18,5	11,7	10,7	9,2	6,5	7,1	8,3

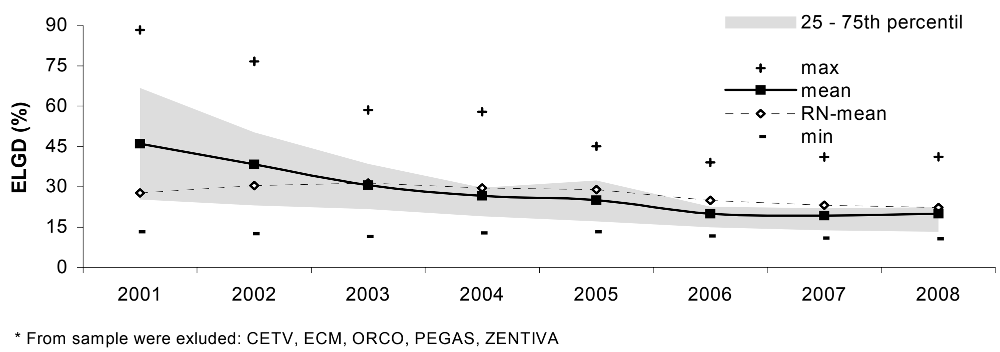
Source: computed from eq. (6.11)

The relatively high ELGD in both measures for ČEZ in 2002 might seem contradictory, since ČEZ ended year 2001 successfully with increase in net profit over 26% to more than 9 bill. CZK. However, the share price drops from initial 101 CZK at the end of 2000 to 77,5 CZK at the end of 2001 what lead to more than 23% decrease in the market value of equity. This development together with high dividend rate was reflected in almost 14% deterioration of assets value and lead to significant increase in ELGD. Similarly, high decrease in market value of equity caused the worsening of predictions for Telefónica in 2002 and 2003. Nonetheless, the sharp rise of ELGD in 2008 is solely incurred by rash increase in assets volatility.

The Figure 17 displays the average ELGD over the period from 2001 to 2008. To provide comparable estimates across time, we excluded companies which were not quoted on PSE during the whole period. The shaded strip covers the quartile range with extending from the 25th to the 75th percentile, which illustrates the variability of ELGD in particular year. From the figure, decreasing trend both in the average physical ELGD and in its variability is also evident. The expected downswing of economic activity due to global and domestic factors (see CNB 2008) was not incorporated enough in the share prices at the end of 2007. Therefore the average ELGD at the beginning of 2008 is relatively small, still capturing good economic development in the recent years. However, expected slowdown in economic growth resulted for some of the analyzed companies drop in the market prices of equity. As a result, the rough average ELGD estimate [1] at the beginning of May 2008 has raised to 24%, which indicates the increase in the credit risk in non-financial corporations sector. A slight increase in the corporate sector's credit risk in 2008 is also indicated by the creditworthiness indicator reported in CNB (2008).[2]

Figure 17

Development of average 5-year ELGD in the period 2001–2008



Source: computed from eq. (6.11)

Risk-neutral estimates are based on the same company's structural values relating to its credit risk, as do physical estimates, except different assumptions about expected growth of company's assets. Kulkarni et al. (2005) even state that since risk-neutral estimates can be calculated without estimating the firm's expected return, they may provide more accurate information. Nevertheless, as it was demonstrated, risk-neutral estimates are not properly characterizing the actual company's riskiness. The more μ_V differs from r_f, the more inaccurate results they provide compared to its physical counterpart. Therefore, creditor trying to appraise its possible recovered amount in the case of obligor's default should consider the real future

[1] The estimate is using all other parameters constant except market value of equity.

[2] This indicator calculates the outlook for the corporate sector's risk at the one-year forecast horizon based on financial indicators of solvency, profitability, liquidity and activity. More details can be found in Jakubík and Teplý (2008).

growth rate of firm's assets μ_V, as the main determinant of the future LGD,[1] even if the average values of physical and risk-neutral measures can be almost identical (Figure 17). From this point hence it is more desirable to use the real physical estimates also for the credit management in the Basel II framework.

3.2.1. Sensitivity analysis

The sensitivity analysis relating to initial Merton's model discussed in the theoretical section assumed that all necessary structural variables are known. However, the value of firm's assets and its volatility are not directly observable and they have to be estimated through the system of two equations, which hold only in the given time. Therefore, the following analysis concentrates on sensitivity of ELGD coming out from potential changes in structural variables of the company influencing also the estimates of σ_V and V. The stress is put especially on the leverage, defined as the ratio between total liabilities and market value of all assets (F/V), since this belongs to the mostly watched indicator affecting the company's creditworthiness.

Before we present the ELGD's sensitivity for individual companies in analyzed sample, we provide a general theoretical discussion based on different scenarios of input parameters. The main difference between the current analysis and the previous one illustrated in Figure 8 is caused by the fact that the change in the leverage influences the estimate of firm's assets volatility σ_V. This was already mentioned in context of dynamics of average assets and equity volatility in the Figure 15. Thus, by leverage's increase the weight of equity in the firm value declines and the volatility is decreasing. The rate of declining is for a given set of parameters presented in the first part of Figure 18. This figure also illustrates the impact of increase in the firm's leverage on the PD and ELGD. However, while the leverage's growth has positive unambiguous effect on PD, the ELGD reaches its maximum value for a particular leverage ratio and then starts to decrease.

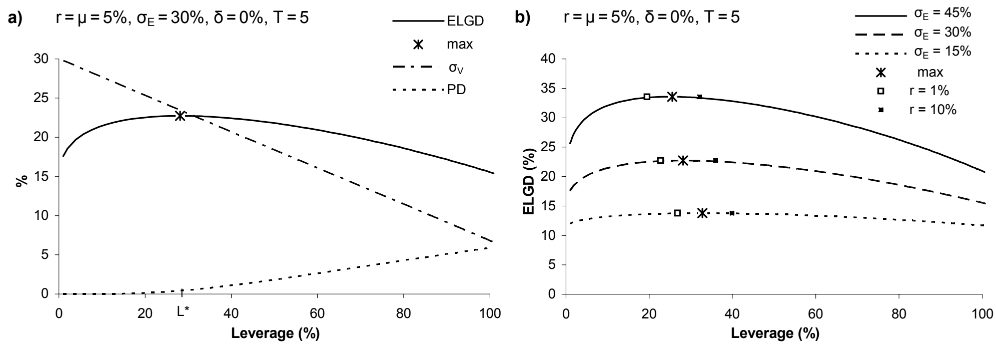
The negative relation between ELGD and leverage may look contra-intuitive; nevertheless, this development is caused by the decreasing assets' volatility σ_V.[2] Although the PD is increasing in leverage, the expected value of firm's assets at the maturity T, conditioned by default ($V_T<F$) has increased with respect to given leverage. In other words, due to lower volatility σ_V is less likely that the expected firm's value will be excessively below the value of default barrier F at time T and therefore the expected recovery ratio (V_T/F) in the case of default has increased.

[1] Also the risk-neutral estimates consider changes in markets value of company's asset through the leverage ratio. Still, as we could see, it does not seem to be sufficient.

[2] The previous analysis reported in the Figure 8 shows the strictly positive correlation between ELGD and leverage. However, the σ_V was taken as a constant and did not change with leverage.

Figure 18

The sensitivity analysis for ELGD – part 1



Source: computed from eq. (6.11) and system (6.3), (6.4)

The result is, that by leaving the initial volatility of equity as a constant,[1] the increase in leverage causes the decline in assets volatility, which from particular leverage ratio (L* – breakpoint) generates a negative correlation between PD and ELGD. Nevertheless, for all presented scenarios the increase in PD outweighs the LGD's decline and expected loss for unit of exposure (PD.ELGD) is hence strictly increasing with leverage.

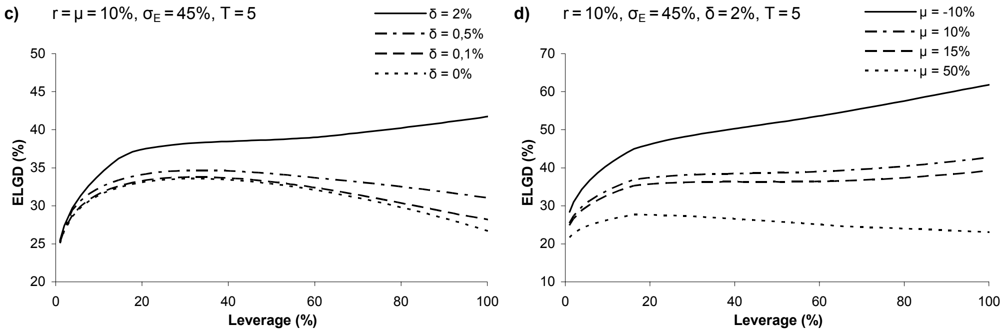
Pursuing the issue further, we analyze the changes in breakpoints with respect to other parameters. The maximum ELGD points are presented for 3 different values of r_f rate and σ_E. As it can be seen, the decline in the risk-free interest rate shifts the max ELGD points to the left, similarly as the increase in the equity's volatility (Figure 18, b). It is evident that any increase in σ_E will lead (because of higher uncertainty) ceteris paribus to higher values of ELGD. However, the figure also presents the variability of potential ELGDs along the whole range of leverages. While for $\sigma_E = 45\%$ ELGDs vary from 22 to 33 percent, the volatility for $\sigma_E = 30\%$ is only 7 percentage points, and in the case of $\sigma_E = 15\%$ is the variability of possible ELGDs minimal. This further highlights the importance of volatility as a crucial variable for LGD predictions and indicates that the companies with identical leverage ratios can have substantially different ELGD's sensitivity.

The existence of dividend rate in the system of equations lowers the estimated market value of the company V, since the part of its value is paid out to the equity holders. Supposing the same value of equity, the presence of dividends increases the estimated assets volatility, compared to the state with zero dividends rate. Thus, dividends offset the initial lowering of σ_V given by increase in leverage, which results in higher ELGD and consequently lower ELGD decrease behind the breakpoint. Moreover, the increase in assets volatility given by sufficiently high dividend rate outweighs the volatility's after breakpoint decline and the overall effect with increase in leverage on ELGD is positive (see Figure 19, c).

[1] The change in leverage will also affect the equity's volatility. However, since we use the long-run volatility σ_E^* for computation, in which does not the sudden short-term changes take effect; the assumption of constant σ_E in the sensitivity analysis is maintainable.

Figure 19

The sensitivity analysis for ELGD – part 2



Source: computed from eq. (6.11) and system (6.3), (6.4)

Till now we did not consider any differences between physical and risk-neutral measure in the analysis of ELGD's sensitivity to leverage. Since the real asset growth μ_V does not figure in V and σ_V estimation, it may seem that physical ELGD will differ for given set of parameters only in the absolute terms, keeping the same rate of change with respect to leverage. The right-hand side of the Figure 19 displays evolution of ELGD for various growth rates relating to the increasing ELGD's sensitivity curve from previous figure (2% dividend rate). As we can see, the μ_V affects also the slope of ELGD's curve, not only its parallel shift. Bad company's performance represented by small and negative μ_V will raise the rate of growth of ELGD, while good development will offset the presence of the dividend payout and the curve will become decreasing from the breakpoint again. The result is that the ELGD in the physical measure has lower growth rate in the leverage for the $\mu_V > r_f$ and for sufficiently high values of μ_V may by even the initial growth rate from some point inverted from increasing to decreasing (see part d, $\mu_V = 50\%$). This holds also vice versa for low and negative values of μ_V.

The empirical results for the analyzed sample are reported in the following table that shows the leverage elasticity of ELGD in both measures at the beginning of 2008. As it can be seen, the most of the analyzed companies have inelastic ELGD with respect to leverage. Only Spolek pro chem. a hut. výrobu has negative elasticity slightly exceeding 1. The lowest elasticity in absolute terms belongs to Pražské služby and the highest positive sensitivity of ELGD to 1% increase in leverage has Pražská plynárenská, both in martingale and physical measure.

Based on our previous discussion we can analyze differences in risk-neutral (ε^Q) and physical (ε^P) elasticity with respect to other parameters. For example, CET or Pr. Služby, companies with zero dividend rate and low leverage at the beginning of 2008, are located on the increasing part of their ELGD's sensitivity curve. However, because μ_V lowers ELGD's rate of growth and the expected assets' rate μ_V is for both companies higher than r_f, their "physical" elasticity is lower than ε^Q. On the contrary, Č. Nám. Plavba or JČ Papírny indicate inverse inequality between ε^P and ε^Q since their $\mu_V < r_f$. Further, if the company's position is at the decreasing part of the sensitivity curve, the high values of expected μ_V will raise the rate of the curve's decline and contrariwise for $\mu_V < r_f$ (ECM, Lázně Teplice, Paramo or Spolana). The dividend payout causes the positive sensitivity behind the breakpoint in the case of JM Plynárenské or Philip Morris, however, ε^P for JM Plynárenská is lowered by $\mu_V > r_f$.[1]

Figure 20

The elasticity of ELGD with respect to Leverage

Company	$\varepsilon^{ELGD^Q}_{Leverage}$	$\varepsilon^{ELGD}_{Leverage}$	Company	$\varepsilon^{ELGD^Q}_{Leverage}$	$\varepsilon^{ELGD}_{Leverage}$	Company	$\varepsilon^{ELGD^Q}_{Leverage}$	$\varepsilon^{ELGD}_{Leverage}$
CETV	0,071	0,022	ORCO	0,344	-0,128	SLEZAN FM	0,432	0,493
Č. NÁM. PLAVBA	0,042	0,045	PARAMO	-0,393	-0,498	SM PLYNÁREN.	0,308	0,228
ČEZ	0,078	-0,034	PEGAS	0,341	0,405	SPOL. CH.HUT.VÝR.	-1,072	-1,095
ECM	-0,607	-0,643	PHILIP MORRIS	0,403	0,403	SPOLANA	-0,647	-0,477
ENERGOAQUA	0,188	0,080	PR. ENERGETIKA	0,268	0,128	TELEFÓNICA	0,175	0,150
JČ PAPÍRNY VĚTŘNÍ	0,116	0,129	PR. PLYNÁREN.	0,856	0,423	TOMA	-0,093	-0,179
JM PLYNÁRENSKÁ	0,198	0,092	PR. SLUŽBY	0,011	0,004	UNIPETROL	-0,025	-0,148
LÁZNĚ TEPLICE	-0,055	-0,047	RM-S HOLDING	0,022	0,024	VČ PLYNÁRENSKÁ	0,271	0,244
LEČ. L. JÁCHYMOV	0,026	0,028	SETUZA	-0,867	-0,890	ZENTIVA	0,012	-0,109

Source: computed from eq. (6.11) and system (6.3), (6.4)

The sensitivity analysis further illustrates already pointed differences between risk-neutral and physical measure. However, the more important finding seems to be that ELGD is quite inelastic in leverage and its sudden changes do not incur significantly high turns in expected LGD. The possible inaccuracies is estimation V and σ_V, mentioned by Crosbie and Bohn (2003), caused by change in leverage might be more relevant for PD estimate, but should not bring important changes to predictions of ELGD. However, the discussion about other limits and shortcomings of presented estimates should be accomplished in more details.

3.2.2. Criticism and limitations

The first implementation of Merton's model applied by Jones et al. (1984), Ogden (1987) or Franks and Torous (1989) suggested that the model generates lower credit spreads than those ones observed on the market do. Similarly more recent studies by Lyden and Saraniti or Helwege et al. (2004) showed that basic Mertonian contingent claim model under predicts actual bond's spread especially for low-leveraged and low-volatility companies. Based on those findings, our ELGD estimates would be undervalued. However, considering the fact that bonds' spreads reflects also market risk, tax or liquidity effects,[2] the mentioned studies only confirmed Merton's inability to capture other components of debt's spread, saying nothing about model's ability to reveal default and recovery risk.

This issue can further be confirmed by Longstaff (2000) who has argued that corporate bond markets are much more illiquid than government bonds and stock market and therefore it seems likely that credit spread is only partly attributed to default risk. In spite of these well known complications and imperfections, majority of the literature empirically test-

[1] The values of leverage and expected assets' growth are reported in the Table 2 in the Appendix C.
[2] See our discussion about corporate spreads in section 2.1.

ing structural models has presumed that the credit spread is primarily attributed to default risk, since the other components are hardly tractable.[1] Sarig and Warga (1989) did not compare absolute values of theoretical corporate bond spreads, but only their rates of change with respect to change in actual bond's default riskiness and approved good predictive power of Merton's model. Further, Dalianedis and Geske (2001) termed the difference between observed and modeled spread the residual spread and empirically confirmed that the spreads estimated with Merton approach correctly evaluates default risk and residual spread is driven by liquidity, tax and other effects.[2] These conclusions move towards the correctness of our LGD estimates, since accuracy of ELGDs is based on the capturing the company's default risk.

If we assume that share prices reflect all relevant information considering future development of the company as well as the expected conditions for given industry or economy, this expectations are also incorporated in our ELGD's, since they are dependent on the development of the stock market. Thus, ELGDs based on market value of equity are forward-looking estimates which may be used to instantaneous watching company's riskiness and may serve as indicator of early-warning. Nevertheless, ELGD's stock market dependence can also embody excessive movements in the share prices caused by market bubbles. Also, the stock market may not efficiently incorporate all publicly-available information about default probability and especially in a case of a young market, such as that one of the Czech Republic, limitations of information given by share prices and particularly by companies, which shares are not so frequently traded, should be considered. [3]

For purposes of Basel II framework, the ELGD's based on equity development are procyclical and due to increase in the minimum required capital in the recession would lead to the credit crunch and contrariwise to the overlending in the time of strong economic growth. The definition of default used in the model corresponds more to the state of bankruptcy than to the obligor's ninety days past due obligation defined under Basel II. Thus, model's definition of default leads to overstated ELGD; however, the analyzed companies should have high capabilities to raise funds. So if the company is past due more than 90 days on its obligation, it has probably exhausted all means to raise the funds and the bankruptcy will follow. The different default definitions hence should not bring significant inaccuracies.

The computations also do not consider any debt's priority, therefore ELGDs for secured and more senior claims should be lower than presented estimates and vice versa higher for subordinated debt, however, the distribution of the value of the bankrupted firm depends

[1] This idea stems from the theoretical assumption that markets for corporate bonds are perfect, complete and trading takes place continuously (see Dalianedis and Geske 2001).

[2] Structural models may also understate spreads in short-run, since the pure diffusion process is not able to capture unpredicted extreme changes in firm's asset value given by shock. Therefore is also possible to add jump process to Brownian motion or to model asset value as a discontinuous Lévy process (see Bhatia 2006, p. 126 and references therein).

[3] Č. nám. plavba, Energoaqua, Jihomoravská plynárenská, Pražské služby, RM-S HOLDING, SLEZAN FM, or Východočeská plynárenská.

also on the violation of APR, what is difficult to predict for single cases. The bankruptcy costs were determined by using other empirical studies, nonetheless, bankruptcy laws and other procedures differ substantially by country, and may therefore differ in the Czech Republic. The calibration on the empirical sample would be needed to obtain more accurate estimates, but the appropriate data sample is not available due to low number of defaults of comparable companies.

The computed ELGDs suffer also by others shortcomings, like the assumption of constant interest rate, no tax shield, and other simplifications coming out from the seminal Mertonian approach. On the other side, more sophisticated models demand higher number of parameters, which have to be estimated. This increases the computation complexity and might therefore produce higher errors. Also, some introduced amendments relating e.g. to stochastic interest rate have unambiguous effects, and sometimes have only little impact on the results (Lyden and Saraniti 2000). Nevertheless, the empirical application of more complex models will be the goal of the further research.

In spite of all mentioned limitations, the presented results are the first estimates of expected LGD based on the market information for single companies listed on Prague Stock Exchange and should therefore serve as the stepping stone for their further improvement. The estimates should not deputize the estimated values of LGD based on historical data, as is requested in Basel II, however, they may serve as the early warning signal and improve thereby the current credit management.

♦ Conclusion

Among intensively studied topics in quantitative finance currently belongs also the concept of Loss Given Default, which is a rather unexplored territory in credit risk area. Especially with the implementation of the New Capital Accord, LGD has obtained increased attention and became a frequent object of empirical and theoretical research. The goal of this thesis was to present to the most recent pieces of knowledge concerning this key input parameter of credit risk analysis. Moreover, the main stress was put on modeling techniques which enable estimation of forward-looking LGDs from market observable data, which we consequently used to empirical estimation of LGD for the sample of companies in the Czech Republic.

We analyzed companies listed on Prague Stock Exchange in the 2000–2008 period and computed expected LGD for every single company at given year. The average LGD of the sample across the time was estimated in the range around 20–45%. We also described the estimation procedures exploiting prices of equity and its volatility and showed, that LGD is relatively inelastic in leverage of the company. We demonstrated that LGD in the physical measure is a more reliable indicator than its risk-neutral counterpart.

The presented results were estimated on the basis of formula derived from asset pricing models. Under structural approach, the dynamics of the value of the firm's assets is usually assumed to follow a geometric Brownian motion and therefore it is log-normally distributed. This enables us to express the likelihood of default, which occurs in the initial Merton's model if the value of the asset is lower than the value of the debt principal at the time of maturity. To gain explicit expression for LGD, we used the formula for conditional mean of log-normally distributed variable, because the recovered amount in the case of default is the rest value of firm's assets. This demonstrates that LGD is even in the initial Merton's framework stochastic since it depends on uncertain development of assets' value.

Furthermore, we presented the extension of the model when the debt is pledged by collateral. The LGD is in this case dependent on two stochastic processes, the first one represents the value of borrower's assets and determines the default event, and the second one is describing the value of the collateral which will be seized by creditor if default occurs. The presented sensitivity analysis revealed that expected LGD increases with correlation between dynamics of collateral's and borrower's assets. We also illustrated how LGD can be extracted in a more complex structural framework, which incorporates stochastic interest rate and default barrier. As the default barrier is in the model determined by the value of riskless bond, the economic recession leads to its decrease, which consequently leads to rise in LGD. Thus, besides dependencies on the same firm's fundamentals, negative correlation between PD and LGD is caused by macroeconomic factors.

Additionally there were also discussed the *reduced-form models,* which are currently hardly applicable for LGD estimation in the Czech Republic because the market with corporate debt is rather illiquid. However, we showed a sensitivity of LGD on *adjusted relative spread*

(ARS), which is a new parameter exploiting the observed prices of bonds with different seniority. ARS gives information independently on the PD of issuer and represents thereby the pure recovery framework. The dynamics of APR can be used in LGD management for particular borrower as an early warning system. With the scope of reduced-form model implementation, we discussed the determinants of corporate spreads and based on the other empirical studies we concluded that market observable spreads are considerably influenced by other effects such as tax impacts or liquidity premium and therefore are not representing information only about credit quality of defaultable security. Therefore we decided to exploit information embedded in the more liquid stock market and utilized structural models for LGD computation.

We also closely analyzed general characteristics of LGD and pointed out that LGD is significantly determined by the value of assets that can be seized in case of default. Therefore, since asset types vary across industries, industry-specificity of borrower is assumed to be a straightforward driving factor of LGD. However, the empirical studies give unambiguous results. Industry-type does not prove any evident pattern for the value of LGD, only utilities were among most of studies identified as the industry with lower LGD. The possible reason is that different stage of economic cycle can influence LGD more than industry-type itself. Besides the cyclical variation, the fact is, that the recession increases default rates and greater supply of pledged assets will lead to their lower prices. It results in positive correlation between default rates and LGD. We also confirmed this correlation by a linear, logarithmic and power univariate regression using Moody's recovery rates from 1982–2006.

The joint dependence implies that PD and LGD should not be in the credit risk modeling treated as independent. This link between PD and LGD should be also considered in IRB approach because neglecting this correlation might lead to understated capital reserves. However, it will lead to even higher procyclical effect of IRB approach and cause overlending during strong economic growth and credit crunch in the time of recession.

Moreover, we illustrated that a significant determinant of LGD after default event is the position of particular claim in the defaulter's capital structure. Obviously, LGD decreases with the seniority and security of the defaulted claim and increases with its degree of subordination. Bank loans have usually lower LGD than bonds since they are typically more senior and have tighter covenants. However, when comparing different studies of the LGD, the definition of default has to be considered. We elaborate default definition as given by Basel II and rating agencies and show that the definition under Basel framework is generally weaker and leads hence to higher observed LGDs. The unification of definitions is therefore desirable.

The thesis presented a broad understanding of the key parameter of current credit risk area, Loss Given Default. In order to do so, we dealt with LGD's properties, possible modeling techniques, and its estimates from market data, respectively. As the main value added of this work are the unique estimates of LGD for the Czech corporate sector. This altogether should bring a perspective on LGD and provide better understanding of difficulties related to this credit risk parameter.

♦ Appendix

Appendix A

- **Wiener process**

Wiener process is a stochastic process that can be characterized by following facts

1. $W(0) = 0$
2. $W(t)$ is almost surely [1] continuous
3. $W(t)$ has independent increments with distribution $W(t) - W(s) \sim \Phi(0, t-s)$ $(0 \leq s < t)$.

$\phi(\mu, \sigma^2)$ denotes the normal c.d.f. with expected value μ and variance σ^2. Independent increments means that for $0 \leq s_1 \leq t_1 \leq s_2 \leq t_2$, $W(t_1) - W(s_1)$ and $W(t_2) - W(s_2)$ are independent random variables (see e.g. Hurt et al. 2003, Shreve 1997).

- **Poisson process**

Let N_t be a standard Poisson process initialized at 0 ($N_t = 0$), increasing by 1 unit at random times t_1, t_2, t_3..., and durations between jump times t_i–t_{i-1} have exponential distribution. If we consider discrete time intervals Δt, then the probability of jump over Δt is approximately

$$P[N_{t+\Delta t} - N_t = 1] \approx \lambda \Delta t$$

where the parameter λ is intensity of the Poisson process.
By dividing the time interval (t, s) into n subintervals of a length Δt and letting $n \to \infty$ and $\Delta t \to dt$, we get the probability of the jump during time interval (t, s) as

$$P[N_s - N_t = 1] = 1 - \exp[-\lambda(s-t)]$$

where the intensity is such that $E[dN] = \lambda dt$.
In *an inhomogeneous Poisson process* λ is no longer constant and is characterized by a deterministic function of time $\lambda(t)$. Probability of jump during (t, s) is then given as

$$P[N_s - N_t = 1] = 1 - \exp[-\int_t^s \lambda(u) du]$$

If the intensity parameter λ is random process (it means that probability of observing jump over a given period is randomly changing), then we speak about *Cox process*. The probability of observing jump during time interval (t, s) is then

$$P[N_s - N_t = 1] = 1 - \exp[-\int_t^s \lambda_u du]$$

where λ_u is stochastic process taking only positive values.
(see Servigny and Renault 2004 or e.g. Hsu 1997).

[1] *Almost surely* means: Let (Ω, A, P) be a probability space. An event x in A happens almost surely if $P(x) = 1$.

- **Itô's lemma (one dimensional)**

Itô's lemma is used to find the differential of a function of another particular stochastic process. Assuming that x is described by a stochastic differential equation of the form

$$dx(t) = A(x,t) + B(x,t)dW_t$$

where $A(x, t)$ is the drift term, $B(X, t)$ is the volatility function and dW_t is a Wiener–Lévy process. The stochastic differential of process $f(x(t),t)$ is then given as

$$df(x(t),t) = \left(A(x,t)\frac{\partial f}{\partial x} + \frac{1}{2}B(x,t)^2\frac{\partial^2 f}{\partial x} + \frac{\partial f}{\partial t} \right)dt + B(x,t)\frac{\partial f}{\partial x}dW_t$$

(see e.g. Hurt et al. (2003), Shreve (1997) or Hull (2002) and references therein).

- **The Bivariate Normal Distribution**

The bivariate normal distribution for two related, normally distributed variables $x \sim \Phi(\mu_x, \sigma_x^2)$ and $y \sim \Phi(\mu_y, \sigma_y^2)$, is defined by the following probability density function

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}\exp[-0,5q(x,y)]$$

where $$q(x,y) = \frac{1}{1-\rho^2}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2 \right]$$

and ρ is the correlation coefficient of x and y (x and y are independent if $\rho = 0$). Conditional probability distribution function for one of the variables, for known value of other variable, is normally distributed with conditional mean and variance given as

$$\mu_{x|y=y_0} = \mu_x + \rho\sigma_x\frac{y-\mu_y}{\sigma_y} \qquad \text{and} \qquad \sigma_{x|y=y_0} = \sigma_x\sqrt{1-\rho^2}$$

if we use mean and variance for $\ln C_T \sim \Phi\left[\ln C_0 + \left(\mu_C - 0,5\sigma_C^2\right)T, \sigma_C^2 T\right]$ and $y \sim \Phi(0,1)$ we get the mean and variance for conditioned $\ln C_T \,|\, y$.

(see e.g. Hsu 1997).

Appendix B

- **The derivation of equation (2.10)** – based on the initial paper by Merton (1974)

As it was said, dynamics of debt security, whose market value is at any time t a function of the value of the firm and time, i.e. $D_t = f(V_t, t)$, can be expressed by stochastic equation as

(2.3) $$dD_t = \left(\mu_D D_t - \delta_D\right) dt + \sigma_D D_t dW_t^D$$

where the symbols are the same as in (2.2). By using Itô's Lemma,[1] it possible to rewrite dynamics for D_t in terms of V_t as the following equation shows

(2.4) $$dD_t = \left(\frac{\partial D_t}{\partial t} + (\mu_V - \delta_V) V_t \frac{\partial D_t}{\partial V_t} + \frac{1}{2} \sigma_V^2 V_t^2 \frac{\partial^2 D_t}{\partial V_t^2} \right) dt + \frac{\partial D_t}{\partial V_t} \sigma_V V_t dW_t^V$$

comparing corresponding variables in (2.3) and (2.4) we get, that

(2.5a) $$\mu_D D_t - \delta_D = \frac{\partial D_t}{\partial t} + (\mu_V - \delta_V) V_t \frac{\partial D_t}{\partial V_t} + \frac{1}{2} \sigma_V^2 V_t^2 \frac{\partial^2 D_t}{\partial V_t^2}$$

(2.5b) $$\sigma_D D_t = \sigma_V V_t \frac{\partial D_t}{\partial V_t}$$

(2.5c) $$dW_t^D = dW_t^V$$

In the last equation, it is evident, that dynamics of D_t and V_t is perfectly correlated as both D and V are affected by the same source of uncertainty $dW_t^{D=V}$. This proves to be important in the following forming of the portfolio that contains x_1 dollars invested in the firm V, x_2 dollars in the security D, and x_3 dollars received by short-selling the riskless debt such that $-x_3 = x_1 + x_2$. The instantaneous return of such portfolio is then

(2.6a) $$d\pi_t = x_1 \frac{dV_t + \delta_V V_t dt}{V_t} + x_2 \frac{dD_t + \delta_D dt}{D_t} + x_3 r dt$$

where r is the riskless interest rate. By substituting (2.2) and (2.3) we get

(2.6b) $$d\pi_t = \left(x_1 \mu_V + x_2 \mu_D\right) dt + x_1 \sigma_V dW_t^V + x_2 \sigma_D dW_t^D - \left(x_1 + x_2\right) r dt$$

Taking into account the equation (2.5c), we can create riskless portfolio by choosing weights $x_{1,2}$ such that the coefficient of dW_t is always zero. It must hold

(2.7a) $$x_1 \sigma_V + x_2 \sigma_D = 0\,, \qquad \text{(no risk condition)}$$

and since such a portfolio is riskless and requires zero net investment, to avoid arbitrage profits, the return of this portfolio must be zero. Formally

(2.7b) $$x_1 \mu_V + x_2 \mu_D - \left(x_1 + x_2\right) r = 0\,, \qquad \text{(no arbitrage condition)}$$

[1] See Appendix A

The weights fulfilling both conditions exist if and only if

(2.8) $$(\mu_V - r)/\sigma_V = (\mu_D - r)/\sigma_D$$

However, from equations (2.5a, b) we can substitute for μ_D, σ_D and after rearranging and simplifying, we get parabolic partial differential equation

(2.9) $$\frac{\partial D_t}{\partial t} + (r - \delta_V)V_t \frac{\partial D_t}{\partial V_t} + \frac{1}{2}\sigma_V^2 V_t^2 \frac{\partial^2 D_t}{\partial V_t^2} + \delta_D - rD_t = 0$$

Since it is given by assumptions, that there are no coupon payments ($\delta_V = \delta_D = 0$), the equation (2.9) can be then rewritten as

(2.10) $$\frac{\partial D_t}{\partial t} + rV_t \frac{\partial D_t}{\partial V_t} + \frac{1}{2}\sigma_V^2 V_t^2 \frac{\partial^2 D_t}{\partial V_t^2} - rD_t = 0$$

Appendix C

- **Companies listed on PSE**

Table 1

Companies listed on Prague Stock Exchange (PSE) at the beginning of June 2008

Name of the company	Abbreviation	ISIN	Observed years
AAA Auto Group N.V. [1)]	AAA	NL0006033375	-
CENTRAL EUROPEAN MEDIA ENTERPRISES LTD.	CETV	BMG200452024	2005 - 2007
Česká námořní plavba, a.s.	Č. NÁM. PLAVBA	CZ0008413556	1999 - 2007
ČEZ, a.s.	ČEZ	CZ0005112300	1999 - 2007
ECM REAL ESTATE INVESTMENTS A.G.	ECM	LU0259919230	2006 - 2007
Energoaqua, a.s.	ENERGOAQUA	CS0008419750	1999 - 2007
Erste Bank der oesterreichischen Sparkassen AG [2)]	ERSTE BANK	AT0000652011	-
Jihočeské papírny, a.s. Větřní	JČ PAPÍRNY VĚTŘNÍ	CZ0005005850	1999 - 2007
Jihomoravská plynárenská, a.s.	JM PLYNÁRENSKÁ	CZ0005078956	1999 - 2007
Komerční banka, a.s. [2)]	KOMERČNÍ BANKA	CZ0008019106	-
Lázně Teplice v Čechách, a.s.	LÁZNĚ TEPLICE	CS0008422853	1999 - 2007
Léčebné Lázně Jáchymov, a.s	LEČ. L. JÁCHYMOV	CS0008446753	1999 - 2007
New World Resources N.V. [1)]	NWR	NL0006282204	-
ORCO PROPERTY GROUP S.A.	ORCO	LU0122624777	2005 - 2007
PARAMO, a.s.	PARAMO	CZ0005091355	1999 - 2007
PEGAS NONWOVENS SA	PEGAS	LU0275164910	2005 - 2007
Philip Morris ČR a.s.	PHILIP MORRIS	CS0008418869	2000 - 2007
Pražská energetika, a.s.	PR. ENERGETIKA	CZ0005078154	1999 - 2007
Pražská plynárenská, a.s.	PR. PLYNÁREN.	CZ0005084350	1999 - 2007
Pražské služby, a.s.	PR. SLUŽBY	CZ0009055158	1999 - 2007
RM-S HOLDING, a.s.	RM-S HOLDING	CS0008416251	1999 - 2007
SETUZA, a.s.	SETUZA	CZ0008460052	1999 - 2007
SLEZAN Frýdek-Místek, a.s.	SLEZAN FM	CZ0005018259	1999 - 2007
Severomoravská plynárenská, a.s.	SM PLYNÁREN.	CZ0005084459	1999 - 2007
SPOLANA, a.s.	SPOLANA	CS0008424958	1999 - 2007
SPOLEK PRO CHEM.A HUT.VÝR.,a.s	SPOL. CH.HUT.VÝR.	CZ0005092858	1999 - 2007
Telefónica O2 Czech Republic,a.s.	TELEFÓNICA	CZ0009093209	1999 - 2007
TOMA, a.s.	TOMA	CZ0005088559	1999 - 2007
UNIPETROL, a.s.	UNIPETROL	CZ0009091500	1999 - 2007
Východočeská plynárenská,a.s.	VČ PLYNÁRENSKÁ	CZ0005092551	1999 - 2007
VGP NV [1)]	VGP	BE0003878957	-
VIENNA INSURANCE GROUP [1)]	VIG	AT0000908504	-
ZENTIVA N.V.	ZENTIVA	NL0000405173	2004 - 2007

[1)] Firm was excluded - insufficient long time series (issud after 2. 1. 2007)
[2)] Firm was excluded - financial institution

Source: Prague Stock Exchange (www.pse.cz)

- **The derivation of equation (6.4)**

Let's assume that the dynamics of equity value can be described by the stochastic differential equation $dE_t = (\mu_E - \delta\ )E_t dt + \sigma_E E_t dW_t^E$, where μ_E is the equity drift, σ_E is the standard deviation of equity's return, and dW_t^E is a standard Gauss–Wiener process. Dynamics of security, whose market value is at any time t a function of the value of the firm and time, i.e. $E_t = f(V_t, t)$, can be expressed by using Itô's Lemma in terms of V_t as follows

$$dE_t = \left(\frac{\partial E_t}{\partial t} + (\mu_V - \delta_V) V_t \frac{\partial E_t}{\partial V_t} + \frac{1}{2}\sigma_V^2 V_t^2 \frac{\partial^2 E_t}{\partial V_t^2} \right) dt + \frac{\partial E_t}{\partial V_t} \sigma_V V_t dW_t^V$$

Comparing corresponding variables in two equations above, we get $\sigma_E E_t = \frac{\partial E_t}{\partial V_t} \sigma_V V_t$,

and since $\frac{\partial E_t}{\partial V_t} = \Phi(d_1)$ [1] (see Helwege et al. 2004), then holds $\sigma_E E_t = \Phi(d_1)\sigma_V V_t$. After considering our discussion about dividend rate that lowers the value of V, we get the relation presented in equation (7.4) $\sigma_E E = \sigma_V \exp[-\delta T] V \Phi(d_1)$.

- **Estimated parameters**

Table 2
All relevant parameters for the sample of analyzed companies
(see the next page)

[1] This relation is also called option (equity) delta (see Hull 2002).

Table 2 - part 1

Company	End of year	Estimates of equity's volatility					Parameters used for ELGD computation						Other used parameters				
		σ_E MA(5y) (%)	σ_E MA(1y) (%)	σ_E EWMA (%)	σ_E GARCH (%)	σ^*_E (%)	r_f (%)	μ^* (%)	δ^* (%)	σ_V (%)	V (bill. CZK)	F (bill. CZK)	δ (%)	μ (%)	Leverage (%)	V´ (bill. CZK)	Equity (bill. CZK)
CETV	2005	22,7	22,7	21,9	22,8	22,7	3,1	-	0,0	17,5	62,94	16,99	0,0	-	27,0	65,35	48,36
	2006	28,2	30,7	27,5	28,7	29,7	3,3	1,2	0,0	23,4	63,69	15,91	0,0	1,2	25,0	66,09	50,18
	2007	28,4	28,6	27,5	28,7	28,7	4,0	17,6	0,0	25,0	82,73	12,74	0,0	29,9	15,4	85,02	72,28
Č. NÁM. PLAVBA	1999	37,8	37,8	33,2	39,3	38,6	6,7	-	0,0	32,3	0,09	0,02	0,0	-	22,9	0,10	0,08
	2000	35,4	32,8	28,4	38,4	36,9	6,8	20,7	0,0	34,2	0,11	0,01	0,0	20,7	10,5	0,12	0,10
	2001	32,2	24,4	23,7	36,7	34,4	4,8	-19,6	0,0	33,0	0,06	0,00	0,0	-49,9	5,5	0,06	0,05
	2002	29,7	20,0	19,2	36,4	33,0	3,2	0,7	0,0	29,7	0,07	0,01	0,0	27,4	11,9	0,07	0,06
	2003	26,6	4,9	12,7	38,3	32,5	3,8	0,2	0,0	30,8	0,07	0,00	0,0	-0,6	6,2	0,07	0,07
	2004	20,5	0,0	6,9	73,5	47,0	3,4	3,6	0,0	40,5	0,08	0,01	0,0	10,6	16,6	0,08	0,07
	2005	16,4	18,0	6,8	21,5	19,7	3,1	4,4	0,0	15,4	0,09	0,02	0,0	9,9	25,9	0,09	0,07
	2006	13,1	9,6	4,7	16,8	15,0	3,3	13,0	0,0	12,0	0,10	0,02	0,0	17,8	23,3	0,11	0,08
	2007	10,3	9,5	4,5	16,2	13,2	4,0	2,3	0,0	12,2	0,09	0,01	0,0	-16,0	9,6	0,09	0,08
ČEZ	1999	35,6	35,6	31,2	39,0	37,3	6,7	-	0,0	17,9	113,76	84,34	0,0	-	74,1	136,83	52,49
	2000	36,0	36,4	33,1	36,7	36,5	6,8	-0,2	0,5	21,1	112,36	81,61	0,8	-0,2	72,6	141,49	59,87
	2001	38,1	41,7	35,2	41,9	41,8	4,8	-8,0	0,8	24,6	95,30	76,45	1,2	-13,9	80,2	122,37	45,92
	2002	36,9	33,3	34,0	39,9	38,4	3,2	-1,3	1,3	26,2	99,83	69,54	2,1	7,5	69,7	124,30	54,76
	2003	34,2	20,9	29,1	41,1	37,7	3,8	14,6	1,8	28,4	137,20	78,22	2,9	42,2	57,0	164,51	86,29
	2004	32,8	27,6	28,1	38,3	35,5	3,4	41,5	2,0	30,2	264,57	79,22	1,9	96,7	29,9	280,99	201,77
	2005	31,9	32,1	28,4	36,4	34,2	3,1	67,1	1,9	30,3	546,00	132,92	1,6	109,7	24,3	568,96	436,04
	2006	29,0	29,6	27,4	39,6	34,6	3,3	61,5	1,9	30,8	701,40	161,00	1,6	30,6	23,0	729,52	568,52
	2007	27,7	27,0	26,7	29,4	28,6	4,0	57,9	2,0	26,9	942,99	169,56	2,4	37,8	18,0	976,15	806,59
ECM	2006	16,0	16,0	16,0	15,0	16,0	3,3	-	0,0	8,7	11,16	5,98	0,0	-	53,6	12,07	6,09
	2007	25,8	26,3	23,6	33,5	29,9	4,0	26,6	0,0	11,1	14,13	10,91	0,0	26,6	77,3	16,03	5,12
ENERGOAQUA	1999	21,5	21,5	15,1	19,6	21,5	6,7	-	0,0	4,4	0,24	0,26	0,0	-	110,9	0,31	0,05
	2000	31,0	37,9	23,3	31,4	34,7	6,8	35,1	0,0	20,2	0,32	0,19	0,0	35,1	59,2	0,37	0,18
	2001	27,2	17,1	16,6	26,5	26,8	4,8	39,1	14,1	26,8	0,33	0,15	32,6	42,2	43,6	0,38	0,23
	2002	25,9	21,5	15,2	25,6	25,8	3,2	41,8	11,3	25,8	0,45	0,20	6,4	45,2	43,6	0,55	0,35
	2003	23,9	12,9	11,9	23,4	23,6	3,8	40,5	8,2	23,6	0,61	0,32	1,8	38,2	52,6	0,76	0,45
	2004	22,2	9,7	8,4	21,6	21,9	3,4	29,1	7,9	21,9	0,61	0,13	5,2	5,8	20,9	0,67	0,54
	2005	15,0	10,8	5,8	20,2	17,6	3,1	29,5	7,3	17,6	0,76	0,13	4,2	31,7	17,3	0,83	0,70
	2006	14,2	13,2	4,9	13,5	13,9	3,3	34,3	3,9	13,9	1,09	0,14	3,0	47,2	12,9	1,16	1,02
	2007	10,8	5,6	3,3	11,7	11,2	4,0	23,3	2,4	11,1	1,12	0,29	0,0	3,5	25,5	1,27	0,98
JČ PAPÍRNY VĚTŘNÍ	1999	44,8	44,8	41,6	45,2	45,0	6,7	-	0,0	22,9	0,43	0,30	0,0	-	71,3	0,51	0,20
	2000	41,1	37,0	35,3	40,7	40,9	6,8	-0,9	0,0	15,7	0,42	0,38	0,0	-0,9	89,1	0,53	0,15
	2001	43,4	47,8	39,0	44,5	46,1	4,8	-15,6	0,0	17,7	0,31	0,25	0,0	-26,7	81,9	0,36	0,11
	2002	40,0	27,2	31,4	40,9	40,4	3,2	-6,1	0,0	20,4	0,33	0,20	0,0	6,4	59,6	0,35	0,16
	2003	39,8	38,6	30,2	42,0	40,9	3,8	-18,0	0,0	12,3	0,20	0,18	0,0	-38,6	86,8	0,23	0,06
	2004	42,0	53,5	35,7	45,7	49,6	3,4	2,2	0,0	26,1	0,29	0,17	0,0	43,5	59,1	0,31	0,14
	2005	45,6	54,2	41,7	49,3	51,8	3,1	43,0	0,0	12,5	0,65	0,61	0,0	126,1	92,5	0,74	0,13
	2006	45,8	48,7	42,9	51,8	50,2	3,3	30,6	0,0	12,7	0,58	0,53	0,0	-11,5	92,2	0,66	0,12
	2007	50,9	56,7	47,6	51,3	54,0	4,0	-8,0	0,0	53,1	0,04	0,00	0,0	-92,8	2,0	0,04	0,04
JM PLYNÁRENSKÁ	1999	32,9	32,9	26,2	32,2	32,9	6,7	-	1,4	25,7	5,41	7,55	1,4	-	139,7	9,78	2,23
	2000	24,1	8,5	13,4	n.a.	18,8	6,8	11,0	1,3	18,8	5,29	7,78	1,2	11,0	146,9	10,43	2,65
	2001	20,5	9,6	9,3	n.a.	15,0	4,8	16,5	1,9	15,0	6,68	8,69	2,8	20,7	130,1	11,90	3,21
	2002	21,4	23,9	11,2	n.a.	22,7	3,2	22,0	2,6	22,7	8,14	10,58	3,6	29,2	130,0	14,52	3,94
	2003	19,1	0,2	7,1	n.a.	13,1	3,8	14,1	3,0	13,1	8,19	9,60	3,9	0,3	117,2	13,53	3,93
	2004	12,1	0,0	3,4	n.a.	7,8	3,4	9,6	3,8	7,8	8,05	8,41	5,0	0,3	104,6	12,35	3,93
	2005	13,7	16,4	5,1	n.a.	15,0	3,1	10,7	5,5	15,0	7,08	2,74	8,4	13,5	38,7	9,79	7,05
	2006	13,4	6,9	3,9	n.a.	10,1	3,3	13,4	6,9	10,1	7,77	2,92	9,1	21,2	37,5	10,69	7,77
	2007	8,1	0,0	1,9	n.a.	5,0	4,0	11,3	6,8	5,0	7,77	3,55	6,0	10,8	45,7	11,33	7,77
LÁZNĚ TEPLICE	1999	27,0	27,0	23,1	29,7	28,3	6,7	-	0,0	10,5	0,13	0,12	0,0	-	88,3	0,17	0,05
	2000	24,3	21,3	18,2	24,4	24,3	6,8	-15,0	0,0	9,8	0,11	0,10	0,0	-15,0	84,0	0,14	0,05
	2001	25,8	28,7	19,8	25,7	27,3	4,8	26,7	0,0	9,5	0,18	0,15	0,0	58,0	83,2	0,21	0,06
	2002	23,5	14,3	15,7	23,3	23,4	3,2	36,1	0,0	8,4	0,27	0,20	0,0	48,5	75,0	0,30	0,10
	2003	23,5	23,3	16,4	23,4	23,5	3,8	30,6	0,0	11,7	0,32	0,20	0,0	21,2	60,7	0,36	0,16
	2004	21,6	17,8	14,1	23,4	22,5	3,4	24,7	0,0	13,0	0,36	0,18	0,0	12,5	50,0	0,39	0,21
	2005	20,6	15,5	11,2	22,6	21,6	3,1	19,7	0,0	13,3	0,36	0,16	0,0	0,0	44,8	0,39	0,22
	2006	17,0	11,9	8,0	21,5	19,2	3,3	10,8	0,0	13,4	0,37	0,13	0,0	1,6	35,8	0,39	0,26
	2007	16,0	6,2	5,9	18,1	17,1	4,0	3,1	0,0	11,2	0,36	0,15	0,0	-3,8	41,8	0,38	0,23
LEČ. L. JÁCHYMOV	1999	42,8	42,8	35,3	42,0	42,8	6,7	-	0,0	26,3	0,58	0,32	0,0	-	55,4	0,67	0,35
	2000	33,9	20,5	18,7	n.a.	27,2	6,8	-41,3	0,0	11,7	0,34	0,28	0,0	-41,3	80,6	0,42	0,15
	2001	30,3	21,3	17,7	48,2	39,2	4,8	-18,0	0,0	20,1	0,34	0,22	0,0	-0,5	63,7	0,38	0,17
	2002	28,0	19,5	15,0	28,1	28,1	3,2	5,3	0,0	16,6	0,46	0,22	0,0	35,8	48,2	0,49	0,27
	2003	25,8	13,3	12,2	25,6	25,7	3,8	4,2	0,0	15,4	0,47	0,23	0,0	2,4	48,3	0,51	0,28
	2004	17,8	9,9	7,4	23,9	20,9	3,4	2,1	0,0	14,3	0,46	0,17	0,0	-2,3	37,4	0,49	0,32
	2005	14,9	0,0	4,7	16,3	15,6	3,1	5,0	0,0	10,6	0,47	0,17	0,0	0,8	37,5	0,49	0,32
	2006	13,4	15,7	4,1	21,4	18,6	3,3	7,9	0,0	13,9	0,52	0,16	0,0	12,4	29,8	0,55	0,39
	2007	11,1	9,8	3,8	17,1	14,1	4,0	3,4	0,0	11,5	0,53	0,12	0,0	0,9	22,7	0,55	0,43
ORCO	2005	21,0	21,0	19,6	35,9	28,5	3,1	-	0,0	18,9	29,57	11,58	0,0	-	39,2	31,18	19,60
	2006	25,8	29,6	24,6	31,7	30,7	3,3	76,4	0,2	18,1	51,91	26,72	0,4	76,4	51,5	56,57	29,86
	2007	26,7	28,1	25,6	28,8	28,4	4,0	34,4	0,4	17,0	53,07	52,69	0,5	3,0	99,3	76,15	23,46
PARAMO	1999	50,1	50,1	48,8	49,9	50,1	6,7	-	0,0	22,4	2,68	2,18	0,0	-	81,5	3,27	1,09
	2000	46,1	40,9	40,3	51,3	48,7	6,8	-24,3	0,0	7,0	2,03	2,51	0,0	-24,3	123,7	2,74	0,24
	2001	40,9	27,4	32,7	42,7	41,8	4,8	-17,1	0,0	6,1	1,79	1,98	0,0	-11,6	110,7	2,21	0,23
	2002	39,1	33,1	28,3	44,0	41,5	3,2	-11,7	0,0	11,1	1,71	1,51	0,0	-4,7	88,2	1,91	0,41
	2003	37,1	27,0	24,6	39,7	38,4	3,8	9,6	0,0	10,5	2,50	2,24	0,0	46,5	89,6	2,87	0,63
	2004	32,2	28,2	22,8	39,3	35,8	3,4	11,3	0,0	18,2	2,87	1,69	0,0	15,0	58,9	3,12	1,42
	2005	33,0	45,5	29,6	39,2	42,4	3,1	16,7	0,0	17,0	3,42	2,47	0,0	19,1	72,0	3,74	1,28
	2006	34,2	33,8	30,6	35,8	35,0	3,3	9,5	0,0	14,2	3,01	2,14	0,0	-12,0	71,1	3,32	1,18
	2007	33,2	27,6	28,8	34,8	34,0	4,0	11,2	0,0	16,6	3,35	2,12	0,0	11,2	63,2	3,71	1,60
PEGAS	2006	28,6	28,6	28,6	25,3	28,6	3,3	-	1,7	23,3	9,63	4,78	1,7	-	49,6	11,73	6,95
	2007	20,9	20,6	20,2	20,6	20,7	4,0	0,3	0,7	15,4	9,66	4,47	0,0	0,3	46,3	11,40	6,93
PHILIP MORRIS	2000	13,8	13,8	10,5	12,5	13,8	6,8	-	12,4	13,8	12,74	3,46	12,4	-	27,2	14,47	11,01
	2001	23,3	24,0	20,3	n.a.	23,7	4,8	63,8	15,5	23,7	17,46	3,21	17,9	63,8	18,4	19,06	15,85
	2002	29,2	34,5	26,8	33,3	33,9	3,2	60,7	15,4	33,9	23,69	4,69	15,3	58,5	19,8	26,03	21,34
	2003	28,3	26,2	25,8	29,5	28,9	3,8	61,0	14,0	28,9	33,90	7,61	11,5	61,4	22,4	37,70	30,10
	2004	28,9	31,0	26,9	29,4	30,2	3,4	44,9	13,2	30,2	35,24	6,27	11,5	16,9	17,8	38,37	32,10
	2005	29,0	28,4	26,9	29,1	29,1	3,1	35,7	11,3	29,1	38,14	6,42	7,4	16,9	16,8	41,34	34,93
	2006	30,6	32,0	27,6	29,9	31,3	3,3	10,6	9,2	31,3	23,39	5,28	6,3	-34,4	22,6	26,03	20,74
	2007	28,6	25,0	26,0	29,9	29,3	4,0	3,9	8,6	29,3	21,37	12,38	8,8	1,7	57,9	27,56	15,18
PR. ENERGETIKA	1999	50,1	50,1	44,0	48,3	50,1	6,7	-	4,7	47,8	6,77	3,24	4,7	-	47,9	7,88	4,64
	2000	38,1	19,5	26,5	28,5	33,3	6,8	-4,4	4,7	33,3	5,86	3,52	4,7	-4,4	60,0	7,86	4,35
	2001	33,0	19,0	20,9	26,7	29,9	4,8	-0,3	5,0	29,9	5,12	3,87	5,5	2,9	75,6	7,79	3,92
	2002	33,5	35,0	21,0	31,0	34,3	3,2	7,0	6,2	34,3	5,92	3,22	8,3	16,6	54,4	7,79	4,57
	2003	30,6	14,3	17,2	25,3	28,0	3,8	21,6	6,4	28,0	7,27	3,56	6,8	46,8	49,0	10,16	6,60
	2004	21,8	14,7	12,7	22,9	22,4	3,4	22,5	6,5	22,4	8,23	3,64	6,3	24,5	44,2	11,52	7,88
	2005	21,8	19,3	13,8	24,7	23,2	3,1	30,5	6,5	23,2	11,31	3,08	5,9	40,4	27,3	14,32	11,24
	2006	21,1	14,7	12,6	18,9	20,0	3,3	25,4	7,6	20,0	10,91	3,20	9,8	8,3	29,4	14,06	10,86
	2007	16,7	19,8	13,5	20,1	20,0	4,0	29,3	5,1	20,0	15,05	3,67	0,0	35,4	24,4	18,71	15,04
PR. PLYNÁREN.	1999	28,0	28,0	19,5	28,1	28,1	6,7	-	1,1	21,2	3,63	3,06	1,1	-	84,1	5,36	2,30
	2000	22,0	13,4	12,2	20,0	21,0	6,8	-4,7	1,0	19,2	3,01	3,48	0,9	-4,7	115,5	5,34	1,86
	2001	21,0	18,9	12,1	19,6	20,3	4,8	7,4	0,9	18,9	3,64	4,21	0,8	16,4	115,7	6,24	2,03
	2002	22,4	26,1	12,1	21,0	24,3	3,2	5,7	1,8	23,7	3,78	3,90	3,2	3,6	103,2	6,04	2,14
	2003	20,8	12,3	9,5	19,4	20,1	3,8	6,5	2,8	20,1	3,57	3,99	4,8	8,0	111,5	6,12	2,13
	2004	16,8	6,8	6,5	17,8	17,3	3,4	11,9	3,2	17,3	4,34	4,88	3,8	22,9	112,5	7,23	2,35
	2005	17,0	14,3	6,2	14,6	15,8	3,1	20,2	3,0	15,8	5,38	5,14	2,0	33,3	95,5	8,77	3,64
	2006	16,2	15,0	5,4	16,1	16,1	3,3	21,9	3,1	16,1	6,21	5,37	2,7	24,6	86,5	10,14	4,77
	2007	11,7	7,2	3,8	13,9	12,8	4,0	16,0	2,1	12,8	6,16	6,24	0,0	-0,4	101,3	10,54	4,31

Table 2 - part 2

Company	End of year	Estimates of equity's volatility					Parameters used for ELGD computation						Other used parameters				
		σ_E MA(5y) (%)	σ_E MA(1y) (%)	σ_E EWMA (%)	σ_E GARCH (%)	σ^*_E (%)	r_f (%)	μ^* (%)	δ^* (%)	σ_V (%)	V (bill. CZK)	F (bill. CZK)	δ (%)	μ (%)	Leverage (%)	V´ (bill. CZK)	Equity (bill. CZK)
PR. SLUŽBY	1999	27,2	27,2	24,1	28,4	27,8	6,7	-	0,0	11,9	0,72	0,58	0,0	-	80,4	0,88	0,30
	2000	33,2	38,3	28,6	33,8	36,1	6,8	44,4	0,0	21,9	1,04	0,58	0,0	44,4	56,0	1,20	0,62
	2001	30,0	22,2	22,3	29,5	29,8	4,8	5,4	0,0	25,8	0,79	0,13	0,0	-23,8	17,0	0,82	0,69
	2002	28,4	22,5	19,5	30,2	29,3	3,2	3,5	0,0	25,1	0,80	0,13	0,0	0,9	16,9	0,82	0,68
	2003	27,3	22,2	16,9	28,7	28,0	3,8	30,4	0,0	25,8	1,42	0,13	0,0	77,1	9,3	1,44	1,31
	2004	24,4	0,0	10,3	n.a.	17,3	3,4	20,8	0,0	15,8	1,43	0,15	0,0	1,3	10,5	1,46	1,31
	2005	17,4	0,0	5,3	n.a.	11,4	3,1	14,8	0,0	9,6	1,55	0,28	0,0	7,9	18,1	1,59	1,31
	2006	14,2	0,0	2,8	n.a.	8,5	3,3	13,1	0,0	7,1	1,56	0,29	0,0	0,6	18,9	1,60	1,31
	2007	10,1	3,0	1,4	n.a.	6,6	4,0	10,6	0,0	5,5	1,59	0,31	0,0	2,4	19,6	1,65	1,34
RM-S HOLDING	1999	38,9	38,9	34,0	39,5	39,2	6,7	-	0,5	36,5	3,47	0,46	0,5	-	13,3	3,60	3,14
	2000	41,7	44,4	31,7	47,4	45,9	6,8	-33,6	0,2	39,1	2,30	0,52	0,0	-33,6	22,5	2,45	1,93
	2001	36,9	24,6	25,8	38,8	37,8	4,8	-26,5	2,4	36,2	1,72	0,35	5,2	-21,2	20,2	1,80	1,45
	2002	33,0	16,7	19,8	35,2	34,1	3,2	-15,6	1,5	33,2	1,70	0,20	0,0	-1,2	11,7	1,73	1,53
	2003	30,2	14,6	15,3	32,9	31,6	3,8	16,7	22,2	31,6	1,73	0,25	64,6	72,6	14,5	1,86	1,61
	2004	26,2	19,7	13,9	30,7	28,4	3,4	10,9	16,6	28,4	1,72	0,46	0,0	-1,0	26,7	1,95	1,49
	2005	18,2	13,1	11,0	25,9	22,0	3,1	3,2	12,4	22,0	1,32	0,29	0,0	-23,1	21,8	1,46	1,18
	2006	14,5	0,0	6,5	16,8	15,6	3,3	0,6	8,9	15,6	1,18	0,01	0,0	-10,4	1,0	1,19	1,18
	2007	12,6	5,1	4,5	13,8	13,2	4,0	-1,1	6,7	13,2	1,12	0,00	0,0	-5,2	0,4	1,12	1,12
SETUZA	1999	37,7	37,7	29,3	37,7	37,7	6,7	-	0,0	18,7	1,41	1,49	0,0	-	105,6	1,82	0,33
	2000	37,0	36,2	25,7	36,5	36,7	6,8	87,4	0,0	18,0	2,64	3,06	0,0	87,4	115,6	3,51	0,46
	2001	33,2	23,7	23,2	33,7	33,4	4,8	44,4	0,0	14,7	2,96	3,38	0,0	12,1	114,2	3,68	0,30
	2002	33,2	33,4	23,9	39,2	36,3	3,2	21,3	0,0	14,0	2,70	2,92	0,0	-9,0	108,4	3,12	0,20
	2003	32,2	27,7	21,5	40,0	36,1	3,8	13,8	0,0	15,1	2,72	2,93	0,0	0,8	107,9	3,21	0,28
	2004	28,4	15,5	17,5	41,2	34,8	3,4	12,1	0,0	17,2	2,95	2,91	0,0	8,6	98,5	3,39	0,48
	2005	24,4	16,5	14,5	38,9	31,6	3,1	1,7	0,0	18,0	2,90	2,66	0,0	-1,6	91,7	3,27	0,61
	2006	23,8	20,7	13,8	27,4	25,6	3,3	0,7	0,0	17,8	2,94	2,56	0,0	1,2	87,2	3,32	0,76
	2007	19,0	9,5	9,7	28,0	23,5	4,0	10,7	0,0	16,0	3,78	3,63	0,0	28,8	95,9	4,43	0,80
SLEZAN FM	1999	21,4	21,4	14,9	22,1	21,7	6,7	-	0,0	13,0	0,72	0,92	0,0	-	127,2	0,98	0,06
	2000	16,9	10,6	8,4	18,7	17,8	6,8	-32,0	0,2	10,6	0,49	0,87	0,4	-32,0	179,0	0,92	0,05
	2001	19,7	23,5	10,4	19,9	21,7	4,8	-16,1	0,4	16,2	0,46	0,62	0,6	-4,1	133,4	0,77	0,15
	2002	22,5	29,5	14,7	30,8	30,2	3,2	8,5	0,5	18,5	0,65	0,65	0,5	40,8	100,0	0,92	0,27
	2003	20,9	12,7	10,9	30,8	25,9	3,8	12,8	0,5	17,3	0,77	0,73	0,5	20,3	94,8	1,12	0,39
	2004	19,6	12,8	9,2	28,1	23,9	3,4	5,7	0,4	16,3	0,70	0,73	0,1	-8,8	103,9	1,05	0,32
	2005	20,8	19,6	10,9	n.a.	20,2	3,1	3,7	0,3	14,0	0,64	0,61	0,1	-9,4	96,1	0,94	0,33
	2006	17,8	0,0	6,8	n.a.	12,3	3,3	3,2	0,2	12,0	0,64	0,61	0,1	0,4	96,4	0,94	0,33
	2007	12,0	0,0	3,5	n.a.	7,7	4,0	2,7	0,1	7,7	0,71	0,75	0,0	10,6	106,3	1,08	0,33
SM PLYNÁREN.	1999	28,1	28,1	18,3	28,4	28,3	6,7	-	0,8	21,2	4,96	4,60	0,8	-	92,6	7,26	2,66
	2000	22,2	13,9	12,3	21,0	21,6	6,8	6,7	0,3	15,8	5,30	5,12	0,0	6,7	96,6	7,86	2,74
	2001	25,8	32,0	17,1	35,3	33,6	4,8	9,3	1,2	24,8	5,69	6,30	2,3	11,2	110,7	8,84	2,54
	2002	26,4	28,0	17,0	36,2	32,1	3,2	22,1	0,7	20,8	7,90	5,75	0,0	38,8	72,7	10,09	4,34
	2003	23,7	3,5	11,1	45,6	34,6	3,8	10,5	1,4	25,4	6,89	4,51	2,9	-9,6	65,4	8,70	4,20
	2004	21,0	14,2	8,8	27,2	24,1	3,4	3,5	2,4	24,1	5,82	3,95	4,2	-10,8	67,8	7,80	3,85
	2005	21,2	15,1	8,6	26,4	23,8	3,1	12,0	4,2	23,8	7,02	1,67	7,3	30,3	23,8	7,85	6,18
	2006	17,5	17,6	8,2	24,8	21,2	3,3	11,6	5,8	21,2	7,08	1,62	8,7	10,6	22,9	7,89	6,27
	2007	12,5	4,9	5,2	22,8	17,6	4,0	12,0	8,1	17,6	7,48	1,76	11,5	19,2	23,6	8,36	6,60
SPOL. CH.HUT.VÝR.	1999	47,3	47,3	44,4	46,6	47,3	6,7	-	0,0	9,7	1,43	1,63	0,0	-	114,6	1,88	0,25
	2000	41,3	34,2	36,9	41,0	41,2	6,8	-18,1	0,0	6,1	1,17	1,42	0,0	-18,1	121,7	1,57	0,15
	2001	41,5	41,5	38,0	41,4	41,5	4,8	-8,0	0,0	14,5	1,16	0,99	0,0	-0,4	85,5	1,37	0,37
	2002	41,7	42,3	37,2	42,5	42,4	3,2	-2,4	0,0	14,8	1,22	0,96	0,0	5,0	78,8	1,35	0,39
	2003	39,3	28,0	31,5	40,4	39,9	3,8	5,5	0,0	17,9	1,45	0,99	0,0	19,1	68,2	1,61	0,62
	2004	35,7	30,1	28,3	40,9	38,3	3,4	23,6	0,0	14,5	2,34	1,76	0,0	60,8	75,1	2,59	0,83
	2005	39,8	52,3	35,4	43,8	48,1	3,1	45,1	0,0	16,2	4,19	3,39	0,0	79,3	80,9	4,64	1,25
	2006	39,1	37,8	35,5	46,4	42,7	3,3	34,8	0,0	12,8	4,31	3,67	0,0	2,8	85,2	4,84	1,16
	2007	36,7	30,1	32,9	40,4	38,5	4,0	30,4	0,0	11,1	4,92	4,38	0,0	14,2	88,9	5,68	1,30
SPOLANA	1999	44,0	44,0	39,4	44,4	44,2	6,7	-	0,0	18,4	5,29	6,79	0,0	-	128,5	7,20	0,40
	2000	39,4	34,2	31,8	40,1	39,8	6,8	-0,7	0,0	19,0	5,25	6,59	0,0	-0,7	125,5	7,13	0,54
	2001	40,2	41,7	34,2	40,5	41,1	4,8	-26,3	0,0	23,0	2,86	2,94	0,0	-45,5	102,6	3,48	0,54
	2002	40,5	41,7	32,9	43,3	42,5	3,2	-13,8	0,0	19,8	2,94	3,05	0,0	2,7	103,7	3,37	0,33
	2003	37,3	19,2	24,8	39,9	38,6	3,8	-4,5	0,0	21,7	3,28	3,26	0,0	11,4	99,4	3,82	0,56
	2004	33,7	25,9	21,7	38,9	36,3	3,4	-5,8	0,0	22,2	3,00	2,82	0,0	-8,4	94,0	3,42	0,60
	2005	37,8	50,9	29,7	51,7	51,3	3,1	1,6	0,0	18,3	3,54	2,80	0,0	18,0	79,2	3,91	1,11
	2006	36,2	34,4	28,9	62,2	49,2	3,3	3,9	0,0	18,6	3,45	2,67	0,0	-2,5	77,3	3,83	1,16
	2007	32,5	21,9	24,9	57,0	44,8	4,0	-0,5	0,0	18,7	3,11	2,29	0,0	-10,0	73,8	3,49	1,20
TELEFÓNICA	1999	31,9	31,9	28,7	32,0	32,0	6,7	-	0,0	26,8	221,55	49,96	0,0	-	22,6	235,65	185,68
	2000	38,2	43,8	36,4	39,9	41,9	6,8	-10,4	0,7	36,2	196,08	45,58	1,2	-10,4	23,2	208,95	163,36
	2001	43,2	51,8	42,9	44,4	48,1	4,8	-18,5	0,4	39,1	147,99	38,95	0,0	-24,5	26,3	155,71	116,76
	2002	42,8	41,7	41,6	43,3	43,1	3,2	-18,0	6,2	32,5	103,75	34,19	16,4	-17,4	33,0	113,01	78,82
	2003	41,8	37,6	39,1	42,3	42,0	3,8	0,5	5,4	28,1	132,06	55,46	3,7	32,6	42,0	149,28	93,82
	2004	40,7	24,0	34,0	40,3	40,5	3,4	4,8	4,0	33,3	149,91	38,74	0,0	13,5	25,8	157,65	118,92
	2005	36,6	17,8	26,6	n.a.	31,6	3,1	16,4	5,3	29,1	190,22	29,24	7,3	36,6	15,4	198,17	168,94
	2006	30,1	22,0	22,8	n.a.	26,4	3,3	13,1	6,9	23,5	168,01	29,40	8,8	-3,2	17,5	182,71	153,31
	2007	25,0	18,2	19,9	80,4	52,7	4,0	20,8	6,5	48,6	200,88	30,76	7,8	29,1	15,3	206,23	175,47
TOMA	1999	28,1	28,1	22,7	27,6	28,1	6,7	-	0,0	20,1	0,22	0,20	0,0	-	94,9	0,27	0,07
	2000	26,3	24,4	21,7	25,9	26,1	6,8	-21,1	0,0	19,5	0,17	0,16	0,0	-21,1	95,0	0,22	0,05
	2001	32,1	41,4	28,9	31,7	36,8	4,8	2,9	0,0	16,2	0,21	0,15	0,0	20,9	72,8	0,24	0,09
	2002	30,9	27,0	24,9	31,6	31,3	3,2	-13,3	0,0	25,8	0,13	0,03	0,0	-34,5	20,6	0,14	0,11
	2003	29,1	20,0	20,5	29,9	29,5	3,8	24,4	0,0	27,3	0,26	0,02	0,0	89,7	8,9	0,26	0,24
	2004	29,3	28,8	21,6	31,2	30,3	3,4	64,0	0,0	29,4	0,63	0,02	0,0	145,3	3,3	0,63	0,61
	2005	29,6	26,4	20,8	32,8	31,2	3,1	54,4	0,0	25,8	0,72	0,15	0,0	14,4	20,3	0,74	0,59
	2006	24,6	18,4	16,9	31,4	28,0	3,3	41,4	0,0	21,7	0,76	0,20	0,0	6,8	26,7	0,80	0,59
	2007	22,9	18,4	16,3	25,8	24,4	4,0	48,6	0,0	16,1	1,11	0,46	0,0	45,5	41,6	1,20	0,73
UNIPETROL	1999	47,5	47,5	42,7	55,0	51,3	6,7	-	0,0	32,7	15,23	8,09	0,0	-	53,1	17,36	9,27
	2000	41,4	34,1	36,6	44,1	42,7	6,8	23,2	0,0	26,5	18,76	10,32	0,0	23,2	55,0	21,59	11,27
	2001	40,2	37,7	36,2	42,4	41,3	4,8	7,0	0,0	20,0	17,80	12,06	0,0	-5,1	67,8	20,23	8,17
	2002	41,1	44,0	37,9	43,5	43,8	3,2	5,4	0,0	16,4	18,38	13,99	0,0	3,3	76,1	20,26	6,27
	2003	38,5	25,2	32,9	40,7	39,6	3,8	14,1	0,0	20,9	23,73	13,88	0,0	29,1	58,5	25,93	12,05
	2004	33,8	23,2	27,1	47,0	40,4	3,4	11,0	0,0	29,3	24,88	8,23	0,0	4,8	33,1	26,04	17,81
	2005	38,6	53,7	33,8	47,9	50,8	3,1	71,9	0,0	30,4	74,49	36,75	0,0	199,4	49,3	78,91	42,16
	2006	37,8	33,8	31,6	64,8	51,3	3,3	52,0	0,0	33,0	69,26	30,75	0,0	-7,0	44,4	73,23	42,49
	2007	34,2	24,8	28,2	50,0	42,1	4,0	44,4	0,0	32,0	81,23	24,00	0,0	17,3	29,5	85,22	61,22
VČ PLYNÁRENSKÁ	1999	25,3	25,3	21,5	23,8	25,3	6,7	-	2,7	25,3	1,97	2,42	2,7	-	122,4	3,61	1,20
	2000	18,4	5,6	10,7	16,4	17,4	6,8	9,4	1,9	17,4	2,07	2,73	1,4	9,4	131,8	3,94	1,21
	2001	20,9	25,1	11,8	19,2	23,0	4,8	15,6	4,0	18,0	2,35	3,44	6,7	20,1	146,1	4,51	1,08
	2002	48,8	90,7	23,9	51,4	71,1	3,2	37,7	4,2	54,5	4,42	3,28	4,5	66,8	74,3	5,45	2,17
	2003	44,2	15,7	18,1	48,4	46,3	3,8	23,4	4,9	42,7	4,00	3,44	6,3	-1,4	85,9	5,52	2,08
	2004	42,8	6,9	12,2	43,2	43,0	3,4	18,1	5,6	43,0	3,96	2,84	6,5	7,2	71,8	5,13	2,29
	2005	43,1	13,5	11,8	49,8	46,4	3,1	19,5	5,7	46,4	4,52	1,45	5,0	20,4	32,0	4,83	3,38
	2006	42,0	11,7	9,1	59,1	50,5	3,3	20,1	5,5	50,5	5,20	1,30	5,3	21,4	24,9	5,42	4,13
	2007	11,1	0,0	3,9	49,7	30,4	4,0	9,3	6,6	30,4	4,53	1,08	8,7	-1,6	23,8	5,20	4,13
ZENTIVA	2004	24,1	24,1	23,2	25,2	24,7	3,4	-	1,0	24,4	30,70	2,14	1,0	-	7,0	31,03	28,89
	2005	27,7	29,4	25,6	28,0	28,7	3,1	68,2	0,8	25,2	51,28	9,29	0,7	68,2	18,1	52,61	43,32
	2006	28,3	29,2	25,4	30,8	30,0	3,3	32,3	0,8	28,2	53,59	6,17	0,8	5,4	11,5	54,53	48,36
	2007	29,6	32,4	9,7	29,8	31,1	4,0	21,0	0,5	21,0	56,84	24,97	0,0	6,1	43,9	62,04	37,07

Source: author's computation, Magnus (2008), Prague Stock Exchange

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Teze diplomové práce

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Název:

Implied Market Loss Given Default

Tržně odhadnutá ztráta z defaultu

Charakteristika tématu:

Nová basilejská dohoda vyžaduje kvalitnější metody pro výpočet kapitálových požadavků souvisejících s kreditním rizikem bankovního portfólia a umožňuje bankám používat své vlastní interní odhady položek kreditního rizika pro jeho výpočet. Jednou z těchto komponent je LGD (loss given default), ztráta banky způsobená defaultem dlužníka.

Existují tři způsoby jeho výpočtu. Jde o tržní LGD, které se odhaduje na základě obchodovatelných cen dluhopisů a půjček po úpadku, workout LGD, snažící se odhadnout cash-flow plynoucí z procesu vymáhání již defaultovaných dluhopisů a půjček, a třetí možností je „tržně implikované LGD" (im-LGD), které představuje zcela odlišný způsob výpočtů ztráty z úpadku na základě cen rizikových dluhopisů a teoretického modelu oceňování aktiv.

Hlavní náplní této práce budou metody výpočtů tržně implikovaného LGD a jejich srovnání s tradičními modely. Práce se pokusí na základě tržních dat odhadnout LGD pro jednotlivá odvětví v ČR a ukázat, že tržně implikované LGD je užitečný odhad pro řízení kreditního rizika, ačkoliv v bankách stále nepatří mezi často využívanou metodu.

Hypotézy:

- Spready rizikových dluhopisů slouží jako dostatečný indikátor pro výpočet im-LGD.
- Průměrné im-LGD leží systematicky nad „fyzickým" LGD (terminologie Altmana a Kishora 1996)

Metody práce:

- matematická statistika
- analýza časových řad
- ekonometrický model

Osnova:

- Úvod
- Charakteristiky Basel II
- Hlavní modely LGD
- Implikovaný LGD
  – teoretický rámec modelu
  – model oceňování aktiv
  – způsoby výpočtu implikovaného LGD
- Výpočet Implikovaného LGD pro různá odvětví ČR
- Další možné aplikace implikovaného LGD
- Závěr

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V Praze dne ……………

Podpis vedoucího diplomové práce   Podpis autora