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Diffusion Processes on Complex Networks

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Diffusion Processes on Complex Networks

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Abstract:

In this paper we apply agent-based methodology on an issue that is fundamental for economic prosperity and growth: the diffusion of innovations. The diffusion of innovations is one of the topics where agent-based simulation is an extremely fruitful method allowing not only the observation of stable states but also the process and development of the diffusion. Furthermore, empirical studies revealed that the topological structure of interactions among individuals importantly influences the diffusion's course and outcomes. We analyze diffusion outcomes for five different topologies, assuming markets where individuals are highly influenced by the adoption decision of their peers and innovations are introduced into the markets in two different ways: mass media campaigns and seeding procedures. Our results indicate that the topology of the relations among individuals importantly influences the speed and development of the diffusion process as well as final market penetration. Scale free topology seems to promote fast innovation diffusion, at the same time being characterized by the high uncertainty of the diffusion outcomes. Less heterogeneous networks (small worlds, two-dimensional lattice and ring) yield a much slower diffusion of the innovation, at the same time being much less unpredictable than scale free topology.

Keywords: innovation diffusion, complex networks, scale-free networks

JEL: O31, O33.

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I. Introduction

The fast growth of the standard of living and production over the last few decades can be attributed to a large extent to the continual flow of innovations – new products and technologies used by companies as well as consumers. Macroeconomic growth models often implicitly assume that after a new superior technology or product is invented it starts to be used by economic agents immediately (e.g. Barro, Sala-i-Martin, 2003). In reality the process of innovation diffusion can take years and even decades (it took 200 years until citrus fruit as scurvy prevention was adopted (Rogers, 1962), and eventually ends with the adoption of inefficient technology or fails completely (the non diffusion of the Dvorak keyboard is among the most popular examples).

Researchers quickly acknowledged that the contribution of innovations to economic growth and welfare is largely determined by the speed and degree to which they propagate through a societal system. Diffusion is therefore an integral part of the innovation process, consisting of the invention of new ideas and their subsequent introduction into practice (commercialization).

Early debates on the diffusion of innovations date back to the 1950s and 1960s when Rural Sociology and other journals published a series of papers about the adoption of hybrid corn in the United States. Ryan and Gross (1943), Griliches (1957, 1960) and others identified basic features of the adoption process later observed for many other types of innovations and

diffusion processes. Griliches described a typical S-shaped pattern in the data and estimated three parameters of the logistic function: the origin, slope and ceiling for empirically collected observations.

There are two basic questions that concern researchers in connection with innovation diffusion. 1) Which factors affect the speed of diffusion (slope of S-curve)? 2) What determines the final share of the innovation on the market (ceilings at which the adoption S-curve asymptotes)? Hall (2004) divided these factors into four groups:

- Those that affect benefits from adoption,
- Those that affect the costs of adoption,
- Those related to an industry or social environment,
- Those related to uncertainty and information problems.

In our paper we focus on the two latter factors, particularly the influence of the structure of social relations among individuals on the spreading of information about innovation and the subsequent decision about adoption.

A mechanism of the diffusion of innovations in a social environment is very similar to the propagation of a contagious disease in the population. People that already adopted the innovation will transfer information about an innovation to their friends and acquaintances that have not adopted it yet. Based on the adopter's recommendation, their friends and acquaintances are introduced to the existence and characteristics of the new product and buy it as well. Thus, models of epidemic spreading can accurately describe the diffusion of an innovation in society. The first epidemiological SIS and SIR models from the 1930s (Kermack a McKendrick, 1927; Bailey, 1957; Anderson a May, 1992) and their refined and extended versions (Anderson, 1988; Grenfell et al., 2001; Hethcote, Yorke, 1984; Keeling, 1997) share one common drawback – an assumption about random mixing. In the real world individuals do not interact with other individuals with the same probability. They only directly interact with a limited number of individuals and their interaction is often repetitive. For the same reason diffusion models that assume random mixing are a too-simplified description of real diffusion processes. If learning occurs on the word of mouth (WOM) basis (direct contact between a potential adopter and adopter is necessary for the transmission of the information to take place) then the structure of interactions between individuals must be taken into account. As shown e.g. by Milgram (1967), social networks are not random at all. They

often display a high clustering coefficient and a short average path-length (see Watts, Strogatz, 1998; Albert, Barabasi, 1999). Therefore, in our paper we include networks with empirically observed characteristics with small-world and scale-free properties to investigate the diffusion of innovations in society following Delre et al. (2007), Delre et al. (2006) etc.

Marketing literature (e.g. Rogers, 1962) is highly concerned with the role of interpersonal (or word-of-mouth) communication in the diffusion process. According to Rogers, "... the heart of the diffusion process is the modeling and imitation by potential adopters of their networks partners who have adopted previously." (Rogers, 1962, pp. 18). Bass (1969) introduced the mathematical model of diffusion, defining the likelihood of purchase at time T in the following way:

$$\frac{f(T)}{1 - F(T)} = p + q \cdot F(T),$$

where p is the probability of the purchase at $T = 0$ (can be interpreted as the influence of mass media) and q is the imitation coefficient (reflecting the propagation of innovation through word-of-mouth communication). Bass's model accurately fits some real data and generates the empirically observed S-shaped pattern of market penetration, but it does not provide sufficient insight into micro-level behavior and the decision making of individual adopters.

Analytical models of diffusion, including the micro perspective (heterogeneous populations), are constrained by the solvability of the model and therefore the amount of heterogeneity included in the model is restricted (see e.g. Chatterjee, Eliashberg, 1990). The agent-based modeling methodology allows the investigation of heterogeneous populations and is highly flexible in its model construction. Furthermore, as pointed out by Wilhite (2006), "Agent-based computational modeling is ideally suited for studying networks and economic activity on networks." Therefore, there is a great deal of diffusion literature using agent-based simulation. Cowan, Jonard (1999) present a model of the diffusion process based on the barter of different types of knowledge among economic agents placed in a network. Structured interactions among economic agents analyzed using computational simulations can be found in Janssen, Jager (2002), Midgley et al. (1992) or Delre et al. (2006). We take the advantage of agent-based modeling as a tool for modeling networked societies and also as a suitable method for modeling heterogeneous populations of agents.

Our main goal is to answer the question as to whether and how the topology of social relations among potential adopters influences the process of the diffusion, whether some of the

topologies are more supportive for fast innovation diffusion and how the social susceptibility of potential adopters influences these results. Compared to existing literature we present experiments for the simultaneous inclusion of two technologies on the market and report comparable results for the exceptionally high number of network topologies.

II. The Model of the Diffusion

Models of innovation diffusion generally incorporate either a potential adopter's heterogeneity, or learning, or both of these assumptions, to explain an S-shaped adoption curve observed by empirical studies. In our model we incorporate both these properties. An agent's heterogeneity in our model stems from two sources: 1) each agent perceives a different suitability of the innovation, implying different benefits from the adoption for different agents; 2) each agent is part of a social network with a unique position in the network, which implies a different influence of the agent's neighborhood on her decision-making about innovation adoption. We assume that agents are boundedly rational and myopic. Their decisions are based on the current state of affairs; they are not able to anticipate future developments of the system. Their ability to receive information is restricted (Simon, 1957) and the channels through which they can obtain information are limited.

In the following section we describe how our boundedly rational agents decide about the adoption of an innovation and the structure of the network through which information is transmitted.

II.1. The Agents

We follow Delre et al. (2006) using the following decision-making procedure: Agent i adopts innovation j if the individual utility from adoption exceeds a certain threshold level:

$$U_{i,j} \geq U_{i,j,MIN} \quad (1)$$

$U_{i,j,MIN}$ specifies the minimum level of satisfaction agent i requires to adopt innovation j . It follows uniform distribution $[0,1]$ and can be also described as the aspiration level of the individual. Heterogeneity in required utility is thus incorporated into the model.

Individual utility $U_{i,j}$ is defined as a weighted utility of individual preference and social influence:

$$U_{i,j} = \beta_{i,j}x_{i,j} + (1 - \beta_{i,j})y_{i,j}$$

Parameter $\beta_{i,j}$ expresses the strength of the social influence in the agent's decision making, and $x_{i,j}$ and $y_{i,j}$ are defined using the following threshold functions:

$$y_{i,j} \begin{cases} q_j \geq p_i \Rightarrow 1 \\ \text{otherwise} \Rightarrow 0 \end{cases}$$

$$x_{i,j} \begin{cases} a_{i,j} \geq h_{i,j} \Rightarrow 1 \\ \text{otherwise} \Rightarrow 0 \end{cases}$$

Individual preference $y_{i,j}$ reflects the suitability of a given product or technology for an individual agent and can also be interpreted as a willingness to use new technologies and products, with some individuals being more innovative and others being less willing to accept innovative products and technologies (Rogers, 1962). The assumption about the heterogeneity of agents in their “natural inclination” or “inherent value of technology” is quite common in diffusion literature (Cowan, Cowan, 1998; Arthur, 1989; Roedenbeck et al. 2008; etc.). The individual preference p_i of agent i is uniformly distributed between 0 and 1. The quality of the product q_j is neutral to the agents' preferences and equal 0.5 for all products. As a consequence, individual preference is equally likely to be assigned 0 or 1.

The social component of the agent's i personal network is equal to 1 if $a_{i,j}$ (the percentage of the agent's neighbors that adopted the product j) exceeds the *exposure threshold* $h_{i,j}$. The exposure threshold is normally distributed with default values $N \square (0.3, 0.01)$.

The use of threshold models has a long tradition in the investigation of social phenomena such as collective or group behavior (Granovetter, 1978). Individuals making binary decisions (in our case the adopt vs. non-adopt) in fact decide whether to be involved in group behavior or not. If the group pressure exceeds a certain threshold then the individual decides to adopt the group behavior. Threshold models introduce a positive feedback mechanism into the model and are able to explain certain interesting patterns in macro behavior. In our model we apply

the threshold mechanism within a local neighborhood (e.g. Schelling, 1978) that better reflects the bounded rationality of individual decision makers.

In case there are more innovations competing on the market the agent follows the procedure described in Eq. 1 for both of them. Three situations can occur: 1) none of the products exceeds a minimum level of satisfaction, in that case no product is adopted; 2) only one of the products exceeds a minimum level of satisfaction, in that case the product that satisfies Eq. 1 is adopted; 3) both innovations exceed a minimum level of satisfaction, in that case the agent compares $U_{i,A}$ with $U_{i,B}$ and chooses the innovation with a higher utility level.

II.2. Social network

Empirical studies on consumer behavior revealed that consumers are involved in different kinds of relationships for different kinds of products (Bearden, Etzel, 1982). Because these relationships between individual consumers seem to be extremely relevant for micro-level decision-making about innovation adoption (Rogers, 1962) the examination of the impact of the topological characteristics of these structures in society as a whole seems to be highly relevant for the discussion about innovation diffusion.

In our model nodes of the network represent consumers and relations between them are represented by links. The existence of a link between two consumers means that these two consumers do communicate with each other and at the same time each of them is socially influenced by the other. We investigate how the unique set of relationships between consumers (network topology) influences the innovation diffusion process as well as the final market penetration of the market.

The topology of the network influences the innovation diffusion process in two ways. First, when information is passed from one node to another in the network, we can observe that networks with different topologies embody a different speed of spreading information. Second, different clustering characteristics of the network topologies create different conditions with regards to the social pressure for innovation adoption. Based on the bulk of network literature we decided to use five network topologies in our paper: random network, two-dimensional (2D) regular lattice, ring topology, small world and a scale free network.

Random networks were the first network topology used to simulate structured markets and represent the principle of random mixing in society when the relations between individuals are completely random. They are characterized by relatively rapid information-spreading

within the network and no clustering among agents (see e.g. Plouraboue, 1998, for a diffusion model with random networks).

Attempts to embody more realistic assumptions about social networks led to the introduction of regular networks (ring and 2D lattice) into the sociological and economic literature (e.g. Eshel et al., 1998). Regular networks embody a highly local structure with each agent being connected to the same number of her nearest neighbors. In our model each agent is connected to the four nearest neighbors, in the case of a ring network to the two nearest neighbors on each side. The resulting network structure with overlapping neighborhoods is typical by the high clustering coefficient and slow information propagation through the network in the case of a ring network, and not overlapping neighborhoods in the case of the two-dimensional regular lattice.

Since Milgram's (1967) experiment it was clear that real social networks exhibit not only high clustering but also a high information spreading velocity and none of the abovementioned topologies fulfill both these attributes. Watts, Strogatz (1998) and others have empirically confirmed the relevance of these empirical findings for many natural and social systems and their rewiring procedure applied on a regular network is the most-often used approach to create a network with small-world characteristics. We use Watts, Strogatz (1998) small-world formation procedure with a rewiring probability of 0.01.

Finally, networks with highly connected, and hence highly influential, individuals have been observed under many circumstances: cellular metabolism and protein regulatory networks, networks of Hollywood movie actors, citation networks or Internet networks (Barabási and Bonabeau, 2003). These highly influential individuals (often professionals or VIPs) are then crucial for innovation diffusion. Rogers (1962) points out that early adopters are very often characterized by many connections. Scale-free topology is thus characteristic of the power law distribution of links, i.e. the probability for each node of having n links decays as a power law ($P(n) \propto n^{-\lambda}$ with $2 \leq \lambda \leq 3$). There exist few highly connected individuals (hubs) together with a large number of individuals with only a very few connections (Barabási, Albert, 1999). We use a scale free network created by the preferential attachment process. Nodes are sequentially added to the network and attached to already existing nodes with a probability proportional to the number of links that these nodes already have.

The average degree of all networks (average number of nodes directly connected to a node) is equal to 4 for all discussed networks.

II.3. The Marketing Strategies

The process of the diffusion can be commenced in two ways. First, the company introducing the product on the market can use a *mass media campaign*. With probability e_2 an agent becomes aware of the existence of the innovation and is involved in decision making about its adoption. The agents start considering the adoption of the new product if at least one of their neighbors has already adopted the product or if they become aware of the existence of the product from the mass media campaign. The second way of launching a product into the population is a *seeding* procedure: a donation of a given product to a certain percentage of potential adopters (e_1) at the beginning of the simulation run. These initial adopters then spread information about the product to their neighbors through word-of-mouth communication (WOM). Their neighbors, in the next step, decide about their own adoption of the innovation using the procedure described by Eq. 1. In this way the diffusion process continues until a stable state is reached in which no other potential adopters are interested in adoption or are not informed about the existence of the product. The diffusion process can be re-established only by another seeding or mass media campaign.

III. Simulation experiments and results

In our simulations we used two launching strategies: the mass media campaign and the seeding procedure. Our main concern was whether and how different structures of interactions among individuals influence the outcomes of the diffusion process with regards to the process itself as well as final market penetration. If not mentioned otherwise all results are based on 20 runs for each parameter setup (for precise specifications of the simulation setups see the Appendix).

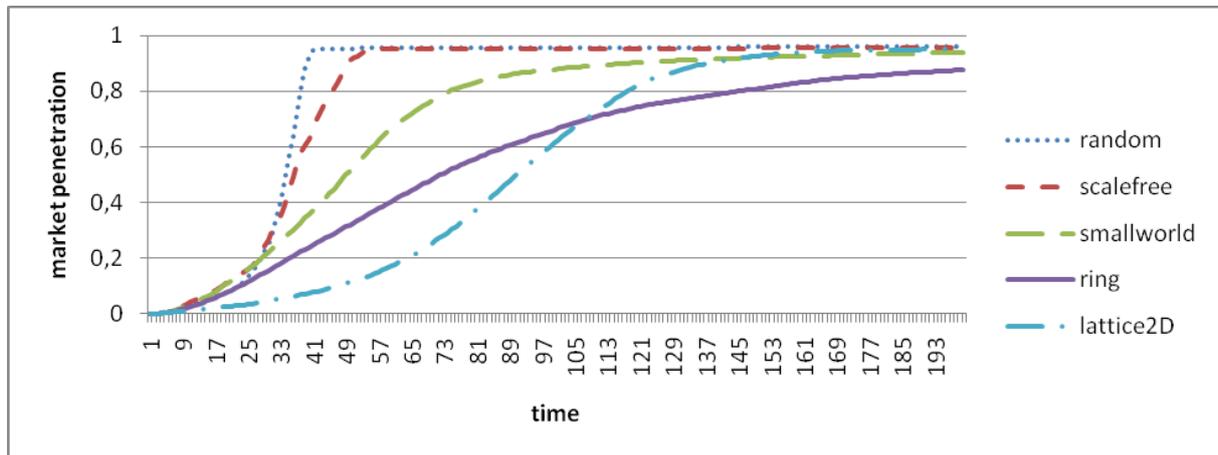
III.1. Mass media campaign

First, we will discuss the situation when a mass media campaign is chosen as a launching strategy ($e_2 = 0.001$) and only one innovation is introduced into the market (individuals choose only among two options: to adopt the innovation or not).

To accentuate the role of the social environment we have chosen a market with strong social influence expressed by a high social preference coefficient $\beta_{i,j} = N \square (0.9, 0.01)$ and

exposure threshold $h_{i,j} = N(0.3,0.01)$. Agents are heterogeneous in their preferences, but on average their decision is strongly influenced by the decisions of their neighbors.

FIGURE 1 Diffusion curves for different network topologies



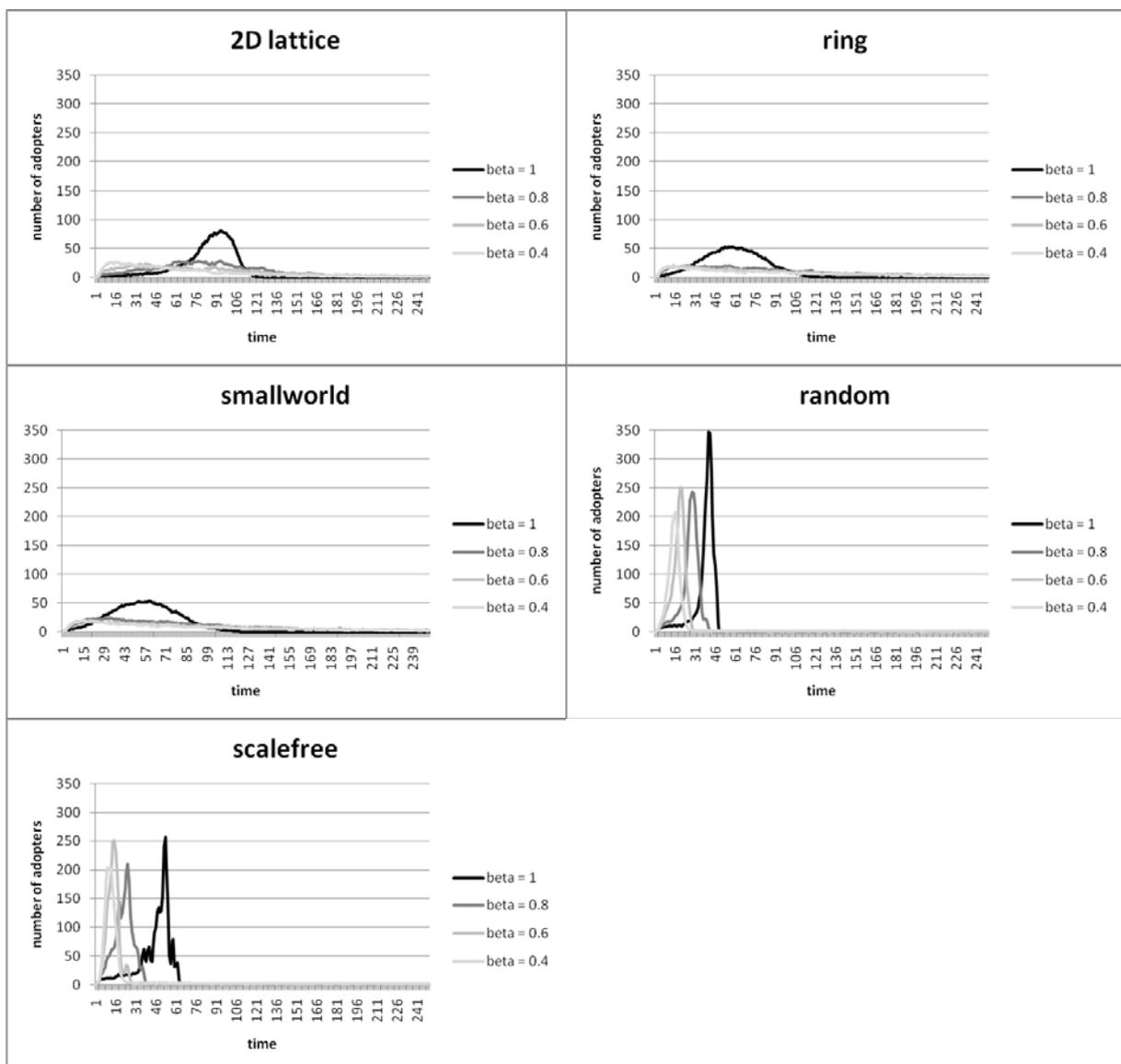
Source: Authors' own calculations.

Figure 1 shows significant differences in the diffusion process for the five examined topologies. Despite the common S-shape pattern, random and scale-free topologies move towards their ceiling market penetration much faster, exceeding 90 percent market penetration within less than 50 periods. Scale free topology is a favorable environment for innovation diffusion because hubs are informed about the existence of an innovation very soon and once hubs adopt the innovation the rest of the network is very easily informed. Taking the random and the ring network as two extreme cases of a small world network with rewiring probabilities 1 and 0, we can conclude that increasing the rewiring probability (more randomness in the network topology) accelerates the diffusion process. Having a completely clustered network leads to slow diffusion because only direct and close neighbors are informed about the existence of an innovation. With more randomness in the network, the information about the existence of an innovation spreads through the rewired links (shortcuts) that bring the information to more distant parts of the network and the diffusion process then accelerates. Ring and lattice topologies are very similar in most of the network characteristics, except the overlapping of their neighborhoods. Whereas the ring has a clustering coefficient $\frac{3}{4}$, the 2D lattice is characterized by a non-overlapping neighborhood with a clustering coefficient equal to 0. We can observe that in initial stages of the diffusion process the

innovation propagates more quickly through the ring network being overtaken by the 2D lattice in the later stages of the diffusion.

Many empirical studies conclude that different markets are characterized with the different social susceptibilities of the participants. Fashionable markets are typically highly socially susceptible, with people being strongly influenced by their peers (e.g. brown good markets), whereas some others are characterized by lower social influence (e.g. white good markets). The question arises to what extent and in which way diffusion in markets with different topologies is influenced by the social susceptibility of market participants.

FIGURE 2 Number of adopters for β equal to 1, 0.8, 0.6 and 0.4



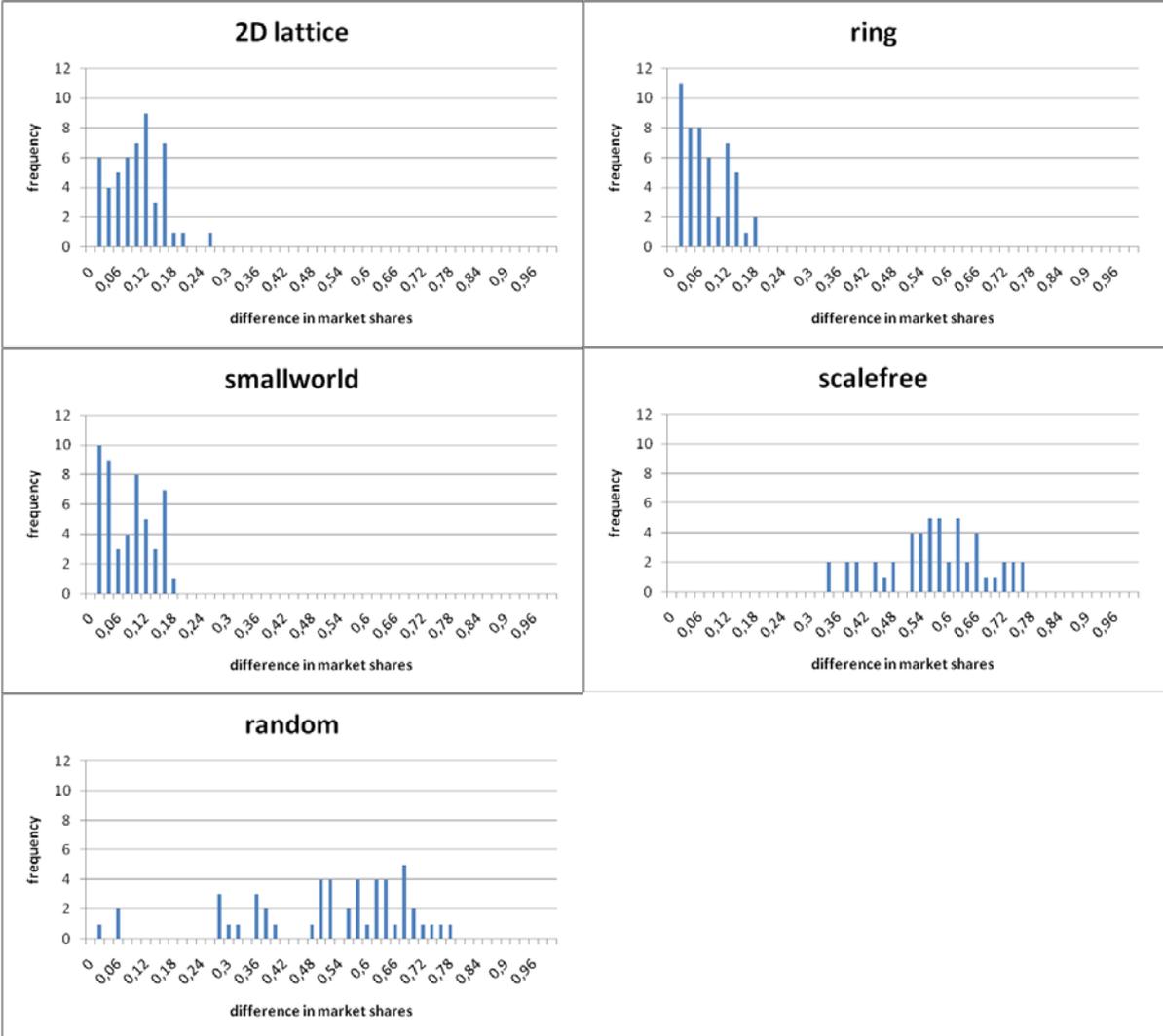
Source: Authors' own calculations.

To answer this question we ran a set of experiments with different β coefficients on five different topologies. Figure 2 depicts the number of adopters in each simulation step for β coefficients equal to 1, 0.8, 0.6 and 0.4. We can see that the underlying structure of the network crucially influences diffusion patterns with higher social susceptibility, causing a shift of the peak of the number of adopters to later periods and the more pronounced adoption peak for 2D lattice, small world and ring networks. The impact of the network structure is clearly visible; networks with a more regular distribution of links (2D lattice, ring, small world) experience very similar adoption curves. Random and scale free networks show very high peaks in the adoption curves being postponed to later periods for more socially susceptible markets. At the same time markets with higher social susceptibility are characterized by a higher variability between trials. Comparing different networks we observed high variability between trails for the scale free network and very low variability for the small world and ring networks.

Very often real markets experience the almost simultaneous introduction of more innovations competing for the favor of potential adopters (e.g. beta vs. VHS videotape formats). In the following part we investigate how the structure of interactions among potential adopters can influence the final market penetration of the competing innovations. We assume there were two competing innovations that were introduced to the market at the same time with a mass media campaign ($e_2 = 0.001$ for both technologies). Figure 3 shows the difference in market penetration between innovation 1 and 2 (in absolute terms) for five network topologies. We can see that networks with a high regularity in network structure (ring, 2D lattice and small world) exhibit low differences in market shares; final shares of both technologies range approximately around 50 percent of the market.

The average difference between market shares of the first and second innovation is 56 percentage point for the scale free network (variance 0.012) and 51 percentage points for the random network (variance 0.032). Figure 3 shows that random and scale free networks exhibit very high volatility in the difference in market shares between the two competing innovations. In this simulation we increased the number of simulation runs for each setting to 50 and high variance indicates that different runs resulted in very different final market share differences, as well as the fact that the market is highly uncertain and the success of particular innovation is very difficult to predict. Initial random development has a strong influence on the final outcomes of the diffusion process.

FIGURE 3 Difference in market shares for two competing technologies and different network topologies



Source: Authors’ own calculations

The possible response of the company introducing the innovation that is competing with another new product or technology is to increase the intensity of the mass media campaign. The marginal effect of additional mass media campaigns on different networks is crucial for the decision making about such investment. In the next section we try to investigate how different markets respond to an increased mass media campaign of one of the competing innovations. We fixed the mass media campaign intensity for the second technology ($e_2 = 0.001$) and varied the mass media campaign intensity for the first technology ranging from 0.001 to 0.02.

Figure 4 shows that an intensified mass media campaign very quickly leads to the prevalence of the first technology, with the difference in market shares exceeding 90 percentage points (the first technology completely dominates the market). This 90 percentage points difference in market shares is achieved for the mass media campaign of the first innovation equal to 0.011 in both small world and scale free networks.

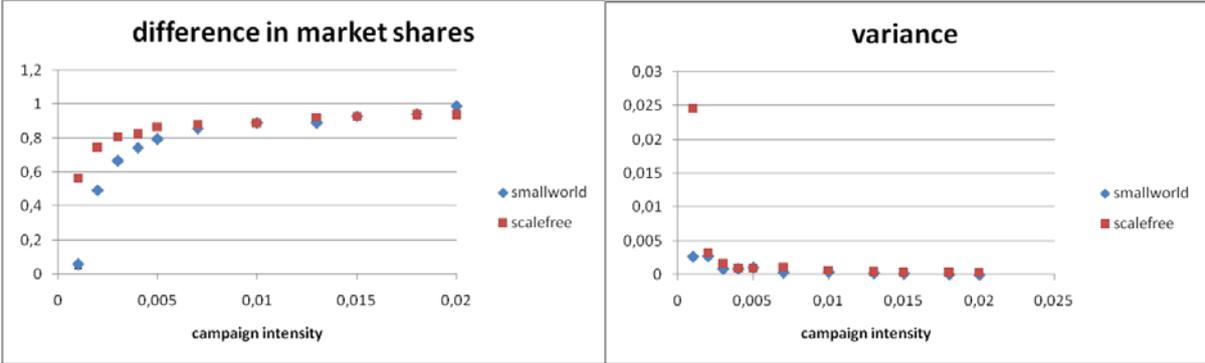
Figure 4 clearly shows that the marginal effect of additional mass media campaigns is strongest for the mass media campaign increased from 0.001 to 0.002.

The outcome of the competition is relatively well predictable for the small world network. Equal mass media campaign intensity will end up with approximately the same market shares for both competing innovations, with an increase in mass media campaigns to 0.002 the company gets around 74 percent of the market with the second competing technology having below 25 percent of the market share. Even for low differences in mass media campaign intensity the variation in the final market shares is relatively modest. Similar results were obtained for the ring and 2D lattice networks.

As mentioned above, markets with a scale free structure of interactions are highly volatile and unpredictable when the mass media campaign intensity is identical for both competing innovations. However, Figure 4 shows that the influence of “historical accidents” can be avoided by the increased intensity of mass media campaigns by one of the competitors. A mass media campaign intensified to 0.002 will increase the market share of the first innovation at 86 percent and more importantly the variance declines to 0.003. An even more radical drop in variance was observed for the random network.

Thus we can conclude that companies operating on markets with scale free or random topologies are the most motivated towards investing in mass media campaign dominance.

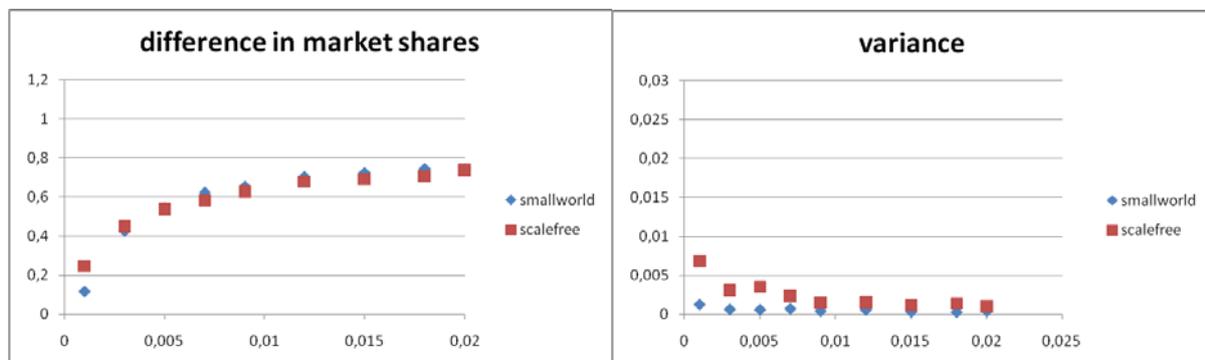
FIGURE 4 Increased mass media campaign of the second innovation



Source: Authors' own calculations

Figure 5 supports the findings of Delre et al. (2006) that the effect of the market structure decreases for more individualistic markets. With $\beta_{i,j} = N \square (0.5, 0.01)$ the difference in market shares is almost identical for both topologies. At the same time the intensification of mass media campaigns has a lower impact on the market shares (when $e_2 = 0.02$ for the first innovation its share exceeds the share of the second innovation by less than 80 percent, compared to over 93 percent for highly socially susceptible markets). Variance remains relatively stable with increasing a campaign's intensity for the small world network but decreases visibly for scale free topology.

FIGURE 5 Experiment 8



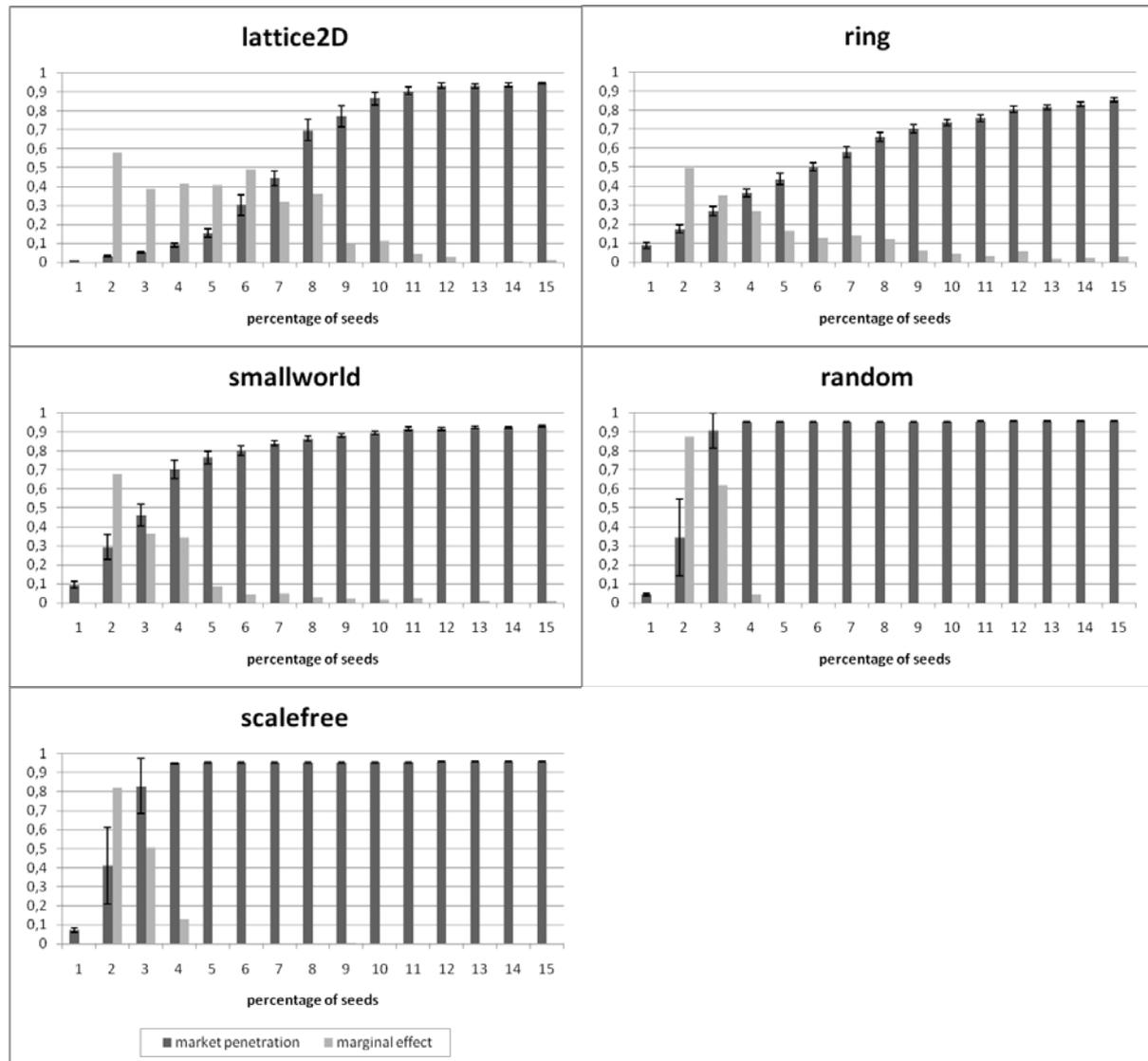
Source: Authors' own calculations

III.2. Seeding

The second method of launching the innovation on the market is to donate the products to chosen individuals, who will subsequently inform their neighbors, who (based on Eq. 1) will then decide whether to adopt the product or not. Libai et al. (2005) investigated the efficiency of concentrated vs. dispersed marketing efforts for different regions. They have found that a dispersed marketing effort is superior to a concentrated marketing effort (focused only on certain regions). We experimented with similar problem on networks with different topologies investigating differences in the efficiency of the seeding procedure for groups of different sizes. Delre (2007) distinguishes between the *throwing gravel* and *throwing rocks* seeding strategies. In both cases a certain share of potential adopters (e_1) obtains the innovation at the beginning of the simulation run. The diffusion process then continues based purely on word-of-mouth communication between linked individuals. In the *throwing gravel* case the seeds (individuals that receive the innovation at the beginning of the simulation) are chosen

randomly. In the case of the throwing rocks strategy the seeding procedure is focused on groups of closely connected individuals of different sizes.

FIGURE 6 Market penetration and marginal effect for the different number of seeds



Source: Authors' own calculations

Let us first examine the throwing gravel strategy with a different number of individuals being chosen as seeds (e_1 ranging between 0.01 and 0.15). Delre (2007) found that for a small world network that it is necessary to select at least 8 percent of the individuals as seeds to achieve a 75 percent market penetration (for $\beta_{i,j} = N \square (0.8, 0.01)$ and $h_{i,j} = N \square (0.35, 0.01)$). Figure 6 shows that for slightly more socially susceptible markets with $\beta_{i,j} = N \square (0.9, 0.01)$ and $h_{i,j} = N \square (0.3, 0.01)$ only 5 percent of seeds is needed to achieve 75 percent market

penetration and 11 percent of seeds is needed to achieve above 90 percent market penetration on a small world network. The ring network does not reach 90 percent penetration even for 15 percent of individuals being seeds. The marginal effect of the additional percentage of seeds declines more rapidly for the small world and ring networks than for the 2D lattice network. Scale free and random topology show a very substantial marginal effect for the low seeding values and reach 90 percent penetration for e_1 equal to 0.04 and 0.03.

To compare the efficiency of the throwing gravel vs. the throwing rocks seeding strategies we decided (based on the previous experiments) to use 60 individuals as seeds (approx. 2 percent of the population). We examined 12 different situations ranging from 1 group consisting of 60 individuals to 60 groups consisting of 1 individual each.

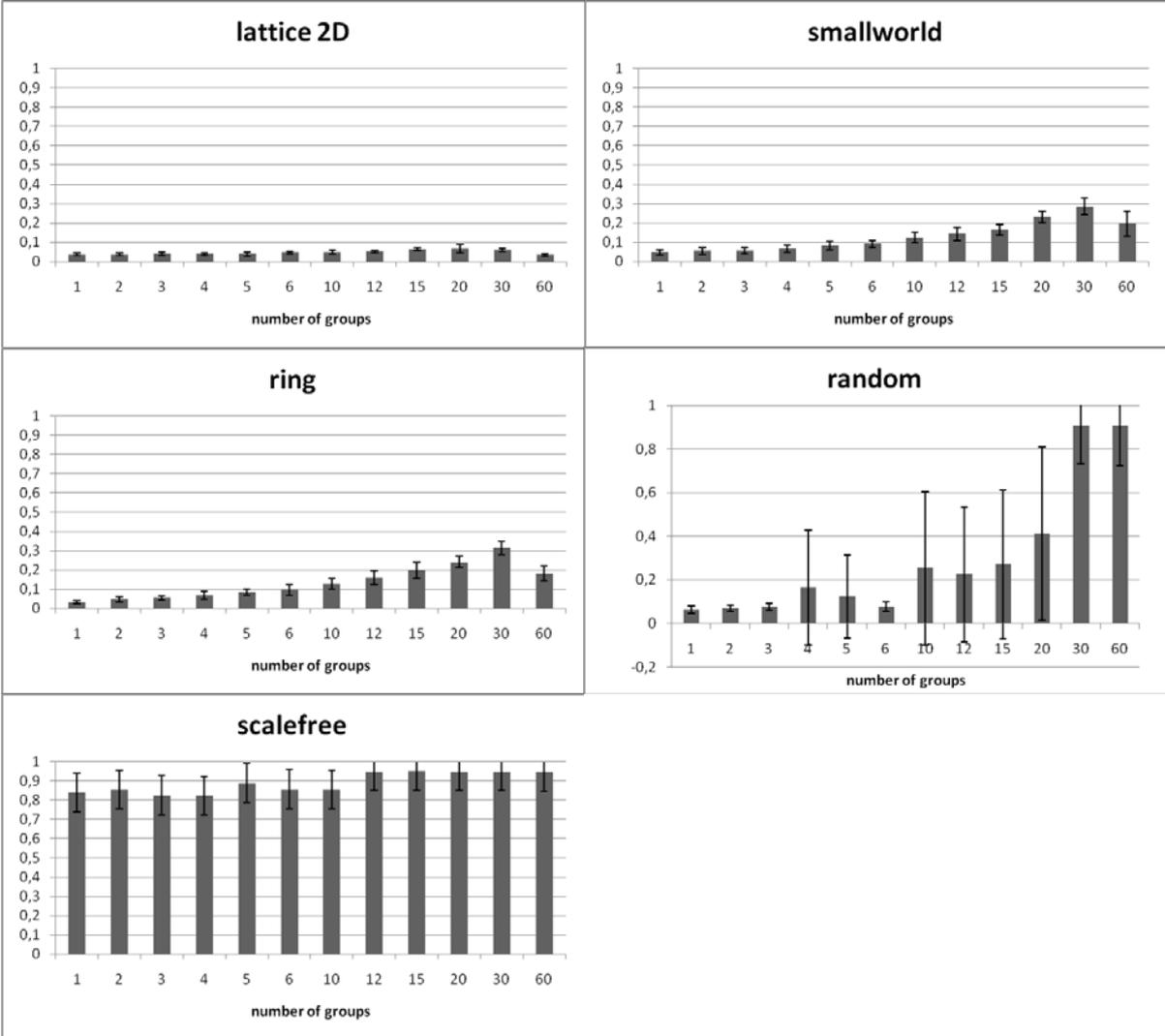
Figure 7 shows the final market penetration for five examined networks. The market penetration for the 2D lattice network peaks for the setting with 20 groups each consisting of 3 individuals. The rationale behind this outcome is given by the structure of the network. 30 groups (consisting of 2 individuals each) are not sufficient to pass the exposure threshold $h_{i,j} = 0.3$ whereas 20 groups (3 individuals each) will create clusters of 3 neighboring adopters which in 50 percent of the cases will be sufficient for surpassing the exposure threshold and neighboring individuals will be likely to adopt the innovation in the next step, contributing to the further diffusion.

Small world and ring networks both reach the highest market penetration for 30 groups each consisting of 2 neighboring individuals. Compared to the 2D lattice network, the neighborhoods in these networks significantly overlap. Therefore, 1 seed consisting of 2 neighboring individuals will (for $h = 0.3$) result with a high likelihood in 1 (or 2) additional adoption(s). Groups consisting of 2 individuals are sufficient to start diffusion in their neighborhood. The final market penetration is relatively low because diffusion is localized in the close neighborhood of the initial seeds and more distant parts of the network are affected only scarcely.

The random network exhibits almost total market penetration for more dispersed seeding (groups consisting of 1 and 2 individuals). Let us compare two extreme cases: 60 groups consisting of 1 individual each and 1 large group consisting of 60 individuals to explain this outcome. The initial seed consisting of one large group affects 60 individuals that are interconnected, but because the low clustering coefficient of the random network neighbors of these individuals with high likelihood are connected only to one of the affected individuals.

Therefore the diffusion process stops relatively soon (the social influence is weak). Only one part of the network, consisting of the initial cluster of seeds, is affected. The development of the diffusion differs from the dispersed seeding because the initial wave of the adoption stops very soon. For dispersed seeding the initial development of the diffusion is much slower, but then the bandwagon of adopters emerges and the diffusion takes off reaching a high total market penetration after approx. 30 periods. A very high standard deviation for groups consisting of between 4 and 15 individuals emerges because in some of the simulation runs diffusion takes off and reaches high market penetration above 90 percent while in others diffusion stops before taking off and final market penetration remains very low below 10 percent of the market.

FIGURE 7 Market penetration for the throwing gravel vs. throwing rocks strategy



Source: Author’s own calculations

The scale free network exhibits very high market penetration for all group sizes, and more dispersed seeding leads to slightly higher market penetration. Interestingly, there is a much higher volatility in market penetration (based on 50 simulation runs) for fewer large groups in the scale free network. In the case of one large group we can observe two scenarios of the diffusion process. Either the diffusion takes off and the market penetration reaches above the 90 percent level or the diffusion does not take off and the market penetration remains very low. Decreasing the number of seeds to 30 individuals yields outcomes very similar to other network types with groups consisting of 2 individuals being the most efficient setting of the marketing effort.

IV. Conclusion and discussion

The set of experiments presented in this study shows that the structure of the interactions among individuals on the markets can be a crucial factor affecting the speed and scope of the diffusion process. On average, higher heterogeneity in degree distribution yields a faster and/or broader diffusion (higher market penetration) of the innovation, at the same time being accompanied by much higher volatility and uncertainty with regards to the diffusion outcomes.

Using different topologies of the underlying networks we have confirmed findings of other authors that the impact of the structure of interactions is more relevant for markets with a high social influence. The vast literature on herding behavior (Banerjee, 1992), conformity, fads and fashion (Karni, Schmeidler, 1990; Bikhchandani, Hirshleifer, Welch, 1992) and group behavior (Cowan et al., 2004) indicates that social influence can be extremely strong on certain markets and these markets especially are therefore the most interesting for the investigation of the structure of underlying interactions.

Our experiments suggest that networks with a less regular structure (higher heterogeneity in the degree of nodes) are more uncertain with regards to diffusion outcomes. David's "historical accidents"¹ play a much more crucial role on scale free and random networks than on highly regular small world, ring and 2D lattice networks. However, high volatility in the diffusion process can be avoided by changes in marketing efforts. Dominance in marketing

¹ David (1985)

efforts is then rewarded by a significantly increased final market share of the given innovation.

Networks with a highly heterogeneous structure, such as the scale free and random network, do exhibit a very high marginal increase in the final market penetration for a small increase in the number of seeds at the beginning of the diffusion process. A company introducing a new product aware of the structure of the interactions among potential adopters can thus reflect this fact in its initial launching strategy. While achieving sufficient market penetration in the 2D lattice or ring network can require a very high initial investment in the seeding, markets with scale free network topology are much less demanding and the successful introduction of the innovation can be accomplished with a much smaller initial investment. Knowledge of the underlying topology is then crucial for an adequate cost/benefit analysis equating the marginal cost and benefits of additional marketing efforts.

Finally, different market structures imply a different optimal dispersion of marketing effort. In this case more than in others, precise knowledge of the underlying topology of the network can importantly increase the efficiency of the launching campaign. In general, the golden mean seems to be again the best solution, with the targeting of small groups of potential adopters bringing highest market shares.

These findings have important implications for many economic actors, from managers of innovating companies to governments trying to support economic growth. Our findings even further emphasize the necessity and urgency of the empirical investigation of the topological structures of social interactions.

V. References

- Anderson, R. M. (1988) The epidemiology of HIV infection: variable incubation plus infectious periods and heterogeneity in sexual activity. *J.R. Stat. Soc. A* 151, 66–98.
- Anderson, R. M., May, R. M. (1992) *Infectious diseases of humans*. Oxford: Oxford University Press.
- Arrow, K.J. (1962) The Economic Implications of Learning by Doing. *The Review of Economic Studies*, Vol. 29, No. 3., pp. 155-173.
- Arthur, W.B. (1989) Competing Technologies, Increasing Returns, and Lock-In by Historical Events. *The Economic Journal*, Vol. 99, No. 394, 116-131.
- Arthur, W.B. (1994) *Increasing Returns and Path Dependence in the Economy*. Ann Arbor: University of Michigan Press.
- Bailey, N. T. J. (1957) *The mathematical theory of epidemics*. London: Griffin.
- Banerjee, AV (1992) A Simple Model of Herd Behavior. *The Quarterly Journal of Economics*, Vol. 107, No. 3. pp. 797-817.
- Barabási, A.-L., Bonabeau, E. (2003) Scale-free networks. *Scientific American* 288. 60-69
- Barabási, A.-L., Albert, R. (1999) Emergence of scaling in random networks, *Science*, 286, 509; <http://arxiv.org/abs/cond-mat/9910332>
- Barro, RJ, Sala-i-Martin X (2003) *Economic Growth*. The MIT Press.
- Bass, Frank M. (1969). A New Product Growth for Model Consumer Durables. *Management Science* 15 (5): 215-227
- Bearden, W.O., Etzel, M.J. (1982) Reference Group Influence on Product and Brand Purchase Decisions. *Journal of Consumer Research*: Vol. 9, Issue 2, 183-194.
- Bikhchandani, S., Hirshleifer, D., Welch, I. (1998) Learning from the behavior of others : conformity, fads, and informational cascades. Working paper University of Michigan No. 98010.

Bikhchandani, S., Hirshleifer, D., Welch, I. (1992) A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades. *Journal of Political Economy* Vol. 100, No. 5: pp. 992.

Brynjolfsson E (2000). Beyond Computation: Information Technology, Organizational Transformation and Business Performance. *Journal of Economic Perspectives* 14: 23-48.

Chatterjee RA, Eliashberg J (1990). The Innovation Diffusion Process in a Heterogeneous Population: A Micromodeling Approach. *Management science* 36: 1057-1079.

Cowan, R., Cowan, W. (1998) Technological Standardization with and without Borders in an Interacting Agents Model. MERIT Research Memoranda. <http://www.merit.unu.edu/publications/rmpdf/1998/rm1998-018.pdf>

Cowan, R., Cowan, W., Swan, PGM (2004) Waves in Consumption with Interdependence among Consumers. *The Canadian Journal of Economics* Vol. 37, No. 1, pp. 149-177.

Cowan, Jonard (2004) Network structure and the diffusion of knowledge. *Journal of Economic Dynamics and Control* Vol. 28, Issue 8, 1557-1575.

David, P.A. (1985) Clio and the Economics of QWERTY. *The American Economic Review*, vo. 75, No. 2, 332-337.

Delre SA, Jager W, Janssen MA (2006). Diffusion dynamics in small-world networks with heterogeneous consumers. *Computational & Mathematical Organization Theory*, Vol. 13, NO. 2, 185-202.

Delre SA, Jager W, Bijmolt THA, Janssen MA (2007). Will It Spread or not? The Effects of Social Influences and Network Topology on Innovation Diffusion. <http://dissertations.ub.rug.nl/FILES/faculties/feb/2007/s.a.delre/c2.pdf>

Eshel, I., Samuelson, L., Shaked, A. (1998) Altruists, Egoists, and Hooligans in a Local Interaction Model. *The American Economic Review*, Vol. 88, No. 1, pp. 157-179

Granovetter, M., (1978) Threshold models of collective behavior. *The American Journal of Sociology*, Vol. 83, No. 6., pp. 1420-1443.

- Granovetter, M., Soong, R. (1986) Threshold Models of Interpersonal Effects in Consumer Demand. *Journal of Economic Behavior and Organization* 7, 83-99.
- Grenfell, B. T., Bjornstad, O. N., Kappey, J. (2001) Travelling waves and spatial hierarchies in measles epidemics. *Nature* 414, 716–723.
- Griliches, Z. (1957). Hybrid Corn: An Exploration in the Economics of Technological Change. *Econometrica* 25: 501-522.
- Griliches, Z. (1960). Hybrid Corn and the Economics of Innovation. *Science* 132: 275-280.
- Hall, B. H. (2004) Innovation and Diffusion. NBER Working Paper 10212. Cambridge.
- Hethcote, H. W., Yorke, J. A. (1984) Gonorrhoea transmission dynamics and control. Springer Lecture Notes in Biomathematics. Berlin: Springer.
- Ireland, N., Stoneman, P. (1986) Technological Diffusion, Expectations and Welfare. *Oxford Economic Papers, New Series*, Vol. 38, No. 2, 283-304.
- Janssen, M.A. and W. Jager (2002), Stimulating diffusion of green products, Co-evolution between firms and consumers, *Journal of Evolutionary Economics*, 12: 283-306
- Karni, E., Schmeidler, D. (1990). “Fixed Preferences and Changing Tastes,” *American Economic Review* 80, 262–267.
- Katz, M. L., Shapiro, C. (1983) Network Externalities, Competition and Compatibility. Woodrow Wilson School Discussion Paper No. 54. Princeton University.
- Katz, M. L., Shapiro, C. (1986) Technology Adoption in the presence of Network Externalities. *The Journal of Political Economy*, Vol. 94, No. 4, 822-841.
- Keeling, M. J. (1997) Modelling the persistence of measles. *Trends Microbiol.* 5, 513–518.
- Kermack, W. O., McKendrick, A. G. (1927) A contribution to the mathematical theory of epidemics. *Proc. R. Soc. A* 115, 700–721.
- Libai B, Muller E, Peres R (2005). The role of seeding in multi-market entry. *International Journal of Research in Marketing* 22, 375–93.

- Liebowitz, S. J., Margolis, S. E. (1995) Path Dependence, Lock-In, and History. *Journal of Law, Economics and Organization* , 11(1), pp. 205-226.
- Leinhardt, S. (1977) *Social networks: a developing paradigm*. New York: Academic Press.
- Midgley, D.F., Morrison, P.D. and Roberts, J.H. (1992) The Effect of Network Structure in Industrial Diffusion Processes. *Research Policy*. 21(6), 533-552.
- Milgram, S. (1967). "The Small World Problem". *Psychology Today* 2: 60-67.
- Moreno, Y., Nekovee, M., Pacheco, A. F. (2006) Dynamics of rumor spreading in complex networks, arXiv:cond-mat/0312131 v2 20 Apr 2004
- Plouraboue F., Steyer, A. Zimmermann, J.B. (1998) Learning Induced Criticality in Consumer's Adoption Pattern: a neural network approach. *Economics of Innovations and New technologies*, 6, pp. 73-90.
- Roedenbeck, M.R.H, Nothnagel, B. (2008) Rethinking Lock-in and Locking: Adopters Facing Network Effects. *Journal of Artificial Social Science Simulation* 11(1)4 <<http://jasss.soc.surrey.ac.uk/11/1/4.html>>.
- Rogers, Everett M. (1962). *Diffusion of Innovations*. Free Press, New York, ISBN 0743222091
- Rosenberg, N. (1976) On Technological Expectations. *The Economic Journal*, Vol. 86, No. 343, pp. 523-535.
- Rosenberg, N. (1985) *Inside the black box: technology and economics*. Cambridge University Press, New York.
- Ryan, B., N. C. Gross (1943) The diffusion of hybrid seed corn in two Iowa communities. *Rural Sociology* 8 (March): 15.
- Simon, H.A. (1957) *Models of Man*. New York: Wiley.
- Tellis, Gerard J., Stremersch, S. and Yin, Eden (2002) The International Takeoff of New Products: The Role of Economics, Culture, and Country Innovativeness. *Marketing Science*, Vol. 22, No. 2, 188-208.

Watts, D.J., Strogatz, S.H. (1998) Collective dynamics of small-world networks, *Nature*, 393, 440

Wilhite, A (2006) Economic Activity on Fixed Networks. In *Handbook of Computational Economics*, Vol. 2, eds. Judd, K. L. & Tesfatsion, L., Elsevier, Amsterdam.

Zanette, D. H., Kuperman, M. (2006) Effects of immunization in small-world epidemics. [arXiv:cond-mat/0109273](https://arxiv.org/abs/cond-mat/0109273) v1 14 Sep 2001

VI. Appendix: Simulation setup

	figure 1	figure 2	figure 3	figure 4	figure 5	figure 6	figure 7
Number of technologies	1	1	2	2	2	1	1
Percent of population as seeds for the first technology (e1)	0	0	0	0	0	0.01....0.15	0.02
Percent of population as seeds for the second technology (e1)	0	0	0	0	0	0	0
Intensity of mass media campaign of the first technology (e2)	0.001	0.001	0.001	0.001.....0.02	0.001....0.02	0	0
Intensity of mass media campaign of the second technology (e2)			0.001	0.001	0.001	0	0
Parameter beta	0.9	1/0.8/ 0.6/0.4	0.9	0.9	0.5	0.9	0.9
Number of groups (in case of throwing rocks) first technology							1.....60
Number of groups (in case of throwing rocks) second technology							
Parameter h	0.3	0.3	0.3				
Number of nodes of the network	3025	3025	3025	3025	3025	3025	3025
Number of time steps	200	200	200	200	200	200	200
Number of simulation runs	20	20	50	20	20	20	20 (50)

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