Dynamical Agents’ Strategies and the Fractal Market Hypothesis

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Abstract: It has been shown in many papers ([11], [18] etc.) that the Efficient Market Hypothesis (EMH) fails as a valid model of financial markets. The Fractal Market Hypothesis (FMH) is in a place as a more general alternative way to the EMH. The FMH can be formed on the following parameter: agents’ investment horizons. It lead us to conclude that a financial market is more stable when we adopt this fractal character in structures of agent’s investment horizons. For computer simulations, the Brock and Hommes model [2] is modified. This adjusted model shows that various frequency distributions on agents’ investment horizons lead to different returns behavior. The FMH focuses on matching of demand and supply of agents’ investment horizons in the financial market. It is the cornerstone that holds financial markets together. The EMH assumes the market is at equilibrium. The FMH on the other hand asserts that investors have an information differently based on temporal attributes. Since all investors in the market have different time investment horizons, the market remains stable. Our simulations of probability distributions of agents’ investment horizons demonstrate that many investment horizons ensure stability of the financial market. The behavior of the model under dynamical changes of agents’ trading strategies is analyzed.

Keywords: Efficient Market Hypothesis, Fractal Market Hypothesis, agents’ investment horizons, agents’ trading strategies

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1. Introduction

The EMH was paradigm of economic and finance theory for the last twenty years. After empirical data analysis on financial markets and after theoretical economic and finance progress this paradigm is gotten over. There are phenomena observed in real data collected from financial markets that cannot be explained by the recent economic and finance theories. One paradigm of recent economic and finance theory asserts that sources of risk and economic fluctuations are exogenous. Therefore the economic system would converge to a steady-state path, which is determined by fundamentals and there are no opportunities for speculative profits in the absence of external shocks prices. It means that the other factors play important role in a construction of real market forces as heterogeneous expectations. Since agents no have sufficient knowledge of a structure of the economy to form correct theoretical expectations, it is impossible for any formal theory to postulate unique value expectations that would be held by all agents [6]. Prices are partly determined by fundamentals and partly by the observed fluctuations endogenously caused by non-linear market forces. This implies that technical trading rules need not be systematically bad and may help in predicting future price changes. Developments in the theory of non-linear dynamic systems have contributed to new approaches in economics and finance theory [3]. Introducing non-linearity in these models may improve research of a mechanism generating the observed movements in the real financial data. The financial market exhibits local randomness but also global determinism. Thus financial markets can be considered as nonlinear dynamic systems of the interacting agents processing new information immediately. Investors with the same investment horizons, and holding similar positions in the market may utilize this information differently. Therefore a financial market has a fractal structure in investment horizons. For an analyzing of behaviour of such market, an adjusted version of the model, introduced by Vacha and Vosvrda in the [14] with two main types of traders, i.e., fundamentalists, and technical traders, is used. Technical traders tend to put little faith in strict efficient markets. Fundamentalists rely on their model employing fundamental information basis for forecasting of the next price period. The traders determine whether current conditions call for the acquisition of fundamental information in a forward looking manners, rather than relying on post performance. This approach relies on heterogeneity in the agent information and subsequent decisions either as fundamentalists or as chartists. Changing of the chartist's profitability and fundamentalist's positions is a basis of the cycles behaviour. A more detailed analysis is introduced in the Brock and Hommes model. This model with memory was analyzed in [15]. The model is presented in a form of evolutionary dynamics of price model. The fundamentalists are considered as traders with more rich structure of memory for price prediction. The chartists are considered as traders with more simple structure of memory for a price prediction. A simulation analysis of this model under changing probability properties of memory shows connections between EMH and FMH. Section 2 is devoted to dynamics of fractions of different traders. Agent's investment horizons with a different form of memory structures in the performance measure are analyzed. Fractal structure of financial markets is shown in Section 3. Results of the simulation analysis are introduced in Section 4.

2. Dynamics of Traders

Let us concentrate on dynamics of the fractions $n_{h,t}$ of different h-trader types, i.e.

$$a \cdot x_t = \sum_h n_{h,\, t-1} \cdot f_h\left(x_{t-1},\dots,x_{t-L}\right) \equiv \sum_h n_{h,\, t-1} \cdot f_{h,\, t}\,, \tag{2.1}$$

where $n_{h,t-1}$ denotes the fraction of trader type h at the beginning of period t, before than the equilibrium price x_t has been observed and a denotes a gross return of a risk free asset which is perfectly elastically supplied, i.e., $a > 1$ and L is a number of lags. Now the **realized excess return** over period t to the period $t+1$ is computed, where $x_t = p_t - p_t^*$, and p_t^* is the price corresponding to the intersection point of demand and supply, by

$$Z_{t+1} = p_{t+1} - a \cdot p_t \tag{2.2}$$

$$Z_{t+1} = x_{t+1} + p_{t+1}^* - a \cdot x_t - a \cdot p_t^* \tag{2.3}$$

$$Z_{t+1} = x_{t+1} - a \cdot x_t + p_{t+1}^* - E_t\left(p_{t+1}^*\right) + E_t\left(p_{t+1}^*\right) - a \cdot p_t^* \tag{2.4}$$

From the equation (2.4) we get

$$E_t\left(p_{t+1}^*\right) - a \cdot p_t^* = 0 \text{ and } \delta_{t+1} = p_{t+1}^* - E_t\left(p_{t+1}^*\right) \tag{2.5}$$

is a martingale difference sequence with respect to $\Im_t$ i.e.,

$$E_t\left(\delta_{t+1} \middle| \Im_t \right) = 0,$$

for all t. So the Eq.(2.4) can be written as follows

$$Z_{t+1} = x_{t+1} - a \cdot x_t + \delta_{t+1} \tag{2.6}$$

The decomposition of the equation (2.6) as separating the 'explanation' part of realized excess returns Z_{t+1} into the contribution $x_{t+1} - a \cdot x_t$ and the additional part δ_{t+1}. Let a performance measure $\pi(Z_{t+1}, \rho_{h,t})$be defined by

$$\begin{aligned} \pi_{h,t} &= \pi\left(Z_{t+1}, \rho_{h,t}\right) = Z_{t+1} \cdot z_t \cdot \rho_{h,t} \\ \pi_{h,t} &= \left(x_{t+1} - a \cdot x_t + \delta_{t+1}\right) \cdot z_t \cdot \rho_{h,t} \end{aligned} \tag{2.7}$$

where

$$\rho_{h,t} = E_{h,t}\left(Z_{t+1}\right) = f_{h,t} - a \cdot x_t$$

and z_t denotes the number of shares of the asset purchased at time t. So the h-performance is given by the realized profits for the h- trader.

Let the updated fractions $n_{h,t}$ be given by the discrete choice probability

$$n_{h,t} = \exp\left(\beta \cdot \pi_{h,t-1}\right) / Y_t \tag{2.8}$$

$$Y_t = \sum_h \exp\left(\beta \cdot \pi_{h,t-1}\right) \tag{2.9}$$

The parameter β is the **intensity of choice** measuring how fast agents switch between different predictors. The parameter β is a measure of trader's rationality. The variable Y_t is just a normalization so that fractions $n_{h,t}$ sum up to 1. If the intensity of choice is infinite ($\beta = +\infty$), the entire mass of traders uses the strategy that has the highest fitness. If the intensity of choice is zero, the mass of traders distributes itself evenly across the set of available strategies.

3. Memory in the performance measure

The performance measure is given by summation m-values of the lagged h-performance measures in the following form

$$n_{h,t} = \exp\left(\beta \cdot \frac{1}{m} \sum_{p=1}^{m} \pi_{h,t-p} \right) / Y_t, \tag{3.1}$$

$$Y_t = \sum_h \exp\left(\beta \cdot \frac{1}{m} \sum_{p=1}^{m} \pi_{h,t-p} \right), \tag{3.2}$$

where the m denotes the memory length, and the η is the realization of the random vector of predictor memory (trading horizons). We assume that the expression $m = E_h[\eta]$ holds. All beliefs or the formation of expectations with different lag lengths will be of the following form

$$f_{h,t} = g_h \cdot x_{t-1} + b_h \tag{3.3}$$

where g_h denotes the trend, b_h the bias of trader type h. If $b_h = 0$, the agent h is called a **pure trend chaser** if $g > 0$ (strong trend chaser if $g > a$) and a **contrarian** if $g < 0$ (strong contrarian if $g < -a$). If $g_h = 0$, type h trader is said to be **purely biased**. He is upward (downward) biased if $b_h > 0$ ($b_h < 0$). In the special case $g_h = b_h = 0$, type h trader is called **fundamentalist** i.e., the trader is believing that prices return to their fundamental value. Fundamentalists do have all past prices and dividends in their information set, but they do not know the fractions $n_{h,t}$ of the other belief types.

4. R/S Analysis of Agent Investment Strategies

For estimating and analysing of correlation structures on capital markets, a nonparametric method of Hurst type is used. H. E. Hurst discovered very robust nonparametric methodology which is called rescaled range, or R/S analysis. The R/S analysis was used for distinguishing random and non-random systems, the persistence of trends, and duration of cycles. This method is very convenient for distinguishing random time series from fractal time series as well. Starting point for the Hurst's coefficient was the Brownian motion as a primary model for random walk processes.

For computation of the R/S coefficients we have to divide the time series of length T, into N intervals of the length n, where $n \cdot N = T$. Values $\{(R/S)_n\}$ are defined in the following form

$$(R/S)_n = \frac{1}{N} \sum_N \frac{R_n}{S_n} \tag{4.1}$$

where

$$R_n = \max_{1 \le k \le n} \left[\sum_{i=1}^{k} (x_i - \overline{x}_n) \right] - \min_{1 \le k \le n} \left[\sum_{i=1}^{k} (x_i - \overline{x}_n) \right] \tag{4.2}$$

where is the range and

$$S_n = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \overline{x}_n)^2} \tag{4.3}$$

is the sample standard deviation. The Hurst exponent H can be approximated by the following equation

$$\log((R/S)_n) = \log(c) + H \log(n), \tag{4.4}$$

where c is a constant, and $n = 9,\ldots,400$. If a system of random variables $\{(R/S)_n\}$ is i.i.d, then $H = 0.5$ The values of Hurst exponent belonging to $0 < H < 0.5$ signifies antipersistent system of variables covering less space than random ones. Such a system must reverse itself more frequently than a random process. With the assumption of a stable mean (which we do not impose here) we can equate this behaviour to a mean-reverting process. Values $0.5 < H < 1$ show persistent process that is characterized by long memory effects. This long memory occurs regardless of time scale, i.e., there is no characteristic time scale which is the key characteristic of fractal time series [11].

For obtaining of the expected $\{(R/S)_n\}$ values we have used the following equation [11]:

$$E(R/S)_n = \left(\frac{n-0.5}{n}\right)\left(n \cdot \frac{\pi}{2}\right)^{-\frac{1}{2}} \cdot \sum_{r=1}^{n-1}\sqrt{\frac{n-r}{r}} \tag{3.3}$$

Moreover, we have still used the statistics V which estimates the breaks in the R/S plot in an easier way. This one is usually used as a good measure of cycle length in the presence of noise [11]. This statistics is defined as

$$V_n = \frac{(R/S)_n}{\sqrt{n}} \tag{3.4}$$

A plot of V_n versus $log(n)$ would be flat if the system of random variables was an independent system, i.e., the R/S statistic was scaling with the square root of time. For $H > 0.5$ (persistent) the R/S was scaling faster than the square root of time so the plot would be upward sloping. On the other hand for $H < 0.5$ (antipersistent) the graph would be downward sloping.

Data used in the analysis are generated by the model outlined above in part 2 and 3. In all the cases we used with twenty belief types (traders), which has the trend g and bias b generated by random numbers generator with the normal distribution $N \sim (0, 0.4)$ and $N \sim (0, 0.3)$. The model generates four thousands observations, but to avoid problems with transients we do not use the first four hundreds observations. The intensity of choice ($\beta = 80$) is the same for all simulations. Memory or trading horizons are also random numbers generated by three distributions (normal, uniform, weibull), the fourth is a given, fixed, value for all traders of memory in a particular simulation. With each probability distribution of predictor memory, we make step by step the four simulations with different expected memory as follows (5, 10, 20, 40).To minimize a linear dependency of raw returns, which can bias estimates of the Hurst exponent significantly [11], we have used AR (1) residuals of these ones. This procedure eliminates serial correlation. Results of those experiments are demonstrated in Tables (4.1-4.4). In these tables there are two estimates of the Hurst exponent. The first one used data from period $n = 9$ to 400. The second one (i.e., Hurst mod) used data from period $n = 9$ to break-even point of the V-statistics. Figures (4.1–4.4) demonstrate R/S and V statistic for four different memory distributions with the same expected memory ($E[\eta] = 20$). There are visible differences in the position of maximal value of the V-statistic (break-even point). Figures (4.1, 4.5-4.7), where different memory lengths are shown for normal distribution, make obvious the importance of the memory length for the pricing behavior of the market.

Table 4.1. Estimated Hurst coefficients and statistic of raw returns for memory mean 5

E(η) = 5	Normal (5,1.25)	Uniform (1,10)	Fixed (5)	Weibull (1.3)
Hurst	0.112	0.122	0.171	0.128
Hurst mod	0.858 (9-20)	0.636 (9-20)	0.851 (9-20)	0.542 (9-25)
Var (x)	0.044	0.071	0.048	0.069
Kurtosis (x)	0.029	0.119	-0.319	0.93
Skewness (x)	0.025	0.197	-0.315	-0.378

Table 4.2. Estimated Hurst coefficients and statistic of raw returns for memory mean 10

E(η) = 10	Normal (10,2.5)	Uniform (1,20)	Fixed (10)	Weibull (1.3)
Hurst	0.186	0.164	0.239	0.146
Hurst mod	0.718 (9-36)	0.727 (9-20)	0.789 (9-36)	0.594 (9 - 25)
Var (x)	0.019	0.029	0.00584	0.027
Kurtosis (x)	0.642	1.915	0.371	1.379
Skewness (x)	0.186	0.41	0.096	-0.163

Table 4.3. Estimated Hurst coefficients and statistic of raw returns for memory mean 20

E(η) = 20	Normal (20,5)	Uniform (1,40)	Fixed (20)	Weibull (1.3)
Hurst	0.387	0.31	0.383	0.318
Hurst mod	0.677 (9-40)	0.732 (9-20)	0.7 (9-45)	0.733 (9-18)
Var (x)	0.00000438	0.00732	0.0000515	0.00778
Kurtosis (x)	1.158	8.811	3.66	19.54
Skewness (x)	-0.571	-1.339	-0.155	1.671

Table 4.4. Estimated Hurst coefficients and statistic of raw returns for memory mean 40

E(η) = 40	Normal (40,10)	Uniform (1,80)	Fixed (40)	Weibull (1.3)
Hurst	0.462	0.421	0.47	0.399
Hurst mod	0.611 (9-72)	0.556 (9-72)	0.606 (9-72)	0.587 (9-45)
Var (x)	0.00000398	0.0052	0.00000354	0.00267
Kurtosis (x)	0.245	24.652	0.203	17.795
Skewness (x)	-0.503	2.328	-0.895	-2.392

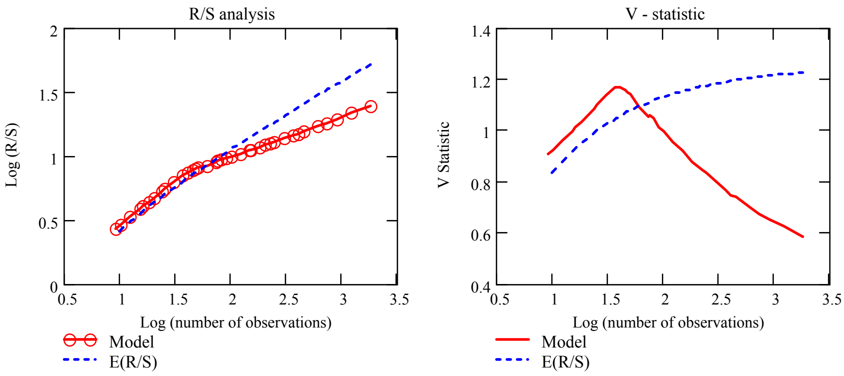

Fig. 4.1. R/S analysis and V– statistic for memory distribution $N \sim (20{,}5)$.

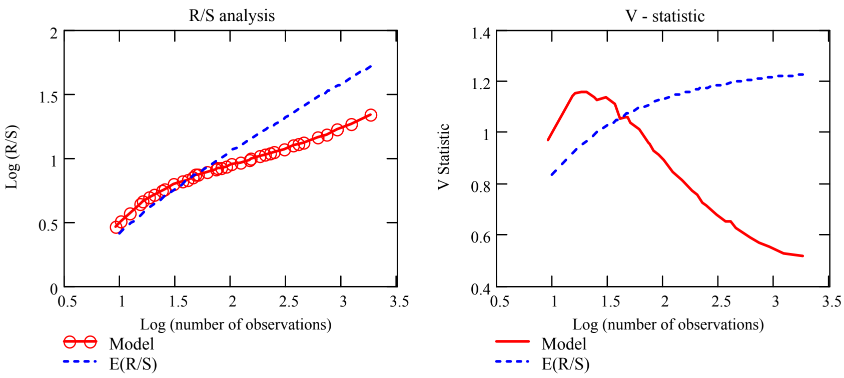

Fig. 4.2. R/S analysis and V– statistic for memory distribution Uniform (1,40).

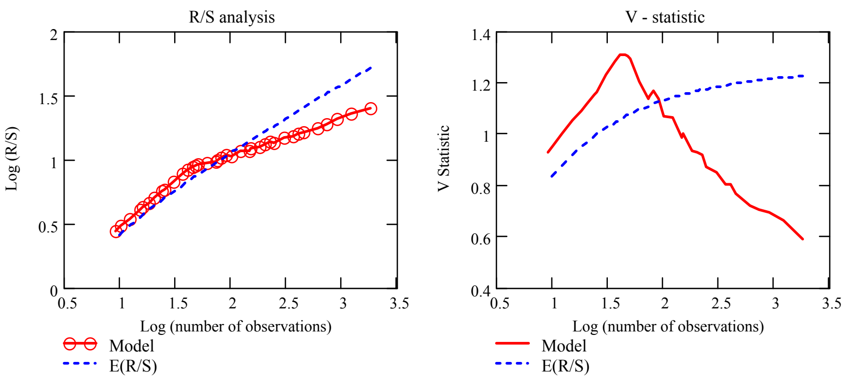

Fig. 4.3. R/S analysis and V– statistic for memory distribution Fixed (20).

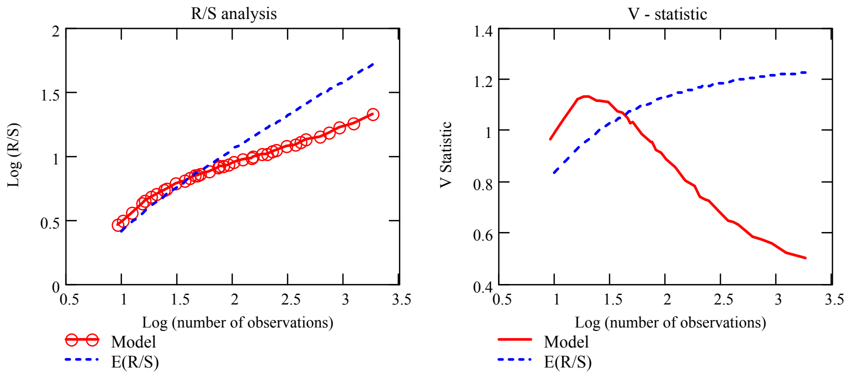

Fig. 4.4. R/S analysis and V– statistic for memory distribution Weibull (1.3), E(η) = 20.

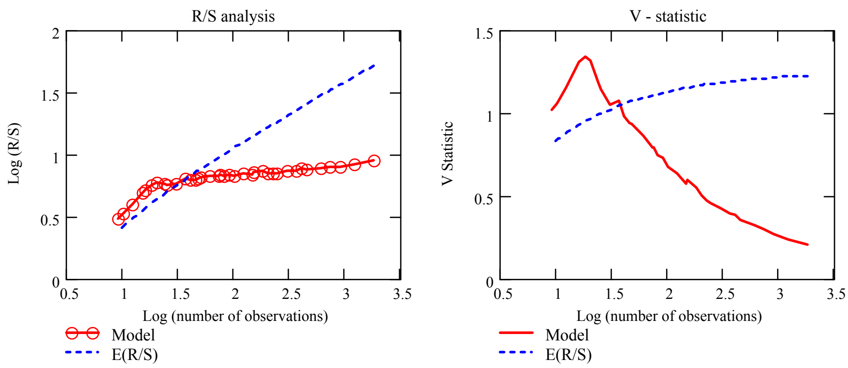

Fig. 4.5. R/S analysis and V– statistic for memory distribution $N \sim (5,1.25)$

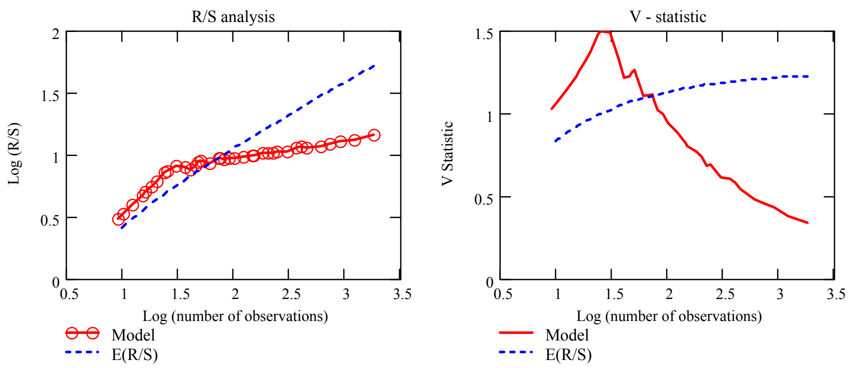

Fig. 4.6. R/S analysis and V– statistic for memory distribution $N\sim(10{,}2.5)$

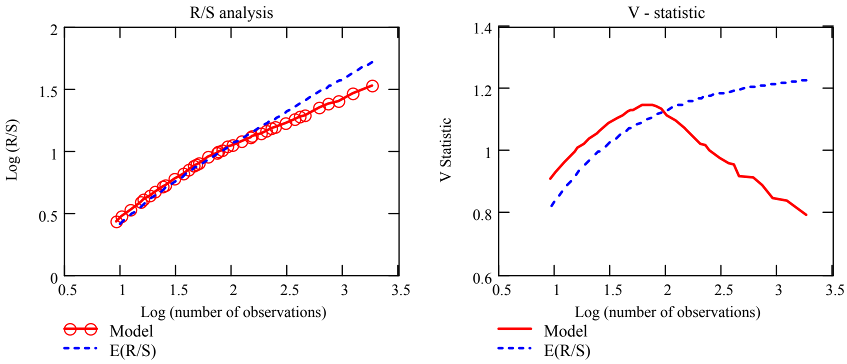

Fig. 4.7. R/S analysis and V– statistic for memory distribution $N\sim(40{,}10)$

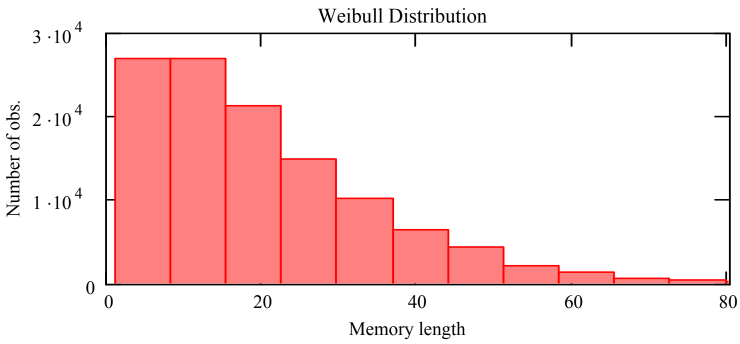

Fig. 4.8. Weibull distribution for memory length with $E[m] = 20$.

Conclusions

Short memories of predictors i.e., short agent's investment horizons cause more volatile of price realizations on capital markets, but by values of the Hurst coefficients there exist possibilities of the price predictions due to the persistence of the fundamental strategy structures. Long memories of predictors i.e., long agent's investment horizons cause more stable behaviour of price realizations on capital markets (see Tables 4.1–4.4). These tables demonstrate dependencies among agent's investment horizons and both local randomness and global determinism.

The FMH is a more general notation than the EMH. The FMH and the EMH are equivalent in the break-even point of the *V*-statistics. Here are equivalent the Brownian motion and the fractional Brownian motion. Therefore financial markets are nonlinear systems with a fractal structure of agent's investment horizons. These markets are unpredictable in the long-term period, but predictable in the short-term period. The key features- the lengths of memory and probability distribution in memory- influence self-similarity properties in agent's investment horizons are demonstrated in tables (4.1–4.4) and figures (4.1–4.7).

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