Institute of Economic Studies, Faculty of Social Sciences
Charles University in Prague

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Michaela Krčílková
Jan Zápal

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Institute of Economic Studies,
Faculty of Social Sciences,
Charles University in Prague

[UK FSV – IES]

Opletalova 26
CZ-110 00, Prague
E-mail : ies@fsv.cuni.cz
http://ies.fsv.cuni.cz

Institut ekonomických studií
Fakulta sociálních věd
Univerzita Karlova v Praze

Opletalova 26
110 00 Praha 1

E-mail : ies@fsv.cuni.cz
http://ies.fsv.cuni.cz

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OCA cubed: Mundell in 3D

Michaela Krčílková*
Jan Zápal#

* Czech University of Life Sciences Prague,
Faculty of Economics and Management

IES, Charles University Prague and
London School of
Economics and Political Science, London

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Abstract:
This paper intends to fill two gaps in the Optimal Currency Area literature. First of all, Mundell's original idea has very little formalmodel theoretical underpinning. Second, it almost exclusively views countries contemplating monetary unification as single economies. We question this view and expand the model to incorporate the division of an economy into three sectors. In the empirical part of the paper, we follow recent OCA empirici literature and investigate the correlation of shocks between the individual new EU member countries and the `EU-core'. Treating the whole economy as one sector this is a standard exercise. However, since the three-sector version of our model provides a natural metric on which to assess the appropriateness of unification, we are able to repeat the exercise treating each country's economy as a collection of three distinct sectors.

Keywords: OCA, supply and demand shocks, VAR decomposition, new EU member states

JEL: E32, F15, F40

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1 Introduction

Optimum Currency Area (OCA) theory has been a framework for discussing monetary integration for over half a century, ever since the seminal Mundell (1961) contribution and further refinements by McKinnon (1963) and Kenen (1969).

At the onset of European monetary unification, OCA theory gave rise to more empirically focused literature which tried to assess the appropriateness of the said unification. One of the directions this strand of the literature took was to estimate supply and demand shocks for a given country and look at their correlation with the supply and demand shocks in a country representing the 'EU core'. This approach, proposed by Bayoumi and Eichengreen (1992), which uses a two-equation VAR model with output growth and inflation as a first step in the estimation of supply and demand shocks, has found widespread use.

Another wave of studies was motivated by the EU eastern enlargement, which came with the obligation of the accession countries to join the European Monetary Union (EMU). Using a similar methodology as Bayoumi and Eichengreen (1992) those studies try to estimate the correlation of supply and demand shocks between the various new EU member states (NMSs) and the existing EMU.

Despite its widespread use, OCA theory has found rather limited treatment on the formal theoretical side. Among the few contributions, Bayoumi (1994) presents a small general equilibrium model with regionally differentiated goods. In this model, monetary unification presents a welfare trade-off between lower transaction costs and the loss of the exchange rate shock-absorbing role.

More recently Cooper and Kempf (2004) present an OLG model with both monetary and fiscal policy. Their motivation is to see whether the traditional OCA trade-off between lower transaction costs and the loss of independent monetary policy holds in a model with fiscal policy present as well.

This paper hopes to contribute to the OCA discussion in two ways. First of all, we present a simple model which focuses on the traditional transaction cost versus independent monetary policy trade-off. Moreover, the model is built with the empirical strategy discussed above in mind. All the studies in this strand of literature treat each country as a single sector, something we view with reservations. That is why we split the economy of each country into a collection of three sectors and derive a simple condition under which monetary unification is welfare improving.

With the model at hand, we are then able to assess the appropriateness of monetary unification for the set of ten NMSs. We repeat the usual exercise of deriving supply and demand shocks and calculating their correlation with the supply and demand shocks in the 'EU core'. However, our empirical strategy differs from the usual approach in a crucial way. Since our three-sector model gives us a natural metric to focus on, we can treat the economy of each country in the exercise as a collection of three sectors. We then compare our results with the results of the single-sector exercise and with the results in Fidrmuc and Korhonen (2006), who present a meta-analysis of the business cycle correlations between the euro area and the Central and Eastern European countries. This is the second contribution we hope to make.

We proceed in the following steps. Section 2 lays out our model first for the one-sector and then for the three-sector version. Section 3 includes the results of our empirical exercise. Section 4 concludes. To keep the main body of the paper brief, we relegate various details to two appendices. Appendix A further investigates the robustness of our model. First, it shows that our model applies to any n-sector division of the economy, not just to the three-sector division we discuss in the main part. Second, it shows that our model conclusions are virtually unchanged if we use more conventional economy and preference equations compared to the model from the main part. Appendix B then spells out details of the supply and demand shock estimation and gives details of the data we use in the empirical part.

2 Model

This section lays out a simple formal model which captures Mundell's (1961) original idea that the correlation of economic shocks in two countries contemplating monetary unification should be sufficiently high for the benefits to outweigh the costs of the unification. We start with the one-sector version of the model.

One-sector model

There are two countries in our model contemplating monetary unification. The economy of each country $j \in \{1, 2\}$ before unification is described by the following equation

$$y_j = y_j^* + \pi_{cb,j} - e_j, \tag{1}$$

where y_j is (log)output, y_j^* is potential output, $\pi_{cb,j}$ is inflation set by the central bank in country j and e_j is a shock hitting the economy observed by the central bank, but not by the citizens in either of the countries. Furthermore, we assume $e_j \sim N(0, \sigma_j^2)$ and $\mathbb{E}(e_1 e_2) = cov(e_1; e_2) = \sigma_{12}$.[1]

The central bank in each country maximizes its utility function of the form

$$\mathbb{W}_{cb,j} = y_j^* - (y_j - y_j^*)^2 \tag{2}$$

by choosing $\pi_{cb,j}$ after observing the shock e_j.[2]

The welfare valuation of citizens constitutes the last building block of our model. We assume a welfare function of citizens of the form

$$\mathbb{W}_j = \mathbb{E}[y_j^* - (y_j - y_j^*)^2] \tag{3}$$

where $\mathbb{E}(\cdot)$ is the expectation operator. A welfare function of this form captures the notion that citizens prefer a stable economic environment (the second term) and high output growth (the first term).

To derive citizens' welfare, we first need to solve for the inflation chosen by the central bank. Substitution of (1) into the central bank's utility function (2) and straightforward maximization with respect to $\pi_{cb,j}$ gives inflation chosen by the central bank

$$\pi_{cb,j}^* = e_j.$$

Using inflation chosen by the central bank along with the economy equation (1) in the welfare function (3) gives

$$\mathbb{W}_j = y_j^*.$$

[1] Note that the structure of the economy outlined in (1) is not fully conventional in that it does not include inflation expected by the citizens (higher-than-expected inflation would increase output). We show in the appendix that the conclusions of our model remain almost identical if we alter the structure of the economy in line with this suggestion.

[2] Again, the utility function we use is not a conventional one in that central bank cares only about output and not about inflation. We show in the appendix that the conclusions of our model do not depend on this assumption.

Next we need to derive the welfare after unification. We assume that unification changes the economy so that it can be described by

$$\tilde{y}_j = \left(y_j^* + \tfrac{b}{2}\right) + \pi_{cb} - e_j \tag{4}$$

where $\frac{b}{2}$ is the benefit of unification, which increases the potential of the economy in both countries. Along with the natural assumption that $b > 0$, equation (4) incorporates the assumption that both countries share the benefits of unification equally. Although restrictive, this assumption is made only for convenience and has no influence on our results. Any assumption about the way the benefits are split will lead to the same conclusions. The second change comes from the loss of independent monetary policy, which is captured by inflation π_{cb}, which is common to both countries.

After unification, the central bank maximizes its utility function of the form

$$\mathbb{W}_{cb} = \sum_{j \in \{1,2\}} \left(y_j^* + \tfrac{b}{2}\right) - \left(\tilde{y}_j - y_j^* - \tfrac{b}{2}\right)^2 \tag{5}$$

by choosing π_{cb} after observing both shocks, e_1 and e_2.[3]

Again, the last component we need is the welfare valuation of the citizens. We assume that the welfare function (3) changes as a result of unification into

$$\mathbb{W}_{j,u} = \mathbb{E}\left[\left(y_j^* + \tfrac{b}{2}\right) - \left(\tilde{y}_j - y_j^* - \tfrac{b}{2}\right)^2\right], \tag{6}$$

which differs from (3) only in that citizens take into account the change in the potential of the economy.

Solving for the welfare of citizens after unification again requires solving for the inflation chosen by the central bank. Substituting the economy equations from (4) into the central bank's utility function (5) and maximizing with respect to π_{cb} gives

$$\pi_{cb}^* = \frac{e_1 + e_2}{2}.$$

Using this along with the economy equation in the welfare function (6) gives

$$\mathbb{W}_{j,u} = y_j^* + \tfrac{b}{2} - \frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{4}.$$

[3] We would get exactly the same conclusion for a central bank utility function of the form

$$\mathbb{W}_{cb} = y^* - (\tilde{y} - y^*)^2$$

where $\tilde{y} = \tilde{y}_1 + \tilde{y}_2$ is the output of the unified economies and $y^* = y_1^* + y_2^* + b$ is the new potential output, which includes the benefit of unification.

For this simple model, monetary unification is welfare improving if $\mathbb{W}_{j,u} - \mathbb{W}_j > 0$, which gives the condition

$$\sigma_{12} > \frac{\sigma_1^2 + \sigma_2^2}{2} - b. \tag{7}$$

Condition (7) has an intuitive explanation. For unification to be welfare improving, the covariance of the shocks between the two countries has to be sufficiently high. The higher the variance of the shock in either of the countries, the higher the covariance has to be. Alternatively, the higher the benefit of the unification, the lower the covariance has to be.

If we further assume $\sigma_j^2 = 1$ for $j \in \{1, 2\}$, which is the assumption made by empirical studies that investigate the correlations of supply and demand shocks derived in the VAR model, the condition (7) becomes

$$\rho_{12} > 1 - b \tag{8}$$

where ρ_{12} is the correlation between the shocks in the two countries. This condition justifies the focus of the aforementioned studies on the correlation of shocks in the relevant economies.

Three-sector model

How does our model change if we split the economy of each country into three sectors? And what is the relevant variable researchers should focus on when empirically investigating the appropriateness of monetary unification? Should they focus on the correlation of shocks between the corresponding sectors? And what about the correlations between the different sectors within and between countries? This section answers those questions and guides our empirical strategy in the next one.

We split the economy of each country j into three sectors. Although we use agriculture, industry and services this is just a matter of interpretation and our model equally applies to any three-sector division. Under this division, the economy of each country is described by the following system of equations

$$\begin{aligned} y_{j,a} &= y_{j,a}^* + \pi_{cb,j} - e_{j,a} \\ y_{j,i} &= y_{j,i}^* + \pi_{cb,j} - e_{j,i} \\ y_{j,s} &= y_{j,s}^* + \pi_{cb,j} - e_{j,s} \end{aligned} \tag{9}$$

which includes sector-specific shocks and inflation set by the central bank, which is unique to the whole economy. As regards the shocks we assume $\mathbf{e}_j = [e_{j,a}\ e_{j,i}\ e_{j,s}]' \sim N(0, V_j)$, where V_j is a variance-covariance matrix of sector-specific shocks within a country. Furthermore, we assume $\mathbb{E}(\mathbf{e}_1\mathbf{e}_2') = cov(\mathbf{e}_1; \mathbf{e}_2) = V$, where V is a matrix of covariances among the shocks across the two countries. Formally

$$V_j = \begin{bmatrix} \sigma^2_{j,a} & & \\ \sigma_{j,ai} & \sigma^2_{j,i} & \\ \sigma_{j,as} & \sigma_{j,is} & \sigma^2_{j,s} \end{bmatrix} \qquad V = \begin{bmatrix} \sigma_{aa} & \sigma_{ai} & \sigma_{as} \\ \sigma_{ia} & \sigma_{ii} & \sigma_{is} \\ \sigma_{sa} & \sigma_{si} & \sigma_{ss} \end{bmatrix}$$

where $\sigma^2_{j,v}$ is the variance of the shocks in sector v of country j, $\sigma_{j,uv}$ is the covariance of the shocks between sectors u and v in country j and σ_{uv} is the covariance of the shocks in sector u of the first country with the shocks in sector v of the second country. Note also that while V_j is symmetric by definition, V is not.

The change in the structure of the economy we work with requires an analogous change in the utility function of the central bank. We assume that the central bank maximizes

$$\mathbb{W}_{cb,j} = \sum_{v \in \{a,i,s\}} y^*_{j,v} - (y_{j,v} - y^*_{j,v})^2 \phi_{j,v} \tag{10}$$

by choosing $\pi_{cb,j}$ after observing all the shocks $e_{j,v}$. Parameter $\phi_{j,v}$ captures the weights the central bank attaches to the different sectors. For notational convenience, we will use $\phi_j = \sum_{v \in \{a,i,s\}} \phi_{j,v}$ for the sum of the weights attached to the different sectors in country j, $\phi = \phi_1 + \phi_2$ for the sum of all weights and $\Phi_j = [\phi_{j,a}\ \phi_{j,i}\ \phi_{j,s}]'$ for the vector of weights for country j.

Citizens' welfare valuation changes similarly to

$$\mathbb{W}_j = \mathbb{E}\left[\sum_{v \in \{a,i,s\}} y^*_{j,v} - (y_{j,v} - y^*_{j,v})^2 \phi_{j,v}\right] \tag{11}$$

which again includes a preference for stability (the second term) and for high output growth (the first term).

Solving for the welfare valuation of citizens requires solving for the inflation chosen by the central bank in the first place. Substitution of the economy equations (9) into the central bank's utility function (10) and straightforward maximization gives

$$\pi^*_{cb,j} = \frac{1}{\phi_j} \sum_{v \in \{a,i,s\}} e_{j,v}\phi_{j,v}$$

which has the intuitive interpretation. Since the central bank has only one instrument to stabilize the three sectors of the economy it will set inflation as a weighted average of the shocks. Note also that setting all the weights but one equal to zero and the remaining one equal to unity we get the result that applies to the one-sector version of the model. Using the inflation set by the central bank along with the economy equations in the citizens' welfare function (11) we get

$$\mathbb{W}_j = \sum_{v \in \{a,i,s\}} y^*_{j,v} - \sum_{v \in \{a,i,s\}} \sigma^2_{j,v}\phi_{j,v} + \frac{1}{\phi_j}\Phi'_j V_j \Phi_j$$

with the intuitive interpretation. Focusing on the very last term, welfare is increasing with the covariance of shocks among the sectors of the economy. The appropriate covariances have to be weighted to reflect the weights that individual sectors share in the utility function. On the other hand, increasing variance of shocks in the sectors of the economy is welfare reducing, since factoring the individual $\sigma^2_{j,v}$s in the expression gives their weight equal to $-\phi_{j,v}(1 - \frac{\phi_{j,v}}{\phi_j})$, which is unambiguously negative. The very first term then captures the positive welfare effect of the potential of each of the sectors.

The effect of monetary unification in the three-sector version of the model is very similar to the effect in the one-sector model. The three sectors of the economy in country j are now described by the system of equations

$$\begin{aligned} \tilde{y}_{j,a} &= \left(y^*_{j,a} + \tfrac{b_a}{2}\right) + \pi_{cb} - e_{j,a} \\ \tilde{y}_{j,i} &= \left(y^*_{j,i} + \tfrac{b_i}{2}\right) + \pi_{cb} - e_{j,i} \\ \tilde{y}_{j,s} &= \left(y^*_{j,s} + \tfrac{b_s}{2}\right) + \pi_{cb} - e_{j,s} \end{aligned} \tag{12}$$

where we assume $b_v > 0$ for $v \in \{a, i, s\}$.

The central bank's utility function changes correspondingly to

$$\mathbb{W}_{cb} = \sum_{j \in \{1,2\}} \sum_{v \in \{a,i,s\}} \left(y^*_{j,v} + \tfrac{b_v}{2}\right) - \left(\tilde{y}_{j,v} - y^*_{j,v} - \tfrac{b_v}{2}\right)^2 \phi_{j,v} \tag{13}$$

which differs from the central bank's utility function before unification (10) in that it takes into account all the sectors in both countries and the change in the potential of all the sectors which stems from unification.

Finally, citizens evaluate welfare based on

$$\mathbb{W}_{j,u} = \mathbb{E}\left[\sum_{v\in\{a,i,s\}} \left(y^*_{j,v} + \tfrac{b_v}{2}\right) - \left(\tilde{y}_{j,v} - y^*_{j,v} - \tfrac{b_v}{2}\right)^2 \phi_{j,v}\right] \tag{14}$$

which differs from the welfare function before unification only in that it takes into account the change in the potential of the different sectors of the economy.

Solving for welfare again includes solving for inflation chosen by the central bank. Substitution of all the economy equations into the central bank's utility function and maximization with respect to π_{cb} gives

$$\pi^*_{cb} = \frac{1}{\phi} \sum_{j\in\{1,2\}} \sum_{v\in\{a,i,s\}} e_{j,v}\phi_{j,v}$$

which again has the interpretation of the weighted average of the shocks. Differently to the case before unification the central bank now weights all the shocks in both of the countries. Substitution of all the economy equations and of inflation chosen by the central bank into the citizens' welfare function (14) gives

$$\begin{aligned}\mathbb{W}_{j,u} = &\sum_{v\in\{a,i,s\}} \left(y^*_{j,v} + \tfrac{b_v}{2}\right) - \sum_{v\in\{a,i,s\}} \sigma^2_{j,v}\phi_{j,v} + \frac{\phi + \phi_{-j}}{\phi^2}\Phi'_j V_j \Phi_j \\ &- \frac{\phi_j}{\phi^2}\Phi'_{-j} V_{-j}\Phi_{-j} + 2\frac{\phi_{-j}}{\phi^2}\Phi'_j V \Phi_{-j}\end{aligned}$$

where we use subscript $-j$ to denote the country other than country j. For the unification to be welfare improving for country j we need $\mathbb{W}_{j,u} - \mathbb{W}_j > 0$, which implies

$$\Phi'_j V \Phi_{-j} > \frac{1}{2}\left(\frac{\phi_{-j}}{\phi_j}\Phi'_j V_j \Phi_j + \frac{\phi_j}{\phi_{-j}}\Phi'_{-j} V_{-j}\Phi_{-j} - \frac{\phi^2}{\phi_{-j}} \sum_{v\in\{a,i,s\}} \tfrac{b_v}{2}\right). \tag{15}$$

Condition (15) is the three-sector counterpart of condition (7) derived for the one-sector model, with very similar meaning. For unification to be welfare improving, the covariance of shocks between the two countries has to be sufficiently high. For the three-sector model, the relevant variable is the sum of all the covariances in the V matrix, weighted by the ϕ coefficients. The chances that unification is welfare improving decrease with the variances of shocks in the two countries captured by the V_1 and V_2 matrices and increase with the benefit of unification captured by the b_v variables.

Note also that if we assume that all the variances are equal to unity, i.e. $\sigma_{j,v}^2 = 1$ for $j \in \{1,2\}$ and $v \in \{a,i,s\}$, which is the assumption made by empirical studies using VAR methodology to decompose supply and demand shocks, all the variance-covariance matrices become matrices of correlations. Formally, under assumption $\sigma_{j,v}^2 = 1$ for $j \in \{1,2\}$ and $v \in \{a,i,s\}$

$$V_j = C_j = \begin{bmatrix} 1 & & \\ \rho_{j,ai} & 1 & \\ \rho_{j,as} & \rho_{j,is} & 1 \end{bmatrix} \qquad V = C = \begin{bmatrix} \rho_{aa} & \rho_{ai} & \rho_{as} \\ \rho_{ia} & \rho_{ii} & \rho_{is} \\ \rho_{sa} & \rho_{si} & \rho_{ss} \end{bmatrix}.$$

If we further assume that the ϕ coefficients are equal to the shares of the individual sectors in the production of the relevant economy, $\phi_j = 1$ for $j \in \{1,2\}$ and $\phi = \phi_1 + \phi_2 = 2$ and condition (15) becomes

$$\Phi_j' C \Phi_{-j} > \frac{\Phi_j' C_j \Phi_j + \Phi_{-j}' C_{-j} \Phi_{-j}}{2} - \sum_{v \in \{a,i,s\}} b_v. \tag{16}$$

3 Empirical results

With the model in hand we can now examine the correlations of supply and demand shocks between the individual NMSs and the EMU. We set Germany (*DE*) and the euro area (*EA12*)[4] as benchmark representatives of the EMU. All ten new EU members from Central and Eastern Europe are included in our study. They are: Bulgaria (*BG*), Czech Republic (*CZ*), Estonia (*EE*), Hungary (*HU*), Latvia (*LV*), Lithuania (*LT*), Poland (*PL*), Romania (*RO*), Slovenia (*SL*) and Slovakia (*SK*).

Our assessment of the correlations starts by applying the VAR algorithm to our dataset.[5] We estimate VAR models for the economy as a whole and for the three different sectors (agriculture, industry and services). This approach

[4] Although the euro area now consists of 15 members we use the EA12 instead since our study focuses on the period beginning in 1995. The fact that the time series for the EA13 and EA15 are very short was also a factor in the decision.

[5] See Appendix B for a detailed description of the VAR method, underlying data and data sources.

enables us firstly to compare our results with those described in the meta-analysis by Fidrmuc and Korhonen (2006) (henceforth F/K) and secondly to see what happens when the three different sectors are considered instead of the usual one. The number of lags was set to three in all the VARs according to the average number of lags indicated by the Akaike information criterion.[6] As we use seasonally unadjusted data, we include seasonal dummies in the estimation as well.[7]

From the estimated VARs we retrieved the underlying supply and demand shocks for all the countries and all the sectors. Having the estimated shocks we calculated the correlation coefficients between the sector-specific shocks in a given country and the sector-specific shock in the euro area. The results obtained for the whole economies are shown in Table 1. Table 2 then presents the results obtained for the three-sector economies.

Table 1: Correlation coefficients for the whole economies

	Estimated correlations				Ranks							
	Supply		Demand		Supply		Demand		Average		Reordered	
	EA12	DE	EA12	DE	EA12	DE	EA12	DE		F/K		F/K
BG	-0.20	-0.18	-0.04	-0.02	10	9	4	5	7.0	7.4	9	7
CZ	0.33	0.20	0.03	-0.10	1	4	3	8	4.0	4.3	3	4
EE	0.18	-0.07	-0.07	0.02	4	8	6	4	5.5	4.8	5	5
HU	-0.03	-0.20	0.10	0.17	9	10	2	2	5.75	1.0	6	1
LT	0.00	-0.06	-0.40	-0.30	8	7	10	10	8.75	10.0	10	10
LV	0.21	-0.06	-0.10	-0.03	3	6	7	6	5.5	6.0	5	6
PL	0.17	0.42	-0.27	-0.11	5	1	9	9	6.0	3.0	7	3
RO	0.23	0.35	-0.06	0.09	2	2	5	3	3.0	8.3	2	8
SL	0.06	0.23	0.23	0.17	6	3	1	1	2.75	2.0	1	2
SK	0.02	0.08	-0.17	-0.09	7	5	8	7	6.75	8.4	8	9
DE	0.74		0.87									

[6] We also estimated VARs with two lags because the Schwarz-Bayesian information criterion suggested the number of lags to be less than three. The results obtained under this specification are similar and thus are not reported in this paper.

[7] The reason we use seasonally unadjusted data instead of the readily available seasonally adjusted ones is the fact that each member country performs seasonal adjustment according to a different (albeit similar) methodology, which raises data comparability issues.

Looking firstly at the first four columns of Table 1, one can see the correlation coefficients of the supply and demand shocks between the individual countries and the EA12 and/or Germany. Focusing on Central and Eastern European countries we can see there is a handful of countries with a considerable correlation of supply shocks with the EA12 and Germany – specifically, the Czech Republic (0.33) with the EA12 and (0.20) with Germany, Romania (0.23) with the EA12 and (0.35) with Germany, and Poland with (0.17) with the EA12 and (0.42) with Germany. Taking demand shocks into consideration the majority of countries exhibit a low or negative correlation. This is not an uncommon result and several studies, for example Whitt (1995), Fidrmuc and Korhonen (2003) and Valentinaite and Snieska (2005), identify a similar phenomenon. It can be explained by the fact that demand shocks are frequently policy induced and thus can arise due to fiscal or monetary policy changes.[8] Slovenia and Hungary are the only exceptions, with positive and fairly high correlation of demand shocks. However, it should be noted that the estimation period was somewhat shorter for those countries, which may bias the results.

Turning to Germany it is not surprising that it exhibits high positive correlations with the EA12 for both supply (0.74) and demand (0.87) shocks, since it is the biggest economy of the euro area, with a one-third share in its GDP.

More interesting is a comparison of the supply and demand shock correlations computed for the whole euro area to those computed for Germany. Although for several countries the results are similar, in some cases the pairs of correlations differ considerably, namely Latvia with a high positive correlation (0.21) of supply shocks with the EA12 but a negative correlation (-0.06) with Germany, and Estonia with (0.18) with the EA12 in contrast to (-0.07) with Germany. A possible explanation of these phenomena can be found in different specialization patterns across countries, but correlations with the remaining EA12 countries would be needed to explain this hypothesis more comprehensively.

To compare our results with other studies we use the paper by Fidrmuc and Korhonen (2006). They present a meta-regression analysis based on 35 studies in an effort to control for factors influencing the estimates of the

[8] This explanation leads to a conjecture that the correlation of demand shocks increases after joining the EMU since the European Central Bank and the Stability and Growth Pact impose limits on fiscal and monetary policies, but the data series are too short to verify this hypothesis.

correlations that decrease the comparability of different studies. We use the results of their estimation as a benchmark. In the comparison we refer to the relative rank of the countries instead of focusing on the correlation coefficients because the estimation methods differ significantly from study to study. The 'F/K rank' column presents the average rank of the countries based on F/K's estimation (see their Table 4). Our rank of countries is set as the average of the ranks based on our estimation of the correlations of supply and demand shocks with the EA12 and Germany. The higher the rank the less appropriate is it for a given country to join the EMU.

To facilitate the comparison we re-ordered the countries once again according to their average position in our as well as in F/K's ranks. Those new ranks are presented in the last two columns of Table 1. From the comparison we can see that several countries display stable results and occupy similar positions in our and in F/K's estimates. Spearman's rank correlation (0.48) confirms this conclusion, but it has to be noted that F/K identify at least six papers that do not follow this pattern.[9]

Table 2: Correlation coefficients for the three-sector economies

	Estimated correlations				Ranks							
	Supply		Demand		Supply		Demand		Average		Reordered	
	EA12	DE	EA12	DE	EA12	DE	EA12	DE		F/K		F/K
BG	0.05	0.10	0.09	0.05	7	4	2	2	3.75	7.4	2	7
CZ	0.01	0.07	0.01	-0.05	9	6	5	5	6.25	4.3	7	4
EE	0.19	0.12	0.01	-0.09	2	3	6	7	4.5	4.8	4	5
HU	0.11	0.07	0.05	0.03	6	5	3	3	4.25	1.0	3	1
LT	0.05	-0.03	-0.23	-0.26	8	9	10	10	9.25	10.0	10	10
LV	0.17	0.06	0.02	-0.03	4	7	4	4	4.75	6.0	5	6
PL	0.18	0.27	-0.09	-0.23	3	2	9	9	5.75	3.0	6	3
RO	0.14	0.06	-0.04	-0.06	5	8	7	6	6.5	8.3	8	8
SL	0.27	0.34	0.17	0.14	1	1	1	1	1.0	2.0	1	2
SK	-0.08	-0.12	-0.06	-0.10	10	10	8	8	9.0	8.4	9	9
DE	0.48		0.37									

[9] Horvath (2000), Horvath and Ratfai (2004), Fidrmuc and Hagara (2004), Korhonen (2003), IMF (2000), EFN (2003).

Turning our attention to Table 2 we can assess what happens with the results when the individual economies are divided into three different sectors. The first four columns present the estimates of the left-hand side of equation (16). Generally speaking the numbers are weighted sums of the between-country sector-specific shock correlations between the individual countries and the EA12 and Germany. The country-specific weights are the shares of the individual sectors in the production of the whole economy.

For most of the countries the three-sector correlations change only slightly in comparison to the previous results, but some exceptions can be found. For the Czech Republic the correlations of supply shocks decrease from (0.33) to (0.01) for the EA12 and from (0.20) to (0.07) for Germany. Also for Romania the supply shocks correlations decrease considerably from (0.23) to (0.14) for the EA12 and from (0.35) to (0.06) for Germany. This finding is consistent with F/K, who report the Czech Republic and Romania (together with Bulgaria) as countries with relatively low average correlations. Slovenia and Hungary are countries for which the supply shock correlations noticeably increase in comparison to the whole-economy estimates. This again brings our results closer to the results of F/K. This conclusion is confirmed by Spearman's rank correlation, which increases from (0.48) to (0.70) when our rank based on three-sector estimates and the rank based on F/K's estimate are compared.

Finally, looking at the correlation of supply and demand shocks for Germany two things are worth noticing. Firstly, Germany again exhibits the largest correlations with the EA12 among the countries considered. Secondly, those correlations are somewhat smaller than for the whole economies. This is due to the fact that the three-sector condition for monetary unification to be welfare improving is more demanding in that it also takes into account correlations between different sectors. Viewing our results in this light, the three-sector results in general point in the direction of unification, as the correlations for the NMSs are closer to the correlations for Germany.

To check the robustness of our results we repeated the whole exercise after dropping several observations either from the beginning or from the end of our dataset.[10] The results do not change considerably when the observations from the beginning of the dataset are dropped. Both the results for the whole economies and the results for the three-sector economies exhibit high correlations with the original estimates. When the observations from the

[10] The results of the robustness exercise are available on request.

end of the dataset are dropped the results for the whole economies become rather volatile and the correlations with the original results decrease. The three-sector estimates remain stable regardless of the missing observations. For the three-sector economies the average correlations of the 'new' and the original results fluctuate between 0.7 and 0.8 for all the estimates.

Lastly, while we focus on the term involving the correlations of shocks between countries in condition (16), we also estimated the remaining two terms in (16), which involve the correlations of shocks within countries. As this additional exercise gives very similar results regarding the relative ranking of the countries compared to the results above, we do not include details of it.

4 Conclusion

This paper assessed the issue of EMU enlargement towards Central and Eastern European countries from the shock asymmetry perspective. In the first part we developed a model which views monetary unification as a trade-off between lower transaction costs and loss of independent monetary policy. Moreover, it views monetary unification as a citizens' decision based on the maximization of their welfare function. We have shown that for monetary unification to be welfare improving, the covariance of the shocks between the two countries has to be sufficiently high. There is no clear cut point defining what the term 'sufficiently high' means because the benefits of unification cannot be explicitly measured. But the higher the covariance the lower the benefits have to be. This condition can be transformed into a condition involving the correlation of shocks when an assumption of unit variance of shocks is made. Later transformation then justifies the widespread focus of empirical studies on the correlation of supply and demand shocks.

An objection to the said condition is that economies are described by aggregates. In reality each economy consists of several sectors which can be hit by different types of shocks. In an effort to make our model more realistic we split each economy into three sectors and reformulate the condition under this specification.

In the empirical part of our study we use both conditions to evaluate the appropriateness of Central and Eastern European countries joining the EMU. Applying the one-sector condition is the usual exercise in empirical studies using the VAR algorithm. The three-sector algorithm is partly similar, but the condition for welfare-improving monetary unification is estimated as a weighted average of the sector-specific correlations.

Although the algorithms differ only slightly, some important changes can be found regarding the results. According to the one-sector condition the Czech Republic and Romania belong to the group of countries with a considerably high correlation of supply shocks. This finding is confirmed neither by other authors nor by the three-sector method. The opposite holds true for Poland and Hungary, which occupy bad positions in our one-sector rank of countries but considerably better positions according to F/K's estimates and also according to our three-sector estimates. In general we can say that our three-sector estimates follow the same pattern as those of F/K, with Bulgaria being the only exception.

In conclusion, two findings make us believe in the virtue of the three-sector methodology. Firstly, the estimates for the three-sector economies are closer to the estimates of F/K compared to the estimates for the whole economies. This and the fact that the F/K results are based on more than a decade of research into the topic raises our hopes. Secondly, our robustness exercise suggests that the three-sector estimates are less sensitive to the estimation period and the length of the data used for the estimation. This is especially important for those countries for which relevant data has become available only recently.

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A Appendix - Model Robustness

This appendix further explores the robustness of our model in two ways. Firstly, we re-derive results from the main part of the paper for an arbitrary number of sectors. Secondly, we specify an alternative structure of the economy, an alternative central bank utility function and an alternative citizens' welfare function and re-derive the inflation chosen by the central bank, the resulting citizens' welfare and the condition for monetary unification to be profitable.

To keep matters simple, we switch to vector notation. Hence, from now on all the variables should be understood as $[n \times 1]$ vectors with a general n^{th} element corresponding to the n^{th} sector of the economy, except in obvious cases. Furthermore, we denote by ι $[n \times 1]$ the vector of ones. The last piece of notation we use is an $[n \times n]$ matrix with sector weights on the main diagonal

$$\phi_j^d = \begin{bmatrix} \phi_{j,1} & 0 & \cdots & \\ 0 & \phi_{j,2} & \ddots & \\ \vdots & \ddots & \ddots & \\ & & & \phi_{j,n} \end{bmatrix}.$$

Notice that with this notation $\phi_j^d \iota = \Phi_j$ and $\iota' \phi_j^d \iota = \phi_j$.

The structure of the economy postulated in the main part of the paper in vector notation is (here and below, the first equation is before and second one after unification)

$$\begin{aligned} y_j &= y_j^* + \pi_{cb,j}\iota - e_j \\ \tilde{y}_j &= y_j^* + \tfrac{b}{2} + \pi_{cb}\iota - e_j. \end{aligned} \tag{EC1}$$

The central bank utility function used in the main part of the paper in vector notation is

$$
\begin{aligned}
\mathbb{W}_{cb,j} &= y_j'^{*}\iota - (y_j - y_j^*)'\phi_j^d(y_j - y_j^*) \\
\mathbb{W}_{cb} &= \sum_{j\in\{1,2\}} y_j'^{*}\iota + b'\iota - \sum_{j\in\{1,2\}} \left(\tilde{y}_j - y_j^* - \tfrac{b}{2}\right)' \phi_j^d \left(\tilde{y}_j - y_j^* - \tfrac{b}{2}\right).
\end{aligned}
\tag{CB1}
$$

Finally, the welfare of the citizens in country j from the main part of the paper in vector notation is

$$
\begin{aligned}
\mathbb{W}_j &= \mathbb{E}[y_j'^{*}\iota - (y_j - y_j^*)'\phi_j^d(y_j - y_j^*)] \\
\mathbb{W}_{j,u} &= \mathbb{E}\left[y_j'^{*}\iota + \tfrac{b}{2}'\iota - \left(\tilde{y}_j - y_j^* - \tfrac{b}{2}\right)' \phi_j^d \left(\tilde{y}_j - y_j^* - \tfrac{b}{2}\right)\right].
\end{aligned}
\tag{W1}
$$

To solve the model, we first need to derive the optimal inflation set by the central bank. Substitution of the economy equations (EC1) into the appropriate central bank utility functions (CB1) and straightforward maximization gives the central bank's optimal inflation[11]

$$
\begin{aligned}
\pi_{cb,j}^* &= \frac{1}{\phi_j} e_j' \Phi_j \\
\pi_{cb}^* &= \frac{1}{\phi} \sum_{j\in\{1,2\}} e_j' \Phi_j,
\end{aligned}
$$

which are n-sector vector counterparts of the expressions given in the main part of the paper.

Finally, substituting the economy equations (EC1) into the appropriate citizens' welfare functions (W1) and using the inflation just derived gives the welfare for country j

$$
\begin{aligned}
\mathbb{W}_j &= y_j'^{*}\iota - \sum_{v\in\{a,i,s\}} \sigma_{j,v}^2 \phi_{j,v} + \frac{1}{\phi_j} \Phi_j' V_j \Phi_j \\
\mathbb{W}_{j,u} &= y_j'^{*}\iota + \tfrac{b}{2}'\iota - \sum_{v\in\{a,i,s\}} \sigma_{j,v}^2 \phi_{j,v} + \frac{\phi + \phi_{-j}}{\phi^2} \Phi_j' V_j \Phi_j \\
&\quad - \frac{\phi_j}{\phi^2} \Phi_{-j}' V_{-j} \Phi_{-j} + 2\frac{\phi_{-j}}{\phi^2} \Phi_j' V \Phi_{-j}.
\end{aligned}
$$

[11] It is easy to confirm that the S.O.C.s for maximization hold for the problem at hand.

The condition for monetary unification to be welfare improving for country j's citizens is then $\mathbb{W}_{j,u} - \mathbb{W}_j > 0$, which can be rewritten as

$$\Phi_j' V \Phi_{-j} > \frac{1}{2}\left(\frac{\phi_{-j}}{\phi_j}\Phi_j' V_j \Phi_j + \frac{\phi_j}{\phi_{-j}}\Phi_{-j}' V_{-j}\Phi_{-j} - \frac{\phi^2}{\phi_{-j}}\frac{b}{2}'\iota\right). \tag{A1}$$

Using the assumptions discussed in the text this simplifies to

$$\Phi_j' C \Phi_{-j} > \frac{\Phi_j' C_j \Phi_j + \Phi_{-j}' C_{-j}\Phi_{-j}}{2} - b'\iota. \tag{A2}$$

As an alternative specification of our model we use economy equations of the form

$$\begin{aligned} y_j &= y_j^* + \pi_{cb,j}\iota - \pi_j^e\iota - e_j \\ \tilde{y}_j &= y_j^* + \tfrac{b}{2} + \pi_{cb}\iota - \pi_{j,u}^e\iota - e_j, \end{aligned} \tag{EC2}$$

where π_j^e and $\pi_{j,u}^e$ denote inflation expected by the citizens in country j before and after unification respectively. The alternative central bank utility functions are

$$\begin{aligned} \mathbb{W}_{cb,j} &= y_j'^*\iota - (y_j - y_j^*)'\phi_j^d(y_j - y_j^*) - (\pi_{cb,j} - \pi_j^*)^2 \\ \mathbb{W}_{cb} &= \sum_{j\in\{1,2\}} y_j'^*\iota + b'\iota - \sum_{j\in\{1,2\}} \tfrac{1}{2}\left(\tilde{y}_j - y_j^* - \tfrac{b}{2}\right)'\phi_j^d\left(\tilde{y}_j - y_j^* - \tfrac{b}{2}\right) \\ &\quad - (\pi_{cb} - \pi^*)^2 \end{aligned} \tag{CB2}$$

where π_j^* and π^* are the central bank's inflation targets before and after unification. Lastly, we modify the citizens' welfare function into

$$\begin{aligned} \mathbb{W}_j &= \mathbb{E}[y_j'^*\iota - (y_j - y_j^*)'\phi_j^d(y_j - y_j^*) - (\pi_{cb,j} - \pi_j^e)^2] \\ \mathbb{W}_{j,u} &= \mathbb{E}\left[\begin{array}{l} y_j'^*\iota + \frac{b}{2}'\iota - \left(\tilde{y}_j - y_j^* - \frac{b}{2}\right)'\phi_j^d\left(\tilde{y}_j - y_j^* - \frac{b}{2}\right) \\ -(\pi_{cb} - \pi_{j,u}^e)^2 \end{array}\right]. \end{aligned} \tag{W2}$$

The rationale for all three alternatives is the following. The alternative economy structure specified in (EC2) is a standard expectations-augmented Phillips equation. The alternative central bank utility function (CB2) is again a more conventional one in that the central bank cares about both output and inflation. A monetary authority with a similar utility function is standard in the literature dealing with the dynamic inconsistency of low-inflation monetary policy. The reason for multiplying the deviations of the economy from its potential by $\frac{1}{2}$ is to achieve a specification which gives equal weight to the economy's fluctuations as to the deviations of inflation from its target.

Lastly, the alternative citizens' welfare function (W2) incorporates the notion that citizens care not only about the level and stability of output, but also about the correctness with which they predict inflation. The rationale for this may be that citizens engage themselves in nominal (wage) contracts which are not continuously renegotiable. In this respect, (W2) can be regarded as a more general version of the welfare function used in the main part of the paper.

To solve our alternative model consisting of (EC2), (CB2) and (W2) one substitutes the economy equations into the appropriate central bank utility functions. The resulting F.O.C.s are

$$\begin{aligned}\pi_{cb,j} &= \frac{1}{1+\phi_j}(\pi_j^* + \pi_j^e\phi_j + e_j'\Phi_j)\\ \pi_{cb} &= \frac{1}{2+\phi}(2\pi^* + \sum_{j\in\{1,2\}} \pi_{j,u}^e\phi_j + e_j'\Phi_j).\end{aligned}$$

The law of iterated expectations implies $\mathbb{E}[\pi_{cb,j}] = \pi_j^e$ and $\mathbb{E}[\pi_{cb}] = \pi_{j,u}^e$ for $j \in \{1,2\}$. Furthermore, since the citizens of both countries have the same information sets $\mathbb{E}[\pi_{-j,u}^e] = \pi_{j,u}^e$ for $j \in \{1,2\}$. Using this we can derive expected inflation

$$\begin{aligned}\pi_j^e &= \pi_j^*\\ \pi_{j,u}^e &= \pi^*\end{aligned}$$

which substituted back into the central bank's F.O.C.s gives

$$\begin{aligned}\pi_{cb,j}^* &= \pi_j^* + \frac{1}{1+\phi_j}e_j'\Phi_j\\ \pi_{cb}^* &= \pi^* + \frac{1}{2+\phi}\sum_{j\in\{1,2\}} e_j'\Phi_j.\end{aligned}$$

Finally, substituting economy equation (EC2) into the appropriate welfare functions (W2) using the inflation set by the central bank and expected inflation gives welfare

$$\mathbb{W}_j = y_j'^{*}\iota - \sum_{v\in\{a,i,s\}} \sigma_{j,v}^2\phi_{j,v} + \frac{1}{1+\phi_j}\Phi_j' V_j \Phi_j$$

$$\mathbb{W}_{j,u} = y_j'^{*}\iota + \tfrac{b}{2}'\iota - \sum_{v\in\{a,i,s\}} \sigma_{j,v}^2\phi_{j,v} + \frac{3+\phi+\phi_{-j}}{(2+\phi)^2}\Phi_j' V_j \Phi_j$$
$$- \frac{1+\phi_j}{(2+\phi)^2}\Phi_{-j}' V_{-j}\Phi_{-j} + 2\frac{1+\phi_{-j}}{(2+\phi)^2}\Phi_j' V \Phi_{-j}.$$

Again, for monetary unification to be welfare improving for country j's citizens we need $\mathbb{W}_{j,u} - \mathbb{W}_j > 0$, which can be rewritten as

$$\Phi_j' V \Phi_{-j} > \frac{1}{2}\left(\frac{1+\phi_{-j}}{1+\phi_j}\Phi_j' V_j \Phi_j + \frac{1+\phi_j}{1+\phi_{-j}}\Phi_{-j}' V_{-j}\Phi_{-j} - \frac{(2+\phi)^2}{1+\phi_{-j}}\tfrac{b}{2}'\iota\right). \quad \text{(A3)}$$

Using the assumptions discussed in the text this simplifies to

$$\Phi_j' C \Phi_{-j} > \frac{\Phi_j' C_j \Phi_j + \Phi_{-j}' C_{-j}\Phi_{-j}}{2} - 2b'\iota. \quad \text{(A4)}$$

Comparing the condition derived from the main model (A1) with the condition derived using the alternative one (A3) reveals that the logic of the two equations is very similar. For unification to be welfare improving, the variance of shocks in both countries has to be sufficiently small. The relevant statistic in this respect is again the variance-covariance matrix of the shocks in the given countries multiplied by the vectors of the weights of the different sectors in the central bank's utility function. Furthermore, the chances that unification is welfare improving increases with the size of the benefits of unification.

Turning to the conditions derived using the assumptions discussed in the text, expressions (A2) and (A4) are almost identical, except for the last term, which is multiplied by 2 in the latter condition.

B Appendix - Data and Empirical Strategy

This appendix outlines the details of our empirical strategy from section 3 and the data we use.

Empirical Strategy

We follow the method used for example in Bayoumi and Eichengreen (1992), Fidrmuc and Korhonen (2003) and Horvath and Ratfai (2004), which decomposes output and inflation shocks into demand and supply shocks.[12] This method starts with a 2-equation p-lag VAR model of the form

$$x_t = B_1 x_{t-1} + \cdots + B_p x_{t-p} + e_t$$

where $x_t = [y_t \; p_t]'$ is a vector of output growth and inflation observations, Bs are $[2 \times 2]$ matrices of the estimated coefficients and $e_t = [e_t^y \; e_t^p]'$ is the estimated vector of output and inflation shocks. Furthermore, estimation of the model gives the variance-covariance matrix of the estimated shocks Ω.

The objective of the exercise is to derive the vector of demand and supply shocks $\xi_t = [\xi_t^d \; \xi_t^s]'$ from e_t. Assuming that the output and inflation shocks are a linear combination of the demand and supply shocks, we get $e_t^y = c_{11}\xi_t^d + c_{12}\xi_t^s$ and $e_t^p = c_{21}\xi_t^d + c_{22}\xi_t^s$ and using matrix notation

$$e_t = \begin{bmatrix} e_t^y \\ e_t^p \end{bmatrix} \qquad \xi_t = \begin{bmatrix} \xi_t^d \\ \xi_t^s \end{bmatrix} \qquad C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

this implies $e_t = C\xi_t$. We need four constraints to pin down the C matrix in order to derive the demand and supply shocks from $\xi_t = C^{-1}e_t$, assuming C is non-singular.

Denoting by σ_d^2 the variance of demand shocks, by σ_s^2 the variance of supply shocks and by σ_{ds} their covariance, first set of constraints is

$$\begin{aligned} A1 &: \sigma_d^2 = \sigma_s^2 = 1 \\ A2 &: \sigma_{ds} = 0. \end{aligned}$$

Using this and the fact that $\Omega = \mathbb{E}[e_t e_t'] = \mathbb{E}[C\xi_t\xi_t'C'] = C\Delta C'$, where Δ is the variance-covariance matrix of the demand and supply shocks and Ω is known, this becomes

$$\Omega = \begin{bmatrix} \sigma_y^2 & \sigma_{yp} \\ \sigma_{yp} & \sigma_p^2 \end{bmatrix} = \begin{bmatrix} c_{11}^2 + c_{12}^2 & c_{11}c_{21} + c_{12}c_{22} \\ c_{11}c_{21} + c_{12}c_{22} & c_{21}^2 + c_{22}^2 \end{bmatrix},$$

where σ_y^2 is the estimated variance of shocks from the output equation, σ_p^2 is the estimated variance of shocks from the inflation equation and σ_{yp} is their covariance.

[12] Bayoumi and Eichengreen (1992) and subsequent papers all use the method originally developed by Blanchard and Quah (1989).

The last constraint comes from the economic theory and states that the demand shock has no long-term impact on output. Bringing all the terms involving x in the VAR equation above to the LHS and using the lag operator we get

$$x_t - B_1 L x_t - \cdots - B_p L^p x_t = e_t.$$

Factoring out x_t from the LHS we get

$$B(L) x_t = e_t$$

where

$$B(L) = \begin{bmatrix} 1 - b_{11}(1) - \cdots - b_{11}(p) & -b_{12}(1) - \cdots - b_{12}(p) \\ -b_{21}(1) - \cdots - b_{21}(p) & 1 - b_{22}(1) - \cdots - b_{22}(p) \end{bmatrix}$$

where $b_{ij}(k)$ is the element from the i-th row and j-th column of matrix B_k.

Since $e_t = C\xi_t$ we get

$$x_t = B(L)^{-1} C \xi_t$$

and because matrix $B(L)$ is polynomial with infinite lags, for our last constraint to hold we need $[B(L)^{-1}C]_{11} = 0$. Inverting $B(L)$ gives

$$B(L)^{-1} = \begin{bmatrix} 1 - b_{22}(1) - \cdots - b_{22}(p) & b_{12}(1) + \cdots + b_{12}(p) \\ b_{21}(1) + \cdots + b_{21}(p) & 1 - b_{11}(1) - \cdots - b_{11}(p) \end{bmatrix} \frac{1}{|B(L)|}$$

where $|B(L)|$ denotes the determinant of $B(L)$. Now $[B(L)^{-1}C]_{11} = 0$ implies

$$[B(L)^{-1}C]_{11} = 0 = c_{11}[1 - b_{22}(1) - \cdots - b_{22}(p)] + c_{21}[b_{12}(1) + \cdots + b_{12}(p)]$$

or simply

$$[B(L)^{-1}C]_{11} = 0 = c_{11} a + c_{21} b$$

where $a = [1 - b_{22}(1) - \cdots - b_{22}(p)]$ and $b = [b_{12}(1) + \cdots + b_{12}(p)]$.

In summary, we get four equations for four unknowns

$$\begin{aligned} c_{11}^2 + c_{12}^2 &= \sigma_y^2 \\ c_{21}^2 + c_{22}^2 &= \sigma_p^2 \\ c_{11}c_{21} + c_{12}c_{22} &= \sigma_{yp} \\ c_{11}a + c_{21}b &= 0. \end{aligned}$$

Solving this system of equations one gets

$$c_{11} = \nu_{11} \left(b\sqrt{\frac{\sigma_y^2\sigma_p^2 - \sigma_{yp}^2}{D}} \right) \quad c_{12} = \nu_{12}(a\sigma_y^2 + b\sigma_{yp})\sqrt{\frac{1}{D}}$$

$$c_{21} = \nu_{21} \left(a\sqrt{\frac{\sigma_y^2\sigma_p^2 - \sigma_{yp}^2}{D}} \right) \quad c_{22} = \nu_{22}(a\sigma_{yp} + b\sigma_p^2)\sqrt{\frac{1}{D}}$$

with $D = a^2\sigma_y^2 + b^2\sigma_p^2 + 2ab\sigma_{yp}$ and all the ν's equal to unity in absolute value. It is easy to check that this system has four solutions depending on the signs of the ν's

	I	II	III	IV
ν_{11}	+	−	+	−
ν_{12}	+	+	−	−
ν_{21}	−	+	−	+
ν_{22}	+	+	−	−

where in our computations we use the solution from the first column.[13]

Lastly, in section 3, where we estimate the correlation of demand and supply shocks treating each country as a collection of three sectors, we estimate three separate VAR models, one for each sector, and derive three sets of demand and supply shocks. When we treat each country as a one-sector one, we simply estimate the single VAR model.

Data

The data we use come from the Eurostat quarterly National Accounts database. To obtain the data for the three sectors, we use the NACE-06 division, which includes information about gross value added in constant prices and price indices for the following branches[14]

[13] As all the sign combinations satisfy the system of equations, this does not alter our results. Choosing different columns results in the same derived demand and supply shocks which differ in sign. Hence using consistently only one of the sign combinations has no impact on the calculated correlation coefficients.

[14] More specifically we use the 'MIO_EUR_CLV2000' variable from the namq_nace06_k.tsv database file downloaded from the Eurostat webpage and 'PCH_PRE_CPI00_EUR' from the namq_nace06_p.tsv file.

I	Agriculture, hunting, forestry and fishing
II	Total industry (excluding construction)
III	Construction
IV	Wholesale and retail trade, repair of motor vehicles, motorcycles and personal and household goods; hotels and restaurants; transport, storage and communication
V	Financial intermediation; real estate, renting and business activities
VI	Public administration and defence, compulsory social security; education; health and social work; other community, social and personal service activities; private households with employed persons

For our purpose, we define branch I as agriculture, branches II and III as industry and branches IV and V as services. We omit the last branch due to the fact that the development of output and prices is in our view given more by political than economic factors.

The output data in constant prices come in volumes and for the estimation we use the percentage changes relative to the previous quarter. For the three sectors, we simply sum the underlying volume data. For the country as a whole, again we simply sum the data for the three sectors.

The price indices data come already in percentage changes relative to the previous quarter. For each sector, we use the weighted average of the relevant branch price changes, where the quarter-specific weights used are calculated from the output data as a share of each branch in the output of the given sector. We use a similar strategy when calculating inflation for the country as a whole.

Having the data about output changes and inflation, we estimate VAR models for the three different sectors in a given country and its whole economy and derive the demand and supply shocks using the approach outlined in the previous section. As we use seasonally unadjusted data, we include seasonal dummies in the estimation. The period we focus on runs from 1995q1 to 2008q1, although in practice the data for some countries are somewhat shorter but always start no later than 2000q1. Obviously, due to the fact that we use percentage changes for the estimation, we lose one quarter at the beginning of the dataset. Table 3 shows the summary statistics of the data we use.

Finally, having the estimated demand and supply shocks we can calculate the matrix of correlation coefficients between the sector-specific shocks in a given country and the sector-specific shock in the euro area. This is matrix C in (16). To estimate the vector of weights Φ_j, we calculate the average share of each of the three sectors in the overall output in country j in our data. This allows us to calculate the LHS of (16) for a given country for both demand and supply shocks.

Table 3: Summary statistics

var	mean	st.d.	min	max	N	mean	st.d.	min	max	N
			EA12					CZ - Czech Republic		
ya	0.018	0.173	-0.33	0.22	52	0.142	0.603	-0.61	1.53	48
yi	0.006	0.049	-0.07	0.09	52	0.013	0.068	-0.13	0.13	48
ys	0.007	0.022	-0.04	0.04	52	0.013	0.073	-0.13	0.14	48
y	0.007	0.033	-0.06	0.06	52	0.011	0.052	-0.07	0.11	48
pa	0.001	0.026	-0.05	0.05	52	0.021	0.098	-0.22	0.28	48
pi	0.003	0.007	-0.02	0.02	52	0.014	0.031	-0.06	0.08	48
ps	0.004	0.005	-0.01	0.01	52	0.017	0.033	-0.09	0.08	48
p	0.004	0.004	-0.01	0.01	52	0.015	0.027	-0.07	0.08	48
			BG - Bulgaria					EE - Estonia		
ya	0.219	0.718	-0.67	1.39	52	0.020	0.144	-0.21	0.37	52
yi	0.011	0.091	-0.18	0.20	52	0.020	0.077	-0.15	0.16	52
ys	0.018	0.146	-0.34	0.23	52	0.021	0.039	-0.04	0.13	52
y	0.022	0.181	-0.29	0.30	52	0.020	0.046	-0.06	0.14	52
pa	0.020	0.159	-0.40	0.73	52	0.018	0.100	-0.13	0.36	52
pi	0.021	0.091	-0.20	0.47	52	0.015	0.033	-0.04	0.12	52
ps	0.023	0.087	-0.27	0.27	52	0.021	0.046	-0.07	0.19	52
p	0.019	0.076	-0.25	0.41	52	0.018	0.037	-0.06	0.15	52
			HU - Hungary					LV - Latvia		
ya	0.059	0.312	-0.34	0.57	32	0.060	0.327	-0.48	0.55	52
yi	0.016	0.112	-0.21	0.18	32	0.020	0.099	-0.14	0.27	52
ys	0.018	0.120	-0.20	0.15	32	0.024	0.057	-0.08	0.16	52
y	0.017	0.117	-0.19	0.17	32	0.020	0.024	-0.04	0.08	52
pa	0.040	0.300	-0.41	0.91	32	0.014	0.118	-0.37	0.24	52
pi	0.017	0.043	-0.09	0.10	32	0.021	0.072	-0.12	0.16	52
ps	0.014	0.032	-0.05	0.10	32	0.027	0.120	-0.22	0.27	52
p	0.016	0.033	-0.04	0.09	32	0.022	0.095	-0.17	0.22	52

y∗ - gross value added quarterly percentage change
p∗ - price index quarterly percentage change
a - agriculture, i - industry, s - services, whole economy

Table 3: Summary statistics

var	mean	st.d.	min	max	N	mean	st.d.	min	max	N
	LT - Lithuania					PL - Poland				
ya	0.202	0.67	-0.67	1.51	52	0.031	0.234	-0.32	0.56	52
yi	0.020	0.092	-0.18	0.15	52	0.025	0.146	-0.25	0.27	52
ys	0.021	0.068	-0.14	0.19	52	0.016	0.078	-0.11	0.16	52
y	0.022	0.105	-0.15	0.23	52	0.017	0.091	-0.16	0.11	52
pa	0.057	0.271	-0.37	0.73	52	0.014	0.123	-0.27	0.24	52
pi	0.021	0.042	-0.09	0.14	52	0.007	0.046	-0.10	0.08	52
ps	0.023	0.047	-0.09	0.12	52	0.015	0.048	-0.09	0.14	52
p	0.021	0.051	-0.09	0.15	52	0.012	0.046	-0.082	0.12	52
	RO - Romania					SL - Slovenia				
ya	0.742	1.290	-0.90	2.45	32	0.008	0.171	-0.26	0.31	32
yi	0.044	0.229	-0.35	0.27	32	0.014	0.048	-0.04	0.14	32
ys	0.026	0.125	-0.25	0.21	32	0.012	0.035	-0.05	0.06	32
y	0.051	0.248	-0.37	0.31	32	0.012	0.041	-0.05	0.10	32
pa	0.055	0.264	-0.35	0.75	32	0.010	0.086	-0.22	0.25	32
pi	0.027	0.045	-0.05	0.12	32	0.003	0.009	-0.02	0.03	32
ps	0.020	0.082	-0.14	0.19	32	0.008	0.029	-0.06	0.06	32
p	0.022	0.053	-0.08	0.17	32	0.006	0.017	-0.03	0.03	32
	SK - Slovakia					DE - Germany				
ya	0.054	0.317	-0.54	0.72	52	0.005	0.071	-0.20	0.25	52
yi	0.020	0.061	-0.17	0.23	52	0.005	0.040	-0.10	0.11	52
ys	0.016	0.118	-0.21	0.40	52	0.006	0.025	-0.06	0.04	52
y	0.016	0.070	-0.16	0.16	52	0.005	0.030	-0.07	0.07	52
pa	0.009	0.067	-0.16	0.18	52	-0.002	0.051	-0.16	0.16	52
pi	0.010	0.035	-0.10	0.09	52	0.000	0.014	-0.04	0.03	52
ps	0.021	0.059	-0.13	0.17	52	0.001	0.009	-0.02	0.02	52
p	0.014	0.034	-0.05	0.10	52	0.000	0.007	-0.02	0.02	52

y∗ - gross value added quarterly percentage change
p∗ - price index quarterly percentage change
a - agriculture, i - industry, s - services, whole economy

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Univerzita Karlova v Praze, Fakulta sociálních věd
Institut ekonomických studií [UK FSV – IES] Praha 1, Opletalova 26
E-mail : ies@fsv.cuni.cz http://ies.fsv.cuni.cz