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Analysis of Interdependencies among Central European Stock Markets

Master Thesis

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Declaration of Authorship

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Prague, May 17, 2011 Jana Mašková

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Abstract

The objective of the thesis is to examine interdependencies among the stock markets of the Czech Republic, Hungary, Poland and Germany in the period 2008-2010. Two main methods are applied in the analysis. The first method is based on the use of high-frequency data and consists in the computation of realized correlations, which are then modeled using the heterogeneous autoregressive (HAR) model. In addition, we employ realized bipower correlations, which should be robust to the presence of jumps in prices. The second method involves modeling of correlations by means of the Dynamic Conditional Correlation GARCH (DCC-GARCH) model, which is applied to daily data. The results indicate that when high-frequency data are used, the correlations are biased towards zero (the so-called "Epps effect"). We also find quite significant differences between the dynamics of the correlations from the DCC-GARCH models and those of the realized correlations. Finally, we show that accuracy of the forecasts of correlations can be improved by combining results obtained from different models (HAR models for realized correlations, HAR models for realized bipower correlations, DCC-GARCH models).

Keywords

Central Europe, stock markets, realized correlation, realized bipower correlation, high-frequency data, heterogeneous autoregressive model, DCC-GARCH model

Abstrakt

Cílem této diplomové práce je prozkoumání závislostí mezi akciovými trhy České republiky, Maďarska, Polska a Německa v období 2008-2010. V analýze jsou aplikovány dvě hlavní metody. První metoda je založena na využití vysokofrekvenčních dat a spočívá ve výpočtu realizovaných korelací a jejich následném modelování pomocí heterogenního autoregresního (HAR) modelu. Kromě toho používáme též realizované bipower korelace, které by neměly být ovlivněny přítomností skoků v cenách. Druhou metodou je modelování korelací pomocí Dynamic Conditional Correlation GARCH (DCC-GARCH) modelu, který aplikujeme na denní data. Výsledky ukazují, že při použití vysokofrekvenčních dat jsou korelace vychýleny směrem k nule (tzv. Epps efekt). Rovněž nacházíme poměrně významné rozdíly mezi dynamikou korelací z DCC-GARCH modelů a realizovaných korelací. Na závěr zjišťujeme, že pro dosažení přesnějších předpovědí korelací je vhodné kombinovat výsledky získané z různých zkoumaných modelů (HAR modely pro realizované korelace, HAR modely pro realizované bipower korelace, DCC-GARCH modely).

Klíčová slova

střední Evropa, akciové trhy, realizovaná korelace, realizovaná bipower korelace, vysokofrekvenční data, heterogenní autoregresní model, DCC-GARCH model

Master Thesis Proposal

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Proposed Topic:

Analysis of Interdependencies among Central European Stock Markets

Topic Characteristics:

The aim of my thesis will be to examine the dependence structure of stock markets in Central Europe. To do this, I would like to apply the Dynamic Conditional Correlation GARCH model, which enables to estimate how the correlation structure of data evolves in time. Therefore, we can explore whether there was any trend in the correlations among markets and whether the correlation structure was subject to any abrupt changes. Of particular interest will be the response of correlations to the financial crisis that started in 2007. I would like to find out if the outbreak of the crisis was followed by any statistically significant increase in correlation coefficients, thus indicating financial contagion. My main data sources will be the statistics published by Central European stock exchanges.

Hypotheses:

1. Correlations of returns in Central European stock markets evolve in time.
2. There has been an upward trend in the correlations among Central European stock markets.
3. The pattern of correlations among Central European stock markets has been significantly influenced by the current economic crisis.

Methodology:

The main method used in the thesis will be the econometric model known as the Dynamic Conditional Correlation GARCH model. It is a relatively new model proposed by Robert Engle in 2001-02 and it can be used to estimate the time-dependent conditional correlation matrix of several time series. The model is estimated in two steps. In the first step, we estimate the conditional variances of the time series residuals. In the second step, we standardise the residuals by their estimated standard deviations and then we use the standardised residuals to estimate the coefficients of the correlation process. This two-step procedure produces consistent maximum likelihood parameter estimates. The model can be also modified by including a trend coefficient. Having estimated the DCC-GARCH model, we can test for differences in correlation coefficients between two different time periods.

Outline:

1. Introduction
2. Description of the Dynamic Conditional Correlation GARCH Models
3. Central European Stock Markets
    3.1. Characteristics of the Markets
    3.2. Review of Literature on Linkages Among the Markets
4. Data and Estimation of the Model
5. Results and Discussion
6. Conclusion

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Supervisor

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List of Abbreviations

ARCH	Autoregressive conditional heteroscedasticity
BUX	BUX index of the Budapest Stock Exchange
CE-3	Czech Republic, Hungary, Poland
CEE	Central and Eastern Europe
DAX	DAX index of Deutsche Börse
DCC	Dynamic conditional correlation
EU	European Union
GARCH	Generalized autoregressive conditional heteroscedasticity
HAR (model)	Heterogeneous autoregressive
OLS	Ordinary least squares
PX	PX index of the Prague Stock Exchange
RBPC	Realized bipower correlation
RBPCOV	Realized bipower covariance
RBPV	Realized bipower variance
RC	Realized correlation
RCOV	Realized covariance
RV	Realized variance
RVOL	Realized volatility
U.S.	United States of America
WIG20/WIG	WIG20 index of the Warsaw Stock Exchange

1. Introduction

One of the key problems in financial econometrics is the estimation, modeling and forecasting of volatility and correlations of asset returns. A large body of literature has been devoted to this topic in recent decades. While the main focus has been on volatility modeling, understanding the comovements of returns is also of great practical importance. Accurate estimates of covariances or correlations are needed in many financial applications, such as risk management, asset allocation or derivative pricing.

Over the past decade, new insights into the behavior of asset returns have been gained. As high-frequency data became widely available, researchers were given the opportunity to exploit the information contained in intraday returns. This opened a whole new chapter in the modeling of volatility and correlations, with the attention being turned to the use of so-called "realized volatility". The realized volatility approach was pioneered by Andersen and Bollerslev (1998) but it took a few years until rigorous theoretical framework was developed. In this respect, the most important paper is that of Andersen et al. (2003). Currently, the realized volatility approach is an active area of research that produces very interesting findings. Considerable progress has already been achieved in the analysis of the univariate case. We can mention for example the papers of Barndorff-Nielsen and Shephard (2004c), Zhang, Mykland and Aït-Sahalia (2005), Andersen, Bollerslev and Diebold (2007) or Corsi (2009). In the multivariate context (realized covariances and correlations) the foundations were laid by the work of Barndorff-Nielsen and Shephard (2004a) but more systematic research began only recently, see for example Barndorff-Nielsen et al. (2010) or Zhang (2011).

Besides that, most researchers (not meaning only those who apply the realized volatility approach) analyze data from the U.S.[1] or Western European markets. In contrast, markets in Central and Eastern Europe have typically received considerably less attention, which is by itself a good reason to examine these markets. Moreover, the stock markets in this region have already attracted the attention of numerous investors, for whom it is of great interest to understand the links among the markets. Further motivation can be provided by the fact that Central European countries are obliged (once they meet the convergence criteria) to join the euro zone, so the degree of comovements among the stock

[1] U.S. = United States of America

markets of these countries and vis-à-vis the markets of the euro area can have implications for the stability of the monetary union.

Generally, the existing literature on the relationships among the Central European stock markets indicates that over the course of their development, the emerging stock markets in this region have become more closely linked to each other, as well as to the developed markets. There is some evidence that the interdependencies among the markets were influenced by the Asian and Russian crises and later by the Central European countries' accession to the European Union. Most of the empirical studies used daily or weekly data and when high-frequency data were employed (see Égert and Kočenda (2007a) and Égert and Kočenda (2007b)), they were analyzed by methods that are usually applied to daily or lower-frequency data.

In the light of what was mentioned above, we now turn to the objectives and contributions of our thesis. We examine linkages among the stock markets of the Czech Republic, Poland, Hungary and Germany in the period 2008-2010, thus employing recent data. Our most important contribution is that we analyze Central European stock markets by means of realized correlations constructed from high-frequency data. To our best knowledge, no study on this topic has been published so far, which means that we present primary results obtained in this field. In addition, we also use so-called realized bipower correlations, which should be robust to jumps in the price process. We study the main characteristics of the realized correlations and investigate the dynamics of the correlations among the analyzed markets. To capture the correlation dynamics, we model the realized correlations using the heterogeneous autoregressive model which was proposed by Corsi (2009) and then used by Audrino and Corsi (2010) in the context of realized correlations.

However, our analysis is not restricted to the use of the realized correlations. The second method that we apply is multivariate GARCH modeling, namely the DCC-GARCH model of Engle and Sheppard (2001) and Engle (2002). Although GARCH models have certain weaknesses, a very good motivation for their use is that sometimes we have to work with data that are simply not available at high frequencies, in which case the realized volatility approach cannot be applied. Even in a situation where we have the high-frequency data at hand, the results given by GARCH models are still worth looking at because they offer an interesting comparison.

Among the most important research questions that we will try to answer in our thesis are the following: What is the nature and dynamics of the interdependencies among the Central European stock markets? Are there significant differences between the results obtained by the two methods (realized correlations, DCC-GARCH model)? Concerning the realized correlations, how much are the results affected by the use of different sampling frequencies or by the use of estimators that should be robust to the presence of jumps? Do the correlations respond to market developments during the recent financial crisis? How do our models perform in forecasting correlations?

The rest of the thesis is organized as follows. In Section 2 we explain the theoretical background of our analysis. Section 3 presents some basic information on the Central European stock markets and also provides a literature review on the linkages among the markets. In Section 4 we describe our data and detail the construction of variables. In Section 5 we report and discuss our empirical results. Section 6 summarizes the main findings and concludes.

2. Theoretical Background

In this chapter we provide the theoretical framework of two different approaches to estimating, modeling and forecasting volatility and correlations.

The first approach is based on the realized variance and the analogous concepts of realized covariances and correlations. Together, the realized variance and the related measures can be referred to as realized measures. In Section 2.1.1. we show how these measures are constructed. By means of the theory of quadratic variation, realized variances and covariances can be connected to conditional variances and covariances of asset returns, which is shown in Section 2.1.2. A considerable advantage of this approach is that it enables us to treat volatility (or co-volatility) as an observable variable. As a consequence, relatively simple and straightforward methods can be used for the modeling and forecasting of volatility and correlations. This is shown in Section 2.1.3., in which we describe the heterogeneous autoregressive (HAR) model.

The second approach presented in this chapter (Section 2.2.) is based on GARCH modeling, thus it is somewhat more traditional. The model that we use is the Dynamic Conditional Correlation GARCH (DCC-GARCH) model, which is currently one of the most popular multivariate GARCH models. The section starts with a brief introduction to multivariate GARCH modeling and then we continue with the specification and estimation procedure of the DCC-GARCH model.

2.1. Realized Measures

The realized volatility approach including its theoretical underpinnings was introduced by Andersen et al. (2001). Later Andersen et al. (2003) and Barndorff-Nielsen and Shephard (2004a) developed a truly rigorous theoretical framework for the realized measures. We will try to present here the main points. However, we should first explain how the realized measures are constructed.

2.1.1. Construction of Realized Measures

Let $p_{i,t}$ denote the logarithmic price of asset i at time t. Suppose that we have a sample of T days and that within each day the prices are sampled at time interval Δ with a total of m such intervals in one trading day. The length of a trading day is normalized to unity and the interval Δ is expressed as a fraction of the trading day, so we have $\Delta = 1/m$. Realized variance (RV) of asset i on day t is defined as

$$RV_{i,t} = \sum_{k=1}^{m} r_{i,t-1+k\Delta}^{2}, \tag{1}$$

where $t = 1, \dots, T$ and $r_{i,t-1+k\Delta} = p_{i,t-1+k\Delta} - p_{i,t-1+(k-1)\Delta}$ are intraday returns for day t. Realized volatility[2] ($RVOL$) is then computed as the square root of realized variance, formally

$$RVOL_{i,t} = \sqrt{RV_{i,t}}. \tag{2}$$

In a similar vein, for two assets i and j whose prices are synchronized, we can construct the daily realized covariance ($RCOV$). This is computed as

$$RCOV_{i,j,t} = \sum_{k=1}^{m} r_{i,t-1+k\Delta}\, r_{j,t-1+k\Delta}. \tag{3}$$

To generalize the concept, we can consider a total of N assets with their logarithmic prices given by the $N \times 1$ vector $\boldsymbol{p}_t = (p_{1,t}, \dots, p_{N,t})^T$, assuming the synchronization of all prices. The $N \times N$ realized covariance matrix on day t is then defined as

$$\boldsymbol{RCOV}_t = \sum_{k=1}^{m} \boldsymbol{r}_{t-1+k\Delta}\, \boldsymbol{r}_{t-1+k\Delta}^{T}, \tag{4}$$

where $\boldsymbol{r}_{t-1+k\Delta} = \boldsymbol{p}_{t-1+k\Delta} - \boldsymbol{p}_{t-1+(k-1)\Delta}$ is the $N \times 1$ vector of intraday returns for day t. The i^{th} element on the main diagonal of $\boldsymbol{RCOV}_t$ is equal to the realized variance of asset i, while the off-diagonal element in the i^{th} row and the j^{th} column represents the realized covariance between assets i and j. For the realized covariance matrix to be positive definite, the number of assets (N) cannot exceed the number of intraday returns for each day (m) (Andersen et al., 2003). Finally, we also define the daily realized correlation (RC) between assets i and j, which is given by

$$RC_{i,j,t} = \frac{RCOV_{i,j,t}}{\sqrt{RV_{i,t}} \cdot \sqrt{RV_{j,t}}} = \frac{RCOV_{i,j,t}}{RVOL_{i,t} \cdot RVOL_{j,t}}. \tag{5}$$

[2] It should be noted that the terminology used in the literature is not consistent. In some papers the term "realized volatility" refers to the quantity defined in equation (1).

In addition, we define the realized bipower variance and covariance, which were introduced by Barndorff-Nielsen and Shephard (2004c) and Barndorff-Nielsen and Shephard (2004b). The realized bipower variance ($RBPV$) of asset i on day t is given by

$$RBPV_{i,t} = \mu_1^{-2} \frac{m}{m-1} \sum_{k=2}^{m} \left| r_{i,t-1+k\Delta} \right| \left| r_{i,t-1+(k-1)\Delta} \right|, \tag{6}$$

where $\mu_1 = \sqrt{2}/\sqrt{\pi} = \mathrm{E}(|u|)$ and $u \sim N(0,1)$. The realized bipower covariance ($RBPCOV$) between assets i and j is defined as

$$\begin{aligned} RBPCOV_{i,j,t} = \frac{\mu_1^{-2}}{4} \frac{m}{m-1} \sum_{k=2}^{m} \Big(& \left| r_{i,t-1+k\Delta} + r_{j,t-1+k\Delta} \right| \left| r_{i,t-1+(k-1)\Delta} + r_{j,t-1+(k-1)\Delta} \right| \\ & - \left| r_{i,t-1+k\Delta} - r_{j,t-1+k\Delta} \right| \left| r_{i,t-1+(k-1)\Delta} - r_{j,t-1+(k-1)\Delta} \right| \Big). \end{aligned} \tag{7}$$

In the general framework of N assets we can construct the $N \times N$ daily realized bipower covariance matrix, which is simply given by

$$\boldsymbol{RBPCOV}_t = \begin{pmatrix} RBPV_{1,t} & RBPCOV_{1,2,t} & \cdots & RBPCOV_{1,N,t} \\ RBPCOV_{2,1,t} & RBPV_{2,t} & & \vdots \\ \vdots & & \ddots & \vdots \\ RBPCOV_{N,1,t} & \cdots & \cdots & RBPV_{N,t} \end{pmatrix}. \tag{8}$$

Finally, the daily realized bipower correlation ($RBPC$) between assets i and j is computed as

$$RBPC_{i,j,t} = \frac{RBPCOV_{i,j,t}}{\sqrt{RBPV_{i,t}} \cdot \sqrt{RBPV_{j,t}}}. \tag{9}$$

Now that we have shown how the realized measures are constructed, we proceed to explain the underlying theory.

2.1.2. Quadratic Variation Theory

Following Andersen et al. (2003), we consider an *N*-dimensional price process defined on a complete probability space $(\Omega, \mathcal{F}, P)$, evolving in continuous time over the interval $[0, T]$, where T is a positive integer. Further, let $\mathcal{F}_t \subseteq \mathcal{F}$ be the σ-field that reflects the information at time t, so that $\mathcal{F}_s \subseteq \mathcal{F}_t$ for $0 \le s \le t \le T$.[3] If the price process is arbitrage-free and has finite mean, then the logarithmic vector price process

[3] It is assumed that the family of σ-fields $(\mathcal{F}_t)_{t\in[0,T]} \subseteq \mathcal{F}$ satisfies the conditions of *P*-completeness and right continuity, which are the usual assumptions for an information filtration.

$\boldsymbol{p} = (\boldsymbol{p}(t))_{t\in[0,T]}$ belongs to the class of special semi-martingales. To avoid confusion, we should stress that the notation introduced in the previous sentence is used for the theoretical continuous-time price process, while the discrete approximation of this process is referred to by subscripts t (see section 2.1.1). The same kind of notation is used for the return process.

To briefly introduce semi-martingales, a process is called a semi-martingale if it can be decomposed as the sum of a finite variation process and a local martingale. Back (1991) further notes that the defining property of a special semi-martingale is that the finite variation process in the decomposition is taken to be predictable, which means that its value at time t is known just before time t. Importantly, the decomposition of a special semi-martingale is unique and it is called the canonical decomposition.

As mentioned above, process $\boldsymbol{p}$ is a special semi-martingale, so it can be decomposed uniquely as the sum of a finite variation and predictable mean component $\boldsymbol{A} = (A_1, \dots A_N)^T$ and a local martingale $\boldsymbol{M} = (M_1, \dots M_N)^T$. These may each be written as the sum of a continuous sample-path part and a jump part. We thus have the following representation for $\boldsymbol{p}(t)$

$$\boldsymbol{p}(t) = \boldsymbol{p}(0) + \boldsymbol{A}(t) + \boldsymbol{M}(t) = \boldsymbol{p}(0) + \boldsymbol{A}^c(t) + \Delta\boldsymbol{A}(t) + \boldsymbol{M}^c(t) + \Delta\boldsymbol{M}(t), \qquad (10)$$

where the finite variation predictable components $\boldsymbol{A}^c(t)$ and $\Delta\boldsymbol{A}(t)$ are respectively continuous and pure jump processes, the local martingales $\boldsymbol{M}^c(t)$ and $\Delta\boldsymbol{M}(t)$ are respectively continuous sample-path and compensated jump processes, and by definition $\boldsymbol{M}(0) \equiv \boldsymbol{A}(0) \equiv 0$. The no-arbitrage condition implies that whenever $\Delta\boldsymbol{A}(t) \neq 0$ (which means that there is a jump whose timing and magnitude is known prior to the jump event and thus an arbitrage opportunity exists), there must be a concurrent jump in the martingale component, i.e. $\Delta\boldsymbol{M}(t) \neq 0$. Furthermore, this martingale jump must be large enough (with strictly positive probability) to change the direction of the jump in the price. Formally, if $\boldsymbol{\Delta} A(t) \neq 0$, then

$$P\left[\operatorname{sgn}\left(\Delta\boldsymbol{A}(t)\right) = -\operatorname{sgn}\left(\Delta\boldsymbol{A}(t) + \Delta\boldsymbol{M}(t)\right)\right] > 0, \qquad (11)$$

where $\operatorname{sgn}(\mathrm{x}) \equiv 1$ for $x \geq 0$ and $\operatorname{sgn}(\mathrm{x}) \equiv -1$ for $x < 0$.

Having discussed the characterization of the price process, let us now focus on the returns. The continuously compounded return over the interval $[t-h, t]$, where $0 \leq t - h \leq t \leq T$, is denoted by

$$\boldsymbol{r}(t,h) = \boldsymbol{p}(t) - \boldsymbol{p}(t-h). \tag{12}$$

We will also assume that for $h = 1$ the interval represents one trading day and $r(t,1)$ is the corresponding daily return. The cumulative return process from $t = 0$ onward, $\boldsymbol{r} = \left(\boldsymbol{r}(t)\right)_{t\in[0,T]}$, is given by

$$\boldsymbol{r}(t) \equiv \boldsymbol{r}(t,t) = \boldsymbol{p}(t) - \boldsymbol{p}(0) = \boldsymbol{A}(t) + \boldsymbol{M}(t). \tag{13}$$

As a result of the properties of $\boldsymbol{p}(t)$, process $\boldsymbol{r}(t)$ is a special semi-martingale with the unique decomposition into the predictable and integrable mean component $\boldsymbol{A}$ and the local martingale $\boldsymbol{M}$. Besides that, the cumulative return process is subject to two types of jumps. First, there are predictable jumps, for which $\Delta\boldsymbol{A}(t) \neq 0$ and equation (11) must hold. Such jumps may occur in case of perfectly anticipated releases of information. In contrast, jumps of the second type are purely unanticipated, i.e. $\Delta\boldsymbol{A}(t) = 0$ but $\Delta\boldsymbol{M}(t) \neq 0$, typically occurring when the market is hit by some unexpected news.

An important property of a semi-martingale (and thus also of every special semi-martingale) is that it has a quadratic variation process. Let $[\boldsymbol{r},\boldsymbol{r}] = \{[\boldsymbol{r},\boldsymbol{r}](t)\}_{t\in[0,T]}$ be the quadratic variation $N \times N$ matrix process of the cumulative return process with its ij^{th} element denoted as $[r_i, r_j]$. Using the definition of quadratic variation employed by Barndorff-Nielsen and Shephard (2004a), we have

$$[\boldsymbol{r},\boldsymbol{r}](t) = \operatorname*{plim}_{m\to\infty} \sum_{k=0}^{m-1} [\boldsymbol{r}(t_{k+1}) - \boldsymbol{r}(t_k)][\boldsymbol{r}(t_{k+1}) - \boldsymbol{r}(t_k)]^T, \tag{14}$$

where $t_0 = 0 < t_1 < \cdots < t_m = t$, $\sup_k(t_{k+1} - t_k) \to 0$ for $m \to \infty$ and plim denotes probability limit. The i^{th} diagonal element of $[\boldsymbol{r},\boldsymbol{r}]$ is the quadratic variation process of the i^{th} asset return, while the ij^{th} off-diagonal element represents the quadratic covariation process between asset returns i and j. Recalling the definition of the cumulative return process, we can also rewrite (14) as

$$[\boldsymbol{r},\boldsymbol{r}](t) = \operatorname*{plim}_{m\to\infty} \sum_{k=0}^{m-1} [\boldsymbol{p}(t_{k+1}) - \boldsymbol{p}(t_k)][\boldsymbol{p}(t_{k+1}) - \boldsymbol{p}(t_k)]^T. \tag{15}$$

Andersen et al. (2003) note that if the finite variation mean component $\boldsymbol{A}$ in equation (10) is continuous (which means that there are no predictable jumps), then the ij^{th} element of the quadratic variation is given by

$$[r_i, r_j](t) = [M_i, M_j](t) = [M_i^c, M_j^c](t) + \sum_{0 \le s \le t} \Delta M_i(\mathrm{s}) \Delta M_j(\mathrm{s}). \tag{16}$$

This is an implication of the fact that the quadratic variation of continuous finite variation processes is zero, so $\boldsymbol{A}$ has no effect on the quadratic variation. Equation (16) also shows that jump components are relevant for the quadratic covariation only if there are simultaneous jumps in the price path for the i^{th} and j^{th} asset.

Given the definition of quadratic variation, we can immediately see how it relates to the realized measures. Equation (15) implies that as $m \to \infty$ (or $\Delta \to 0$) and for all $t = 1, \ldots, T$,

$$\boldsymbol{RCOV}_t \xrightarrow{p} [\boldsymbol{r}, \boldsymbol{r}](t) - [\boldsymbol{r}, \boldsymbol{r}](t-1), \tag{17}$$

where $\xrightarrow{p}$ denotes convergence in probability. It means that the daily realized covariance matrix consistently estimates daily increments to the quadratic return variation process. By the same reasoning, the following convergence result is obtained for the daily realized correlation between assets i and j

$$RC_{i,j,t} \xrightarrow{p} \frac{[r_i, r_j](t) - [r_i, r_j](t-1)}{\sqrt{([r_i, r_i](t) - [r_i, r_i](t-1))([r_j, r_j](t) - [r_j, r_j](t-1))}}. \tag{18}$$

Next, we show the connection between the quadratic variation and the conditional return covariance matrix, as developed by Andersen et al. (2003).

We assume that the arbitrage-free logarithmic price process $\boldsymbol{p}$ is square-integrable and that the mean component $\boldsymbol{A}$ is continuous. The conditional return covariance matrix at time $t-h$ over $[t-h, t]$ is then given by

$$\begin{aligned} \operatorname{cov}(\boldsymbol{r}(t,h)|\mathcal{F}_{t-h}) = \mathrm{E}([\boldsymbol{r}, \boldsymbol{r}](t) - [\boldsymbol{r}, \boldsymbol{r}](t-h)|\mathcal{F}_{t-h}) + \boldsymbol{\Gamma}_{\boldsymbol{A}}(t,h) \\ + \boldsymbol{\Gamma}_{\boldsymbol{AM}}(t,h) + \boldsymbol{\Gamma}_{\boldsymbol{AM}}^T(t,h), \end{aligned} \tag{19}$$

where $0 \le t-h \le t \le T$, $\boldsymbol{\Gamma}_{\boldsymbol{A}}(t,h) = \operatorname{cov}(\boldsymbol{A}(t) - \boldsymbol{A}(t-h)|\mathcal{F}_{t-h})$ and $\boldsymbol{\Gamma}_{\boldsymbol{AM}}(t,h) = \mathrm{E}(\boldsymbol{A}(t)[\boldsymbol{M}(t) - \boldsymbol{M}(t-h)]^T|\mathcal{F}_{t-h})$. By imposing certain additional conditions, we can simplify the expression on the right-hand side of (19). Specifically, if the mean process, $\{\boldsymbol{A}(s) - \boldsymbol{A}(t-h)\}_{s \in [t-h,t]}$, conditional on information at time $t-h$ is independent of the return innovation process, $\{\boldsymbol{M}(u)\}_{u\in[t-h,t]}$, then the last two terms on the right-hand side of (19) are both zero. Furthermore, if the mean process, $\{\boldsymbol{A}(s)-\boldsymbol{A}(t-h)\}_{s\in[t-h,t]}$, conditional on information at time $t-h$ is a predetermined function over $[t-h,t]$, then we get rid of the second term on the right-hand side of (19) and we are thus left with

$$\operatorname{cov}(\boldsymbol{r}(t,h)|\mathcal{F}_{\mathrm{t-h}}) = \mathrm{E}([\boldsymbol{r},\boldsymbol{r}](t)-[\boldsymbol{r},\boldsymbol{r}](t-h)|\mathcal{F}_{\mathrm{t-h}}). \tag{20}$$

Andersen et al. (2003) argue that the conditions leading to equation (20) are satisfied for a wide range of commonly used models. Focusing on the daily horizon, i.e. $h=1$, equation (20) says that the time $t-1$ conditional covariance matrix of the daily returns, $\boldsymbol{r}(t,1)$, equals the time $t-1$ conditional expectation of the daily increments to the quadratic return variation process, $[\boldsymbol{r},\boldsymbol{r}](t)-[\boldsymbol{r},\boldsymbol{r}](t-1)$. Another interpretation is that the time t ex-post value of the daily increment to the quadratic variation is an unbiased estimator for the daily return covariance matrix conditional on information at time $t-1$.

Now we will consider a somewhat less general framework, in which we can obtain more specific results. In addition to the absence of arbitrage and the square integrability of the logarithmic price process $\boldsymbol{p}$, we also assume that $\boldsymbol{p}$ has continuous sample path, i.e. with no jumps, and that the associated quadratic return variation process $[\boldsymbol{r},\boldsymbol{r}](t)$ is of full rank (which implies that no asset is redundant). Under these conditions, we have the following representation for returns

$$\boldsymbol{r}(t,h)=\boldsymbol{p}(t)-\boldsymbol{p}(t-h)=\int_{t-h}^{t}\boldsymbol{\mu}(s)ds+\int_{t-h}^{t}\boldsymbol{\sigma}(s)\,d\boldsymbol{W}(s), \tag{21}$$

where $0\le t-h\le t\le T$, $\boldsymbol{\mu}(s)$ is an integrable predictable vector of dimension $N\times 1$, $\boldsymbol{\sigma}(s)=\left(\sigma_{i,j}(s)\right)_{i,j=1,\dots,N}$ is an $N\times N$ matrix, $\boldsymbol{W}(s)$ is an $N\times 1$ dimensional standard Brownian motion, integration of a matrix or vector with respect to a scalar denotes component-wise integration, so that

$$\int_{t-h}^{t}\boldsymbol{\mu}(s)\,ds=\left(\int_{t-h}^{t}\mu_1(s)\,ds,\dots,\int_{t-h}^{t}\mu_N(s)\,ds\right)^{T}, \tag{22}$$

and integration of a matrix with respect to a vector denotes component-wise integration of the associated vector, so that

$$\int_{t-h}^{t} \boldsymbol{\sigma}(s)\, d\boldsymbol{W}(s) = \left(\int_{t-h}^{t} \sum_{j=1}^{N} \sigma_{1,j}(s) dW_j(s)\,, \dots, \int_{t-h}^{t} \sum_{j=1}^{N} \sigma_{N,j}(s) dW_j(s) \right)^T. \qquad (23)$$

Furthermore, we have

$$P\left[\int_{t-h}^{t} \left(\sigma_{i,j}(s)\right)^2 ds < \infty\right] = 1, \quad 1 \leq i,j \leq N. \qquad (24)$$

Defining the $N \times N$ matrix $\boldsymbol{\Omega}(s) = \left(\Omega_{i,j}(s)\right)_{i,j=1,\dots,N}$ as $\boldsymbol{\Omega}(s) = \boldsymbol{\sigma}(s)\boldsymbol{\sigma}(s)^T$, the increments to the quadratic return variation process have the following form

$$[\boldsymbol{r},\boldsymbol{r}](t) - [\boldsymbol{r},\boldsymbol{r}](t-h) = \int_{t-h}^{t} \boldsymbol{\Omega}\,(s) ds. \qquad (25)$$

The expression $\int_{t-h}^{t} \boldsymbol{\Omega}(s)\, ds$ is the so-called integrated covariance matrix over the interval $[t-h, t]$. Recalling the relationship expressed by (17), it follows that as $m \to \infty$ and for all $t = 1, \dots, T$,

$$\boldsymbol{RCOV}_t \xrightarrow{p} \int_{t-1}^{t} \boldsymbol{\Omega}(s)\, ds, \qquad (26)$$

meaning that the daily realized covariance matrix is a consistent estimator of the daily integrated covariance matrix. Similarly, for the realized correlation between assets i and j we have

$$RC_{i,j,t} \xrightarrow{p} \frac{\int_{t-1}^{t} \Omega_{i,j}(s)\, ds}{\sqrt{\int_{t-1}^{t} \Omega_{i,i}(s)\, ds \int_{t-1}^{t} \Omega_{j,j}(s)\, ds}}. \qquad (27)$$

Finally, if the mean process $\boldsymbol{\mu}(s)$ and the covolatility process $\boldsymbol{\sigma}(s)$ are independent of the Brownian motion $\boldsymbol{W}(s)$ over $[t-h, t]$, then

$$\boldsymbol{r}(t,h) |\, \sigma\{\boldsymbol{\mu}(s), \boldsymbol{\sigma}(s)\}_{s\in[t-h,t]} \sim N\left(\int_{t-h}^{t} \boldsymbol{\mu}(s)\, ds, \int_{t-h}^{t} \boldsymbol{\Omega}(s)\, ds\right), \qquad (28)$$

where $\sigma\{\boldsymbol{\mu}(s), \boldsymbol{\sigma}(s)\}_{s\in[t-h,t]}$ denotes the σ-field generated by $\left(\boldsymbol{\mu}(s), \boldsymbol{\sigma}(s)\right)_{s\in[t-h,t]}$. The result in (28) implies that daily returns are conditionally (on the sample path of $\boldsymbol{\mu}(s)$ and $\boldsymbol{\sigma}(s)$) normally distributed with mean $\int_{t-1}^{t} \boldsymbol{\mu}(s)\, ds$ and covariance matrix $\int_{t-1}^{t} \boldsymbol{\Omega}(s)\, ds$, where the mean is usually very small and $\int_{t-1}^{t} \boldsymbol{\Omega}\,(s)ds$ can be approximated by the realized covariance matrix, as suggested by (26).

So far we have focused on providing the theoretical underpinnings for the realized covariance matrix (and also the realized correlation) so it remains to explain the connection to the realized bipower measures. We will not go deep into technical details, but rather point out the similarities and differences. As clarified by Barndorff-Nielsen and Shephard (2004b), if the price process has continuous sample paths (like in equation (21)), then the realized bipower covariance matrix has the same probability limit as the realized covariance matrix. However, the convergence results differ if the price process exhibits jumps. Assuming finite activity jumps, the price process could be then expressed as

$$\boldsymbol{p}(t) = \int_0^t \boldsymbol{\mu}(s)\,ds + \int_0^t \boldsymbol{\sigma}(s)\,d\boldsymbol{W}(s) + \sum_{k=1}^{C_t} \boldsymbol{J}_k, \tag{29}$$

where C is the simple counting process satisfying $C_t < \infty$ for all t and we also assume that $\sum_{k=1}^{C_t} J_{i,k}^2 < \infty$ for $i = 1, \dots, N$ and all t. In this case the probability limit of the realized covariance matrix is affected by the presence of jumps. To be more specific, for all $t = 1, \dots, T$ and for $m \to \infty$ we have

$$\boldsymbol{RCOV_t} \xrightarrow{p} \int_{t-1}^{t} \boldsymbol{\Omega}(s)\,ds + \sum_{C_{t-1} < k \le C_t} \boldsymbol{J}_k \boldsymbol{J}_k^T \tag{30}$$

In contrast, the limit of the bipower realized covariance matrix does not change with the addition of jumps, i.e. $\boldsymbol{RBPCOV}_t$ still converges to the daily integrated covariance matrix (for all $t = 1, \dots, T$).

Finally, it should be noted that Barndorff-Nielsen and Shephard (2004a) provided the asymptotic distribution theory for the realized covariance matrix (as well as the realized correlation) and that Barndorff-Nielsen and Shephard (2004c) and Barndorff-Nielsen and Shephard (2004b) discussed the asymptotic distribution of the realized bipower variance and covariance. It turns out that realized bipower variance and covariance are less efficient than realized variance and covariance when there are no jumps. Therefore, on the one hand, the realized bipower measures are robust to jumps. On the other hand, if the price process is not subject to jumps, then the robustness to jumps comes at the expense of higher variance of the estimator.

2.1.3. HAR Model

In the previous sections we introduced the realized measures and explained the theory that underlies their use in estimating variances, covariances and correlations. Now we will describe a simple model for the realized measures. The model is called the heterogeneous autoregressive (HAR) model and it was proposed by Corsi (2009). The HAR model was originally derived for realized volatility but it can also be applied to model realized correlations.

Let us explain the derivation of the model in the realized volatility framework. It means that we are now in a univariate setting with only one asset i. Yet, for notational ease we will suppress the subscript i in this subsection. We consider the following continuous time process

$$dp(t) = \mu(t)dt + \sigma(t)dW(t), \tag{31}$$

where $p(t)$ is the logarithm of instantaneous price, $\mu(t)$ is a càdlàg (right continuous with left limits) finite variation process, $W(t)$ is a standard Brownian motion and $\sigma(t)$ is a stochastic process independent of $W(t)$. A full trading day is represented by the time interval $1d$ and the integrated volatility associated with day t is defined as

$$\sigma_t^{(d)} = \left(\int_{t-1d}^{t} \sigma^2\,(\omega) d\omega \right)^{1/2}. \tag{32}$$

The daily realized volatility is denoted as $RVOL_t^{(d)}$ and given by

$$RVOL_t^{(d)} = \left(\sum_{k=0}^{M-1} r_{t-k\Delta}^2 \right)^{1/2}, \tag{33}$$

where $\Delta = 1d/M$ and $r_{t-k\Delta} = p_{t-k\Delta} - p_{t-(k+1)\Delta}$. Besides that, we will also consider volatilities viewed over longer time horizons, namely one week (5 working days) and one month (22 working days). These multi-period volatilities are computed as simple averages of the daily quantities, with the weekly and monthly aggregations indicated by superscripts (w) and (m), respectively. For example, the weekly realized volatility at time t is given by

$$RVOL_t^{(w)} = \frac{1}{5}\left(RVOL_t^{(d)} + RVOL_{t-1d}^{(d)} + \ldots + RVOL_{t-4d}^{(d)} \right). \tag{34}$$

The model is based on the idea that market participants are heterogeneous in terms of their time horizons of trading. It is assumed that participants with different time horizons perceive and create different types of volatility components. Furthermore, an important feature of volatility is its asymmetric propagation, meaning that volatility over shorter time intervals is influenced by volatility over longer time intervals rather than conversely.

We define the latent partial volatility $\tilde{\sigma}_t^{(\cdot)}$ as the volatility generated by a certain market component and for simplicity we consider only three volatility components related to time horizons of one day, one week and one month. The daily, weekly and monthly partial volatilities are then denoted as $\tilde{\sigma}_t^{(d)}$, $\tilde{\sigma}_t^{(w)}$ and $\tilde{\sigma}_t^{(m)}$, respectively. Moreover, we connect the partial volatility to the integrated volatility by assuming that $\tilde{\sigma}_t^{(d)} = \sigma_t^{(d)}$. Each partial volatility is assumed to depend on the past realized volatility corresponding to the same time horizon and the expected value of the next-period longer-term partial volatility. Since the longest time interval that we consider is one month, the monthly partial volatility is determined only by the past monthly realized volatility. The model is thus characterized by the following three equations

$$\tilde{\sigma}_{t+1m}^{(m)} = c^{(m)} + \phi^{(m)} RVOL_t^{(m)} + \tilde{\omega}_{t+1m}^{(m)}, \tag{35}$$

$$\tilde{\sigma}_{t+1w}^{(w)} = c^{(w)} + \phi^{(w)} RVOL_t^{(w)} + \gamma^{(w)} \mathrm{E}_t\left[\tilde{\sigma}_{t+1m}^{(m)}\right] + \tilde{\omega}_{t+1w}^{(w)}, \tag{36}$$

$$\tilde{\sigma}_{t+1d}^{(d)} = c^{(d)} + \phi^{(d)} RVOL_t^{(d)} + \gamma^{(d)} \mathrm{E}_t\left[\tilde{\sigma}_{t+1w}^{(w)}\right] + \tilde{\omega}_{t+1d}^{(d)}, \tag{37}$$

where $\tilde{\omega}_{t+1m}^{(m)}$, $\tilde{\omega}_{t+1w}^{(w)}$ and $\tilde{\omega}_{t+1d}^{(d)}$ are contemporaneously and serially independent zero-mean error terms with an appropriately truncated left tail in order to guarantee the positivity of partial volatilities.

Substituting (35) into (36) and then (36) into (37) while recalling that $\tilde{\sigma}_t^{(d)} = \sigma_t^{(d)}$, we arrive at

$$\sigma_{t+1d}^{(d)} = c + \beta^{(d)} RVOL_t^{(d)} + \beta^{(w)} RVOL_t^{(w)} + \beta^{(m)} RVOL_t^{(m)} + \tilde{\omega}_{t+1d}^{(d)}. \tag{38}$$

Finally, we use the fact that the ex post value of $\sigma_{t+1d}^{(d)}$ can be expressed as

$$\sigma_{t+1d}^{(d)} = RVOL_{t+1d}^{(d)} + \omega_{t+1d}^{(d)}, \tag{39}$$

where $\omega_t^{(d)}$ represents both latent daily volatility measurement and estimation errors.

Substituting (39) into (38), we obtain

$$RVOL_{t+1d}^{(d)} = c + \beta^{(d)} RVOL_t^{(d)} + \beta^{(w)} RVOL_t^{(w)} + \beta^{(m)} RVOL_t^{(m)} + \omega_{t+1d}, \qquad (40)$$

where $\omega_{t+1d} = \widetilde{\omega}_{t+1d}^{(d)} - \omega_{t+1d}^{(d)}$. We have thus obtained a simple time series model of realized volatility. The model is estimated using the ordinary least squares (OLS) method.

Importantly for our case, Audrino and Corsi (2010) suggest that the model can be also used for realized correlations between two assets (for notational ease we do not use the subscripts i and j). Plugging $1d = 1$ and suppressing the superscript for daily correlations, we have

$$RC_{t+1} = c + \beta^{(d)} RC_t + \beta^{(w)} RC_t^{(w)} + \beta^{(m)} RC_t^{(m)} + \omega_{t+1}, \qquad (41)$$

where RC_t, $RC_t^{(w)}$ and $RC_t^{(m)}$ are respectively the daily, weekly and monthly realized correlations. It is quite obvious that besides direct modeling of correlations, the model can be also used to generate forecasts of correlations. Given the information at time t, the one-step ahead forecast is simply obtained as

$$\mathrm{E}_t(RC_{t+1}) = c + \beta^{(d)} RC_t + \beta^{(w)} RC_t^{(w)} + \beta^{(m)} RC_t^{(m)}. \qquad (42)$$

2.2. DCC-GARCH Model

Before formulating the DCC-GARCH model, we will first briefly describe the general framework of multivariate Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models. The ARCH-GARCH modeling dates back to 1980s, when Engle (1982) first introduced an ARCH model and later Bollerslev (1986) proposed its extension to a GARCH model. Since that time GARCH models have become common tools in the analysis of time series data. Using univariate GARCH models, we can model the conditional variance of a single time series, while with multivariate GARCH models we can analyze the conditional variances and covariances of N time series. In financial applications, the analyzed series are usually the daily returns of assets. However, GARCH models can be also used for various other types of time series data.

Following Bauwens et al. (2006), we consider an $N \times 1$ vector of daily returns $\boldsymbol{r}_t$, where $t = 1, \ldots, T$. Further, let $\mathcal{F}_{t-1}$ denote the σ-field generated by the past information until time $t-1$. We can express $\boldsymbol{r}_t$ as

$$\boldsymbol{r}_t = \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t, \tag{43}$$

where $\boldsymbol{\mu}_t = E(\boldsymbol{r}_t|\mathcal{F}_{t-1})$ is the conditional mean vector of $\boldsymbol{r}_t$ and vector $\boldsymbol{\varepsilon}_t$ can be written as

$$\boldsymbol{\varepsilon}_t = \boldsymbol{H}_t^{1/2}\boldsymbol{z}_t. \tag{44}$$

Matrix $\boldsymbol{H}_t^{1/2}$ in (44) is a positive definite matrix of dimension $N \times N$ and $\boldsymbol{z}_t$ is an $N \times 1$ random vector satisfying

$$\begin{aligned} \mathrm{E}(\boldsymbol{z}_t) &= 0 \\ \mathrm{cov}(\boldsymbol{z}_t) &= \boldsymbol{I}_N, \end{aligned} \tag{45}$$

where $\boldsymbol{I}_N$ is the identity matrix of order N.

To make clear what exactly $\boldsymbol{H}_t^{1/2}$ is, we compute the conditional variance-covariance matrix of $\boldsymbol{r}_t$:

$$\begin{aligned} \mathrm{cov}(\boldsymbol{r}_t|\mathcal{F}_{t-1}) = \mathrm{cov}_{t-1}(\boldsymbol{r}_t) &= \mathrm{cov}_{t-1}(\boldsymbol{\varepsilon}_t) \\ &= \boldsymbol{H}_t^{1/2}\mathrm{cov}_{t-1}(\boldsymbol{z}_t)\left(\boldsymbol{H}_t^{1/2}\right)^T \\ &= \boldsymbol{H}_t. \end{aligned} \tag{46}$$

The matrix $\boldsymbol{H}_t^{1/2}$ thus can be defined as any $N \times N$ positive definite matrix such that $\boldsymbol{H}_t$ is the conditional variance-covariance matrix of $\boldsymbol{r}_t$ (as well as $\boldsymbol{\varepsilon}_t$).[4] Denoting the elements of $\boldsymbol{H}_t$ as $h_{i,j,t}$[5], $i,j = 1,\dots,N$, the element $h_{i,i,t}$ is the conditional variance of $r_{i,t}$ and the element $h_{i,j,t} = h_{j,i,t}$, $i \neq j$, is the conditional covariance between $r_{i,t}$ and $r_{j,t}$. Various specifications of $\boldsymbol{H}_t$ were proposed in the literature. In general, one of the main problems with multivariate GARCH models is to find a reasonable balance between flexibility and parsimony. Another issue that has to be taken into account is imposing positive definiteness of $\boldsymbol{H}_t$. The approach taken in the DCC-GARCH model is to specify separately the individual conditional variances and the conditional correlation matrix, using a two-step procedure to estimate the parameters of the model. Let us now describe the DCC-GARCH model in detail.

[4] Matrix $\boldsymbol{H}_t^{1/2}$ can be thought of as the Cholesky decomposition of $\boldsymbol{H}_t$. Given a symmetric positive definite matrix $\boldsymbol{X}$, the Cholesky decomposition is a lower triangular matrix $\boldsymbol{U}$ with strictly positive diagonal entries such that $\boldsymbol{X} = \boldsymbol{U}\boldsymbol{U}^T$.

[5] The same kind of notation will be used for all matrices in this subsection.

The Dynamic Conditional Correlation GARCH (DCC-GARCH) model was proposed by Engle and Sheppard (2001) and Engle (2002). The model can be regarded as a generalization of the constant conditional correlation GARCH model developed by Bollerslev (1990). It should be noted that Tse and Tsui (2002) also proposed a multivariate GARCH model with time-varying correlations and their model thus can be seen as an alternative to the Engle's and Sheppard's model. However, we will focus here on the model of Engle and Sheppard. The model is explained below[6].

Consider an $N \times 1$ random process $\boldsymbol{\varepsilon}_t$ such that

$$\boldsymbol{\varepsilon}_t | \mathcal{F}_{t-1} \sim N(0, \boldsymbol{H}_t), \tag{47}$$

Where

$$\boldsymbol{H}_t \equiv \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t. \tag{48}$$

Since $\boldsymbol{\varepsilon}_t$ is already assumed to have zero mean, it is usually a vector of residuals from some simple model for the conditional mean of the time series. Matrix $\boldsymbol{D}_t$ in (48) is the $N \times N$ diagonal matrix of conditional standard deviations of series $\varepsilon_{1,t}, \dots, \varepsilon_{N,t}$. The i^{th} element on the main diagonal of $\boldsymbol{D}_t$ is thus equal to the square root of the i^{th} element on the main diagonal of $\boldsymbol{H}_t$, while all other elements of $\boldsymbol{D}_t$ are zero, formally $d_{i,i,t} = \sqrt{h_{i,i,t}}$ and $d_{i,j,t} = 0,\ i \neq j, i,j = 1, \dots, N$. Matrix $\boldsymbol{R}_t$ in (48) is the $N \times N$ matrix of conditional correlations, so the elements on its main diagonal are equal to 1. The assumption of multivariate normality in (47) enables us to formulate a likelihood function, using which we estimate the parameters governing the dynamics of $\boldsymbol{H}_t$. However, as noted by Engle and Sheppard (2001), normality of $\boldsymbol{\varepsilon}_t$ is not needed for consistency and asymptotic normality of the estimator. If the assumption of normality is not satisfied, the estimator can be interpreted as a quasi-maximum likelihood estimator.

The log-likelihood can be written as

$$L = -\frac{1}{2}\sum_{t=1}^{T}\left(N\log(2\pi) + \log|\boldsymbol{H}_t| + \boldsymbol{\varepsilon}_t^T \boldsymbol{H}_t^{-1} \boldsymbol{\varepsilon}_t\right)$$

$$L = -\frac{1}{2}\sum_{t=1}^{T}\left(N\log(2\pi) + \log|\boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t| + \boldsymbol{\varepsilon}_t^T \boldsymbol{D}_t^{-1} \boldsymbol{R}_t^{-1} \boldsymbol{D}_t^{-1} \boldsymbol{\varepsilon}_t\right)$$

[6] Please note that the notation used here is slightly different from that used in Engle (2002) and Engle and Sheppard (2001).

$$L = -\frac{1}{2}\sum_{t=1}^{T}(N\log(2\pi) + 2\log|\boldsymbol{D}_t| + \log|\boldsymbol{R}_t| + \boldsymbol{u}_t^T\boldsymbol{R}_t^{-1}\boldsymbol{u}_t) \tag{49}$$

$$L = -\frac{1}{2}\sum_{t=1}^{T}(N\log(2\pi) + 2\log|\boldsymbol{D}_t| + \boldsymbol{\varepsilon}_t{}^T\boldsymbol{D}_t^{-1}\boldsymbol{D}_t^{-1}\boldsymbol{\varepsilon}_t - \boldsymbol{u}_t^T\boldsymbol{u}_t + \log|\boldsymbol{R}_t| + \boldsymbol{u}_t^T\boldsymbol{R}_t^{-1}\boldsymbol{u}_t), \tag{50}$$

where $\boldsymbol{u}_t = \boldsymbol{D}_t^{-1}\boldsymbol{\varepsilon}_t$ are the residuals standardized by their conditional standard deviations. The function in (50) can be split into two parts. The first part is composed of terms containing $\boldsymbol{D}_t$, while the second one is composed of terms containing $\boldsymbol{R}_t$. Let us denote the parameters in $\boldsymbol{D}_t$ as $\boldsymbol{\theta}$ and the additional parameters in $\boldsymbol{R}_t$ as $\boldsymbol{\phi}$. We can then write the log-likelihood as follows:

$$L(\boldsymbol{\theta},\boldsymbol{\phi}) = L_V(\boldsymbol{\theta}) + L_C(\boldsymbol{\theta},\boldsymbol{\phi}), \tag{51}$$

where $L_V(\boldsymbol{\theta})$ is the volatility component given by

$$L_V(\boldsymbol{\theta}) = -\frac{1}{2}\sum_{t=1}^{T}(N\log(2\pi) + \log|\boldsymbol{D}_t|^2 + \boldsymbol{\varepsilon}_t{}^T\boldsymbol{D}_t^{-2}\boldsymbol{\varepsilon}_t) \tag{52}$$

and $L_C(\boldsymbol{\theta},\boldsymbol{\phi})$ is the correlation part, which has the form

$$L_C(\boldsymbol{\theta},\boldsymbol{\phi}) = -\frac{1}{2}\sum_{t=1}^{T}(\log|\boldsymbol{R}_t| + \boldsymbol{u}_t^T\boldsymbol{R}_t^{-1}\boldsymbol{u}_t - \boldsymbol{u}_t^T\boldsymbol{u}_t). \tag{53}$$

The log-likelihood, as formulated in (51), can be maximized in two steps. In the first step we focus on the volatility part. Our aim is to find

$$\widehat{\boldsymbol{\theta}} = \arg\max\{L_V(\boldsymbol{\theta})\}. \tag{54}$$

Note that the maximization of (52) can be also viewed as the maximization of (49) with $\boldsymbol{R}_t$ replaced by $\boldsymbol{I}_N$ (identity matrix of order N). It is convenient to rewrite (52) as

$$L_V(\boldsymbol{\theta}) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\log(2\pi) + \log\left(d_{i,i,t}^2\right) + \frac{\varepsilon_{i,t}^2}{d_{i,i,t}^2}\right). \tag{55}$$

We can see that (55) is the sum of log-likelihoods of the individual series, hence it can be maximized by separately maximizing each of the N terms. Each series is assumed to follow a univariate GARCH process. The most widely used model is the GARCH(1,1) model, in which case the conditional variances are given by

$$d_{i,i,t}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i d_{i,i,t-1}^2 \tag{56}$$

where $\omega_i > 0$, $\alpha_i \geq 0$, $\beta_i \geq 0$, $\alpha_i + \beta_i < 1$, $d_{i,i,0}^2 > 0$, $i = 1, \ldots, N$. We could of course include more lags in the model (where the lag lengths chosen for different series need not be the same) but the GARCH(1,1) is by far the most common choice. In general, the specification is not even restricted to the standard GARCH(p,q) model. The univariate models can be specified as any GARCH process that has normally distributed errors and satisfies appropriate stationarity conditions, as well as non-negativity constraints.

Once we have estimated the volatility parameters, we can proceed to the second step in maximizing (51). We now take $\widehat{\boldsymbol{\theta}}$ as given and maximize (53) with respect to $\boldsymbol{\phi}$, formally

$$\max_{\phi}\{L_C(\widehat{\boldsymbol{\theta}}, \boldsymbol{\phi})\}. \tag{57}$$

The second step thus consists in standardizing the residuals $\boldsymbol{\varepsilon}_t$ by their estimated conditional standard deviations and then using the standardized residuals $\boldsymbol{u}_t$ to estimate the parameters that govern the process of $\boldsymbol{R}_t$. The correlation structure is specified as follows. Consider an $N \times N$ matrix $\boldsymbol{Q}_t$ given by

$$\boldsymbol{Q}_t = (1 - a - b)\overline{\boldsymbol{Q}} + a\boldsymbol{u}_{t-1}\boldsymbol{u}_{t-1}^T + b\boldsymbol{Q}_{t-1}, \tag{58}$$

where $a \geq 0$, $b \geq 0$, $a + b < 1, \overline{\boldsymbol{Q}}$ is the unconditional covariance matrix of the standardized residuals and $\boldsymbol{Q}_0$ is positive definite. Equation (58) could be also generalized to include more lags. Using matrix $\boldsymbol{Q}_t$, matrix $\boldsymbol{R}_t$ can be obtained as

$$\boldsymbol{R}_t = \mathrm{diag}\left(q_{1,1,t}^{-1/2}, \ldots, q_{N,N,t}^{-1/2}\right) \boldsymbol{Q}_t \, \mathrm{diag}\left(q_{1,1,t}^{-1/2}, \ldots, q_{N,N,t}^{-1/2}\right), \tag{59}$$

where $\mathrm{diag}\left(q_{1,1,t}^{-1/2}, \ldots, q_{N,N,t}^{-1/2}\right)$ is a diagonal matrix with elements $q_{1,1,t}^{-1/2}, \ldots, q_{N,N,t}^{-1/2}$ on the main diagonal. The elements of $\boldsymbol{R}_t$ are thus of the form $r_{ij,t} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t}q_{j,j,t}}}$, $i, j = 1, \ldots, N$.

Under some reasonable regularity conditions formulated by Engle and Sheppard (2001), the two-stage maximum likelihood estimator will be consistent and asymptotically normal. Moreover, the model is formulated in such a way that it ensures positive definiteness of $\boldsymbol{H}_t$. To be more specific, $\boldsymbol{Q}_t$ is positive definite for all t because it is a weighted average of a positive semi-definite matrix $(\boldsymbol{u}_{t-1}\boldsymbol{u}_{t-1}^T)$ and positive definite matrices. Positive definiteness of $\boldsymbol{Q}_t$ then implies positive definiteness of $\boldsymbol{R}_t$ and given the restrictions on the parameters of the univariate GARCH models, $\boldsymbol{H}_t$ is positive definite as well. The exact formulations and proofs of the propositions that establish positive definiteness of $\boldsymbol{H}_t$ (for the more general case when (56) and (58) are of higher orders) can be found in Engle and Sheppard (2001).

Finally, let us show how we can make forecasts using the DCC-GARCH model. The one-step ahead forecast can be obtained easily. If all the information at time t is known, the equations of the model directly provide the forecast for the very next point in time, i.e. $t+1$. We have

$$\mathrm{E}_t\left(d_{i,i,t+1}^2\right) = \omega_i + \alpha_i \varepsilon_{i,t}^2 + \beta_i d_{i,i,t}^2, \tag{60}$$

$$\mathrm{E}_t(\boldsymbol{Q}_{t+1}) = (1-a-b)\overline{\boldsymbol{Q}} + a\boldsymbol{D}_t^{-1}\boldsymbol{\varepsilon}_t(\boldsymbol{D}_t^{-1}\boldsymbol{\varepsilon}_t)^T + b\boldsymbol{Q}_t, \tag{61}$$

$$\mathrm{E}_t(\boldsymbol{R}_{t+1}) = \mathrm{E}_t\left(\mathrm{diag}\left(q_{1,1,t+1}^{-1/2}, \dots, q_{N,N,t+1}^{-1/2}\right) \boldsymbol{Q}_{t+1}\, \mathrm{diag}\left(q_{1,1,t+1}^{-1/2}, \dots, q_{N,N,t+1}^{-1/2}\right)\right). \tag{62}$$

3. Central European Stock markets

The purpose of this chapter is to give a short overview of the stock markets in the Central European region. We focus on those stock markets whose relationships we examine in the empirical part. These are the Czech, Polish, Hungarian and German stock markets. Germany, although geographically a part of Central Europe, is taken as a benchmark for Western Europe. We first briefly summarize the development of the three emerging markets (Czech, Polish, Hungarian) and present some key figures of the four markets in question. In the second part of the chapter we provide a literature review on the relationships among the markets. We tried to select several empirical studies which vary in the methods and data used but the review is not intended to be exhaustive.

3.1. Characteristics of the Markets

Although the beginnings of the stock exchanges in Prague, Warsaw and Budapest date back to the nineteenth century, the World War II and the subsequent developments brought an end to trading at these exchanges. Following the collapse of the communist regime in Central and Eastern Europe, the exchanges started to write their modern history. In 1990 the Budapest Stock Exchange was the first one to reopen, followed by the Warsaw Stock exchange in 1991. The Prague Stock Exchange was established in 1992 and the first trading session took place in the following year.

The development of the emerging stock markets was significantly influenced by the privatization strategies of the individual countries. The mass privatization scheme adopted by the Czech Republic initially led to a dramatic increase in the number of companies listed on the Prague Stock Exchange. However, most of the firms were eventually delisted due to a lack of liquidity, which undermined confidence in the market. For example, the number of listed companies decreased by more than 80% between 1996 and 1997. In contrast, the approach chosen by Poland and Hungary was to first establish a framework for securities trading and after that list the companies through initial public offerings, thus ensuring a smoother development of the market[7] (Caviglia, Krause and Thimann, 2002).

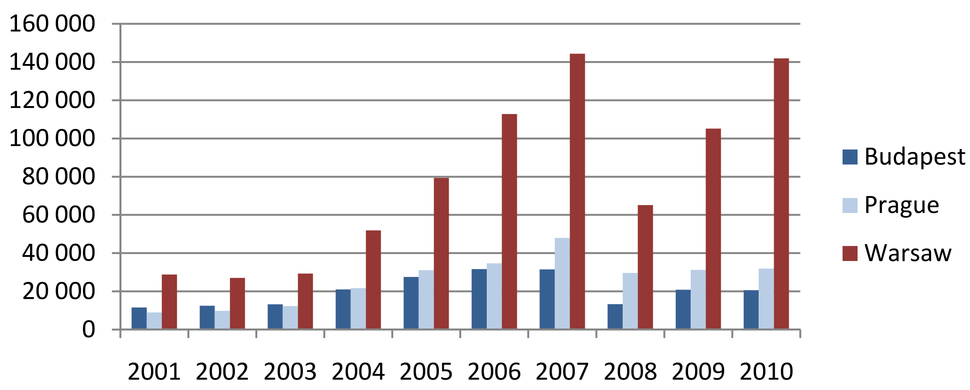
With the approaching accession to the European Union (EU), the Central European stock markets strengthened their credibility and started to attract foreign investors. In

[7] Yet, it should be noted that Poland switched to a mass privatization strategy in 1996.

connection with joining the EU, the three exchanges were also granted full membership in the Federation of European Securities Exchanges. In the following years the markets saw an increase in size, as well as in trading activity but this favorable development was interrupted in 2008, when the markets were hit by the worldwide financial crisis. Finally, we can add that the Budapest and Prague stock exchanges underwent some major changes in their ownership structure and consequently became members of the CEE Stock Exchange Group. The Group was officially launched in 2009 and besides the two members mentioned, it includes the stock exchanges of Vienna and Ljubljana.

To get a better idea of the development of the markets, Figure 3-1 and Figure 3-2 show the market capitalizations and the values of share trading at the Budapest, Prague and Warsaw stock exchanges between the years 2001 and 2010. Several observations can be made from these figures. First, we can notice the overall upward trend in the period 2004-2007 and the subsequent changes caused by the crisis. Second, during the whole ten-year period the Warsaw Stock Exchange had a significantly higher market capitalization than the other two stock exchanges. As for the values of share trading, the differences were much smaller and in two years (2004 and 2005) the Warsaw Stock Exchange was even surpassed by the Prague Stock Exchange. This can be interpreted as an indication of a relatively lower liquidity of the Polish stock market compared to the other two markets. However, it should be also noted that before the markets were hit by the crisis, the trading values at all the three exchanges grew proportionally more (on average) than the corresponding market capitalizations, thus indicating increasing liquidity. Finally, out of the three exchanges the Warsaw Stock Exchange seems to be the most successful in recovering its pre-crisis figures.

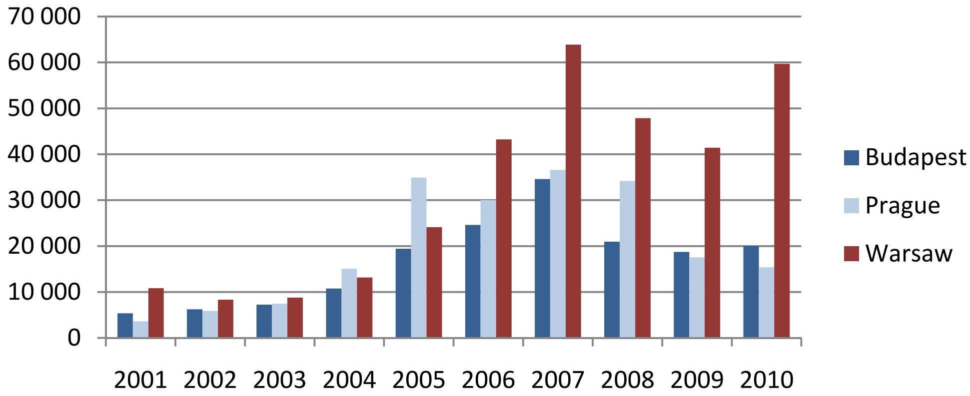
Figure 3-1: Year-end market capitalizations (EUR mil.)



Source: Federation of European Securities Exchanges

Note: The figures exclude foreign companies other than those exclusively listed on the exchange.

Figure 3-2: Values of share trading (EUR mil.)



Source: Federation of European Securities Exchanges

Notes: The figures include all trades, irrespective of the type of shares traded (domestic or foreign) and the mechanism by which the transaction occurred (electronic order book transaction, off-electronic order book transaction, dark pool transaction or reporting transaction). If we considered only trading of domestic shares, there would be some noticeable differences in case of the Prague Stock Exchange but the overall pattern would be very similar.

Table 3-1 summarizes some key data for the three exchanges on which we focused above and also for Deutsche Börse. This enables us to compare the characteristics of all the four markets that we will analyze in the empirical part. We can immediately notice that most of the figures for Deutsche Börse are one or two orders of magnitude higher than the corresponding figures for the other three exchanges. The Warsaw Stock Exchange, which is the largest of the three, has a 7.5 times lower market capitalization than Deutsche Börse. The differences are even more pronounced in case of trading values. For example, the annual value of share trading at the Budapest Stock Exchange is approximately equivalent to a three-day trading value at Deutsche Börse. It is obvious that the value of share trading at Deutsche Börse is not only considerably higher in absolute terms but also relative to market capitalization. This clearly demonstrates that the German stock market is much more liquid than the other three markets.

Another interesting comparison can be made by looking at the numbers of transactions and the implied average values of a transaction. We can, for example, notice that trading at the Warsaw Stock Exchange is characterized by a relatively large number of rather small transactions. An average transaction at Deutsche Börse or even at the Prague Stock Exchange is roughly three times larger than an average transaction concluded at the Warsaw Stock Exchange. This may be connected with the fact that there are quite a lot of companies listed on the Warsaw Stock Exchange relative to its market capitalization and the transaction is thus likely to involve shares that have low market value. Concerning the number of listed companies, we should also point out that the Prague Stock Exchange has a much higher percentage of foreign companies listed (approx. 40%) than the other exchanges. Note that this is also reflected in the value of foreign shares traded.

Table 3-1: Main market indicators for 2010

	Budapest SE	Prague SE	Warsaw SE	Deutsche Börse
Market capitalization at year-end (EUR mil.)	20 624.40	31 922.18	141 918.41	1 065 712.58
Value of share trading (EUR mil.)	20 006.6	15 391.0	59 693.0	1 744 015.9
Domestic shares	19 971.1	10 629.1	58 581.8	1 491 542.9
Foreign shares	35.5	4 761.9	1 111.1	252 472.8
Average daily value of share trading (EUR mil.)	78.2	61.1	235.9	6 812.6
Number of transactions	2 613 895	1 162 643	13 123 810	117 234 113
Average value of a transaction (EUR)	7 653.9	13 237.9	4 548.5	14 876.4
Listed companies at year-end	52	27	584	765
Domestic companies	48	16	569	690
Foreign companies	4	11	15	75

Source: Federation of European Securities Exchanges, own calculations

Notes: SE = Stock Exchange. The market capitalization figures exclude foreign companies other than those exclusively listed on the exchange. If not specified explicitly, the trading figures include all trades, irrespective of the type of shares traded (domestic or foreign) and the mechanism by which the transaction occurred (electronic order book transaction, off-electronic order book transaction, dark pool transaction or reporting transaction). Average daily trading value is the trading value divided by the number of days for which the stock exchange was open. Average value of a transaction is the trading value divided by the number of transactions. Exclusively listed foreign companies are included in domestic companies.

3.2. Review of the Literature

Let us now present the results of a few empirical studies which examined linkages among the Central European stock markets.

Gelos and Sahay (2000) investigated financial market comovements across European transition economies with a special focus on the Czech Republic, Hungary, Poland (hereinafter referred to as CE-3) and Russia. First, weekly stock return correlations computed over different time windows spanning the period 1994-1999 exhibited an upward trend. The authors then employed daily data to analyze the behavior of stock markets during three crisis periods, namely the Czech crisis (1997), the Asian crisis (1997-1998) and the Russian crisis (1998). A vector autoregression analysis was carried out, including impulse response functions and Granger causality tests, and it was also tested whether correlations (adjusted for an increase in variance) between the originating country's stock market and markets of the other countries significantly increased during the crises. To summarize the results, while stock market interactions were weak during the Czech crisis (except for an increase in the correlation between the Czech and Hungarian markets), there was a stronger response of the markets during the Asian crisis and quite substantial shock transmission during the Russian crisis. The Russian crisis was the only one during which the returns in the originating country "Granger caused" those in the other countries. However, there was no significant increase in correlations.

Scheicher (2001) analyzed the regional and global integration of CE-3 stock markets during 1995-1997. Employing daily returns, the author estimated a vector autoregression in which the errors were modeled using a multivariate GARCH model with constant correlations. A number of tests were performed to support the results. Overall, statistically significant spillovers of shocks were found in both returns and volatilities. However, there was a lack of global influences in volatilities (i.e. only regional spillover effects were found) and moreover, the estimated correlation coefficients were low and in most cases insignificant.

Cappiello et al. (2006) examined the financial integration of selected new EU member states (including CE-3) with the euro area and among themselves. The analysis relied on daily data for the period from 1994 to 2005. The whole sample was divided into two sub-samples, the first one covering the pre-convergence period (up to the end of 1999) and the second one the convergence period (from the beginning of 2000). The dependence between markets was measured by the conditional probability of comovements, i.e. the probability that, at time t, the returns on market i were lower (or higher) than the θ-quantile of the return distribution, conditional on the same event occurring on market j. The probabilities were estimated using the regression quantile-based methodology. Comparing the results for the two periods, the stock markets of CE-3 exhibited a significant increase in the probabilities of comovements both among themselves and vis-à-vis the euro zone (with the exception of the couple euro area-Hungary). The authors also assessed the extent to which these changes were driven by global factors, concluding that although in some cases the impact of global factors was significant, they could not entirely explain the increase in comovements.

The stock market integration of selected new EU members (CE-3 and Slovakia) was also investigated by Babetskii, Komárek and Komárková (2007), who, however, focused only on the integration with the euro area (i.e. not among the new member states). Using weekly data for the period 1995-2006, the authors applied the concepts of beta convergence (to measure the speed of convergence) and sigma convergence (to evaluate the degree of integration). The results revealed the existence of relatively fast beta convergence and in the case of the Czech Republic and Hungary there was evidence of an increased pace of convergence in the period 2001-2006 compared to the period 1995-2000. Nevertheless, the analysis also showed that neither the announcement of EU enlargement nor the enlargement itself had a major impact on beta convergence. Regarding sigma convergence, the markets exhibited an overall increase in the degree of integration during the period 1995-2004, yet divergence from the euro area was observed since 2005, which the authors explained by the fact that the examined stock markets experienced high growth (higher than that of the benchmark euro zone index).

Syllignakis and Kouretas (2006) used several different techniques to analyze daily and weekly data of seven Central and Eastern European (CEE) stock markets (including CE-3), the German stock market and the US stock market over the period 1995-2005. Applying the cointegration and common trends methodology, the markets were found to be partially integrated (with the number of cointegrating relations being less than the number of common trends). Moreover, the results indicated that five of the CEE markets (CE-3, Slovakia and Slovenia) together with the two developed markets had a significant common permanent component. An alternative insight into the relationships between the CEE markets and the developed ones was provided by the estimation of bivariate DCC-GARCH models. As for the stock markets of CE-3, there was evidence of an increasing trend in their conditional correlations with the two developed markets. The analysis suggested that the dependencies strengthened during the Asian and Russian crises and that afterwards the correlations declined but remained at relatively high levels until the end of the examined period.

Égert and Kočenda (2007b) used intraday (5-minute) data for the period from mid-2003 to early 2006 and estimated a series of bivariate DCC-GARCH models for stock markets of CE-3 and for three developed stock markets (France, Germany and the UK). The estimated correlations between the CE-3 markets and the French market (taken as a benchmark for Western Europe) were positive but very low (lower than 0.05). Similar values were obtained for the correlations among the CE-3 markets but in this case all the three correlation series at least showed an increasing trend. These results were in sharp contrast with the correlations among the developed markets. However, it should be probably said that the application of the DCC-GARCH model to intraday data is a little bit problematic due to the existence of intraday seasonalities. To overcome this difficulty, the authors considered an appropriately shortened time window for each day (11:00-14:40), thus leaving a relatively large amount of data unexploited.

The study of Égert and Kočenda (2007a) was similar in that it also used 5-minute data and it focused on the same stock markets. However, the analyzed period was shorter (mid-2003 to early 2005) and different methods were employed, namely cointegration tests, Granger causality tests (applied to returns and also to volatilities estimated using univariate GARCH models) and estimation of a vector autoregression model which included both returns and volatilities. While no robust cointegration relationship was found, there was evidence of short-term spillover effects. Granger causality tests revealed the existence of bidirectional causal relationships in returns, as well as in volatilities. Yet, the vector autoregression suggested that there were fewer interactions among the markets. We should add that the authors again considered the relatively short time window (11:00-14:40) but they also discussed the results for a longer time window (10:00-15:55), sometimes finding noteworthy differences.

Finally, Savva and Aslanidis (2010) examined comovements among five CEE stock markets (including CE-3) and the euro zone during the period from 1997 to 2008. Employing weekly data, the authors estimated bivariate smooth transition conditional correlation GARCH models, which assume the existence of different regimes with regime-specific constant correlations and allow for a smooth change between the correlation regimes. The Czech and Polish markets showed an increase in their correlations with the euro zone, while for Hungary there was no evidence of a significant change in correlation. In the case of Poland the shift in the correlation occurred before the EU accession, whereas for the Czech Republic it started before the accession date and gradually continued after the country joined the EU. Besides that, the results revealed that there was a significant increase in correlations among the CE-3 markets and that in general the shifts occurred after the increase in correlations with the euro zone. For some correlation pairs the analysis indicated the presence of a second change in correlations but the double transition models seemed to be just a refinement of the single transition ones. The authors also found that the increase in the correlations with the euro zone mostly reflected EU-related developments rather than the world-wide financial integration.

4. Data

In this chapter we first provide basic information on our data, mention some problems that we encounter and explain the steps that we take in order to obtain the final dataset. Next we discuss the issue of selecting the appropriate sampling frequency, which is one of the foremost problems in the practical application of realized measures. Finally we detail the construction of the realized measures and also explain the way in which we obtain the time series for the DCC-GARCH analysis.

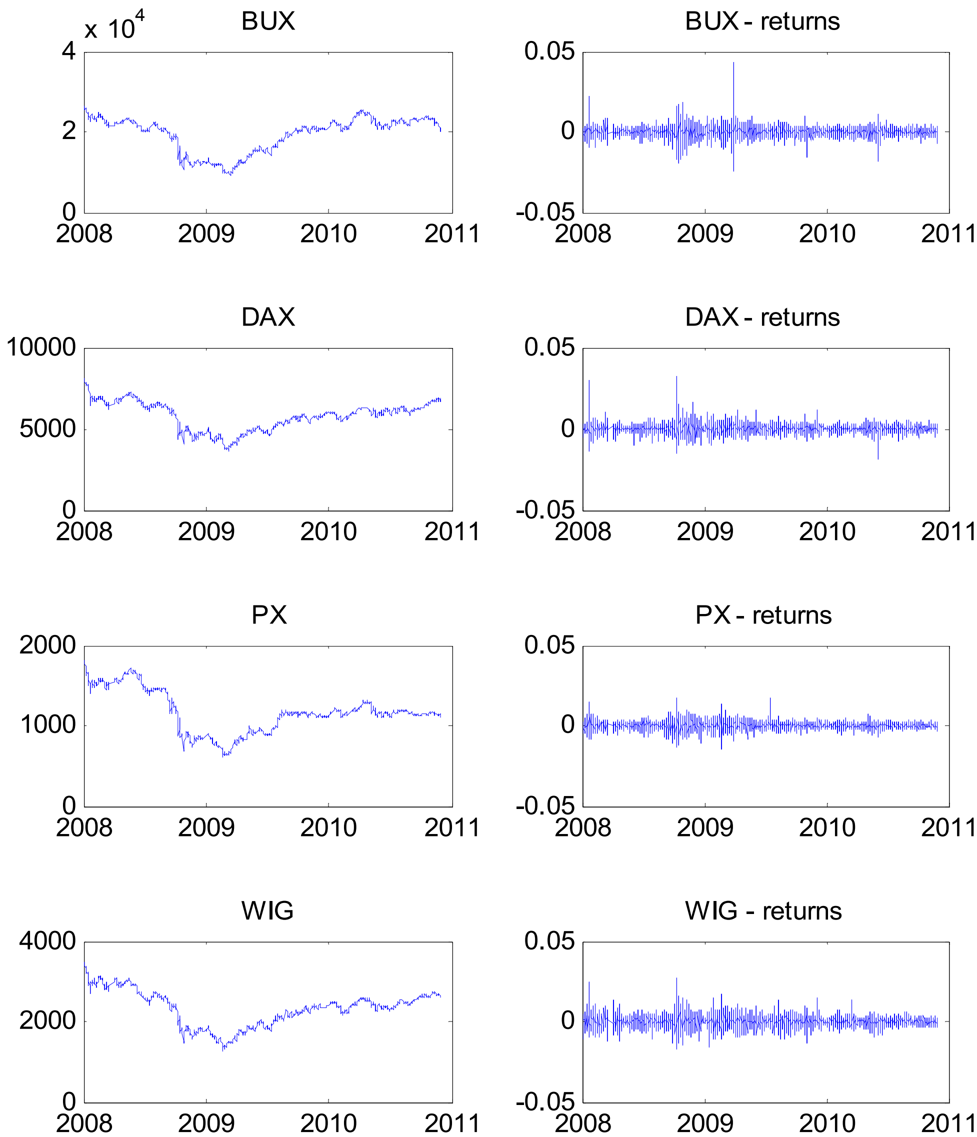
4.1. Description of the Dataset

As mentioned in the previous chapter, we focus on the stock markets of the Czech Republic, Hungary, Poland and Germany (which is taken as a benchmark for Western Europe). Each of the analyzed stock markets is represented by one stock index. The indices are the following: BUX for the Hungarian market, DAX for the German market, PX for the Czech market and WIG20 for the Polish market. Our sample covers the period from January 2, 2008 to November 30, 2010. For each index we have its values recorded at 5-minute intervals throughout each trading day (close prices of the 5-minute intervals are used). All the data were obtained from Tick Data, Inc. It should be noted that the data had been cleaned by the proprietary algorithms of Tick Data. For further information on the issues associated with the data filtering, we refer to Falkenberry (2002).

Since we want to examine interdependencies among the markets, we need the time series to be comparable across the different stock indices. Two issues have to be taken into account. First, there are days on which one stock exchange is open, while another one is closed due to a national holiday or other reasons. This is reflected in the fact that our sample period includes 726 trading days for BUX, 743 for DAX, 724 for PX and 732 for WIG20. Second, the stock exchanges have different trading hours. To be more specific, the 5-minute close prices of BUX, DAX, PX and WIG20 are available for the time windows of 9:05 to 16:30, 9:05 to 17:35, 9:30 to 16:00 and 9:35 to 16:10, respectively. Our solution to the first problem is to include only those days for which we have data on all the four stock indices. This condition is satisfied for 696 days. To overcome the second difficulty, for each day we consider only the time interval 9:35-16:00, which leaves us with 78 observations per day for each of the indices.

An additional practical problem is that there are missing values for some days. The approach that we take in dealing with this issue can be summarized as follows: If more than five observations are missing for some of the series on a given day, we remove the day from the sample. This procedure reduces the number of days to 691.[8] In this smaller sample there are still four days with missing values but in all cases it concerns only one or two observations, which should not have a significant influence on our results. These days are therefore retained in the sample. However, it has to be noted that for each missing observation we also remove the corresponding observations in the other series in order to ensure full comparability.

Figure 4-1: 5-minute index values and 5-minute logarithmic returns



Note: The return series do not include overnight returns.

[8] The removed days are the following: 30/12/2008, 19/05/2009, 13/08/2009, 30/12/2009 and 25/10/2010.

The final time series of the 5-minute index values and the corresponding 5-minute returns (first differences of logarithmic index values) are shown in Figure 4-1. Looking at the graphs, we can observe the effects of the global financial crisis. All the four indices declined sharply in the second half of 2008, reached their lowest points in the first quarter of 2009 and then started to rise again. A noteworthy difference is that while the PX index more or less stagnated from the last quarter of 2009 till the end of the examined period, the other three indices still showed an upward trend. The return plots provide evidence that the period of late 2008 and early 2009 was characterized by high volatility. Later on volatility returned to lower levels.

4.2. Construction of Variables

Let us first explain the choice of the 5-minute sampling scheme for index values. The asymptotic results derived in Section 2.1.2. suggest that prices should be sampled as frequently as possible in order to obtain accurate estimates of variances, covariances and correlations. However, the reality is more complicated. It is a well known fact that if data are sampled at very high frequencies, they are contaminated by the so-called market microstructure noise. The noise arises from various market frictions, such as discreteness of prices, bid-ask spreads or simultaneous quoting of different prices by competing market makers (Andersen, Bollerslev and Meddahi, 2011). Most researchers deal with this problem by sampling relatively sparsely, i.e. they use such frequencies at which the bias caused by microstructure noise is not a major concern.[9] The most common choice in the literature is to sample data at 5-minute intervals. Some studies use even lower frequencies, for example 30 minutes. It should be also noted that Zhang, Mykland and Aït-Sahalia (2005) proposed a methodology for determining the optimal sampling frequency.

It is very important to say that matters are even more complicated in a multivariate setting due to the problem of non-synchronous trading or nontrading, as pointed out for example by de Pooter, Martens and van Dijk (2008) or Barndorff-Nielsen et al. (2010). Non-synchronous trading refers to the fact that different assets do not usually trade at exactly the same instants. Nontrading occurs when one asset trades frequently over a certain period, while another one does not trade. If very high sampling frequencies are

[9] An alternative approach is to explicitly include noise in the price process and then design procedures that reduce its impact on the estimation results.

used, these phenomena can induce quite a significant bias in the measures of dependence (covariance, correlation) between assets.

Clearly, the selection of the appropriate sampling frequency is a nontrivial issue since we face a trade-off between the above mentioned biases and a potentially large stochastic error resulting from using low number of observations per day. As argued by Andersen et al. (2001), a key factor that has to be taken into account is the liquidity of the particular market (where lower frequencies should be used for less liquid markets). We follow the common practice in the literature and use the 5-minute sampling frequency. However, in light of the complexity of the problem and given the fact that the examined markets do not belong to the most liquid ones, we do not want to restrict our analysis to only one frequency. Therefore, we also sample prices (in our case index values) at 30-minute and 1-hour intervals.

The 30-minute and 1-hour price series are obtained from the 5-minute series. Two things should be clarified in this respect. Concerning the 30-minute series, the interval between the first two intraday observations is slightly shorter than 30 minutes since it runs from 9:35 to 10:00.[10] As for the 1-hour series, we disregard the interval 9:35-10:00, which means that the first price on each day is the one at 10:00. For all three frequencies we compute the realized measures using the formulas given in Section 2.1.1. However, it has to be stressed that overnight returns are excluded from our analysis. Taking the 5-minute series as an example, the first return on each day is calculated as the difference between the logarithmic prices at 9:40 and at 9:35 (providing both values are available).

So far we have focused on intraday data and the construction of daily realized measures. The second approach used in our analysis is the estimation of the DCC-GARCH model, for which we need daily returns. Since we want to allow for direct comparison of the results given by the two different methods, the daily returns should be computed over the same time intervals that we use for the construction of realized measures. Therefore, we calculate the return on day t by subtracting the day t logarithmic price at 9:35 from the day t logarithmic price at 16:00. This is equivalent to calculating the sum of all intraday returns for day t. It is thus important to bear in mind that when speaking about daily returns, we will always mean returns computed in the way described above (i.e. not as the

[10] For one day (14/07/2009) the observation at 9:35 is missing, so we use the value at 9:40 instead. Similarly, for one day (08/01/2010) the observation at 11:30 is missing and therefore we use the value at 11:35 instead.

first differences of daily closing logarithmic prices, in which case the daily returns would include the overnight returns).

The last issue we have to deal with is that the DCC-GARCH model should be applied to zero mean data (see equation (47)). It would be possible to directly assume that the returns have zero mean but to ensure that the condition is satisfied, we filter the daily return series by an AR(1) model. Formally, for each of the four series (BUX, DAX, PX, WIG20) we estimate the model

$$r_{i,t} = c_i + \varphi_i r_{i,t-1} + \varepsilon_{i,t}, \tag{63}$$

where $r_{i,t}$ stands for the individual daily return series. By this procedure we obtain the residuals $\varepsilon_{i,t}$ and these are then used as the input data in the DCC-GARCH estimation.

5. Empirical Findings

We analyze four markets, each represented by one stock index, which means that there are a total of six index pairs. Note that for the sake of simplicity, the WIG20 index will be referred to only as WIG, so the notation for the index pairs will be as follows: BUX-DAX, BUX-PX, BUX-WIG, DAX-PX, DAX-WIG and PX-WIG. Each part of our analysis is carried out for all the six pairs. The DCC-GARCH models are estimated in the bivariate form in order to allow the parameters to vary across the index pairs.

In the first part of this chapter we present the main statistics for correlations (as well as for covariances), make a comparison and explain the observed differences. In the second part we report detailed results for the DCC-GARCH and HAR models and discuss the dynamics of the correlations. In the last subsection we examine the forecasting performance of the models. All computations and estimations were carried out in MATLAB, version 7.10.0.499 (R2010a). For the estimation of the DCC-GARCH models we used the code from the UCSD GARCH Toolbox, which was developed by Kevin Sheppard.

5.1. Main Characteristics of Correlations and Covariances

Table 5-1 shows the means and standard deviations of the correlations from the DCC-GARCH models, the realized correlations and the realized bipower correlations. We also report the unconditional correlations of daily returns.

Table 5-1: Main statistics for correlations

	BUX-DAX		**BUX-PX**		**BUX-WIG**		**DAX-PX**		**DAX-WIG**		**PX-WIG**	
Uncond	0.594		0.621		0.627		0.649		0.694		0.680	
	Mean	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**
DCC	0.605	0.021	0.601	0.080	0.605	0.047	0.662	0.033	0.696	0.035	0.640	0.037
RC 1h	0.545	0.340	0.481	0.369	0.508	0.344	0.585	0.304	0.630	0.299	0.505	0.345
RC 30m	0.499	0.257	0.424	0.270	0.455	0.259	0.528	0.239	0.611	0.216	0.454	0.268
RC 5m	0.275	0.171	0.199	0.150	0.250	0.159	0.359	0.144	0.410	0.145	0.253	0.140
RBPC 1h	0.523	0.434	0.494	0.469	0.506	0.445	0.591	0.384	0.645	0.403	0.520	0.429
RBPC 30m	0.489	0.302	0.411	0.319	0.439	0.312	0.508	0.281	0.603	0.258	0.453	0.310
RBPC 5m	0.255	0.176	0.208	0.173	0.230	0.173	0.347	0.164	0.383	0.155	0.248	0.169

Notes: Uncond = Unconditional correlation of daily returns, SD = Standard Deviation

Several important observations can be made from the table. Let us first list the most distinctive features of the correlations and then discuss some of them in more detail. The unconditional correlations of daily returns, as well as the means of the correlations from the DCC-GARCH models, are all quite high. This is in accordance with strengthening of the linkages among the examined markets over the previous years, as documented in some of the studies presented in Section 3.2. Looking at the realized correlations and the realized bipower correlations, we notice that the means decrease when prices are sampled more frequently and that this downward bias is quite substantial when we move from the 30-minute frequency to the 5-minute frequency. On the other hand, the use of higher sampling frequencies leads to a considerable reduction in the standard deviation of the realized correlations, as well as the realized bipower correlations.[11] As for the relationship between the realized correlations and the corresponding realized bipower correlations, we can see that the means are usually very similar (in most cases slightly lower for the bipower correlations) but the bipower correlations have higher standard deviations, with the difference being more pronounced for lower frequencies.

Comparing the results for the different index pairs, all methods (unconditional correlations, DCC-GARCH models, realized correlations and realized bipower correlations) suggest that the strongest linkage is the one between DAX and WIG. The ordering of the other pairs depends on the method used. Overall, the realized correlations and the realized bipower correlations indicate that the three emerging markets (represented by BUX, PX and WIG) are more correlated with the German market than among themselves. However, we should add that sometimes the differences are very small (BUX-DAX compared to BUX-WIG or PX-WIG). In case of the DCC-GARCH models, the PX-WIG pair exhibits higher correlation than the BUX-DAX pair. According to the unconditional correlations, the dependence between PX and WIG is even stronger that between DAX and PX, while the BUX-DAX pair appears to be the least correlated one.

[11] The standard deviations of the realized correlations or the realized bipower correlations cannot be directly compared to the standard deviations of the correlations from the DCC-GARCH models because we use completely different methods to obtain the correlations.

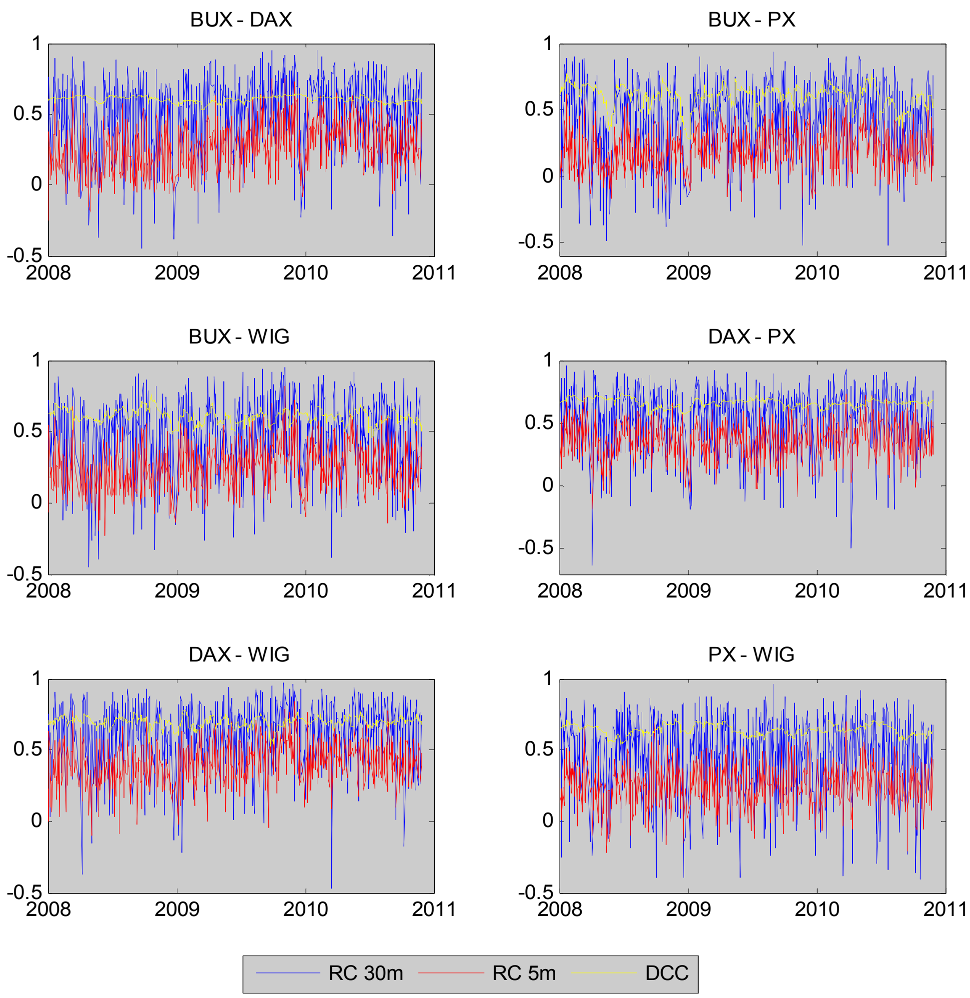
Table 5-2: Main statistics for covariances

	BUX-DAX		BUX-PX		BUX-WIG		DAX-PX		DAX-WIG		PX-WIG	
Uncond	9.16E-05		1.28E-04		1.36E-04		9.61E-05		1.08E-04		1.41E-04	
	Mean	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**
DCC	9.04E-05	5.38E-05	1.27E-04	1.41E-04	1.32E-04	1.15E-04	9.49E-05	7.07E-05	1.07E-04	6.52E-05	1.36E-04	1.39E-04
RCOV 1h	7.84E-05	1.42E-04	7.49E-05	1.34E-04	9.28E-05	2.20E-04	7.90E-05	1.56E-04	9.28E-05	1.58E-04	8.25E-05	1.60E-04
RCOV 30m	7.78E-05	1.39E-04	7.29E-05	1.56E-04	9.12E-05	1.69E-04	7.94E-05	1.62E-04	1.00E-04	1.79E-04	8.54E-05	1.83E-04
RCOV 5m	4.30E-05	7.85E-05	2.56E-05	5.19E-05	4.78E-05	7.79E-05	4.34E-05	8.27E-05	6.85E-05	1.15E-04	3.68E-05	7.13E-05
RBPCOV 1h	7.16E-05	1.94E-04	7.25E-05	1.79E-04	8.40E-05	2.34E-04	7.41E-05	1.83E-04	8.58E-05	1.90E-04	7.37E-05	1.96E-04
RBPCOV 30m	6.29E-05	1.12E-04	6.17E-05	1.35E-04	7.58E-05	1.48E-04	6.61E-05	1.56E-04	8.27E-05	1.44E-04	7.17E-05	1.68E-04
RBPCOV 5m	3.61E-05	6.77E-05	2.41E-05	5.56E-05	4.04E-05	6.74E-05	3.73E-05	6.73E-05	5.61E-05	8.09E-05	3.26E-05	5.79E-05

Note: Uncond = Unconditional covariance of daily returns, SD = Standard Deviation

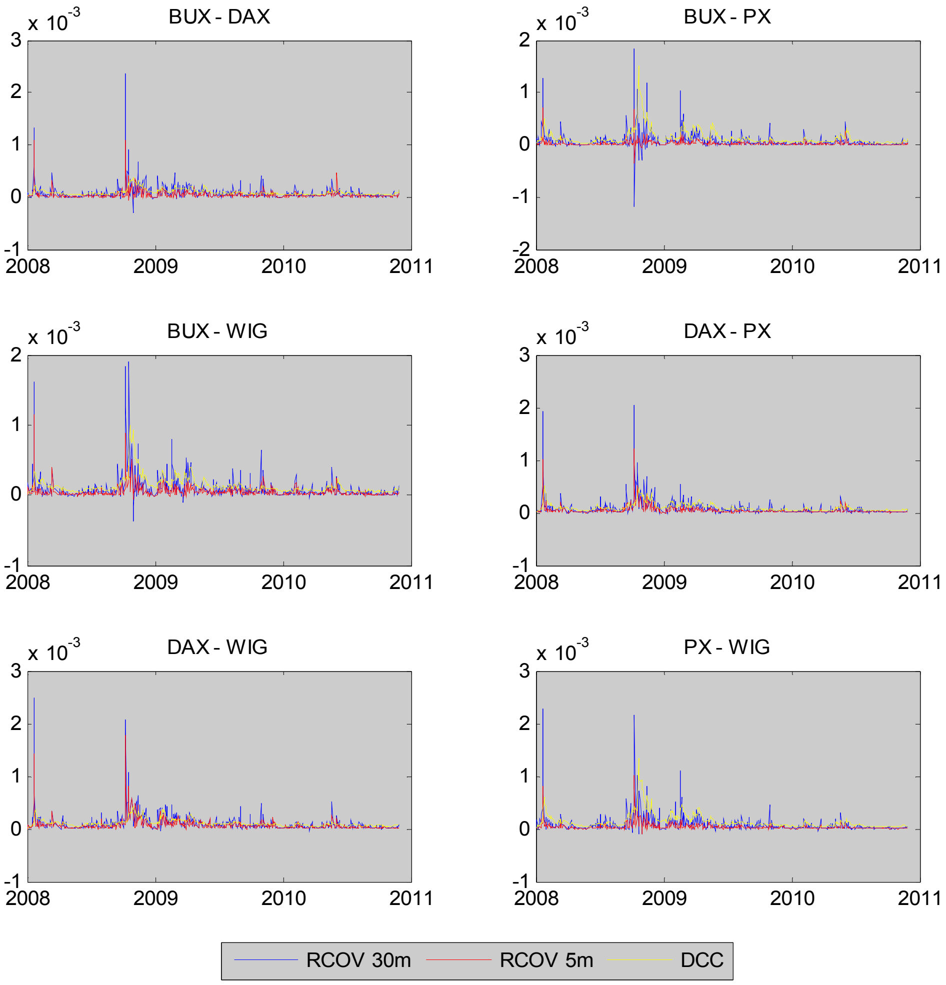
To have the full picture, we also report the results for covariances (Table 5-2). We can notice that the most important characteristics are similar to those of the correlations. When increasing the sampling frequency, the covariances are subject to downward bias (although in some cases the mean of the realized correlations slightly increases when moving from the 1-hour frequency to the 30-minute frequency), while the standard deviations of the realized covariances and the realized bipower covariances decrease (again with some minor exceptions). Concerning the differences between the realized covariances and the corresponding realized bipower covariances, the table shows that the bipower covariances have lower means but there is no universal relationship between the standard deviations (for the 1-hour frequency the bipower covariances have higher standard deviations while for the lower frequencies it is usually the other way round). Note that unlike correlations, covariances cannot be used to compare the degree of dependence across the different index pairs because correlations are dimensionless quantities while covariances are not.

Figure 5-1: Correlations from the DCC-GARCH models vs. realized correlations



Let us now focus on the above mentioned issue of the downward bias of the correlations, as well as the covariances. To illustrate the problem graphically, in Figure 5-1 we plot the correlations from the DCC-GARCH models together with the 5-minute and 30-minute realized correlations. The 1-hour realized correlations are not shown in order to avoid clutter in the figure. Analogously, Figure 5-2 provides a graphical comparison of covariances but we can see that the plots of the correlations make it easier to observe the bias associated with higher sampling frequencies. Similar figures for the realized bipower correlations and covariances can be found in the Appendix (Figure A-1 and Figure A-2).

Figure 5-2: Covariances from the DCC-GARCH models vs. realized covariances



The finding that correlations between stock returns decrease when the sampling frequency increases was first reported by Epps (1979). For this reason the phenomenon is usually referred to as the "Epps effect" in the later literature. Although the effect was observed as early as at the end of the 1970s, it started to receive greater attention only after high-frequency financial data became widely available. In the context of stock market realized covariances and correlations, empirical evidence for the Epps effect was provided for example by de Pooter, Martens and van Dijk (2008) or Barndorff-Nielsen et al. (2010). These researchers analyzed data for stocks traded on the New York Stock Exchange. It has to be pointed out that we can find a noteworthy difference between their results and the results reported here. While we observe a considerable drop in covariances and correlations when increasing the sampling frequency from 30 minutes to 5 minutes, the above mentioned authors report a substantial decrease in covariances[12] at such frequencies as 1 minute or 15 seconds, whereas the bias associated with the 5-minute frequency is relatively small.

De Pooter, Martens and van Dijk (2008) and Barndorff-Nielsen et al. (2010) attribute the observed bias to non-synchronous trading. This is broadly confirmed by Renò (2003) who investigated the determinants of the Epps effect.[13] More recently Zhang (2011) provided an analytic characterization of the Epps effect, formally showing that for positively related assets, non-synchronous trading induces a negative bias in the realized covariance and that the magnitude of the bias increases with the sampling frequency. Importantly, the theory developed by Zhang (2011) implies that the bias due to non-synchronization is more pronounced if the traded assets are less liquid. This provides an explanation for the differences between our results and those reported by de Pooter, Martens and van Dijk (2008) and Barndorff-Nielsen et al. (2010), as the Central European stock markets are clearly characterized by lower liquidity than the US stock market. It should be noted that our estimates can be also affected by microstructure noise but since the bias due to noise should not be large even for the 5-minute frequency, it seems that non-synchronous trading is of much greater importance.

A natural question that arises is whether the observed biases are statistically significant. To find this out, we perform paired t-tests, i.e. we test the significance of the difference between the means of two dependent samples. For each of the six index pairs, the test is performed for the correlations from the DCC-GARCH models versus the different realized correlations and realized bipower correlations, for the realized

[12] Barndorff-Nielsen et al. (2010) report the results for both covariances and correlations, the overall pattern being similar. De Pooter, Martens and van Dijk (2008) focus only on covariances but they also compute realized variances, which exhibit an upward bias for high frequencies. Therefore, it is quite clear that the correlations would suffer from a downward bias as well and that its magnitude would be even larger (in relative terms) than for the covariances.

[13] In fact, Renò (2003) identifies two factors that can explain the Epps effect, namely non-synchronous trading and lead-lag relationships. However, it is shown that non-synchronicity plays the main role and furthermore, the author remarks that non-synchronous trading itself can induce spurious lead-lag relations, as argued by Chan (1992) and Chan (1993).

correlations among themselves, for the realized bipower correlations among themselves and finally also for the realized correlations versus the corresponding bipower correlations. The same is done for the covariances. To give an example, we take the BUX-DAX correlations from the DCC-GARCH model and the BUX-DAX 5-minute realized correlations, calculate their difference for each day and then test the null hypothesis that the mean of the difference is equal to zero against the alternative that it is not equal to zero.

The p-values from the tests are reported in the Appendix (Table A-1 for the correlations and Table A-2 for the covariances). Overall, we can say that the differences among the correlations, as well as among the covariances, are statistically significant. Testing at the 5% level of significance, for most index pairs we fail to reject the null only when we compare either the lower-frequency realized correlations with the corresponding bipower correlations or the 1-hour covariances (realized or realized bipower) with the 30-minute covariances.

However, it is also necessary to say that the paired t-test relies on the assumption that the differences between the two samples are normally distributed, which is violated in some cases. To assess the possible impact on the results for correlations, we also try to use the Fisher-transformed correlations, for which there is evidence that the normality assumption is more likely to be satisfied (especially when one of the tested samples is the 1-hour realized correlations). The Fisher transformation is given by

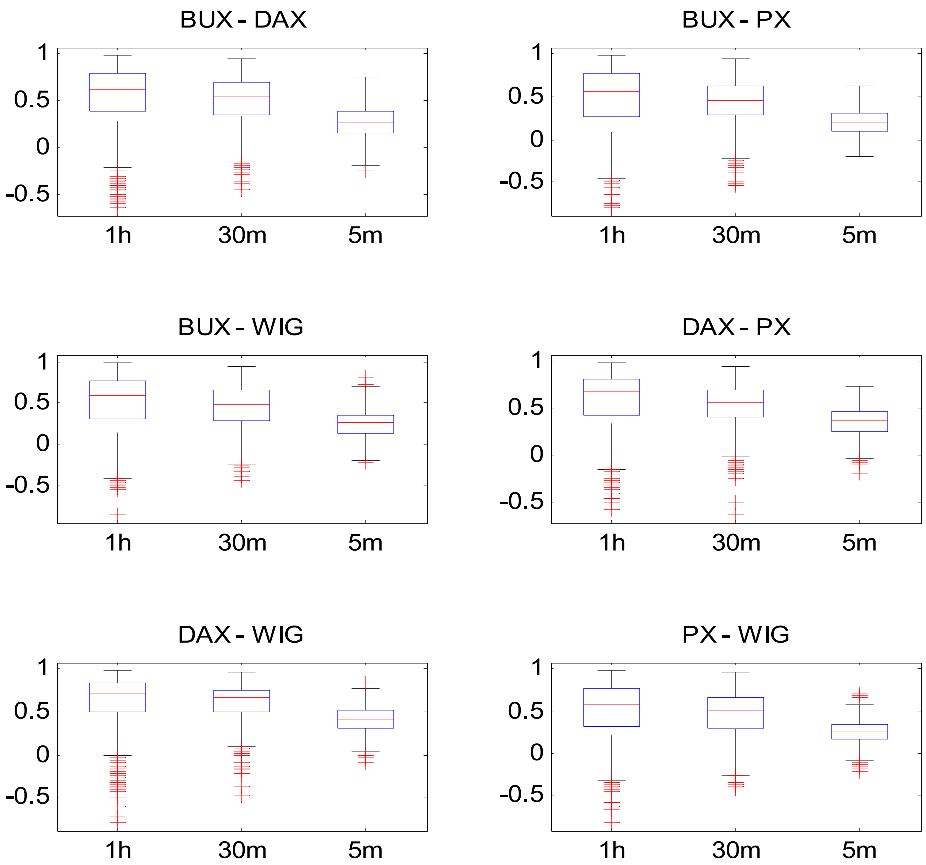
$$z_t = \frac{1}{2} \log \frac{1 + r_t}{1 - r_t}, \qquad (64)$$

where r_t stands for the correlation on day t and z_t is the Fisher-transformed correlation. A disadvantage of this approach is that we cannot perform the tests for all the combinations of correlations because in some cases (namely for the 1-hour and 30-minute bipower correlations) the estimated correlations occasionally fall outside of the [-1,1] interval and hence the Fisher transformation cannot be applied. The p-values from the paired t-tests for Fisher transformed correlations can be found in Table A-3 in the Appendix. The only noteworthy difference from the previous results is that for some index pairs we fail to reject the null when comparing the correlations from the DCC-GARCH models with the 1-hour realized correlations.

Finally, Figure 5-3 shows the boxplots of the 1-hour, 30-minute and 5-minute realized correlations. These pictures enable us to graphically compare the overall distributional characteristics of the realized correlations. Besides the downward bias discussed above, we can observe the reduction in the dispersion of correlations as we increase the sampling frequency. This is simply caused by the fact that we use more observations per day. For example, in the case of the 1-hour sampling frequency only six returns per day are available, resulting in a very high variance of the realized correlations. In the Appendix we show the boxplots of the realized bipower correlations (Figure A-3). The overall pattern is similar but in addition, Figure A-3 demonstrates the problem of the lower frequency (especially the 1-hour) bipower correlations not always falling in the [-1,1] interval, which was already mentioned above in connection with the Fisher transformation.

Taking into consideration all the characteristics of the realized correlations and realized bipower correlations, in the following analysis we focus only on the 5-minute and 30-minute correlations (and of course also on the correlations from the DCC-GARCH models).

Figure 5-3: Boxplots of realized correlations



5.2. Estimation Results for the DCC-GARCH and HAR Models

5.2.1. DCC-GARCH Models

In the previous subsection we already discussed some characteristics of the correlations and covariances from the DCC-GARCH models in comparison with the realized measures. Let us now present more detailed results for the DCC-GARCH models. In Table 5-3 we report the parameter estimates obtained by the two-step maximization of the log-likelihood function. All model equations include only the first lags (i.e. they are in the form of (56) and (58)). In addition, Table 5-3 shows the R^2 of the regressions in which the correlations from the DCC-GARCH models are used as an explanatory variable for the 5-minute and 30-minute realized correlations and realized bipower correlations. The purpose of these regressions is to examine whether the correlations from the DCC-GARCH models develop in a similar way to the realized correlations/realized bipower correlations.

Table 5-3: DCC-GARCH models

	BUX	DAX	PX	WIG
ω	7.85E-06 *	6.53E-06 **	5.31E-06 **	6.04E-06 **
	(4.22E-06)	(2.89E-06)	(2.60E-06)	(3.02E-06)
α	0.133 ***	0.100 ***	0.173 ***	0.108 ***
	(0.033)	(0.022)	(0.044)	(0.027)
β	0.835 ***	0.843 ***	0.812 ***	0.867 ***
	(0.042)	(0.034)	(0.041)	(0.029)

	BUX-DAX	BUX-PX	BUX-WIG	DAX-PX	DAX-WIG	PX-WIG
a	0.009	0.050 *	0.030	0.017	0.038	0.017
	(0.013)	(0.026)	(0.020)	(0.011)	(0.028)	(0.011)
b	0.965 ***	0.878 ***	0.888 ***	0.963 ***	0.844 ***	0.943 ***
	(0.068)	(0.052)	(0.053)	(0.026)	(0.080)	(0.032)
R^2						
RC 5m	0.003	0.031	< 0.001	< 0.001	0.002	0.011
RBPC 5m	0.002	0.022	< 0.001	< 0.001	< 0.001	0.016
RC 30m	0.003	0.034	0.003	0.003	0.005	0.005
RBPC 30m	< 0.001	0.030	0.003	0.001	0.005	0.003

*Note: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively. Significance was assessed using z-tests.*

The parameters of the univariate GARCH models take values that can be commonly found in the literature. The α parameters, which measure the impact of innovations, are relatively small, while the β parameters, which capture the persistence of volatility, are all higher than 0.8. All alphas and betas are significantly different from zero (at the 1% level). However, our focus is rather on the second set of parameters, i.e. a and b, as these describe the dynamics of the correlations. The innovation parameters (a) are considerably lower than the α parameters of the univariate models and in most cases the impact of innovations is not statistically significant even at the 10% level. On the other hand, the persistence parameters (b) are mostly higher than the β parameters of the univariate models and not surprisingly, all of them are significant at the 1% level. Overall, these results indicate a strong persistence of the correlations. A closer look at the parameter estimates reveals some differences among the index pairs. To be more specific, while the BUX-DAX, DAX-PX and PX-WIG correlations are characterized by a particularly strong persistence, the correlations for the DAX-WIG, BUX-PX and BUX-WIG pairs seem to be somewhat less persistent. Looking at the R^2 values reported in Table 5-3, we can say that the dynamics of the correlations from the DCC-GARCH models are quite different from those of the realized correlations and the realized bipower correlations. With only a few exceptions (the BUX-PX pair and partly the PX-WIG pair), the R^2 are lower than 1%.

5.2.2. HAR Models

Now we proceed to discuss the results for the HAR models. We use two different frequencies (5 minutes and 30 minutes) and two different estimators (realized correlations and realized bipower correlations), so combining these, we get four types of HAR models. Besides the parameter estimates and the R^2 of the models, we also report the p-values of several tests performed on the residuals. The tests that we use are the following: the Ljung-Box test for autocorrelation (H_0: no autocorrelation up to a specified lag), the Engle test for conditional heteroscedasticity, i.e. the presence of ARCH effects (H_0: no ARCH effects up to a specified order), and the Jarque-Bera test for normality (H_0: normality).

Let us start with the HAR models for the 5-minute realized correlations (Table 5-4). Concerning the significance of the lagged daily, weekly and monthly correlations, the index pairs differ in terms of which of the variables are significant and how strongly significant they are. Interestingly, none of the variables is significant for all index pairs. Recalling the results obtained for the DCC-GARCH models, one would expect that for the index pairs with the particularly persistent correlations, we should find a strong significance of the monthly and/or the weekly realized correlations. This is confirmed for the BUX-DAX pair but not in case of the other pairs. For the DAX-PX pair we find only a weak significance of the monthly correlations and the weekly correlations are not significant even at the 10% level. In case of the PX-WIG pair the monthly and the weekly correlations just switch their roles. In contrast, we find, for example, a strong significance of the monthly correlations for the DAX-WIG pair, which represents the group with the relatively less persistent correlations according to the DCC-GARCH models.

The R^2 values reported in Table 5-4 are quite low but this is not very surprising given the relatively high variation in the realized correlations. In any case, modeling the realized correlations by means of their lagged daily, weekly and monthly values yields higher R^2 than regressing them on the correlations from the DCC-GARCH models. As for the tests performed on the residuals, we can say that the residuals are well behaved. There is no evidence of autocorrelation or ARCH effects and in four cases we do not even reject (at the 5% level of significance) the hypothesis that the residuals are normally distributed. These results justify the use of the OLS method and indicate that the models provide an adequate fit to the data.

Table 5-4: HAR models for the 5-minute realized correlations

	BUX-DAX		BUX-PX		BUX-WIG		DAX-PX		DAX-WIG		PX-WIG	
c	0.060	***	0.098	***	0.096	***	0.189	***	0.130	***	0.168	***
	(0.021)		(0.022)		(0.023)		(0.043)		(0.037)		(0.033)	
$\beta^{(d)}$	0.028		-0.014		0.093	**	0.091	**	0.094	**	0.084	*
	(0.045)		(0.045)		(0.045)		(0.045)		(0.045)		(0.044)	
$\beta^{(w)}$	0.383	***	0.423	***	0.334	***	0.145		0.156		0.172	*
	(0.094)		(0.094)		(0.091)		(0.095)		(0.098)		(0.097)	
$\beta^{(m)}$	0.374	***	0.103		0.194	*	0.239	*	0.436	***	0.079	
	(0.106)		(0.131)		(0.114)		(0.142)		(0.123)		(0.153)	
R^2	0.173		0.064		0.102		0.039		0.094		0.026	
LB 10	0.942		0.928		0.271		0.311		0.683		0.971	
ARCH 5	0.750		0.709		0.714		0.250		0.127		0.871	
JB	0.191		0.303		0.185		0.005		0.002		0.073	

*Notes: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively. LB 10 = p-value of the Ljung-Box test for residual autocorrelation up to lag 10, ARCH 5 = p-value of the Engle test for the presence of fifth order ARCH effects in residuals, JB = p-value of the Jarque-Bera test for normality of residuals*

Now we move on to the HAR models for the 5-minute realized bipower correlations (Table 5-5). Comparing the results to those reported in Table 5-4, we can observe some differences in the significance of the explanatory variables, especially in case of the lagged daily correlations. While in the HAR models for the 5-minute realized correlations the lagged daily correlations were significant (at the 5% or 10% level) for four pairs, in the models for the bipower correlations they are significant just in one case (and the significance is only weak). A noteworthy consequence of the changes in the significance of regressors is that for one index pair (PX-WIG) none of the explanatory variables is significant. Nevertheless, those variables that were significant at the 1% level in the models for the realized correlations remain strongly significant also when the models are estimated for the bipower correlations. Notice that there are no newly significant variables in the models for the bipower correlations, i.e. if some variable was not significant in Table 5-4, it is not significant in Table 5-5 either. The R^2 values of the models for the realized bipower correlations are lower than those of the models for the realized correlations. Concerning the differences in the properties of the residuals, there is some improvement in the satisfaction of the normality assumption but for one pair (DAX-PX) we reject the null hypothesis of no ARCH effects at the 5% level of significance.

We have to add that all the HAR models discussed so far were also estimated for the Fisher-transformed correlations (see equation (64)). The results for these models are reported in the Appendix (Table A-4 and Table A-5). Overall, there are only very minor differences between the models discussed here and those for the Fisher-transformed correlations.

Table 5-5: HAR models for the 5-minute realized bipower correlations

	BUX-DAX		BUX-PX		BUX-WIG		DAX-PX		DAX-WIG		PX-WIG	
c	0.060	***	0.104	***	0.093	***	0.224	***	0.150	***	0.156	***
	(0.020)		(0.024)		(0.023)		(0.046)		(0.041)		(0.033)	
$\beta^{(d)}$	0.052		-0.041		0.024		0.081	*	0.048		0.056	
	(0.046)		(0.045)		(0.045)		(0.044)		(0.044)		(0.044)	
$\beta^{(w)}$	0.318	***	0.426	***	0.308	***	0.123		0.112		0.142	
	(0.095)		(0.097)		(0.097)		(0.097)		(0.102)		(0.101)	
$\beta^{(m)}$	0.402	***	0.114		0.268	**	0.152		0.451	***	0.173	
	(0.109)		(0.134)		(0.124)		(0.157)		(0.138)		(0.157)	
R^2	0.158		0.057		0.073		0.023		0.052		0.020	
LB 10	0.555		0.808		0.653		0.910		0.793		0.675	
ARCH 5	0.612		0.739		0.425		0.013		0.735		0.511	
JB	0.067		0.136		0.818		0.059		0.050		0.661	

*Notes: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively. LB 10 = p-value of the Ljung-Box test for residual autocorrelation up to lag 10, ARCH 5 = p-value of the Engle test for the presence of fifth order ARCH effects in residuals, JB = p-value of the Jarque-Bera test for normality of residuals*

Let us now devote a few lines to the HAR models for the 30-minute realized correlations, focusing mainly on the differences from the results reported above. As shown in Table 5-6, those variables that were significant at the 1% level in the previous models are still significant but in some cases only weakly. All other significant variables become now insignificant, with the notable exception of the lagged daily BUX-WIG correlations. Interestingly, some previously insignificant variables gain statistical significance, namely the lagged daily BUX-DAX correlations and also the lagged weekly DAX-PX correlations (although in the latter case the significance is only weak). The models for the 30-minute realized correlations have considerably lower R^2 than those for the 5-minute correlations, which we can attribute to the higher variance of the 30-minute correlations. The residuals do not appear to exhibit autocorrelation or ARCH effects but in all cases we strongly reject the hypothesis that they are normally distributed.

It is necessary to say that the non-normality of residuals can have an impact on the results of the t-tests for regression coefficients. For the purpose of comparison, Table A-6 in the Appendix summarizes the results for the HAR models estimated for the 30-minute Fisher-transformed realized correlations. It can be seen that when the Fisher transformation is applied on the correlations, then for most pairs we do not reject the normality of the residuals (at the 5% level). We can also notice that there are some slight changes in the significance of variables compared to the models for the non-transformed correlations.

Table 5-6: HAR models for the 30-minute realized correlations

	BUX-DAX		BUX-PX		BUX-WIG		DAX-PX		DAX-WIG		PX-WIG	
c	0.227	***	0.232	***	0.264	***	0.475	***	0.349	***	0.397	***
	(0.053)		(0.049)		(0.054)		(0.083)		(0.083)		(0.073)	
$\beta^{(d)}$	0.102	**	0.035		0.121	***	0.017		0.038		0.071	
	(0.045)		(0.045)		(0.044)		(0.044)		(0.044)		(0.044)	
$\beta^{(w)}$	0.191	**	0.272	***	0.205	**	0.175	*	0.099		0.077	
	(0.096)		(0.098)		(0.093)		(0.101)		(0.103)		(0.096)	
$\beta^{(m)}$	0.252	*	0.140		0.091		-0.095		0.290	*	-0.025	
	(0.132)		(0.136)		(0.138)		(0.174)		(0.165)		(0.179)	
R^2	0.058		0.038		0.047		0.007		0.019		0.009	
LB 10	0.223		0.795		0.554		0.292		0.635		0.483	
ARCH 5	0.974		0.093		0.520		0.234		0.949		0.247	
JB	< 0.001		< 0.001		< 0.001		< 0.001		< 0.001		< 0.001	

*Notes: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively. LB 10 = p-value of the Ljung-Box test for residual autocorrelation up to lag 10, ARCH 5 = p-value of the Engle test for the presence of fifth order ARCH effects in residuals, JB = p-value of the Jarque-Bera test for normality of residuals*

The HAR models for the 30-minute realized bipower correlations (Table 5-7) can be described as follows: In most cases the explanatory variables become less significant or insignificant (with only one or no significant variable in the individual regressions), the R^2 values further decrease and for all pairs we strongly reject the normality of the residuals. An interesting finding is that we find no significance of the monthly correlations and that out of the five variables that were significant at the 1% level in the models for the 5-minute correlations, only two remain significant. It should be reminded that we cannot apply the Fisher transformation on the 30-minute realized bipower correlations (due to the fact that they occasionally exceed the value of 1), so it is not possible to make a comparison with the models for the transformed correlations.

Table 5-7: HAR models for the 30-minute realized bipower correlations

	BUX-DAX		BUX-PX		BUX-WIG		DAX-PX		DAX-WIG		PX-WIG	
c	0.316	***	0.251	***	0.319	***	0.413	***	0.492	***	0.472	***
	(0.060)		(0.053)		(0.060)		(0.075)		(0.105)		(0.084)	
$\beta^{(d)}$	0.050		0.049		0.122	***	0.021		0.026		0.039	
	(0.044)		(0.045)		(0.044)		(0.044)		(0.043)		(0.043)	
$\beta^{(w)}$	0.223	**	0.176	*	0.098		0.211	**	-0.100		-0.020	
	(0.098)		(0.101)		(0.095)		(0.101)		(0.112)		(0.103)	
$\beta^{(m)}$	0.084		0.157		0.051		-0.050		0.260		-0.061	
	(0.143)		(0.148)		(0.154)		(0.168)		(0.203)		(0.206)	
R^2	0.026		0.023		0.025		0.012		0.003		0.001	
LB 10	0.205		0.595		0.421		0.678		0.549		0.793	
ARCH 5	0.636		0.107		0.411		0.435		0.838		0.589	
JB	< 0.001		< 0.001		< 0.001		< 0.001		< 0.001		< 0.001	

*Notes: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively. LB 10 = p-value of the Ljung-Box test for residual autocorrelation up to lag 10, ARCH 5 = p-value of the Engle test for the presence of fifth order ARCH effects in residuals, JB = p-value of the Jarque-Bera test for normality of residuals*

5.2.3. Comparison of Correlation Dynamics

To conclude the subsection on the estimation results, we plot the correlations from the DCC-GARCH models together with the fitted values from the HAR models. Figure 5-4 shows the fitted values from the HAR models for the 5-minute and 30-minute realized correlations, while in Figure 5-5 we focus on the models for the realized bipower correlations. Besides again demonstrating the bias discussed in the previous subsection, these pictures enable us to compare the correlation dynamics suggested by the various models that we estimated. It should be noted that we could of course directly compare the dynamics of the correlations from the DCC-GARCH models with those of the realized correlations and realized bipower correlations using for example Figure 5-1 and Figure A-1. However, such comparison is made a bit difficult by the large variation of the 30-minute correlations. The HAR models should capture the main dynamics of the realized correlations/realized bipower correlations (recall that in almost all cases there was no evidence of autocorrelation or ARCH effects in the residuals) and plotting of the fitted values allows for an easy visual inspection of the similarities and differences. We thus find this comparison useful.

Looking at Figure 5-4, one of the most interesting findings is that despite the bias and the differences in the HAR models (significance of variables, R^2) the dynamics of the 5-minute realized correlations are generally very similar to those of the 30-minute realized correlations. As for the dynamics of the 5-minute bipower correlations versus the 30-minute ones, sometimes they are characterized by a relatively high degree of similarity (e.g. the BUX-DAX pair) but in some cases there are considerable differences (e.g. the DAX-WIG pair). If we look at the dynamics of the realized correlations in comparison with the corresponding bipower correlations, we find out that the differences are only minor at the 5-minute frequency, but become somewhat more pronounced (at least for some pairs) at the 30-minute frequency. Still, we can conclude that there is a general similarity in the dynamics of the 5-minute and 30-minute correlations.

In contrast, the DCC-GARCH models often suggest different correlation dynamics, which was already indicated by the low R^2 values reported in Table 5-3. There are some index pairs and periods of time for which all the correlations seem to follow similar time paths (e.g. the BUX-WIG pair in 2008 and 2009) but in most cases the contrast is quite striking (the BUX-DAX pair can serve as a good example). In spite of this, we are still able to identify some general tendencies in the development of correlations during the analyzed period. We can usually observe an initial drop in correlations, after which the correlations increased, reflecting the downturn in stock markets. Afterwards, there is a decrease in correlations at the end of 2008/beginning of 2009, i.e. around the time when the markets reached the bottom. During 2009 the correlations increased again and then another fall can be observed at the end of 2009/beginning of 2010, typically followed by a temporary rise in correlations and a further decrease later in 2010. This can be interpreted as an indication that the markets recovered from the global financial crisis and began to be more influenced by domestic events.

Figure 5-4: Correlations from the DCC-GARCH models vs. fitted values from HAR models for realized correlations

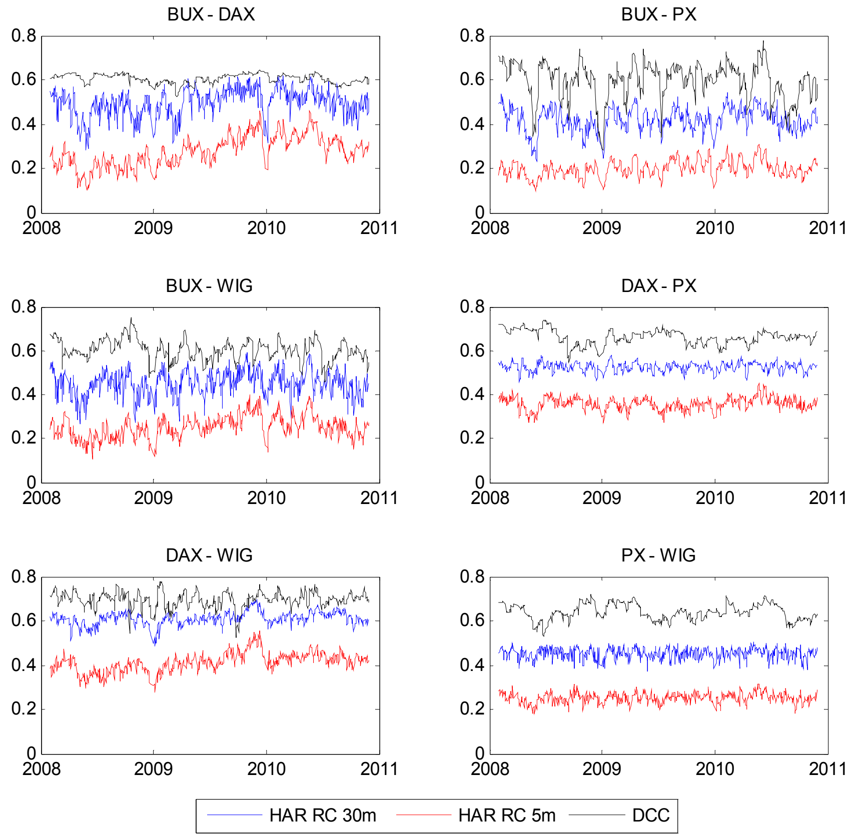
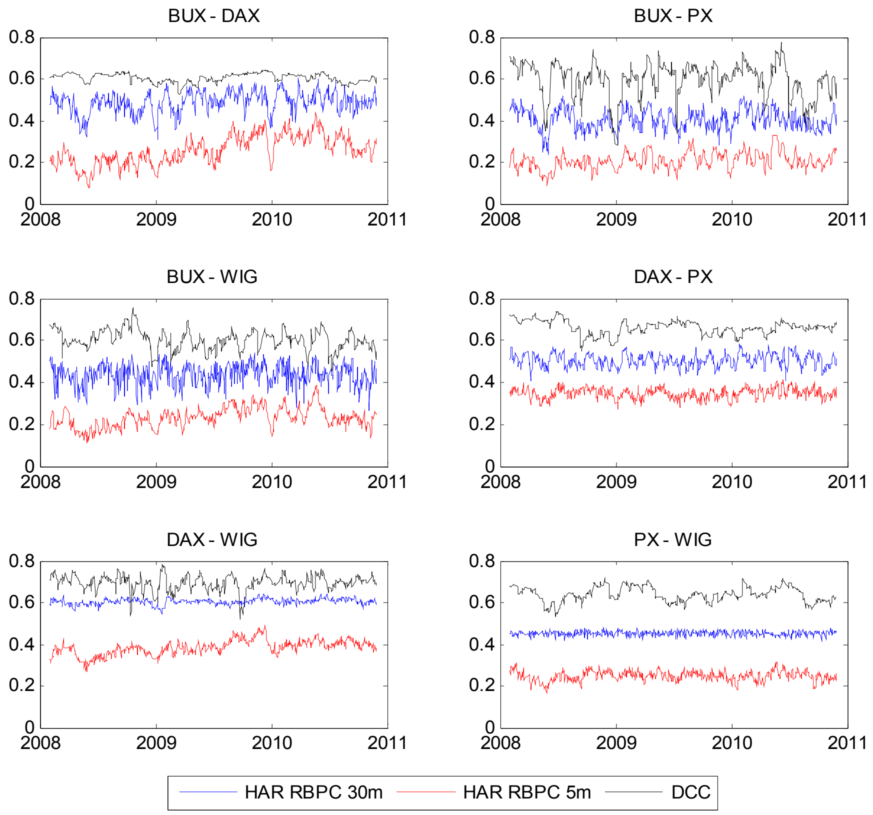


Figure 5-5: Correlations from the DCC-GARCH models vs. fitted values from HAR models for realized bipower correlations



5.3. Forecasting Exercise

5.3.1. Description of the Setting

So far, we have focused on analyzing the dynamics of correlations during the whole period under study, thus estimating the models on all the available data (January 2008 to November 2010). Now we would like to find out whether the models are able to predict the future development of correlations. For this purpose, we have to divide the sample into two parts, where the first part will contain the information that we know and the second part will represent "the future". Therefore, we divide the analyzed period into an in-sample period of 550 days and an out-of-sample period of the remaining 141 days. We presume that the in-sample period is long enough to allow the markets to absorb the effects of the crisis. The out-of sample period covers seven months[14] and its length relative to the length of the in-sample period is approximately equal to 1:4 (i.e. the lengths of the out-of-sample period and of the whole period under study are in the ratio of 1:5). All the models described in the previous subsection are reestimated using only the data for the in-sample period and based on the obtained parameter estimates, the models produce one-step ahead forecasts of correlations for the out-of-sample period. In other words, we keep the estimated parameters fixed and taking the more recent observations one by one[15], each time we generate the forecast for the next day.

To evaluate the forecasts, we employ the approach introduced by Mincer and Zarnowitz (1969), who suggest to regress the realizations of a given time series on a constant and the forecasts. Such a regression is commonly referred to as the Mincer-Zarnowitz regression and for the one-step ahead forecasts of realized correlations it takes the form

$$RC_{t+1} = b_0 + b_1 \mathrm{E}_t(RC_{t+1}) + u_{t+1}. \tag{65}$$

As argued by Mincer and Zarnowitz (1969), if the forecast is unbiased and efficient, the coefficients b_0 and b_1 are equal to 0 and 1, respectively. Moreover, the higher the R^2 of the regression, the better is the predictive power of the forecast.

One question that arises is what we should use as the dependent variable in the Mincer-Zarnowitz regressions. In our analysis we decided to employ the 5-minute realized

[14] The in-sample period ends on April 29, 2010.

[15] In case of the DCC-GARCH models the observations are first filtered by the same AR(1) model that is used for the filtration of in-sample returns.

correlations. First, we regress them on the forecast from the HAR models for the 5-minute realized correlations, by which we obtain a certain kind of benchmark. We will thus refer to this regression and the forecast used in the regression as the benchmark regression and the benchmark forecast, respectively. It is then possible to make a comparison by running similar regressions for the forecasts from the other models, i.e. using the same response variable and changing the explanatory variable. However, even more interesting is to include the alternative forecast in the benchmark regression (thus having two explanatory variables) and observe how it affects the coefficient estimates, the significance of explanatory variables and the R^2 of the regression. Such an analysis can help us to find out whether the particular alternative forecast contains valuable information that is not embodied in the benchmark forecast and whether the inclusion of the alternative forecast significantly improves the predictive power of the forecast.

5.3.2. Results

The key results are summarized in the following six tables (Table 5-8 to Table 5-13). For each index pair the first row shows the benchmark regression, then we report the results for the regressions that additionally include one alternative forecast and finally we include all the forecasts in one regression.

Table 5-8: Evaluation of forecasts for the BUX-DAX correlations

const	RC 5m		RBPC 5m		RC 30m	RBPC 30m	DCC	R^2
0.054	0.870	***						0.096
(0.072)	(0.227)							
0.044	-0.174		1.145	**				0.124
(0.072)	(0.546)		(0.546)					
0.071	0.904	***			-0.054			0.096
(0.120)	(0.301)				(0.313)			
0.130	0.907	***				-0.176		0.098
(0.142)	(0.235)					(0.283)		
-0.051	0.804	***					0.211	0.097
(0.277)	(0.282)						(0.538)	
-0.246	-0.636		1.341	**	0.190	-0.298	0.722	0.136
(0.299)	(0.691)		(0.580)		(0.435)	(0.401)	(0.575)	

*Note: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively.*

Table 5-9: Evaluation of forecasts for the BUX-PX correlations

const	RC 5m	RBPC 5m	RC 30m	RBPC 30m	DCC	R^2
-0.013	1.126 ***					0.077
(0.069)	(0.331)					
-0.016	-0.052	1.133				0.090
(0.069)	(0.899)	(0.805)				
-0.109	0.687		0.440			0.085
(0.109)	(0.509)		(0.387)			
-0.031	1.071 ***			0.074		0.077
(0.105)	(0.408)			(0.319)		
-0.095	0.711 *				0.287	0.092
(0.087)	(0.427)				(0.188)	
-0.151	-1.238	1.479 *	0.951	-0.486	0.164	0.119
(0.121)	(1.096)	(0.861)	(0.607)	(0.479)	(0.199)	

*Note: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively.*

Table 5-10: Evaluation of forecasts for the BUX-WIG correlations

const	RC 5m	RBPC 5m	RC 30m	RBPC 30m	DCC	R^2
0.014	0.977 ***					0.096
(0.067)	(0.254)					
0.010	0.601	0.420				0.100
(0.067)	(0.545)	(0.539)				
-0.090	0.636		0.422			0.105
(0.110)	(0.384)		(0.358)			
-0.228	0.651 **			0.747 *		0.116
(0.151)	(0.312)			(0.421)		
0.245	1.053 ***				-0.413	0.103
(0.227)	(0.264)				(0.388)	
0.018	0.156	0.684	-0.105	0.875	-0.480	0.132
(0.271)	(0.632)	(0.550)	(0.493)	(0.584)	(0.397)	

*Note: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively.*

Table 5-11: Evaluation of forecasts for the DAX-PX correlations

const		RC 5m	RBPC 5m	RC 30m	RBPC 30m	DCC		R^2
0.261		0.346						0.004
(0.162)		(0.441)						
0.296	*	0.794	-0.564					0.008
(0.171)		(0.808)	(0.852)					
0.458		0.481		-0.471				0.008
(0.324)		(0.482)		(0.671)				
0.198		0.276			0.177			0.005
(0.230)		(0.477)			(0.452)			
-0.229		0.088				0.884	**	0.038
(0.275)		(0.451)				(0.403)		
0.064		0.778	-0.840	-1.333	0.942	0.842	**	0.057
(0.389)		(0.840)	(0.945)	(0.925)	(0.674)	(0.406)		

*Note: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively.*

Table 5-12: Evaluation of forecasts for the DAX-WIG correlations

const		RC 5m	RBPC 5m	RC 30m	RBPC 30m	DCC	R^2
0.304	*	0.321					0.004
(0.178)		(0.411)					
0.472	**	0.803	-0.928				0.018
(0.214)		(0.534)	(0.661)				
0.402		0.500		-0.283			0.006
(0.254)		(0.529)		(0.526)			
0.540		0.351			-0.411		0.006
(0.589)		(0.418)			(0.978)		
0.076		0.271				0.361	0.009
(0.344)		(0.417)				(0.465)	
0.428		0.864	-0.918	-0.159	-0.233	0.365	0.025
(0.703)		(0.616)	(0.671)	(0.541)	(1.005)	(1.005)	

*Note: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively.*

Table 5-13: Evaluation of forecasts for the PX-WIG correlations

const	RC 5m	RBPC 5m	RC 30m	RBPC 30m	DCC	R^2
-0.279 *	2.191 ***					0.078
(0.161)	(0.640)					
-0.282 *	2.683 **	-0.483				0.080
(0.162)	(1.173)	(0.965)				
-0.252	2.235 ***		-0.084			0.078
(0.236)	(0.702)		(0.540)			
0.320	1.514 *			-0.968		0.092
(0.437)	(0.785)			(0.657)		
-0.983 ***	1.394 **				1.392 **	0.120
(0.315)	(0.699)				(0.540)	
-0.478	2.761 **	-1.811	-0.141	-0.773	1.408 **	0.139
(0.696)	(1.261)	(1.097)	(0.544)	(0.783)	(0.607)	

*Note: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively.*

To provide a comparison and supporting evidence for our findings, Table A-7 in the Appendix presents the results for the regressions in which we use only one forecast, i.e. we simply change the explanatory variable instead of including the alternative forecast in the benchmark regression. One important thing to note is that when we use forecasts obtained from lower-frequency data (30-minute, daily), we cannot generally expect the constant term in the regression to be close to 0 due to the bias discussed in Section 5.1.

A very interesting finding is that the results differ across the index pairs. Let us comment on each of the six cases. Concerning the BUX-DAX pair (Table 5-8), the benchmark forecast performs quite well (b_0 and b_1 are close to 0 and 1, respectively, and the regressor is strongly significant). However, when we include the forecast from the model for the 5-minute bipower correlations, the newly added variable is significant (with the coefficient not far from 1), while the benchmark forecast becomes insignificant. Moreover we observe quite a significant increase in the R^2 of the regression. Note that if the 5-minute bipower correlation forecast is used as the only explanatory variable (see Table A-7), the R^2 remains almost the same and b_0 and b_1 are even closer to the desired values that in the benchmark regeression. The rest of Table 5-8 shows that when we include other forecasts in the benchmark regression, they turn out to be insignificant and the increase in the R^2 is very small. Also, the 5-minute bipower correlation forecast remains significant even in the regression that includes all the forecasts.

As for the BUX-PX case (Table 5-9), the results for the benchmark regression are again satisfactory but we observe a slightly different pattern when including the alternative forecasts. With the exception of the 30-minute bipower correlation forecast, which is clearly inferior, the inclusion of the alternative forecasts leads to a situation where either both explanatory variables are insignificant, or one of the variables is insignificant and the other one is only weakly significant (at the 10% level). This indicates that all these forecasts embody a similar kind of information, which is also confirmed by similar R^2 values reported in Table A-7. Nevertheless, we can notice that in the regression that includes the 5-minute bipower correlation forecast, the coefficients for the benchmark and for the alternative forecast are close to 0 and 1, respectively. Moreover, when we include all forecasts in one regression, the 5-minute bipower correlation forecast is weakly significant, while all other variables are insignificant. In the light of these findings, it is quite interesting that out of the regressions with two explanatory variables, the regression with the highest R^2 is not the one that includes the 5-minute bipower correlation forecast but the one with the DCC-GARCH forecast (although the difference is small).

Turning to the BUX-WIG pair (Table 5-10), the results for the benchmark regression do not differ much from the previous two cases, except for the fact that the b_1 coefficient is even closer to 1. Similarly to the BUX-PX case, if the 5-minute bipower correlation forecast or the 30-minute correlation forecast is included in the benchmark regression, the explanatory variables make each other insignificant. In contrast, the DCC-GARCH forecast does not have any influence on the significance of the benchmark forecast and is itself insignificant. The most interesting result is the one obtained for the regression that includes the 30-minute bipower correlation forecast. In this regression both explanatory variables are significant (at least at the 10% level), which has not occurred in any of the cases discussed so far. Moreover, the regression with the 30-minute bipower correlation forecast has a higher R^2 than those which include the other alternative BUX-WIG forecasts. However, if all variables are included in one regression, none of them is significant.

Unlike the previous benchmark forecasts, the forecast for the DAX-PX pair has very little predictive power (see Table 5-11). The inclusion of the alternative forecasts does not lead to any significant improvement, with one notable exception, namely the

DCC-GARCH forecast.[16] The coefficient for the DCC-GARCH forecast is close to 1 and the variable is significant at the 5% level. Yet, the R^2 is quite low compared to the previous index pairs. Concerning the DAX-WIG pair (Table 5-12), we again obtain poor results for the benchmark regression but this time none of the alternative forecasts significantly changes the situation.

Turning to the results for the last pair, i.e. PX-WIG (Table 5-13), we can see that the R^2 of the benchmark regression is comparable to those obtained for the first three index pairs but the forecast is biased and inefficient. When we include the alternative forecasts, two different outcomes can be observed. The forecasts from the HAR models are insignificant and even though in some cases they slightly change the significance of the benchmark forecast, it has to be pointed out that the coefficients for these forecasts are negative. On the other hand, the DCC-GARCH forecast turns out to be significant and its inclusion is associated with relatively large increase in R^2. Nevertheless, we must also add that similarly to the benchmark forecast, the DCC-GARCH forecast is biased and inefficient (see Table A-7). When we include all variables in one regression, both the benchmark and the DCC-GARCH forecast remain significant.

Finally, we should say that we also experimented with the forecasts for the Fisher-transformed correlations, using the 5-minute Fisher-transformed correlations as the response variable in the regressions. In the Appendix we report six tables (Table A-8 to Table A-13) analogical to those shown above.[17] Generally, the regressions for the Fisher-transformed correlations have slightly higher R^2 values but the overall pattern of results is very similar.

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[16] It is probably worth mentioning that for the DAX-PX pair there is a noteworthy difference between the results reported in Table 5-3 and those that we obtain if the model is estimated using only the data for the in-sample period. In the latter case the a and b parameters are equal to 0.064 and 0.68, respectively.

[17] Note that the realized correlation and realized bipower correlation forecasts are obtained from the HAR models for the Fisher-transformed correlations. The DCC-GARCH forecasts are generated in the same way as before and we only apply the Fisher transformation on the forecasts.

6. Conclusion

In this thesis we studied the interdependencies among the stock markets of the Czech Republic, Poland, Hungary and Germany in the period 2008-2010. We first described the theories underlying our calculations and provided an overview of the Central European stock markets. Afterwards we devoted a short chapter to some data issues and finally we presented our empirical findings. We contribute to the research on the Central European stock markets by analyzing their interdependencies with the use of high-frequency data. We studied the main characteristics and dynamics of realized correlations and compared the results to those given by the DCC-GARCH models. There are several factors that make our analysis particularly interesting, namely (i) the use of both the realized correlations and the realized bipower correlations (ii) the computation of realized measures for different sampling frequencies, and (iii) the fact that the period under study includes the recent financial crisis.

When comparing the main characteristics of the correlations, we observed the so-called Epps effect, i.e. the decrease in correlations for higher sampling frequencies, which is attributable to non-synchronous trading. Interestingly, we found a considerably larger downward bias for the 5-minute frequency than the researchers who investigated the US stock market. This illustrates the role that market liquidity can play in affecting the correlation results. Another distinct feature of the realized correlations, as well as the realized bipower correlations, is the decrease in variance for higher sampling frequencies. Overall, these findings show the difficulty of selecting the appropriate sampling frequency and point to the importance of developing such estimators that would be able to handle non-synchronous trading. In this respect, we can mention for example the recent works of Zhang (2011) and Barndorff-Nielsen et al. (2010). Concerning the differences among the examined index pairs, it is probably worth repeating that all methods used in our analysis indicated that the strongest dependence is the one between the stock markets of Germany and Poland.

The parameter estimates of the DCC-GARCH models implied quite strong persistence of the correlations. This was only partly confirmed by the results for the HAR models, since for some pairs we found no or only weak significance of the lagged weekly and monthly realized correlations. Compared to the models for the 5-minute realized correlations, the use of the bipower correlations and/or of the 30-minute frequency generally resulted in weaker significance of regressors. Nevertheless, it should be also added that we found no evidence of autocorrelation or ARCH effects (with one exception) in the residuals of the HAR models. Importantly, the visual inspection of the fitted values revealed a relatively high degree of similarity in the correlation dynamics suggested by the different kinds of HAR models (sometimes with the exception of the model for the 30-minute bipower correlations). This correlation pattern often contrasted with the dynamics suggested by the DCC-GARCH models but on the whole, it was possible to find some common tendencies in the development of correlations, apparently reflecting the responses of the markets to the global financial crisis.

Finally, several interesting findings emerged from the forecasting exercise, taking the 5-minute realized correlations as a benchmark. The results differed across the index pairs. First, for three pairs the benchmark forecast could be considered unbiased and efficient. Second, in most cases (regardless of whether the benchmark forecast was unbiased and efficient or not), the predictive power of the forecast could be significantly improved by including either the bipower correlation forecast (5-minute or 30-minute) or the DCC-GARCH forecast. Concerning the DCC-GARCH forecast, the result is particularly interesting. Recall that the DCC-GARCH model was estimated for daily data (i.e. only one observation per day), so it is a little bit surprising that the forecast from the model contained valuable information that was not yet embodied in the benchmark forecast. In any case, our findings indicate that when making a forecast, it may be useful to consider different models.

Our results have important implications for risk management, for example the calculation of beta or value at risk of a portfolio. This could be an interesting extension of our analysis and a possible topic for further research.

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Data sources:

FEDERATION OF EUROPEAN SECURITIES EXCHANGES: http://www.fese.eu

TICK DATA, INC.: http://www.tickdata.com

Appendix

Figure A-1: Correlations from the DCC-GARCH models vs. realized bipower correlations

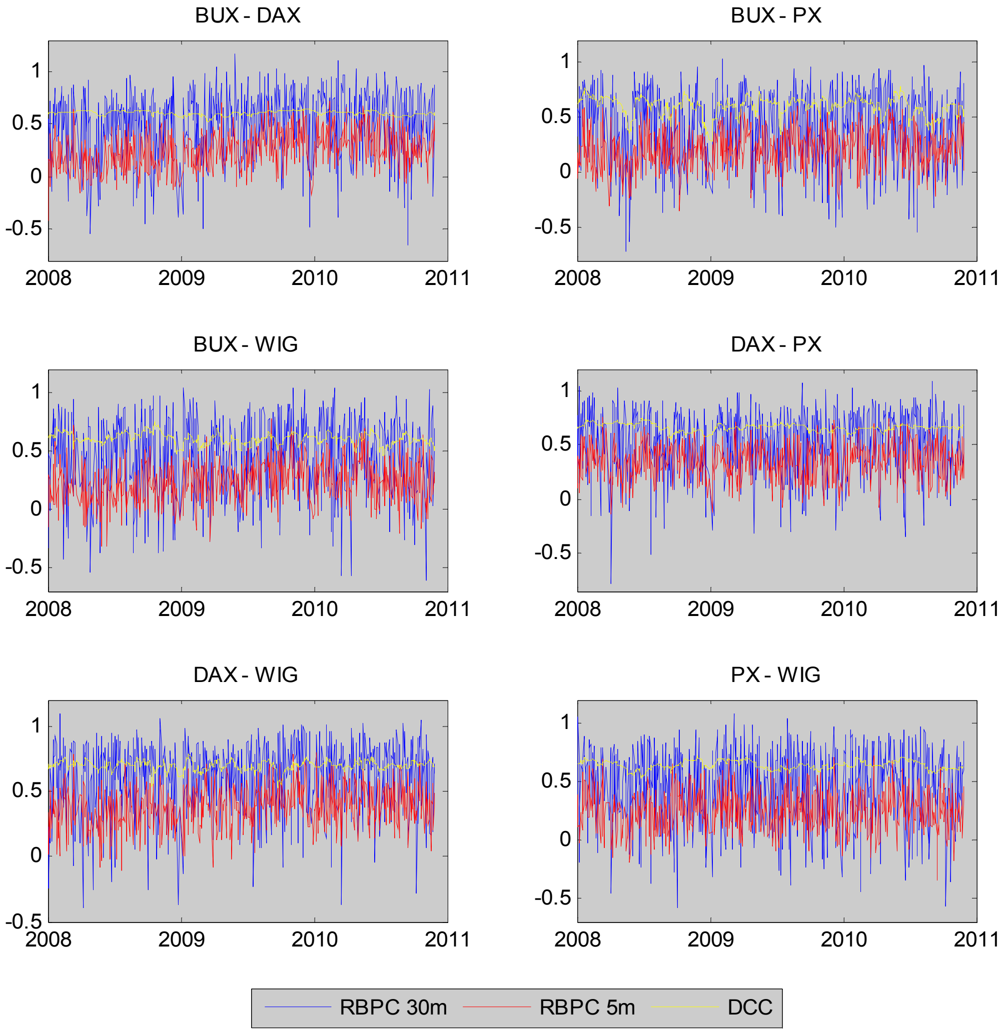

Figure A-2: Covariances from the DCC-GARCH models vs. realized bipower covariances

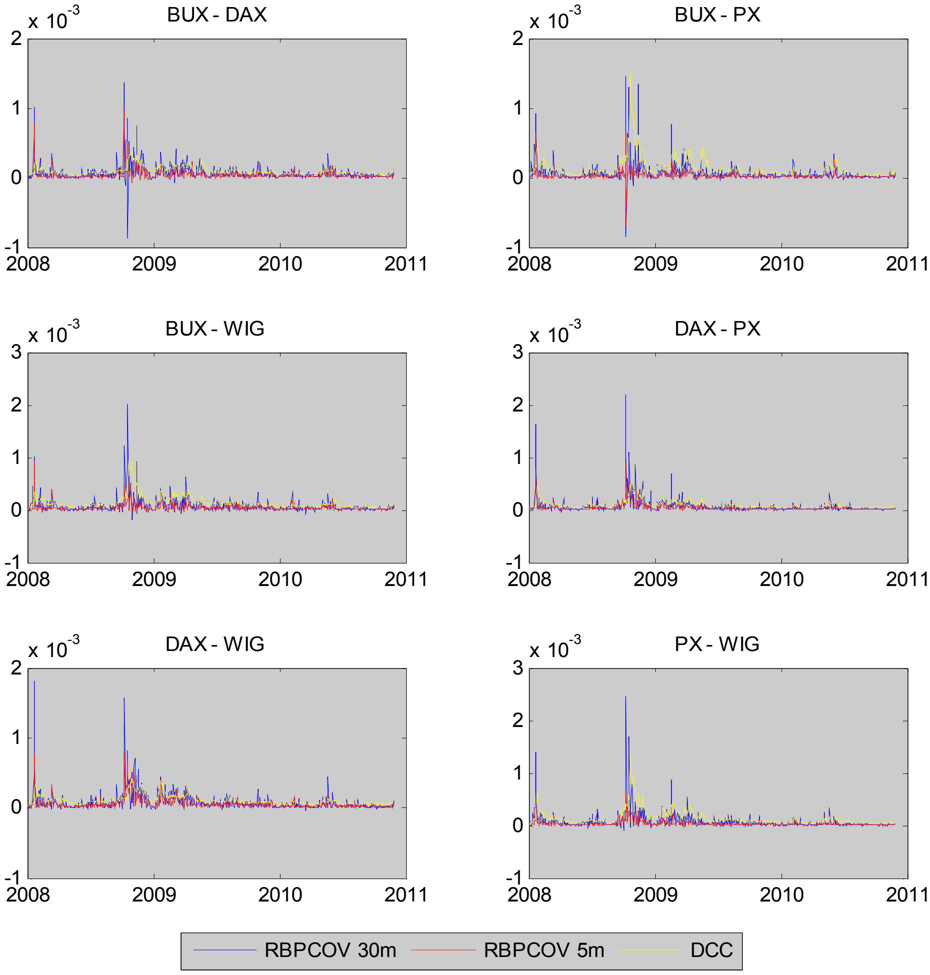

Table A-1: P-values from paired t-tests for correlations

	BUX-DAX	BUX-PX	BUX-WIG	DAX-PX	DAX-WIG	PX-WIG
DCC vs. RC 1h	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
DCC vs. RC 30m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
DCC vs. RC 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
DCC vs. RBPC 1h	< 0.001	< 0.001	< 0.001	< 0.001	0.001	< 0.001
DCC vs. RBPC 30m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
DCC vs. RBPC 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RC 1h vs. RC 30m	< 0.001	< 0.001	< 0.001	< 0.001	0.021	< 0.001
RC 1h vs. RC 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RC 30m vs. RC 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RBPC 1h vs. RBPC 30m	0.034	< 0.001	< 0.001	< 0.001	0.005	< 0.001
RBPC 1h vs. RBPC 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RBPC 30m vs. RBPC 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RC 1h vs. RBPC1h	0.051	0.297	0.810	0.567	0.161	0.196
RC 30m vs. RBPC 30m	0.188	0.076	0.027	0.008	0.283	0.948
RC 5m vs. RBPC 5m	< 0.001	0.007	< 0.001	< 0.001	< 0.001	0.268

Table A-2: P-values from paired t-tests for covariances

	BUX-DAX	BUX-PX	BUX-WIG	DAX-PX	DAX-WIG	PX-WIG
DCC vs. RCOV 1h	0.021	< 0.001	< 0.001	0.004	0.010	< 0.001
DCC vs. RCOV 30m	0.014	< 0.001	< 0.001	0.006	0.320	< 0.001
DCC vs. RCOV 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
DCC vs. RBPCOV 1h	0.009	< 0.001	< 0.001	0.001	0.002	< 0.001
DCC vs. RBPCOV 30m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
DCC vs. RBPCOV 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RCOV 1h vs. RCOV 30m	0.848	0.564	0.765	0.921	0.085	0.565
RCOV 1h vs. RCOV 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RCOV 30m vs. RCOV 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RBPCOV 1h vs. RBPCOV 30m	0.115	0.062	0.168	0.079	0.563	0.741
RBPCOV 1h vs. RBPCOV 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RBPCOV 30m vs. RBPCOV 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RCOV 1h vs. RBPCOV 1h	0.039	0.565	0.009	0.041	0.015	0.006
RCOV 30m vs. RBPCOV 30m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RCOV 5m vs. RBPCOV 5m	< 0.001	0.077	< 0.001	< 0.001	< 0.001	< 0.001

Table A-3: P-values from paired t-tests for Fisher-transformed correlations

	BUX-DAX	BUX-PX	BUX-WIG	DAX-PX	DAX-WIG	PX-WIG
DCC vs. RC 1h	0.020	0.014	0.447	0.933	0.151	< 0.001
DCC vs. RC 30m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
DCC vs. RC 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
DCC vs. RBPC 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RC 1h vs. RC 30m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RC 1h vs. RC 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RC 30m vs. RC 5m	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
RC 5m vs. RBPC 5m	< 0.001	0.002	< 0.001	0.009	< 0.001	0.785

Figure A-3: Boxplots of realized bipower correlations

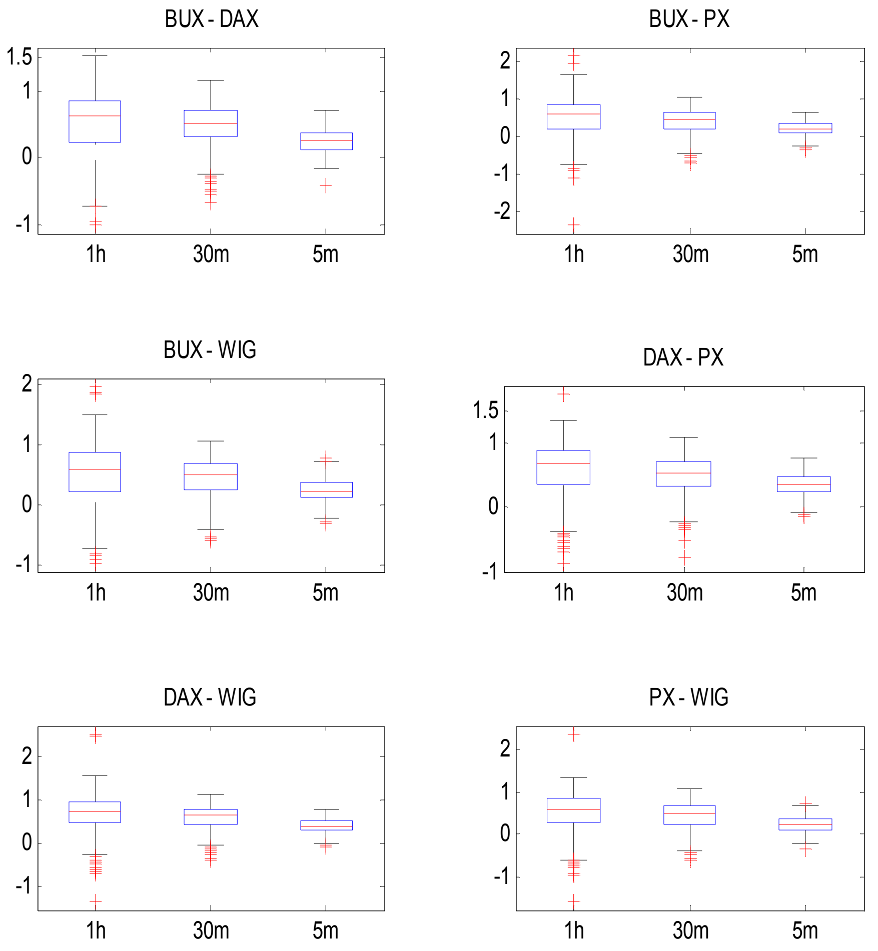

Table A-4: HAR models for the 5-minute Fisher-transformed realized correlations

	BUX-DAX		BUX-PX		BUX-WIG		DAX-PX		DAX-WIG		PX-WIG	
c	0.064	***	0.101	***	0.100	***	0.202	***	0.140	***	0.179	***
	(0.022)		(0.023)		(0.024)		(0.046)		(0.040)		(0.035)	
$\beta^{(d)}$	0.026		-0.012		0.089	**	0.093	**	0.098	**	0.082	*
	(0.045)		(0.045)		(0.045)		(0.044)		(0.045)		(0.044)	
$\beta^{(w)}$	0.379	***	0.417	***	0.323	***	0.122		0.153		0.170	*
	(0.095)		(0.094)		(0.091)		(0.095)		(0.098)		(0.098)	
$\beta^{(m)}$	0.380	***	0.110		0.212	*	0.260	*	0.441	***	0.071	
	(0.107)		(0.131)		(0.115)		(0.143)		(0.122)		(0.154)	
R^2	0.172		0.064		0.100		0.037		0.098		0.025	
LB 10	0.926		0.935		0.218		0.259		0.653		0.956	
ARCH 5	0.672		0.786		0.721		0.359		0.270		0.823	
JB	0.011		0.284		< 0.001		0.595		0.002		0.004	

*Notes: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively. LB 10 = p-value of the Ljung-Box test for residual autocorrelation up to lag 10, ARCH 5 = p-value of the Engle test for the presence of fifth order ARCH effects in residuals, JB = p-value of the Jarque-Bera test for normality of residuals*

Table A-5: HAR models for the 5-minute Fisher-transformed realized bipower correlations

	BUX-DAX		BUX-PX		BUX-WIG		DAX-PX		DAX-WIG		PX-WIG	
c	0.063	***	0.110	***	0.098	***	0.248	***	0.166	***	0.168	***
	(0.021)		(0.025)		(0.024)		(0.051)		(0.045)		(0.036)	
$\beta^{(d)}$	0.050		-0.035		0.027		0.075	*	0.058		0.049	
	(0.046)		(0.045)		(0.045)		(0.044)		(0.044)		(0.044)	
$\beta^{(w)}$	0.306	***	0.428	***	0.299	***	0.119		0.108		0.132	
	(0.095)		(0.097)		(0.097)		(0.097)		(0.101)		(0.102)	
$\beta^{(m)}$	0.416	***	0.106		0.277	**	0.146		0.440	***	0.177	
	(0.109)		(0.133)		(0.124)		(0.159)		(0.138)		(0.160)	
R^2	0.156		0.058		0.073		0.020		0.052		0.018	
LB 10	0.473		0.792		0.653		0.886		0.851		0.685	
ARCH 5	0.793		0.626		0.600		0.024		0.580		0.608	
JB	0.043		0.864		0.003		0.142		0.049		< 0.001	

*Notes: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively. LB 10 = p-value of the Ljung-Box test for residual autocorrelation up to lag 10, ARCH 5 = p-value of the Engle test for the presence of fifth order ARCH effects in residuals, JB = p-value of the Jarque-Bera test for normality of residuals*

Table A-6: HAR models for the 30-minute Fisher-transformed realized correlations

	BUX-DAX		BUX-PX		BUX-WIG		DAX-PX		DAX-WIG		PX-WIG	
c	0.263	***	0.267	***	0.304	***	0.569	***	0.409	***	0.460	***
	(0.066)		(0.058)		(0.064)		(0.102)		(0.100)		(0.088)	
$\beta^{(d)}$	0.088	**	0.035		0.115	***	0.022		0.014		0.061	
	(0.045)		(0.045)		(0.044)		(0.044)		(0.044)		(0.044)	
$\beta^{(w)}$	0.184	*	0.265	***	0.209	**	0.142		0.145		0.073	
	(0.097)		(0.098)		(0.093)		(0.100)		(0.103)		(0.097)	
$\beta^{(m)}$	0.294	**	0.161		0.117		-0.049		0.318	**	0.017	
	(0.133)		(0.136)		(0.135)		(0.176)		(0.156)		(0.178)	
R^2	0.058		0.039		0.049		0.006		0.025		0.007	
LB 10	0.180		0.894		0.211		0.332		0.699		0.515	
ARCH 5	0.118		0.242		0.597		0.487		0.914		0.238	
JB	0.961		0.152		0.021		0.055		0.124		0.909	

*Notes: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively. LB 10 = p-value of the Ljung-Box test for residual autocorrelation up to lag 10, ARCH 5 = p-value of the Engle test for the presence of fifth order ARCH effects in residuals, JB = p-value of the Jarque-Bera test for normality of residuals*

Table A-7: Mincer-Zarnowitz regressions for individual forecasts

BUX-DAX

	RC 5m	RBPC 5m	RC 30m	RBPC 30m	DCC
b_0	0.054	0.036	0.046	0.279 *	-0.338
	(0.072)	(0.067)	(0.123)	(0.143)	(0.264)
b_1	0.870 ***	0.986 ***	0.562 **	0.098	1.120 **
	(0.227)	(0.223)	(0.243)	(0.287)	(0.444)
R^2	0.096	0.123	0.037	< 0.001	0.044

BUX-PX

	RC 5m	RBPC 5m	RC 30m	RBPC 30m	DCC
b_0	-0.013	-0.018	-0.136	-0.006	-0.065
	(0.069)	(0.065)	(0.107)	(0.107)	(0.086)
b_1	1.126 ***	1.090 ***	0.836 ***	0.558 **	0.486 ***
	(0.331)	(0.295)	(0.252)	(0.265)	(0.146)
R^2	0.077	0.090	0.073	0.031	0.074

BUX-WIG

	RC 5m	RBPC 5m	RC 30m	RBPC 30m	DCC
b_0	0.014	0.038	-0.127	-0.286 *	0.263
	(0.067)	(0.062)	(0.108)	(0.151)	(0.239)
b_1	0.977 ***	0.946 ***	0.867 ***	1.264 ***	0.005
	(0.254)	(0.252)	(0.238)	(0.345)	(0.393)
R^2	0.096	0.092	0.087	0.088	< 0.001

DAX-PX

	RC 5m	RBPC 5m	RC 30m	RBPC 30m	DCC
b_0	0.261	0.340 **	0.494	0.249	-0.210
	(0.162)	(0.165)	(0.322)	(0.211)	(0.257)
b_1	0.346	0.136	-0.202	0.274	0.904 **
	(0.441)	(0.466)	(0.615)	(0.418)	(0.388)
R^2	0.004	< 0.001	< 0.001	0.003	0.038

DAX-WIG

	RC 5m	RBPC 5m	RC 30m	RBPC 30m	DCC
b_0	0.304 *	0.561	0.426 *	0.608	0.161
	(0.178)	(0.206)	(0.253)	(0.583)	(0.318)
b_1	0.321	-0.290	0.029	-0.273	0.408
	(0.411)	(0.509)	(0.410)	(0.963)	(0.458)
R^2	0.004	0.002	< 0.001	< 0.001	0.006

PX-WIG

	RC 5m	RBPC 5m	RC 30m	RBPC 30m	DCC
b_0	-0.279 *	-0.069	-0.003	1.029 ***	-0.941 ***
	(0.161)	(0.134)	(0.230)	(0.239)	(0.318)
b_1	2.191 ***	1.365 **	0.611	-1.710 ***	1.868 ***
	(0.640)	(0.536)	(0.510)	(0.239)	(0.489)
R^2	0.078	0.045	0.010	0.068	0.095

*Note: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively.*

Table A-8: Evaluation of forecasts for the BUX-DAX Fisher-transformed correlations

const	RC 5m		RBPC 5m		RC 30m	DCC	R^2
0.054	0.882	***					0.100
(0.077)	(0.224)						
0.044	-0.161		1.136	**			0.126
(0.076)	(0.565)		(0.566)				
0.073	0.928	***			-0.057		0.100
(0.120)	(0.319)				(0.277)		
0.006	0.844	***				0.088	0.100
(0.230)	(0.285)					(0.401)	
-0.181	-0.522		1.363	**	-0.050	0.445	0.132
(0.253)	(0.722)		(0.610)		(0.276)	(0.429)	

Comparison with the regression for non-transformed correlations excluding RBPC 30m

const	RC 5m	RBPC 5m		RC 30m	DCC	R^2
-0.267	-0.543	1.354	**	-0.035	0.644	0.132
(0.297)	(0.679)	(0.579)		(0.312)	(0.564)	

*Note: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively.*

Table A-9: Evaluation of forecasts for the BUX-PX Fisher-transformed correlations

const	RC 5m		RBPC 5m	RC 30m	DCC	R^2
-0.016	1.136	***				0.079
(0.071)	(0.329)					
-0.016	-0.082		1.146			0.093
(0.071)	(0.900)		(0.788)			
-0.109	0.644			0.398		0.091
(0.099)	(0.492)			(0.297)		
-0.056	0.720				0.191	0.093
(0.076)	(0.435)				(0.131)	
-0.151	-1.122		1.294	0.443	0.153	0.119
(0.101)	(1.046)		(0.798)	(0.302)	(0.132)	

Comparison with the regression for non-transformed correlations excluding RBPC 30m

const	RC 5m	RBPC 5m	RC 30m	DCC	R^2
-0.184	-0.956	1.236	0.487	0.216	0.112
(0.116)	(1.060)	(0.827)	(0.400)	(0.192)	

*Note: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively.*

Table A-10: Evaluation of forecasts for the BUX-WIG Fisher-transformed correlations

const	RC 5m	RBPC 5m	RC 30m	DCC	R^2
0.009	0.997 ***				0.099
(0.071)	(0.256)				
0.007	0.623	0.405			0.102
(0.071)	(0.575)	(0.557)			
-0.083	0.596		0.369		0.108
(0.103)	(0.418)		(0.304)		
0.213	1.090 ***			-0.325	0.108
(0.186)	(0.267)			(0.274)	
0.150	0.251	0.523	0.337	-0.362	0.122
(0.207)	(0.650)	(0.567)	(0.305)	(0.280)	

Comparison with the regression for non-transformed correlations excluding RBPC 30m

const	RC 5m	RBPC 5m	RC 30m	DCC	R^2
0.157	0.211	0.585	0.402	-0.442	0.118
(0.256)	(0.634)	(0.549)	(0.361)	(0.398)	

*Note: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively.*

Table A-11: Evaluation of forecasts for the DAX-PX Fisher-transformed correlations

const	RC 5m	RBPC 5m	RC 30m	DCC	R^2
0.259	0.412				0.006
(0.181)	(0.456)				
0.301	0.808	-0.520			0.008
(0.195)	(0.819)	(0.892)			
0.633 *	0.686		-0.751		0.017
(0.346)	(0.504)		(0.595)		
-0.132	0.096			0.648 **	0.046
(0.241)	(0.467)			(0.269)	
0.259	0.401	-0.026	-0.800	0.661 **	0.059
(0.374)	(0.824)	(0.917)	(0.613)	(0.270)	

Comparison with the regression for non-transformed correlations excluding RBPC 30m

const	RC 5m	RBPC 5m	RC 30m	DCC	R^2
-0.018	0.474	-0.320	-0.462	0.886 **	0.043
(0.386)	(0.814)	(0.872)	(0.686)	(0.406)	

*Note: Standard errors of the parameter estimates are reported in parentheses.***, ** and * denote significance at the 1%, 5% and 10% level, respectively.*

Table A-12: Evaluation of forecasts for the DAX-WIG Fisher-transformed correlations

const		RC 5m		RBPC 5m	RC 30m	DCC		R^2
0.317		0.356						0.005
(0.194)		(0.407)						
0.472	**	0.778		-0.804				0.016
(0.230)		(0.530)		(0.649)				
0.350		0.462			-0.105			0.006
(0.222)		(0.533)			(0.339)			
0.132		0.307				0.244		0.010
(0.301)		(0.412)				(0.303)		
0.310		0.838		-0.805	-0.110	0.260		0.022
(0.335)		(0.625)		(0.653)	(0.341)	(0.305)		

Comparison with the regression for non-transformed correlations excluding RBPC 30m

const		RC 5m		RBPC 5m	RC 30m	DCC		R^2
0.294		0.860		-0.918	-0.183	0.378		0.024
(0.402)		(0.614)		(0.669)	(0.529)	(0.465)		

*Note: Standard errors of the parameter estimates are reported in parentheses.***, ** and * denote significance at the 1%, 5% and 10% level, respectively.*

Table A-13: Evaluation of forecasts for the PX-WIG Fisher-transformed correlations

const		RC 5m		RBPC 5m	RC 30m	DCC		R^2
-0.331	*	2.347	***					0.080
(0.177)		(0.675)						
-0.328	*	2.652	**	-0.315				0.081
(0.178)		(1.149)		(0.960)				
-0.361		2.294	***		0.082			0.080
(0.283)		(0.779)			(0.595)			
-0.789	***	1.552	**			0.860	**	0.122
(0.250)		(0.731)				(0.336)		
-0.833	**	2.862	**	-1.459	-0.128	1.060	***	0.135
(0.326)		(1.236)		(1.028)	(0.590)	(0.364)		

Comparison with the regression for non-transformed correlations excluding RBPC 30m

const		RC 5m		RBPC 5m	RC 30m	DCC		R^2
-1.060	***	2.771	**	-1.406	-0.151	1.623	***	0.132
(0.371)		(1.261)		(1.017)	(0.544)	(0.566)		

*Note: Standard errors of the parameter estimates are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% level, respectively.*