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DIPLOMOVÁ PRÁCE

STRESS TESTING OF BANK RISKS

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Prohlášení

Prohlašuji, že jsem diplomovou práci vypracovala samostatně a použila pouze uvedené prameny a literaturu

V Praze dne   podpis studentky

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Project of the thesis

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Title:

Stress testing of bank risks

Aim:

This thesis will deal with the topic of stress testing, mainly of two bank risks – the market risk and the credit risk. The first question is what exactly the definition of stress testing is. For example, the 1999 BIS document *Framework for Supervising Information about Derivatives and Trading Activities* says that stress scenarios need to cover "a range of factors that can create extraordinary losses or gains in trading portfolios" and that they should "provide insights into the impact of such event on positions." The original Market Risk Amendment to the Accord contained two full pages on the importance of stress-testing and recommended quantitative and qualitative criteria to "identify plausible stress scenarios to which banks could be exposed."

The key point is that stress testing provides information of value that may not be available from other risk measurement tools like Value at risk, particularly if VaR models focus on "normal" market risks rather than the risks associated with rare or extreme events.

Generally, stress testing means choosing scenarios that are costly and rare, and then putting them to a valuation model. The problem of course is that choosing stress-test scenarios is by its very nature subjective, although some typical scenarios are prescribed by the regulatory institution.

I am convinced that stress-testing techniques, although still in a developing stage in the area of credit risk, can play an important role in the future for assisting banks, and to some extent regulators, in the process of capital adequacy assessment.

In my opinion many working papers exist that deal minutely only with some special problem of the stress testing process, that is why I would like to explain the whole process of stress testing in my thesis. The aim of this thesis is to give a solid overview of problematic of risk factors identification, scenario building and overview of suitable techniques for market and credit risk stress testing; even the stress testing techniques are sophisticated, using comprehensive econometric techniques. At the end I would like also to mention possibilities of stress testing of other risk, in order the topic is complete.

Plan:

- Definition of the stress testing
- Reasons for stress testing
- Features of previous stress events and their relevance
- Explain elements of stress tests
- Give general overview of stress scenarios and suitable stress testing techniques
    - Single-factor vs. multi-factor stress tests
    - Historical vs. hypothetical scenarios
- Identify appropriate techniques for market risk stress testing
- Identify appropriate techniques for credit risk stress testing
- Mention other types of risks

Literature:

Official BIS publications:

- Amendment to the Capital Accord to incorporate market risks, Basel Committee for international settlement, January 1996.
- An Internal model-based approach to market risk capital requirements, April 1995.

Other:

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Table of abbreviations

ARCH .... Auto Regressive Conditional Heteroscedasticity

BCBS......................... Basel Committee on Banking Supervision

BCGFS .... BIS Committee on the Global Financial System

BIS............................. Bank for International Settlement

BM .... Block Maxima

CAR........................... Capital / risk weighted assets

DPG........................... Derivatives Policy Group

EaR .... Earnings at Risk

EVT .... Extreme Value Theory

EWMA .... Exponentially Weighted Moving Average

GARCH...................... Generalized Auto Regressive Conditional Heteroscedasticity

GPD........................... Generalized Pareto Distribution

IRB .... Internal Rating Based

LGD .... Loss given default

NBCA .... The New Basle Capital Accord

PC .... Principal Component

PD .... Probability of default

POT .... Peak Over Threshold

RR .... Recovery rate

RWA .... Risk weighted assets

VaR .... Value at Risk

I INTRODUCTION

Increasing complexity and diversity of activities of large, internationally active financial institutions have been accompanied by a process of innovation in how these institutions measure and monitor their exposures to different kinds of risk. One of risk measurement techniques that have attracted much attention over the past several years, both among practitioners and regulators, is stress testing. It can be loosely defined as the examination of the potential effects on a bank's financial condition of a set of specified changes in risk factors, corresponding to exceptional but plausible events.[1]

Stress tests enable managers to track a bank's exposure to price changes during events that are considered plausible, and allow senior management and supervisors to determine a bank's risk profile. Further, they are used to set limits on the size of trades and asset positions, and lead to trading positions adjustment.

In general, stress testing plays a complementary role in risk management practices of financial institutions even if value-at-risk (VaR) seems to be the dominant methodology. VaR calculations have become a routine exercise for risk managers, and banks and regulators are committed to act upon VaR results. Stress testing, however, is vaguely defined, and when it is defined, the definition is rather specific to the institution.

The concept of stress testing per se can appear to be straightforward, but the specification, implementation, and interpretation of the tests is difficult. Stress tests often require a number of practical choices as to what risk factors to stress, how to combine factors stressed, what range of values to consider, and what time frame to analyze. Having addressed all these questions, a risk manager still stands before sifting through results and finding out implications that stress test results might have for risk-taking activities.

[1] Stress testing is a general term. It is possible to test whole financial systems, public debt, etc. We will deal only with a subgroup comprising internal risks of banks.

In my opinion, while there is an extensive professional and academic literature on VaR, stress testing has not attracted as much interest among academicians, although practitioners, regulators and central banks have been paying more attention in recent years. Thus, stress testing could be considered as not very explored and nowadays very challenging topic. In the last few years, an abundance of different studies has appeared, that deals with identifying risk factors that should be used, proposing „right" model and methodology of computation. Unfortunately, they are usually too specific, dealing with only a part of stress testing process or are applicable only to some of market data. This is why I chose the issue of stress testing. I would like to provide an overview of stress testing approaches, models and methodology of processing the inputs. My aim is to give a compact presentation of stress testing that would give a reader more than a basic notion.

The aim of the second part of this thesis is to identify what the stress testing is and how does the stress testing process look like. At the beginning, I will summarize the stress testing requirement given by international regulators and compare them with the practices applied in practice. In addition to basics of stress testing, comparison and distinction between VaR and stress tests will be made. It also contains the description of stress testing process inclusive overview of scenarios used. It is possible to say that the identification of appropriate stress scenarios is both art and science. A quantum of possibilities begins with several supervisor-specified scenarios, which are usually historical events. Also incorporation of hypothetical scenarios is required from regulators. Generally, the number and nature of the hypothetical scenarios employed varies in response to changes in forecasts of economic and political factors considered as key in driving national economies. There are also many types of scenarios depending on the number of risk factors employed and on the technique of choice. However, most common in financial institutions (also in the Czech banks), are simple single-factor sensitivity tests.

The third part is devoted to market risk stress testing. After specifying the market risk factors, suitable scenarios are identified and models described.

The fourth part is devoted to credit risk stress testing. This field is less examined and less literature has been published on this issue. The Czech literature almost does not exist. Moreover, the credit risk model and the more credit risk stress tests are part of internal bank management and the information about processes exchange only in frame of narrow professional groups. In this part I discuss characteristics of credit risk, models that can be used to measure the credit risk and I try to show how the models could be adjusted to incorporate stress testing. I will also mention problems that arise in connection with credit risk stress testing in the Czech Republic. Additionally I will try to identify if and how stress testing in Czech banks is applied.

The fifth part focuses on other types of risks on which stress testing may also be applied. We briefly concern with liquidity risk and operational risk. Final part is devoted to the recent trends in the risk modeling and stress testing.

The sixth chapter concludes the whole issue and summarizes what was realized about current stress testing practices.

II COMMON FEATURES OF STRESS TESTING

II.1 What is stress testing

The BIS Committee on the Global Financial System (BCGFS) (2000) defines 'Stress testing' as – "a generic term describing various techniques used by financial firms to gauge their potential vulnerability to exceptional, extreme or simply unexpected but plausible events".

Exceptional event is one that happens once or only a few times and can have dire consequences. For example, the bankruptcy of Argentina in 2001 was an exceptional event in relation to credit risk. An example of an extreme event in a market variable is the stock market crash of October 1987, which was 14 standard deviations away from the expected value based on normal distribution. Another case of event not expected by analysts was the collapse of Enron in 2001. It can be tested how a portfolio would suffer under such events.

II.1.1 Stress testing requirement

Basle Committee on Banking Supervision (BCBS) introduces stress testing in the Amendment to the Capital Accord to incorporate market risks (1996, Part B.5, page 46)[2].

„Banks that use the internal models approach for meeting market risk capital requirements must have in place a rigorous and comprehensive stress testing program. Stress testing to identify events or influences that could greatly impact banks is a key component of a bank's assessment of its capital position. "

The BCBS further specified that stress testing should cover „a range of factors that can create extraordinary loses or gains in trading portfolios" and should show the impact of such low-probability events. Banks´ stress tests must fulfill both qualitative and quantitative criteria defined. From qualitative side they should evaluate the bank's absorbsion potential of large losses and „identify steps the bank can take to reduce its risk and conserve capital". Further, a bank must be able to provide the supervisory institution with information relating to three broad areas that can be briefly expressed by following questions:

- What are the largest losses during reporting period and what proportion has been covered by internal measurement system?
- What is the result of testing past period's disturbances and how sensitive is the bank to changes in the assumptions about volatilities and correlations?
- What is the methodology of creating the bank's scenarios and what are the results of the tests?

The approach to credit risk is less developed, as only qualitative criteria were introduced by the BCBS. However, the second consultative document issued by the BCBS on the New Basel Capital Accord (2001) ("Basel II"), also specifically mentions

[2]The original Capital Accord was developed in 1988 by the Basel Committee on Banking Supervision and later endorsed by the central bank governors on the Group of Ten (G-10) countries. In April 1995 the BCBS issued a consultative proposal to amend the Accord known as the "1996 Amendment". It was implemented in 1998.

that banks that adopt the internal based approach for calculating capital requirements must undertake stress testing[3]:

„A bank must have in place sound stress testing processes for use in the assessment of capital adequacy. Stress testing should involve identifying possible events or future changes in economic conditions that could have unfavorable effects on a bank's credit exposures and assessment of the bank's ability to withstand such changes. Three areas that banks could usefully examine are: (i) economic or industry downturns; (ii) market-risk events; and (iii) liquidity conditions. "

The aim of the New Basel Capital Accord (NBCA) concept is to increase the security and stability of financial systems and to enable implementation of more complex approaches to risk management for regulatory purposes.

The BCBC issued in 2001 the second and in 2003 the third consultative document on the NBCA. The mean by which the EU will implement the NBCA, Basel II, into legislation will be the new Capital Adequacy Directive[4] known as CAD 3, proposed by the European Commission. The directive should be adopted by the European Parliament by the end of 2005 and the transposition into national laws should be finished by the end of 2006. Then the implementation will be binding for all EU countries, including new entrants that joined the EU on May 1.[5]

Even though the incompleteness of the NBCA and the CAD hinders from the beginning of legislative work in the Czech Republic, there already is a scope for preparation on the implementation, as it requires adoption of more sophisticated approaches. Dealing with capital ratios of Pillar 1, storage of huge databases, and other sources for risk modeling and testing will mean large enhancements to IT infrastructure. It entails both initial investment and significant maintenance cost. The preparation concerns both the financial institutions and regulators.

[3] (viii) Use of internal ratings, c) , paragraph 297

[4] The current EU rules on capital adequacy to a large extent result from the Basel I, implemented into EU legislation via the Solvency Ratios Directive (now incorporated into the Consolidated Banking Directive), CAD 1 and CAD 2.

[5] All EU member states will have to apply Cad 3 to all banks and investment firms within their borders. That's in contrast to the US, where banking regulators expect that in total some 20 of the country's largest banks will operate under Basel II rules, and those banks will be allowed to use only the most advanced approaches to assessing their credit and operational risks. The rest of the US banking system will remain on the current, and simpler, Basel I capital adequacy rules that date from 1988.

As far as the impact of NBCA on banks concerns, the technical demands will rise and this will be reflected in changes in existing risk management processes and increase role of internal controls and audit. Implementation of Basel II will also take stress testing issue further. Until now the regulators were in position that it is good practice for banks to conduct stress scenarios, but it was not the firm requirement.

At first, the main focus will be on formulating the internal rating based approach under Pillar One, but later will definitely increase the focus on stress testing. Stress testing of credit portfolio will become important not only for banks, but also from regulatory point from view and probably some common tests will be approved by regulators in order to assure greater comparability of banks.

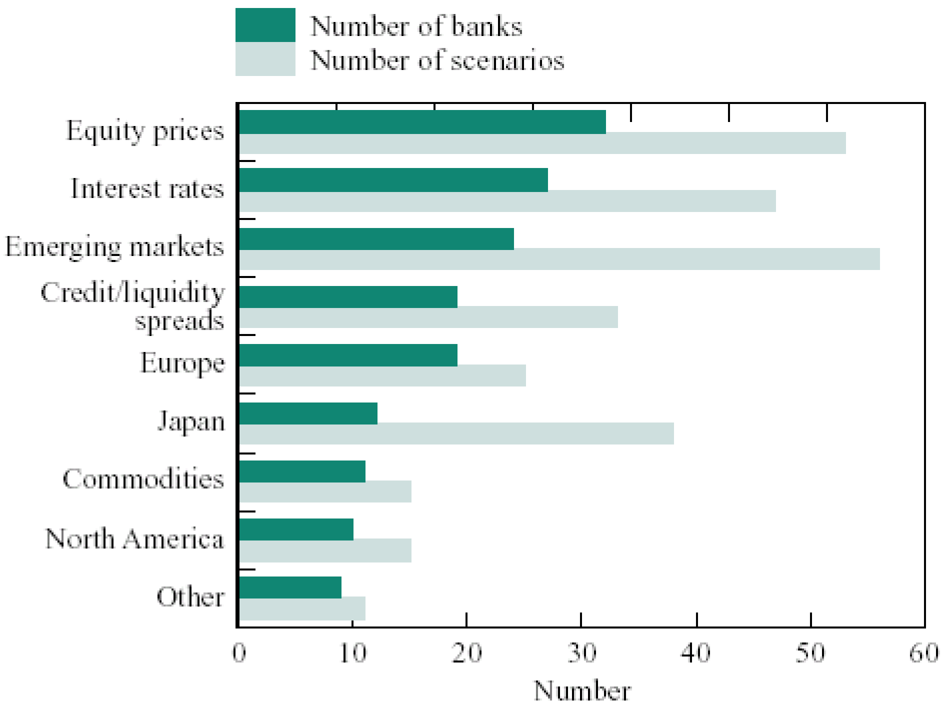
II.1.2 Stress testing in practice

The impacts of new regulations were examined in several surveys. In 2000 the CGFS[6] made a global census of stress tests in use at major financial institutions. Forty-three major commercial and investment banks from 10 countries were asked to report their firm-wide scenarios and key risk factors or asset prices. Further, they were asked about how firms conduct stress tests and how they use results.

These banks and securities firms submitted a total of 293 stress test scenarios and 131 sensitivity stress tests. Stress test scenarios were classified into themes based on their dominant asset class or geographical region. The four most common areas stress-tested were equity prices, interest rates, emerging markets and credit/liquidity spreads, followed by those focused on stress events in particular regions (including stress to foreign exchange rates). Only a few stress tests focused on commodities and related risk factors or on stress in options markets.

[6] CGFS(2000)

Figure 1: Stress tests by theme



Source: Fender, Gibson, Moser (2001)

It showed up that different banks regard different risks for important. This is because the bank's stress tests are related to its asset and derivatives positions and because the perception of likelihood may differ. Other interesting result is that the negative evolvement of risk factors (increase of interest rates or spreads, depreciation of a currency) was stressed more frequently than positive.

If we want to look separately on the state of stress testing in the Czech banks, we cannot find out much, as no survey was published. Following table introduces information about stress testing activities in the Czech banks that is detectable from their annual reports. Unfortunately, it seems from this accessible material that stress testing is not developed and widely practiced.

Brief survey: What do we know about stress testing in the Czech banking sector?

I tried to find information about stress testing practices in 2003 annual reports of seven large and medium-sized banks operating in the Czech Republic (Česká spořitelna, Komerční banka, ČSOB, IC banka, Citibank, E-banka, Volksbank). Unfortunately, the only stress testing practices that are connected with the loan (structural) book concern interest rates, credit risk as it does not mention any of them.

In the Komerční banka interest rate risk within the structural book is monitored and measured using a gap analysis, sensitivity of interest income to a parallel shift of the yield curve, and Earnings at Risk ("EaR") for net interest income. The calculation of EaR to net interest income involves a stress-testing approach to interest rate risk within the Structural Book.

The EaR indicator shows the maximum departure of the planned net interest income from the initial value over a one year period attributable to the movements in interest rates. In KB, EaR is set using stochastic simulations of random scenarios of interest rate developments and a change in interest income relative to the initial value is established for each scenario. In scenarios also the stress scenario is included. In Citibank, where EaR shows the potential change in net interest income before taxation if interest rates change by 2 standard deviations during the fixed period, stress testing is performed through modeling the change in interest rates is higher than 2 standard deviations.

IC-banka, E-banka and ČSOB shortly inform about running stress test for interest rates, but only in connection with trading book.

II.2 Connection with VaR models

The issue of stress testing often appears in connection with VaR models. The algorithm of value at risk (VaR) was developed in early 1990s and it enables that managers can be informed about exposures on a daily basis. Although it is a quick, widely used test, it does not usually work with extreme or unexpected market conditions. Therefore, according to the BCBS requirements above, institutions that employ VaR models (as part of the internal models approach) to compute their capital requirements are obliged to conduct stress testing as a complement.

II.2.1 VaR

VaR is a measure of potential loss, where the potential loss is linked directly to the probability of occurrence of adverse movements in market prices. There are three different methods that are used in calculating it: the variance-covariance, historical-simulation and Monte-Carlo simulation methods.

The application of VaR techniques is usually limited to assessing the risks being run in banks' treasury or trading operations (such as securities, foreign exchange and equities trading).

To quantify potential loss (and the severity of the adverse price move to be used), two underlying parameters must be specified – the holding period under consideration and the desired statistical confidence interval. The holding period refers to the time frame over which changes in portfolio value are measured. The Basle Committee's standards require that banks use a ten-day holding period – thus requiring banks to apply ten-day price movements to their portfolios. The confidence level defines the proportion of trading losses that are covered by the VaR amount. For example, if a bank calculates its VaR assuming a one-day holding period and a 99 per cent confidence interval then it is to be expected that, on average, trading losses will exceed the VaR figure on one occasion in one hundred trading days.

Estimation of a VaR figure is based on the historical behavior of those market prices that affect the value of the portfolio. In line with the Basle Committee's requirements we use 250 days of historical data. The starting point of all three VaR approaches is to revalue the portfolio at current market prices.

II.2.2 Stress testing as a complement

While VaR is used by numerous financial institutions, it is not without shortcomings. Comparing publications dealing with stress testing issue and VaR literature[7], we can notice the following two limitations of VaR that support the application of stress tests.

First, since the VaR estimate is based solely on historical data, to the extent that the past may not be a good predictor of the future, the VaR measure may under or overestimate risk. However, real markets don't remain constant over time and VaR provides no information about losses that may arise if more adverse price movement occurs than dictated by chosen confidence level. To address this shortcoming more subjective approaches such as stress testing are being adopted in addition to the statistically based VaR approach.

Second, there is a problem with the standard assumption of VaR models that risk factors are normally distributed. Under stressful conditions, distribution tends to have "fat tails", which means the extreme values occur with much higher probability than under the assumption of normal distribution.[8] Therefore, it is suitable to use stress

[7] Schachter (1998), Breuer, Krenn(1999).

[8] Österreichische Nationalbank: The slump in stock prices triggered by the equity crash of 1987, for instance, amounted to something between 10 and 20 standard deviations. Considering that under normality a 7 standard

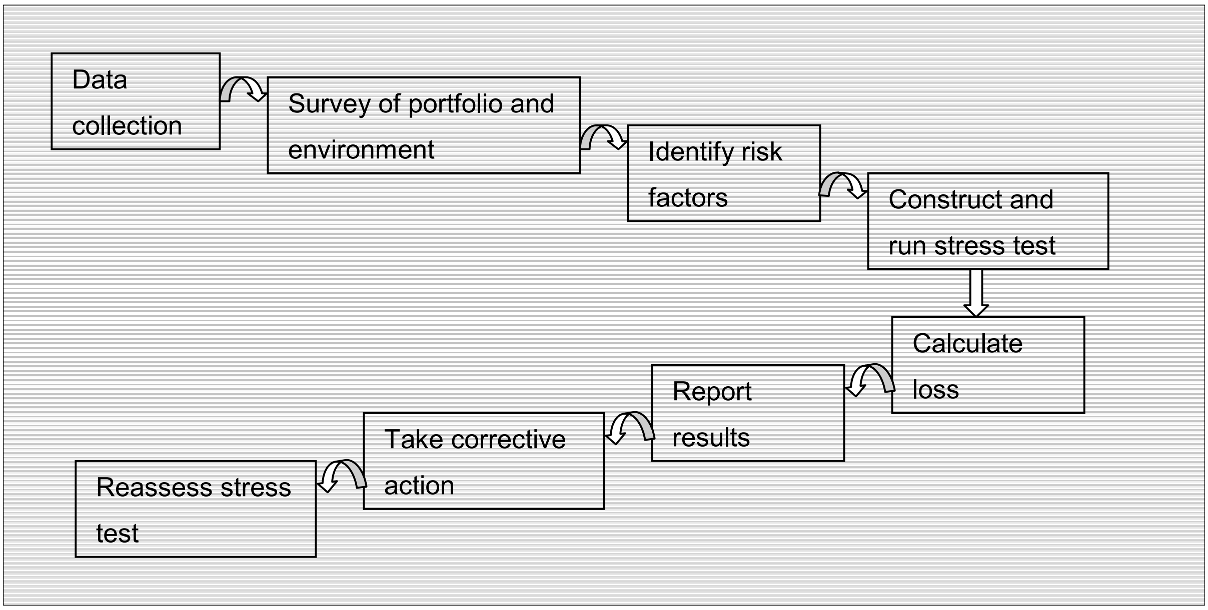
tests as a complement, which is not based on assumption about risk factors´ distribution.

However, VaR understood as a statement “We are X percent certain that we will not lose more than V units of money in the next N days” is not necessary based on assumption about normality of returns. As Berkowitz (1999) shows, the distinction between VaR and stress testing model is more or less artificial. In the next chapter we will see that stress testing can be carried out in the value at risk framework. This is usual for example in extreme value approaches or correlations base approaches. Then the distinction between the VaR and stress testing might appear to be only theoretical. In practice, however, there is a strong motive to handle unusual scenarios outside basic model and to have two predicted returns distribution. The reason is that the preparing for possible financial shocks and treating of the resulting losses is different from management practices under normal circumstances.

II.3 Stress testing process

The stress testing program of a bank is illustrated in Figure 2.

Figure 2: Scheme of stress testing process



deviations change should on average occur at one day in three billion years, the assumption of normality seems inadequate.

Source: Author

II.3.1 Data and survey of portfolio and environment

To detect potential stressful events, it is important to study the environment including economic, regulatory, financial market and other factors.

By data collection, the first and very important step in stress testing is to ensure that the data being used in risk management are accurate and timely. The data include all aspects of the bank's credit portfolio for purposes of credit stress testing and instruments of trading book for purposes of market stress test, as well as market data relating to the risk factors.

Secondly, all kinds of financial instruments included in the portfolio should be listed and it should be identified which risk factors influence each instrument. However, it is important to know that the identification process has not fixed rules. Different banks then usually run different stress tests. The surveys[9] confirmed the substantial heterogeneity across scenarios that look rather similar, even by modeling identical historical events the magnitude of shocks varies. It follows that the choice of risk factors, their combination, range of values considered and of what time frame to analyze depends on subjective judgment of an analyst. Different choices of time horizons or considering differences in banks´ portfolios will give different shock sizes in most historical episodes. Also the decision about the importance of risks is a subjective notion. It may be important to monitor large exposures, monitor hedge or check that a bank is not exposed to a particular event.

II.3.2 Risk factors

The stress testing concept is based on the notion that the value of a portfolio depends on the behavior of risk factors. The following figure sums up the main risk factors that can be used in stress analysis:

[9] CGFS (April 2001), CGFS (April 2000).

Figure 3: Risk factors

Counterparty	Environmental	Model	Analytics
• Probability of default • Loss given default • Credit Spreads	• Fin. market factors • Industry • Geographical • Economic • Political • Sociological • Regulatory • Ecological	• Assumptions • Holding period	• Correlation • Transition Matrices • Volatility

Generally we can mark the risk factors chosen from four groups above r_1 , r_2 ,..., r_n. The vector m=(r_1 , r_2 ,..., r_n) then describes a specific market situation. Under these conditions, the value of portfolio is given as a function P(r) of risk factors. Note that different portfolios have different functions P, as the valuation process is not the same. Employing stress testing we ask, what happens if market situation m occurs, and therefore we construct market scenarios m_1, m_2,..., m_k. After evaluating the portfolio values $P(m_1)$,...,$P(m_k)$, we compare them with current value of the portfolio $P(m_c)$ to set losses that would a bank suffer, if market situation m_i occurs.

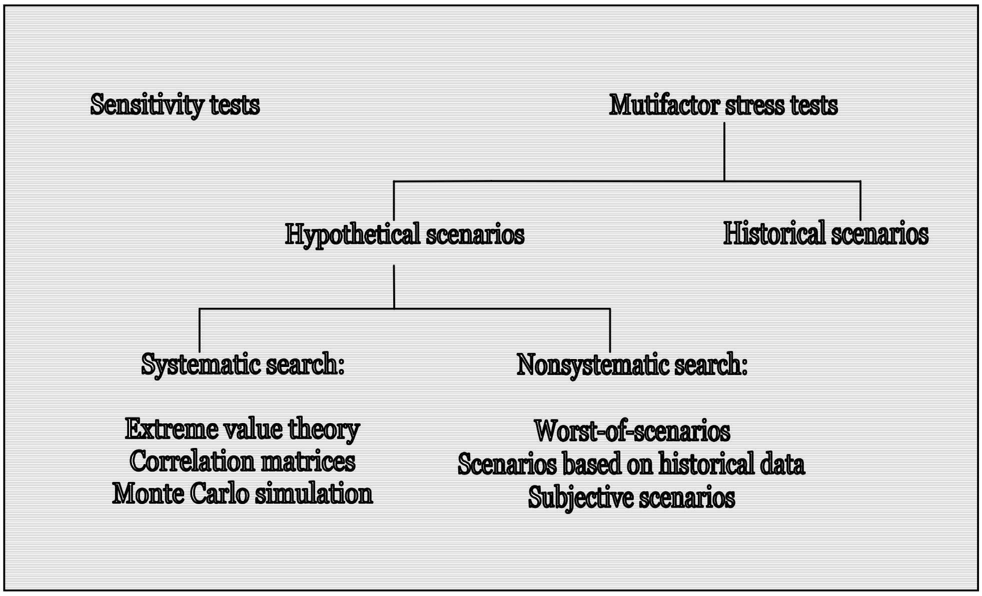
Using any type of model, it is necessary to check that the model itself is appropriate. By the market risk modeling there arise a risk that the model is used, which cannot accurately evaluate market prices. Similarly the credibility of debtors and credit risk must be evaluated well. Sources of such risk include (1) use of wrong assumptions, (2) errors in estimations of parameters, (3) errors resulting from discretization, and (4) errors in market or credit data. Very important for risk modeling and testing is the risk of not accurately estimated probability of future losses. This includes (1) difference between assumed and actual distribution and (2) errors in the logical framework of the model.

Between risk factors relevant to credit portfolios and trading portfolios we will distinguish in corresponding sections.

II.3.3 Methods for construction of stress tests

There is a number of stress testing techniques that measure the magnitude of risk factor changes, matching either to testing credit risk or market risk. One possibility to categorize stress tests is according to the number of risk factors incorporated. Namely, one can distinguish one-factor sensitivity tests and multi-factor stress tests.

Figure 4: Types of scenarios



Source: Author

II.3.3.1 Sensitivity tests

A sensitivity test is a single-factor technique that isolates the short-term impact of predefined moves in a particular market risk factor on portfolio's value. Such a test is appropriate when a trader wants to realize the effect of a large move in a risk factor on his position or portfolio. However, when assessing a portfolio's exposure to stress events, a single factor shock is rarely appropriate. Standardized single-factor stress tests have been issued by various organizations and can be adopted off-the-shelf. One example of standardized single -shock stress tests are those prescribed by the Derivatives Policy Group (1995).

II.3.3.2 Multi-factor stress tests

Multi-factor stress tests involve stressing several risk factors at the same time. They are not so simplistic as the simple-factor stress test, however, one have to chose among risk factors, as comprising all of them is mot possible. The choice my be inspired by a historical event or new scenario is constructed. The criterion is the relevancy for portfolio.

1) **Historical stress tests** simulate extreme stress events, which have occurred in the past (e.g. Russian crisis, the Mexican peso crisis, Asian or Brazilian crisis). Risk managers are likely to find at least a few episodes that have relevance to their portfolios.

2) **Construction of hypothetical events** by stressing of a group of risk factors, which is also called **scenario building,** risk managers use when no historical event matches the special feature of their portfolio. In this case risk managers may construct scenarios including hypothetical movements of risk factors. The search for the risk factors influencing a portfolio may be systematic or nonsystematic. Scenario testing can be constructed as a top-down method, where we move from a definition of a stress event to identification of change in risk factors, or as bottom-up process by deciding the change in risk factors without specifying particular event.

    a) **Nonsystematic search for hypothetical scenarios**

    - **Worst-of-scenarios** or a **maximum loss** approach finds combinations of movements in the various risk factors within a specified period that have the maximum adverse impact on the portfolio without taking correlations among risk factors into account. Such kind of a stress test ignores the correlation among risk factors and the created scenarios thus may not make economic sense.

    - **Scenarios based on historical data** combine maximum changes of risk factors that had occurred within chosen historical period. It is the simplest way how to combine historical data and it do not take into account correlations within individual risk factors. It differs from the historical scenarios, because it is not based on one concrete historical stress event. The difference from the worst-of-scenarios is that the combination of maximum changes does not have to lead to extreme adverse impact on the portfolio. The plausibility and the economic sense are similarly unclear as by worst-of-method.

- **Subjective scenarios** are usually created using expert inputs from people outside the bank, such as traders or consultants. However, despite their experience, experts can omit some risk factors or misspecify their correlation, as it is not possible subjectively construct a precise correlation matrix.

b) **Systematic search for scenarios** enables to minimize the above weaknesses by trying to encompass all the relevant risk factors and ensure that the simulation of their changes makes an economic sense. There are several approaches in this group, namely:

- **Extreme Value theory** is the statistical theory of the tails of probability distributions that attempts to better capture risk of loss in extreme circumstances. As the only stress test technique, it attaches the probability to stress test results. The advantage is that this method is not limited by the assumption of the normal distribution, and thus can accommodate skewed and fat-tailed distributions of portfolio changes. However, the approach retains the assumption that extreme events are uncorrelated, which may not be true in reality, and thus the consequences for the portfolio may be distorted.
- **Correlation (covariance) matrix** needs to be modeled and forecasted correctly in many practical applications; for instance in option pricing or modeling stock market volatilities.
- **Monte-Carlo simulation** consists of repeatedly simulating the random process that governs market prices and rates. Each simulation (scenario) generates a possible value for the portfolio at the target horizon. If enough scenarios are generated, the simulated distribution of the portfolio's value converges to the true distribution.

If we want to choose one of possibilities stated above, we have to decide what is the desired statistics or the result. For example, VaR can be estimated from historical scenarios, single- (multi-)step Monte Carlo scenarios, each having its pros and cons. The decision process on the choice of scenarios can be divided in several stages:

1) Determine what the purpose is, if we want to compute VaR, risk exposure, etc.

2) What are the risk factors and do we want to examine single risk factors or do they need to be grouped?

Sometimes the examination of influence of single factor change is enough (for example, if a bank expects that the central bank will decrease long term interest rates). By simulation for example some historical event, more risk factors have to be incorporated, although it is not possible to include all of them each time. The decision about number of risk factors is always the compromise between the accuracy of results and processing and computational demands. Usually more scenarios and more types of stress test are used to access risks in different instruments in portfolio.

3) What is the statistical representation (distribution) of risk factors?

The important thing is to set statistical properties of risk factors and distribution, when probability is taken into account. For example in case of market risk examination, we can often work with normal distribution, by credit risk fat tail have to be taken into account.

4) Are we able to manage such analysis computationally? Should we not work with some simplifying models?

In modeling generally, simplified models are used at expense of precision of results. In stress testing, it is often recommended to use Monte Carlo method, but many banks are not able to do so, as the computation may take several days.

After addressing these four issues, we know the scenario and risk factors se want to include. Then the stress test can be run and the portfolio revalued.

II.3.4 Reporting results and corrective action

Senior management that set policies and limits has to be included in the stress testing procedures. At least management have to get the results of stress tests periodically and incorporate them in their decisions. Stress test serve primarily for the assessment of a bank’s capital situation and the identification of measures to minimize risk. In interpreting the results management have to decide whether the bank is able to cope with the losses incurred in a stress scenario, eventually if it is able to cover also losses other than from stress scenario in case they were incurred simultaneously.

In order that management is able to judge the results and plausibility of stress scenarios, they should have active knowledge of or better should be involved in designing stress tests.

The Survey of Stress Tests by GCFS (2001) showed that the results are communicated to the firm's senior management in 100% and they are used to understand the nature of the firm's risk profile in 95%. In 60% the results are used to set limits and in 49% to conduct contingency planning. They influence monitoring liquidity risk only in 26% and the capital allocation in 19%.

Taking corrective action is not always necessary. This is the case when a bank is able to absorb losses incurred in scenario or when the scenario results do not allow immediate conclusion. For example when risk factors were changed in a large number of different markets, the hedging strategy to reduce potential losses may not be clear.

In markets in which a bank's exposure is large, it is frequently monitored and usually worst case scenarios are constructed. The loss resulting from such scenario is good measure of the exposure in the respective market. Then the risk factors contributing most to losses in a scenario can be identifies and management can take a countermeasures. The urgent action includes restructuring of positions or portfolios, hedging strategies, etc.

From the survey by GCFS (2001) follows that in interpreting the results of stress tests, banks take into account their position in the market and strategic aspects of risk management. Thus, the response is not the same by different banks and individual banks do not apply strict, mechanistic policies to unwind the position if given limits are breached. Decisions are rather made on case by case basis.

III MARKET RISK

III.1 Market risk characteristics

Market risk is the risk that a bank may experience loss due to unfavorable movements in market prices. A broader definition encompasses also losses from change in liquidity, rates, indices, volatilities, correlation, and others. Exposure to such risk arises from deliberate speculative positions (proprietary trading) or from the bank's market-making (dealer) activities.

Market risk results from changes in the prices of equity instruments, commodities, money and currencies, therefore its main components are equity position risk, commodities risk, interest rate risk and foreign exchange risk. In addition to standard instruments, market risk also applies to derivatives, such as options, equity derivatives, or interest rate derivatives.

III.1.1 Exchange rate risk

Both on- and off-balance sheet items can be influenced by exchange rate changes, if a bank takes a position in foreign currency (or a position in local currency that is indexed to the exchange rate). Financial institutions can be exposed to exchange rate risk also indirectly; when counter-parties´ creditworthiness depends on exchange rates.

Foreign exchange exposure is measured as net open foreign exchange position calculated according to methodology by BCBS[10] , and then both on- and off-balance sheet positions are included into stress tests. Net open position in each currency can be stressed against variation in the exchange rate (sensitivity analysis), or, in case of significant exposure to several currencies, simultaneous stress tests are applied (scenario analysis). Institutions using internal model usually conduct scenario analysis, taking account of correlations between currencies, although these may break down during crises. Models that test significant short positions in foreign currency options against large price movements should use second order approximation of exchange rate sensitivity (gamma), instead of simple linear

[10] Detailed methodology in BCBS(1996).

approximation (delta). This is because the delta is a linear approximation of a non linear relationship between the value of the exchange rate and the price of the option. The gamma term accounts for nonlinear effects of changes in the spot rate.

The type of scenario used depends on the history of exchange rate, for example a sharp depreciation in the past can be a base for testing impact of future moves on portfolio. However, in rapidly developing financial markets, crises different from the past one may occur. Therefore historical scenarios are combined with hypothetical, including "worst-case" scenarios. The size of shocks applied to exchange rates, volatilities and correlations depends on economic environment in which a financial institution operates. Some minimum size of shock for stress test is again given by The Derivative Policy Group (1995):

- Increase and decrease in the exchange value (relative to the USD) of foreign currencies by 6% in the case of major currencies and 20% of prevailing levels
- Increase and decrease in foreign exchange rate volatilities by 20% of prevailing levels

Other possible type of simulation is through Monte Carlo method. However, only a few financial institutions use this method due to its computational complexity.

III.1.2 Interest rate risk

A financial institution is exposed to the interest rate risk when the interest rate sensitivity of its assets and liabilities (and its off-balance sheet positions) is mismatched. Changes in interest rate can affect both interest income and expenses and market value of balance and off-balance sheet items. Therefore it is related to both trading book and loan book (also called banking book).[11]

[11] Issues relating to trading book and market risks are worked out in the Amendment to capital accord (BCBS(1996)) the interest rate. Interest rate risk in the loan book is treated in the NBCA, which states that it is most appropriate to treat interest rate risk in the banking book under the Pillar 2 –Supervisory Review process. However, in the countries where is a sufficient homogeneity within the banking populations regarding the nature and methods for monitoring and measuring this risk, regulators could establish a mandatory minimum capital requirement. (Bank for International Settlement (2001,p.145))

There are two common approaches used for interest rate analysis: the maturity gap analysis and the duration gap model[12]. Once the exposure of the portfolio is estimated, the nature and size of the shocks needs to be specified. The most common shocks are a parallel shift in the yield curve, a change in the slope of the yield curve, and a change in the interest rate spread. Except for stressing the level of interest rate, underlying volatilities and correlations can also be shocked. The assessment of the size of the shock can be based on historical experience, or on a hypothetical scenario. The standard tests recommended by DPG[13] are:

- Parallel yield curve shifts of 100 basis points up and down
- Steepening and flattening of the yield curves (for maturities of 2 to 10 years) by 25 basis points
- Each of the four permutations of a parallel yield curve shift of 100 basis points concurrent with a tilting of the yield curve (for maturities of 2 to 10 years) by 25 basis points
- Increase and decrease in all 3-month yield volatilities by 20% of prevailing levels

III.1.3 Equity price risk

Equity price risk is a risk that a change in stock prices affects a financial institution's balance and off-balance items. It consists of general equity price risk, associated with movements of the whole stock market, and specific price risk, associated with movements in price of individual stock. The exposure is measured as a net open position, and then both the on- and off-balance sheet equity positions should be included into stress tests. Commonly, stress tests are conducted for general market risk; basic type of shock is a shock to main stock market index.

The DPG proposals of single-factor scenarios are :

- Increase and decrease in equity index values by 10%

[12] Both are explained in detail in chapter about sensitivity stress tests of market risk.

[13] DPG (1995; section 4 no.4)

- Increase and decrease in equity index volatilities by 20% of prevailing levels.

When a financial institution has a significant exposure in several equities, scenario analysis is more appropriate than sensitivity analysis. Stress tests for specific risk are used only in case of highly concentrated trading portfolio of equities. Comprehensive internal models an also implement scenario analysis, e.g. breakdown of correlations among stock prices (indices) during crises or volatility correlations.

Bank are obliged to test historical stock market crashes, but useful are combinations of historical simulation with various hypothetical scenarios, taking future stock market development (such as increases in liquidity, introduction of equity derivatives, changes in regulation and supervision, etc.) into account. Shock to volatilities is dealt with where the stock options position is significant. Impact of different combinations of variables on complex equity option portfolios can be simulated using Monte Carlo methods but here holds facts stated above.

III.1.4 Commodity price risk

Commodity price risk refers to the potential losses resulting from changes in market price of bank's on- and off-balance sheet items due to commodity price changes. Similarly as by the exchange rate risk, commodity prices indirectly affects bank's credit portfolios, when the borrower's repayment ability is affected by commodity price changes (connection to bank's credit risk).

Net positions in the most relevant commodities are usually stressed using historical scenarios, since many commodities has been volatile in the past and therefore size of possible future price swings is quite well assessable. Commodity options are held by banks only seldom.

III.1.5 Principal component analysis[14]

From the previous section, it can be concluded that market risk is commonly defined as the susceptibility of portfolio values to changes in asset prices, volatilities of

[14] Loretan (1997) gives technical exposition of PCA and presents the PCA analysis for data from nine countries on spot exchange rates, stock market indexes and long-term and short-term interest rates.

prices, and related functions of asset prices. However, in practice many asset prices and volatility movements are highly correlated and it is therefore not worth of concentrating at such multidimensional risk. One way how to extract market risk factors from observed data is the Principal Component Analysis (PCA). Although the greater number of risk factors increases the descriptive accuracy (greater fraction of data variability is captured by a model), it on the other hand requires a very complex methodology and the analysis becomes impalpable.

Consider T observations of N asset returns and denote X the resulting TxN matrix. The aim is to find a linear combination of the observed returns that explains best the observed variability of the data. If we denote P a matrix of the eigenvectors of XX´ that corresponds to N nonzero eigenvalues sorted in descending order. Then the first column of P is the "fist principal component" of X, which explains the highest fraction of variability. The second column – the second principal component (PC) - also maximizes the explained variability, but now the explanation given by first principal component. Loadings of the data on each principal component have the property that the sum of their squares for each factor is 1.

Note that PCs are not directly observed, but are constructed as a linear combination of the data.

In a majority of cases, one or two principal components suffice to capture most variability of the data. These are considered as an effective dimensionality of data. Since PC is the transformation of the observed data, it is possible to recalculate from corresponding value of the e.g. first PC the values of each of series back. We may also pick tail quantiles of the empirical distribution of PC and generate corresponding tail events of the observable series. In case two or more PCs are needed to explain sufficiently the total variance of data, we may consider separate shock in each direction or we may create a linear combination of relevant PCs to estimated combined shock. It is also possible to generate scenario with a large realization of the series, which is highly correlated with PCs.

The following chapters will explain nearer the different types of stress scenarios. We will begin with the simplest one based on change of a single risk factor and proceed with historical and hypothetical scenarios (structured above), etc. The aim is to introduce models that may be used to asses the changes in the portfolio.

III.2 Sensitivity stress tests

In order to illustrate these single-factor stress tests nearer, I explain several approaches dealing with interest rate risk. Its modeling is the most important for most fixed-income portfolios.

III.2.1 Maturity/repricing gap approach

Measurement of interest rate risk (see definition in III.1.2) is usually done separately for banking and trading book. The simplest technique for measurement interest rate risk begins with a maturity/reprising schedule that distribute interest-sensitive assets and liabilities into few time categories or “buckets”.[15] This method is called GAP analysis. In the repricing gap model the gap is the difference in the flow of earnings on assets and liabilities hold in each bucket. In the maturity gap model, the gap is defined as a difference between maturity of its assets and liabilities, where the weights correspond to proportion of an individual asset on total market value of portfolio.

The following example shows how the repricing gap approach may be applied. Table 1 contains the shows buckets of interest rate sensitive assets and liabilities and basic gap analysis. The table provides information on the extent of the bank’s interest rate exposure based either on the contractual maturity date of its financial instruments or, in the case of instruments that reprice to a market rate of interest before maturity, the next repricing date. Off balance sheet assets and liabilities reflect amounts receivable and payable arising from interest rate derivatives which include interest rate swaps, interest rate forwards, interest rate options and cross currency swaps.

[15] The length of time for which the rate of interest is fixed on a financial instrument, therefore, indicates to what extent it is exposed to interest rate risk.

Table 1: Gap analysis - example (data in CZK million)

	assets (A)	liabilities (L)	GAP (A-L)	off-balance A	off-balance L	GAP A_{off}-B_{off}	GAP total	Weights[1]	weighted GAP total
To 3M	287790	150257	137533	69158	144857	-75699	61834	0,2	12366,8
3M-1Y	57587	26453	31134	134153	129056	5097	36231	0,6	21738,6
1Y-5Y	44734	1215	43519	89827	60216	29611	73130	2	146260
Over 5Y	25662	3046	22616	50934	9943	40991	63607	4,6	292592,2
						sum:	234802	sum:	472957,6

[1]The weights for each time band are those from BCBS(1998, table 1) or were calculated by linear approximation.

Source: Komercni Banka Annual Report 2003 and the author's calculations

The gap can be multiplied by an assumed change in interest rates to yield an approximation of the change in net interest income that would result from such an interest rate movement. A positive gap implies that the bank's net interest income could decline as a result of a decrease in the level of interest rates.[16]

The size of the interest rate movement used in the analysis can be based on a variety of factors, including historical experience, simulation of potential future interest rate movements, and the judgment of bank management or supervisory requirement.

The following table shows, how stress scenarios based on shift of yield curve of 100 points and on rotation of yield curve by 25 basis points for maturities over one year (combination of standard sensitivity tests recommended by DPG). In more sophisticated analysis, positions may be weighted by a factor that is designed to reflect the sensitivity of the positions in different maturity bands to an assumed change in interest rate. In the second and third column are analyzed the consequences of interest rate changes without considering the weights, in the fourth and fifth column with considering the weights. The results of analysis using the weighted gap are than better than of analysis using unweighted gap.

The impact on risk-weighted assets (RWA) and on capital is assumed to be 100%, the highest possible in order to model extreme situation, but can be chosen less.

[16] Note that a substantial simplification is involved when just adding A/L balance gap and A/L off-balance gap.

Table 2: Examples of stress scenarios

CZK million

capital adequacy	15,4%			
regulatory capital	15319,00			
RWA	99668,00			
	Unweighted	Unweighted	Weighted	Weighted
Shift of yield curve (Δr)	0,01	-0,01	0,01	-0,01
net interest income impact = GAP*Δr	2348	-2348	4730	-4730
capital after shock	17667	12971	20049	10589
Impact on RWA/impact on capital (%)	100,0	100,0	100,0	100,0
RWA after-shock	102016,0	97320,0	104397,6	94938,4
CAR after-shock (percent)	17,3%	13,3%	19,2%	11,2%
Change in CAR after-shock (pct points)	**1,95**	**-2,04**	**3,83**	**-4,22**
rotation of yield curve				
Change in long term interest rate	0,0025	-0,0025	0,0025	-0,0025
net interest impact	195,6	-195,6	4096,1	-4096,1
capital after shock	15514,6	15123,4	19415,1	11222,9
impact on RWA/impact on capital	100,0	100,0	100,0	100,0
RWA after shock	99863,6	99472,4	103764,1	95571,9
CAR after shock	16%	15%	19%	12%
Change in CAR after-shock (pct points)	**0,17**	**-0,17**	**3,34**	**-3,63**

RWA.....Risk Weighted Assets, CAR=regulatory capital/RWA.
Note that for simplicity no profit is assumed. Considering it would improve the results.

Source: Data from Komercni banka Annual Report 2003, calculations by the author.

Although the gap analysis is a very commonly used approach to assessing interest rate risk exposure, it has a number of shortcomings. First, gap analysis does not take account variation in the characteristics of different positions within a time band (all are assumed to mature simultaneously). Second, gap analysis ignores differences in spreads between interest rates that could arise as the level of market interest rates changes (basis risk). Third, it does not include the impact of interest rate changes on market prices of assets. In addition, matching the maturity of assets and liabilities, a bank may still remain exposed to losses from interest rate changes, e.g. if teh timing of the cash flows in assets and liabilities is different. This is the main reason, why the following approach based on duration is a more accurate measure of exposure to interest rate risk.

III.2.2 Duration gap approach

The second approach analyzing the sensitivity of cash flows to changes in interest rates is a duration model. Duration is a measure of the percent change in the economic value of a position that will occur given a small change in the level of interest rates (interest elasticity of the economic value). It is calculated as the weighted average time-to-maturity, where the weights are present values of cash flows. Thus duration reflects the timing and size of cash flows that occur before the instrument's contractual maturity.

Once the duration of analyzed group of assets is derived, a bank can use duration gap analysis to determine the exposure to interest rate risk. The duration gap is used by portfolio immunization – matching the gains and losses in the assets´ values form the changes in interest rates with gains and losses in the liabilities´ values.

Using a single discount factor (based on single interest rate r) in calculation of duration of duration, the scenarios constructed through changes in r are limited on the parallel shifts of a flat yield curve. However, the choice of specific discount factors for different maturities enables simulation of changes in the shape of the yield curve.

The main weakness is that duration is a good measure of the change in the economic value of a position only for small changes in interest rates and it does not take into account changes in the shape on the yield curve. However, stress often involves large movements in interest rate yield. In such case the incorporation of second order approximation – the convexity- allows to estimate the price of a position more accurately.

Following model introduces the possibility how to account for curve exposure in the portfolio. Different shapes on the yield curve are dealt through term structure model.

III.2.3 Yield curve model[17]

In this section, a simple model for term structure of interest rates will be introduced, which can be helpful for stress testing under different interest rate scenarios. The yield curve is modeled by a function that changes rapidly at the beginning and then flattens at the end:

$$y(t) = y_l - (y_l - y_s) \cdot e^{-\alpha t}$$

[17] Simozar(1998).

where t means time, α is a positive constant and y(t) represents the spot yield at time t. Long term and short term parts of the yield curve are expressed by y_l and y_s, as obviously $y(\infty)=y_l$ and $y(0)=y_s$. Coefficient α here sets the transition time, t_{sl}, between short and long term, for great α transition occurs very quickly and the long term rates dominate and vice versa. If we define a yield in the t_{sl} as an average of short term and long term rate and insert it into the previous equation, we get:

$y(t_{sl}) = \dfrac{y_l + y_s}{2}$ and $\alpha t_{sl} = \ln(2) = 0.693$.

For further computation, we choose α=0.2, which corresponds to 3.5 years transition time, but any other value between 0.11 and 0.35 (≈2 to 6 years transition time) would also be reasonable without greater impact on the final relative risk measures.

Values of short term and long term rates are obtained from minimization of yield error, i.e. the difference between the market yield of a zero coupon bonds $y_m(t_i)$ with maturity of t_i and actual yield,

$$Z = \sum_i \left(y_m(t_i) - y(t) \right)^2$$

Further, the price for security with a cash flow stream, using continuous compounding method can be written:

$$p = \sum_i c_i e^{-yt_i} \quad ,$$

The duration can be written as the weighted average time to cash flow, discounted by the market price:

$D = \dfrac{1}{P_m} \sum c_i t_i e^{-yt_i}$. Taking derivatives of the duration with respect to y_l and y_s, it is possible to divide it into short and long rate parts (Ds, Dl), the sum of which gives D:

$D_s = \dfrac{1}{P_m} \sum c_i t_i e^{-\alpha t_i} e^{-yt_i}$ and $D_l = \dfrac{1}{P_m} \sum c_i t_i (1 - e^{-\alpha t_i}) e^{-yt_i}$

Every security can be calibrated such that its calculated price is identical to its market price. This can be done through subtracting (or adding) a yield spread for every security as follows:

$p_m = \sum_i c_i e^{=(y_c + x)t_i}$ where p_m is the market price, x the yield spread and y_c the calculated yield based on term structure model. X is calculated either by iterative process or simplified from following equation:

$x = \frac{p_c - p_m}{D \cdot p_m}$ where p_c is calculated price for security.

Let us see how to manage interest rate stress testing. First, from equation for p_m the yield spread of every security in portfolio relative to the term structure model is calculated. Stress scenario is made through changing one parameter of term structure (e.g. in accordance with some historical change having occurred in markets). Then the new prices for all securities are recalculated.

This simple term structure model seems to be an improvement of the duration approach, but it does not account well for changes in the middle of the yield.

III.3 Historical scenarios

The Basle Committee[18] requires the construction of stress scenarios on the basis of historical crises:

"Banks should subject their portfolios to a series of simulated stress scenarios and provide supervisory authorities with the results. These scenarios could include testing the current portfolio against past periods of significant disturbance, for example, the 1987 equity crash, the ERM crises of 1992 and 1993 or the fall in bond markets in the first quarter of 1994, incorporating both the large price movements and the sharp reduction in liquidity associated with these events."

Example: A scenario replicating the stock market crash of October 1987

- Worldwide drop of equity markets by 20 percent on average, Asian markets declining by 30 percent and increase in volatilities from 20 to 50 percent
- Appreciation of U.S. dollar as consequence of flight to quality (up to 10 percent against Asian currencies)

[18] Amendment to the Capital Accord to incorporate market risks, Basle Committee (1996; section B.5 no 6)

- Drop of interest rates in Western markets and rise in Asia by 100 bp in short term and 40 bp in long term
- Expectations of recession result in commodity prices fall (oil prices decline by 5 percent)

Historical scenarios can be conducted by re-valuing portfolios using values of risk factors that existed during historical stress events. In the background of this approach is an assumption that past crises are similar to future ones. Risk managers therefore cannot ignore the results of testing arguing with improbability of tested scenarios. In comparison with VaR models that use only recent data, historical models work also with events in distant past. However, the analysis of past stress events must not show the worst possible loss, as this method does not reflect the portfolio composition. Some portfolios do not have to be influenced by maximum of some other factors. The challenge in using historical scenarios is to choose a scenario that is appropriate for the bank's portfolio. This may be difficult because of the changed nature of financial markets or because of the introduction of new financial instruments that did not exist at the time of the historical stress event.
For the scenario building choice of the values of several parameters is crucial. It is the choice of an observation period, the choice of a duration window (1-day changes, 10-day changes, etc.) and of change parameters follows. The wider the sample the greater is the probability that it will incorporate data about more extreme events (changes). On the other, a wider sample also includes data from the distant past that are irrelevant to the present situation.

III.4 Hypothetical scenarios

III.4.1 Maximum Loss

Going back to previous notation, by comparing P(m) with the current value of the portfolio $P(m_c)$, one can identify the losses that would occur if market changes from m_c to m without realizing portfolio rebalancing. Allowing for rebalancing, the possible loss will be smaller, thus the computed value creates the upper bound for the loss. As neither the BCBS nor any other regulatory institutions provide the method, the question is how to identify scenarios to model worst case loss that would at the same time contain market state at the end of holding period with some high probability. Denoting fixed set of scenarios – admissible domain- as A, maximum loss within such set is:

$$Max\ loss_A(P) = P(m_c) - \min_{r \in A} P(m)$$

The domain A can be defined as all scenarios above certain plausibility threshold expressed by probability. The higher is the probability of movement from mc to m, the higher the plausibility of m. This implies that scenarios which are more distant from the present market state will be less plausible.

A relatively simple method how to at least roughly identify a worst-case scenario is a factor push method. The basic process is to take each individual risk factor and change it in the direction that will reduce the portfolio value most.

Firstly the portfolio values after the positive and negative risk factor changes are computed – as a multiple k of risk factor's standard deviation σi:

$$P_{1,2} = P\left(r_{c,1},\ldots,r_{c,i}\left(1 \pm k\,\sigma_i\right),\ldots,r_{c,n}\right).$$

The second step is a transformation using the function sign:

$$S(i) = sign(P_1 - P_2)\,,$$

thus S=1 it the upward movement results in higher portfolio value than downward movement and S=-1vice versa. The worst case scenario now can be written as:

$$m_{wc} = \left(r_{c,1}\cdot\left[1 - S(1)k\sigma_1\right],\ldots,r_{c,n}\cdot\left[1 - S(n)k\sigma_n\right]\right)\ .$$

Because of choosing the multiple k constant for all risk factors, this method is suitable mainly for portfolios with linear valuation functions, for non-linear functions (portfolios containing derivatives) applying several values of k is more suitable. Note that scenario with simple k lies on the surface of the n-dimensional cuboid.

III.4.2 Scenarios based on historical data

Some sources[19] introduce scenarios based on historical data, which are, however, constructed as combinations of the maximum changes for each risk factor. These

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[19] Breuer,Krenn (1999)

changes are then combined into a scenario. Thus the scenario is does not copy fluctuations of single historical event.

Going back to the notation from chapter II.3.2, the resulting stress scenario is:

$$m = (r_{c,1} \pm \Delta r_1, r_{c,2} \pm \Delta r_2, ..., r_{c,n} \pm \Delta r_n)$$

A problem of this equation is that it gives too many possible results (2^n). Adding or subtracting the change may be conform with the direction of real greatest jump[20], or similar risk factors may be decided to move in one direction while creating the scenario or only movement of some risk factors may be reflected other leaving unchanged.

However, including all extreme movements observed within chosen period at the same time might lead to very implausible scenarios. The larger number the risk factors is involved, the smaller is the plausibility of resulting scenario. This argument favors the techniques respecting correlations between risk factors or historical scenarios based on actual historical events. However, neither the scenarios based on historical event nor this approach based on combination of maximum movements of historical data does not specify the likelihood of occurrence of specified extreme movements.

III.4.3 Subjective scenario search

In this technique, neither the determination of stress events and selection of triggering events nor the determination of relevant risk factors is clearly defined. In the background stays one's anticipation of an adverse political or economic event that could cause large losses to a bank. The quality and plausibility of such scenario depends on the quality of the economic expertise and reasoning.

Subjective search is very demanding on experience in regional politics, industry specifics, banking, etc., has to involve number of experts and still the deduced triggering events or risk factors can be incomplete or wrong.

[20] Similarly as by simulating the historical event, the historical observation period and the time window are chosen.

III.4.4 Scenarios with covariance matrix forecasting

For a bank, the forecasting of covariances between asset returns is important in addition to the forecasting of variances. The forecasted covariance matrix is important in many practical applications; for instance in option pricing and calculation of VaR measures and thus also in stress tests.

For example, there are several possibilities how to deal with peripheral risk factors. The simplest specification for peripheral asset moves is to assume no change (recall scenarios ignoring peripheral risk factors). The second specification applies moves in the peripheral assets that have coincided with large moves in the core assets historically. The third specification utilizes estimates of volatility and correlation to estimate the conditional expectation of peripheral asset moves given the stress moves in the core assets. Of the three methods, the latest one appears the most attractive; however, we have to justify the contention that standard volatility and correlation estimates will produce good stress forecasts of the peripheral asset moves. This Kupiec(1998) shows, who has developed a simplified distribution model.

In this model, the stress tests can be performed using the characteristics of conditional multivariate normal distribution in the framework of VaR. Assume that there are N assets in the portfolio and that first N-k are non-core assets and k remaining are core assets. We can then partion the return vector $R_t = \begin{bmatrix} R_{1t} \\ R_{2t} \end{bmatrix}$, where R_{1t} is (N-k)x1 vector and R_{2t} is kx1 vector. The return vector follows an N-dimensional normal distribution $R_t \approx N^N \left(\frac{\mu_{1t}}{\mu_{2t}}, \begin{matrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{matrix} \right)$.

If we denote $X_t=(x_{1t}, x_{2t}, x_{3t},... x_{nt})$ a vector of total cash flows from all portfolio positions, the portfolio value change can be written:

$$\Delta V_t = \begin{bmatrix} X_{1t} & X_{2t} \end{bmatrix} \begin{bmatrix} R_{1t} \\ R_{2t} \end{bmatrix}$$

and the expected value of portfolio value change:

$E(\Delta V_t) = X_{2t} R_{2t} + X_{1t} \mu_c$, where μ_c is a mean value of other factors´ conditional distribution: $R_{1t}\big|_{R_{2t}} \approx N(\mu_c, \Sigma_c)$ and can be expressed as $\mu_c = \Sigma_{12} \Sigma_{22}^{-1} R_2$, while the conditional variance as $\Sigma_c = \Sigma_{11} - (\Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21})$.

The portfolio risk exposure, using for example 5% critical value, can be then counted as 95% stress scenario VaR:

$$StressVaR(95) = X_{2t}R_{2t} + X_{1t}\mu_c - 1.65\sqrt{X_{1t}\Sigma_c X_{1t}^T}\ .$$

The last element, $-1.65\sqrt{X_{1t}\Sigma_c X_{1t}^T}$, expresses the unexpected variation in the pricing factors in R_{1t}. If we decide to use the expected value of the stress test portfolio value, we just omit it.

Now it is to show how volatilities and correlations can be stressed. Let's define Ω_t, the factor return correlation matrix and D_t, a diagonal matrix with factor return standard deviations as elements. The covariance matrix used for VaR computation is then possible to rewrite as $\Sigma_t = D_t\Omega_t D_t$.

Using this decomposition, volatility shock can be easily involved. We define a matrix Δ, the elements of which are differences of standard deviation desired in the stress test scenario from historical standard deviations. It is zero whenever the factor's standard deviation stays the same in the stress test. Now it is possible to rewrite covariance matrix as $\Sigma_t^c = (D_t + \Delta)\Omega_t(D_t + \Delta)$.

Let's assume a stress scenario, where correlations among *h* factors are different from the historical ones. We write these *h* pricing factors in h first rows of the factor matrix and divide the pricing factor return vector into two parts. R_{at} is a (hx1) vector of a pricing vector returns that will experience correlation shocks in the chosen stress scenario and R_{bt} is a vector of remaining returns. Now, the distribution of factor pricing return is:

$$\begin{bmatrix} R_{at} \\ R_{bt} \end{bmatrix} \approx N\left(\begin{bmatrix} 0_h \\ 0_{N-h} \end{bmatrix}, \begin{bmatrix} D_{at} & 0 \\ 0 & D_{bt} \end{bmatrix} \times \begin{bmatrix} \Omega_{aat} & \Omega_{abt} \\ \Omega_{abt} & \Omega_{bbt} \end{bmatrix} \times \begin{bmatrix} D_{at} & 0 \\ 0 & D_{bt} \end{bmatrix}\right)$$

The last factor is the portioned correlation matrix, so that substituting it into a covariance matrix discussed in the previous section for Ω_t, previous procedure can be applied in conducting stress test. Note, that a sub-matrix Ω_{aat} is the correlation matrix that comprises all the in the stress test changed correlation. The StressVaR can be computed in the same way as before (only using this modified correlation matrix).

We can show such portioned correlation matrix for three assets, to make the method more comprehensible. For example, we may expect that the volatility of the third asset will increase and other volatilities will not change. Further we will simulate the change in correlations between first and second asset.

$$\underset{D_b}{\overset{D_a}{\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_{21} & 0 \\ 0 & 0 & \boxed{\sigma_{31}+\Delta} \end{bmatrix}}} \times \overset{\Omega_{aa}}{\begin{bmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{21} & 1 & \rho_{23} \\ \rho_{31} & \rho_{32} & 1 \end{bmatrix}} \times \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_{21} & 0 \\ 0 & 0 & \sigma_{31}+\Delta \end{bmatrix}$$

III.4.4.1 **Recent approaches**

Traditionally, variances and covariances were described by very simple models that often relied on historical data and extrapolated into the future in an unconditional way. However, the need to access possible impact of large shocks led to a search for more elaborate, usually conditional, models. Kupiec (1998) believes that standard VaR normality assumption and the use of historical correlations and volatilities do not cause any unacceptable bias in stress event loss measure, at least for portfolios with exposures broadly distributed among risk factors. He showed how to parameterize stress test scenarios that use conditional probability distribution and shock risk factors, risk factor volatilities and risk factor correlations.

Nowadays the strong belief prevails that financial return distributions have fat tails. Looking for alternative modeling that would be conform with the evidence that time series of returns often exhibit time-dependent volatility, the unconditional (time-independent) distribution of returns is substituted by conditional distributions (time-dependent). Unconditional distribution of returns assumes that returns are independent of each other and that the return-generating process is linear with parameters that are independent of past realizations. An example is the standard normal distribution, t-distribution or the compound normal model.

The first conditional model was ARCH (Auto Regressive Conditional Heteroscedasticity) model developed by Engle(1982). This and other ARCH-type models[21] were univariate and their extension to multivariate framework was problematic, as the number of parameters that have to be estimated rises dramatically. It seems that GARCH-type models have gained the most attention recently. The multivariate estimation problem solves e.g. orthogonal GARCH model.[22]

The least computationally demanding procedures for estimating volatility are the extreme value and regression methods.

In practice usually quite simple approaches are used. RiskMetrics uses the exponentially weighted moving average model (EWMA) to forecast variances and covariances (volatilities and correlations) of the multivariate normal distribution. It is on improvement of volatility forecasting method that relies on moving averages with fixed weights. The advantage of implying variable weights is that the volatility reacts faster to shocks in the market as recent data carry more weight than data in the distant past. Further, after a shock, the volatility declines exponentially as the weight of the shock observation falls.

III.4.5 Extreme value theory

As expressed by the Basle Committee, using VaR models in banks is associated with an obligation to conduct a rigorous stress testing program that meets certain quantitative criteria. Extreme Value Theory (EVT) is a quantitative tool that provides a unified framework for both VaR and stress testing.

The EVT was introduced[23] because of extraordinary events such as the stock market crash of October 1987, the breakdown of the European Monetary System in September 1992, the turmoil in the bond market in February 1994 and the recent

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[21] Since the introduction of the basic ARCH model, extensions include generalized ARCH (GARCH), Integrated GARCH (IGARCH), Exponential GARCH (EGARCH), and the others.

[22] Byström(2000) use this method to forecast covariance matrix in stress scenario represented by Nordic stock market during Asian financial crises.

[23] This chapter is based on Longin(2000), Bekiros, Georgoutsos(2003) and Gencay, Selcuk(2000). A comprehensive treatment of EVT can be found in Embrechts, Kluppelberg, and Mikosh (1997).

crisis in emerging markets are a central issue in finance and particularly in risk management and financial regulation.

The performance of a financial institution over a year is often the result of a few exceptional trading days as most of the other days contribute only marginally to the bottom line. Regulators are also interested in market conditions during a crisis because they are concerned with the protection of the financial system against catastrophic events which can be a source of systemic risk. From a regulatory point of view, the capital put aside by a bank has to cover the largest losses with a given level of plausibility such that it can stay in business even after a great market shock. In statistics, extremes of a random process refer to the lowest observation (the minimum) and to the highest observation (the maximum) over a given time period. In financial markets, extreme price movements correspond to market corrections during ordinary periods, and also to stock market crashes, bond market collapses or foreign exchange crises during extraordinary periods.

Extreme price movements can be observed during usual periods corresponding to the normal functioning of financial markets and during highly volatile periods corresponding to financial crises. An approach based on extreme values then covers market conditions ranging from the usual environment considered by the existing VaR methods to the crises which are the focus of stress testing.

Generally, extreme value approach uses a parametric method, in which the distribution of extreme returns instead of all returns is considered and then computes the VaR of a market position. The VaR computation is conducted either for market position decomposed on risk factors (stable portfolio with few assets) or for aggregated market position (complex unstable portfolios). The latter case is treated through univariate distribution of extreme returns while the decomposed position through the multivariate distribution.

It is possible to distinguish two alternative methods for generating extreme returns. The older one is called *Block Maxima* (BM), the newer one the *Peak over Threshold* (POT).

III.4.5.1 **Block maxima**

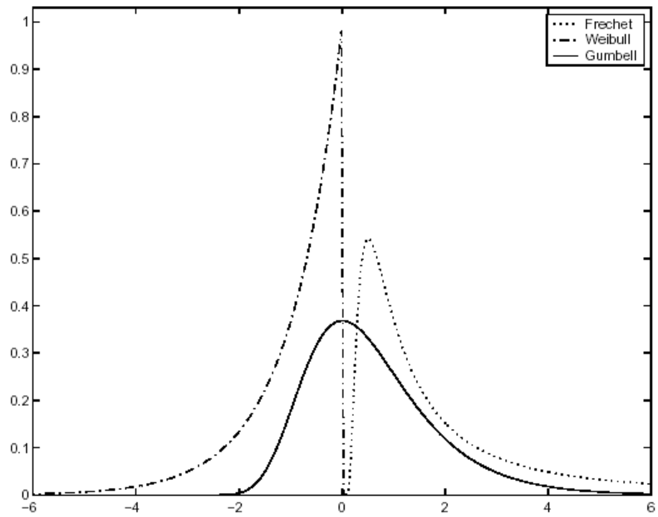
Firstly, we divide the sample of m observation into time-intervals of length n (corresponding to n trading intervals, e.g. n trading days), over which we observe returns R_1, R_2,..., R_n. Supposing Z_n is a maximum of n random variables $X_1, X_2, ..., X_n$, and under the assumption that there exists normalizing constants a_n and b_n such that $\frac{Z_n - b_n}{a_n}$ is non-degenerated, that means if

$\lim_{n\to\infty} P\{(Z_n - a_n)/b_n \le x\} = \lim_{n\to\infty} F^n(b_n x + a_n) = H(x)$ for some nondegenerated limit distribution H, then H is of Generalized Extreme value Distribution (GEV):

$$H(x) = \begin{cases} \exp\left(-(1+\xi \cdot x)^{1/\xi}\right) & if\ \xi \neq 0 \\ \exp(-e^{-x}) & if\ \xi = 0 \end{cases}$$ where ξ is the slope parameter.

For financial series is most relevant the case when ξ>0, the ordinary Pareto distribution, which is a heavy (fat) tailed one and is known as Fréchet distribution. The case ξ=0, so called Gumbel distribution, is a thin-tailed distribution, where cumulative distribution function declines exponentially. ξ<0 corresponds to thin-tailed distribution with finite endpoint, also called Weibull distribution.

Figure 5: Densities of Fréchet, Weibull, and the Gumbel distributions



Source: Gencay, Selcuk(2000; p.6)

If we believe that the distribution of the series of return is heavy tailed, thus of a Fréchet type, we fit the GEV distribution $H_{\xi,\mu,\sigma} = H[(Z_n - \mu)/\sigma]$ on standardized data of block maxima, $Z_n - \mu/\sigma$, where μ is the location parameter and σ a scale parameter, that take care of the unknown sequences of normalizing constants a_n and b_n.

Now we can derive VaR, thus a level, that we expect to be exceeded in one block for every k blocks (of n observations each), on average. Here VaR is a quantile of the generalized extreme value distribution:

$VaR = H^{-1}_{\xi,\mu,\sigma}(1-1/k) = \hat{\mu} - \frac{\hat{\sigma}}{\hat{\xi}}\left(1-(-\log(1-1/k)^{1/n})^{-\xi}\right)$, where the sheltered variables are the maximum likelihood estimates of parameters. For example, if we have weekly blocks (n=5 days) and we want to estimate a value to be exceeded once every 100 weeks, we compute the $(0.99)^{1/5}$ = 0.998 quantile.

III.4.5.2 **Peak over Threshold**

The POT method is based on selecting a high threshold *u* and analyzing the values exceeding the threshold. Let us denote a X_t a sample of observation with distribution function F(x)=Pr{ $X_t \leq x$}. An exceedence over *u* occurs when X_t >u and we define it y= X_t-u. Probability distribution of the excess values can be now written as

$$\mathrm{F_u(y) = Pr\{X - u \leq y \mid X > u\} = \frac{F(y+u) - F(u)}{1 - F(u)}}.$$

Since x=y+u for X_t >u, we have the following representation of excess distribution function: F(x)=[1-F(u)]F_u(y)+F(n).

According to Balkema and de Hahn(1984), Pickands(1975) theorem, for sufficiently high u the limiting distribution of $\mathrm{F_u(y)}$ is the Generalized Pareto Distribution (GPD) which is defines as

$$\mathrm{G}_{\xi,\sigma,\mathrm{v}}(x) = \begin{cases} 1-\left(1+\xi\dfrac{x-v}{\sigma}\right)^{-1/\xi} & if\ \xi \neq 0 \\ 1-\exp\left(-(x-v)/\sigma\right) & if\ \xi = 0 \end{cases} \quad \text{with } \mathrm{x} \in \begin{cases} [\mathrm{v},\infty] & if\ \xi \geq 0 \\ [\mathrm{v}, \mathrm{v} - \sigma/\xi] & if\ \xi \prec 0 \end{cases},$$

where $\xi=1/\alpha$ is the shape parameter, α is the tail index, σ is the scale parameter and v is the location parameter.

The GPD can be estimated by maximum likelihood method and the estimated parameters are then asymptotically normal distributed.

Tail estimation can be rewritten using the maximum likelihood estimates of the shape and the scale parameters ξ and σ:

$$F(x)=\left[1-F(u)\right]\cdot G_{\xi,\sigma,u}(x-u)+F(u)=(1-\frac{u-n_u}{n})G_{\xi,\sigma,u(x-u)}+\frac{u-n_u}{n}=1-\frac{n_u}{n}(1+\hat{\xi}\frac{x-u}{\hat{\sigma}})^{-1/\xi}$$

Now, the EVT can be used to obtain VaR estimate. For a given probability p, an estimate of the VaR can be calculated by inverting the tail estimator:

$$Var(1-p)=u+\frac{\hat{\sigma}}{\hat{\xi}}\left[\left(\frac{n(1-p)}{N_u}\right)^{-\xi}-1\right]$$

, where N_u is the number of data in the tail and u is the chosen threshold.

The extreme value distribution, similarly as empirical (historical) distribution or normal distribution, are used for unconditional estimates that give the same results whatever the market conditions at the time of estimation. All the methods give also similar VaR estimates for low probability levels. However, if we concern with high confidence level (e.g. of 99.9%), the VaR computed from normal distribution may underestimate the risk, especially in the presence of fat-tailed time-series. Historical simulation does not suffer from the tail-bias problem, because it does not rely on normality. However, its disadvantage is that high quantiles are calculated only from few observations and are therefore not reliable.

Variance-covariance analysis relies on the assumption that financial market returns follow a multivariate normal distribution. It is easy to implement, because the VaR can be computed from a simple linear formula with variances and covariances of returns as the only inputs. Its major drawback is that financial market returns may not be normally distributed, having fatter tails than the normal. This means that losses are much more frequent than predicted by variance-covariance analysis.

Conditional models such as the GARCH process and the EWMA process use normal distribution with time-varying mean and variance and thereby account for time-varying conditions of the market. They lead to VaR that reflects the degree of market volatility at the time of estimation. VaR estimates increase during high volatility periods and decrease during low volatility ones. If the conditional distribution is assumed normal, the VaR estimate is similar to those from unconditional models using normal distribution during periods with low volatility. The occurrence of an extreme return immediately influences conditional estimates.

Several studies[24] tried to evaluate which of mentioned methods gives most accurate VaR estimates. The research was made on stock indices data in different periods and all came to conclusion, that the extreme value method gives the best risk estimates on high risk levels as 0.1%.

III.4.6 Monte Carlo simulation

Monte Carlo simulation is a special comprehensive method of generating the probability distribution for change in value of portfolio. It consists of repeatedly simulating the random processes that govern market prices and rates. Here I will only very briefly describe the process:

i. The portfolio is evaluated at present day using the current values of market variables.

ii. Then once is sampled from the multivariate normal probability distribution of changes in asset values.

iii. The values of changes in asset values (returns) that are sampled are used to determine the value of each market variable at the end of the day.

iv. The portfolio is revalued at the end of the day in the usual way.

v. The value calculated in step one is subtracted from the value in step four to determine change in value of portfolio (dP).

vi. Then steps two to five are repeated many times to build up probability distribution of portfolio value.

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[24] Longin(1999), Bekiros, Georgoutsos(2003), Danielsson,de Vries(1997).

After such procedure, it is possible to calculate VaR as the appropriate percentile of the probability distribution of portfolio value change. For example, we can suppose that we calculate 10 000 different sample values of the dP in the way described by previous procedure. The 1-day 99% VaR is the value of dP for the 100^{th} worst outcome. The N-day VaR is usually assumed to be 1-day VaR multiplied by $\sqrt{N}$.

Monte Carlo simulation is a powerful and flexible approach. It can accommodate any distribution of risk factors including "fat tail" distribution with extreme events and "jumps" in price processes. Although the Monte Carlo simulation allows stress tests to simulate the impact of a wide variety of different combinations of variables, and to include the effect on the portfolios with non-linear characteristics (as complex option portfolios), it is computationally very intensive and requires high level risk management. Because of complexity of portfolios that have to be revalued many times it tends to be slow method. Additionally, supervisors have to dispose by sufficient experts in order to be able to verify the accuracy, and to interpret correctly the results of the simulation. For all these reasons it can be found only in the most sophisticated financial institutions and it can be only an additional tool.

IV CREDIT RISK

IV.1 Credit risk characteristics

IV.1.1 Definition, differences and difficulties

Credit risk is the risk that a counterparty or obligor will default on their contractual obligations. It refers to the risk that the cash flows of an asset may not be paid in full according to contractual agreements, which may affect bank's liquidity and potentially also solvency. A default may occur due to problems such as bankruptcy, illiquidity, or bad faith.[25]

Credit risk models cannot be created through simple extension of their market risk counterparts, because of data limitation. Unlike the market variables, many credit instruments are not traded or marked-to-market, so there is very little information on the underlying value of a particular instrument. The predictive nature of a credit risk model is not derived from a statistical projection of future prices based on a comprehensive record of historical prices. The scarcity of the data required to estimate credit risk models also stems from the infrequent nature of default events[26] (at least in comparison with changes in market prices) and the longer-term time horizons used in measuring credit risk. The distribution of returns is usually asymmetric (Figure 6). The long downside tail of the distribution of credit returns is caused by defaults. Credit returns are characterized by a fairly large likelihood of earning a (relatively) small profit through net interest earnings (NIE), coupled with a (relatively) small chance of losing a fairly large amount of investment. Across a large portfolio, there is likely to be a blend of these two forces creating the smooth but skewed distribution shape below.

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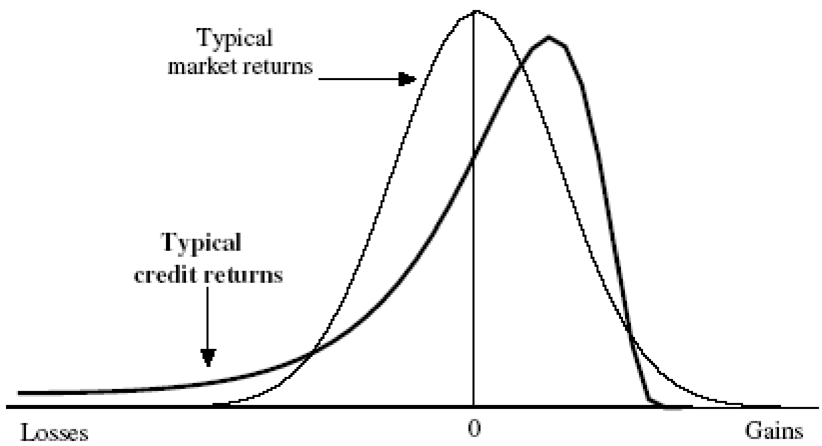
[25] Counterparty risk can be handled as an extension of the concept of credit risk, as it goes beyond financial failure and includes other things such as delays in execution caused by the counterparty and the financial environment. However, this separation of counterparty and credit risk in not always used. After all, independent rating agencies grade not only the likelihood of counterparty default, but usually take into account also country risk or legal risk.

[26] Therefore, it is up banks (among others also in the Czech Republic) to begin with data collection. This will only enable to implement advanced credit risk models and stress testing.

Figure 6: Comparison of market return and credit return distribution



Source: CreditMetrics™ -Technical Document

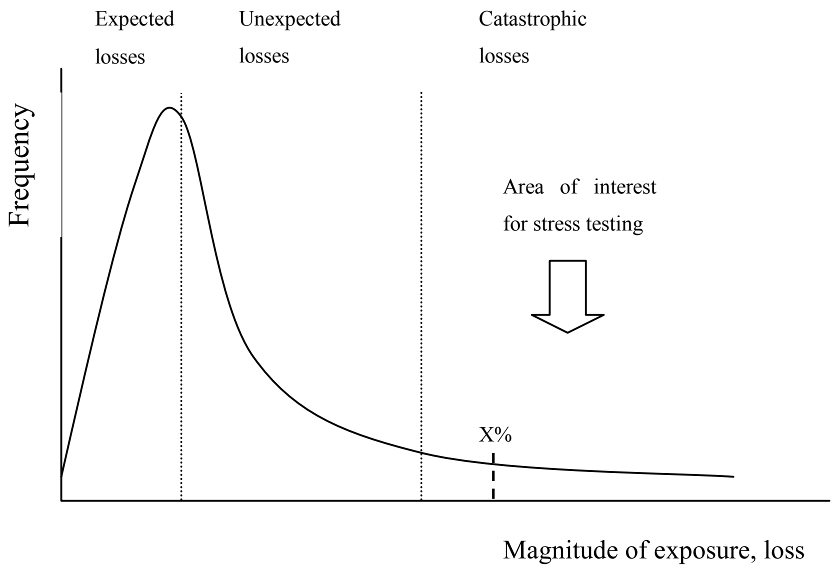
The combination of insufficient data, infrequent observations, and asymmetric distributions makes the modeling credit risk extremely difficult, both analytically and empirically, and also limits the possibilities of backward validation of models and examination of consequences of extreme events (stress testing).

IV.1.2 Probability distribution

The internal rating based method (New Basle Capital Accord) requires the regulatory capital to cover both expected and unexpected losses. Expected credit losses must be covered through income received by properly pricing the transactions, unexpected credit losses are usually confronted through capital adequacy and reserves. Additionally to these two categories, there are extra losses, catastrophic losses, which must be estimated by means of historical and hypothetical events. Their occurrence might even oblige a bank to stop doing business and therefore it is useful to assess them through stress testing. The economic capital needed to support credit risk activities of a financial institution is determined so that the estimated probability of unexpected credit loss exhausting economic capital is less than some target insolvency rate (x%).[27]

[27] Insolvency rate is often consistent with desired credit rating of a bank and might be equal to the historical one-year default rate (e.g. 0,02% for A-rated bank).

Figure 7: Probability density function of credit losses



Using the probability distribution, one can calculate the likely loss at any level of probability. For example, deciding *x%*=99th%ile level, the loss is in 99% lower or equal to value of this percentile (or VaR).

Previously, the increasing convergence of regulatory and economic capital was mentioned. In last years, well-known financial institutions released credit risk models to the public, the aim of which is to make reliable predictions of the economic capital banks have to maintain. These will be shortly described in this chapter. We name the underlying assumptions of models, sketch the process of computation, give summary of advantages and limitations and finally we mention the applicability in the Czech Republic. Such characterization of individual models is useful, as the stress testing possibilities are given by the parameters of models that can be changed to simulate various situations.

Generally, the models can be divided into three groups: default-mode models (DM), model of marked-to-market type, and models based on option theory. The default mode paradigm only recognizes a loss in the portfolio if the obligor has defaulted on its legal obligations within the modeled time horizon. The mark-to-market paradigm recognizes any gains or losses in the value of a debt security caused by changes in the credit quality of the obligor over the measured time horizon. If the credit of the obligors in a portfolio deteriorates as a result of recession, for example, the portfolio value will be lower, even without any defaults. A market price for each debt security is obtained by discounting cash flows on the obligor’s credit curve. The discrete mark-to-market loss paradigm is used in CreditMetrics, the default mode is used in CreditRisk+.

IV.2 Credit risk models

IV.2.1 CreditMetrics

CreditMetrics by JP Morgan, first published in 1997[28], is based on the analysis of credit migration – the probability that a credit moves from one quality grade to another (including default) within given time horizon[29]. It models the full distribution of future values of any credit portfolio, where the changes in values are caused by credit migration, upwards and downwards, as well as by default. Therefore CreditMetrics is a model of a market-to-market type.

The first step is to divide debtors into rating categories and then determine the probabilities of migration into other ratings and probability of staying identically rated. Further follows the calculation of expected values of a loan or bond after individual migration alternatives. If we weight these expected values by probabilities we get the expected value of the credit in one year and we can set the deviations of expected values from the mean. In next step we compute the variance and standard deviation. The VaR is then calculated in the same manner as by market risk, e.g. it is a distance from the mean of the percentile of the forward distribution at desired confidence level.

The reliance on ratings transition probabilities is also a major weakness of the CreditMetrics approach, as this transition probabilities are based on average historical frequencies of defaults and credit migration. Therefore the accuracy of calculation depends on two critical assumptions. First, that firms within the same rating class have the same default rate and the same spread curve even when

[28] J.P. Morgan (1997)

[29] The risk horizon is usually set at one year; such transition matrix is also used in this thesis. But this horizon is arbitrary, mostly given by the availability of financial reports used by rating agencies. In KMV´s framework, which relies on market data, any horizon can be chosen.

recovery rates differ among obligors. Second, that the actual default rate is equal to the historical average default rate. Next weakness is the assumption that the default free interest rates are deterministic and thus the model is insensitive to market risk and underlying economic changes. Finally, the model proxies asset return correlations by equity return correlations.

IV.2.2 KMV model

The KMV model based on the asset value model of the firm originally proposed by Merton(1974) and it is a representative of a group of models based on the option theory. KMV stands for the fact that the historical average default rate and transition probabilities can deviate significantly from the actual rates. In addition, substantial differences in default rates may exist within the same bond rating class. Therefore, KMV does not use Moody's or Standard & Poor's statistical data to assign a probability of default which only depends on the rating of the obligor. Instead, KMV derives the actual probability of default (PD), the Expected Default Frequency (EDF), for each obligor.

It is assumed that the firm's capital structure consists only of equity, short-term and long-term debt, convertible and preferred shares. If the value of firm's assets falls under certain value (the default point), it is more advantageous for shareholders to hand the firm over to debt holders. To derive the loss distribution, it is assumed that the portfolio is highly diversified.

The model estimates three basic elements: the probability of default (in KMV called EDF) of individual obligors, the present value of the future cash flows and the loss distribution. The probability of default is a function of the current asset value[30], the volatility of the asset returns and estimated distance to default (DD), which is a measure of default risk. The transition from DD to actual probabilities is made through scaling of the DD using a default database[31]. Last step lies in mapping procedure from DD to EDF based on large cross-sectional data.

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[30] KMV best applies to publicly traded companies for which the value of equity is market determined.

[31] Mapping procedure from DD to EDF requires historical information on a large sample of firms from different sectors, which includes defaulted firms. Only then one can estimate the proportion of firms of given ranking for each time horizon.

An advantage of this model is that the default probability is more related to the firm's characteristics than to initial ratings as in CreditMetrics. It is thus more sensitive to changes in credit quality of obligors. Additionally, it contains market information present in firm's equity. On the other hand, the assumed firm structure is too simplistic and the assumption of well diversified portfolio unrealistic. Also, the mapping procedure is made on US data and the process is not precisely explained.

For the latter reason, it is hardly applicable to conditions outside the US. In the Czech Republic is thereto a problem with the determination of market value of equity. Stock market data are often not available even for large companies; for medium and small businesses that are the bulk of banks´ clients are very seldom.

IV.2.3 CreditRisk+

CreditRisk+ is an approach released by Credit Suisse Financial Products in 1997[32]. It is a model of a default-mode type, thus, it focuses on default alone rather than credit migration. In this model, causes of defaults are not taken into account; therefore, default events are not connected to the capital structure or cross-sectional database as previously. The default rate of an obligor is modeled either as constant or as continuous variable. The economic capital is assessed within the VaR framework (similarly to CreditMetrics).

It is assumed that exposures to individual obligors have small probabilities of default, these probabilities are mutually independent. Under these circumstances, the probability distribution for the number of defaults, during a given period of time (say 1 year) is well represented by a Poisson distribution.

What is important for stress testing, CreditRisk+ allows extension of the basic model. It enables differentiating among sectors, such as countries or industries, whose members are influenced by common "background" factor. Unfortunately, the mapping of obligors to different sectors is not specified closer. The systemic risk of each obligor is reflected by default rate volatilities.

The advantage of this model is that is not very demanding as far as data and computation. However, the determination of default rates of individual obligors is

[32] Credit Suisse First Boston (1997)

rather problematic. Banks are assumed to know both the default probabilities and volatilities. The limitation is that CreditRisk+ ignores migration risk so that the exposure for each obligor is fixed and does not depend on changes in the credit quality. Even in the extended form where the probability of default depends upon background factors, exposures are still constant and not related to changes in these factors.

IV.2.4 Limitations of models

It follows from the overview above that all the methods do not take market risk into account. Both market and credit risk are at current practice analyzed separately and in my opinion the greatest challenge for the future is to develop a combined stress test that would simultaneously test for market and credit risk.

Further shortage may be in correlations modeling. Mutual correlations between PD, LGD and exposure are assumed to be zero. In practice, however, the exposure and PD are rather positively correlated. Moreover, assessment of correlations across obligors is based on evaluation of correlations among underlying (background) factors (e.g. CreditMetrics or KMV). The deviation in the industrial structure of a country from historical pattern may lead to biased estimates. Similarly the duplication of relationship among indices from one country to another may lead to biases.

Finally, none of the models above also does deal with nonlinear products such as, e.g., options and foreign currency swaps.

IV.3 Why manage credit stress tests

According to the New Basel Capital Accord, independent units have to conduct stress tests, which must be properly documented, at least every six months. Further, it orders regular reporting of results to senior management that should take appropriate corrective actions. However, the Accord does not bring any closer specification of stress testing methodology, but only specifies[33]:

„Stress testing should include specific scenarios that quantitatively assess the impact of broad rating migration of exposures to lower rating grades. Such analysis should

[33] (viii) Use of internal ratings , c) , paragraph 298

also examine the impact of higher default rates and lower recovery rates than a bank’s predicted PD, LGD and exposure measurement. “

The BIS (1999) in its report recommends stress testing and notices that:

„Stress tests aim to overcome some of the major uncertainties in credit risk models – such as the estimation of default rates or the joint probability distribution of risk factors – by specifying particular economic scenarios and judging the adequacy of bank capital against those scenarios, regardless of the probability that such events may occur. Stress tests could cover a range of scenarios, including the performance of certain sectors during crises, or the magnitude of losses at extreme points of the credit cycle. “

Implementation of stress tests is likely to have positive impacts also for the Czech banking system. Supplementing the traditional analysis, currently mainly based on borrowers´ reputation and previous credit discipline, with more objective methods including stress testing may lead to better assessment of potential risks and of the capital reserves needed to handle with losses. Next positive impact may have the implementation of predefined stress test by all banks for regulators that would then be able to measure the relative performance of banks better, to asses the risks within the whole banking system. As the implementation of stress testing requires the collection of bulk of data, central database could be created through cooperation of banks. Such central databases always assure higher and quicker access to information and make analysis more transparent.

IV.4 Data required for credit stress testing

Generally, a bank’s credit activity can be divided into two parts, instruments of the trading book and of the loan book. Trading book consists of instruments such as bonds and swaps, the trading activity of which is rapid. Loan book, in contrast contains credit portfolios with much slower transactional speed. For both banking books, data for stress tests should include both on both balance- and off-balance sheet exposures. Except for processing all credit position data, for stress testing credit risk it is also necessary to have access to actual market data about risk factors such as interest rate, exchange rate, equity indices, and swap rates. These are important especially for trading book portfolio, which is in fact part of market portfolio and thus the previous market risk approach is relevant. Additionally, some important risk analytics for calculation of risk like transition matrices or default correlations have to be in time at the banks´ disposal.

IV.4.1 Loan book risk factors

Firstly, obligor specific risk factors can be stressed, namely PDs and LGDs. PD means the probability of default and depends on counterparty's credit rating. LGD or loss given default is the percentage of exposure at default that would be lost if default take place. Thus if a client's ability or willingness to repay his debt decreases, his PD has to be increased (the credit rating lowered) and it is to calculate a new LGD. In order to be able to evaluate PDs and LGDs the information about credit distribution by loan quality, provisions, collateral, etc. is needed.

Secondly, industry factors should be considered for calculation of risk of the credit portfolio – namely correlations between the industries. On the base of identified correlations, only one industry can be stressed and then accordingly adjusted credit ratings of counterparties belonging to correlated industries.

Similarly different regions or countries that are influenced by the same geographical or geo-political factors can be dealt, for example a war or just suspicion from contagion behavior in some region.

Usually, a bank has a system of internal limits for individual countries, industries and debtors in order to prevent a significant concentration of credit risk. It arises from the existence of loans with similar economic characteristics influencing the debtor's ability to meet obligations.

Also macroeconomic factors, such as interest rates or foreign exchange rates, are used in some credit risk models. Either for calculating PDs, which enables to revaluate the portfolio, or for estimating influence of macroeconomic variables in counterparties´ credit ratings.

Political factors, such as stability of a system, the extent of state regulation, and the regulatory and institutional environment are often very relevant, especially in emerging market countries. However, these factors are unfortunately very difficult to handle.

IV.4.2 Trading book risk factors

Fixed income instruments, such as bonds and swaps, are amenable to both credit and market stress tests, because these instruments are liable to both credit risk (risk of default by the issuer of the bond) and market risk (risk arising from change in market prices of fixed income instruments). In practice, however, credit and market risk factors are indistinguishable. Therefore there has been a large effort lately to come up with integrated market and credit stress tests.

The main risk factor affecting bonds and swaps is the credit spread - the difference between the yield or swap curves for a particular rating class and the benchmark curve (usually the government curve). The higher is the creditworthiness of a company, the higher credit rating and lower credit spread.

The following example shows how market and credit risk are interconnected. If the price of a bond begins to be more volatile, which means higher market risk, it lowers the credit rating of a company simultaneously and thereby increases credit risk too.

Shocks to volatility including credit spreads and FX rates are also appropriate for portfolios containing options.

IV.4.3 Other Risk Factors

Many stress tests are aimed at testing changes in the correlation structure such as correlation breakdown or reversal. Breakdown means the distortion of historical correlation caused for example by a natural disaster or political turnover, while the reversal denotes change in a group of instruments with high correlation such, that a part begins to behave differently resulting in inverse correlation. An example can be the flight to quality bonds during a crisis, while under normal business conditions all bonds respond similarly.

Also transition matrices used in credit risk models are often stressed. A transition or migration matrix gives the probability of change in a credit rating over a chosen time interval. The rows correspond to the existing risk grades, while the columns are the risk grades a particular risk grade can migrate to at the end of the time interval. Each cell in the matrix shows the probability of the existing rating in the row becoming a rating in the respective column. This way, the migration matrix describes the probability distribution of grades at time t+1 given the grade at t. The following figure shows an example of a migration matrix for 1-year migration horizon, which is standard for the calculation of credit risk exposures.

Table 3: One-year transition matrix

Initial Rating	Rating at year-end (%)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90.81	8.33	0.68	0.06	0.12	0	0	0
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.30	1.17	0.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC	0.22	0	0.22	1.30	2.38	11.24	64.86	19.79

Source: Standard & Poor's CreditWeek (15 April 96)[34]

IV.4.4 Risk factors covered in practice

In looking for risk factors really used in stress tests, we may look for inspiration into the BIS (1999) survey on current stress testing practices. Even though it came to the conclusion that the procedure of stress testing is not formally developed or is carried out only sporadically[35], it describes scenarios covered. These are:

- deterioration in credit ratings
- deterioration in market spreads
- changes in LGDs
- shifts in default probabilities
- changes in correlation structures

Which risk factors are stresses or included into stress scenarios depends on the choice of model used. Banks that apply any of credit models described above could extend them to stress risk factors that are cardinal in individual models and on which

[34] Although the more recent transition matrices exist, the one from year 1996 is the last one that does not contain the column "non-rated". This column makes the analysis more complicated and would make the example below less understandable.

[35] „Some institutions are doing work in this area; however, to date, we have not seen comprehensive work. " (BIS 1999, Credit risk modeling..., p.60)

models are sensitive. The choice of the most adequate model depends mainly on availability of data and on the bank's portfolio specifics. In my opinions, from the described models CreditMetrics could be the most suitable. Below is discussed how the problem of almost non-existing ratings could be solved and what parameters could be modified to simulate stress scenario.

Another possibility for Czech banks is to create own model for stress testing, which would be simpler, use available data and would be more costly.

IV.5 CreditMetrics approach

IV.5.1 From rating to transition matrix determination

The starting point is the categorization of individual obligors into rating groups. The rating assign either external rating agencies, as the standard method of NBCA prescribes, or can be set by a bank itself, according to regulation of internal rating based approach (IRB approach) of NBCA.

In the Czech Republic, a rating has been so far assigned only to a few companies. This is partly given by the short history of ratings in Europe as a whole, because companies do not use financing through bonds frequently.[36] By credit financing, the rating assignment, which can be very expensive, is not so important. Moreover, the cost of rating does not decrease proportionally to the size of a company. Therefore it is not worth, mainly for relatively smaller companies (the Czech Republic case), to pay for rating assignment. For these reasons, I would see the method of internal ratings[37] as crucial for Czech banks. It would come out from internal scoring functions. The awarded scoring can then be mapped on the standard rating system, e.g. Standard & Poor's basic scale.

[36] Even in the U.S. is the coverage by rating so high, as could be expected. Therefore FED do not deal with recognition of rating agencies and lets the usage of external ratings on banks´ own consideration. Source: Bauerová(2004).

[37] In order that a bank could use internal ratings, it has to satisfy strict rules, such as the division into 6-9 classes for standard loans and 2 classes for non-performing loan. Criteria of mapping into groups have to be precisely specified, each applicant have to obtain rating before being award a credit, etc. For more information see NBCA.

The NBCA by differentiates six categories of exposures in the method of internal ratings. In the Czech banks the majority of exposures represented are the corporate exposures. All six categories distinguish only one bank out of 37 banks, as from the results of survey of the Czech national bank.[38]

The method of internal ratings is relatively widespread. According to information from mentioned survey of the Czech national bank internal rating conduct 30 banks out of 37 surveyed. All banks use expert evaluation of internal specialists and thus the mapping to rating grade may influenced by subjective view. On the other hand it is possible to reproduce and retrospectively reassess the process of ratings assignment. This enables the audit of internal rating, which is one of NBCA requirements.

The next step is to derive transition probabilities for all rating groups and then to construct a transition matrix. This requires enough large portfolio and longer time series (about five years at least). Transition matrices are compiled for each year separately and then is the "average" one-year matrix derived (remember Table 3).[39] For further description we will assume we derived just this matrix.

In this place offers the first stress test application – testing of transition matrices - offers. In chapters IV.6.1 and IV.6.2 it will be shown how transition matrices can be split according to the states of the business cycle or how the migration probabilities and probabilities of default can be adjusted for macroeconomic conditions.

IV.5.2 Forward pricing

Having determined the likelihoods of migration to any possible credit quality state at the risk horizon (one year), we further determine the values at the risk horizon for these credit quality states. Value is calculated once for each migration state. These valuations fall into two categories. First, in the event of up(down)grades, we estimate the change in credit spread that results from the rating migration. Second, in the

[38] ČNB(2004)

[39] The strong assumption of all approaches using migration matrices is that all issuers within the same rating class are homogenous credit risks. They have the same transition probabilities and default probability.

event of a default, we estimate the recovery rate, given as percentage of face value, based on the seniority classification. We then perform a present value calculation of the remaining cash flows at the new yield to estimate its new value.

The information about discount rate give for loans credit spreads for each rating category and maturity, while for bonds the forward zero curves.

Let us illustrate the above steps with the help of AA bond with maturity of five years, paying annual coupon rate of 4%. Assume that the forward zero curves for each rating category are those from following table.

Table 4: Example one-year forward zero curves by credit rating category (%)

Category	Year 1	Year 2	Year 3	Year 4
AAA	3.60	4.17	4.73	5.12
AA	3.65	4.22	4.78	5.17
A	3.72	4.32	4.93	5.32
BBB	4.10	4.67	5.25	5.63
BB	5.55	6.02	6.78	7.27
B	6.05	7.02	8.03	8.52
CCC	15.05	15.02	14.03	13.52

Source: CreditMetrics, Technical Document

Therefore assuming the face value of 100CZK, at the end of one year under assumption the bond downgrades to BBB is:

$$V = 4 + \frac{4}{(1+4{,}1\%)} + \frac{4}{(1+4{,}67\%)^2} + \frac{4}{(1+5{,}25\%)^3} + \frac{104}{(1+5{,}63\%)^4} = 94{,}46$$

In the above formula, we use the forward zero rates for the BBB rating category. To calculate the value of the bond in a rating category other than BBB, we would substitute the appropriate zero rates from the table. After completing these calculations for different rating categories, we obtain the values in third column of Table 5.

Table 5: Calculating volatility due to credit quality changes

	Probability of state	Bond value in one year (CZK)	Probability weighted value	Difference from mean	Probability weighted difference squared
AAA	0.09	104.78	0.09	0.87	0.00
AA	2.27	104.60	2.37	0.69	0.01
A	91.05	104.08	94.77	0.17	0.03
BBB	5.52	103.00	5.69	-0.91	0.05
BB	0.74	95.38	0.71	-8.52	0.54
B	0.26	93.76	0.24	-10.15	0.27
CCC	0.01	79.72	0.01	-24.18	0.06
Default	0.06	50.00	0.03	-53.91	1.74
		Mean=	103.91	Variance=	2.69
				St. deviation=	1.64
Normal distribution					
	99.00%	2.33	VaR	3.82	
	99.50%	2.58	VaR	4.23	
	99.90%	3.09	VaR	5.07	

The expected value in case of default of 50 is because the recovery rate of 50% was assumed, as NBCA for basic approach of IRB has recommended. Of course, it is possible to incorporate various recovery rates, as will be shown in IV.6.4

To establish the fist percentile of the distribution of bond values, it is necessary to sum up probabilities of default, CCC, etc. rating up to value of 1%.The combined likelihood of default, CCC, B and BB is 1,07%, corresponding to 95,38, which is 8,61CZK below average. This exhibits the long downside tail, because the VaR calculated from normal distribution at the 99 percent confidence level is -3,82.

IV.5.3 Credit VaR for a portfolio

In this section we return back to phase, where we have computed migration probabilities for individual rating categories and we want to construct transition matrix containing joint migration probabilities. Joint probabilities in Table 3 were constructed under the assumption of zero correlations, i.e. just as the product of probability of chosen state of bond 1 and probability of chosen state of bond 2.

To assess the portfolio risk with some acceptable precision, it necessary to estimate joint movements in credit quality, usually represented by correlation parameters.

There are several alternative how to treat correlations among obligors. Correlations might be expected to be higher for firms within the same industry or region than for companies in unrelated sectors. Moreover, correlations vary with the state of economy. In recession, most assets of obligors will decline in value and quality and the probability of multiple defaults rises. Their accurate estimation is therefore one of the key determinants of portfolio optimization.

The simplest alternative is to suppose some fixed value of firm asset correlations, such as the average correlation. Such approach, however, does not provide information about concentration of credits, e.g. in a particular industry. The most precise approach is to evaluate obligor-by-obligor asset correlations, often by proxy of equity returns correlations. The problem is that the scarcity of data for many obligors does not allow producing correlations for any pair of obligor and even if it would be possible, the storage of huge correlation matrices seems impossible. Therefore the approach that resorts obligors and assesses their correlations on basis of index correlations is used.[40] The degree to which it gives good information depends on the strength of correlations between the sectors.

Large bank portfolios do not cause problems only in case of correlations assessment, but also in credit risk estimation as a whole. The analytic approach outlined above for a portfolio with two obligors is not practicable in practice for large portfolios. Instead, a Monte Carlo simulation is usually implemented (e.g. by CreditMetrics) to generate the full distribution of the portfolio values at the credit horizon of one year.

IV.6 Stress testing applications

IV.6.1 Migration matrices –stress test application

In this chapter we will try to incorporate systematic risk into a model, because changes in the economic environment influence, among others, also the credit quality of portfolio.

[40] Chapter IV.6.5 gives an example demonstrating how correlations in CreditMetrics are treated. It also sketches stress testing possibility.

Table 3 is an example of unconditional migration matrix, which averages across stages of business cycle. By separating the economy into two states – expansion and contraction- and conditioning the transition matrix on them[41], it is possible to asses the loss distribution of credit portfolios depending on the business cycle.

Table 6: Conditional transition matrices

1/4-year US Expansion matrix

Initial rating	Terminal rating AAA	AA	A	BBB	BB	B	CCC	D
AAA	98.21%	1.66%	0.11%	0.02%	0.02%	0%	0%	0%
A	0.15%	98.08%	1.61%	0.12%	0.01%	0.03%	0.01%	0%
A	0.02%	0.53%	98.06%	1.21%	0.11%	0.06%	0.00%	0.00%
BBB	0.01%	0.07%	1.47%	96.94%	1.25%	0.22%	0.02%	0.02%
BB	0.01%	0.03%	0.19%	1.93%	95.31%	2.25%	0.16%	0.12%
B	0%	0.02%	0.07%	0.10%	1.70%	95.91%	1.31%	0.88%
CCC	0.05%	0%	0.19%	0.23%	0.47%	3.57%	87.32%	8.17%

Source: Bangia et al (2002), based on S&P data 1981–1998.

1/4-year US Recession matrix

Initial rating	Terminal rating AAA	AA	A	BBB	BB	B	CCC	D
AAA	97.99%	1.76%	0.25%	0%	0%	0%	0%	0%
A	0.18%	96.89%	2.79%	0.05%	0.09%	0%	0%	0%
A	0.02%	0.88%	96.44%	2.59%	0.07%	0%	0%	0%
BBB	0.04%	0.04%	1.11%	96.31%	2.33%	0.07%	0%	0.11%
BB	0%	0.06%	0.06%	1.39%	94.98%	2.72%	0.42%	0.36%
B	0%	0.06%	0.06%	0.11%	0.72%	95.02%	2.27%	1.77%
CCC	0%	0%	0%	0%	0%	1.20%	85.60%	13.20%

Source: Bangia, Diebold, Kronimus, Schagen, Schuermann (2002), based on S&P data 1981–1998.

There is a striking difference between these two matrices, mainly in default probabilities that increase significantly in contraction. For example the PD of BB rated obligor rises three times from 0.12% in expansion to 0.36% in contraction.

However, this application has a problem that the future state of the economy over the transition horizon is not known. Therefore so called regime switching matrices are used, which depict the probability of being in expansion or contraction next period conditional upon the current regime. The following example is the simplest (2x2) matrix that uses NBER 1981-1998 data.

[41] To be able to construct conditional transition matrices with the same plausibility we need much more data

Table 7: Quaterly regime switching matrix

	Expansion	Recession
Expansion	85%	15%
Recession	69,2%	30,8%

Source: Bangia, Diebold, Kronimus, Schagen, Schuermann(2002)

We can ask: what will be the portfolio value distribution if next period is an expansion or contraction. The analysis process then continues in the same way, only it is worked with two conditional matrices weighted by probabilities instead of unconditional one.

IV.6.2 *Regression-based transition probabilities*

The distribution of default and migration probabilities for various rating groups in different industries and for each country is conditional on the value of macroeconomic factors. When the economy worsens, number of both downgrades and defaults increase. The contrary holds in economic expansion. It is possible to construct a regression model that relates the default and migration probabilities to macroeconomic variables such as unemployment rate, the rate of economic growth, the level of interest rates, foreign exchange rates, government expenditures, aggregate savings, etc. Such methodology may be applied to various classes of obligors (from different industries and countries).

On the above named assumptions CreditPortfolioView[42] is based, a model used for simulation of the joint conditional distribution of default and migration probabilities. To calibrate the model, following system of equations has to be solved:

$$P_{j,t} = \frac{1}{1+e^{-Y_{j,t,}}} \qquad (1)$$

$$Y_{j,t} = \beta_{j,0} + \beta_{j,1} X_{j,1,t} + \ldots + \beta_{j,m} X_{j,m,t} + \nu_{j,t} \qquad (2)$$

$$X_{j,i,t} = \gamma_{j,i,0} + \gamma_{j,i,1} X_{j,i,t-1} + \gamma_{j,i,2} X_{j,i,t-2} + e_{j,i,t} \qquad (3)$$

(longer time series).

[42] Wilson (1997a,1997b).

The first equation models conditional probability of default in period by a logit function, where the independent variable *Y* is a country/industry speculative grade specific index. The second equation shows how this index depends on the state of economy; X_js are period values of the macroeconomic variables in period t and β_js are coefficients to be estimated. As the third equation shows, each macroeconomic variable is assumed to follow a univariate, auto-regressive model of order 2.

When the model is calibrated, a Monte Carlo simulation is applied to determine the distribution of the default probabilities conditional on the state of economy. Then the VaR can be estimated. The calibration can be made at the country/industry level, however, the higher the segmentation, the scarcer the data are.

IV.6.3 Macroeconomic approach

The assumption of dependence between macroeconomic factors and credit quality use also simple models that do not use default probabilities and transition matrices. Based on linear regression like equation (2) above, different items of bank's balance sheet may be stressed on macroeconomic factors. Most often is applied the regression of NPL[43]/Total assets ratio on macroeconomic factors. The coefficients of such regression provide estimate of sensitivity of bank borrowers to the applied macroeconomic factors. Forming extreme scenarios for these factors, the changes in bank's portfolio under stress situation may be estimated.

Blaschke, Jones, Majnoni, Peria(2001) introduces following regression:

$$\left(\frac{NPL}{TotalAssets}\right)_{i,t} = \alpha + \beta \cdot i_{i,t} + \gamma \cdot p_{i,t} + \delta \cdot \Delta GDP_{i,t} + \lambda \cdot \Delta ToT_{i,t} + \varepsilon_{i,t}$$

where *i* is nominal interest rate, *p* is inflation rate, ΔGDP is percentage change of real GDP and ΔToT is percentage change of terms of trade.

The Hong Kong monetary authority shows in its stress testing manual[44] how banks can use the scenario of a domestic economic downturn to assess the impact on its financial position. This includes changes in asset quality (asset positions before provisioning, asset quality of selected sectors, collateral value for classified loans), in

[43] NPL, nonperforming loans, is sum of substandard, doubtful and loss loans.

[44] Hong Kong Monetary authority (2003)

provisioning, profitability and capital adequacy. Creating the scenario, a number of macro-economic indicators is applied; like unemployment rate, real GDP growth, real interest rate, number of bankruptcy petitions and property price and equity indices. The expected development of these indicators is taken as a baseline and then the impact of more severe development is examined.[45]

The advantage of this approach is that it enables an integrated treatment of credit and market risk, as relevant macroeconomic variables typically contain also exchange rate and interest rates or other market indices. Such approach also allows assessing credit loss of individual banks or of banks in aggregate. It could be launched also in transition countries, because it is not very demanding on data. However, it relies only on macroeconomic factors, which do not have to be the only cause of bank's portfolio deterioration. Therefore, the results should be confronted with microeconomic information.

IV.6.4 Recovery rate simulation

Recovery rate can significantly differ for financial assets with different maturities, collaterals and guarantees. Also the seniority of an instrument and the institutional environment including e.g. the effectiveness of juridical system play an important role. In practice, fixed recovery rates, like in our example above, are used, or are assumed zero.

In advanced approach or for purposes of stress testing can a bank estimates the recovery rate by itself. Generally holds that the recovery rates rise with the seniority and security, as can be seen in Figure 8[46].

[45] Such approach might be applied also in the Czech Republic, taking the macroeconomic predictions by ČNB as a baseline.

[46] When the loss distribution is derived from a Monte Carlo simulation, it is generally assumed that the recovery rates are distributed according to a beta distribution with standard deviation and mean showed in this figure.

Figure 8: Recovery rates for loans by seniority class

Seniority Class	Mean (%)	Standard Deviation (%)
Senior Secured	53.80	26.86
Senior Unsecured	51.13	25.45
Senior Subordinated	38.52	23.81
Subordinated	32.74	20.18
Junior Subordinated	17.09	10.90

Source: Creditmetrics Technical Document

IV.6.5 Asset return correlation

It is possible to utilize country and industry indices in different countries to construct a correlation matrix for these industries. Next, we can assigns to each obligor country and industry weights.

In this example[47], we will work only with two bonds for simplicity. Let us assume, that these are exposed to Czech food industry (CZF) and German banking (GB) and insurance (GI), respectively. Further we assume that country and industry exposures of each firm are as in following table.

Table 8: Country and industry exposure of the two firms (%)

	Czech food (CZF)	German banking (GB)	German insurance (GI)
Firm 1		50	40
Firm 2	80		

This means that 50% of the firm 1´s volatility of equity returns is explained by the German banking index and 40% by the German insurance index and the rest of the 10% is due to firm-specific factors; similarly for the firm 2.

Further we need to know correlations of the country- and industry-specific indices.[48] For example they could look as follows.

[47] The example is only illustrative, the data do not come out from reality or any research.

[48] Long cross-sectional data sets are especially for transitional countries not available. Therefore, relationships among industry-specific indices and their attributes are often transferred from other countries. They, however, do not have to correspond to structure of industry in transition country at all.

Table 9: Volatilities and correlations of country- and industry-specific indices

	CZF	GB	GI
CZF	1	0,18	0,2
GB	0,18	1	0,24
GI	0,2	0,24	1

Now we can compute the firm's standardized asset return R:

$$R^1 = w^1{}_1 x_{GB} + w^1{}_2 x_{GI} + w^1 \varepsilon^1$$
$$R^2 = w^2{}_1 x_{CZF} + w^2 \varepsilon^2$$

where X_i s are sets if standardized returns on a country and industry equity indices (Table 9) with known pairwise correlations w_i s (values set in Table 8). The weight of firm-specific term *w* is determined as to make the asset return *R* equal to one.

The correlation of the asset returns of the two firms is then:

$$\rho(R^1, R^2) = corr(0{,}5x_{GB} + 0{,}4x_{GI} + 0{,}33\varepsilon^1,\ 0{,}8x_{CZF} + 0{,}6\varepsilon^2) = 0{,}5 \cdot 0{,}8 \cdot 0{,}18 + 0{,}4 \cdot 0{,}8 \cdot 0{,}2 = 0{,}136$$
i.e. 13,6%.

Stress testing of correlations may rest on testing the sensitivity of the portfolio risk to the level of correlations. For example we may change the level of correlation in steps of 0.1%. Such simulation is appropriate mainly when constant correlations within the portfolio we set. In addition to testing the sensitivity to overall levels of correlation in the portfolio, it is also tests the effect of a change in the specific correlation among a set of countries and industries.

V OTHER RISKS AND RISK AGGREGATION

Having covered credit risk in the last chapter, and market risk in earlier chapter, we turn to consider the other main types of risk: liquidity risk and operational risk.

V.1 Liquidity risk

Liquidity risk comprises two closely related dimensions: funding liquidity risk and trading liquidity risk.

V.1.1 Funding liquidity risk

Funding liquidity risk relates to a financial institution's ability to raise necessary cash to roll over its debt, to meet the cash, margin and collateral requirements of counterparties[49], and to satisfy capital withdrawals. It is affected by various factors such as the maturity structure of liabilities, reliance on secured sources, by the access to public markets (e.g. with commercial papers) and by various counterparty arrangements (withdrawal rights, lines of credit).

Stress tests for this type of liquidity risk involve assessing the impact on the liquidity gap of a shock to liquid assets and liabilities. Typically banks are endangered on side of liabilities-by sudden extensive withdrawals. Stress tests may help to asses how long the bank is able to survive in case of such withdrawals. Other possibility is to examine the impact of certain percentage change in liquid liabilities or assets, which may be based on past bank runs.

V.1.2 Trading-related liquidity risk

Trading-related liquidity risk, closely related to market risk, is the risk of loss arising from the cost of liquidating a position. It arises where markets are less than perfectly liquid during market crises. Under normal circumstances liquidity risk arises from dealing with markets that are most of time less than perfectly liquid. The degree of liquidity, of course, varies very greatly from one market to another.

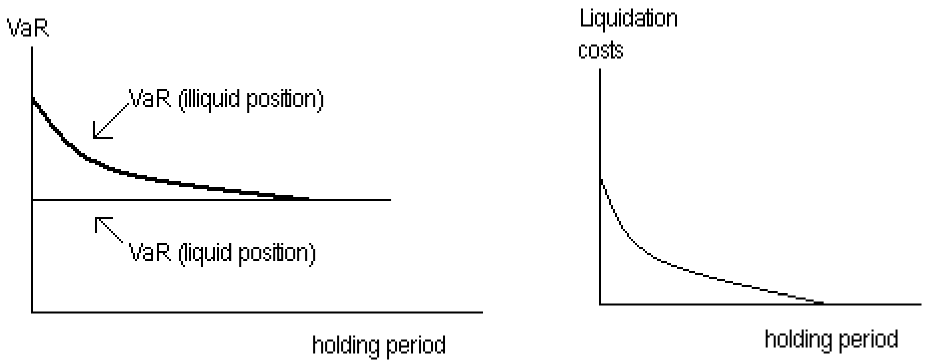
Unlike market risk, where the loss is caused by the adverse market price movements, dealing with liquidity risk, the loss may arise because we may not get the market price at time we want to sell. Typically, market illiquidity manifests in the form of significant transaction costs, low market turnover, a relatively small number of traders and significant bid-ask spread. During market turmoil or in crises, it is more dangerous and brings great liquidity costs, as liquidation of a position is possible only by taking much larger losses than under normal circumstances. Bid-ask spread increases dramatically, quantities that can be traded at those prices (bid and ask) become smaller and smaller and large quantities, if needed to be traded, widen the bid-ask even further. Then securities can be traded only for prices that are below

[49] This part of liquidity risk can be considered as a version of concentration risk on banks' liabilities.

economic value of securities. At some point, it may not have sense to trade further, as opportunity cost are too high.

Figure 9 shows a highly liquid position and the illiquid one. It is possible to sell the liquid position at short notice and obtain the market price, without any significant liquidation cost. However, the illiquid position can be sold only by paying some liquidation costs. These liquidation costs can be taken into account through modifying standard VaR. Other things equal, the illiquid asset has a higher VaR comprising cost of liquidation. The VaR and liquidation cost also depend on period involved: the longer we are prepared to wait (longer holding period), the lower the cost and VaR.

Figure 9: Dependence of VaR and liquidation cost on holding period



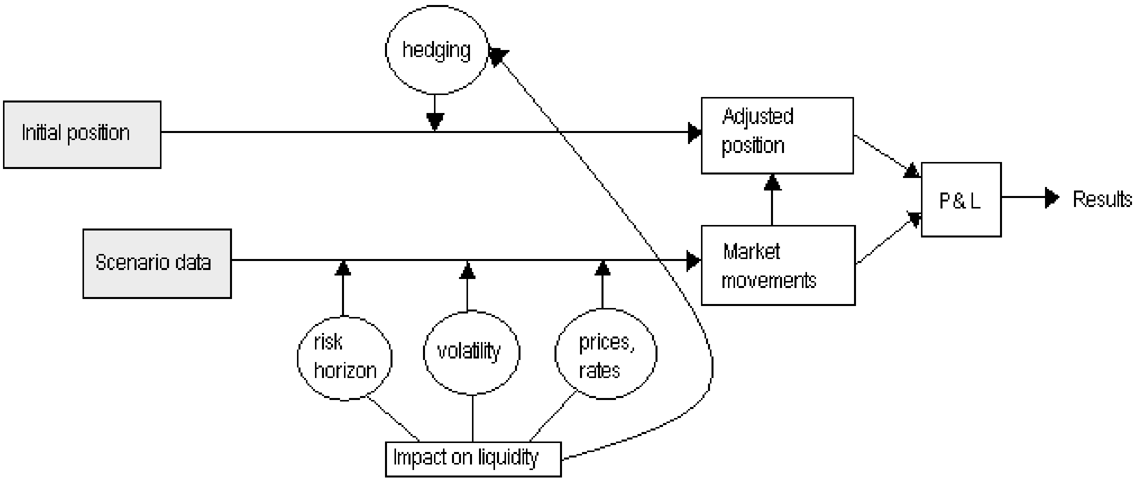
Source: Down(1998, p.188)

The importance of liquidity costs vary across positions, but obviously the VaR, inclusive liquidation cost, can be much higher than standard market-price VaR. These, therefore, may be very poor estimates and may underestimate the risk considerably. Therefore, it is necessary to adjust standard VaR, forexample in the way introduced in following chapter.

V.1.2.1 **Scenario analysis incorporation**

The traditional VaR model is static, by construction and cannot therefore encompass liquidity risk. To do this a framework incorporating intraday decisions to change the portfolio´ structure is needed. One way is to employ a multi-period framework for VaR calculations and add scenario analysis. Such model is described in following paragraphs and illustrated in the following figure:

Figure 10: Liquidity risk in multi-period VaR



Source: Author (based on Crouhy, Galai, Mark(2001,p.242))

At first, trading rules are specified; for example risk limits are set and whenever they should be breached a hedging strategy must be implemented to reduce the risk. Initial portfolio position is thus adjusted through hedging, or, alternatively through adding new exposures. To assess possible position changes, scenario analysis simulating market environment changes can be implemented. It follows, that liquidity risk is closely related to market risk and advanced market risk models should not omit liquidity issues, as liquidity risk may reduce an institution's ability to manage and hedge market risk as well as its capacity to satisfy deficiency in the funding side through asset liquidation.

Scenarios are simulated over specified risk horizon, e.g. a quarter. In the process, simulation allows for jumps in risk factors, rather than count on standard VaR assumption of "smooth" behavior, i.e. stationary stochastic process for risk factors. Liquidity crises may occur after sharp drop in equity or commodity prices, huge interest rate or exchange rate movements, volatility changes or any combination of mentioned factors. Here, probabilistic scenarios as well as predetermined path for all the risk factors can be used. Each simulation leads to average bid-ask prices. It is then possible to work with different degrees of liquidity crises, considering different levels of bid-ask spread.

The simulation is repeated many times and after each adjustment the daily profit and losses (P&L) are counted. Then the distribution of the daily P&L can be produced.

V.2 Operational risk

Operational risk is not well-defined concept. In the financial instruction's context, it refers to a range of failures in the operation of the firm that are not related directly to market or credit risk. These failures include unauthorized trading, human error, communication failures, computer breakdown, fraud and many others. Such risks may be less visible and it is often difficult to make a distinction between them and other risks. For example, if a client failed to pay back a loan, it was either due to credit risk or due to error of an officer that had approved the credit.

While many operational risks are clearly impossible to quantify, there are also some that can be measured and thus the loss estimates and the likelihood of occurring can be attached to particular risk events. The most difficult part is to collect and categorize relevant data. Then the database can be used to quantify expected loss and VaR as the relevant quantile of the PDF. Risk estimates can also indicate where the bank is vulnerable and in this field it would be reasonable to test some extremal events.

However, stress tests in this area are not developed, as the modeling and quantifying of the operational risk itself causes difficulties.[50]

V.3 Aggregate stress scenarios

In current practice the risk management is based on separate calculation of amount of market risk and credit risk separately without considering their interaction with one another. The results, however, may not give true picture. For example, for the high rated debtor credit risk model would quantify a high probability of repayment. However, if the debtor is from an emerging country, such model would neglect the influence of exchange rate (market risk) - the significant potential that an emerging market will devalue its currency as a mild form of default. Similarly, liquidity problems may cause the delay in liquidating the position; the changed duration window will then significantly influence the calculated VaR (either under normal circumstances or in frame of stress testing process).

[50] More bout management of operational risk can be found in Crouhy, Galai, Mark (2001).

Also neglecting correlations among different kinds of risk may cause problems, not only to an individual institution, but problems might spill over into the entire financial system when, simultaneously, market prices fall and market liquidity dries up. Such a situation could make for many institutions impossible meet their obligations. [51]

The models in the future will capture in a consistent framework the market component, credit component and the liquidity component of bonds, loans and other instruments and the operational risk component. Such integration is logically next steps in development, as all named components are not independent. Moreover the innovative process, for instance development of credit derivatives, diminishes the distance between trading and banking book.

VI CONCLUSIONS

We showed that commonly used VaR models are based on several limiting assumptions. It can provide a substantial cushion against losses caused by a range of market moves but will fall under many extreme shocks. Risk measures such as VAR provide useful baseline information. Stress testing provides a tool that cannot prevent losses but tries to minimize surprises. It is a powerful means of anticipating, understanding, and preparing for shocks and the resulting potential losses.

The thesis showed why stress testing belongs to critical components of effective risk management. After we had discussed reasons for stress testing and the process has been described generally, we comprised a fairly comprehensive set of approaches that deal with a large number of risk factors relevant in market, credit and liquidity stress. We identified the key attributes of effective stress testing and outlined actions to consider, given the results of stress testing.

Taken together, the stress tests first ensure that the bank can survive the stress events (which include the impact on capital adequacy, reported earnings, liquidity and customer and investor confidence). In addition, they aim to preserve enough

[51] Note that even if this thesis deals only with stress tests conducted within a bank, also interbank stress tests should be applied (e.g. by regulators) in order to identify possible channels of interbank contagion.

resilience in distressed market conditions and to enable the firm to take the offensive and move quickly.

In each part we also looked at an existing application in practice. It showed up that only the stress testing of market risk is well developed and widely a conducted. The implementation of stress tests for credit risk brings many problems mentioned above. Also transition countries, including Czech Republic, must start to build a credit stress-testing programme. Even if the scarcity of reliable data will limit the possibility of reliable interpretation, the benefits from conducting even rudimentary stress-tests are still substantial.

In the final part we depicted other types of risk that are usually not incorporated into stress testing models, but are also relevant for the bank's performance and we sketched possible future development toward integrated risk measurement.

I hope this thesis convinced about the importance of stress testing and the summary of recent developments in literature will contribute to understanding of this topic, which is interesting and more and more topical also for the Czech banks.

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Abstract

Previous decades brought to financial institutions process of diversification and extensive innovations on the one hand, but also a number of financial crises on the other hand. Increasing frequency of financial distress and its tendency to spread out over international markets lead to expansion in risk management practices.

One of risk management activities that have attracted much attention over past several years among regulators and practitioners is stress testing. This thesis is an comprehensive interpretation of the issue of stress testing. Firstly, it is explained what stress testing is, from what the legal background it comes out and in what extent it is managed in practice. Also the role of stress testing in relation to standard risk management measure of VaR is discussed. Secondly, the banks´ risks are divided into market, credit and other risks including liquidity and operational risk. Then parameters and techniques suitable for these areas are analyzed separately. In the part devoted to credit risk we meet with difficulties, as stress testing in this area is much less examined in comparison with market risk stress testing. We will also mention additional problems that arise in connection with stress testing in transitional countries, including the Czech Republic. In order not to focus only on the theoretical aspects of stress testing, also the currently used practices will be studied. The final part is devoted to presumable future development toward risk aggregation and to new trends in stress testing.