

Institute of Economic Studies, Faculty of Social Sciences
Charles University in Prague

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František Turnovec

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Institute of Economic Studies,
Faculty of Social Sciences,
Charles University in Prague

[UK FSV – IES]

Opletalova 26
CZ-110 00, Prague
E-mail : ies@fsv.cuni.cz
<http://ies.fsv.cuni.cz>

Institut ekonomických studií
Fakulta sociálních věd
Univerzita Karlova v Praze

Opletalova 26
110 00 Praha 1

E-mail : ies@fsv.cuni.cz
<http://ies.fsv.cuni.cz>

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Quota Manipulation and Fair Voting Rules in Committees

František Turnovec*

* IES, Charles University Prague
E-mail: turnovec@fsv.cuni.cz

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Abstract:

The concept of fairness is being discussed related to the distribution of voting power among different actors of voting. We are using the following, intuitively natural principle of fairness: each unit of resources (shares, seats) should yield the same voting power (influence). Voting power is not directly observable: as a proxy for it voting weights are used. Therefore, fairness is usually defined in terms of voting weights (e.g. voting weights are proportional to the results of an election). Assuming that a principle of fair distribution of voting weights is selected, we are addressing the question of how to achieve equality of relative voting power (at least approximately) to relative voting weights.

Keywords: Fairness, optimal quota, simple weighted committee, strict proportional power, voting and power indices.

JEL: C71, D72, H77

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1. Introduction

Let us consider a committee with n members. Each member has some voting weight (number of votes, shares etc.) and a voting rule is defined by a minimal number of weights required for passing a proposal. Given a voting rule, voting weights provide committee members with voting power. Voting power means an ability to influence the outcome of voting. Voting power indices are used to quantify the voting power.

The concept of fairness is being discussed related to the distribution of voting power among different actors of voting. This problem was clearly formulated by Nurmi (1982, p. 204): *“If one aims at designing collective decision-making bodies which are democratic in the sense of reflecting the popular support in terms of the voting power, we need indices of the latter which enable us to calculate for any given distribution of support and for any decision rule the distribution of seats that is ‘just’. Alternatively, we may want to design decision rules that – given the distribution of seats and support – lead to a distribution of voting power which is identical with the distribution of support.”*

Voting power is not directly observable: as a proxy for it voting weights are used. Therefore, fairness is usually defined in terms of voting weights (e.g. voting weights are proportional to the results of an election). Assuming that a principle of fair distribution of voting weights is selected, we are addressing the question of how to achieve equality of voting power (at least approximately) to relative voting weights. The concepts of strict proportional power and the randomized decision rule introduced by Holler (1982a, 1985, 1987), of optimal quota of Słomczyński and Życzkowski (2007), and of intervals of stable power (Turnovec, 2008b) are used to find, given voting weights, a voting rule minimizing the distance between actors’ voting weights and their voting power.

Concept of fairness is frequently associated with so-called square root rule, attributed to British statistician Lionel Penrose (1946). The square root rule is closely related to indirect voting power measured by the Penrose-Banzhaf power index.¹ Different aspects of the square root rule have been analysed in Felsenthal and Machover (1998, 2004), Laruelle and Widgrén (1998), Baldwin and Widgrén (2004), Turnovec (2009). The square root rule of “fairness” in the EU Council of Ministers voting was discussed and evaluated in Felsenthal and Machover (2007), Słomczyński and Życzkowski (2006, 2007), Hosli (2008), Leech and Aziz (2008), Turnovec (2008a) and others. Nurmi (1997a) used it to evaluate the representation of voters’ groups in the European Parliament.

In the second section basic definitions are introduced and the applied power indices methodology is shortly resumed. The third section introduces concepts of quota intervals of stable power, index of fairness and exact optimal quota. While the framework of the analysis of fairness is usually restricted to the Penrose-Banzhaf concept of power, we are treating it in a

¹ The square root rule is based on the following propositions: Let us assume n units with population p_1, p_2, \dots, p_n , and the system of representation by a super-unit committee with voting weights w_1, w_2, \dots, w_n of units’ representations. It can be rigorously proved that for sufficiently large p_i the absolute Penrose-Banzhaf power of individual citizen of unit i in unit’s referendum is proportional to the square root of p_i . If the relative Penrose-Banzhaf voting power of unit i representation is proportional to its voting weight, then indirect voting power of each individual citizen of unit i is proportional to the product of voting weight w_i and square root of population p_i . Based on the conjecture (not rigorously proved) that for n large enough the relative voting power is proportional to the voting weights, the square root rule concludes that the voting weights of units’ representations in the super-unit committee proportional to square roots of units’ population lead to the same indirect voting power of each citizen independently of the unit she is affiliated with.

more general setting and our results are relevant for any power index based on pivots or swings and for any concept of fairness.

2. Committees and Voting Power

A *simple weighted committee* is a pair $[N, \mathbf{w}]$, where N be a finite set of n committee members $i = 1, 2, \dots, n$, and $\mathbf{w} = (w_1, w_2, \dots, w_n)$ be a nonnegative vector of committee members' voting weights (e.g. votes or shares). By 2^N we denote the power set of N (set of all subsets of N). By voting coalition we mean an element $S \in 2^N$, the subset of committee members voting uniformly (YES or NO), and $w(S) = \sum_{i \in S} w_i$ denotes the voting weight of coalition S . The voting rule is defined by quota q satisfying $0 < q \leq w(N)$, where q represents the minimal total weight necessary to approve the proposal. Triple $[N, q, \mathbf{w}]$ we call a *simple quota weighted committee*. The voting coalition S in committee $[N, q, \mathbf{w}]$ is called a winning one if $w(S) \geq q$ and a losing one in the opposite case. The winning voting coalition S is called critical if there exists at least one member $k \in S$ such that $w(S \setminus k) < q$ (we say that k is critical in S). The winning voting coalition S is called minimal if any of its members is critical in S .

A priori voting power analysis seeks an answer to the following question: Given a simple quota weighted committee $[N, q, \mathbf{w}]$, what is an influence of its members over the outcome of voting? The absolute voting power of a member i is defined as a probability $\Pi_i(N, q, \mathbf{w})$ that i will be decisive in the sense that such a situation appears in which she would be able to decide the outcome of voting by her vote (Nurmi, 1997b and Turnovec, 1997), and a relative voting power as:

$$\pi_i(N, q, \mathbf{w}) = \frac{\Pi_i(N, q, \mathbf{w})}{\sum_{k \in N} \Pi_k(N, q, \mathbf{w})}.$$

Three basic concepts of decisiveness are used: swing position, pivotal position and membership in a minimal winning coalition (MWC position). The *swing position* is an ability of an individual voter to change the outcome of voting by a unilateral switch from YES to NO (if member j is critical with respect to a coalition S , we say that he has a swing in S). The *pivotal position* is such a position of an individual voter in a permutation of voters expressing a ranking of attitudes of members to the voted issue (from the most preferable to the least preferable) and the corresponding order of forming of the winning coalition, in which her vote YES means a YES outcome of voting and her vote NO means a NO outcome of voting (we say that j is pivotal in the permutation considered). The MWC position is an ability of an individual voter to contribute to a minimal winning coalition (membership in the minimal winning coalition).

Let us denote by $W(N, q, \mathbf{w})$ the set of all winning coalitions and by $W_i(N, q, \mathbf{w})$ the set of all winning coalitions with i , $C(N, q, \mathbf{w})$ as the set of all critical winning coalitions, and by $C_i(N, q, \mathbf{w})$ the set of all critical winning coalitions i has the swing in, by $P(N, q, \mathbf{w})$ the set of all permutations of N and $P_i(N, q, \mathbf{w})$, the set of all permutations i is pivotal in, $M(N, q, \mathbf{w})$ the set of all minimal winning coalitions, and $M_i(N, q, \mathbf{w})$ the set of all minimal winning coalitions with i . By $\text{card}(S)$ we denote the cardinality of S , $\text{card}(\emptyset) = 0$.

Assuming many voting acts and all coalitions equally likely, it makes sense to evaluate the a priori voting power of each member of the committee by the probability to have a swing, measured by the absolute Penrose-Banzhaf (PB) power index (Penrose, 1946; Banzhaf, 1965):

$$\Pi_i^{PB}(N, q, \mathbf{w}) = \frac{\text{card}(C_i)}{2^{n-1}},$$

($\text{card}(C_i)$ is the number of all winning coalitions the member i has the swing in and 2^{n-1} is the number of all possible coalitions with i). To compare the relative power of different committee members, the relative form of the PB power index is used:

$$\pi_i^{PB}(N, q, \mathbf{w}) = \frac{\text{card}(C_i)}{\sum_{k \in N} \text{card}(C_k)}.$$

While the absolute PB is based on a well-established probability model (see e.g. Owen, 1972), its normalization (relative PB index) destroys this probabilistic interpretation, the relative PB index simply answers the question of what is the voter i 's share in all possible swings.

Assuming many voting acts and all possible preference orderings equally likely, it makes sense to evaluate an a priori voting power of each committee member by the probability of being in pivotal situation, measured by the Shaply-Shubik (SS) power index (Shapley and Shubik, 1954):

$$\Pi_i^{SS}(N, q, \mathbf{w}) = \frac{\text{card}(P_i)}{n!},$$

($\text{card}(P_i)$ is the number of all permutations in which the committee member i is pivotal, and $n!$ is the number of all possible permutations of committee members). Since $\sum_{i \in N} \text{card}(P_i) = n!$ it holds

that:

$$\pi_i^{SS}(N, q, \mathbf{w}) = \frac{\text{card}(P_i)}{\sum_{k \in N} \text{card}(P_k)} = \frac{\text{card}(P_i)}{n!},$$

i.e. the absolute and relative form of the SS-power index is the same.

Assuming many voting acts and all possible coalitions equally likely, it makes sense to evaluate the voting power of each committee member by the probability of membership in a minimal winning coalition, measured by the absolute Holler-Packel (HP) power index:

$$\Pi_i^{HP}(N, q, \mathbf{w}) = \frac{\text{card}(M_i)}{2^n},$$

($\text{card}(M_i)$ is the number of all minimal winning coalitions with i , and 2^n is the number of all possible coalitions).² Originally the HP index was defined and is usually being presented in its relative form (Holler, 1982b; Holler and Packel, 1983)

$$\pi_i^{HP}(N, q, \mathbf{w}) = \frac{\text{card}(M_i)}{\sum_{k \in N} \text{card}(M_k)}.$$

Above definition of the absolute HP index allows a clear probabilistic interpretation. Multiplying and dividing it by the $\text{card}(M)$, we obtain:

$$\frac{\text{card}(M_i)}{\text{card}(M)} \frac{\text{card}(M)}{2^n}.$$

In this breakdown the first term gives the probability of being a member of a minimal winning coalition, provided the MWC is formed, and the second term the probability of forming a

² The definition of an absolute HP power index is provided by the author (a similar definition of absolute PB power can be found in Brueckner (2001), the only difference is that we relate the number of MWC positions of member i to the total number of coalitions, not to the number of coalitions of which i is a member).

minimal winning coalition assuming that all voting coalitions are equally likely. The relative HP index has the same problem with a probabilistic interpretation as the relative PB index.³

In the literature there are still two other concepts of power indices: the Johnston (J) power index based on swings, and the Deegan-Packel (DP) power index, based on membership in minimal winning coalitions.

The Johnston power index (Johnston, 1978) measures the power of a member of a committee as a normalized weighted average of the number of her swings, using as weights the reciprocals of the total number of swings in each critical winning coalition (the swing members of the same winning coalition have the same power, which is equal to $1/(\text{number of swing members})$).

The Deegan-Packel power index (Deegan and Packel, 1978) measures the power of a member of a committee as a normalized weighted average of the number of minimal critical winning coalitions he is a member of, using as weights the reciprocals of the size of each MCWC

It is difficult to provide some intuitively acceptable probabilistic interpretation for relative J and DP power indices. They provide a normative scheme of the division of rents in the committee rather than a measure of an a priori power (and in the sense of Felsenthal and Machover (1998) classification they can be considered as measures of P power).

It can be easily seen that for any $\alpha > 0$ and any power index based on swings, pivots or MWC positions it holds that $\Pi_i(N, \alpha q, \alpha \mathbf{w}) = \Pi_i(N, q, \mathbf{w})$. Therefore, without the loss of generality, we shall assume throughout the text that $\sum_{i \in N} w_i = 1$ and $0 < q \leq 1$, using only relative weights and relative quotas in the analysis.

3. Quota Intervals of Stable Power, Fairness and Strict Proportional Power

Let us formally define a few concepts we shall use later in this paper:

Definition 1: A simple weighted committee $[N, \mathbf{w}]$ has a property of *strict proportional power* with respect to a power index π , if there exists a voting rule q^* such that $\pi[N, q^*, \mathbf{w}] = \mathbf{w}$, i.e. the relative voting power of committee members is equal to their relative voting weights.

In general, there is no reason to expect that such a voting rule exists. Holler and Berg extended the concept of a strict proportional power in the model with randomized voting rule.

Definition 2: Let $[N, \mathbf{w}]$ be a simple weighted committee, $\mathbf{q} = (q_1, q_2, \dots, q_m)$ be a vector of different quotas, π^k be a relative power index for quota q_k , and $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_m)$ be a probability distribution over elements of \mathbf{q} . The *randomized voting rule* (\mathbf{q}, λ) selects within different voting acts by random mechanism quotas from \mathbf{q} by the probability distribution λ . Then $[N, \mathbf{w}]$ has a property of *strict proportional expected power* with respect to a relative power index π , if there exists a randomized voting rule $(\mathbf{q}^*, \lambda^*)$ such that the vector of the mathematical expectations of relative voting power is equal to the vector of voting weights:

³ For a discussion about the possible probabilistic interpretation of the relative PB and HP see Widgrén (2001).

$$\pi(N, (\mathbf{q}, \lambda), \mathbf{w}) = \sum_{k=1}^m \lambda_k \pi^k(N, q_k, \mathbf{w}) = \mathbf{w}.$$

The concept of randomized voting rule and strict proportional expected power was introduced by Holler (1982, 1985), and studied by Berg and Holler (1986).

Definition 3: Let $\mathbf{w} = (w_1, w_2, \dots, w_n)$ be a fair distribution of voting weights (with whatever principle is used to justify it) in a simple weighted committee $[N, \mathbf{w}]$, π is a relative power index, $(\pi(N, q, \mathbf{w}))$ is a vector valued function of q , and d is a distance function, then the voting rule q_1 is said to be *at least as fair* as voting rule q_2 with respect to the selected π if $d(\mathbf{w}, \pi(N, q_1, \mathbf{w})) \leq d(\mathbf{w}, \pi(N, q_2, \mathbf{w}))$.

Intuitively, given \mathbf{w} , the voting rule q_1 is preferred to the voting rule q_2 if q_1 generates a distribution of power closer to the distribution of weights than q_2 .

Definition 4: The voting rule q^* that minimizes a distance d between $\pi(N, q, \mathbf{w})$ and \mathbf{w} is called an optimal voting rule (*optimal quota*) for the power index π .

Proposition 1: Let $[N, q, \mathbf{w}]$ be a simple weighted quota committee and C_{is} be the set of critical winning coalitions of the size s in which i has a swing, then

$$\text{card}(P_i) = \sum_{s \in N} \text{card}(C_{is})(s-1)!(n-s)!$$

is the number of permutations with the pivotal position of i in $[N, q, \mathbf{w}]$.

Proof follows directly from Shapley and Shubik (1954). \square

From Proposition 1 it follows that the number of pivotal positions corresponds to the number and structure of swings. If in two different committees sets of swing coalitions are identical, then the sets of pivotal positions are also the same.

Proposition 2: Let $[N, q_1, \mathbf{w}]$ and $[N, q_2, \mathbf{w}]$, $q_1 \neq q_2$, be two simple quota-weighted committees such that $W(N, q_1, \mathbf{w}) = W(N, q_2, \mathbf{w})$, then

$$C_i(N, q_1, \mathbf{w}) = C_i(N, q_2, \mathbf{w})$$

$$P_i(N, q_1, \mathbf{w}) = P_i(N, q_2, \mathbf{w}),$$

and

$$M_i(N, q_1, \mathbf{w}) = M_i(N, q_2, \mathbf{w})$$

for all $i \in N$.

Proof. Without a loss of generality suppose $q_1 > q_2$. Assuming that the statement of the proposition is not true, i.e. a member k from S has a swing in S for quota q_2 and does not have a swing for quota q_1 , we obtain $w(S \setminus k) - q_2 < 0$ and $w(S \setminus k) - q_1 \geq 0$, hence $S \setminus k \notin W(N, q_2, \mathbf{w})$ and $S \setminus k \in W(N, q_1, \mathbf{w})$, which contradicts the assumption of Proposition 2 that the sets of winning coalitions are equal. From Proposition 1 it follows that $C_i(N, q_1, \mathbf{w}) = C_i(N, q_2, \mathbf{w})$ implies $P_i(N, q_1, \mathbf{w}) = P_i(N, q_2, \mathbf{w})$. In a similar way we can prove identity for sets of minimal winning coalitions. Assuming that S is the minimal winning coalition for q_2 but it is not a minimal winning coalition for q_1 , we have $w(S) - q_2 \geq 0$ and $w(S \setminus i) - q_2 < 0$ for any $i \in S$, and there exists at least one $k \in S$ such that $w(S \setminus k) - q_1 \geq 0$. But in this case $S \setminus k$ is the losing coalition for q_2 and the winning coalition for q_1 what contradicts the assumption of

Proposition 2 that the sets of winning coalitions are the same, hence $M_i(N, q_1, \mathbf{w}) = M_i(N, q_2, \mathbf{w})$. \square

From Proposition 2 it follows that in two different committees with the same set of members, the same weights and the same sets of winning coalitions, the PB-power indices, SS-power indices and HP-power indices are the same in both committees, independently of quotas. Moreover, since the J-index is based on the concept of swing and the DP power index is based on membership in minimal winning coalitions, the J and DP indices are also the same.

Proposition 3: Let $[N, q, \mathbf{w}]$ be a simple quota weighted committee with a quota q ,

$$\mu^+(q) = \min_{S \in W[N, q, \mathbf{w}]} (w(S) - q)$$

and

$$\mu^-(q) = \min_{S \in 2^N \setminus W(N, q, \mathbf{w})} (q - w(S)).$$

Then for any particular quota $q > 0$ we have $W(N, q, \mathbf{w}) = W(N, \gamma, \mathbf{w})$ for all $\gamma \in (q - \mu^-(q), q + \mu^+(q)]$.

Proof. a) Let $S \in W(N, q, \mathbf{w})$, then $w(S) - q \geq 0$ and from the definition of $\mu^+(q)$

$$w(S) - q \geq \mu^+(q) \geq 0 \Rightarrow w(S) - q - \mu^+(q) \geq 0 \Rightarrow S \in W(N, q + \mu^+(q), \mathbf{w}),$$

hence S is winning for quota $q + \mu^+(q)$. If S is winning for q and $q + \mu^+(q)$, then it is winning for any quota $\gamma \leq q + \mu^+(q)$.

b) Let $S \in 2^N \setminus W(N, q, \mathbf{w})$, then $q - w(S) > 0$ and from the definition of $\mu^-(q)$

$$q - w(S) \geq \mu^-(q) \geq 0 \Rightarrow q - \mu^-(q) - w(S) \geq 0 \Rightarrow S \in 2^N \setminus W(N, q - \mu^-(q), \mathbf{w}),$$

hence S is losing for quota $q - \mu^-(q)$. If S is losing for $q - \mu^-(q)$, then it is losing for any quota $\gamma \leq q - \mu^-(q)$.

From (a) and (b) it follows that for any $\gamma \in (q - \mu^-(q), (q - \mu^+(q))]$

$$S \in W(N, q, \mathbf{w}) \Rightarrow S \in W(N, \gamma, \mathbf{w})$$

$$S \in \{2^N \setminus W(N, \gamma, \mathbf{w})\} \Rightarrow S \in \{2^N \setminus W(N, q, \mathbf{w})\}$$

which implies that $W(N, q, \mathbf{w}) = W(N, \gamma, \mathbf{w})$. \square

From Propositions 2 and 3 it follows that swing, pivot and MWC-based power indices are the same for all quotas $\gamma \in (q - \mu^-(q), q + \mu^+(q)]$. Therefore the interval of quotas $(q - \mu^-(q), q + \mu^+(q)]$ we call an *interval of stable power* for quota q . Quota $\gamma^* \in (q - \mu^-(q), q + \mu^+(q)]$ is called the marginal quota for q if $\mu^+(\gamma^*) = 0$.

Example 1. Consider a committee $[N, q, \mathbf{w}]$ where $n = 3$, $w_1 = 0.1$, $w_2 = 0.4$, $w_3 = 0.5$. Then $2^N = (\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\})$. Consider a simple majority quota $q = 0.51$. Then:

$$W(N, q, \mathbf{w}) = (\{1,3\}, \{2,3\}, \{1,2,3\}), 2^N \setminus W(N, q, \mathbf{w}) = (\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}).$$

$$\mu^+(q) = \min \{w_1 + w_3 - 0.51 = 0.09, w_2 + w_3 - 0.51 = 0.39, w_1 + w_2 + w_3 - 0.51 = 0.49\} = 0.09$$

$$\mu^-(q) = \min \{0.51 - w_1 = 0.41, 0.51 - w_2 = 0.11, 0.51 - w_3 = 0.01, 0.51 - w_1 - w_2 = 0.01\} = 0.01$$

The quota interval of stable power for quota $q = 0.51$ is $(0.5, 0.6]$. The marginal quota for $q = 0.51$ is $\gamma^* = 0.6$.

Now we define a partition of the power set 2^N into equal weight classes $\Omega^{(0)}, \Omega^{(1)}, \dots, \Omega^{(r)}$ (such that the weight of different coalitions from the same class is the same and the weights of different coalitions from different classes are different). For the completeness set $\Omega^{(0)} = \emptyset$, $w(\emptyset) = 0$. Consider the weight-increasing ordering of equal weight classes $\Omega^{(0)}, \Omega^{(1)}, \dots, \Omega^{(r)}$ such that for any $t < k$ and $S \in \Omega^{(t)}, R \in \Omega^{(k)}$ it holds that $w(S) < w(R)$. Denote $q_t = w(S)$ for any $S \in \Omega^{(t)}, t = 1, 2, \dots, r$.

Proposition 4: Let $\Omega^{(0)}, \Omega^{(1)}, \dots, \Omega^{(r)}$ be the weight-increasing ordering of the equal weight partition of 2^N . Set $q_t = w(S)$ for any $S \in \Omega^{(t)}, t = 0, 1, 2, \dots, r$. Then there is a finite number $r \leq 2^n - 1$ of marginal quotas q_t and corresponding intervals of stable power $(q_{t-1}, q_t]$ such that $W[N, q_t, \mathbf{w}] \subset W[N, q_{t-1}, \mathbf{w}]$.

Proof follows from the fact that $\text{card}(2^N) = 2^n$ and an increasing series of k real numbers a_1, \dots, a_k subdivides interval $(a_1, a_k]$ into $k-1$ segments. An analysis of voting power as a function of the quota (given voting weights) can be substituted by an analysis of voting power in a finite number of marginal quotas. \square

From Proposition 4 it follows that there exists at most r distinct voting situations generating r vectors of power indices.

Example 2. In the committee from Example 1 enumerate equal weight classes of voting coalitions in the weight-increasing order (see Table 1):

Table 1. Equal weight classes of coalitions

t	$\Omega^{(t)}$	$w(S)$	Intervals	marginal quotas
0	\emptyset	0.		
1	{1}	0.1	(0, 0.1]	0.1
2	{2}	0.4	(0.1, 0.4]	0.4
3	{1,2}, {3}	0.5	(0.4, 0.5]	0.5
4	{1,3}	0.6	(0.5, 0.6]	0.6
5	{2,3}	0.9	(0.6, 0.9]	0.9
6	{1,2,3}	1	(0.9, 1]	1

In this committee we have seven classes of equal weight voting coalitions ordered by weights and $r = 6$ intervals of stable power and corresponding marginal quotas.

Proposition 5: Let $[N, q, \mathbf{w}]$ be a simple quota weighted committee and $(q_{t-1}, q_t]$ is the interval of stable power for quota q . Then for any $\gamma = 1 - q_t + \varepsilon$, where $\varepsilon \in (0, q_t - q_{t-1}]$ and for all $i \in N$

$$\text{card}(C_i(N, q, \mathbf{w})) = \text{card}(C_i(N, \gamma, \mathbf{w}))$$

and

$$\text{card}(P_i(N, q, \mathbf{w})) = \text{card}(P_i(N, \gamma, \mathbf{w}))$$

Proof. Let S be a winning coalition, member k has the swing in S and $(q_{t-1}, q_t]$ is an interval of stable power for q . Then it is easy to show that $N \setminus S \cup k$ is a winning coalition, k has a swing in

$N \setminus S \cup k$ and $(1 - q_t, 1 - q_{t-1}]$ is an interval of stable power for any quota $\gamma = 1 - q_t + \varepsilon$ ($0 < \varepsilon \leq q_t - q_{t-1}$). Let R be a winning coalition, j has a swing in R , and $(1 - q_t, 1 - q_{t-1}]$ is an interval of stable power for quota $\gamma = 1 - q_t + \varepsilon$ ($0 < \varepsilon \leq q_t - q_{t-1}$). Then $N \setminus R \cup j$ is a winning coalition, j has a swing in $N \setminus R \cup j$ and $(q_{t-1}, q_t]$ is an interval of stable power for any quota $q = q_{t-1} + \tau$ where $0 < \tau \leq q_t - q_{t-1}$. \square

While in $[N, q, \mathbf{w}]$ the quota q means the total weight necessary to pass a proposal (and therefore we can call it a *winning quota*), the *blocking quota* means the total weight necessary to block a proposal. If q is a winning quota and $(q_{t-1}, q_t]$ is the quota interval of stable power for q , then any voting quota $1 - q_{t-1} + \varepsilon$ (where $0 < \varepsilon \leq q_t - q_{t-1}$), is a blocking quota. From Proposition 4 it follows that the blocking power of the committee members, measured by swing and pivot-based power indices, is equal to their voting power. It is easy to show that voting power and blocking power might not be the same for power indices based on membership in minimal winning coalitions (HP and DP power indices). Let r be the number of marginal quotas, then from Proposition 5 it follows that for power indices based on swings and pivots the number of majority power indices does not exceed $\text{int}(r/2) + 1$.

Example 3. In Table 2 we provide the numbers of the decisive positions of committee members for different intervals of stable power in the committee from Example 1 (swings, pivots and MWC memberships), in Table 3 we show the corresponding relative PB, SS and HP power indices.

Table 2. Swings, pivots and MWC memberships in quota intervals of stable power

Quota interval of stable power	Members			Σ
	1	2	3	
SWINGS				
(0, 0.1]	1	1	1	3
(0.1, 0.4]	0	2	2	4
(0.4, 0.5]	1	1	3	5
(0.5, 0.6]	1	1	3	5
(0.6, 0.9]	0	2	2	4
(0.9, 1]	1	1	1	3
PIVOTS				
(0, 0.1]	2	2	2	6
(0.1, 0.4]	0	3	3	6
(0.4, 0.5]	1	1	4	6
(0.5, 0.6]	1	1	4	6
(0.6, 0.9]	0	3	3	6
(0.9, 1]	2	2	2	6
MWC memberships				
(0, 0.1]	1	1	1	3
(0.1, 0.4]	0	1	1	2
(0.4, 0.5]	1	1	1	3
(0.5, 0.6]	0	1	1	2
(0.6, 0.9]	0	1	1	2
(0.9, 1]	1	1	1	3

Table 3. SS, PB and HP relative power indices

t	Interval	marginal quota	SS-power	PB-power	HP-power
1	(0, 0.1]	0.1	(1/3,1/3,1/3)	(1/3,1/3,1/3)	(1/3,1/3,1/3)
2	(0.1,0.4]	0.4	(0,1/2,1/2)	(0,1/2,1/2)	(0,1/2,1/2)
3	(0.4,0.5]	0.5	(1/6,1/6,4/6)	(1/5,1/5,3/5)	(1/3,1/3,1/3)
4	(0.5,0.6]	0.6	(1/6,1/6,4/6)	(1/5,1/5,3/5)	(1/4,1/4,1/2)
5	(0.6,0.9]	0.9	(0,1/2,1/2)	(0,1/2,1/2)	(0,1/2,1/2)
6	(0.9,1]	1	(1/3,1/3,1/3)	(1/3,1/3,1/3)	(1/3,1/3,1/3)

Proposition 6: Let $\mathbf{q} = (q_1, q_2, \dots, q_m)$ be the set of all majority marginal quotas in a simple weighted committee $[N, \mathbf{w}]$, and $\boldsymbol{\pi}^k$ be a vector of Shapley-Shubik relative power indices corresponding to a marginal quota q_k , then there exists a vector $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_m)$ such that:

$$\sum_{k=1}^m \lambda_k = 1, \lambda_k \geq 0, \sum_{k=1}^m \lambda_k \boldsymbol{\pi}^k = \mathbf{w}$$

Proof follows from Berg and Holler (1986, p. 426), who, using geometrical arguments, showed that there exists a finite number of different relative power indices corresponding to different majority quotas such that the vector of relative voting weights is interior with respect to the simplex generated by them. The randomized voting rule $(\mathbf{q}, \boldsymbol{\lambda})$ leads to the strict proportional expected SS power. Clearly, if there exists an exact quota q^* such that $\pi_i(N, q^*, \mathbf{w}) = w_i$, we can find it among finite number of marginal majority quotas.

Example 4. The randomized voting rule in the committee from Example 1 applied to the SS power index:

$$\begin{aligned} \frac{1}{6}\lambda_1 + \frac{1}{3}\lambda_3 &= \frac{1}{10} \\ \frac{1}{6}\lambda_1 + \frac{1}{2}\lambda_2 + \frac{1}{3}\lambda_3 &= \frac{4}{10} \\ \frac{4}{6}\lambda_1 + \frac{1}{2}\lambda_2 + \frac{1}{3}\lambda_3 &= \frac{5}{10} \\ \lambda_1 + \lambda_2 + \lambda_3 &= 1 \\ \lambda_j &\geq 0 \end{aligned}$$

The system has a unique solution $\lambda_1 = \frac{1}{5}, \lambda_2 = \frac{3}{5}, \lambda_3 = \frac{1}{5}$. If there is a random mechanism selecting marginal quotas $q_1=0.6, q_2=0.8, q_3=1$ with probabilities $\lambda_1 = \frac{1}{5}, \lambda_2 = \frac{3}{5}, \lambda_3 = \frac{1}{5}$, then the mathematical expectation of the SS-power of the members of the committee will be equal to their relative weights (we obtain the case of strict proportional power).

Although we can apply a randomized voting rule to any relative power index, based on pivots and swings, the problem is with the interpretation of what we get. For instance, the

relative PB index has no probabilistic interpretation, and the randomized voting rule calculated for it by Proposition 6 does not provide the mathematical expectation of the number of swings, leading to a relative PB power equal to weights.

Example 5. We can easily verify that

$$\lambda_1 = \frac{5}{20}, \lambda_2 = \frac{12}{20}, \lambda_3 = \frac{3}{20}$$

are the probabilities for a randomized voting rule equalizing relative PB power to relative voting weights in the committee from Example 1. The swings for majority quotas and PB absolute power see in Table 4.

Table 4, Absolute PB power

SWINGS				
Majority quotas	Member 1	Member 2	Member 3	Σ
(0,5, 0,6]	1	1	3	5
(0,6, 0,9]	0	2	2	4
(0,9, 1]	1	1	1	3
ABSOLUTE PB POWER				
(0,5, 0,6]	1/4	1/4	3/4	5/4
(0,6, 0,9]	0	2/4	2/4	4/4
(0,9, 1]	1/4	1/4	1/4	3/4

Applying a randomized voting rule equalizing the relative PB power to voting weights we obtain the mathematical expectation of absolute power

$$\begin{pmatrix} 1/4 \\ 1/4 \\ 3/4 \end{pmatrix} \frac{5}{20} + \begin{pmatrix} 0 \\ 2/4 \\ 2/4 \end{pmatrix} \frac{12}{20} + \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \end{pmatrix} \frac{3}{20} = \begin{pmatrix} 8/80 \\ 32/80 \\ 42/80 \end{pmatrix},$$

which yields an expected relative power

$$\begin{pmatrix} 8/80 \\ 32/80 \\ 42/80 \end{pmatrix} \frac{1}{82/80} = \begin{pmatrix} 8/82 \\ 32/82 \\ 42/82 \end{pmatrix} \neq \begin{pmatrix} 1/10 \\ 4/10 \\ 5/10 \end{pmatrix}$$

different from voting weights.

In general, the number of majority power indices can be greater than the number of committee members, and the system

$$\sum_{k=1}^r \lambda_k = 1, \lambda_k \geq 0, \sum_{k=1}^r \lambda_k \pi^k = \mathbf{w}$$

might not have the unique solution. To find the randomized voting rule leading to strict proportional power we can use the following optimization problem:

minimize

$$\sum_{i=1}^n \text{abs} \left(\sum_{k=1}^r \pi_i^k \lambda_k - w_i \right)$$

subject to

$$\sum_{k=1}^r \lambda_k = 1, \lambda_k \geq 0$$

that can be transformed into the equivalent linear programming problem (Gale, 1960):

minimize

$$\sum_{i=1}^n y_i$$

subject to

$$\sum_{k=1}^r \pi_i^k \lambda_k - y_i \leq w_i \quad \text{for } i = 1, \dots, n$$

$$\sum_{k=1}^r \pi_i^k \lambda_k + y_i \geq w_i \quad \text{for } i = 1, \dots, n$$

$$\sum_{k=1}^r \lambda_k = 1$$

$$\lambda_k, y_i \geq 0 \quad \text{for } k = 1, \dots, r, \quad i = 1, \dots, n$$

This problem is easy to solve by standard linear programming simplex methods.

One can hardly expect that randomized voting rules leading to the strict proportional power would be adopted by actors in real voting systems. However, the design of a “fair” voting system can be based on an approximation provided by the quota generating the minimal distance between vectors of power indices and weights, which is called an *optimal quota*.

The optimal quota was introduced by Słomczyński and Życzkowski (2006, 2007) as a quota minimizing the sum of square residuals between the power indices and the voting weights by $q \in (0.5, 1]$

$$\sigma^2(q) = \sum_{i \in N} (\pi_i[N, q, \mathbf{w}] - w_i)^2.$$

Słomczyński and Życzkowski introduced the optimal quota concept within the framework of the so-called Penrose voting system as a principle of fairness in the EU Council of Ministers voting. Here power is measured by the Penrose-Banzhaf power index. The system consists of two rules (Słomczyński and Życzkowski, 2007, p. 393):

- a) *The voting weight attributed to each member of the voting body of size n is proportional to the square root of the population he or she represents;*
- b) *The decision of the voting body is taken if the sum of the weights of members supporting it is not less than the optimal quota.*

Looking for a quota providing a priori voting power “as close as possible” to the normalized voting weights, Słomczyński and Życzkowski (2007) are minimizing the sum of square residuals between the power indices and voting power for $q \in (0.5, 1]$. Using heuristic minimization of the sum of square residuals based on simulation (with the data obtained numerically by averaging quotas over a sample of random weights distributions), they propose the following heuristic approximation of the solution for relative PB index:

$$\underline{q} = \frac{1}{2} \left(1 + \frac{1}{\sqrt{n}} \right) \leq q \leq \frac{1}{2} \left(1 + \sqrt{\sum_{i \in N} w_i^2} \right) = \bar{q}.$$

Clearly $\underline{q} = \bar{q}$ if and only if all the weights are equal, but in this case any majority quota is optimal.

By the index of the fairness of a voting rule q in $[N, q, \mathbf{w}]$ we call:

$$\varphi(N, q, \mathbf{w}) = 1 - \sqrt{\frac{1}{2} \sum_i (\pi_i[N, q, \mathbf{w}] - w_i)^2} ..$$

It is easy to see that $0 \leq \sqrt{\frac{1}{2} \sum_i (\pi_i[N, q, \mathbf{w}] - w_i)^2} \leq 1$ (zero in the case of the equality of weights and power, e.g. $w_1 = 1/2, w_2 = 1/2, \pi_1 = 1/2, \pi_2 = 1/2$, and one in the case of an extreme inequality of weights and power, e.g. $w_1 = 1, w_2 = 0, \pi_1 = 0, \pi_2 = 1$), hence $0 \leq \varphi(N, q, \mathbf{w}) \leq 1$. We say that a voting rule q_1 is “at least as fair” as a voting rule q_2 if $\varphi(N, q_1, \mathbf{w}) \geq \varphi(N, q_2, \mathbf{w})$.⁴

Looking for a “fair” voting rule we can maximize φ , which is the same as to minimize $\sigma^2(q)$. Using marginal quotas and intervals of stable power we do not need any simulation.

Proposition 7. *Let $[N, q, \mathbf{w}]$ be a simple quota-weighted committee and $\pi_i(N, q, \mathbf{w})$ be relative power indices for marginal quotas q_i , and q_{t^*} be the majority marginal quota minimizing*

$$\sum_{i \in N} (\pi_i(N, q_j, \mathbf{w}) - w_i)^2$$

($j = 1, 2, \dots, r$, r is the number of intervals of stable power such that q_j are marginal majority quotas), then the exact solution of Słomczyński and Życzkowski’s optimal quota (SZ optimal quota) problem for a particular power index used is any $\gamma \in (q_{t^-1}, q_{t^*}]$ from the quota interval of stable power for q_{t^*} .*

Proof follows from the finite number of quota intervals of stable power (Proposition 5). The quota q_{t^*} provides the best approximation of strict proportional power that is related neither to a particular power measure nor to a specific principle of fairness.□

Example 5: In Table 5 we provide the values of the index of fairness for the majority marginal quotas in the committee from Example 1.

⁴ The index of fairness follows the same logic as measures of deviation from proportionality used in political science to evaluate the difference between results of an election and the composition of an elected body - e.g. Loosemore and Hanby (1971) is based on the absolute values of the deviation metric, or Gallagher (1991) using the square roots metric.

Table 5. Index of fairness

Shapley-Shubik relative power					
Member		Weight	SS for $q_4=0.6$	SS for $q_5=0.9$	SS for $q_6=1$
	1	0.1	0.166667	0	0.333333
	2	0.4	0.166667	0.5	0.333333
	3	0.5	0.666667	0.5	0.333333
Σ		1	1	1	1
Index of fairness φ			0.791833	0.9	0.791833

Penrose-Banzhaf relative power					
Member		Weight	PB for $q_4=0.6$	PB for $q_5=0.9$	PB for $q_6=1$
	1	0.1	0.2	0	0.333333
	2	0.4	0.2	0.5	0.333333
	3	0.5	0.6	0.5	0.333333
Σ		1	1	1	1
Index of fairness φ			0.826795	0.9	0.791833

The exact optimal marginal quota for the PB and SS power index is the same: $q_{t^*} = q_5 = 0.9$ (and all quotas $q \in (0.6, 0.9)$).

The Słomczyński and Życzkowski heuristic approximation $\underline{q} = 0.79 \leq q \leq 0.82 = \bar{q}$ is a part of the exact interval of the optimal quota, but the next example illustrates that it might not be the case.

Example 6. Consider a committee of 4 members with weights (100/183, 49/183, 25/183, 9/183) and majority marginal quotas $q_1 = 100/183$, $q_2 = 109/183$, $q_3 = 125/183$, $q_4 = 135/183$, $q_5 = 149/183$, $q_6 = 158/183$, $q_7 = 174/183$, $q_8 = 183/183$. In Table 6 we provide the PB power indices for majority marginal quotas and corresponding values of index of fairness.

Table 6. Heuristic optimal quota might not work

PB relative power indices for marginal quotas								
Members	$q_1=100/183$	$q_2=109/183$	$q_3=125/183$	$q_4=134/183$	$q_5=149/183$	$q_6=158/183$	$q_7=174/183$	$q_8=183/183$
1	1	0.7	0.6	0.5	0.5	0.375	0.333	0.25
2	0	0.1	0.2	0.3	0.5	0.375	0.333	0.25
3	0	0.1	0.2	0.1	0	0.125	0.333	0.25
4	0	0.1	0	0.1	0	0.125	0	0.25
index of fairness φ	0.7268310	0.88205	0.94106	0.95781	0.8611	0.89186	0.84933	0.812

Comparing the values of index of fairness, we can see that optimal marginal quota in this case is $q_4 = 134/183 \approx 0.3224$, and the interval of stable power (optimal quotas) is $(125/183, 134/183] \approx (0.68306, 0.73224)$, while the Słomczyński and Życzkowski heuristic estimation gives an interval $[0.75, 0.81283]$ that is completely outside of the exact solution.⁵

4. Concluding Remarks

In simple quota weighted committees with a fixed number of members and voting weights there exists a finite number r of different quota intervals of stable power ($r \leq 2^n - 1$) generating a finite number of power indices vectors. For power indices with a voting power equal to

⁵ Clearly Słomczyński and Życzkowski heuristic estimate is meant to work for large n , but the authors do not specify from what critical size of the committees it can be applied.

blocking power the number of different power indices vectors corresponding to majority quotas is equal at most to $\text{int}(r/2) + 1$.

If the fair distribution of voting weights is defined, then the fair distribution of voting power is achieved by the quota that maximizes the index of fairness (minimizes the distance between relative voting weights and relative voting power). The index of fairness is not a monotonic function of the quota.

The problem of optimal quota has an exact solution via the finite number of majority marginal quotas. Słomczyński and Życzkowski introduced an optimal quota concept within the framework of the so called Penrose voting system as a principle of fairness in the EU Council of Ministers voting and related it exclusively to the Penrose-Banzhaf power index and the square root rule. However, the fairness in voting systems and approximation of strict proportional power is not exclusively related to the Penrose square-root rule and the Penrose-Banzhaf definition of power, as it is usually done in discussions about EU voting rules. In this paper it is treated in a more general setting as a property of any simple quota weighted committee and any well-defined power measure. Fairness and its approximation by optimal quota are not specific properties of the Penrose-Banzhaf power index and square root rule.

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