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A Relative Efficiency Measure Based on Stock Market Index Data

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IES Working Paper: 13/2012

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Bibliographic information:
Ivanková, K. (2012). "A Relative Efficiency Measure Based on Stock Market Index Data" IES Working Paper 13/2012. IES FSV. Charles University.

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A Relative Efficiency Measure Based on Stock Market Index Data

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May 2012

Abstract:
This article introduces a new measure of stock market efficiency. The measure specifies how much a stock market index deviates from Brownian motion and is computed from frequency representations of isoquantile shapes estimated from lagged index returns. We describe the theory behind the approach, discuss parameter choices and apply the novel measure on chosen indices.

Keywords: isoquantile, Efficient Market Hypothesis, stock market index, efficiency measure

JEL: C14; G14

Acknowledgements:
I'd like to thank my supervisor Prof. Ing. Miloslav Vošvrda, CSc. and the unknown referee.

1 Introduction

In this article we extend our previous work on isoquantile applications in economics [15]. The main result of this article is the introduction of a novel measure of stock market efficiency that agrees with previously-used visual examinations.

The notion of an *efficient market* was introduced in Fama [8]. In an efficient market investors exercise rational behavior and important information is available to all of them, which means that no one has an advantage over the others. The efficient market hypothesis (EMH) describes the case of an "ideal" stock market where actual prices fully reflect all relevant information. Consequently, the price (and corresponding return) fluctuations are not predictable and it's impossible to make gains systematically.

The efficient market hypothesis has three forms according to the degree of reflected information:

- The strong form of the EMH states that all information (public or private) in a market is reflected in current stock prices. No investor can profit from any new information.
- The semi-strong form of the EMH states that all public information is reflected in current stock prices. Investors cannot benefit by trading on publicly available new information, thus, examining related economic, financial and other qualitative and quantitative factors cannot help.
- The weak form of the EMH states that all past prices of a stock are reflected in the current stock price. Returns should be independent, which means that the history of stocks cannot be used for predictions of their future performance.

Chuvakhin [3] and Akintoye [1] summarize the development of currently used efficient market hypothesis varieties and of their evaluation.

Weak-form EMH tests can be divided into trading tests (i.e. whether rules that traders follow yield profit) and tests of return predictability (such as classic randomness tests, runs tests, autocorrelation tests and variance ratio tests).

Lagoarde-Segot and Lucey [19] uses unit root analysis, non-parametric multiple variance ratio analysis and trading simulations (variable moving average rule, trade range breaking rule) to study efficiency in emerging Middle-Eastern North African stock markets. Chordia et al. [2] applies regression and variance ratios to study short-horizon returns on NYSE stocks; they find out that decreasing the minimum tick size and periods of greater liquidity reduces predictability. Griffin et al. [12] use trading strategies over a wide range of emerging and developed markets and show that applying common tests without accounting for private and public informational efficiency aspects can give misleading results. McPherson and Palardy [13] develops generalized spectral tests for testing of linear and non-linear predictability and examines predictability of data from large markets.

Another method of disproving the weak EMH is to discover systematic empirical periodic deviations (anomalies). Rozeff and Kinney [22] introduced a seasonal effect called the January effect: in the beginning of every year, small stocks returns tend to be higher than in any other month. As a result of tax-related moves, it has been shown that investors profit by buying stocks in December when they are sold at a lower price and then selling them again in January. Gibbons and Hess [10] found the weekly Monday effect: stock prices tend to go down on Mondays and there is a negative average return because of weekend non-trading days.

Finally, articles [9, 11, 23, 16] discuss alternative contemporary approaches for testing the EMH. Formosa [9] introduces a composite index of market efficiency with particular reference to the goods market, the labour market and the financial market. Giglio et al. [11] and Shmilovici et al. [23] each develop a method for efficiency testing by computing compressibility with universal coding methods. For testing long-term memory of returns one can also use various methods for estimating the Hurst exponent [16].

In our work we study the particular version of the weak form of the efficient market hypohesis stating that returns of indices inferred from efficient stock markets follow the behaviour of Brownian motion. As for this article, closeness to this ideal will be understood as a measure of efficiency.

The first section discusses isoquantiles (originally called *isobars*). Isoquantiles were concieved [4] as a tool for dimensional reduction: they represent an ordering on multidimensional data based on quantile functions defined in polar coordinates. Following [17], we use nonparametric regression to estimate the isoquantile shapes. We make use of further theoretical extensions presented in [18]. Further research on isoquantiles has been performed in [5, 6, 7].

In the second section we describe the analysis of isoquantile shapes using the Fourier transform, a fundamental tool in signal processing. We compare various ways of frequency magnitude weighting and combining into a single measure characterizing the efficiency of a single stock market index.

We apply the approach on seven chosen indices and discuss the results in the third section.

2 Isoquantiles

Isoquantiles are defined in polar coordinates. The transformation of a non-zero vector $\mathbf{x} \in \mathbb{R}^d$ to generalized polar coordinates is given by

$$r = \|\mathbf{x}\|_2 \,, \quad \theta = \frac{\mathbf{x}}{\|\mathbf{x}\|_2},$$

where $\|\mathbf{x}\|_2$ is the Euclidean norm of the vector $\mathbf{x}$. Observe that the generalized angle θ is just a point on S^{d-1}, the sphere of unit radius in $\mathbb{R}^d$.

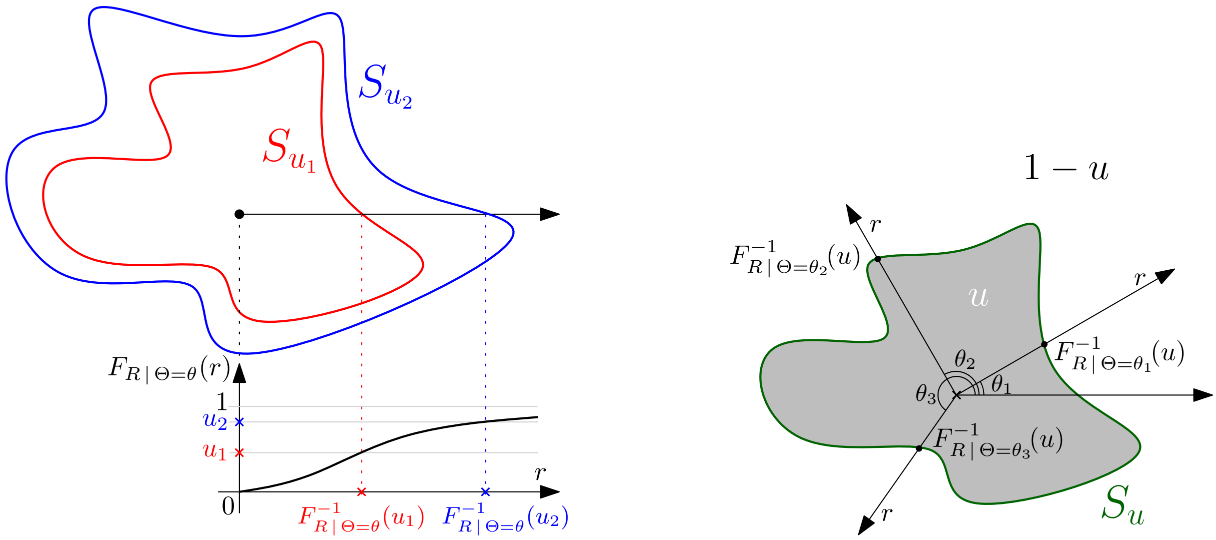

Figure 1: Illustrations of both isoquantile definitions.
The mapping $(\theta \mapsto r)$ on the left: we compute r from the inverse distribution function along a fixed direction θ as $r = F^{-1}_{R\,|\,\Theta=\theta}(u)$.
The surface S_u on the right is formed by the images of all direction mappings with fixed u. The portion of the distribution enclosed within the isoquantile is u.

We'll use the definition of isoquantile as it appears in Delcroix [4]: For every $u \in (0,1)$, the u-level isoquantile is defined as a mapping of a fixed θ to the value of the inverse distribution function of the Euclidean distance from the origin: $\theta \to F^{-1}_{R\,|\,\Theta}(u)$. The name "$u$-level isoquantile" will also be used interchangeably for the image of the mapping—the surface $S_u = F^{-1}_{R\,|\,\Theta}(u)$. See Figure 1.

We'll assume our sample to originate from the random variable $X = (R, \Theta)$ where R is a random variable with range $\mathbb{R}^+$ and Θ is a random variable with range S^{d-1}. We'll futher assume continuity of the marginal density $f_\Theta(\theta)$, conditional density $f_{R\,|\,\Theta}(r\,|\,\theta)$ and the conditional distribution function $F_{R\,|\,\Theta}(r\,|\,\theta)$, invertibility of the distribution function and continuity and strict positivity of the introduced mapping.

The mapping can be used to order multidimensional data in its domain by the levels of isoquantiles containing each datum. Formally: given a sample of n independent realizations of the random variable X, e.g. $X_i = (R_i, \Theta_i)$, $1 \le i \le n$, for each i there exists an unique isoquantile containing X_i. Denoting $X_{i,n}$ the realizations ordered by their respective isoquantile levels u_i, the maximum value is given by the point $X_{n,n}$ which belongs to the upper-level isoquantile with level $\max_{1\le i\le n} u_i$.

In practice, we'll assess the 1-level isoquantile on the grounds of the *asymptotical location property* as described in [18]. For large n, the furthest points from the origin lie near the $\frac{n-1}{n}$-level isoquantile. The 1-level isoquantile is then simply the edge of the bounded support.

Isoquantile estimation is performed by the non-parametric regression of [17, 18]. For the estimation we'll assume *homotheticity* of isoquantiles, e.g. for some strictly positive continuous function $v(\theta)$ and a distribution function G,

$$F_{R\,|\,\Theta}(r\,|\,\theta) = G\left(\frac{r}{v(\theta)}\right) \quad \text{for } r \in [0, v(\theta)].$$

The function $v(\theta)$ corresponds to the 1-level isoquantile and unambiguously describes the shape of all isoquantiles. The distribution of $\frac{\mathbf{x}}{v(\theta)}$ is spherically symmetric and it can be fully described by G on $[0,1]$.

We estimate $v(\theta)$ using radial regression:

$$w(\theta) = E(R\,|\,\Theta = \theta) = \int_0^{v(\theta)} \left(1 - G\Big(\frac{r}{v(\theta)}\Big)\right) dr = c\,v(\theta),$$

where c is the expected value of G. The estimate of the expected value of R given $\Theta = \theta$ describes the shape of 1-level isoquantile up to a multiplicative constant. This constant is chosen in a way that the estimated expected value shape $\hat{w}(\theta)$ contains the whole data after scaling:

$$\hat{v}(\theta) = \frac{\hat{w}(\theta)}{\hat{c}}, \quad \text{where } 1/\hat{c} = \max_{1 \le i \le n} \frac{R_i}{\hat{w}(\Theta_i)}$$

For practical estimation we'll use the two parametrizations introduced in [17] (hyperspherical) and [15] (unit sphere projection)[1]. For details and rationale see our previous work [15], where we develop the methodology using a simulation study, apply it on the NASDAQ and PX stock market indices and visually assess the isobar shapes.

3 The Fourier transform and power means

In this section we compute a relative efficiency measure from weighted frequency coefficient magnitudes of an isoquantile shape. Given a periodic sequence $f(j)$, $j \in \{0, \dots, m-1\}$, where m is the sequence length, the result of applying the discrete Fourier transform to $f(j)$ are complex frequency coefficients $F(k)$, $k \in \{0, \dots, m-1\}$, defined by

$$F(k) = \sum_{j=0}^{m-1} f(j)\, e^{-2\pi i jk/m}.$$

In our case we sample $f(j)$ from $\hat{w}(\theta)$,

$$f(j) = \hat{w}(2\pi j/m) \text{ for } m = 2^{11},\ j \in \{0, \dots, m-1\},$$

and compute the discrete Fourier transform using the `R` project for scientific computing [21]. The chosen value of m represents a sweet spot for our data sizes.

Since Brownian motion has independent and normally distributed increments, the isoquantile shape for ideally efficient markets converges to a circle. This is reflected by constant $w(\theta)$ and vanishing frequencies $F(k)$, $k > 0$. We take the magnitudes of frequency coefficients (normalized to cancel out the effect of different market volumes),

$$h_k = \frac{|F(k)|}{F(0)},$$

and average the remaining normalized coefficients using power (Hölder) means

$$M_p(\mathbf{h}) = \lim_{\alpha \to p} \left(\frac{1}{m} \sum_{k=1}^{m} h_k^\alpha\right)^{1/\alpha}.$$

These quantify the similarity of the isoquantile shape to a circle; values of $p = \{-\infty, 0, 1, 2, \infty\}$ correspond respectively to the minimum, geometric mean, arithmetic mean, root mean squared and maximum of h_k [2].

[1] For computations we make use of the nonparametric regression package `np` for the `R` project [14, 21].

[2] We've also tried the weighted median and other order statistics, but power means have proven to be more adequate for

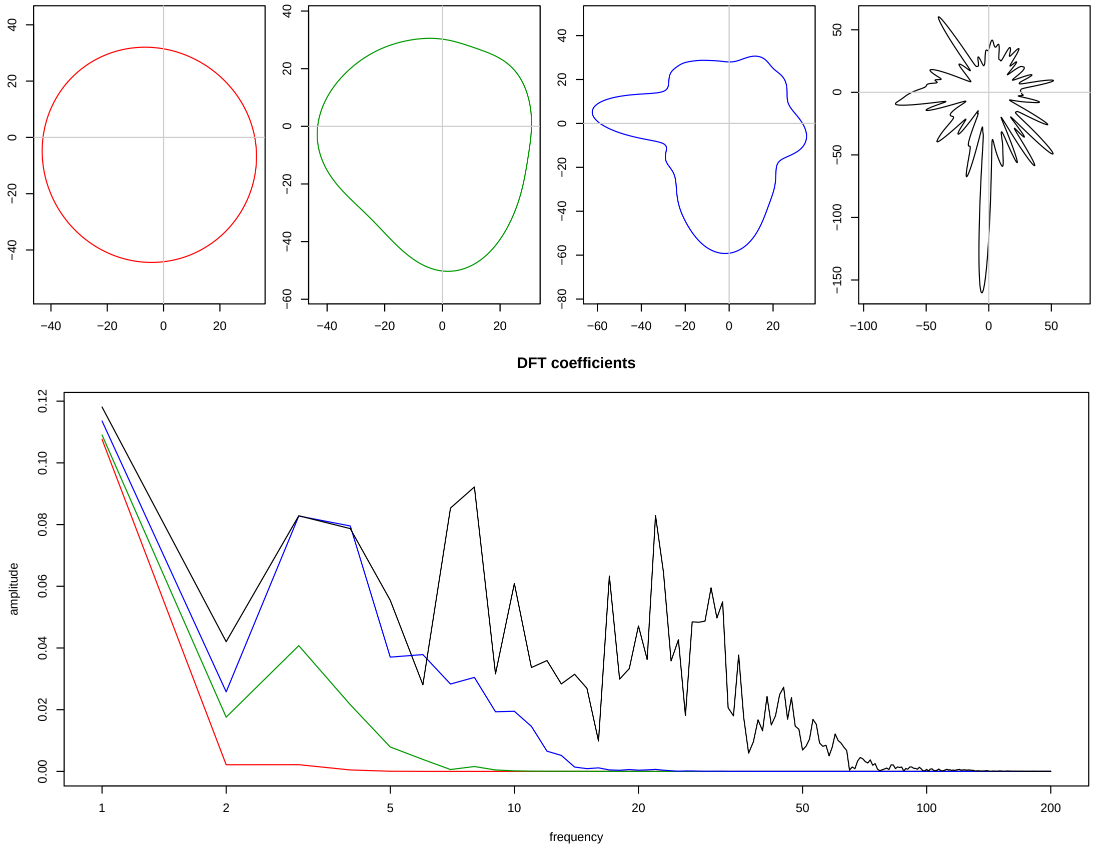

Figure 2: Examples of isoquantile shapes along with their lowest discrete Fourier coefficients. Observe that coefficients of smooth shapes vanish quicker than the coefficients of explosions.

4 Application

According to Osborne [20], the Efficient Market Hypothesis states that returns (the opening price subtracted from the closing price) of market indices in efficient markets behave like Brownian motion. In practice, this assumption is violated mostly by the periodic structure (day, week, quarter, year) of agent behaviour. Further bias reveals non-rational behaviour, non-zero information costs or delayed reactions.

Our data consists of weekly closing and opening prices for the past ten years (sample size around 500) obtained from the Reuters Wealth Manager service. We will follow the usual practice and compute weekly returns as differences between the opening price on Monday and Friday's closing price, which eliminates the influence of non-trading days (information obtained during weekends can't be reflected in prices).

In all isoquantile figures, the vertical axis will denote the current value of stock market index returns and the horizontal axis their lagged values (we've considered lags from one to sixteen weeks). As an example, Figure 2 shows examples of various isoquantile shapes for the assessed stock market indices and their discrete Fourier coefficients (from representative lags of the PX index). Additionally, the shapes for all sixteen lags for the most and least effecient index (ASPI and JSX Composite, respectively) can be inspected in Figures 5 and 6.

We've applied the methods on seven stock market indices. We'll shortly summarize them before presenting the results of visual examination:

- The All Share Price Index (ASPI): 241 Sri Lankan stocks of the Colombo Stock Exchange
- The BET Index: 10 Romanian stocks of the Bucharest Stock Exchange.
- The BUX Index: 13 Hungarian stocks of the Budapest Stock Exchange.

the evaluation of isoquantile shapes.

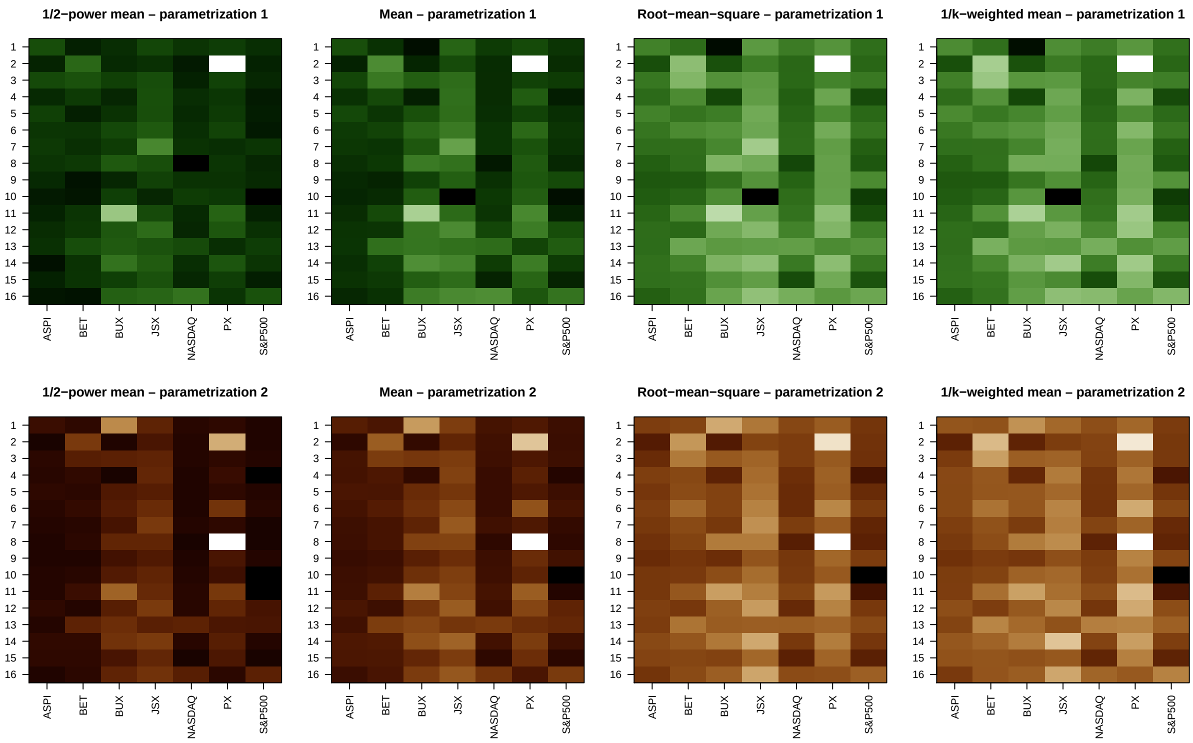

Figure 3: Measuring similarity to circles with different power means for lags 1–16. Darker cells correspond to smaller measure values, which indicates that the corresponding shape is closer to a circle.

- The JSX Composite Index: 379 Indonesian stocks of the Indonesia Stock Exchange.
- The NASDAQ Composite Index is comprised of 2742 stocks of the NASDAQ Stock Market.
- The PX Index is comprised of 14 stocks of the Prague Stock Exchange (only five of which are Czech).
- S&P500: 500 stocks traded on NYSE or NASDAQ.

Isoquantile shapes for the All Share Price Index, NASDAQ Composite Index and S&P500 are very close to circles. Small deviations from the circle shape can be observed in ASPI (lags 1, 3 and 5), NASDAQ (lags 12, 13 and 16) and in S&P500 (lags 12, 13 and 16). Deviations in the 13-week lag can be explained by the expected quarterly periodicity of agent behaviour. Based on visual examination, the underlying markets of ASPI, NASDAQ and S&P500 may follow the efficient market hypothesis.

Isoquantile shapes for BET differ from circles in multiple lags (of 2, 3, 4, 11 and 13 weeks): the deviations are distinctive, which suggests short-time dependency in the data.

The isoquantile shapes of the PX Index, BUX and JSX Composite Index deviate from circles constantly: for PX it's the longer lags of 4, 7, and 9–15 weeks, for BUX it's 3 and 5–16 weeks. Isoquantiles for the JSX Composite Index don't resemble a circle for any lag. Observing a systematic deviation from independence between current values and lagged ones, we can postulate that the efficient market hypothesis doesn't apply to markets described by these three indices.

After computing intermediate isoquantile scores for all lags using the procedure described in Section 2, we have to combine them into a single value. To this end we've chosen to use another power mean (possibly with a different p).

We've evaluated many power means for both stages augmented with various coefficient weighting schemes. For closer examination we've chosen $M_{1/2}$, M_1, M_2 and M_1 weighted by $1/k$ for the first stage (isoquantile shape evaluation), and M_0, $M_{1/2}$, M_1 for the second stage (lag combination). Finer quantization of p didn't produce measurable distincions and powers outside of these intervals were not robust enough for our purposes.

The results for both isoquantile parametrization methods can be seen in Figures 3 and 4; green refers to the first (hyperspherical) and brown to the second (unit sphere projection) parametrization. Figure 3

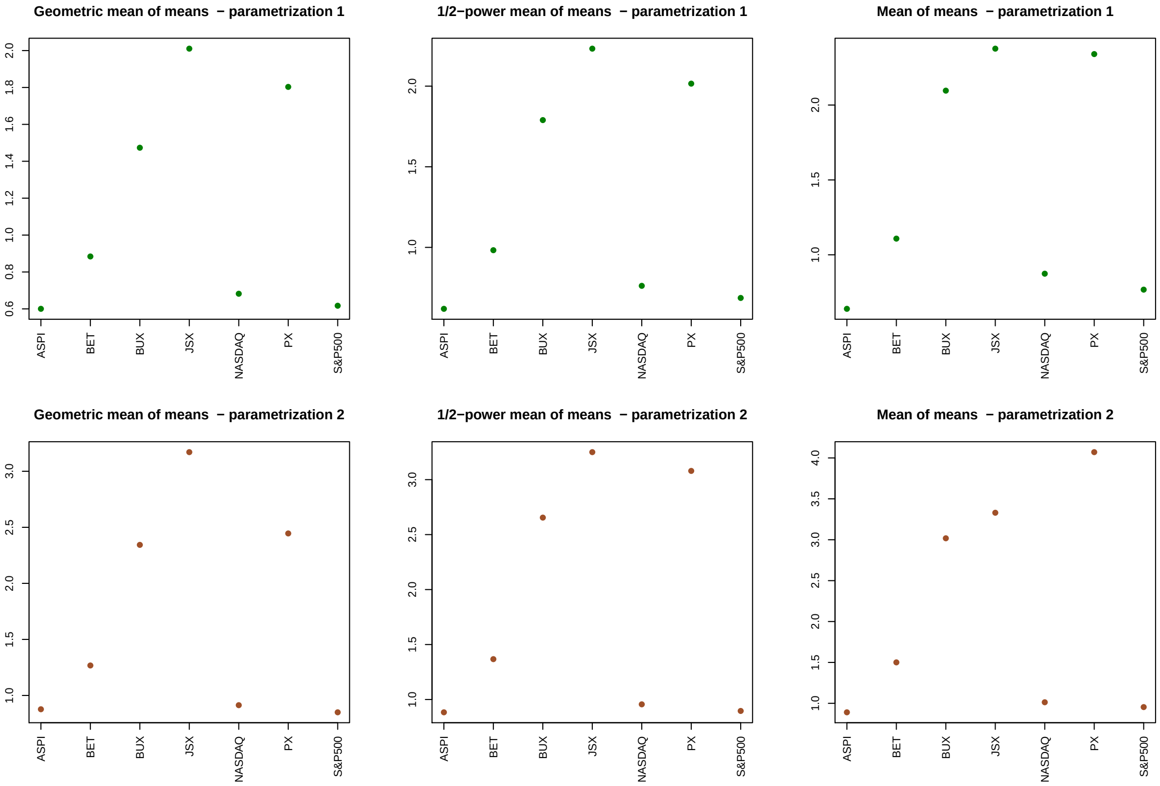

Figure 4: Measures for lags 1–16 combined into measures for the whole index, using different power means. Similarity to circles was computed via the arithmetic mean (M_1): smaller values correspond to circle isoquantiles.

depicts the intermediate scores for all combinations of lag and index—a darker shade represents a value closer to zero. Figure 4 lays out the relative combined scores for each index and method.

The first parametrization prefers rounder shapes; isoquantiles resemble a circle more often. The second parametrization follows the data shape better. Thus, for the final measure of stock market efficiency we've chosen the second parametrization, the arithmetic M_1 mean for combining normalized frequency coefficients and the $M_{1/2}$ power mean for mixing all sixteen lags, both means with uniform coefficient weights. This combination is robust to isoquantile shape estimation errors and closely follows visual shape assessment.

5 Conclusion

We've introduced a novel measure of stock market efficiency. This measure is inferred from frequency representations of homothetic isoquantile shapes estimated from lagged index returns. We've described the algorithm to compute this measure, rationalizing the choice of parameters for each step: nonparametric regression for the estimation of isoquantile shapes, computing a numerical representation of "circle-ness" of the shapes, and combining these representations from various lags to form a final measure.

We've evaluated seven indices and ascertained that the measure corresponds well to visual assessment: while the All Share Price Index, NASDAQ Composite Index and S&P500 follow the EMH closely, the PX, BUX and JSX Composite indices do not. The BET index exhibited some degree of dependency, but only in shorter lags. Our future research can now focus on quantitative comparisons to other methods of evaluating the Efficient Market Hypothesis and pinpointing the relative strenghts and weaknesses of the isoquantile efficiency measure.

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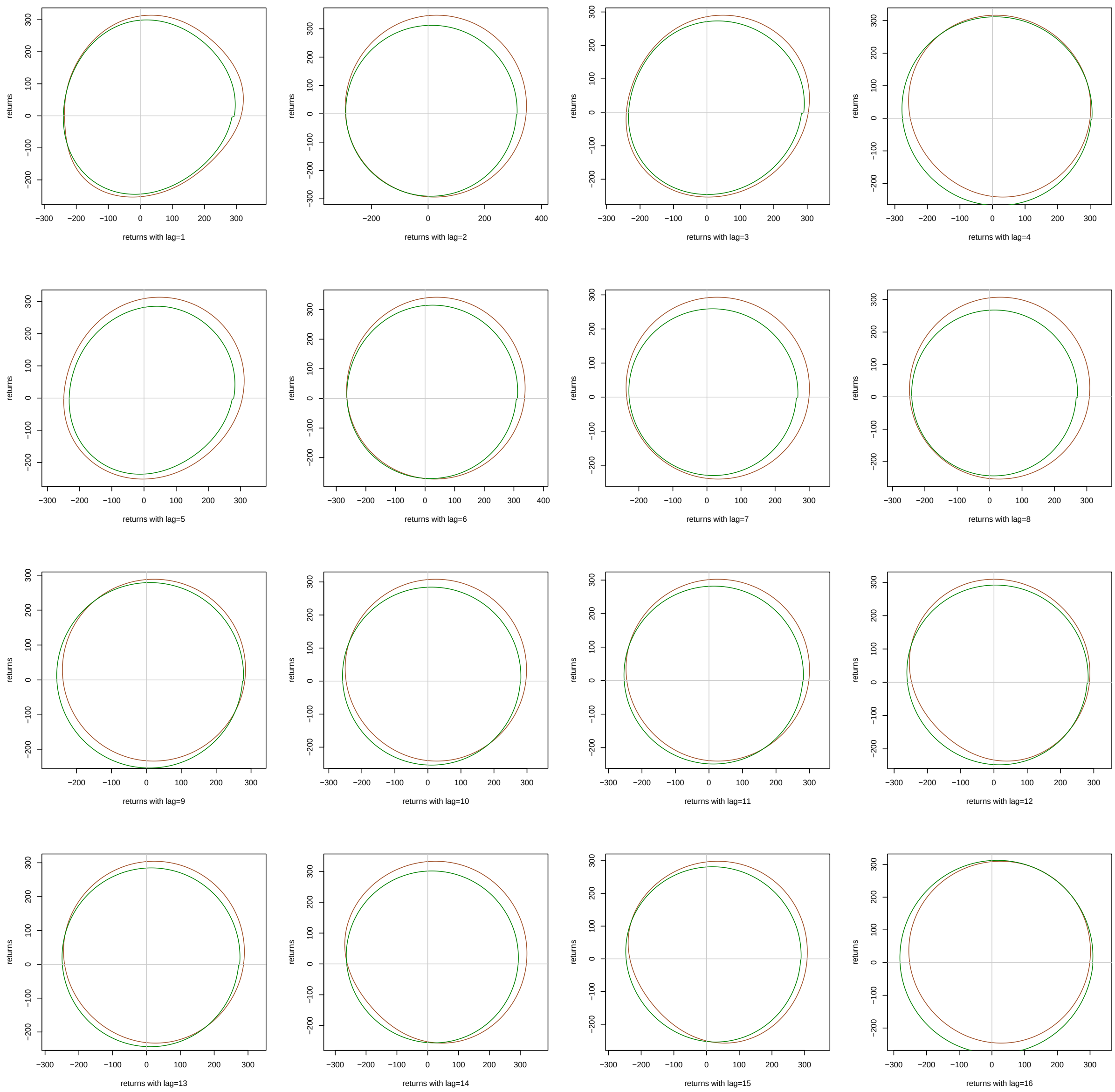

Figure 5: Estimated isoquantile shapes for the All Share Price Index, lags from one to sixteen weeks. Green curves are the first parametrization (hyperspherical), brown curves the second one (unit sphere projection). This is the most effective index from the data set (isoquantiles are closer to circles).

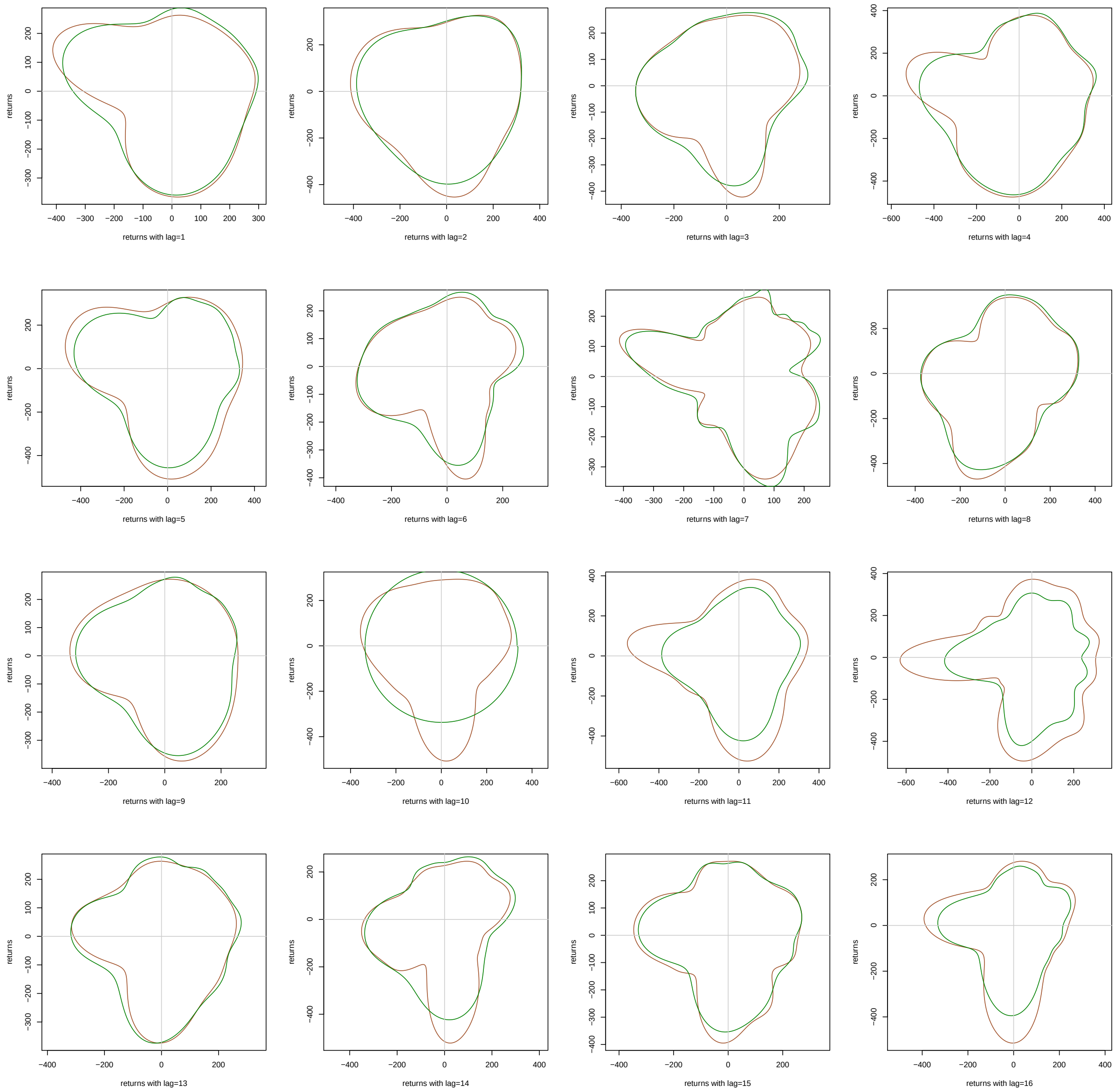

Figure 6: Estimated isoquantile shapes for the JSX index, lags from one to sixteen weeks. Green curves are the first parametrization (hyperspherical), brown curves the second one (unit sphere projection). This is the least effective index from the data set (isoquantiles differ from circles the most).

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