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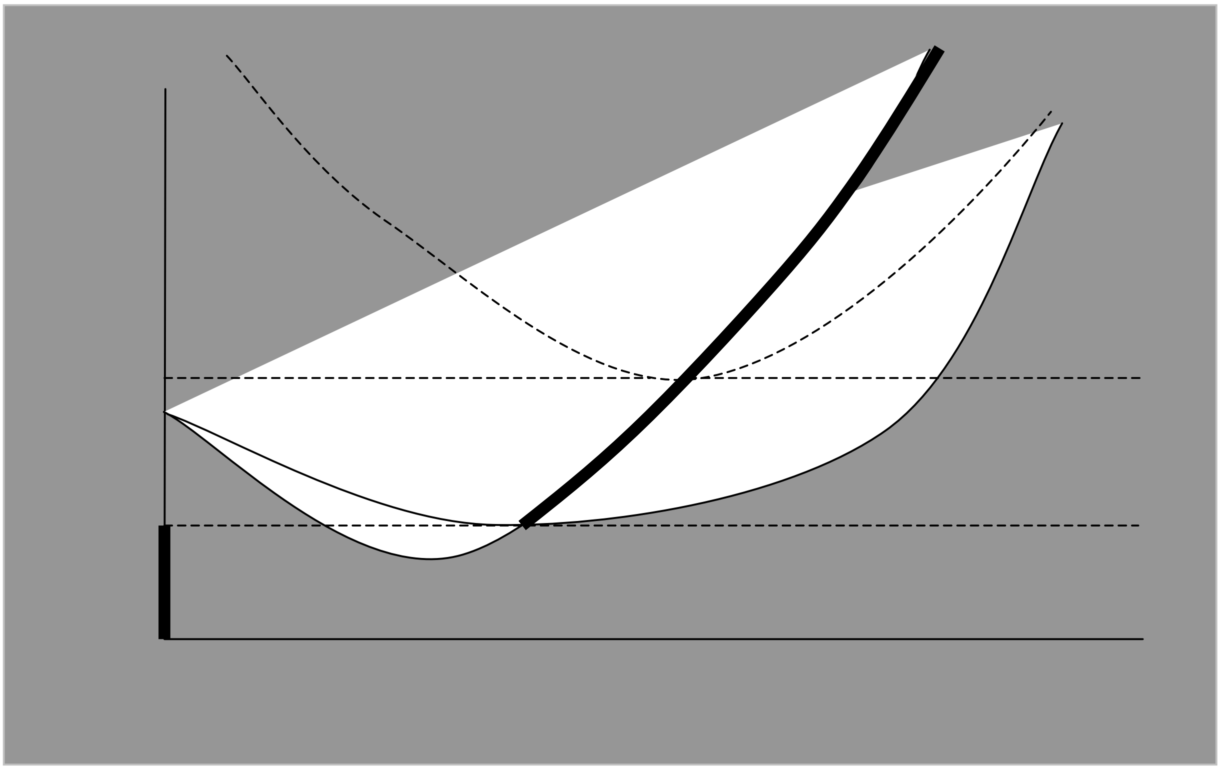

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Tolerable Intolerance: An Evolutionary Model

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Abstract

A cornerstone of liberal-democratic regimes is the right of free speech, granted even to non-liberals who manifestly oppose it. Communism and political Islamism are two primary examples of ideologies which are tolerated in spite of calls for the limits on the right of expression. Not surprisingly, it is often argued that a tolerant society needs laws preventing non-tolerant beliefs from attacking tolerance.

Yet, does intolerance necessarily prosper in a tolerant society, or is deemed to decay? To address the question, I build an evolutionary model of competing (political and/or religious) beliefs. In the model, individuals are assumed to gain from having beliefs. The gain may increase with intolerance of the belief (premium). High intolerance, however, makes strong believers fragile in a society of tolerant people.

Having examined evolutionarily stable states in two specifications, I demonstrate that (for any positive premium) heterogeneity cannot prevent intolerant beliefs from spreading out. A sufficiently small increase in intolerance, when premium exceeds losses from fragility, allows intolerance to spread. Intolerance is vulnerable only as long as the premium is non-positive. This finding can also be interpreted as follows: unless fundamentalist confessions are proved to be vital for individual human existence (positive premium), a tolerant society needs no intervention to preserve tolerance.

Keywords: Evolutionary stability, Religion, Political ideology

JEL Classification: A13, C79, Z10

1. Introduction

Man needs to confess in order to make life meaningful. The subject of confession (or adoration) can be religious, ideological, or social (group identity). Not surprisingly, a large variety of confessions is present in liberal societies. We may broadly classify them by the strength of beliefs. The strength is to be interpreted as the range of personal activities regulated by the confession; whilst some confessions regulate only Sunday morning time, and marriage, some take all wealth, free time, as well as your children, and some even demand life to be sacrificed.

All confessions are costly, for the essence of confession is to defend unverifiable beliefs. The stronger the confession is (the more regulated life), the higher cost is borne. Because of trade-off between gains and losses from various confession strengths, we can identify an "ideal confession", with maximum total utility.

However, confessions differ also in the level of intolerance. Intolerance brings two effects. First, intolerance may provide extra feelings, which can be seen on as utility premium, so in effect the "ideal confession" level may increase comparing to the ideal confession in the case of full tolerance. The second effect goes rather differently: intolerant believers suffer extra utility loss from living in a population of unequally strong believers. All in all, intolerant believers are disadvantaged in heterogeneous societies, but may have a comparative advantage in a homogeneous society.

Is this intuition applicable to real-world phenomena? I suppose the general setup is acceptable on microeconomic grounds (for further defense see ensuing Section 2). It generally covers a wide range of confessions. First of all, it describes political ideologies as secular counterparts of religious confessions. A lot of political ideologies are intolerant and openly hostile to liberal democracy, and consist of strong believers. In Eastern Europe, for instance, Communist parties overtly celebrate totalitarian predecessors. In Germany, neo-Nazists commemorate legacy of "heroic Wehrmacht soldiers". The setup further describes religious fundamentalists with political aspiration, such as Islam fundamentalists, who don't hesitate to reveal intolerance. In Western Europe for example, some Muslim clerics teach children that Western society is a devil-governed society.

The analytical question is whether tolerance can survive in the world of intolerant believers. In an evolutionarily setup, it happens in cases when tolerant believers exploit their difference from intolerant believers. This difference diminishes utility of intolerant believers. Section 3 reveals how the difference can be exploited. On the other hand, Section 4 proves that in the case of (any) premium from intolerance, tolerance cannot survive. In conclusion, I discuss possible interpretations of the premium; I argue we have no reason to assume a positive premium for fundamentalist confessions.

2. Assumptions

Confessions

Two variables describe a confession $\mathbf{C} = (s_C, t_C)$ – strength s_C and intolerance t_C, where $s_C \in \langle 0, s^{max} \rangle$ and $t_C \in \langle 0, s_C \rangle$. The minimal strength ($s_C = 0$) denotes total skepticism. The maximum s^{max} is set as there is obviously a limit of the confession strength (the maximum value can be achieved only by a "saint"). Inbetween the bounds, anything is possible – one can be silent Buddhist, in-born Catholic, persuaded Communist or suicidal attacker fighting jihad.

The minimum in $t_C = 0$ denotes total tolerance, i.e. confession is expressed and enjoyed only privately. In contrast, high t denotes a "social" confession, where you need public for

salvation (Communism constituting a classic case). Intolerance is limited by strength ($t_C < s_C$) since we rarely observe intolerant liberal confessions, whereas cases of high intolerance are frequent among the more intensive believers.
The technical question is how to view a choice between confessions. I think the best approach here is evolutionary game theory. Let us have an initial distribution of confessions in population. This distribution is not necessarily stable; people change confession by meetings with other people. Like in biological evolution, high payoff means high probability of survival, and the survival of confession depends on whether utility from meetings is relatively higher than the average utility in the population. Only those confessions which do perform well can survive.
How can we interpret this standard evolutionary notion for confessions? Ex ante, people cannot know what alternative confessions bring, since the choice of confession is beyond the boundaries of reason (unverifiable beliefs). But in each period, they interact with randomly assigned partner. The interaction may represent any opportunity to compare confession and adjust it, such as discussion with friends, watching TV, or time shared in co-workers. Based on their utility relative to the others, they adjust their confession.

Evolutionary game

Assume a society populated by a big number of players, largely exceeding a number of confessions. An evolutionary game is a repeated game with infinite number of rounds, where players meet randomly in bilateral interactions. High payoff means high probability of repeating the confession in the next round. In the paper, dynamics is not explored; we will only use the static (steady state) concept of evolutionary stability.
A population is said to be evolutionarily stable as long as mutants cannot invade the population by earning higher or equal payoff. Maynard-Smith (1982) established *evolutionary stable strategy* (henceforth ESS) as a strategy $\sigma^* \in \Delta S$[1] satisfying two conditions for all $\sigma \neq \sigma^*$:

Condition 1 (Nash equilibrium): $U_1(\sigma^*, \sigma^*) \geq U_1(\sigma, \sigma^*)$
Condition 2 (Stability property): $U_1(\sigma^*, \sigma^*) = U_1(\sigma, \sigma^*) \Rightarrow U_2(\sigma, \sigma^*) > U_2(\sigma, \sigma)$

Utility

The crucial part is to establish utility functions in these interactions. Suppose we have symmetric bilateral interactions:

$$U_i = U_i(s_i, t_i, s_{-i}) \qquad i \in \{1, 2\}$$

To make the model as realistic as possible, let us impose the following requirements:

1. **Confession strength is a good with diminishing marginal utility.**

In order to distinguish between pros and cons of stronger beliefs, let us define benefits of confession (A) and costs of confession (B), such that:

$$U = U(A, B), \qquad \text{where } \frac{\partial U}{\partial A} > 0, \frac{\partial U}{\partial B} < 0 .$$

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[1] *Note* S *is a finite set of pure strategies, and* ΔS *the set of probability measures on* S. *This allows to interpret any element* $\sigma \in \Delta$S *as a mixed strategy.*

Now, a stronger confession brings extra utility, and the marginal utility diminishes.

$$\frac{\partial \mathrm{A_i}}{\partial s_i} \geq 0, \frac{\partial^2 \mathrm{A_i}}{\partial s_i^2} \leq 0$$

2. Strength is costly.

Political ideologies, for instance, are tremendous shortcuts of policy alternatives often with drastic consequences.

$$\frac{\partial \mathrm{B_i}}{\partial s_i} > 0, \frac{\partial^2 \mathrm{B_i}}{\partial s_i^2} \geq 0$$

Furthermore, the function U (A, B) shall be restricted such that a trade-off between A and B gives a unique $s_i^{opt}(t_i)$ for a homogeneous population ($s_{-i} = s_i$).

$$s_i^{opt} = \arg\max \mathrm{U_i}(\mathrm{A_i}, \mathrm{B_i})$$

$$\frac{\partial \mathrm{U_i}(s_i^{opt}, s_i^{opt}, t_i)}{\partial s_i} = 0$$

$$\frac{\partial \mathrm{U_i}(s_i^{opt}, s_i^{opt}, t_i)}{\partial \mathrm{A_i}} \frac{\partial \mathrm{A_i}}{\partial s_i} = -\frac{\partial \mathrm{U_i}(s_i^{opt}, s_i^{opt}, t_i)}{\partial \mathrm{B_i}} \frac{\partial \mathrm{B_i}}{\partial s_i}$$

3. Intolerant confessions attract stronger believers.

Intolerant believers can have a less diminishing marginal utility:

$$\frac{\partial^2 \mathrm{A_i}}{\partial s_i \partial t_i} \geq 0 \qquad \text{or} \qquad \frac{\partial s_i^{opt}(t_i, s_{-i})}{\partial t_i} \geq 0$$

4. Intolerant confessions suffer in meetings with differently strong confessions.

This is to capture two effects. The first is *persuasion effect*: to face someone strikingly different means to lose as it is obviously unpleasant to meet someone believing in something completely different. The second is the *political success effect*; for confession with political ambitions, widespread support is a must.

This loss is zero for $t_i = 0$ – absolutely tolerant believers neither care about confession of the others, nor exert political influence (their confessions are private anyway).

$$s_i < s_{-i}: \qquad \frac{\partial \mathrm{U_i}}{\partial s_{-i}} \leq 0, \frac{\partial^2 \mathrm{U_i}}{\partial s_{-i} \partial t_i} < 0$$

$$s_i > s_{-i}: \qquad \frac{\partial \mathrm{U_i}}{\partial s_{-i}} \geq 0, \frac{\partial^2 \mathrm{U_i}}{\partial s_{-i} \partial t_i} > 0$$

3. Benchmark case: no premium for intolerance

For clarity of exposition, consider a special case when only 3 strengths with 3 levels of tolerance are possible (in total 9 confessions).

$$s \in \{0, 1, 2\}$$
$$t \in \{0, 1, 2\}$$

Now we can express the payoffs (utilities) in the strategic form. Since utility depends on three variables, the 3-dimensional set of payoffs is decomposed as follows:

Player 1 (choosing horizontally):

Table 1: $U_1(s_1, s_2, 0)$

s_1/s_2	0	1	2
0	5	5	5
1	4	4	4
2	3	3	3

Table 2: $U_1(s_1, s_2, 1)$

s_1/s_2	0	1	2
0	4	3	2
1	4	5	4
2	2	3	4

Table 3: $U_1(s_1, s_2, 2)$

s_1/s_2	0	1	2
0	3	2	1
1	3	4	3
2	3	4	5

Player 2 (choosing vertically): since the game is symmetric, his payoffs just mirror payoffs of Player 1.

Table 4: $U_2(s_1, s_2, 0)$

s_1/s_2	0	1	2
0	5	4	3
1	5	4	3
2	5	4	3

Table 5: $U_2(s_1, s_2, 1)$

s_1/s_2	0	1	2
0	4	4	2
1	3	5	3
2	2	4	4

Table 6: $U_2(s_1, s_2, 2)$

s_1/s_2	0	1	2
0	3	3	3
1	2	4	4
2	1	3	5

This benchmark case satisfies all requirements; Requirement 3 is satisfied with equality, i.e. no premium utility is given to intolerant believers in a homogeneous society. Pure-strategy Nash equilibria can be revealed in Table 7 as a comprehensive strategic representation of the game:

Table 7: U_1 (t_1, s_1); U_2 (t_2, s_2)

$t_1, s_1/t_2, s_2$	0, 0	0, 1	0, 2	1, 0	1, 1	1, 2	2, 0	2, 1	2, 2
0, 0	**5, 5**	5, 4	5, 3	5, 4	5, 4	5, 2	5, 3	5, 3	5, 3
0, 1	4, 5	4, 4	4, 3	4, 3	4, 5	4, 3	4, 2	4, 4	4, 4
0, 2	3, 5	3, 4	3, 3	3, 2	3, 4	3, 4	3, 1	3, 3	3, 5
1, 0	4, 5	3, 4	2, 3	4, 4	3, 4	2, 2	4, 3	3, 3	2, 3
1, 1	4, 5	5, 4	4, 3	4, 3	**5, 5**	4, 3	4, 2	5, 4	4, 4
1, 2	2, 5	3, 4	4, 3	2, 2	3, 4	4, 4	2, 1	3, 3	4, 5
2, 0	3, 5	2, 4	1, 3	3, 4	2, 4	1, 2	3, 3	2, 3	1, 3
2, 1	3, 5	4, 4	3, 3	3,3	4, 5	3, 3	3, 2	4, 4	3, 4
2, 2	3, 5	4, 4	5, 3	3, 2	4, 4	5, 4	3, 1	4, 3	**5, 5**

The maximal payoff in the matrix achieves M = 5. Three pure-strategy Nash equilibria exist (all symmetric), which are efficient in a sense of providing maximal payoffs. Can we have any mixed-strategy NE with maximal payoffs?

Proposition 1. *In benchmark case, only pure-strategies achieve maximal payoff.*

Proof. In a symmetric evolutionary game, we can consider only symmetric strategies. We look for a symmetric mixed strategy (**p**, **q**), where vector $\mathbf{p} = (p_1, p_2, p_3)$ denotes likelihood p_j of playing $t = j$ ($j \in \{0, 1, 2\}$) and $\mathbf{q} = (q_1, q_2, q_3)$ denotes the likelihoods q_k of playing $s = k$ ($k \in \{0, 1, 2\}$).

$$\mathbf{p} = (p_1, p_2, p_3)$$
$$\mathbf{q} = (q_1, q_2, q_3)$$
$$0 \le p_{\mathrm{j}}, q_k \le 1$$
$$\textstyle\sum_j p_j = 1; \sum_k q_k = 1$$

For convenience, let us denote an expected utility conditional on the fixed action of the counter-player s_2 as $U_c(s_2)$.

$$U_c(s_2) \equiv E[U(s_2)] = \sum_{j=1}^{3} \sum_{k=1}^{3} p_j q_k \, U_1(s_1^k, t_1^j, s_2)$$

Expected utility of the mixed strategy is as follows:

$$E[U(\mathbf{p}, \mathbf{q})] = \sum_{m=1}^{3} q_m \, U_c(s_2^m)$$

We know:

a) $U_i \le M$.
b) $U_c(s_2)$ is a weighted average of values of U_1 (s_1^k, t_1^j, s_2)
c) $U_c(s_2) = M$

Obviously, a weighted average of a set of values can achieve maximum from the set only and only if all values equal the maximum value.

$$U_c\,(s_2) = M \quad \Leftrightarrow \quad \forall\, j, k{:}\ p_j > 0,\ q_k > 0{:}\ U_1\,(s_1^{\,k}, t_1^{\,j}, s_2) = M$$

Can we guarantee $U_c(s_2) = M$ for non-pure strategies? By inspection of Tables 1-3 above, we observe M can be received only from the following actions:

$$A_0 \equiv (s, t) = (0, 0)$$
$$A_1 \equiv (s, t) = (1, 1)$$
$$A_2 \equiv (s, t) = (2, 2)$$

Table 8: Utilities from interaction of mixed-strategy actions

	A_0	A_1	A_2
A_0	5	5	5
A_1	4	5	4
A_2	3	4	5

Now, by mixing these three actions, can we make the weighted average equal maximum? No, since if we mix A_1 with either A_0 or A_2, we get payoff 4. If we mix A_2 with A_0 or A_1, we get payoffs 3 (respectively 4). This implies that none of A_1 nor A_2 is susceptible to mixing, and we can have only pure strategies achieving payoff M = 5. Q.E.D.

Proposition 2. *In benchmark case, only* $(s, t) = (0, 0)$ *is ESS.*

Proof. Recall we have identified three pure strategies as Nash equilibria – A_0, A_1 and A_2. As Nash equilibria, they satisfy Condition 1 (see Assumptions in Section 2). Does any of them satisfy also Condition 2? From Table 7, we can extract these three strategies and see the payoffs of interactions in Table 9.

Table 9: Utilities from interaction of pure strategies

	A_0	A_1	A_2
A_0	5, 5	5, 4	5, 3
A_1	4, 5	5, 5	4, 4
A_2	3, 5	4, 4	5, 5

Table 10: Stability property examined

σ^*	σ	$U_1\,(\sigma, \sigma^*)$	$U_2\,(\sigma, \sigma^*)$	$U_2\,(\sigma, \sigma)$	Stability	Overall
A_0	A_1	4			Yes	Yes
A_0	A_2	3			Yes	
A_1	A_0	5	4	5	No	No
A_1	A_2	4			Yes	
A_2	A_0	5	3	5	No	No
A_2	A_1	4			Yes	

I have checked stability property in pair-wise comparisons (see results in Table 10). Only A_0 appears to satisfy the stability condition for all potential “invaders”. Thus, $A_0 = (0, 0)$ is the only ESS. Q.E.D.

4. Utility premium from intolerance

The benchmark case illuminated how tolerant attitudes survive in a society where intolerance attempts to invade tolerance. Whenever maximum utilities for all levels of intolerance are equal (constant s_i^{opt}), full tolerance has an advantage of invading *any* symmetric Nash-equilibrium strategy with above-zero intolerance. This is why tolerance is self-enforcing even in the face of intolerant "mutations".

In another model, we shall consider cases when the intolerant strategies receive utility premium, and thereby constitute a threat to tolerant strategies. Can the threat be eliminated?

Assumptions

Assume the following utility function and check whether it conforms to requirements set in Section 2.

$$\mathrm{A_i} = (t_i + 1)\sqrt{s_i}$$

$$\mathrm{B_i} = s_i$$

$$\mathrm{U_i} = \mathrm{A} - \mathrm{B} - t_i \left| s_i - s_{-i} \right| = (t_i + 1)\sqrt{s_i} - s_i - t_i \left| s_i - s_{-i} \right|$$

a) General requirement

$$\frac{\partial \mathrm{U}}{\partial \mathrm{A}} = 1 > 0, \frac{\partial \mathrm{U}}{\partial \mathrm{B}} = -1 < 0$$

b) Marginal utility from benefits

$$\frac{\partial \mathrm{A_i}}{\partial s_i} = \frac{t_i + 1}{2\sqrt{s_i}} \geq 0, \frac{\partial^2 \mathrm{A_i}}{\partial s_i^2} = \frac{-t_i - 1}{4\sqrt{s_i^3}} \leq 0$$

c) Costly strength

$$\frac{\partial \mathrm{B_i}}{\partial s_i} = 1 > 0, \frac{\partial^2 \mathrm{B_i}}{\partial s_i^2} = 0$$

d) Unique $s_i^{opt}(t_i)$ for a homogeneous population

$$\frac{\partial \mathrm{U_i}}{\partial s_i} = \frac{t_i + 1}{2\sqrt{s_i^{opt}}} - 1 = 0 \Rightarrow s_i^{opt} = \frac{(t_i + 1)^2}{4}$$

e) Intolerance attracts stronger believers.

$$\frac{\partial^2 \mathrm{A_i}}{\partial s_i \partial t_i} = \frac{1}{2\sqrt{s_i}} \geq 0 \qquad \text{or} \qquad \frac{\partial s_i^{opt}(t_i, s_{-i})}{\partial t_i} = \frac{t_i + 1}{2} \geq 0$$

f) Intolerant believers suffer in heterogeneous societies.

$s_i < s_{-i}$: $\quad \mathrm{U_i} = (t_i + 1)\sqrt{s_i} - s_i + t_i(s_i - s_{-i}) = (t_i + 1)\sqrt{s_i} - s_i + t_i s_i - t_i s_{-i}$

$$\frac{\partial \mathrm{U_i}}{\partial s_{-i}} = -t_i \leq 0, \frac{\partial^2 \mathrm{U_i}}{\partial s_{-i} \partial t_i} = -1 < 0$$

$s_i > s_{-i}$: $\quad \mathrm{U_i} = (t_i + 1)\sqrt{s_i} - s_i - t_i(s_i - s_{-i}) = (t_i + 1)\sqrt{s_i} - s_i - t_i s_i + t_i s_{-i}$

$$\frac{\partial \mathrm{U_i}}{\partial s_{-i}} = t_i \geq 0, \frac{\partial^2 \mathrm{U_i}}{\partial s_{-i} \partial t_i} = 1 > 0$$

In hunt for evolutionary stability

A sufficient condition for ESS is strict dominance (Condition 2 does not apply for strict dominance); strict Nash equilibria as the most easily recognizable evolutionarily stable states. To identify them, strictly dominated pure strategies for each player must be eliminated. Doing this, we receive best-response functions whose intersections are strict Nash equilibria. If best-response functions intersect in one point, we have only one pure-strategy strict Nash equilibrium and the hunt for ESS can cease. If best-response functions intersect in more than one point, evolutionary stability must be examined by checking stability property, and computing polymorphic equilibria.

Since selection of confession is out of rational choice and subject to evolutionary development, players cannot link strategies from different rounds. Each round of the game is thus a bilateral strategic game of players i and $-i$. The strategy $\mathbf{C_i} = (s_i, t_i)$ interacts with $\mathbf{C_{-i}} = (s_{-i}, t_{-i})$, and the payoffs are given as:

$$[\mathrm{U_i}, \mathrm{U_{-i}}] = [\mathrm{U_i}(\mathbf{C_i}, \mathbf{C_{-i}}), \mathrm{U_{-i}}(\mathbf{C_i}, \mathbf{C_{-i}})]$$
$$0 \leq t_i \leq s_i \leq s^{max};\ 0 \leq t_{-i} \leq s_{-i} \leq s^{max}$$

The game is four-dimensional (R^4), because single strategies are two-dimensional ($\mathrm{R} \times \mathrm{R}$). Fortunately, $\mathrm{U_i}$ is independent on t_{-i} from definition, so the space of possible payoffs $\mathrm{U_i}$ can be reduced to three dimensions, given by $s_i \times t_i \times s_{-i}$. This allows us to seek best response functions $\mathbf{C_i}^*$ and $\mathbf{C_{-i}}^*$:

$$\mathbf{C_i}^* (s_{-i}) = \arg\max \mathrm{U_i} (s_{-i})$$
$$\mathbf{C_{-i}}^* (s_i) = \arg\max \mathrm{U_{-i}} (s_i)$$

As the game is symmetric, suffice to find only $\mathbf{C_i}^*$.

Eliminating dominated strategies

For the utility function (payoff) includes sign function with the difference $\Delta = s_i - s_{-i}$ in the argument, we shall distinguish between two cases.

Case 1: $\Delta \geq 0 \Rightarrow s_i \geq s_{-i}$

Case 2: $\Delta < 0 \Rightarrow s_i < s_{-i}$

We search for not one, but two best response sub-functions, for each range subset of s separately:

$$\mathbf{C_{i,1}} (s_{-i}) = \arg\max \mathrm{U_i} (s_{-i});\ s_i \geq s_{-i}$$
$$\mathbf{C_{i,2}} (s_{-i}) = \arg\max \mathrm{U_i} (s_{-i});\ s_i < s_{-i}$$
$$\mathrm{U_i} [\mathbf{C_{i,1}} (s_{-i}), s_{-i}] > \mathrm{U_i} [\mathbf{C_{i,2}} (s_{-i}), s_{-i}] \quad \Rightarrow \quad \mathbf{C_i}^*(s_{-i}) = \mathbf{C_{i,1}} (s_{-i})$$
$$\mathrm{U_i} [\mathbf{C_{i,2}} (s_{-i}), s_{-i}] \geq \mathrm{U_i} [\mathbf{C_{i,1}} (s_{-i}), s_{-i}] \quad \Rightarrow \quad \mathbf{C_i}^*(s_{-i}) = \mathbf{C_{i,2}} (s_{-i})$$

For convenience, let us hereafter use the following substitution:

$$s \equiv s_i,\ t \equiv t_i,\ z \equiv s_{-i},\ \mathbf{C_1} \equiv \mathbf{C_{i,1}} (s_{-i}),\ \mathbf{C_2} \equiv \mathbf{C_{i,2}} (s_{-i}),\ \mathrm{U} \equiv \mathrm{U_i}$$

Case 1: Identification of $\mathbf{C_1}$

$$\text{U} = (t+1)(\sqrt{s} - s) + tz$$

The best way to find optimum is to find optimal t^* for each s, and then find optimal s^* as the maximum of utility constrained to t^*. The first order necessary condition reveals:

$$\frac{\partial \text{U}}{\partial t} = \sqrt{s} - s + z; s \geq z$$

Denote s^{crit} to be the critical point where the function of slope equals zero (i.e. the root of given polynomial function). The feasible critical point is given as follows:

$$s^{crit} = z + \frac{1}{2} + \sqrt{z + \frac{1}{4}}; s^{crit} \geq z$$

In order to check whether the slope is negative or positive for values below s^{crit} (above respectively), we can derivate:

$$\text{U}_{\text{s,t}} \equiv \frac{\partial^2 \text{U}}{\partial s \partial t} = \frac{1}{2\sqrt{s}} - 1$$

$$s > \frac{1}{4} \Rightarrow \text{U}_{\text{s,t}} < 0$$

$$s < \frac{1}{4} \Rightarrow \text{U}_{\text{s,t}} > 0$$

In close proximity to s^{crit} from the left (note that min s^{crit} = 1), the slope always decreases ($\text{U}_{\text{s,t}}$ is negative for s^{crit} and in close neighborhood), approaching zero in s^{crit}. Therefore, the slope must be positive. For s = ¼, the slope is in maximum, and declines to zero at $s = z = 0$. All in all, for all values below s^{crit}, the slope is positive. In consequence, the optimal t^* must be (for given s):

1. Below critical s^{crit}: $z \leq s \leq s^{crit} \Rightarrow \frac{\partial \text{U}}{\partial t} \geq 0 \Rightarrow t^* = s$
2. Above critical s^{crit}: $s > s^{crit} \Rightarrow t^* = 0$

Now, we receive s^* by maximizing utility constrained to t^* and z.

1. Below critical value (s^*_{bc})

   $z \leq s \leq s^{crit} : U = \sqrt{s}(s+1) + s(z-1) - s^2$

   In this interval, the utility grows for values closely above z, but only up to a specific s^o. There it reaches the maximum and begins to decline. The optimum s^o can be described implicitly:

   $3s^O - 2\sqrt{s^O}(2s^O + 1 - z) + 1 = 0$

   Two properties of s^o:

   A) $z \geq 0$: $s^o < s^{crit}$ (s^o always dominates s^{crit})

   B) $z > 0$: $s^o > z$ (s^o always dominates z)

$$s^*_{bc} = s^o$$

2. Above critical value (s^*_{ac})
   $s > s^{crit} \geq 1: U = \sqrt{s} - s$
   Here, the utility is always decreasing (recall the F.O.C. holds for s = ¼), thus:

$$s^*_{ac} = s^{crit}$$

Now, since s^o always dominates s^{crit} (see Property A), we have s^* = max (s^o, z) and $t^* = s^*$. In short:

$$\mathbf{C_1} = [s^o, s^o]$$

Case 2: Identification of C_2

$\text{U} = (t+1)\sqrt{s} - s + ts - tz$

Let us examine the first-order condition and the second-order condition.

$$\frac{\partial \text{U}}{\partial t} = \sqrt{s} - s + z; s < z \qquad \frac{\partial^2 \text{U}}{\partial t \partial s} = \frac{1}{2\sqrt{s}} + 1 > 0$$

From the second-order condition, we can see the permanently increasing slope of the first derivative (up to zero in s^{crit}). Among feasible s, the first derivative therefore either:

a) permanently grows (corner solution), or
b) permanently falls (corner solution), or
c) declines and then grows, i.e. there is a minimum (corner solution).

As a result, we need to investigate only corner solutions (for each s). One is $t = 0$. Suppose we maximize $\text{U} = \sqrt{s} - s$ for these corner solution; by maximization we get s = ¼.

$$\text{U}\,(¼, 0, z) = ¼$$

We can call this value the *reservation utility*. Now let us use a fact stating that U (z, z, z) > ¼ for all $z > 0$ (not proved here). This states that **R** = (¼, 0) may beat **S** = (s, z), but it is always worse than **Z** = (z, z).

$$\mathbf{C_2} = [z, z]$$

Global best-response function and strict Nash equilibria

Now we need to determine between $\mathbf{C_1} = [s^o, s^o]$ and $\mathbf{C_2} = [z, z]$. We know that s^o dominates all strategies $s \in \langle z, s^{crit} \rangle$ (see Property B) above). We also know that s^o is limited by $s \leq s^{max}$, hence:

$$\mathbf{C}^* = [\min(s^o, s^{max}), \min(s^o, s^{max})]$$

Obviously, such best response function in a bilateral game leads to a sole symmetric strict Nash equilibrium (N):

$$\text{N} = (s^{max}, s^{max}, s^{max}, s^{max})$$

Evolutionary stability

Strict dominance is sufficient for Nash equilibrium to be evolutionarily stable. There are nevertheless three more cases when further ESS may exist:

1. Weakly-dominant Nash equilibria (non-strict monomorphic ESS).
2. Multiple pure-strategy Nash equilibria (polymorphic ESS).
3. Asymmetric payoff exceeds payoff in strict Nash equilibrium (polymorphic ESS).

None of the three conditions are satisfied here. We end by stating the evolutionary stability in the game is given by total dominance of strong and intolerant confessions $\mathbf{C}^{\mathbf{max}} = (s^{max}, t^{max})$.

5. The extra chance for tolerance

The case with positive premium justifies policy intervention in order to preserve tolerance. However, we can empirically observe that intolerance does not dominate tolerant societies. The question is: which assumption in our simplified framework is most likely violated?

a) We are still in the course of evolution (*out-of-equilibrium state*).
b) *Confessions must be recognizable as different*. Only very different confessions are on offer, so one cannot make infinitesimal adjustments. If one can choose only from z and $z + a$, where $z + a >> s^{crit}$, then keeping z may be the best option. In that instance, we can find polymorphic mixed strategy, i.e. an equilibrium proportion of various confessions.
c) The choice of confessions is from discrete variables also because *to abandon confession by small deviation can be penalized* by special cost (e.g. heresy punishment). This cost prevents confession from being invaded by largely similar mutations.

6. Conclusion

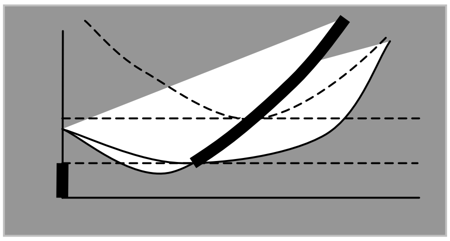
By comparing benchmark case and the case with positive premium, we found strikingly different outcomes. In the former model, heterogeneity made all intolerant beliefs vulnerable. In stark contrast, heterogeneity was not an obstacle for intolerant beliefs in the latter model. This difference points to the crucial importance of Requirement 3 (i.e. sign of utility premium for strong beliefs in intolerant homogenous societies).

Now, should we assume that gains from extreme intolerance always grow more than losses abound? This assumption seems not to be entirely justified; most importantly, extremely intolerant confessions (sects, personality cults) renege on flexibility and adaptiveness, and their comparative disadvantages (relative to open societies) rather increase. We can conclude that if the case for positive premium is not established, intolerance cannot drive tolerance out of society and is tolerable.

The models elaborated here do not capture all political effects of intolerant confessions, thereby do not explain all risks attached to political intolerance. In particular, they don't capture cases when an intolerant confession changes the "rules of evolutionary game" by suppressing tolerant confessions with political power. But the possibility to change the rules of the game would mean to introduce a super-game, which is beyond the scope of this paper.

Reference

Maynard Smith, John (1982). *Evolution and the theory of games*. Cambridge: Cambridge University Press.



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