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Public Employment Effects over the Business Cycle: the Czech Case

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Public Employment Effects over the Business Cycle: the Czech Case

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Abstract:
The paper contributes to understanding the effects stemming from the public sector employment changes in the Czech Republic and their impacts on the labor market through the lens of a New Keynesian dynamic stochastic general equilibrium model with search and matching frictions. The size of the public sector has been generally expanding over the last decade contrary to many other European countries with the exception of the years 2011 - 2012 when the economic crisis became more evident even in the otherwise financially stable Czech Republic. We model the labor market dynamics across the business cycle and examine the impacts of the varying number of public workers on the labor market variables as private employment, unemployment rate and market tightness as well as on the overall economic growth. We aim at determining whether a portion of unemployment can be explained by either the increased public hiring or shrinking of the number of public employees in the last decade. As the results suggest, in recessionary times the expansion of the public sector managed to keep the unemployment rate from attaining higher values. However, the following turnover of government size development threw the labor market into a deeper crisis than it would have been if the public sector size had remained unaltered.

Keywords: search and matching frictions, DSGE, labor market, public employment
JEL: E37, J21, J45, J48, J64

Introduction

This paper provides an insight into the role public employment plays in the labor market and its contribution to the economic growth through the lens of the New Keynesian dynamic stochastic general equilibrium (DSGE) model. The contribution of this paper resides in improving the understanding of effects stemming from the public employment with focus on the Czech Republic where the public sector is relatively rich and thus its changes inevitably influence the whole labor market. An enriched labor market structure permits us to analyze the effects which the changes in the public sector employment ultimately have on the unemployment in the economy as well as on the number of vacancies and the private employment. Also, the model simulations let us examine what impacts the hiring-prone policy might have had if it had been carried out even during the times of the economic crisis. Such knowledge is relevant in particular for countries that have the option to decide about timing of consolidation effort as sustainability of their fiscal policies is not under concern.

As the literature in the field suggests, economies tend not to be robust against public employment policies whose implications on the labor market might be rather various and inducing overall economic changes. Presumably one of the most discussed is the crowding out of the private employment with a potential of mitigating the overall employment benefits public jobs can deliver (Behar & Mok 2013). Even if public jobs did not cause a significant crowding out leading to an increase in unemployment, there still might be present a pronounced private job structural change, where the nontradable sector can benefit from public job creations while the tradable sector may shrink (Faggio & Overman 2012). Particularly over the last years, the generally perceived image of the public sector being stable and secure when it comes to employment (Kopelman & Rosen 2014) has been shattered and the ongoing public layoffs policy in many European countries under extreme financial stress has been turning the perceptions around. Even though the Czech Republic generally does not belong among such countries, some public sector shrinking occurred during the financial crisis as well. Thus, the ultimate objective of this paper is to examine the macroeconomic effects of varying government employment in the Czech Republic.

The rest of the paper is organized in the following way. In the first part, we provide an overview of different points of view on the government behavior when it comes to the labor market. Section 3 describes the labor market model and its incorporation into the New Keynesian DSGE model. Model parameters and steady state levels of model variables for the Czech Republic are calibrated in section 4. Finally, section 5 presents the results and section 6 concludes.

2 Public Employment Impacts on the Labor Market

Fiscal policy undeniably influences the whole macroeconomic development and its impact is even stronger after the global economic crisis when interest rates have reached the zero lower bound and thus its importance for stimulating the economy has risen. Therefore, a number of research papers has been focusing lately on the fiscal policy implications regarding the labor market and particularly the accelerating unemployment. The government can influence the labor market through multiple channels either intentionally or as a consequence of other measures not directly aiming at changing the situation on the labor market.

An example of a measure directly influencing the behavior of the economically active population are unemployment benefits which are a tool oriented towards financially supporting individuals who lost their jobs. However, as Andersen (2014) suggests, it is appropriate to adapt the unemployment insurance to the business cycle phase with lower benefits offered during expansionary stages of the economy in order to incentive the unemployed to search for a job and conversely more generous benefits in recessions when there is generally a shortage of vacancies. Similar findings have been confirmed by Kiley (2003). Despite the positive effects, difficulties are appearing as well. The main complication connected with the unemployment benefits adjustments, besides administrative complications, lies in the fact that it might be rather arduous to find triggers for launching such policy changes. The Czech government released in 2009 a stimulative package aiming at downsizing the negative impacts of the global economic crisis and increased the unemployment benefits by 1 billion CZK (Ambriško *et al.* (2012)).

The public sector employment, which is the focus of this paper, besides being an indivisible part of every economy, it is also a potentially powerful mean to alternate the situation on the labor market. Jobs in the government sector are generally perceived as more stable and secure by the public as stated by Kopelman & Rosen (2014) who found out that the probability of loosing a job is higher in private sector over the whole business cycle with widened advantage during the recessionary stage. The lower job loss probability concerned all races, genders and education groups. However, the report by EPSU (2013) using the Eurostat data revealed that after the global economic crisis offset, a part of publicly employed people lost their jobs in the Czech Republic. This is motivates our analysis to find out whether the layoffs carried out by the Czech government during the years 2011 - 2012 had a negative overall impact on the Czech economy and whether the continuation of performing of the opposite action of creating new vacancies and hiring as in the previous period would not have been more beneficial for the economy. Choulet (2006) focused on the implications of public job creation for unemployment and the number of new private jobs. The results of the SVAR model showed that in the short run the unemployment drops significantly due to newly created vacancies immediately taken by unemployed and also part of workers from the private sector. However, the private sector decreases its offer of vacancies which acts against the further unemployment decrease. In the long run, the results were dependent on the public wage premium. The public wages effects on the labor market were explored by Gomes (2010) with the result that they ought to follow the path of private wages thus being procyclical in order to smooth the demand for public jobs over the business cycle. One of the most important, potentially negative impacts, which the public employment might have, is the crowding out effect of the private employment that can even outweigh the advantages of newly created public jobs and bring the overall unemployment lower in case the public employment rates are substantially high as pointed out by Behar & Mok (2013). While public employment might influence the level of the private one, it might also affect its structure. Faggio & Overman (2012) estimated that in the short run each new public vacancy creates 0.5 jobs in the nontradable sector and crowds out 0.4 jobs in tradable sector while in the long run the crowding out for tradables outweighs and drags the overall private employment down.

3 Model

This paper utilizes a NK DSGE model with search and matching frictions. Contrary to standard New Keynesian DSGE models where the government expends on private goods, in the current setting its expenditures comes mainly from paying wages to public sector employees. Therefore, the government contributes to the aggregate labor demand instead of being a part of the resource constraint. Other specialty of the model lies in the fact that the labor market is not perfectly competitive but admits frictions regarding the job search and matching of employees with vacancies. This setting induces a change in the labor supply which becomes the quasi-labor supply. The existence of the labor market tightness parameter determining the share between vacancies and unemployed then arises from the evenness between the quasi-labor supply and the aggregate labor demand. Moreover, firms have to account for hiring costs when looking for new employees.

The rest of this section introduces model constituents and derives their optimal conditions.

The domestic economy is populated by a representative rational household composed of a continuum of individuals of mass one and representative domestic firms and domestic retailers both of mass one as well. Besides them, the central bank and the fiscal sector represented by the government are present. The model assumes rational behavior of economic agents. The model is based on Michaillat (2014) but due to the varying size of public employment in the Czech Republic an alteration was made and a layoff case during the economic slowdown period was added.

3.1 Labor market

The labor market is the most important feature of the model since the focus is put on the unemployment issues. The key equation playing essential role is the matching function which is later used to derive other labor market notions. The matching function is closely connected to the Beveridge curve since the relation between unemployment and vacancies is essential for the labor market dynamics.

The concept of the matching function introduced by Petrongolo & Pissarides (2011) can be considered as key for the analysis of mismatch and thus structural changes in the labor market. The matching function relates the inflow to employment, also called new hires, h_t with the stock of homogeneous vacancies v_{t-1} and pool of unemployed persons u_{t-1} at the beginning of a time period t.

The inflow into employment h_t used in the matching function is measured by the number of previously unemployed and registered persons who found a job. However, they are only part of the employment inflow group. Other two subgroups are workers changing their job and persons previously not participating in the labor market (i.e. students, persons taking parental leave, handicapped) who got employed.

The matching function is often found in the Cobb-Douglas form which assumes constant returns to scale, is differentiable and increasing in both arguments.

$$h_t(u_{t-1}, v_{t-1}) = \omega u_{t-1}^{\eta_1} v_{t-1}^{\eta_2} \tag{1}$$

where ω denotes the efficiency of the matching process otherwise called also the level of mismatch on the labor market. The exponential coefficients η_1 and η_2 determine the sensitivity of new matches to the number of unemployed and the number of vacancies. According to Borowczyk-Martins *et al.* (2011) constant returns to scale of the matching function can be assumed. Thus $\eta_2 + \eta_1 = 1$ and I set $\eta_1 = \eta$. The matching function then becomes $h_t = \omega u_{t-1}^{\eta} v_{t-1}^{1-\eta}$.

Multiple notions essential for the labor market modeling can be derived from the matching function. One of them is the probability of finding a job $f(\theta)$ which is a function of the labor market tightness $\theta_t = \dfrac{v_{t-1}}{u_{t-1}}$ and is related to the Cobb-Douglas matching function as

$$f(\theta_t) = \frac{h_t}{u_{t-1}} = \omega \theta_t^{1-\eta} \tag{2}$$

A tight labor market means that it is difficult and costly for firms to fill a vacancy with a suitable worker. The overall level of employment in period t is given as the sum of non-separated workers form the previous period and newly employed

$$n_t = (1-s)n_{t-1} + h_t \tag{3}$$

Each household member can be either employed or unemployed and looking for a job. The number of unemployed workers u_{t-1} searching for a job at the time t is

$$u_{t-1} = 1 - (1-s)n_{t-1} \tag{4}$$

where n_{t-1} denotes the aggregate level of employment and s stands for the separation rate defined as the proportion of the total number of employment terminations to the total number of workers employed.

Next important concept is the probability of filling a vacancy defined as

$$q(\theta_t) = \frac{1}{v_{t-1}} h_t = \omega \theta_t^{-\eta} \tag{5}$$

Based on Diamond (2013), job-finding probability, separation rate, employment and unemployment can be linked by the relation

$$f(\theta_t) u_{t-1} = s n_t \tag{6}$$

This relation indicates the assumption of equality between flows into and out of employment. Plugging equations (3) and (2) into the equation (6) and combining with the equation (4) leads to the expression for the Beveridge curve

$$u_t = \frac{s}{s + (1-s) f(\theta_t)} \tag{7}$$

Due to the presence of frictions, the employment rate is obtained after plugging the relation for unemployment and employment (4) into (6) as

$$n_t = (1-s) n_{t-1} + [1 - (1-s) n_{t-1}] f(\theta_t) \tag{8}$$

In a steady state $n_t = n_{t-1}$ and this relation therefore becomes a quasi-labor supply which replaces the conventional labor supply and determines the stationary employment.

$$n^s(\theta) = \frac{f(\theta)}{s + (1-s) f(\theta)} \tag{9}$$

In the environment where there are search and matching frictions present, workers are not free to choose the amount of working hours they will supply. However, they can choose the intensity of job search in case of their unemployment. Thus, the quasi-labor supply translates the workers optimal job searching decision in the steady state into the employment rate. In this model setting, the search decisions of workers is set exogenously to 1. However, it can be endogenized being the function of unemployment and work flow values. Then, the quasi-labor supply expresses directly the search decisions through the employment rate and is in this sense akin to the conventional labor supply in the sense that it expresses the amount of labor supplied by workers stemming from worker's optimum dependent on the economic environment (Michaillat 2014).

Moreover, The properties of $f(\theta)$ induce that in the environment of balanced flows on the labor market and high market tightness, the level of employment is also high therefore currently unemployed workers can find a job quickly.

3.1.1 Overall employment level and Government employment

The overall employment n_t is a combination of public g_t and private l_t employment. The Eurostat database provides data on the employment in the general government for all European countries under the module Non-financial transactions. The average number of public employees in the Czech Republic according to this database is 680 000 over the last 10 years. However, studies on the public sector as for instance (Bouchal & Janský 2014) work with the number reaching 950 000 public employees, which suggests that the Eurostat data take into account the core of public workers but a large number of employees in health care, education and public services in general are influenced by the government political decisions to a large extent too even though some subsectors might not be officially classified as *the General government*. Not accounting for this fact and using the more restricted version of data, as is the Eurostat table, might prevent us from observing the full impact of public measures regarding the labor market. Therefore, to define the public employment, we use a sum of number of employees in physical unit in public administration and defense and compulsory social security, education and human health and social work activities (CZ-NACE sections O, P and Q respectively) provided by the Czech Statistical Office to measure the public employment in the Czech Republic.

Over the last 10 years, the overall employment level in the Czech Republic underwent one major increase in 2009 and one decline during the global economic crisis in 2011. The development of the public employment did not fully followed the same path as the overall employment and on Figures 1 and 2 we can observe a delay of government sector reaction. In 2009, the employment dropped by 1.4% and continued to decrease up to 2011 but the public employment turned its trend only later in 2010 and dropped by 0.8%. It remained low also in 2012 when the overall employment had already been increasing which then remained also a common trend of both up till now.

Overall, the size of the Czech public sector increased almost by 8% during the observed period 2005 - 2014. The variations of the government sector size are the primary factor which motivates our analysis since the share of government

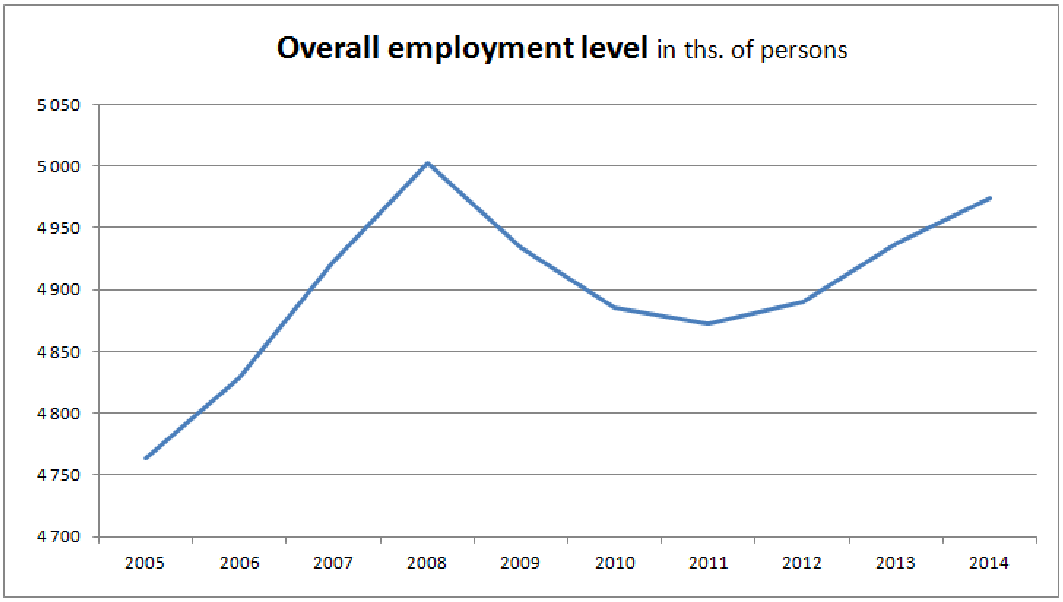

Figure 1: Employment level

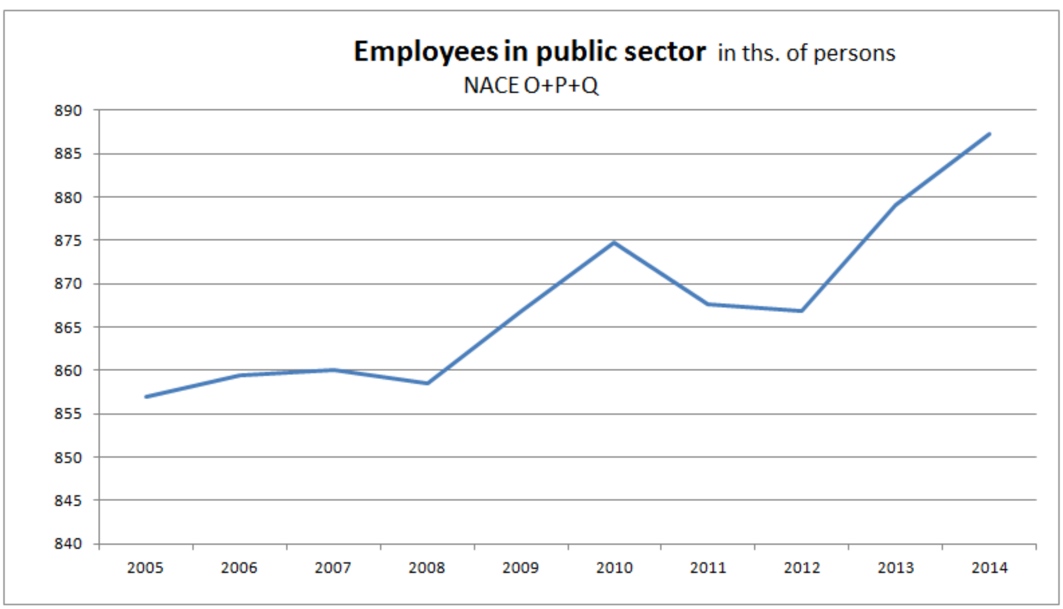

Figure 2: Public sector employees

employees on the total employment fluctuated around 17% making it large enough to induce overall macroeconomic changes when altered. Thus, we are focusing both on increases and declines of public employment g_t and examine the macroeconomic impacts of its interactions with the economy.

The parameter ζ is assigned to the share of public employment on the overall employment n_t and since n_t is composed of private and public employees we can express it as a sum of shares of workers working in either sector

$$n_t = (1 - \zeta)l_t + \zeta g_t \tag{10}$$

We model the change in government employment level as a shock with value σ_g representing the reaction of the government to the change of economic conditions consisting of either hiring an additional percentage of workers or of laying off a part of them at the time of the shock to the economy. After the first period,σ_g is equal to zero. The public hiring law of motion is thus in the following form

$$g_t = s\bar{g} + (1 - s)g_{t-1} + \sigma_g \tag{11}$$

where $\bar{g}$ is the steady state public employment.

3.2 Households

The household problem consists of maximizing the expected utility in the form proposed by Blanchard & Galí (2010)

$$E_0 \left[\sum_{t=0}^{+\infty} \beta^t [ln(c_t) + \chi \frac{n_t^{1+\Phi}}{1 + \Phi} \right] \tag{12}$$

with the discount factor $\beta < 1$ where c_t denotes the aggregate consumption and n_t the fraction of employed household members. Household consumes the final good and public good thanks to its income from working, thus maximizes (12) subject to the budget constraint

$$p_t c_t + b_t = (1 - \tau_t) w_t n_t p_t + b_{t-1} r_{t-1} + T_t p_t \tag{13}$$

where p_t denotes the price level, c_t consumption, n_t employment, b_{t-1} bonds purchased in previous time period $t - 1$, w_t wage, b_t bonds purchased in current time period t, r_{t-1} the gross interest rate, τ_t the income tax and T_t transfers to households. The usual no-Ponzi game constraint applies to the optimization problem.

Households choose the set of stochastic processes $\{c_t, b_t\}_0^{+\infty}$ in order to solve the maximization problem. The optimization problem yields the Euler equation in the usual form

$$\frac{1}{\beta} = E_t \left[\frac{r_t}{1+\pi_{t+1}} \frac{c_t}{c_{t+1}} \right] \tag{14}$$

3.3 Final good firms

The total final goods y_t are given by the CES aggregator of the different quantities of intermediate goods produced:

$$y_t = \left(\int_0^1 y_t(k)^{\frac{\psi-1}{\psi}} dk \right)^{\frac{\psi}{\psi-1}} \tag{15}$$

where $\psi > 1$ stands for the final goods substitution elasticity. Final goods firms operate on a perfectly competitive market, buy intermediate goods $y_t(k)$ and produce the final good y_t in order to maximize profits. In other words, firms try to minimize expenditure given the production constraint. The Lagrangian can be written using the modified version of the equation (15) as:

$$\mathcal{L} = \int_0^1 p_t(k) y_t(k) dk + \lambda_t \left(y_t^{\frac{\psi-1}{\psi}} - \int_0^1 y_t(k)^{\frac{\psi-1}{\psi}} dk \right) \tag{16}$$

Optimal choice of $y_t(k)$ solves $\dfrac{\partial \mathcal{L}}{\partial y_t(k)} = 0$, that is

$$y_t(k) = p_t(k)^{-\psi} \left(\lambda_t \frac{\psi-1}{\psi} \right)^{\psi} \tag{17}$$

After plugging into (15) we get

$$y_t^{\frac{\psi-1}{\psi}} = \left(\lambda_t \frac{\psi-1}{\psi} \right)^{\psi-1} p_t^{1-\psi} \tag{18}$$

utilizing assumption, that $p_t = \left[\int_0^1 p_t(k)^{1-\psi} dk \right]^{\frac{1}{1-\psi}}$.

Thus, the individual demand is in the following form

$$y_t(k) = p_t(k)^{-\psi}\left(p_t y_t^{\frac{1}{\psi}}\right)^{\psi} = \left(\frac{p_t(k)}{p_t}\right)^{-\psi} \cdot y_t \tag{19}$$

Such result is conditioned on the price assumption, that

$$p_t = \left(\int_0^1 p_t(k)^{1-\psi} dk\right)^{\frac{1}{1-\psi}} \tag{20}$$

3.4 Intermediate goods firms

There is a continuum of intermediate goods monopolist firms indexed by $k \in [0, 1]$ and each firm employs $l_t(k)$ workers during time period t to produce output $y_t(k)$. Thus the aggregate number of employed workers in a particular time period t is $l_t = \int_0^1 l_t(k)dk$.

Each firm has the same production function of the form:

$$y_t(k) = a_t l_t^{\alpha}(k) \tag{21}$$

where a_t is the technology process same for all firms and $\alpha \in (0, 1)$ denotes diminishing marginal returns to labor. The aggregate number of recruited employees each period is

$$l_t = (1 - s)l_{t-1} \tag{22}$$

and firms pay them wage w_t. The price setting mechanism is subject to the adjustment costs given by Rotemberg (1982) as

$$\frac{\phi}{2}\left(\frac{p_t(k)}{p_{t-1}(k)} - 1\right)^2 c_t \tag{23}$$

where $\phi > 0$ denotes adjustment costs coefficient.

The intermediate good firm faces also ra_t, a cost for holding a vacancy open in time period t. Thus we can express hiring costs as

$$[l_t(k) - (1 - s)l_{t-1}(k)]\frac{ra_t}{q(\theta_t)} \tag{24}$$

Firm's profit in period t is equal to

$$\begin{aligned}\pi_t =& y_t(k)\frac{p_t(k)}{p_t} - w_t l_t(k) - \frac{\phi}{2}\left(\frac{p_t(k)}{p_{t-1}(k)} - 1\right)^2 c_t \\ & - \left[l_t(k) - (1-s)l_{t-1}(k)\right]\frac{r a_t}{q(\theta_t)}\end{aligned} \tag{25}$$

Firm solves the optimization problem where it seeks to maximize the discounted sum of expected profits subject to (19) and (21).

Solving the firms' optimization problem yields the firm's labor demand and the Phillips curve.

$$l_t^d = \left[\frac{1}{\Lambda_t \alpha}\left[\frac{w_t}{a_t} + \frac{r}{q(\theta_t)} - \beta(1-s)E_t\left[\frac{r_t}{q(\theta_{t+1})}\frac{c_t}{c_{t+1}}\frac{a_{t+1}}{a_t}\right]\right]\right]^{\frac{1}{\alpha-1}} \tag{26}$$

$$\begin{aligned}\pi_t(\pi_t+1) =& \frac{1}{\phi}\frac{y_t}{c_t}\left[\frac{\psi}{\alpha a_t l_t^{\alpha-1}}\left(w_t + \frac{a_t r}{q(\theta_t)} - \beta(1-s)\frac{c_t}{c_{t+1}}\frac{a_{t+1} r}{q(\theta_{t+1})}\right) + (1-\psi)\right] + \\ & \beta E_t\left[\pi_{t+1}(\pi_{t+1}+1)\right]\end{aligned} \tag{27}$$

Since the labor market is a combination of private and public employment, also the aggregate labor demand is a sum of private and public labor demand.

$$n^d = l^d + g \tag{28}$$

3.5 Wage schedule

We follow the standard literature and set wages as in (Blanchard & Galí 2010) where they are assumed to be partially rigid and the wage schedule has the following form:

$$w_t = w_0 a_t^{\gamma} \tag{29}$$

where w_0 is the stationary wage and γ determines the wages elasticity with respect to productivity.

3.6 Central Bank

The central bank follows a gross nominal interest rate rule set as

$$r_t = \frac{1}{\beta}(1+\pi_t)^{\mu_\pi(1-\mu_R)}(\beta r_{t-1})^{\mu_R} \tag{30}$$

where π_t is the time t inflation rate, μ_π measures the monetary policy to inflation response and μ_R is the interest rate smoothing parameter. The steady state inflation $\bar{\pi}$ is assumed to be zero and steady state interest rate $\bar{r} = \dfrac{1}{\beta}$.

3.7 Government

The number of workers employed in public sector in time period t is g_t. Government is exposed to hiring costs similarly to private sector $\dfrac{ra_t}{q(\theta)}[g_t - (1-s)g_{t-1}]$. Public wages are equal to private wages w_t. Each period the government is obliged to repay its debt from the previous period which costs $r_{t-1}b_{t-1}$ and a new debt b_t is produced. Public income is given by the labor taxation.

$$n_t\tau_t\ w_t + \frac{b_t}{p_t} = g_t w_t + \frac{ra_t}{q(\theta)}[g_t - (1-s)g_{t-1}] + \frac{r_{t-1}}{p_t}b_{t-1} \tag{31}$$

After plugging in (13) and the expression for firm's profits (25) the budget constraint becomes

$$y_t = c_t\left(1 + \frac{\phi}{2}\pi^2\right) + \frac{ra_t}{q(\theta)}[n_t - (1-s)n_{t-1}] \tag{32}$$

The meaning behind the equation (32) is that the produced output is either consumed by households or spend on new employees recruitment or price adjustment mechanism.

3.8 Productivity

The driving force of the business cycle in this model is the technology shock ϵ_t, which follows random walk and is *i.i.d.* This shock enters into the technology AR(1) process defined as

$$a_t = \rho a_{t-1} + \epsilon_t \tag{33}$$

The technology series a_t is defined as a ratio of the output y_t to the employment level.

4 Parameter calibration

For the analysis we choose three subperiods dependent on the development of n_t and g_t. Thus the parameter ζ and steady state values of n_t, θ_t, g_t and u_t vary based on the chosen subperiod. Other model parameters remain constant in order to be able to examine scenarios of alternative development of the labor market. The first period taken into account represented by the years 2005-2008 constitutes the benchmark case when the overall employment increased relatively steadily and the public employment was almost constant. Second period comprises the economic crisis offsetting in 2009 when the government sector size still increased and thus went against the decreasing n_t. We set it as Scenario 1. Lastly, we are interested in Scenario 2 which is a period containing the year 2011 when the public employment trend turned around and followed n_t. After the economic crisis the trend of both n_t and g_t changed back to pre-crisis development therefore the qualitative results drawn for the Scenario 1 apply.

First, we calibrate parameters with constant values. The separation rate s is calibrated according to Hobijna & Şahin (2009) as 0.0024. The per-period vacancy cost c estimation varies with different sources. Pissarides (2009) estimated it as 0.357 of worker's wage. Shimer (2005) as 0.213 and Hall & Milgrom (2008) as 0.433. Therefore a geometric mean of these values was used and c calibrated as $c = 0.3205w_0$. The production function parameter α is calibrated to 0.53 according to Aliyev *et al.* (2014). The elasticity of wages with respect to labor productivity γ is set to value 0.7 as estimated by Haefke *et al.* (2008).
The intermediate goods substitution elasticity ψ is calibrated to 11 which is a fairly standard value found in related literature (Michaillat (2014)). The discount factor β is set to 0.99 according to Aliyev *et al.* (2014). The parameters introduced by the interest rate rule are: the responsiveness of monetary policy to inflation denoted by μ_π and the interest rate smoothing parameter μ_R which are set respectively to values 1.5 and 0.52 which is in line with Štork *et al.* (2009). The Rotemberg price adjustment cost ϕ is set to a standard value of 50 as in Bergin *et al.* (2007).
The steady state wage w_0 is calibrated using the firms' labor demand 26 in

steady state to get $w_0 = 0.5228$ which yields the per-period vacancy cost as $r = 0.3205 \cdot w_0 = 0.1676$.
The matching coefficient is obtained using the equality relation for steady state unemployment inflows and outflows $f(\bar{\theta})\bar{u} = \bar{n}s$ where $f(\bar{\theta}) = \bar{h}\bar{u}^{-1}$ and $\bar{h} = \omega\bar{\theta}^{-\eta}$. Combining these three relations yields $\omega = s\bar{n}\bar{\theta}^{\eta-1}\bar{u}^{-1}$ and after plugging in numerical values I get the matching efficiency equal to 0.0484.
The job-filling elasticity parameter η of the matching function is calibrated based on Petrongolo & Pissarides (2011) as 0.7.

For calibrating the varying steady states of variables $\bar{n}$, $\bar{g}$, $\bar{u}$, $\bar{\theta}$ and the parameter ζ we use the data from the Czech Statistical Office. For calibrating $\bar{u}$ we use the yearly average shares of unemployed persons, which is the ratio of job applicants aged 15-64 in the population of the same age (MoLSA 2016). For $\bar{g}$ we use the proxy consisting of the sum of workers in public administration, defense and compulsory social security, education and human health and social work activities (CZ-NACE sections O, P and Q) divided by the overall number of employees in the economy coming from the same data source (CZSO 2016b). The steady state employment rate $\bar{n}$ is computed based on the equation (4) as

$$\bar{n} = \frac{1 - \bar{u}}{1 - s}$$

The labor market tightness $\bar{\theta}$ is calculated as a ratio of number of registered vacancies to the number of job applicants registered at the employment offices. Both time series come from the Czech Statistical Office (CZSO 2016a).

The parameter ζ denotes the share of employees working in the public sector thus is computed as $\zeta = \frac{\bar{g}}{\bar{n}}$. The steady state private employment rate is finally calculated as $\bar{l} = (1 - \zeta)\bar{n}$.

The shock to the public employment σ_g is calibrated based on the time series for public employment from the CZSO (CZSO 2016b).

The stochastic process $a_t = \rho a_{t-1} + \epsilon_t$ comprises two parameters to calibrate - productivity persistence factor ρ and standard deviation of the error component σ_ϵ. ϵ_t is independent identically distributed with zero mean and variance σ_ϵ^2. ρ is estimated as a parameter of the AR (1) model. Before estimation, the productivity series is computed as a ratio of quarterly time series of output to the quarterly average number of employees (headcount). The series is then detrended using Hodrick-Prescott filter with the smoothing parameter $\lambda = 1600$. For the quarterly output time series we take the gross domestic product identity from the production side provided by the CZSO as well (CZSO 2016c).

The Table 1 shows the calibrated values of varying parameters while Table 2 summarizes constant parameters.

symbol	parameter meaning	Benchmark	Scenario 1	Scenario 2
ζ	public employment share	0.1776	0.1774	0.1781
$\bar{g}$	steady state public employment rate	0.1653	0.1681	0.1664
$\bar{l}$	steady state private employment rate	0.7656	0.7795	0.7683
$\bar{u}$	steady state unemployment rate	0.0534	0.0658	0.0667
$\bar{n}$	steady state employment rate	0.9309	0.9475	0.9348
$\bar{\theta}$	steady state labor market tightness	0.2163	0.0561	0.0704
σ_g	public employment shock	0.0006	0.0189	-0.0081

Table 1: Varying parameters calibration

symbol	parameter meaning	value
ψ	intermediate goods substitution elasticity	11
β	discount factor	0.99
μ_π	monetary policy response to inflation	1.5
μ_R	interest rate smoothing parameter	0.52
ϕ	price adjustment cost	50
ρ	productivity autocorrelation	0.9827
σ_ϵ	standard deviation of productivity	0.0031
w_0	steady state wage	0.5228
r	per-period vacancy cost	0.1676
s	separation rate	0.0024
c	per-period vacancy cost	0.1739
ω	matching efficiency coefficient	0.0484
η	job-filling elasticity	0.7
α	production function parameter	0.53
$\bar{a}$	steady state labor productivity	1
γ	elasticity of wages	0.7

Table 2: Constant parameters calibration

5 Results

The conditions on the Czech labor market have been changing over the last decade. We identified the period 2005-2008 as relatively stable with steadily growing employment level and almost constant public employment. Therefore, we set it as the benchmark scenario and compare it to two alternative scenarios based on real labor market conditions prevailing in the Czech economy at later time periods. The objective is to determine quantitatively the impact of varying public employment on the economy ceteris paribus. After the economic crisis receded the trend of both n_t and g_t changed back to the pre-crisis development therefore the qualitative results drawn for the Scenario 1 apply. The rest of the section examines the impulse responses of the three set scenarios and quantifies the impacts of the public sector size changes.

The figure 3 depicts impulse responses in the benchmark case of labor productivity, public employment, labor market tightness, private employment, unemployment, gross domestic product, consumption and inflation rate in this order. In the growing stage of the economy, which the period 2005-2008 was, the public sector size was varied only very slightly, the overall change over the period was 0.06 %. The labor market remained in fact stable which is the reason we set this case as the benchmark.

In the first alternative scenario, captured in the figure 4, the economic slowdown set off. The solid line represents the situation based on the observed development when the public employment increased by 1.9 % despite the declining state of the overall employment. The dashed line represents the benchmark scenario and thus reveals how the macroeconomic conditions would have developed if the public sector had not increased its size but rather remained constant. The increase in public employment is connected with the stimulative package as identified by (Ambriško *et al.* 2012). This package aimed at support the economic activity and among other measures brought increased unemployment benefits or increased children benefits.

After the economic slowdown sets off, the negative shock to the economy induce a decrease in wages. However, wages cannot adjust completely due to

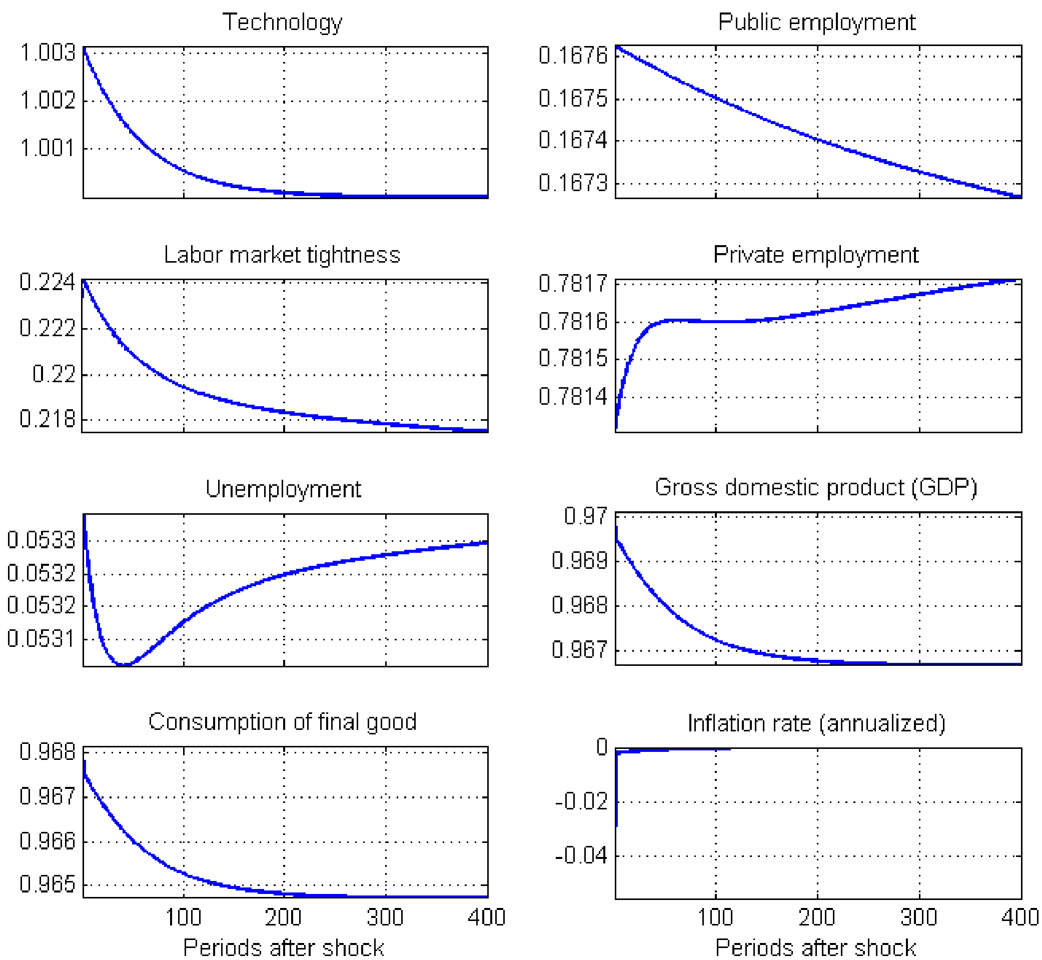

Figure 3: Benchmark scenario

their rigidity. Thus, the marginal cost of labor increases and firms are forced to diminish the number of offered vacancies. Consequently, private employment declines. However, the expansion of the private sector outweighs the decrease in private vacancies and therefore the labor market became actually tighter than it would have been had not the public employment increased. The unemployment got lower by almost 0.7 % compared to the hypothetical benchmark situation if the government had not expanded its size. The gross domestic product which is computed as a sum of output plus government expenses composed of public workers' wage costs and public hiring costs

$$gdp = y_t + g_t w_t + \frac{r a_t}{q(\theta)}[g_t - (1-s)g_{t-1}] \tag{34}$$

after the initial decline converges back to its steady state. The behavior of the household consumption of final goods copies the behavior of the GDP. The inflation rate stationary value is set to 0 and the technology shock influences its behavior only marginally. The impact of the expanding public sector on the consumption and GDP was however only very minor and the main impact centered around the labor market itself.

The second scenario depicted in the Figure 5 introduces a reaction of the government to the economic slowdown consisting of laying off 0.8% of its employees in 2011. The context of this measure was identified by once again by (Ambriško *et al.* 2012). The government released an austerity package aiming at reducing the public budget deficit. Measures taken related to the labor marker included a salary cut in the central government sector, a decrease of sickness benefits, freezing salaries for government employees and overall cuts in wages and salaries, freezing of pensions, decreasing state social support benefits and reductions of non mandatory government expenditure cuts in general.

The initial laying off causes the public employment to drop. Lower wages bring lower recruitment costs for the private sector and thus firms are able to actually increase their hiring but this does not compensate for the rather profound loss of public vacancies and results in a sharp drop in the labor market tightness. Consequently, the overall unemployment rises far more than it would given the benchmark case occurred, thus neutralizing the positive impact on the private employment. The unemployment rate got higher by almost 0.5 percentage points

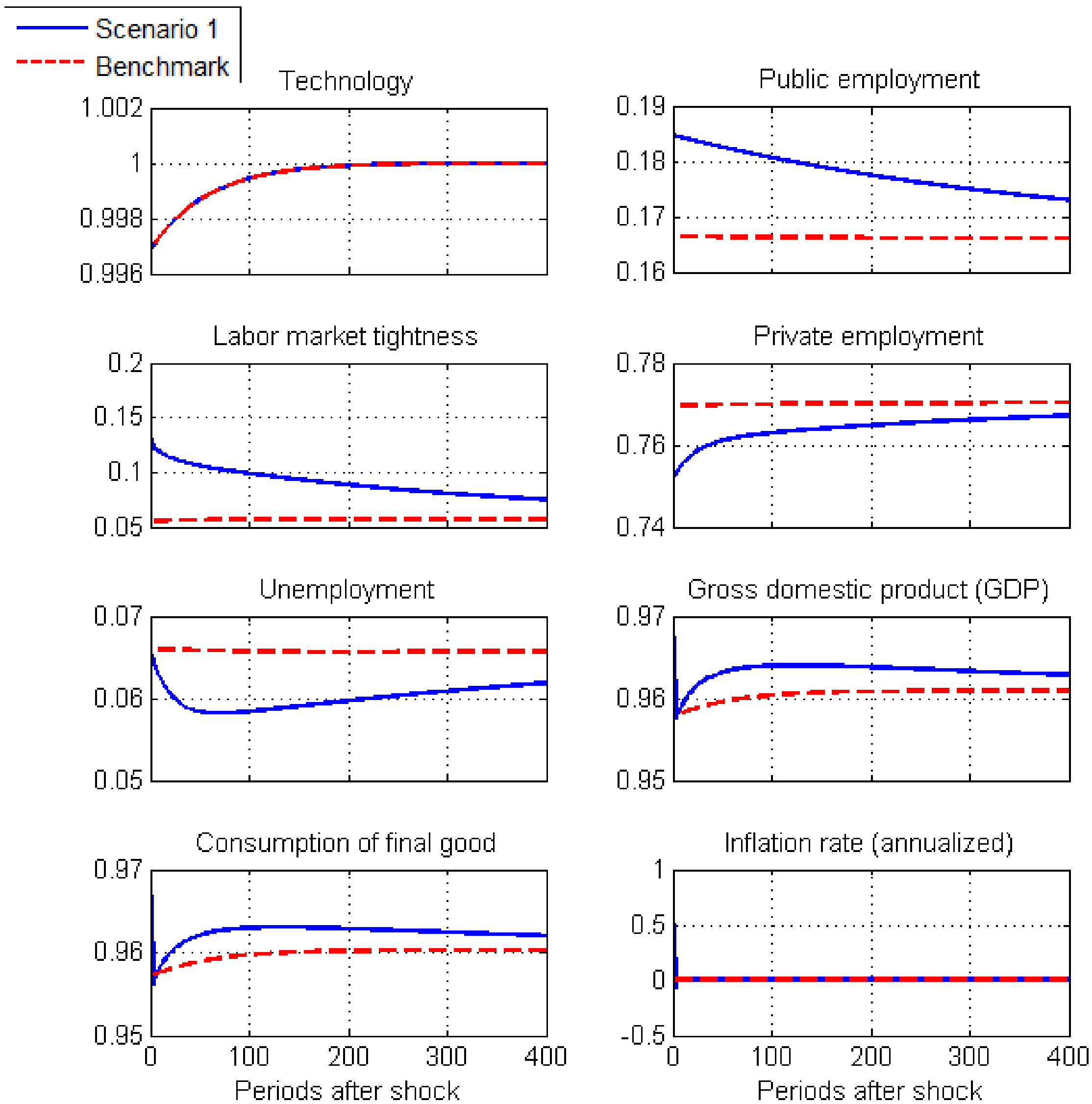

Figure 4: Scenario 1

due to public sector size reduction. As expected, the laying off of public employees deepened the economic downturn and intensified the diminution of consumption and GDP.

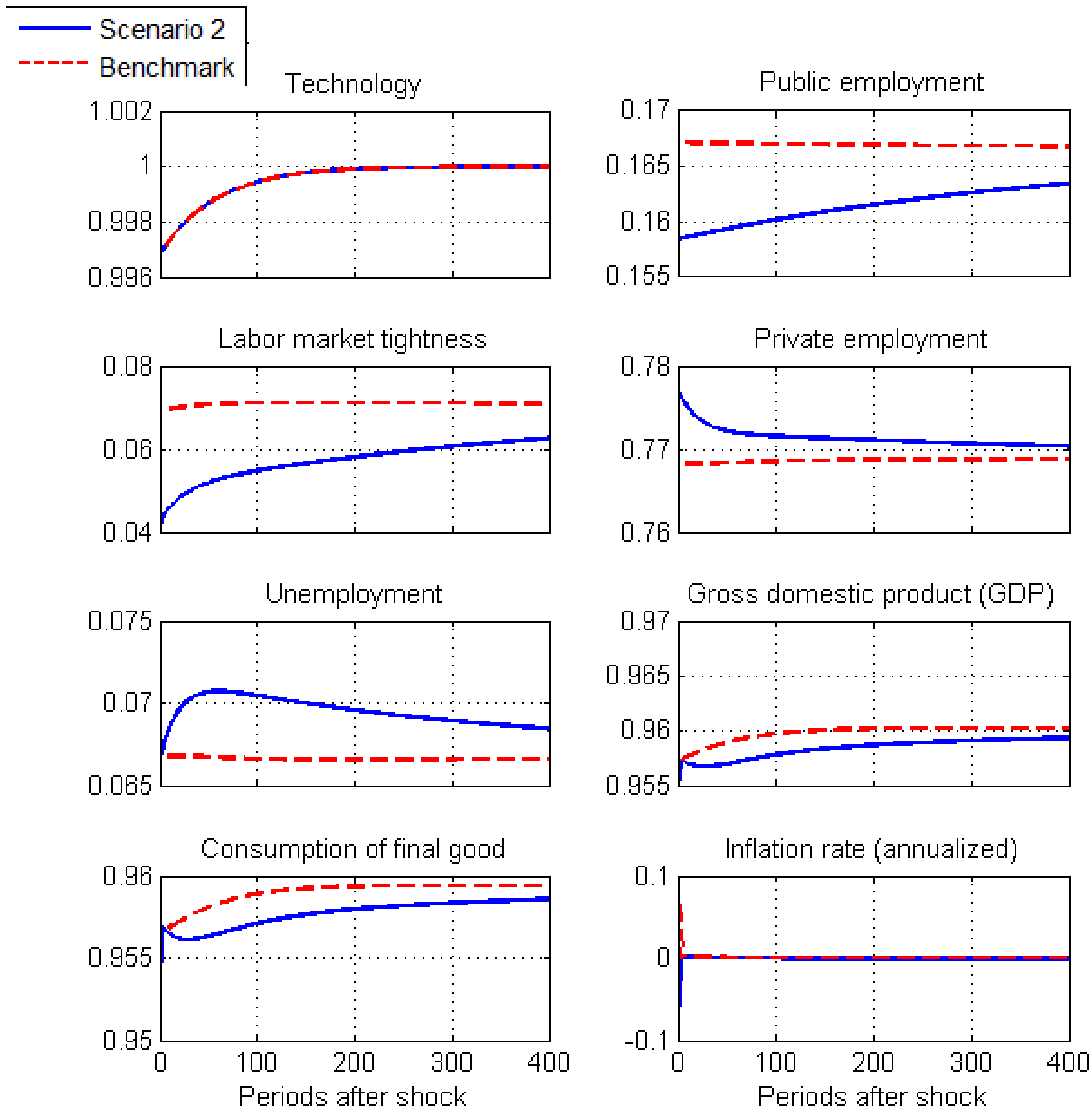

Figure 5: Scenario 2

6 Conclusion

This paper presented a New Keynesian DSGE model with search and matching frictions and public employment. Due to the fact than public employees constitute roughly one fifth of all employees in the Czech economy, any alterations in the government sector employment have potentially large impacts and in this paper we quantified the portion of the unemployment explained by public sector size variations. Due to the fact that the public employment constitutes almost one fifth of the overall employment in the Czech Republic the impacts were expected to be significant. The model results suggest that the public employment really has the capacity to influence the macroeconomic conditions.

We examined the development of the labor market based on time series from the Czech Statistical Office and identified three distinct periods. The benchmark case was set as the period where the government kept the public employment almost stable over time. This period occurred between 2005 and 2008. In 2009 the economic crisis set off and plunged the labor market into a worsened condition. However, the government expanded despite overall unfavorable economic conditions and employed by as many as 1.9 % workers more compared to the 2008 stock. This trend turned around in the following period and just in one year the government employment shrunk by 0.8 % intensifying considerably the downturn of the labor market since the positive effect on the private employment was mitigated by a large drop in the labor market tightness and consequently the overall unemployment increased more than it could if the government had preserved the constant state of its size.

Based on the results, the government might consider to conduct the recruitment process when filling public vacancies thoughtfully with regard to the overall economic conditions since the public hiring might be successfully used as an countermeasure during the economic slowdown when the labor market needs to be supported in overcoming the general slackening.

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Appendix

A The full model

A.1 Labor market

The matching function is in the usual Cobb-Douglas form as follows

$$h_t = \omega u_{t-1}^{\eta} v_{t-1}^{1-\eta} \tag{35}$$

From the matching function we derive the probability of finding a job

$$f(\theta_t) = \frac{h_t}{u_{t-1}} = \omega \theta_t^{1-\eta} \tag{36}$$

and the probability of filling a vacancy

$$q(\theta_t) = \frac{1}{v_{t-1}} h_t = \omega \theta_t^{-\eta} \tag{37}$$

The overall level of employment in period t is given as the sum of non-separated workers form the previous period and newly employed

$$n_t = (1 - s) n_{t-1} + h_t \tag{38}$$

The number of unemployed workers u_{t-1} searching for a job at the time t is

$$u_{t-1} = 1 - (1 - s) n_{t-1} \tag{39}$$

The job-finding probability, separation rate, employment and unemployment can be linked by the relation

$$f(\theta_t) u_{t-1} = s n_t \tag{40}$$

which indicates the assumption of equality between flows into and out of employment. Plugging equations (38) and (36) into the equation (40) and combining with the equation (39) leads to the expression for the Beveridge curve

$$u_t = \frac{s}{s + (1 - s) f(\theta_t)} \tag{41}$$

Due to the presence of frictions, the employment rate is obtained after plugging the relation for unemployment and employment (39) into (40) as

$$n_t = (1-s)n_{t-1} + [1-(1-s)n_{t-1}]f(\theta_t) \tag{42}$$

In a steady state $n_t = n_{t-1}$ and this relation therefore becomes a quasi-labor supply which replaces the conventional labor supply and determines the stationary employment.

$$n^s(\theta) = \frac{f(\theta)}{s+(1-s)f(\theta)} \tag{43}$$

A.2 Households

The household maximizes the expected utility in the form

$$E_0\left[\sum_{t=0}^{+\infty}\beta^t[ln(c_t) + \chi\frac{n_t^{1+\Phi}}{1+\Phi}\right] \tag{44}$$

subject to the budget constraint

$$p_t c_t + b_t = (1-\tau_t)w_t n_t p_t + b_{t-1}r_{t-1} + T_t p_t \tag{45}$$

The usual no-Ponzi game constraint applies to the optimization problem

$$E_0\left[\lim_{t=+\infty}\frac{b_t}{\prod_{i=0}^{t} r_{t-1}}\right] \geq 0 \tag{46}$$

We denote $\pi_t = \left(\frac{p_t}{p_{t-1}} - 1\right)$ as the inflation rate in time period t.
The Lagrangian is formed as

$$\mathcal{L} = E_t\sum_{t=0}^{+\infty}\beta^t\left\{\left[ln(c_t) + \chi\frac{n_t^{1+\Phi}}{1+\Phi}\right] + \lambda_t\left(p_t c_t + b_t - (1-\tau_t)w_t n_t p_t - b_{t-1}r_{t-1} - T_t p_t\right)\right\} \tag{47}$$

Households choose the set of stochastic processes $\{c_t, b_t\}_0^{+\infty}$ in order to solve the maximization problem. The inflow rate into employment is equal to the outflow rate from unemployment. Plugging the relation for unemployment and employment (39) into (40) yields the household employment law of motion

$$n_t = (1-s)n_{t-1} + [1-(1-s)n_{t-1}]f(\theta_t) \tag{48}$$

In a steady state $n_t = n_{t-1}$ and this relation becomes the quasi-labor supply which replaces the conventional labor supply and determines the stationary employment.

$$n^s(\theta) = \frac{f(\theta)}{s+(1-s)f(\theta)} \tag{49}$$

The first order conditions are formed as follows

$$\begin{aligned} &\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Rightarrow \frac{1}{c_t} + \lambda_t p_t = 0 \\ &\lambda_t = -\frac{1}{p_t c_t} \end{aligned} \tag{50}$$

$$\frac{\partial \mathcal{L}}{\partial b_t} = 0 \Rightarrow \lambda_t = \beta \lambda_{t+1} r_t \tag{51}$$

The Euler equation is obtained after combining the first order condition (50) with (51)

$$\frac{1}{\beta} = E_t\left[\frac{r_t}{1+\pi_{t+1}}\frac{c_t}{c_{t+1}}\right] \tag{52}$$

A.3 Final good firms

The total final goods Y_t are given by the CES aggregator of the different quantities of intermediate goods produced:

$$y_t = \left(\int_0^1 y_t(k)^{\frac{\psi-1}{\psi}} dk\right)^{\frac{\psi}{\psi-1}} \tag{53}$$

Final goods firms operate on a perfectly competitive market, buy intermediate goods $Y_t(k)$ and produce the final good Y_t in order to maximize profits. The Lagrangian can be written as follows

$$\mathcal{L} = \int_0^1 p_t(k) y_t(k) dk + \lambda_t \left(y_t^{\frac{\psi-1}{\psi}} - \int_0^1 y_t(k)^{\frac{\psi-1}{\psi}} dk\right) \tag{54}$$

Optimal choice of $y_t(k)$ solves $\dfrac{\partial \mathcal{L}}{\partial y_t(k)} = 0$, that is

$$p_t(k) = \lambda_t \frac{\psi - 1}{\psi} y_t(k)^{\frac{-1}{\psi}} \tag{55}$$

$$y_t(k) = p_t(k)^{-\psi} \left(\lambda_t \frac{\psi - 1}{\psi} \right)^{\psi} \tag{56}$$

Afterwards, this can be plugged into (53):

$$y_t = \left(\int_0^1 \left[P_t(k)^{-\psi} \left(\lambda_t \frac{\psi - 1}{\psi} \right)^{\psi} \right]^{\frac{\psi-1}{\psi}} dk \right)^{\frac{\psi}{\psi-1}} \tag{57}$$

$$y_t^{\frac{\psi-1}{\psi}} = \int_0^1 p_t(k)^{1-\psi} \left(\lambda_t \frac{\psi - 1}{\psi} \right)^{\psi-1} dk \tag{58}$$

$$y_t^{\frac{\psi-1}{\psi}} = \left(\lambda_t \frac{\psi - 1}{\psi} \right)^{\psi-1} \int_0^1 p_t(k)^{1-\psi} dk \tag{59}$$

$$y_t^{\frac{\psi-1}{\psi}} = \left(\lambda_t \frac{\psi - 1}{\psi} \right)^{\psi-1} p_t^{1-\psi} \tag{60}$$

utilizing assumption, that $p_t = \left[\int_0^1 p_t(k)^{1-\psi} dk \right]^{\frac{1}{1-\psi}}$.

Consequently,

$$\left(\lambda_t \frac{\psi - 1}{\psi} \right) = p_t y_t^{\frac{1}{\psi}} \tag{61}$$

And after plugging (61) into (56), the individual demand becomes:

$$y_t(k) = p_t(k)^{-\psi} \left(p_t y_t^{\frac{1}{\psi}} \right)^{\psi} = \left(\frac{p_t(k)}{p_t} \right)^{-\psi} \cdot y_t \tag{62}$$

Such result is conditioned on the price assumption, that

$$p_t = \left(\int_0^1 p_t(k)^{1-\psi} dk \right)^{\frac{1}{1-\psi}} \tag{63}$$

A.4 Intermediate goods firms

A continuum $k \in [0,1]$ of monopolist firms, each employing $l_t(k)$ workers to produce output $y_t(k)$ has the same production function of the form

$$y_t(k) = a_t l_t^{\alpha}(k) \tag{64}$$

The aggregate number of recruited employees each period is

$$l_t = (1-s) l_{t-1} \tag{65}$$

and firms pay them wage w_t. The price setting mechanism is subject to the adjustment costs

$$\frac{\phi}{2}\left(\frac{p_t(k)}{p_{t-1}(k)} - 1\right)^2 c_t \tag{66}$$

The intermediate good firm faces also ra_t, a cost for holding a vacancy open in time period t. Thus we can express hiring costs as

$$[l_t(k) - (1-s) l_{t-1}(k)] \frac{ra_t}{q(\theta_t)} \tag{67}$$

Firm's profit in period t is equal to

$$\begin{aligned}\pi_t =& y_t(k)\left(\frac{p_t(k)}{p_t}\right) - w_t l_t(k) - \frac{\phi}{2}\left(\frac{p_t(k)}{p_{t-1}(k)} - 1\right)^2 c_t \\ &- [l_t(k) - (1-s) l_{t-1}(k)] \frac{ra_t}{q(\theta_t)}\end{aligned} \tag{68}$$

Firm solves the optimization problem where it seeks to maximize the discounted sum of expected profits

$$\begin{aligned}E_0 \sum_{t=0}^{+\infty} \frac{\beta^t}{c_t} \Bigg\{ & \frac{p_t(k)}{p_{t-1}(k)} y_t(k) - w_t l_t(k) - \frac{\phi}{2}\left(\frac{p_t(k)}{p_{t-1}(k)} - 1\right)^2 c_t \\ & - \frac{r \cdot a_t}{q(\theta_t)} [l_t(k) - (1-s) l_{t-1}(k)] \Bigg\}\end{aligned} \tag{69}$$

subject to (62) and (64).

We construct the Lagrangian as

$$\mathcal{L} = E_0 \sum_{t=0}^{+\infty} \frac{\beta^t}{c_t} \Bigg\{ \left(\frac{p_t(k)}{p_{t-1}(k)} \right)^{1-\psi} y_t - w_t l_t(k) - \frac{\phi}{2} \left(\frac{p_t(k)}{p_{t-1}(k)} - 1 \right)^2 c_t$$
$$- \frac{r a_t}{q(\theta_t)} [l_t(k) - (1-s) l_{t-1}(k)] + \Lambda_t(k) \left[a_t l_t^{\alpha}(k) - \left(\frac{p_t(k)}{p_{t-1}(k)} \right)^{-\psi} y_t \right] \Bigg\} \tag{70}$$

The first order conditions yields the firm's labor demand and the Phillips curve.

$$\frac{\partial \mathcal{L}}{\partial l_t(k)} = 0 \Rightarrow$$
$$\Lambda_t \alpha l_t(k)^{\alpha-1} = \frac{w_t}{a_t} + \frac{r}{q(\theta_t)} - \beta(1-s) E_t \left[\frac{r_t}{q(\theta_{t+1})} \frac{c_t}{c_{t+1}} \frac{a_{t+1}}{a_t} \right] \tag{71}$$

The aggregate labor demand has the following form

$$l_t^d = \left[\frac{1}{\Lambda_t \alpha} \left[\frac{w_t}{a_t} + \frac{r}{q(\theta_t)} - \beta(1-s) E_t \left[\frac{r_t}{q(\theta_{t+1})} \frac{c_t}{c_{t+1}} \frac{a_{t+1}}{a_t} \right] \right] \right]^{\frac{1}{\alpha-1}} \tag{72}$$

$$\frac{\partial \mathcal{L}}{\partial p_t(k)} = 0 \Rightarrow$$
$$\frac{p_t(k)}{p_t} = \frac{\psi}{\psi-1} \Lambda_t(k) + \frac{\phi}{\psi-1} \frac{c_t}{y_t} \left(\frac{p_t(k)}{p_t} \right)^{\psi}$$
$$\left[\beta E_t \left[\left(\frac{p_{t+1}(k)}{p_t(k)} - 1 \right) \frac{p_{t+1}(k)}{p_t(k)} \right] - \left(\frac{p_t(k)}{p_{t-1}(k)} - 1 \right) \frac{p_t(k)}{p_{t-1}(k)} \right] \tag{73}$$

In the symmetric equilibrium, the relations $p_t(k) = p_t$ and $\Lambda_t(k) = \Lambda_t$ hold, therefore I first substitute $\left(\frac{p_t}{p_{t-1}} - 1 \right)$ for π_t and we rewrite (73) as

$$1 = \frac{\psi}{\psi-1} \cdot \Lambda_t + \frac{\phi}{\psi-1} \frac{c_t}{y_t} \left[\beta E_t \left[\pi_{t+1}(\pi_{t+1}+1) \right] - \pi_t(\pi_t+1) \right]$$

Next, multiplying the expression above by $(\psi - 1)$ and rearranging to obtain

$$1 = \frac{\psi}{\psi-1} \Lambda_t + \frac{\phi}{\psi-1} \frac{c_t}{y_t} \left[\beta E_t \left[\pi_{t+1}(\pi_{t+1}+1) \right] - \pi_t(\pi_t+1) \right]$$

$$\Leftrightarrow \psi - 1 + \phi \frac{c_t}{y_t} \pi_t \cdot (\pi_t + 1) = \psi \Lambda_t + \phi \cdot \frac{c_t}{y_t} \cdot \left[\beta E_t \left[\pi_{t+1}(\pi_{t+1}+1) \right] \right]$$

$$\Leftrightarrow \pi_t(\pi_t + 1) = \frac{1}{\phi}\frac{y_t}{c_t}[\psi\Lambda_t - (\psi - 1)] + \beta E_t\left[\pi_{t+1}(\pi_{t+1} + 1)\right] \tag{74}$$

Λ_t is expressed from the firm's labor demand (71) and then plugged into (74) in order to get an expression for the intermediate good firms optimal condition.

$$\begin{aligned}\pi_t(\pi_t + 1) =& \frac{1}{\phi}\frac{y_t}{c_t}\left[\frac{\psi}{\alpha a_t l_t^{\alpha-1}}\left(w_t + \frac{a_t r}{q(\theta_t)} - \beta(1-s)\frac{c_t}{c_{t+1}}\frac{a_{t+1} r}{q(\theta_{t+1})}\right) + (1-\psi)\right] + \\ & \beta E_t\left[\pi_{t+1}(\pi_{t+1} + 1)\right]\end{aligned} \tag{75}$$

Since the labor market is a combination of private and public employment, also the aggregate labor demand is a sum of private and public labor demand.

$$n^d = l^d + g \tag{76}$$

A.5 Wage schedule

Partially rigid wages are assumed to exist and the aggregated wage schedule has the following form:

$$w_t = w_0 a_t^{\gamma} \tag{77}$$

A.6 Monetary policy

The gross nominal interest rate rule is set as

$$r_t = \frac{1}{\beta}(1 + \pi_t)^{\mu_\pi(1-\mu_R)}(\beta r_{t-1})^{\mu_R} \tag{78}$$

A.7 Government budget constraint and resource constraint

The government budget constraint is as follows

$$n_t \cdot \tau_t\ w_t + \frac{b_t}{p_t} = g_t w_t + \frac{r a_t}{q(\theta)}[g_t - (1-s)g_{t-1}] + \frac{r_{t-1}}{p_t} b_{t-1} \tag{79}$$

and after plugging in (45) and the expression for firm's profits (69) the budget constraint becomes

$$y_t = c_t\left(1 + \frac{\phi}{2}\pi^2\right) + \frac{r a_t}{q(\theta)}[n_t - (1-s)n_{t-1}] \tag{80}$$

A.8 Equilibrium

In the symmetric equilibrium relations $y_t(k) = y_t$, $n_t(k) = n_t$, $l_t(k) = l_t$, $p_t(k) = p_t$ holds and thus the set of stochastic processes

$$\{w_t, \theta_t, n_t, l_t, \pi_t, c_t, y_t, r_t, a_t\}_{t=0}^{+\infty}$$

satisfies the following system of equations

- the wage schedule

$$w_t = w_0 a_t^{\gamma} \tag{81}$$

- the quasi-labor supply

$$n_t = (1-s)n_{t-1} + (1-(1-s)n_{t-1})f(\theta_t) \tag{82}$$

- the aggregate labor demand

$$n_t = (1-\zeta)l_t + \zeta g_t \tag{83}$$

- the intermediate good firms optimal condition

$$\begin{aligned}\pi_t(\pi_t+1) =& \frac{1}{\phi}\frac{y_t}{c_t}\left[\frac{\psi}{\alpha a_t l_t^{\alpha-1}}\left(w_t + \frac{a_t r}{q(\theta_t)} - \beta(1-s)\frac{c_t}{c_{t+1}}\frac{a_{t+1}r}{q(\theta_{t+1})}\right) + (1-\psi)\right] \\ &+ \beta E_t\left[\pi_{t+1}(\pi_{t+1}+1)\right]\end{aligned} \tag{84}$$

- the Euler equation

$$1 = \beta E_t\left[\frac{r_t}{1+\pi_{t+1}}\frac{c_t}{c_{t+1}}\right] \tag{85}$$

- the monetary policy rule

$$r_t = \frac{1}{\beta}(1+\pi_t)^{\mu_\pi(1-\mu_R)}(\beta r_{t-1})^{\mu_R} \tag{86}$$

- the production function

$$y_t = a_t \cdot l_t^{\alpha} \tag{87}$$

- the resource constraint

$$y_t = c_t \left(1 + \frac{\phi}{2}\pi^2\right) + \frac{ra_t}{q(\theta)}[n_t - (1-s)n_{t-1}] \tag{88}$$

- the labor productivity process

$$a_t = \rho a_{t-1} + \epsilon_t \tag{89}$$

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