The Application of Extreme Value Theory in Operational Risk Management

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Abstract

This paper focuses on modeling the real operational data of an anonymous Central European bank. We have applied the Extreme Value Theory, in which we have used two estimation methods – the standard maximum likelihood estimation method and the probability weighted moments (PWM). Our results proved a heavy-tailed pattern of operational risk data as documented by many researchers. Additionally, we showed that the PWM is quite consistent when the data is limited as it was able to provide reasonable and consistent capital estimates. Our findings show that when using the Advanced Measurement Approach rather than the Basic Indicator Approach used in Basel II, the researched bank might save approx. 6 – 8% of its capital requirement on operational risk.

Keywords: *operational risk, economic capital, bank, extreme value theory, probability weighted method*

JEL Classification: G18, G21, G32

Introduction

Operational risk has become one of the most discussed topics by both academics and practitioners in the financial industry in the recent years as documented by recent works by Carrillo-Menéndez and Suárez (2012), Cope, Piche and Walter (2012) or Plunus, Hübner and Peters (2012). The reasons for this attention can be attributed to higher investments in information systems and technology, the increasing wave of mergers and acquisitions, emergence of new financial instruments, and the growth of electronic dealing. BCBS (2006) defines operational risk as the risk of loss resulting from inadequate or failed internal

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processes, people and systems or from external events failures. This definition was incorporated in banking regulation Basel II that newly demanded a capital requirement for operational risk and further motivates financial institutions to more precisely measure and manage this type of risk (Matějašák, Černohorský and Teplý, 2009).

In the past years financial institutions have experienced several large operational loss events including recent scandals in JPMorgan Chase & Co. in 2012 (USD 6 billion), UBS in 2011 (USD 2.3 billion), Societe Generale in 2008 (USD 7.3 billion), Allied Irish Bank in 2002 (USD 700 million), Sumitomo Corporation in 1996 (USD 2.9 billion), Daiwa Bank in 1995 (USD 1.1 billion), Barings Bank in 1995 (USD 1 billion) and Orange County in 1994 (USD 1.7 billion). Operational risk also materialized during the US subprime mortgage crisis in 2007, when mortgage frauds became a serious issue. As noted by Dilley (2008), “mortgage applicants with weak financial standing or poor credit history have an obvious temptation to exaggerate their income or assets in order to secure a loan”. However, not only some applicants but also some mortgage dealers cheated as they intentionally offered mortgages to the people with a low creditworthiness. These dealers preferred own interests to adhering to prudence rules set by a financial institution, what could be considered as a fraud. We should also mention three operational risk failures materialized during the 2008 crisis: 65 billion USD swindle by Mr. Bernard Madoff, 8 billion USD fraud of Sir Allen Stanford or non-existence of 1 billion USD in a balance sheet of Indian company Satyam.

Moreover, there have also been several instances in the Central Europe when operational risk occurred. For instance, in 2000 a trader and his supervisor in one of the biggest Czech banks exceeded their trading limits when selling US treasury bonds and caused a USD 53 million loss to the bank. In the late 1990s another Central European bank suffered a USD 180 million loss as a result of providing financing to a company based on forged documents. Other general instances of operational risks in the Central European banks such as cash theft, fee rounding errors in IT systems or breakdowns of internet banking can be listed similarly to other banks around the world.

Although large operational losses are extreme events occurring very rarely (therefore called as black swan events), a bank – or a financial institution in general – has to consider the probability of their occurrence when identifying and managing future risks. In order to have reasonable estimates of possible future risks a bank needs an in-depth understanding of its past operational loss experience. As a result, a bank may create provisions for expected losses and set aside capital for unexpected ones (i.e. for the possibility that the actual loss might exceed the expected loss). In this paper we focus on modelling of the economic capital that should be set aside to cover unexpected losses resulting from operational risk failures.

The contribution of this study is threefold. The first contribution is the presentation of a possible methodology for operational risk management. Banks in Central Europe generally do not possess a methodology to model operational risk since they rely on the competence of their parent companies to calculate operational risk requirement on the consolidated basis of the whole group. Therefore, our study that proposes the complete methodology might be beneficial for banks willing to model their operational risk but not selected a sophisticated methodology yet.

Secondly, our study is an empirical study which uses real operational risk data from an anonymous Central European bank (the "Bank"). We are going to test various approaches and methods that are being used to model operational risk and calculate capital requirements based on the results. The final outcome of our study is to propose the model of operational risk that could be implemented by the Bank. Our estimates ought to be consistent with the real capital requirement of this bank.

Lastly, our analysis provides important results and conclusions. We have found out that even a general class distribution is not able to fit the whole distribution of operational losses. On the other hand, extreme value theory (EVT) appears more suitable to model extreme events. Additionally, we have discovered that traditional estimation using maximum likelihood does not provide consistent results while estimation based on probability weighted moments proved to be more coherent. We attribute it to limited dataset and conclude that probability weighted moments estimation that assign more weight to observations further in the tail of a distribution might be more appropriate to model operational loss events.

This paper is organised as follows: the first part provides a literature review; the second part presents a theoretical background : the modelling issues of operational risk and implications for economic capital. The methodology is described in the third part, while the fourth part describes the data used and the results of exploratory data analysis. The fifth part presents the results of our research and compares them with the findings of other studies. Finally, the last part concludes the paper and state final remarks.

1. Literature Overview

Until Basel II requirements in the mid 1990s, operational risk was largely a residual category for risks and uncertainties that were difficult to quantify, insure and manage in traditional ways. For this reasons one cannot find many studies focused primarily on operational risk until the late 1990s, although the term ‘operations risk’ already existed in 1991 as a generic concept of Committee of Sponsoring Organizations of the Treadway Commission.

Operational risk management methods differ from those of credit and market risk management. The reason is that operational risk management focuses mainly on low severity/high impact events (tail events) rather than central projections or tendencies. As a result, the operational risk modelling should also reflect these tail events which are harder to model (Jobst, 2007a). Operational risk can build ideas from insurance mathematics in the methodological development (Cruz, 2002). Hence one of the first studies on operational risk management was done by Embrechts, Klüpperberg and Mikosch (1997) who did the modelling of extreme events for insurance and finance. Later, P. Embrechts conducted further research in the field of operational risk (e.g. Embrechts, Furrer and Kaufmann, 2003; or Embrechts, Kaufmann and Samorodnitsky, 2006) and his work has become classic in the operational risk literature. Subsequently, Moscadelli (2004), de Fontnouvelle, Jordan and Rosengren (2005), Nešlehová, Embrechts and Chavez-Demoulin (2006) or Dutta and Perry (2007) experimented with operational loss data over the past few years. To this date Moscadelli (2004) is probably the most important operational risk study. He performed a detailed Extreme Value Theory analysis of the full QIS data set of more than 47,000 operational losses and concluded that the loss distribution functions are well fitted by generalised Pareto distributions in the upper-tail area. Operational risk modelling helps the risk managers to better anticipate operational risk and hence it supports more efficient risk management. There are several techniques and methodological tools developed to fit frequency and severity models including the already-mentioned EVT (Cruz, 2002; Embrechts, Kaufmann and Samorodnitsky, 2006; or Chernobai, Rachev and Fabozzi, 2007), Bayesian inference (Schevchenko and Wuthrich, 2006; or Cruz, 2002), dynamic Bayesian networks and expectation maximisation algorithms (Bee, 2006). The Bayesian approach to analysing operational risk caused by wilful default of a borrower is used by Janda (2007; 2009).

When modelling operational risk, other methods that change the number of researched data of operational risk events are used. The first ones are the robust statistic methods used Chernobai, Rachev and Fabozzi (2007) that exclude outliers from a data sample. On the other hand, a stress-testing method adds more data to a data sample and is widely used by financial institutions (Arai, 2006; Rosengren, 2006; or Rippel and Teplý, 2011). More recently, Hess (2011) analyze operational risk in the context of the 2007 – 2009 financial crisis with a primary focus on different business lines in a bank. Carrillo-Menéndez and Suárez (2012) discusses an issue of economic capital related to robust quantification of the exposure to operational risk. Cope, Piche and Walter (2012) research a data sample of over 57,000 losses incurred in more than 130 countries reported and investigate the relationships between the severity of operational loss events reported in the banking sector and various regulatory, legal, geographical, and economic indicators. Plunus, Hübner and Peters (2012) introduced a structural operational risk model named OpRisk+ applicable in financial institutions. Last but not least, Rippel, Suchánková and Teplý (2012) provide empirical analysis of the role of insurance in operational risk management in an anonymous bank located in Central Europe.

2. Theoretical Background

In this section we present basic terms related to operational risk and economic capital, what will serve a solid basis for our research. Moreover, we will discuss two main principles of modelling operational risk: top-down and bottom-up approaches.

2.1. Basics of Operational Risk

The above-mentioned definition of operational risk provided by BCBS (2006) encompasses a relatively broad area of risks, with the inclusion of for instance, transaction or legal risk (Table 1). However, the reputation risk (damage to an organisation through loss of its reputational or standing) and strategic risk (the risk of a loss arising from a poor strategic business decision) are excluded from the Basel II definition. The reason is that the term "loss" under this definition includes only those losses that have a discrete and measurable financial impact on the firm. Hence strategic and reputational risks are excluded, as they would not typically result in a discrete financial loss (Fontnouvelle et al., 2005). Other significant risks such as market risk and credit risk are treated separately in the Basel II.

Some peculiarities of operational risk exist compared to market and credit risks.[1] The main difference is the fact that operational risk is not taken on a voluntary basis but is a natural consequence of the activities performed by a financial institution (Sironi and Resti, 2007). In addition, from a view of risk management it is important that operational risk suffers from a lack of hedging instruments. Teplý and Vejdovec (2012) analysed economic capital distribution of

[1] For more details on risk management during the 2008 – 2009 global crisis we refer to, for instance, Černohorská, Teplý and Vrábel (2012), Buzková and Teplý (2012), Jakubík and Teplý (2011~~/2012~~), Teplý and Vejdovec (2012) or Vodová (2011).

TOP world banks and calculated the following shares of particular risks on total economic capital of the observed banks in the year of 2010: credit risk (60.9%) followed by market (18.5%), business risk (10.4%) and operational (10.1%). On a relate note, Nagafuji (2011) states that original Basel II proposals were targeted to a 12% operational risk on total regulatory capital. Despite these relative low shares on both types of capital in banking, operational risk events should not be underestimated (see operational risk events listed in Introduction).

Table 1

Operational Risk and Main Factors

People	Systems	Processes	External Events
Fraud, collusion and other criminal activities	IT problems (hardware or software failures, computer hacking or viruses etc.)	Execution, registration, settlement and documentation errors (transaction risk)	Criminal activities (theft, terrorism or vandalism)
Violation of internal or external rules (unauthorized trading, insider dealing etc.)	Unauthorized access to information and systems security	Errors in models, methodologies and mark to market (model risk)	Political and military events (wars or international sanctions)
Errors related to management incompetence or negligence	Unavailability and questionable integrity of data	Accounting and taxation errors Inadequate formalization of internal procedures	Change in the political, legal, regulatory and tax environment (strategic risk)
Loss of important employees (illness, injury, problems in retaining staff etc.)	Telecommunications failure	Compliance issues Breach of mandate	Natural events (fire, earthquake, flood etc.)
Violations of systems security	Utility outages	Inadequate definition and attribution of responsibilities	Operational failure at suppliers or outsourced operations

Source: Author based on Sironi and Resti (2007).

2.2. Economic Capital

A concept of economic capital is used for modelling operational risk through the Advanced Measurement Approach (AMA) under Basel II. Despite the term economic capital was firstly used in Merton and Perold (1993), no unique definition of economic capital still exists, however. In our paper we follow Chorafas (2006) who defines economical capital as "*the amount necessary to be in business – at a 99% or better level of confidence – in regard to assume risks*". While regulatory capital should cover (e.g. in the form of provisions) both expected losses and unexpected losses (but excluding extreme events) while economic capital should cover unexpected losses only. In addition, economic capital should cover both risk capital with 99.9% scenarios and capital for extreme events. The latter is important for modelling operational risk as "low frequency/high severity” losses often occur, what is supported by many researchers such as Chernobai, Rachev and Fabozzi (2007), Dutta and Perry (2007) or as it will be shown later, by our results. As the examples of extreme events, we can list 9/11 events in 2001, flooding in the Czech Republic in 2002, Hurricane Katrina in 2005, Hurricane Gustav in 2008 or the Fukushima disaster in 2011.

2.3. Modelling Operational Risk

There are two main ways to assess operational risk – the top-down approach and the bottom-up approach. Under the top-down approach, operational losses are quantified on a macro level only, without attempting to identify the events or causes of losses (Chernobai, Rachev and Fabozzi, 2007). The main advantage of these models is their relative simplicity and no requirement for collecting data. Top-down models include multifactor equity price models, capital asset pricing model, income-based models, expense-based models, operating leverage models, scenario analysis and stress testing and risk indicator models.

On the other hand, bottom-up models quantify operational risk on a micro level and are based on the identification of internal events. Their advantages lie in a profound understanding of operational risk events (the way how and why are these events formed). Bottom-up models encompass three main subcategories: process-based models (causal models and Bayesian belief networks, reliability models, multifactor causal factors), actuarial models (empirical loss distribution based models, parametric loss distribution based models, models based on extreme value theory) and proprietary models.[2] As recommended by many authors such as Chernobai, Rachev and Fabozzi, (2007) or van Lelyveld (2006), the best way for operational risk management is a combination of both approaches. In the paper we follow this best practice and employ bottom-up approaches for operational risk modelling and compare the results.

2.3.1. Top-down Approach of Modelling Operational Risk

Basel II provides an operational risk framework for banks and financial institutions. The framework includes identification, measurement, monitoring, reporting, control and mitigation of operational risk. Stated differently, it requires procedures for proper measurement of operational risk losses (i.e. ex post activities such as reporting and monitoring) as well as for active management of operational risk (i.e. ex ante activities such as planning and controlling). The Basel Committee distinguishes seven main categories of operational risk and eight

[2] For more detailed description of these models see Chernobai, Rachev and Fabozzi (2007), pp. 67 – 75.

business lines for operational risk measurement as depicted in the following table (Table 2).

Table 2

Business Lines and Event Types According to Basel II

Business line	Beta	Event type
1. Corporate Finance	18%	1. Internal Fraud
2. Trading & Sales	18%	2. External Fraud
3. Retail Banking	12%	3. Employment Practices and Workplace Safety
4. Commercial Banking	15%	4. Clients, Products and Business Practices
5. Payment & Settlement	18%	5. Damage to Physical Assets
6. Agency Services	15%	6. Business Disruption and System Failure
7. Asset Management	12%	7. Execution, Delivery and Process Management
8. Retail Brokerage	12%	

Source: BCBS (2006).

Basel II is based on three main pillars. Pillar I of Basel II provides guidelines for measurement of operational risk, Pillar II requires adequate procedures for managing operational risk and Pillar III sets up requirements on information disclosure of the risk. Basel II distinguishes three main approaches to operational risk measurement:

- Basic indicator approach (BIA),
- Standardised approach (SA),
- Advanced measurement approach (AMA).

Under the BIA, the simplest approach, gross income serves as a proxy for the scale of operational risk of the bank. Hence the bank must hold capital for operational risk equal to the average over the previous three years of a fixed percentage (denoted as alpha, α) of positive annual gross income of the bank. Alpha was set at 15%.

The SA[3] is very similar to the BIA, only the activities of banks are dividend into eight business lines. Within each business line, gross income is a broad indicator of operational risk exposure. Capital requirement ranges from 12 to 18% (denoted as beta, β) of gross income in the respective business line (see Table 2). Different business lines pose different risks, however. Hess (2011) found a significant impact on the riskiness of the loss severity in different lines. Based on the financial crisis data in the 2007 – 2009 period, he computed a 150% higher value-at-risk (VaR) for the business line trading and sales and a 50% higher VaR for the business retail brokerage.

The last method, AMA, ranks to advanced methods applied in operational risk management according to Basel II and as a bottom-up approach will be dis-

[3] An alternative approach to the SA exists – the Alternative Standardised Approach (ASA), which uses for the Retail Banking and the Commercial Banking total loans and advances as a proxy for the scale of operational risk of the bank (instead of gross income).

cussed in the following part (sees also BCBS, 2006; or more recently BCBS, 2011).

2.3.2. Bottom-up Approaches of Modelling Operational Risk

Under the AMA, the regulatory capital requirement shall equal the risk measure generated by the bank's internal operational risk measurement system. The bank must meet certain qualitative (e.g. quality and independence of operational risk management, documentation of loss events, regular audit) and quantitative (internal and external data collection, scenario analysis) standards to qualify for using the AMA. For instance, a bank must demonstrate that its operational risk measure is evaluated for one-year holding period and a high confidence level (99.9% under Basel II). The use of the AMA is subject to supervisory approval, what makes it similar to internal rating models (IRB) for credit risk management under Basel II. Theoretically, IRB and AMA models should reflect real risks of bank better than basic models for calculating regulatory capital, what should converge to economic capital. We argue that these internal models sometimes serves as a reason for lower bank´s capital rather than a mirror of the true risk of a bank, what was documented in the case of a UK bank Northern Rock in 2007.

The above-mentioned description of three approaches indicates that the BIA is the simplest while the AMA is the most advanced. The idea behind Basel II requirements lies in the assumption that the AMA capital charge (K_{AMA}) should be lower than K_{BIA} and K_{SA}. Therefore banks should be motivated to use the most advanced approach (AMA), since the lower capital charge hold by a bank should imply, ceteris paribus, its higher profitability.

At present most banks use a combination of two AMA approaches to measure operational risk: (i) the loss distribution approach (LDA), which is a quantitative statistical method analysing historical loss data, and (ii) the scorecard approach, which focuses on qualitative risk management in a financial institution. The above-mentioned approaches complement each other. As a historical data analysis is backward-looking and quantitative, the scorecard approach encompasses forward-looking and qualitative indicators. In our analysis we concentrate on the first approach because of the data availability. However, a combination of both approaches is necessary for successful operational risk management (Chalupka and Teplý, 2008).

Several caveats of the AMA within Basel II exist. Jobst (2010) lists the following three main shortcomings: first, the use of VAR concepts in the LDA is questionable (VAR is an incoherent risk measure and hence cannot reflect properly the heavy-tail pattern of operational risk events). Second, capital adjustment of operational risk estimates can underestimate true operational risk in case of the lack of data (maximum 20% capital deduction resulting from diversification effects might work only if dependencies are robust and fair, what is not always the case). Finally, the lack of underlying of operational risk events is causing a bias created by divergent loss reporting and data collection.

3. Methodology

In this part we describe applied methodology in our research: Extreme value theory (EVT), for details on the LDA method we refer to Nešlehová, Embrechts and Chavez-Demoulin (2006) or Jobst (2007b). Extreme value theory is a promising class of approaches to modelling of operational risk. Although originally utilised in other fields such as hydrology or non-life insurance, EVT is capable of modelling low frequency, high severity instances of operational losses. There are two main kinds of models in EVT: block maxima models and peak over threshold (POT) models (Chernobai, Rachev and Fabozzi, 2007). More traditional models are block maxima models which are for the largest observations collected from large samples of identically distributed observations. The whole sample is divided into equal non-overlapping time intervals and the biggest loss from each interval is used for modelling. In the POT model (or the threshold exceedances model, a more-modern approach, the large enough threshold is determined and the observations above are considered. For both block maxima and POT there is a theorem regarding limiting distribution.

In this section we will discuss block maxima models in more detail. Using the Fisher-Tippet and Gnenenko theorem the limiting distribution for normalised maxima is the GEV distribution (for more details see e.g. Embrechts, Kaufmann and Samorodnitsky, 2006). The distribution function of the (standard) GEV distribution is given by

$$F(x) = \begin{cases} \exp\left\{ -\left(1+\xi \dfrac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}} \right\} & if \quad \xi \neq 0 \\ \exp\left\{ -e^{-\frac{x-\mu}{\sigma}} \right\} & if \quad \xi = 0 \end{cases}$$

where (following Chernobai, Rachev and Fabozzi, 2007)

$$1+\xi \frac{x-\mu}{\sigma} > 0 \quad x > \mu - \frac{\sigma}{\xi} \quad if\ \xi > 0 \quad x < \mu - \frac{\sigma}{\xi} \quad if\ \xi < 0 \quad x \in R \quad if\ \xi > 0\ ;$$

where

x	– refers to the maxima,
$\mu \in R$, and $\sigma > 0$, μ	– the location parameter,
σ	– the scale parameter,

ξ – the shape parameter.

The GEV distribution can be divided into three cases based on the value of the shape parameter. For $\xi > 0$, the GEV is of the *Fréchet* case which is particularly suitable for operational losses as the tail of the distribution is slowly varying (power decay), hence it is able to account for high operational losses. It may be further shown that $E(X^k)=\infty$ for $k > 1/\xi$, thus for instance if $\xi \geq 1/2$ a distribution has infinite variance and higher moments (Embrechts, Klüppelberg and Mikosch, 1997).

The *Gumbel* case ($\xi = 0$) is also plausible for operational losses, although a tail is decreasing faster (exponential decay), it has a heavier tail than the normal distribution. The moments are always finite ($E(X^k) < \infty$ for $k > 0$). The *Weibull* case ($\xi < 0$) is of the least importance as the right endpoint is finite, hence unable to model heavy tails of operational losses. The GEV distribution can be fitted using various methods, we are going to describe and use the two most commonly used, maximum likelihood and probability-weighted moments. Denoting $f_{\xi,\mu,\sigma}$ the density of the GEV distribution, and $M_1 \ldots M_m$ being the block maxima, the log-likelihood is calculated to be

$$l(\xi, \mu, \sigma; M_1 \ldots M_m)$$

$$= \sum_{i=1}^{m} \ln f_{\xi,\mu,\sigma}(M_i) = -m \ln \sigma - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^{m} \ln\left(1 + \xi \frac{M_i - \mu}{\sigma}\right) - \sum_{i=1}^{m} \ln\left(1 + \xi \frac{M_i - \mu}{\sigma}\right)^{-\frac{1}{\xi}}$$

which must be maximised subject to the parameter constraints that $\sigma > 0$ and $1 + \xi(M_i - \mu)/\sigma > 0$ for all i. (for more details see Embrechts, Kaufmann and Samorodnitsky, 2006).

Probability weighted moments (PWM), the second used approach to estimate parameters of GEV, has better applicability to small samples than maximum likelihood (ML) method (Landwehr et al., 1979). Following Hosking, Wallis and Wood (1985), although probability weighted estimators are asymptotically inefficient compared to ML estimators, no deficiency is detectable in samples of 100 or less. As the number of extreme observations is typically limited, this property of PWM makes it very valuable in operational risk modelling.

4. Data Analysis

In this study we have used data from an internal database of the Bank.[4] Altogether the dataset consists of more than 600 operational losses over the period 2001 – 2007 (Table 3). However, there are disproportionally fewer observations in the beginning of the sample (January 2001 – November 2003) signaling lower quality of data when the process of collecting operational losses data was just starting. In order to remove possible bias, we have left out 14 observations of this period.

T a b l e 3

Data Sample Statistics of the Whole Sample (in EUR)

Mean	Median	Standard deviation	Skewness	Kurtosis
41.738	3.114	280.538	14	225

Source: Author´s calculations based on the Bank´s database.

Moreover, the threshold for collecting the data in the Bank (about 1,000 USD) is set quite low compared to other studies, the threshold is typically of the order of 10,000 USD, hence we further cut some of the observations from the beginning as commonly applied in the operational risk literature (de Fontnouvelle, Jordan and Rosengren, 2005; or Chernobai, Rachev and Fabozzi, 2007). By setting the threshold up to 10,000 USD we have left out many small losses, hence the number of observation in our dataset further decreased up to 236.[5] Observations across years starting from December 2004 are by a simple graphical inspection quite stationary and hence can be considered to be collected by consistent methodology. However, there is a significant variation across months; particularly losses in December are significantly more frequent. This can be explained by the end of fiscal year when all possible unrecorded losses up to a date finally appear on the books. This is not a problem when losses are treated on annual basis or independent of time, however, it hinders the possibility to take into account monthly information. Generally, our dataset is not very big, but it is satisfactory enough for operational risk analysis at the level of the whole bank. For analysis focusing on particular business lines and/or particular type of loss events we would need more observations, however.

5. Empirical Results

[4] This bank is located in the Czech Republic but we keep its name secret as to preserve its confidentiality.

[5] Although the number of observations left out is high, they account only for about 2.5% of the sum of total operational losses in the sample. A 10,000 USD threshold is commonly used in operational risk modelling (see Duta and Perry, 2007; or Chernobai, Rachev and Fabozzi, 2007).

In this part we present empirical results our research. First, we present the LDA results using simple parametric distributions such as exponential, gamma, lognormal and log-logistic. Second, we show our results based on block maxima models. Finally, we provide a summary of the results and discuss further research opportunities.

5.1. Loss Distribution Approach

As would be expected, the simple parametric distributions with one or 2-parameters are far too simple to model operational loss data reporting a heavy tail pattern. Although moving from exponential to a gamma distribution and from a gamma to a lognormal or a log-logistic distribution somewhat improves the fit and the test statistics, we reject the hypothesis that the data follow any of these distributions (Table 4). The reason is that the losses in the end of the tail of the distribution are significantly underpredicted and the analyzed distributions failed to capture the unusual (heavy-tailed) pattern of operational risk data as documented by many researchers such as Cruz (2002), Moscadelli (2004), de Fontnouvelle, Jordan and Rosengren (2005) or Duta and Perry (2007).

T a b l e 4

Simple Parametric Distributions – the Goodness-of-fit Statistics (p-values)

Distribution	MLE*	
	√nD	√nV
Exponential	<0.01	<0.01
Gamma	<0.01	<0.01
Lognormal	<0.01	<0.01
Log-logistic	<0.01	<0.01

Note: *MLE – maximum likelihood estimation; √nD – stands for the Kolmogorov-Smirnov; (√nV) – the Kuiper statistic.

Source: Author´s calculations.

Although none of the parametric distributions got close to a reasonable fit, we have still calculated a risk informative indicator recognised by Basel II requirements, VAR,[6] for these models (Table 5). The first three distributions provide relatively low capital requirements in the range (2.0 – 2.7%). Based on the log-logistic distribution the calculated capital requirement is much higher as this distribution allow for higher losses. Finally, the GH distribution provides unreasonably high capital requirement owning to the high shape parameter and over-prediction of the highest losses.

T a b l e 5

Summary of Calculated VAR – Parametric Distributions

Distribution	VAR (99.9%) – Monte-Carlo	
	MLE*	QE**
Exponential	2.7%	
Gamma	2.1%	
Lognormal	2.0%	
Log-logistic	9.5%	
GH distribution		>100%

Notes: *MLE – maximum likelihood estimation, **QE – quantile estimation.

Source: Author´s calculations.

5.2. Block Maxima Models

Two different scenarios have been employed when applying the block maxima model, the highest losses in each month and the highest dozen (twelve) of losses.[7] For each scenario the parameters were estimated by maximum likelihood estimation (MLE) and probability weighted moments (PWM) methods. Although both estimation methods indicate a heavy tail of the distribution, MLE and PWM yield quite different results for both block maxima models. While for PWM the parameters are less than one, (even less than 0.5 for the second model indicating finite variance) the parameters derived from MLE are well above one (infinite mean), indicating extremely heavy tailed data.

Table 6 depicts the goodness-of-fit statistics, the Kolmogorov-Smirnov (√nD) and the Kuiper statistic (√nV), if the p-value is below 1%, the hypothesis of a good fit of the model is rejected on the 1% significance level. On the contrary, if it is above 10%, the model appears as very appropriate to model the data. The other cases are in-between these two boundary cases.

T a b l e 6

Block Maxima Models – the Goodness-of-fit Statistics (p-values)

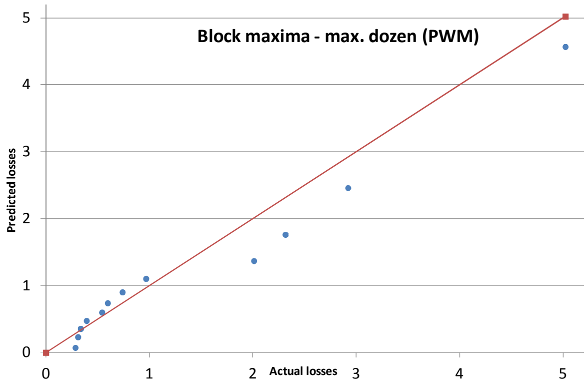
	MLE		PWM	
	√nD	√nV	√nD	√nV
Max. each month	<0.01	<0.01	>0.01	<0.01
Max. dozen	<0.01	>0.10	>0.10	>0.10

Source: Author´s calculations

[6] Jorion (2007, p. 2) defines VAR as “the maximum loss over a target horizon such that there is a low, perspecified probability that the actual loss will be higher”.

Figure 1

Block Maxima Model – QQ-plot for Max. Dozen Model Fitted by PWM



Note: The actual numbers on the axes were replaced by normalised numbers as to preserve the confidentiality of the data.

Source: Author´s calculations.

From the above table we can conclude that the second model (the maximum dozen model) fitted by PWM produces the best results, while the use of MLE for the first model can be rejected. The other two cases deliver mixed results. Figure 1 shows that although the maximum dozen model estimated by PWM slightly underpredicts the highest losses, the fit of the data is very good, supporting the adequacy of this model.

5.3. Summary of Results and Further Research Opportunities

Our result proves an unusual (heavy-tailed) pattern of operational risk data as documented by many researchers. The LDA, in which parametric distributions are fitted to the whole data sample, was not able to capture the pattern of the data and was rejected based on the goodness-of-fit statistics. Hence we conclude that the parametric distributions like exponential, gamma, log-normal, log-logistic and GH do not fit well the data. This result proves an unusual (heavy-tailed) pattern of operational risk data.

The EVT, on the other hand, for both block maxima and POT proved to fit the data in the tail of the distribution. We have used two estimation methods in the EVT approach, the standard MLE in which all the observation have the same weight and the PWM in which the observations higher in the tail have a higher weight. When applying the block maxima model we have found out that the maximum dozen model fitted by PWM produces the best results. Cruz (2002)

[7] As the twelve losses are not the maximas as defined in the theorem for the limiting distribution, there is no assurance that this scenario will even in the limit follow the GEV distribution. However, the GEV can still be a good model that fits the data well.

used PWM to analyse fraud loss data on an undisclosed source for the 1992 – 1996 period and deduced that the data in 1994 and 1996 recorded a heavy-tailed GEV distribution. POT models are frequently used for application of EVT to operational loss data. We observed that the high shape parameters for some of the MLE models bring unreasonable high capital estimates, what is consistent with Moscadelli (2004) or de Fontnouvelle, Jordan and Rosengren (2005). These authors also mention the estimates are highly sensitive to the chosen threshold, what again underpins our conclusions.

Table 7 presents a summary of our research. As we indicated earlier, EVT shows the best statistical fit when estimating capital of the Bank on a 99.9% confidence level. The EVT methods imply capital requirements for operational risk of the Bank in the range of 7.2 – 9.2%. For a comparison, Basel II requires banks to hold a capital requirement for operational risk at 15% of banking income in case of using the BIA. As a result, when using the AMA rather than the BIA, the Bank might save approx. 6 – 7% of the capital requirement on operational risk.

Table 7

Summary of Results – LDA and Selected EVT Models

Body	**Tail**	**Statistical fit**	**Capital estimate (99.9%)***
Exponential	Exponential	very poor	2.7%
Gamma	Gamma	very poor	2.1%
Lognormal	Lognormal	poor	2.0%
Log-logistic	Log-logistic	poor	9.5%
GH distribution	GH distribution	poor	>100%
Empirical sampling	EVT (block maxima, max. dozen, PWM)	excellent	7.2%
Empirical sampling	EVT (block maxima, max. 2%, PWM)	excellent	9.2%

Note: * As % of gross income of the Bank.

Source: Author´s calculations.

As we have mentioned earlier, Central European banks usually do not possess a methodology to model operational risk since they rely on the competence of their parent companies to calculate operational risk requirement on the consolidated basis of the whole group. The question is, if there is anything to gain from shifting the calculation of operational risk capital requirement to the subsidiary level. Although the PWM methodology might give reasonable results for a subsidiary, parent companies need to consolidate capital requirements of their subsidiaries (not only operational risk but also other risks such as credit, market and other risks). Therefore the parent companies use their models and the subsidiaries usually provide these models only with some modifications (e.g. more data or scenario analysis). As documented both in the theory (OWC, 2001) and practice (Deutsche Bank, 2007; or BBVA, 2007), this portfolio approach brings a diversification effect resulting in a lower capital requirement. For instance, Deutsche

Bank (2007) recorded a 20% positive diversification effect of an overall economic capital requirement in the year 2007. Similarly, Banco Bilbao Vizcaya Argentaria estimated a 45 – 58% positive diversification effect for operational risk capital requirement in 2007. However, some authors raise doubt over the possibility decrease the total capital requirement on operational risk due to diversification effects. For instance, Jobst (2010) argues that capital deduction resulting from diversification effects might work only if statistical dependencies in the underlying data are robust and fair, what is not always the case for registered operational risk events.

Despite the conclusions cited above, there are still several ways in which our research can be improved. Firstly, a similar study can be done on a larger sample of data (we used the data from one Central European Bank). Secondly, the research provided on all eight business lines recognised by Basel II may reveal interesting facts about different operational risk features among various business lines (see Table 2). Finally, other research might include other results derives from modelling operational risk using such techniques as robust statistics, stress-testing, Bayesian inference, dynamic Bayesian networks and expectation maximisation algorithms.

Conclusion

In this paper we analysed and modelled real operational data of an anonymous Central European Bank. We have utilised two approaches currently described in the literature. First, the LDA, in which parametric distributions are fitted to the whole data sample, was not able to capture the pattern of the data and was rejected based on the goodness-of-fit statistics. Second, we employed the EVT and showed that both block maxima and POT models proved to fit the data in the tail of the distribution. We have used two estimation methods in the EVT approach, the standard MLE in which all the observation have the same weight and the PWM in which the observations higher in the tail have a higher weight. When applying the block maxima model we have found out that the maximum dozen model fitted by PWM produces the best results. In addition, the Kuiper statistics for PWM showed the best results in all four years, which confirms our findings.

Unlike the others, our research showed that PWM are quite consistent from a practical point of view and they might be suitable in the estimation of operational risk when data is limited. This result might be useful for the banks that have limited data series of operational risk events, what is typical for many Central European banks. Our findings show that when using the AMA rather than the BIA used in Basel II, the researched bank might save approx. 6 – 7% of its capital requirement on operational risk.

From a policy perspective it should be hence noted that banks from emerging markets such as the Central Europe are also able to register operational risk events. Data from the Bank showed an improvement in time, what could be attributed to more attention devoted to recording operational risk events. Moreover, as we have demonstrated, the distribution of these risk events can be estimated with a similar success than those from more mature markets.

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