Institute of Economic Studies, Faculty of Social Sciences
Charles University in Prague

Wavelet Applications to Heterogeneous Agents Model

Lukáš Vácha
Miloslav Vošvrda

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Institute of Economic Studies,
Faculty of Social Sciences,
Charles University in Prague

[UK FSV – IES]

Opletalova 26
CZ-110 00, Prague
E-mail : ies@fsv.cuni.cz
http://ies.fsv.cuni.cz

Institut ekonomických studií
Fakulta sociálních věd
Univerzita Karlova v Praze

Opletalova 26
110 00 Praha 1

E-mail : ies@fsv.cuni.cz
http://ies.fsv.cuni.cz

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Wavelet Applications to Heterogeneous Agents Model

Lukáš Vácha[*]
Miloslav Vošvrda[*#]

[*] IES, Charles University Prague

[#] Institute of Information Theory and Automation,
Academy of Sciences of the Czech Republic,
E-mail: vosvrda@utia.cas.cz

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Abstract:
A heterogeneous agent model with the WOA was considered for obtaining more realistic market conditions. The WOA replaces periodically the trading strategies that have the lowest performance level of all strategies presented on the market by the new ones. New strategies that enter on the market have the same stochastic structure as an initial set of strategies. This paper shows, by wavelets applications, strata influences of the trading strategies with the WOA.

Keywords: agents' trading strategies, heterogeneous agent model with stochastic memory, worst out algorithm, wavelet

JEL: C061; G014; D084

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1. Introduction

Modern finance undergoes an important change of an economic agent perceiving i.e., from a representative, rational agent approach towards a behavioral, agent-based approach in which markets represented by boundedly rational, heterogeneous agents using rule of thumb strategies. In the traditional approach, simple analytically tractable models with a representative, perfectly rational agent have been the main corner stones and mathematics has been the main tool of analysis. The new behavioral approach fits much better with agent-based simulation models and computational and numerical methods have become an important tool of analysis [7]. The new behavioral, heterogeneous agents approach challenges the traditional representative, rational agent framework. Heterogeneity in expectations can lead to market instability and complicated dynamics of prices. Prices are driven by endogenous market forces. In a heterogeneous agent model typically two types of agents are distinguished, fundamentalists and chartists. Fundamentalists base their expectations about future asset prices and their trading strategies upon market fundamentals and economic factors, such as dividends, earnings, macroeconomic growth, unemployment rates, etc. Chartists or technical analysts try to extrapolate observed price patterns, such as trends, and exploit these patterns in their investment decisions. One of the expressive models was developed by Brock and Hommes in 1998 (see [2]). In our early work (see [13]) we were focused on a simple heterogeneous agent model with two or four types of beliefs. These beliefs were fixed for all our simulations. In our previous papers (see [14, 15, 16]), we employed a memory and some learning schemes in the Brock and Hommes's model. In this paper we use the core of the Brock and Hommes's model on which we base further extensions, such as a stochastic formation of beliefs and parameters including a memory length. Another extension is in an application of the WOA. In [17] we show how a memory length distribution in the agents' performance measure affects a persistence of the generated price time series. Our motivation is to trace a memory length in the price time series with different replacement ratios of the improved WOA. The implementation of the WOA should increase a persistence of returns. A wavelet analysis is a convenient tool for an activity detection on various scales of the simulated price time series. This one is more convenient tool for the frequency detection in price time series than the Fourier analysis because the price changes in the market mood are better detected in time. The wavelet analysis uses time-scale domain instead of time-frequency domain. Capital markets have a typical stylized fact in existence clusters of both high positive returns and low negative returns in the realizations of the price time series. We can retrospectively analyze which a part of the trading strategies set was used on the capital market and we can estimate their statistical properties.

2. Model

Financial markets are considered as systems of the interacting agents processing new information immediately. Prices are driven by endogenous market forces. Agents adapt their predictions by choosing among a finite number of predictors (see [1]). Each predictor has a performance measure. Based on this performance measure, agents realize a rational choice among the predictors (see [2]). This approach relied on heterogeneity in the agent information and subsequent decisions either as fundamentalists or as chartists (see [4], [5]). Let we consider an asset-pricing model with one risky asset and one risk-free asset. Let pt be the share price (ex dividend) of the risky asset at time t, and let {yt} be an i.i.d. stochastic dividend process of the risky asset. The risk free asset is perfectly elastically supplied and pays a fixed rate of return r. The gross return rg is equal 1+ r. The risky asset pays a random dividend. The dynamics of wealth can then be written as

$$\mathbf{W}_{t+1} = r^g \cdot W_t + \left(\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t\right) \cdot z_t \quad , \tag{1.1}$$

where z_t denotes the number of shares of the asset purchased at time t, and a bold face type denotes random variables at date t. Let Et and Vt denote the conditional expectation and conditional variance operators, based on the publicly available information set consisting of past prices and dividends, i.e., on the information set □t = {pt, pt-1,…; yt, yt-1 ,…}. Let Eh,t, Vh,t denote forecasts of investor of type h about a conditional expectation and conditional variance. Investors are supposed to be a myopic mean-variance maximizer so that the demand zh,t. for risky asset is obtained by solving the following criterion

$$\max_{z_t} \left\{ E_{h,t}\left[\mathbf{W}_{t+1}\right] - (a/2) \cdot V_{h,t}\left[\mathbf{W}_{t+1}\right] \right\} \quad , \tag{1.2}$$

where a risk aversion, a, is here assumed to be the same for all traders. Thus the demand zh,t. of type h for risky asset has the following form

$$E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t\right] - a \cdot \sigma^2 \cdot z_{h,t} = 0 \quad , \tag{1.3}$$

$$z_{h,t} = E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t\right] / \left(a \cdot \sigma^2\right)$$

assuming that the conditional variance of excess returns is a constant for all investor types

$$V_{h,t}\left(\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t\right) \equiv \sigma_h^{\,2} = \sigma^2 \quad . \tag{1.4}$$

Let zs be a supply of outside risky shares per investor. Let nh,t be a fraction of type h at date t. The equilibrium of demand and supply is

$$\sum_{h=1}^{H} n_{h,t} \left\{ E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t\right] / a \cdot \sigma^2 \right\} = z^s \tag{1.5}$$

where H is the number of different investor types. For the special case of zero supply, i.e., zs = 0, the market equilibrium is as follows

$$r^g \cdot p_t = \sum_{h=1}^{H} n_{h,t} \cdot E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1}\right] \tag{1.6}$$

If there is only one investor type, the market equilibrium yields the following pricing equation

$$r^g \cdot p_t = E_t\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1}\right] \tag{1.7}$$

It is well known that, using the arbitrage (1.7) repeatedly and assuming that the transversality condition

$$\lim_{k \to \infty} \frac{E_t\left[\mathbf{p}_{t+1}\right]}{\left(r^g\right)^k} = 0 \tag{1.8}$$

holds, the fundamental price of the risky asset is uniquely obtained by

$$p_t^* = \sum_{k=1}^{\infty} \frac{E_t\left[\mathbf{y}_{t+k}\right]}{\left(r^g\right)^k} \tag{1.9}$$

Thus the fundamental price pt* depends on the stochastic dividend process {yt}. From the equation (1.7) we obtain the following price equation

$$p_t = p_t^* + \left(r^g\right)^t \cdot \left(p_0 - p_0^*\right) \tag{1.10}$$

For our purpose, it is better to work with the deviation xt from the benchmark fundamental price pt*, i.e., $x_t = p_t - p_t^*$.

3. Evolutionary Dynamics of Investors

Let us admit the following assumptions:

A1)

$$E_{h,t}\left[\mathbf{y}_{t+1}\right] = E_t\left[\mathbf{y}_{t+1}\right], \tag{2.1}$$

A2)

$$V_{h,t}\left(\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t\right) = V_t\left(\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t\right) = \sigma_t^{\ 2}, \tag{2.2}$$

A3) all forecasts $E_{h,t}\left[\mathbf{p}_{t+1}\right]$ have the following form

$$E_{h,t}\left[\mathbf{p}_{t+1}\right] = E_t\left[\mathbf{p}^*_{t+1}\right] + f_h^{\mathbf{L}}\left(x_{t-1},\ldots,x_{t-\mathbf{L}}\right). \tag{2.3}$$

Each forecast $f_h^{\mathbf{L}}$ represents a model of the market for which type h believes that prices deviate from the fundamental price. Let us concentrate on the evolutionary dynamics of the fractions nh,t of different h-investor types, i.e.

$$r^g \cdot x_t = \sum_{h=1}^{H} n_{h,\,t-1} \cdot f_h^{\mathbf{L}}\left(x_{t-1},\ldots,x_{t-\mathbf{L}}\right) \equiv \sum_{h=1}^{H} n_{h,\,t-1} \cdot f_{h,\,t}^{\mathbf{L}} \tag{2.4}$$

where nh,t-1 denotes the fraction of investor type h at the beginning of period t, before than the equilibrium price xt has been observed and L is a random number of lags. Now the realized excess return over period t to the period t+1 is computed by

$$\mathbf{Z}_{t+1} = \mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t \tag{2.5}$$

We need now a measure of profits generated by forecasts $f_{h,\,t}^{\mathbf{L}}$. Let a performance measure $\pi_{h,t}$ be defined by

$$\pi_{h,t} = E_t\left[\frac{\mathbf{Z}_{t+1} \cdot \rho_{h,t}}{a \cdot \sigma_t^2}\right] \tag{2.6}$$

where

$$\rho_{h,t} = E_{h,t}\left[\mathbf{Z}_{t+1}\right] = f_{h,t}^{\mathbf{L}} - r^g \cdot x_t = f_{h,t}^{\mathbf{L}} - \sum_{j=1}^{H} n_{j,t} \cdot f_{j,t}^{\mathbf{L}} = f_{h,t}^{\mathbf{L}} \cdot \left(1 - \sum_{j \neq h}^{H} n_{j,t} \cdot f_{j,t}^{\mathbf{L}}\right). \tag{2.7}$$

So the π-performance is given by the realized performance for the h-investor. Let the updated fractions nh,t be given by the discrete choice probability (Gibb's distribution)

$$n_{h,t} = \exp\left(\beta \cdot \pi_{h,t-1}\right) / Y_{t-1} \tag{2.8}$$

where

$$Y_t = \sum_{j=1}^{H} \exp\left(\beta \cdot \pi_{j,t}\right) \tag{2.9}$$

The parameter β is the intensity of choice. The parameter β is a measure of investor's rationality. If the intensity of choice is infinite (β = +∞), the entire mass of investors uses the strategy that has the highest performance. If the intensity of choice is zero, the mass of investors distributes itself evenly across the set of available strategies. All forecasts will have the following form

$$f_t^{\mathbf{L}} = g \cdot \left(x_{t-1} + \cdots + x_{t-l}\right) + b \tag{2.10}$$

where the g denotes the trend of investor and the b denotes the bias of investor. If b = 0, the investor is called a pure trend chaser if g > 0 and a contrarian if g < 0. If g = 0, investor is called purely biased. Investor is upward (downward) biased if b > 0 (b < 0). In the special case g = b = 0, the investor is called fundamentalist, i.e., the investor believes that price return to their fundamental value. Fundamentalists strategy is based on all past prices and dividends in their information set, but they do not know the fractions nh,t of the other belief types.

4. Simulations and the WOA

For simulation, an updated version of the WOA is used. The algorithm replaces zero, one, two, three, four, five, six, and eight strategies with the lowest performance by sequel 0, 1, 2, 3, 4, 5, 6, and 8 (i.e. 0WOA, 1WOA, 2WOA, 3WOA, 4WOA, 5WOA, 6WOA, and 8WOA). A set of strategies is composed from fifteen different strategies, i.e., the replacement ratio of the market strategies are from 0% to 53.3%. The high replacement ratio is implemented for a simulation of dramatic changes in the mood on the market. From such conditions on market, there is a bigger chance of the price turbulence emergence. The WOA replaces periodically the trading strategies that have the lowest performance level of strategies presented on the market by the new ones. The new strategies that enter on the market are taken from the set that has the same stochastic parameters as the initial strategies, i.e., the trend g~N(0,0.16), the bias b~N(0,0.09), the memory length m~U(1,100). Simulations are performed with fifteen agents or beliefs represented by trading strategies, the intensity of choice, β, are set to 120. The WOA makes the replacement after 40 iterations. For example, when we want to replace four strategies with the lowest performance (4WOA, replacement ratio is 26.6%) the algorithm after every 40 iterations evaluate and arrange in descending order the performance of all fifteen strategies in the market and the last four replaces by the new ones. Number of observations in our simulations is 8192.

For a better understanding of the evolution dynamics with the WOA we compare eight cases that differ in the replacement ratio. The first one is without WOA (0WOA, replacement ratio 0%), the last one replaces eight strategies (8WOA, replacement ratio 53.3%).

4.1. Analysis of the Price Returns Time Series

For estimating and analyzing of correlation structures on capital markets, a nonparametric method is used. H. E. Hurst discovered very robust nonparametric methodology, which is called rescaled range, or R/S analysis that is used for estimating the Hurst exponent (see [12]). The R/S analysis was used for distinguishing random and non-random systems, the persistence of trends, and duration of cycles. This method is very convenient for distinguishing random time series from fractal time series as well. Starting point for the Hurst's coefficient was the Brownian motion as a primary model for random walk processes. If a system of random variables is an independently, identically distributed, then $H = 0.5$. The values of Hurst exponent belonging to $0 < H < 0.5$ signifies an anti-persistent system of variables covering less space than random ones. Such a system must reverse itself more frequently than a random process; we can equate this behavior to a mean-reverting process. The values of Hurst exponent belonging to $0.5 < H < 1$ signifies a persistent process that is characterized by long memory effects. This long memory occurs regardless of time scale, i.e., there is no characteristic time scale, which is the key characteristic of fractal time series (see [12], [18]).

For the 0WOA case (no replacement of strategies), presented market strategies are generated randomly and the Hurst exponent, as we expect, is close to the Efficient Market Hypothesis (EMH) case, i.e., 0.5. When the WOA is implemented, we can see a strong learning effect that is transformed to long-memory (persistent) behavior of price returns.

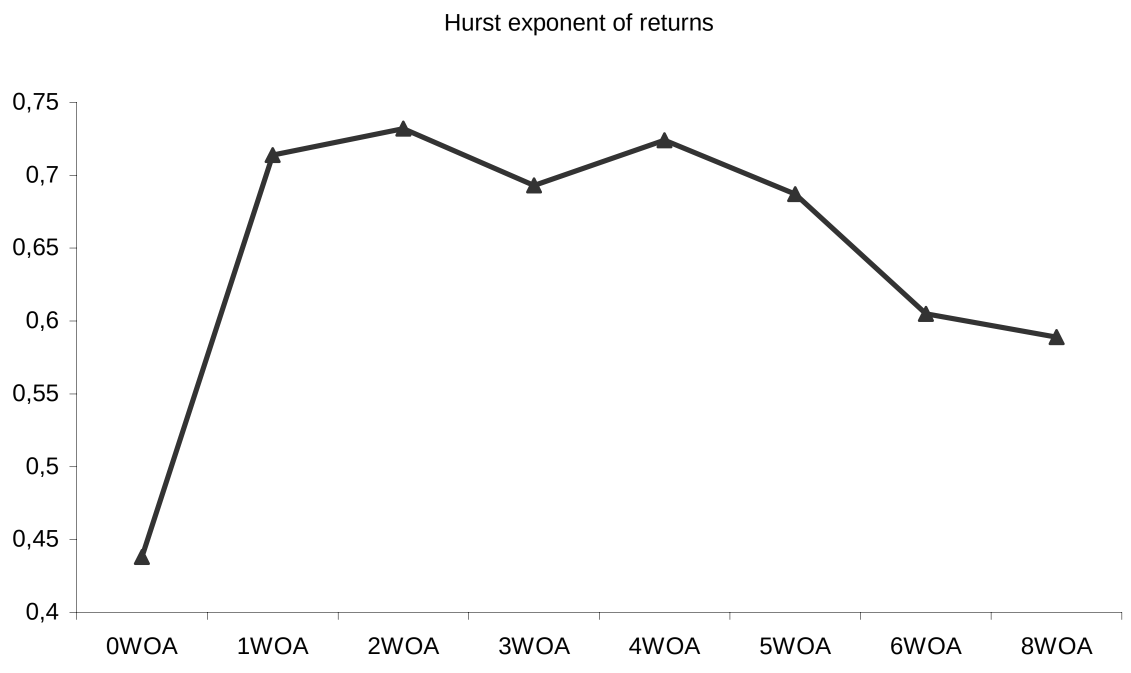

Figure 1: The value of the Hurst exponent with different replacement rate of the WOA.

The highest level of persistence is in the 2WOA case (13.3% replacement rate) where the market has enough time to learn. When the number of replaced strategies is higher the learning effect is weaken by the randomly chosen new strategies that appear on the market.

With higher replacement ratio, the value of the Hurst exponent declines as the learning is “diluted” by new strategies that enter randomly the market. This phenomenon takes place from the 5WOA case to the 8WOA case (33 % - 53 %replacement rate); see figure 1 and table 1. Figure 2 shows the kurtosis of the price returns time series. The highest kurtosis is in the 1WOA case followed by a falling trend in the kurtosis value as the replacement ratio increases. The reason why we can observe a decreasing trend of the kurtosis value is a fall of the learning effect, which is caused by a higher proportion of new incoming strategies (randomly generated) on the market. Similar, but not so strong results are in the R/S analysis, Figure 1. Variance of the price returns time series is depicted in Figure 3. We can observe rising trend as the replacement ratio increases. The higher is a number of incoming strategies the higher is price volatility. Such a high fluctuation of strategies causes that the dynamic system representing the simulated financial market has not time to stabilize.

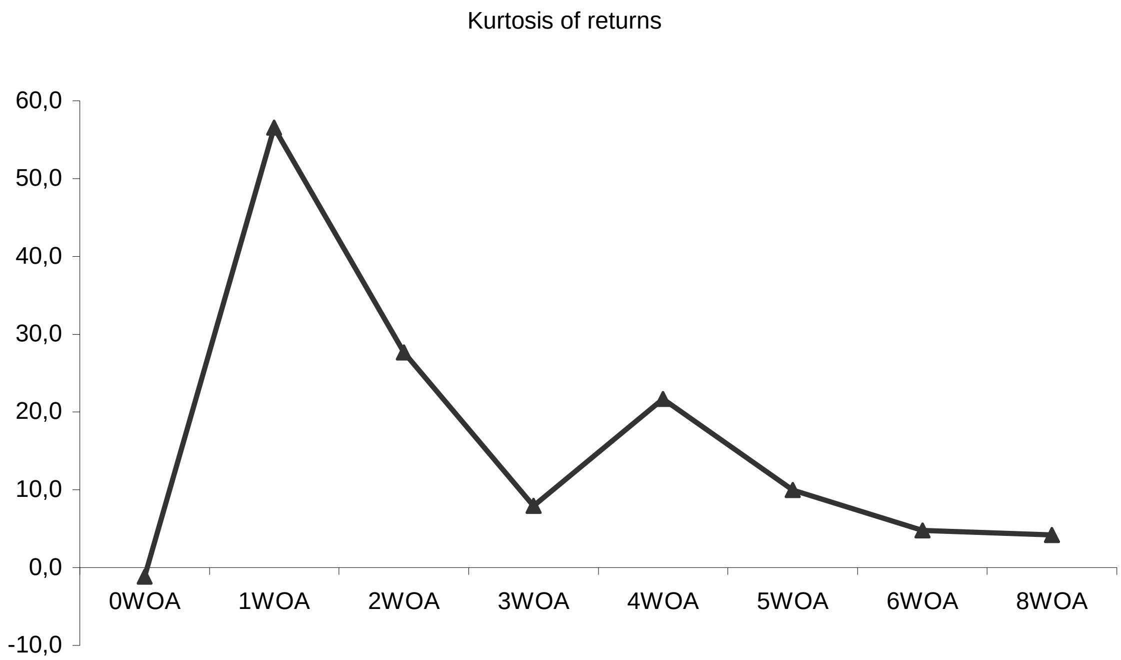

Figure 2: The value of the kurtosis of price returns time series with different replacement rate of the WOA.

Table 1: Hurst exponent, kurtosis and variance of the simulated price returns time series

	0WOA	1WOA	2WOA	3WOA	4WOA	5WOA	6WOA	8WOA
Hurst	0,438	0,714	0,732	0,693	0,724	0,687	0,605	0,589
Kurtosis	-1,2	56,6	27,7	7,9	21,7	10,0	4,8	4,2
Variance	0,016	0,017	0,024	0,034	0,025	0,036	0,066	0,064

3.2 Wavelet Decomposition of the Price Returns Time Series

The wavelet transform decomposes 1-dimensional time series into 2-dimensional time-scale (frequency-) space. In particular, while Fourier analysis breaks down a time series into constituent orthogonal sinusoids of different frequencies (constant periodicities), wavelet analysis breaks down such a time series into constituent orthogonal wavelets of different scales. The wavelet transform replaces the basic sinusoidal waves by a family of basic wavelets generated by translations and dilatations of one particular wavelet atom $\psi_{\tau,a}(t)$ (see [18]). The higher scales correspond to the most dilated (stretched) wavelets. The more dilated the wavelet, the longer the portion of the time series with which it is being compared, and thus the coarser time series features being measured by the wavelet resonance coefficients W. There is an inverse relationship between scale and frequency (see [18] [19]):

low scale ↔ compressed wavelet ↔ rapidly changing time series details ↔ **high frequency**

high scale ↔ dilated wavelet ↔ slowly changing, coarser time series ↔ **low frequency**

Figure 3: The value of the kurtosis of price returns time series with different replacement rate of the WOA.

We use a specific version of the discrete wavelet transform (DWT) that can be directly applied to a time series observed over a discrete set of times. Times series x(t) can be completely decomposed in terms of approximations, provided by so-called scaling functions, and details, provided by the wavelets. The detailed wavelet resonance coefficients, which correlate wavelets with particular segments of the time series x(t) as

$$W(j,n) = \int_{-\infty}^{+\infty} x(t)\psi_{j,n}(t)dt, \quad j,n \in Z \tag{3.1}$$

where $\psi_{j,n}(t)$ is a dyadic wavelet basis

$$\psi_{j,n}(t) = \frac{1}{\sqrt{2^j}}\psi\left(\frac{t-2^j n}{2^j}\right). \tag{3.2}$$

The discrete dyadic scale parameter aj = 2j, while the translation interval is τn = 2jn. This procedure is also called wavelet multiresolution analysis (MRA). In summary, the DWT is an important practical tool for financial time series analysis. The basic reasons are: ability to re-express a time series in terms of coefficients that are associated with a particular time and a particular dyadic scale, we can also reconstruct a time series form its DWT coefficients; the DWT allows us to partition the energy in a time series into pieces that are associated with different scales and time (see [12, 19]).

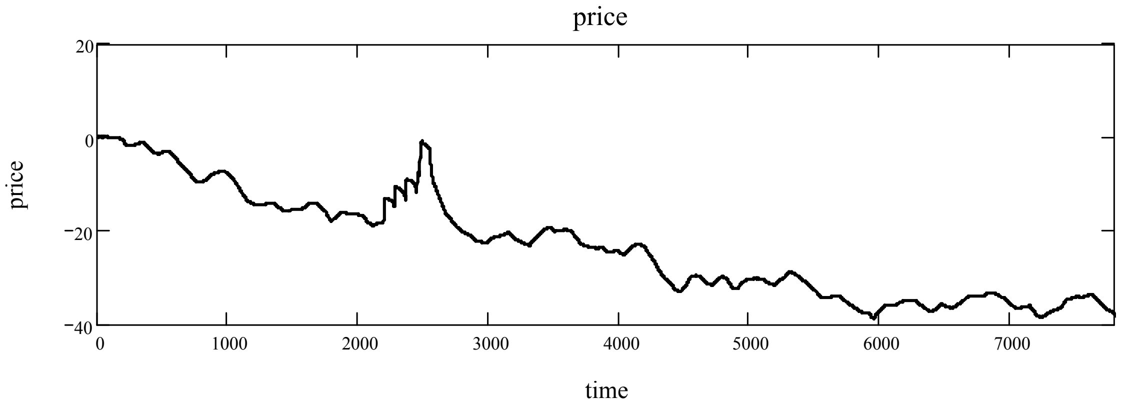

Figure 4: Price time series, the 1WOA case.

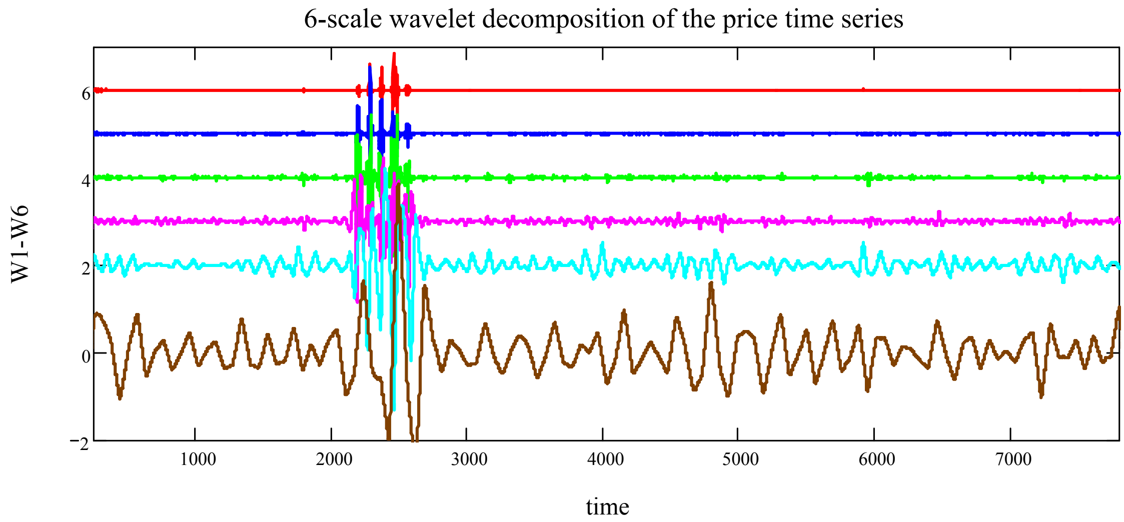

Figure 5: The 1WOA case, a 6-scale wavelet decomposition of the price time series, where the high frequency wavelet resonance coefficients are at the top and the low frequency resonance coefficient are at the bottom.

As an illustrative example we compare the 1WOA, 6WOA and 8WOA case. In Figure 4 we have a simulated price time series. The simulation were performed with the 1WOA i.e., low replacement ratio. At the left side of the Figure 4, around 2500 iteration there is a positive price jump. This price behavior is analyzed in a 6-scale wavelet resonance coefficient sequence decomposition, which shows the dynamic phenomena are identifiable at all six scales, see Figure 5. This is in contrast to the 6WOA case (Figures 6, 7) where the price turbulence (2000-3000 iteration) is mainly noticeable at the low scales. The 6WOA case and the 8WOA case, Figure 8, 9, have, in comparison to the lower replacement ratio cases higher occurrence of significant price swings. This represents higher standard deviation of the detail wavelet resonance coefficients depicted in Figure 10. This is also evident in Figures 4-9.

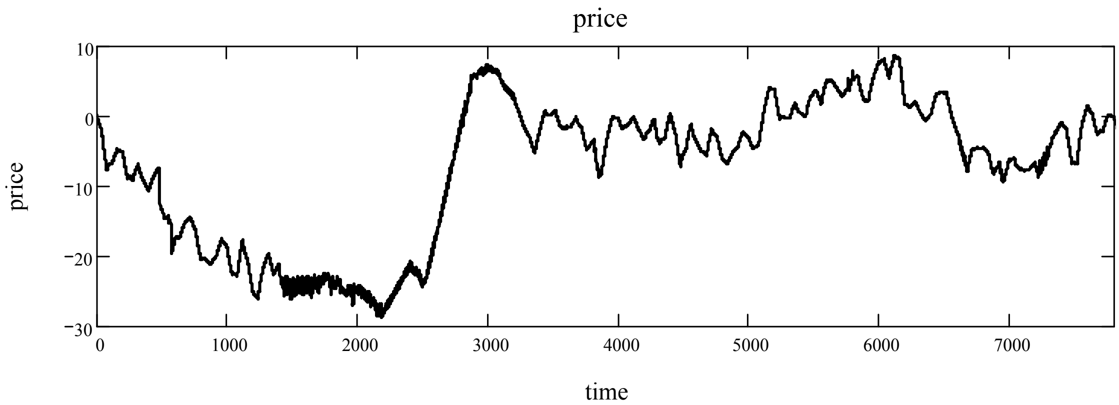

Figure 6: Price time series, the 6WOA case.

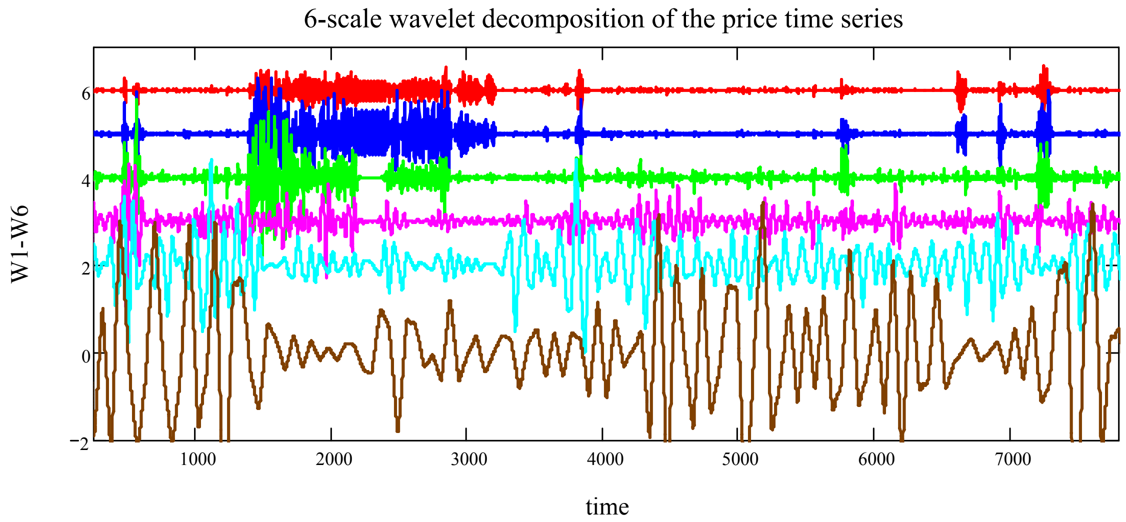

Figure 7: The 6WOA case, a 6-scale wavelet decomposition of the price time series, where the high frequency wavelet resonance coefficients are at the top and the low frequency resonance coefficient are at the bottom.

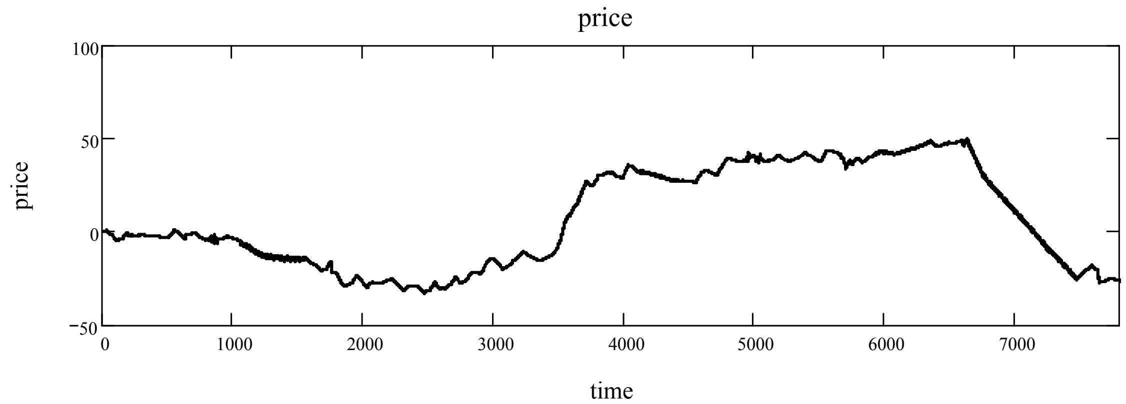

Figure 8: Price time series, the 8WOA case.

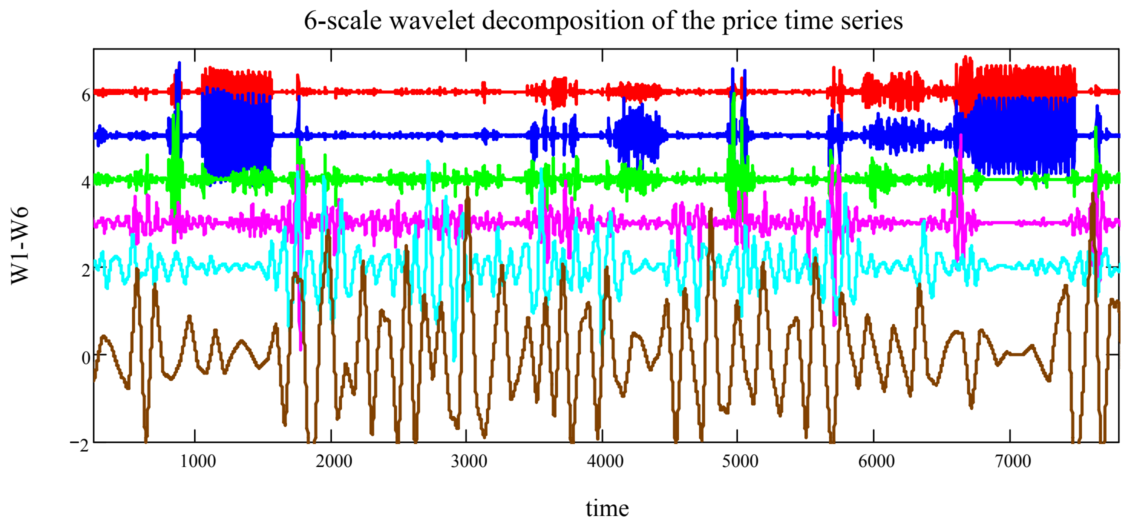

Figure 9: The 8WOA case, a 6-scale wavelet decomposition of the price time series, where the high frequency wavelet resonance coefficients are at the top and the low frequency resonance coefficient are at the bottom.

3.3 Wavelet variance

The wavelet variance decomposes a variance of stochastic processes on scale basis and hence is important in financial time series processing. The wavelet variance is a succinct alternative to the power spectrum based on the Fourier transform, yielding a scale-based analysis that is often easier to interpret than the frequency-based spectrum (see [19]). Such decomposition helps us to track an evolution of the energy contribution at various scales, which is related to traders' investment horizons.

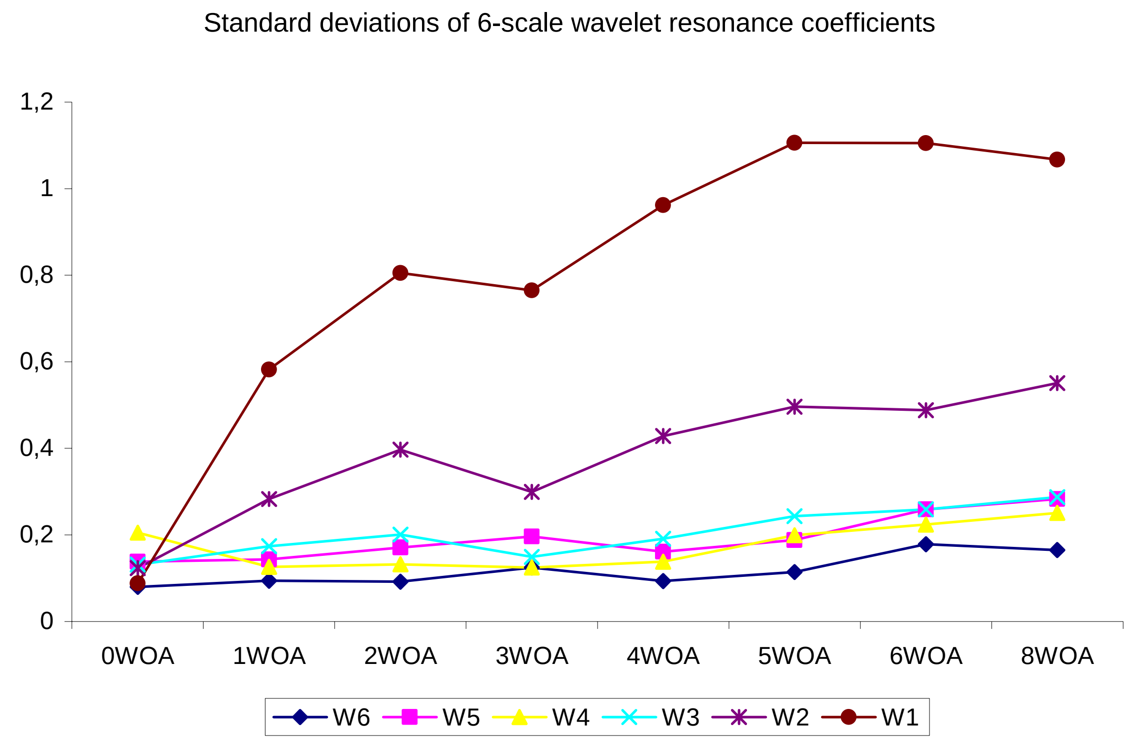

Figure 10: Standard deviations of 6-scale wavelet resonance coefficients of the price returns time series, where the highest frequency (the lowest scale) wavelet resonance coefficients are W6 and the lowest frequency W1.

The 0WOA case has the lowest the wavelet variance (except W4) from all cases, Table 2. When we compare the 0WOA case with the 1WOA case, we see a significant increase in the wavelet variance. This refers to a considerable increase in energy at high scales W1, W2, see Figure 10. With higher replacement ratio the wavelet variance is raising, which indicates higher activity levels at all scales, except for the W1 where from the 5WOA case the wavelet variance of W1 drops slightly.

Table 2: Wavelet standard deviations

Stdev	0WOA	1WOA	2WOA	3WOA	4WOA	5WOA	6WOA	8WOA
W1	0,088	0,582	0,805	0,765	0,962	1,106	1,105	1,067
W2	0,123	0,283	0,397	0,299	0,428	0,496	0,488	0,551
W3	0,131	0,174	0,201	0,149	0,191	0,243	0,259	0,287
W4	0,205	0,126	0,132	0,125	0,138	0,199	0,224	0,251
W5	0,138	0,143	0,171	0,196	0,161	0,188	0,259	0,283
W6	0,08	0,094	0,092	0,125	0,093	0,114	0,178	0,165

5. Conclusions

We demonstrate that the heterogeneous agent model considerably changes its behavior when we implement the WOA. An implementation of the WOA increases a persistence of the price time series considerably, but when we are increasing the number of replaced strategies beyond some point, then the value of the Hurst exponent declines as the learning is "diluted" by new strategies that enter randomly the market. We can also observe higher price time series volatility as the replacement ratio rises.

An application of the wavelet variance is a very convenient tool for activity (energy) decomposition on scale basis. Our simulations show, that the higher replacement ratio (it causes an increment in the wavelet variance) the higher activity levels at all scales.

The R/S analysis and wavelet transforms enable to reformulate the efficient market hypothesis by a behavior of the Hurst exponent, the kurtosis of the price returns, the variance of the price returns, and the standard deviations of wavelet resonance coefficients of the price returns time series.

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