BACHELOR THESIS

Asset Pricing: Theory and Evidence

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Field of study: Economic theory

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Pronouncement

I confirm that I have made this bachelor's thesis independently and that I have used only the sources indicated in the thesis.

Eisenstadt, May 26. 2004 Filip Hájek

Acknowledgement:

I would like to thank all who have helped me with my thesis and supported me during my studies.

BACHELOR'S THESIS PROPOSAL

Submission deadline: Spring semester 2003/2004
Thesis author: Filip Hájek
Thesis advisor: Tomáš Jaroš

Title: ASSETS VALUATION: THEORY AND EVIDENCE

Statement of objectives:

Ever since William Sharpe and John Lintner came up with the capital asset pricing model (CAPM) there has been an academic debate over its validity. Yet over 40 years later, the model is still widely used in academia as well as practice.

The model was the first apparently successful attempt to show how to assess the risk of the cash flow from a potential investment project and how to estimate the project's cost of capital, which is the expected rate of return that investors will demand if they are to invest in the project. In this model, a stock's risk is summarized by its beta with the market portfolio of all invested wealth.

Over the years CAPM has come under heavy research about the validity of the assumption as well as on the validity of the predictions and the thesis will try to highlight the most important papers that has shaped this rich academic debate.

Lately research has also focused on bringing alternatives to Sharpe and Lintner's CAPM. The market-derived capital-pricing model (MCPM) is based on financial market instruments, namely bond yields and options prices and will be introduced and used for empirical estimates.

The purpose of the thesis is to bring comprehensive summary of the CAPM debate and the MCPM and compare models empirically.

Thesis outline:

1. Capital Asset Pricing Model
   a. CAPM description and theoretical framework
   b. CAPM debate
2. Market-derived Pricing Model
3. Empirical evidence
4. Conclusion

Preliminary literature:

Brealey, R.A., Myers, S.C. (2000): "Principles of Corporate Finance", 6th edition
Fama, E. F., French, K.R. (2003): *„The CAPM: Theory and Evidence"*
Fama, E. F., MacBeth, J.D. (1972): *„Risk, Return and Equilibrium: Empirical Tests"*
Jogannathan, R., McGrattan, E.R. (1995): *„The CAPM Debate"*

In Prague
October 28, 2003 ***Thesis Advisor*** ***Thesis Author***
Tomáš Jaroš Filip Hájek

Abstrakt

Bakalářská práce na téma Oceňování aktiv : Teorie a Praxe se skládá ze dvou vzájemně doplňujících se částí. V první teoretické části je představen CAPM a nejdůležitější práce z akademické debaty o jeho platnosti. Dále je vysvětlen teoretický základ konkurenčního modelu MCPM, který čerpá informace z opčních a dluhopisových trhů. Následuje kapitola věnující se kritice tohoto postupu.

Navazující část je empirická. Jsou zde odhadnuty ceny aktiv čtyř společností obchodovaných na amerických burzách a to dvěma alternativními způsoby. Nejprve pomocí odhadu koeficientu beta daných společností prostřednictvím metody nejmenších čtverců, dodatečně pomocí metodiky MCPM modelu.

Abstract

Bachelor thesis on the topic Asset Pricing: Theory and Evidence consists of two complementary parts. In the first part, CAPM is introduced and the most important articles of the ongoing academic debate of its validity are followed. Framework of rival model Market-derived Asset Pricing Model (MCPM) based on the information from bond and option markets is also introduced in this chapter and its critique is provided.

Following part is empirical. There are estimations of asset prices of four companies traded on the US stock exchanges employing both models. Firstly beta estimation via ordinary least square method for CAPM is done and subsequently asset prices computed, secondly estimates employing MCPM framework are performed.

Content

1. Introduction

In the last four decades asset pricing models and Capital asset pricing model in particular have been dominating financial textbooks. Based on the ingenious observations, reasonable assumptions and due to the seductive simplicity, we are today not surprised to see that CAPM has still not found too many competitors.

Nonetheless without the possibility of testing it against other models the research focused on testing its validity and the validity of its assumptions. CAPM came under heavy testing, which consequently led to the debate that had brought together advocates and opponents of the model. During last four decades the debate grew but the crucial question is still not settled.

In recent years nonetheless new innovative models have emerged. Some of them are based on information from derivative markets such as Market-derived asset pricing model (MCPM) which utilizes information from bond and option markets. These models try to bypass the problems of CAPM via totally different approaches in calculations of asset prices and thus challenge the prevailing models.

The aim of this thesis is to follow in the second chapter the birth of CAPM as well as to map the long and very interesting debate that has started, and further, to reveal the theoretical framework of MCPM. Subsequently, in the third chapter this theoretical base will be used for estimates of asset prices of four companies operating on the US market employing both models. Observations from the practical application of MCPM are concentrated in the subchapter 2.3.5. Critique, which for the sake of structure is presented in the second i.e. theoretical chapter.

This thesis is following the previous research on the Institute of economic studies at Charles University in Prague done by Robert Keller (Keller 2003) Since Keller has focused on the models that stem from original CAPM, this thesis will not cover this part of CAPM debate, but rather when it will be appropriate refer to his work.

Hopefully this effort to enrich the previous study will provide more space for further research and consequently lead to more works in this field of financial economics.

2. Asset valuation models

2.1. Capital asset pricing model (CAPM)

2.1.1. CAPM

The CAPM builds on the groundbreaking paper of Harry Markowitz (1952) in which he describes how to select a portfolio using the mean-variance rule (E-V rule) and a latter paper of Tobin (1958) which introduces the idea of linear efficient set and also implies two steps in selection of desirable portfolio, later known as *separation theorem*. In order to derive a CAPM we have to present their findings.

2.1.1.1 Markowitz portfolio theory

Markowitz is one of the first to attack until then prevailing theory of only maximizing the expected return that was set among others by Williams (1939). Given Williams' theoretical framework, investor would invest in an asset that bears the highest expected return without taking into account a risk associated with it, assuming that *"the law of large numbers will insure that the actual yield to the portfolio will be almost the same as the expected yields"* (Williams 1939, pp. 68-69). Markowitz acknowledges the need of using an expected returns and allowing the returns for risk[1] and a trade-off between them. *"There is a rate at which investor can gain expected return by taking on variance, or reduce variance by giving up expected return."* (Markowitz 1952, pp. 79)

Markowitz builds his theory on the following assumptions (Jensen 1983):

(i) Investors select portfolio at time t-1 that produces random return R at t.

(ii) The quantities of all assets are given and short sales are restricted ($X_i \geq 0$) for all *i*.

(iii) Investors are risk averse (i.e. they minimize portfolio return variance given expected return)

[1] Following the work of J.R. Hicks in *Value and Capital* (1938). Hicks however applies this rule to a firm rather then a portfolio.

(iv) Investors are not saturated (i.e. they maximize portfolio expected return given variance)

(v) All assets are perfectly divisible and perfectly liquid, i.e. all assets are marketable and there are no transaction costs

(vi) There are no taxes

In order to solve the problem of efficient portfolio selection Markowitz considers expected return as a random variable, that can be described by two moments: mean (μ_i) and variance (var = σ^2).

Using the probability theory we can derive for the expected return and variance of the portfolio the following formulas.

For expected return of portfolio:

$$E_p = \sum_{i=1}^{n} x_i \mu_i \tag{2.1}$$

and for variance of the return of portfolio

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j \tag{2.2}$$

where E_p is expected return of portfolio, σ_p^2 is variance of the portfolio, x_i is the percentage of the investor's asset which are allocated to the i^{th} security, x_j is the percentage of the investor's asset which are allocated to the j^{th} security, and μ_i is the expected return of i^{th} security. σ_{ij} is covariance between i^{th} and j^{th} security.

Covariance lies in the very heart of the portfolio selection theory and therefore needs to be defined properly. Covariance is defined as follows:

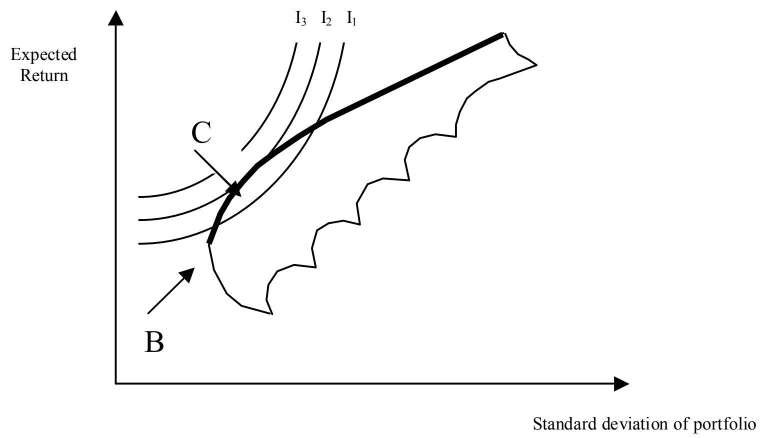
$$\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j \tag{2.3}$$

where σ_{ij} is covariance between i^{th} and j^{th} security, σ_i is standard deviation of i^{th} security ($\sigma_i = \sqrt{\text{var}}$), σ_j is standard deviation of jth security and ρ_{ij} is a correlation coefficient between securities *i* and *j*. Correlation coefficient varies between -1 and +1. The value of +1 implies perfect correlation i.e. fully align movement, -1 implies perfect negative correlation i.e. securities move inversely of each other and 0 implies independent movement of both securities.

Due to the imperfect correlation between securities, diversification is rational and desirable, because portfolio of two imperfectly correlated securities will achieve weighted average of expected returns together with lower than weighted variance of two securities, with weights being the portions of wealth invested. As to the selection of reasonable μ_i and σ_{ij} Markowitz suggests *"as to tentative μ_i and σ_{ij} is to use the observed μ_i, σ_{ij} for some period of the past."*(Markowitz 1952, pp. 91)

The E-V rule restricts the desirable portfolios to the ones on the efficiency frontier shown on Figure 1. Mean-variance efficient portfolio lie above (B) in the Figure1 and investors will select portfolio from this frontier according to their aversion to the risk, in here represented by the indifference curves. The optimal portfolio will lie on the point (C) where efficient set of portfolios touches with the indifference curves of investor, giving him the highest possible utility with given restrains.

Figure 1 : Markowitz efficiency set



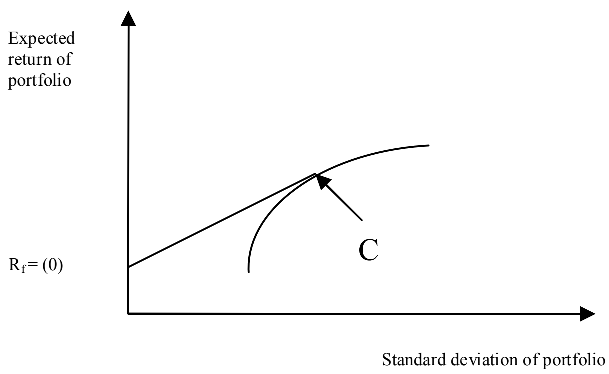
Source: Sharpe, Alexander 1994, pp. 131

2.1.1.2. Tobin's separation theorem

The next ground breaking work that marks the evolution of CAPM is J. Tobin's paper *Liquidity preference as behavior towards risk* (1958) in which the author discusses the liquidity preference in respect to the mean and variance.

He allows investors to choose between holding their wealth in cash or in consoles, which are in the absence of inflation risk-free in nominal sense, and further shows that given these two assets, different investors, with regard to their risk awareness, choose a different optimum point of investment on the investment opportunity line (0,C) in Figure 2, i.e. different portions in the two assets.

Figure 2: Tobin's Capital Market Line



Source: Reilly, Brown 1997, pp. 283

Adding risk-free borrowing (cash) and lending greatly simplifies Markowitz's efficient set.

Consider a portfolio that invests the proportion *x* of portfolio funds in a risk-free security and *1-x* in portfolio *g*

$$R_p = xR_f + (1-x)R_g \tag{2.4}$$

than the expected return is simplified (due to var (R_f) = 0 by definition)

$$ER_p = xR_f + (1-x)R_m \tag{2.5}$$

as well as the variance

$$\text{var}(R_p) = (1-x)^2 \ \text{var}(R_m) \qquad (2.6)$$

These equations imply that efficient portfolios can be obtained by varying x. They then plot a straight line (CML) in Figure 2. The line starts at R_f (x = 1), runs to the point C (x = 0, i.e. all funds invested in C) and continues on for portfolios that involve borrowing at the risk-free rate (x < 0). The key result is that with unrestricted risk-free borrowing and lending, all efficient portfolios are combination of the single tangency portfolio (C) and risk-free asset (0) with either lending at the risk-free rate (points below C along the line from R_f) or risk-free borrowing (points above C along the line from R_f).

More importantly when elaborating on the Market portfolio (C) Tobin proves *"that the proportionate composition of the non-cash assets is independent of their aggregate share of the investment balance. This fact makes it possible to describe investor's decisions as if there were a single non-cash asset, a composite formed by combining the multitude of actual non-cash assets in fixed proportions."* (Tobin 1958, pp. 84) which restricts Markowitz's efficient portfolio set into linear combination of risk-free rate and one optimal portfolio as seen in Figure 2 creating linear efficient set (0,C).

This implies that process of choosing optimal investment *"...can be broken down into two phases: first, the choice of a unique optimum combination of risky assets; and second, a separate choice concerning the allocation of funds between such a combination and a single riskless asset"* (Sharpe 1964, pp.426), which has later become known as Tobin's separation theorem.

These two papers were the theoretical cornerstones necessary for the birth of Sharpe-Linter's CAPM.

2.1.1.3. Sharpe- Lintner's CAPM

In 1964 Sharpe and in 1965 Linter[2] have published papers concerning the prices of assets under conditions of risk. In their work they tried to establish a theory that would answer the crucial question: how to precisely evaluate the relationship between higher risk (variance) and expected return and how to distinguish the part of the risk that market values, because Sharpe observes that *"through diversification, some of the risk inherent in an asset can be avoided so that its total risk is obviously not the relevant influence on its price."* (Sharpe 1964, pp. 426)

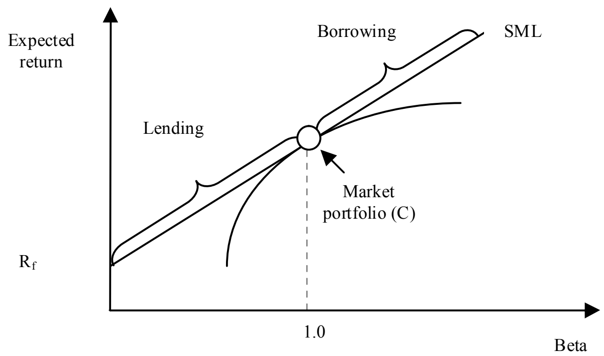
Sharpe and Lintner extended assumptions of Markowitz' model with the following

(i) All investors can borrow and lend an unlimited amount at an exogenously given risk-free rate of interest R_f and there are no restrictions on short sales of any asset. [3]

(ii) Perfect capital markets.

(iii) All investors have identical subjective estimates of the means, variances, and covariances of return among all assets.

Figure 3 : Security Market Line



Source: own graph based on Reilly, Brown 1997, pp. 283 and Elton, Gruber 1991, pp. 290, 306

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[2] Similar unpublished work of J. Treynor : "Towards a theory of market value of risky asset" (1961) comes to the similar conclusions.

[3] This assumption is however not new, but we can see a clear analogy in Tobin's work.

With complete agreement about distributions of returns, all investors form the same tangency portfolio C with R_f. In order for the market to clear assets in the market portfolio (C) must be priced so its weight in given time is its total market value divided by the total value of all risky assets and such portfolio includes all assets.

In such situation we can derive the familiar Sharpe-Lintner CAPM risk-return relation

$$ER_i = R_f + (ER_m - R_f)\beta_i \tag{2.7}$$

where $\beta_i = \dfrac{\mathrm{cov}(R_i, R_m)}{\mathrm{var}(R_m)}$ can be verbally interpreted as the covariance risk of asset *i* in portfolio C, measured relative to the risk of the portfolio, which is just an average of the covariance risks of all assets i.e. the measure of asset's contribution to overall portfolio risk..

Therefore covariance is the right measure of risk as Jensen notes: "*Although investors take the variance (or equivalently the standard deviation) of their portfolio returns as an appropriate measure of risk, there results imply that the appropriate measure of the risk of any individual asset is its covariance with the market portfolio,* $\mathrm{cov}(R_i, R_m)$*, and not its own variance,* $\sigma^2(R_i)$*.*" (Jensen 1983)

2.1.1.4. Black's version of the model

Assumption that any investor may borrow or lend any amounts he wants at the risk less rate of interest was felt as the most restrictive one and there was a feeling that the model would be changed substantially if this assumption would be dropped. Also research was showing that model using risk-free rate doesn't hold very good the empirical findings (Black, Jensen, and Scholes 1972), but rather the two factor model (2.8) holds.

In the very same year Black published a paper in which he proves that in equilibrium the portfolios of all investors consist of a linear combination of two basic portfolios. This paper is one of the first contributions into the CAPM debate on which we will focus in the following sub-chapter, but for the sake of clarity and structure we will present it here.

While this point was recognized earlier[4], Black showed that under absence of risk-free borrowing and lending (but without restrictions on short selling) *"every efficient portfolio may be written as a weighted combination of the market portfolio m and the minimum variance zero-* β *portfolio z."* (Black 1972, pp. 452)

Black demonstrates that in equilibrium the expected return on any asset will be given by

$$ER_i = (1 - \beta_i)ER_z + \beta_i ER_m \qquad (2.8)$$

where ER_z is the expected return on the zero beta portfolio, other defined as previously. Therefore the expected return on all assets is still a linear function of their systematic risk and the equation is similar to the Sharpe-Lintner, only with the exception that ER_z plays the role of R_f.

[4] Sharpe (1970) chap. 4 in Jensen

2.2. CAPM Debate

The Sharpe-Lintner asset pricing model has received since its introduction in mid 60's widespread attention and has been heavily tested during the last 40 years. This section will try to describe this long academic debate about the model's validity. There has been large amount of papers published that are connected to the topic, however in this study only the major papers are used.[5]

Included are studies that support CAPM (Black, Jensen and Scholes; Fama and MacBeth), studies that challenge it (Banz; Fama and French) and studies that challenge these challenges (Black; Amihud, Christensen and Mendelson; Jagannathan and Wang)

2.2.1. Methodology

Equation (2.7) has three testable implications[6] (Fama and MacBeth 1973):

(i) The relationship between the expected return on a security and its risk in any efficient portfolio *m* is linear and upward sloping

(ii) β_i is a complete measure of the risk of security *i* in the efficient portfolio *m*, i.e. no other measure of the risk of *i* appears in the equation (2.7)

(iii) In a market of risk averse investors, higher risk should be associated with higher expected return; i.e. $ER_m - ER_f > 0$

Equation (2.7) is in terms of expected returns, but its implications must be tested with data on period-by-period security and portfolio returns. In order to test the implications the following stochastic equation is examined.

$$r_p = \gamma_0 + \gamma_1 \beta_p + \sum_{j=2}^{J} \gamma_j Z_{jp} + \varepsilon_p \qquad (2.9)$$

[5] The significance of the papers is subjective, and author apologizes if the reader is missing some works, that are believed to be important as well. Such is the nature of subjective-ness.
[6] As for the Black's version of the model, third implication must be modified to $ER_m - ER_z > 0$.

where r_p is an estimate of the expected excess return on portfolio ($ER_p - ER_f$), β_p is an estimate of beta for portfolio p, γ_1 is the market risk premium for bearing one unit of beta risk ($ER_m - ER_f$); γ_0 is the zero beta rate, the expected return on an asset which has a beta of zero; Z_{jp} for j = 2,..,j are non-beta explanatory variables assumed to be relevant for asset pricing and ε_p is a random disturbance term in the regression equation.

In such a regression equation we are trying to determine which explanatory variables are significant. To do so we use the econometric theory framework and we examine the t-statistics of the respected γ coefficients looking for significance.

The estimated coefficients γ_0 and γ_1 obtained from the regression are then compared to R_f (Sharpe-Lintner version) or return on zero-beta security (Black version) and $R_m - R_f$ and $R_m - ER_z$ respectively. R_m is an average return on the market index, which is usually taken as a proxy for market return, over the period.[7]

2.2.2. Early tests

The first test of CAPM was published by Douglas (1969). Douglas regressed the returns on a large cross-sectional sample of common stocks on their variance and on their covariance with an index constructed for the sample. He came to the same results as Linter (unpublished). According to their findings for seven separate five-year periods from 1926 to 1960, the average realized return was significantly positively related to the variance of the security's return over the time but not to the covariance as the model predicted[8].

Miller and Scholes[9] reviewed the theory and Douglas-Lintner evidence and came to the conclusion that these results are mainly due to omitted variable bias,

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[7] Arithmetic averages of annual, quarterly, or monthly returns have generally been used for both R_f and R_m, but a number of authors have also used average continuously compounded rates as well. (Jensen 1983)

[8] In Lintner's test the coefficient on the residual variance was positive and as significant as the β_i term, (both having t-values greater then 6)

[9] Rates of Return in Relation to Risk: A Reexamination of Some Recent Findings, in*Studies in the Theory of Capital Markets*, Editor M. Jensen, Praeger, New York, 1972

heteroscedasticity and errors in variable. Therefore nothing can be said about the validity of the results with certainty.

2.2.2.1 Black, Jensen and Scholes's study

Black, Jensen and Scholes (BJS) (1972) present some additional tests of Sharpe-Lintner (SL) asset pricing model. When testing the original SL model, they conclude that risk-free rate doesn't hold to the empirical findings.

They find that the average returns on these portfolios are not consistent with equation (2.7) especially in the postwar period 1946-66. Their estimates of the expected returns on portfolios of stocks at low levels of β_i are consistently higher than predicted by equation (2.7), and their estimates of the expected returns on portfolios of stocks at high levels of β_i are consistently lower the predicted by the same equation.

According to their findings "two-factor model" (2.8) does a better job in explaining the risk-return relationship and further work of Black also provides theoretical backing of these results.

2.2.2.2. Fama and MacBeth's study

Fama and MacBeth (FM) in their study *Risk, Return, and Equilibrium: Empirical Tests* (1972) following the BJS work, have used the following regression

$$r_i = \gamma_0 + \gamma_1 \beta_i + \gamma_2 \overline{\beta}_i^2 + \gamma_3 \overline{\sigma_i(u)} + \varepsilon_i \qquad (2.10)$$

where r_i is an expected return on security *i,* γ_0 is an expected return on a zero-beta portfolio, γ_1 is an expected market premium, $\overline{\beta}_i^2$ is the average of the β^2 for all individual securities in portfolio *i*, $\overline{\sigma_i(u)}$ is the average of the residual standard deviations and ε_i is a random disturbance term in the regression equation.

Their tests were carried out on the 20 portfolios constructed from all securities on NYSE in the period January 1935-June 1968. The portfolios were constructed in a way that would minimize the measurement error bias problem in cross-sectional estimates of coefficient $\widetilde{\gamma}_{0t}, \widetilde{\gamma}_{1t}, \widetilde{\gamma}_{2t}$ and $\widetilde{\gamma}_{3t}$ in equation (2.10).

Specifically FM first estimated beta of each stock during one period and then formed portfolios on the basis of these estimated betas. Next, they re-estimated the beta of each portfolio by using the returns in the subsequent period. This ensured that the estimated betas for each portfolio were largely unbiased and free from error. Finally, these portfolio betas were plotted against returns in an even later period.(Brealey and Mayers 1988)

The FM tests are based on the examination of time series of coefficients in the equation (2.10), estimated from the cross-sectional regression for each month. The focus was put on tests of three major testable implications of equilibrium properties of the two-parameter model:

(H1) The risk return relationship should be linear - i.e. $E(\gamma_2) = 0$

(H2) No measure of risk in additional to β should be systematically related to expected returns - i.e. $E(\gamma_3) = 0$

(H3) The expected risk-return tradeoff should be positive - i.e. $E(\gamma_1) = ER_m - ER_o > 0$

Apart from these the test for the implication of Sharpe-Linter model was done

(H4) $E(\gamma_0) = R_f$

In addition they provide tests of the proposition that each of the period-by-period coefficient are equal to zero, $\tilde{\gamma}_{0t} = \tilde{\gamma}_{1t} = \tilde{\gamma}_{2t} = \tilde{\gamma}_{3t} = 0$ and the implication of capital market efficiency that each of the coefficient must behave as a fair game through time. If the latter requirement wouldn't hold, there would be a trading rules based on the past values of the γ_j's which would yield above normal profits – thus a violation of the efficient markets hypothesis. (Jensen 1972, pp. 22) [10]

From the examination of the average values and t statistics for the estimated coefficients γ from the equation (2.10) for the entire period and for various sub-periods

[10] Citation from Fama, Eugene F. 1970 "Efficient Capital Markets: A Review of Theory and Empirical Work." in Jensen (1972)

FM conclude that on the 5 % level of significance: ”*one cannot reject the hypothesis (H1-H3) that the pricing of securities is in line with the implications of the two-parameter model for expected returns. And given a two-parameter pricing model, the behavior of returns through time is consistent with an efficient capital market*” (F-M 1973, pp. 625) since the average values of γ_2 and γ_3 are generally small and insignificantly different from zero.

As for the H4, Fama and MacBeth confirm the BJS results, that $E(\gamma_0)$ does not equal R_f. Therefore the SL hypothesis is inconsistent with the data, on the other hand is not contradictory to the Black’s version of the CAPM.

The FM tests of the period-by-period values of the γ_j indicate, that while we cannot reject the H1 and H2, we can reject them in each month. Therefore while there are no (systematic) nonlinearities and effects of non-portfolio risk on security returns, such effects appear in a random fashion from period to period. They conclude that even though adding these two variables to the regression raises its average adjusted coefficient of determination for the entire period, “*the knowledge of these effects is of no help to the investor since the coefficients themselves behave as a fair game through time. Thus, the investor can apparently do no better to act as if the two-factor model, suggested by BJS, is valid*.” (Jensen 1983, pp. 24)

2.2.3. Challenges

In 70’s the CAPM passed its first major empirical tests. However in 1981 came one of the first studies suggesting that CAPM might be missing something, only to be followed decade later by study of Fama and French (1992) that was warning that CAPM might be missing everything.

2.2.3.1. Banz study on size effect

In 1981 Banz tests whether the size of the firm (market value was used as a proxy) cannot explain the residual variation in the average returns across assets that is not explained by CAPM’s beta.

The empirical tests are based on the regression equation

$$r_i = \gamma_0 + \gamma_1\beta_i + \gamma_2[(\phi_i - \phi_m)/\phi_m] + \varepsilon_i \qquad (2.11)$$

where r_i is an expected return on security *i,* γ_0 is an expected return on a zero-beta portfolio, γ_1 is an expected market premium, ϕ_i is a market value of security *i*, ϕ_m is a market value, and ε_i is a random disturbance term in the regression equation.

Banz uses similar procedure to the portfolio-grouping of FM (1972)[11]. He creates 25 portfolios containing similar number of securities, first to one of five on the basis of the market value of the stock, then the securities in each of those five are in turn assigned to one of five portfolios on the basis of their beta.(Banz 1981, pp.7)

For the entire period, 1936-1975, Banz obtains the following estimates (and *t*-statistics): $[\gamma_0 - R_f]$ = 0.0045 (2.76), $[\gamma_1 - (R_m - R_f)]$ = -0.00092 (-1.0), and γ_2= -0.00052 (-2.92).

Because the *t*-statistic for γ_2 is large in absolute value the size effect is statistically significant. The negativity of the estimated value implies that the shares of firms with large market values have had smaller returns, on average, than similar small firms.

He finds that the "*average return on small stocks are too high given their β estimates, and average returns on large stocks are too low*." (Fama and French 1992, pp. 427) His estimates of γ_0 are consistent with Fama and MacBeth (1973) findings, i.e. contradicting the Sharpe-Lintner model.

To assess the importance of these results, Banz tried to specify the "size premium". He constructs two portfolios, each with 20 assets. One portfolio containing only stocks of small firms, the other containing only large firms. The portfolios are constructed in such a way that they have same beta. During the 1936-1975 he concludes that *"the average excess return (from holding very small firms long and very large firms short) is, on average 1.52 percent per month or 19.8 percent on an annual basis."* (Banz 1981, pp. 15-16)

Thus, the CAPM seems to be missing a significant factor: firm size. This observation has become known as the *size effect.*

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[11] In length described in Black, Fisher and Scholes, Myron. (1974)*:" The effects of dividend yield and dividend policy on common stock prices and returns"*, Journal of Financial Economics 1, May, 1-22.

2.2.3.2. Fama & French : Is beta dead ?

In their paper *"The Cross-Section of Expected Stock Returns"* (1992) Fama and French estimated the relations in the equation (2.9) for the period from July 1963 to December 1990. Using this cross-sectional regression they confirm that size, earnings-price, debt-equity, and book-to market ratios add to the explanation of expected returns provided by market beta.

They grouped stocks for firms listed on the NYSE, AMEX and NASDAQ into 10 size classes and then into 10 beta classes, for a total of 100 portfolios. Analyzing firstly the beta and market size they obtained the estimates of γ_1 = -0.37 with *t*-statistic of -1.21 and γ_2 = -0.17 with *t*-statistic of -3.41 respectively as seen in the table 1.

As for the size itself, the average slope from monthly regression of returns on size is -0.15 %, with *t*-statistic of -2.58. This negative relation persist no matter what variables are in the regression equation and more importantly the *t*-statistic is always close to or more than 2 in absolute terms and thus size effect is robust to the inclusion of other variables in the 1963-1990 returns.

Beta is doing quite poorly in respect to size, however when left alone as the only explanatory variable it does even worse. The average slope from the regressions of returns is 0.15% per month with *t*-statistic of mere 0.46. Fama and French conclude: *"In contrast to the consistent explanatory power of size, the regression show that market β does not help explain average stock returns for 1963-1990"* and *"we can also report that β shows no power to explain average returns in regression that use various combinations of β with size, book-to-market equity, leverage, and E/P."* (Fama and French 1992, pp. 438)

As for the other explanatory variables, FF find that the combination of size and book-to-market equity seems to absorb the roles of leverage and E/P in average stock returns.

The consequences of their findings are severe. If market beta fails to explain expected returns, then the market portfolio is not efficient, and the evidence on the size of market premium is irrelevant. The punch line is simple: if beta is dead, so is the CAPM.

Table 1.:Average Slopes (t-statistics) from Month-by-Month Regression of Stock Returns on β, Size, Book-to-Market Equity, Leverage, and E/P: July 1963 to December 1990

β	ln(ME)	ln(BE/ME)	ln(A/ME)	ln(A/BE)	E/P Dummy	E(+)/P
0.15						
(0.46)						
	−0.15					
	(−2.58)					
−0.37	−0.17					
(−1.21)	(−3.41)					
		0.50				
		(5.71)				
			0.50	−0.57		
			(5.69)	(−5.34)		
					0.57	4.72
					(2.28)	(4.57)
	−0.11	0.35				
	(−1.99)	(4.44)				
	−0.11		0.35	−0.50		
	(−2.06)		(4.32)	(−4.56)		
	−0.16				0.06	2.99
	(−3.06)				(0.38)	(3.04)
	−0.13	0.33			−0.14	0.87
	(−2.47)	(4.46)			(−0.90)	(1.23)
	−0.13		0.32	−0.46	−0.08	1.15
	(−2.47)		(4.28)	(−4.45)	(−0.56)	(1.57)

Source: Fama and French (1992), pp.439

2.2.4. Responses

The Fama and French (1992) study has itself been challenged. Obviously most claims have been towards F-F findings that beta has no role for explaining cross-sectional variation in returns, that size has an important role as well as book-to-market equity.

Another line of arguments against their findings stems from the disagreement on whether value-weighted portfolio of all stocks listed in NYSE and AMEX is a reasonable proxy for the return on the market portfolio of all assets, and whether assuming that betas are constant over time is reasonable.

2.2.4.1. Black

One of the first who stood up against FF study and pointed out that they are in some aspects contradicting their own finding and even that some of them are mostly due to data mining was Black (1993). As a clear example he chose the “size effect”.

In 1981 Benz found that size matters and favors small companies. Fama and French extended the time up to 1990 and found no size effect at all in the period of (1981-1990) whether or not they control for beta. However that did not stop them from concluding that size is one of the variables that "captures" the cross-sectional variation in average stock return.

According to Black such anomalies as "size effect" might be just as well just a result of data mining. Since there are literally thousands of researchers looking for profit opportunities in security and all of them looking at roughly the same data, *"once in a while, just by chance, a strategy will seem to have worked consistently in the past. The researcher who finds it writes it up, and we have a new anomaly. But it generally vanishes as soon as it's discovered"* (Black 1993, pp. 9) which seems to be the case, since after the paper was published small firms had tame and inconsistent performance.

The lack of theoretical backing of their findings also points out to the possibility of data mining as is the example of ratio book value to the market value of the firm. According to Black *"the past success of this ratio may be due more to the market inefficiencies than "priced factors" of the kind that Fama and French favor"* (Black 1993, pp. 10) Announcements of the death of beta thus to him seemed premature.

2.2.4.2.Amihud, Christensen and Mendelson

Amihud, Christensen and Mendelson (ACM) took a different approach and tried to use more efficient statistical methods. Using the same data as FF they *"obtained a significantly positive coefficient of average return on* β*. An additional improvement in statistical significance was obtained when GLS was applied to account for heteroskedasticity over time and across portfolios as well as cross-portfolio correlations."* (Amihud, Christensen and Mendelson 1992, pp. 16) The results were also robust to the data used.

Another source of possible invalidity of FF findings stems from the data used and the way portfolios are selected. FF used the portfolio selection method established by FM, where stocks are selected only in they had return data for the year of study and for the preceding seven years. The deletion of stocks that were delisted during the year of study leads to "survivorship bias" - the tendency for failed companies to be excluded from performance studies due to the fact that they no longer exist or due to the merge and acquisition.

If, for example, high-risk stocks are more likely to be delisted due to bankruptcy, excluding them from the sample makes the average return of the surviving stocks higher than the average return of all stocks in that risk group, when accounting for the loss due to bankruptcy. In such a case, survivorship bias may create the appearance of a positive risk-return relationship where none exist.

ACM develop a method that tries to eliminate this bias by adjusting the returns and employing GLS methodology. This approach also implies that return- β relationship was positive and significant.

2.2.4.3. Jagannathan and Wang

Jagannathan and Wang argue that poor empirical support of the CAPM might be due to the inappropriate proxy for the aggregate wealth portfolio of all agents in the economy. Most empirical studies of the CAPM assume that the return on a broad stock market index, like NYSE and AMEX, might be a reasonable proxy for the true market portfolio.

The study of Diaz-Gimenez, Presscott, Fitzgerald and Alvarez (1992) points out that, "*almost two-thirds of non-government tangible assets are owned by the household sector, and only one-third of the corporate assets are financed by equity.*" Furthermore, intangible assets, such as human capital, are omitted in stock market indexes. An evidence that strongly undermines the well spread believe.

Jagannathan and Wang (1993) try to tackle this problem by broadening the market index and including human capital in their measure of wealth.[12] Since human capital is unobservable they proxy it with the growth in per capita income.

Their version of CAPM then degenerate into

$$ER_i = \alpha_0 + \alpha_1 \beta_p^{vw} + \alpha_2 \beta_{lp}^{labour} \qquad (2.12)$$

[12] Following Mayers (1973). Mayers was the first to point out the importance of including return to human capital in measuring the return to the aggregate wealth portfolio. (taken from Jagannathan and Wang, 1993)

where ER_i is the expected return on portfolio p, β_p^{vw} is the risk of portfolio p relative to the value-weighted portfolio of all stocks traded on NYSE and AMEX, and β_{lp}^{labour} is the risk of the portfolio p relative to wealth due to the human capital.[13]

With human capital included in this way Jagannathan and Wang show that the CAPM model is able to explain 28 percent of the cross-sectional variation in average returns in the 100 portfolios examined by Fama and French (1992). This is a very significant raise from 1.35 % that can be done with β alone.

2.2.5. Conclusion

The Sharpe-Lintner CAPM has gone over the course of 40 years under heavy research about its validity which we have followed in this sub-chapter. This line of debate however also led to parallel research.

Relaxation of the basic CAPM's assumptions have been examined and led to many versions of the original CAPM :

i. The heterogeneous-expectations version of CAPM by Lintner (1969)
ii. The CAPM with taxes implication by Brennan (1970)
iii. The zero-beta CAPM by Black (1972) assuming that riskless borrowing is not possible
iv. The non-marketable (i.e., human capital) variation of CAPM by Mayers (1972)
v. The intertemporal CAPM by Merton (1973) to include investors' liquidity preference
vi. The skewness-preference CAPM by Kraus and Litzenberger (1976)
vii. The consumption CAPM by Breeden (1979) based on aggregate real consumption rate
viii. The utility-based CAPM by Brown and Gibbons (1985) for testing Breeden's model
ix. The contingent-claims CAPM by Lo (1986) based on the contingent claims analysis

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13 $\beta_p^{vw} = \frac{\text{cov}(R_{p,} R_{vw})}{\text{var}(R_{vw})}$ and $\beta_{lp}^{labour} = \frac{\text{cov}(R_{p,} R_{labor})}{\text{var}(R_{labor})}$ respectively.

x. The behavioral CAPM by Statman and Shefrin (1994) incorporating the noise traders' beliefs

of which we have elaborated on the Black's relaxation of riskless borrowing assumption and some others are dealt with in Keller (2003). We have not been following this line of CAPM debate for the reasons we have already mentioned, but it is important to highlight the consequences of this debate.

Even though many subsequent models are either the variants or extensions of the original theory of portfolio selection of Markowitz and they are often not empirically robust they have enriched the CAPM debate. They however provide a very valuable benchmark, so we can see if the assets are correctly valued and economically justified in terms of their *ex ante* fundamentals as well as their *ex post* performance.

No matter the length of the debate, it has never lost on intensity as well as quality. Over the course of forty years we have learned about CAPM greatly however debate's outcome is still uncertain.

2.3. Market-derived asset pricing model (MCPM)

Given the fierce academic debate about the usefulness of CAPM in reality, research has been done in the field in order to come up with a new model of asset pricing that would overcome the problems bedeviled in the CAPM formula.

One of the newly introduced models is the market-derived asset pricing model (MCPM) by McNulty, Yeh, Schulze and Lubatkin (2002) that will be introduced in this section.

Instead of focusing on drawing information on the historical stock-to-market correlation, it tries to derive the information from the options market and yields on government and corporate bonds. Such approach has an advantage over beta since it enables investor to use ex ante data and also give the possibility to derive different rates according to the time period of the investment, compared to a “fit-for-all” beta. However it also has some bottlenecks, which will be discussed later.

2.3.1. Assumptions about the risk

MCPM is trying to define the risk premiums that investors value, and give them a proper price tag. In order to come up with the risk premium that investors require for their investment, MCPM breaks the overall risk down into three parts.

(i) National confiscation risk

- which measures the risk that an investor will lose the value of investment due to the national policy

(ii) Corporate default risk

- which measures the risk that a company will default as a result of mismanagement independent of macroeconomic considerations

(iii) Equity returns risk

- which reflects the risk that equity investor has because of the residual claim on the company’s earnings is secondary to debt holders’ claim in bankruptcy

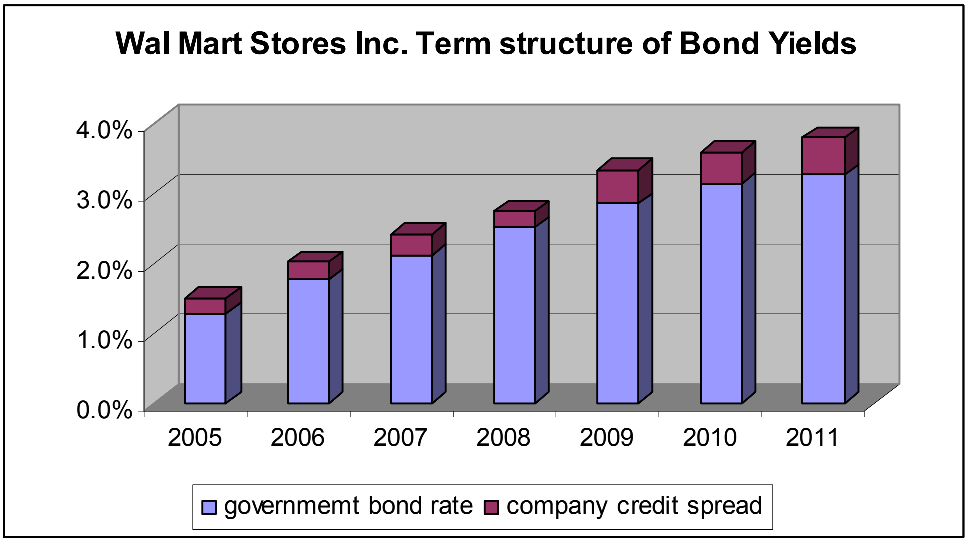
2.3.2. National confiscation and corporate default risk

Information needed for valuing both these risks are obtained from the bond market.

By definition bond is certificate issued by a company or government that promises repayment of borrowed money at a set rate of interest on a particular date. What interests us is the rate of interest that government or corporation has to pay to investors and its logic. Even though U.S. Treasury Bills are often referred to as a risk-free investment they bear an interest, in order to compensate investor for the risk of default, confiscation or runaway inflation. In the case of the corporation, part of the bond yield is due to the risk associated with the mismanagement of the company that is beyond the powers of managers.

Valuing these two types of risk is fairly simple. Compensation for the national confiscation risk is reflected in the government bond rate for a given period of time. The corporate default risk is company credit spread - the premium that a corporation has to pay to its investors on the top of government bonds.

Chart 1. : Term Structure of Bond Yields



Source: www.bondpage.com quotes from 29.3.2004

As seen on the example of Wal Mart Stores Inc. in the Chart 1 national confiscation risk constitute for the majority of the overall risk compensated by the yield on corporate bond and overall rate is rising in time. For companies that have no bonds outstanding, yields can be determined by looking at the debt of companies with equivalent credit ratings, as done in our case[14].

2.3.3. Black-Scholes Formula

Valuation of the equity return risk, which derives most of the information from options market, is based on the Black-Scholes Formula that was presented in the paper *"The pricing of options and corporate liabilities"* (1971). In order to present its computation basics of the formula have to be provided.

The essential idea behind the pricing formula is to set up a package of investment in the stock and loan that will exactly replicate the payoffs from the option. Once we are able to price the stock and the loan, we can price the option.

Even though valuating of such a package seems tedious, Black and Scholes came up with the following formula that calculates the price of put option:

$$P = Xe^{-rT} N(-d_2) - SN(-d_1) \tag{2.13}$$

where

$$d_1 = \frac{\ln(S/X) + rt}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t} \tag{2.14}$$

and

$$d_2 = d_1 - \sigma\sqrt{t} \tag{2.15}$$

and

$$N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt \tag{2.16}$$

and finally

$$\sigma^2 = \text{var}(\ln S_t / S_{t-1}) \tag{2.17}$$

P is the put option price, S is the stock price, N(…) is the cumulative normal distribution function, X is the strike price, r is the annualized risk-free interest rate (as a decimal), t is

[14] In this case quote for the Ontario Prov Cda bond was used for year 2008 and Citigroup Inc bond for year 2010. Both companies have the same rating as Wal Mart Stores. Inc.

the time to expiry (in years) and σ is the annualized standard deviation of stock returns as a decimal.

2.3.4. Equity return risk

Equity returns risk is the risk associated with the position of shareholders. In case the company goes bankrupt, they will stay with their claims at the very end and therefore require for such risk a premium. In order to calculate it; MCPM uses the following four step calculation. Once such premium is calculated it can be added to the previous two and overall risk premium is achieved.

I. Calculation of the forward break-even price

At first it needs to be calculated what is the minimal price of the stock in the future that would compensate investor for holding a share in the company rather than bond. In order to achieve that we begin by determining the minimal capital gain that investors require.

By definition return on equity is sum of rate of capital gain and the dividend yield

$$r_{equity} = r_{capital\ gains} + r_{dividends} \tag{2.18}$$

and also the return on equity should be greater than the one on debt, due to the extra risk shareholders are bearing. Thus the minimal capital gain cannot be lower than the difference between return on debt and the dividend yield.

The return on debt is easily obtained from the yield of bond and the dividend yield is can be easily calculated using the following formula (Brealey and Myers 2000, pp.68)

$$r_{dividend} = \frac{DIV}{P_0} + g \tag{2.19}$$

where DIV is current dividend, P_0 is current share price and g is the expected rate of dividend growth, which authors of MCPM assume to be zero.[15] Once the minimal required capital gain rate is obtained, the price of the stock which needs to be reached at the end of the period in order to persuade the investor to invest in stock rather than bond can be calculated using the standard future value formula (Brealey and Myers 2000, pp. 43)

$$FV = P_0 * (1+i)^t \qquad (2.20)$$

where FV is future value, i is minimal capital gain rate derived earlier and t is the length in years of the investor's holding period.

II. Estimation of the stock's future volatility

Given the calculated minimal price investor requires, our focus shifts to the likelihood that such price will be achieved and what are the costs of ensuring such outcome. Options market and option valuation formula are used in order to derive such an information.

Prices of options reflect truly the markets level of uncertainty about the ability of the company to deliver the expected cash flows. High degree of uncertainty is tied closely with the high volatility of the stock option, until the actual cash flow proves otherwise. In order to obtain this information from the market Black-Scholes option valuation formula is employed.

Taken the quotes for stock option for given period of time from the market we have all the necessary information required to calculate variability. Such estimate reflects the true value in case of the option market efficiency which we assume to hold.

III. Calculate the cost of downside insurance

[15] This assumption is legitimized by reference to the research done in the field, which shows that the current dividend is a good predictor of a company's long-term dividend stream, due probably to the fact that companies make a conscious effort not to surprise the markets with unexpected changes in their financial distribution policies. This assumption seems to be quite dubious. It would be natural to expect that not the dividend but div/profit ratio should be constant and therefore the dividend would at least grow with inflation. However, since the authors don't document this part of the research, we don't have the means to verify their claim.

When we have the estimate of volatility for a given time period, we combine it with the calculated forward breakeven price to determine the price investors would be prepared to pay in order to insure against the chances that their shares will fall below the forward breakeven price. This is the premium that reflects the extra risk of equity over debt. However it is nothing else than the price of the put option – the right to sell in particular time the shares an investor holds – for given stock. Such option protects the investor from the possibility of a drop in stocks price under a given strike price.

This calculation can be done very simply by, again, employment of Black-Scholes Formula in this case only with the exception that the unknown variable is not variability (which we have just calculated), but rather the price of the option.

IV. Derive the annualized excess equity return

In the final stage the price of an option needs to be expressed in terms of annual premium. This can be achieved in a very straightforward way. Ratio of calculated option price and spot price of the stock reveals what percentage of the actual price investor is willing to pay in a form of "insurance" in order to secure minimal price of stock in particular time in the future.

In order to transform this ratio into annualized premium, we employ the standard annuity formula (Brealey and Myers 2000, pp. 42)

$$Present\ value\ of\ annuity = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right] \tag{2.21}$$

where r is the opportunity cost of investing the money for the period of one year, t is the number of years investor will receive annuity payment and C is the coupon. In our case we plug the ratio as a PV, bond rate as r and time period as t. In this way we calculate the annual excess equity return.

Once the last part of the risk is calculated it can be simply added to the two preceding risks and the overall cost of equity is achieved.

2.3.5. Critique

Even though MCPM is one of the first models that attempts to derive information about the asset pricing from the derivatives market, rather than from analysis of ex post stock performance, it is not invincible and posses some bottlenecks.

It is obvious that MCPM does not stand on such a strong theoretical basis as CAPM, but rather is just a set of formulas put together in order to come to a desirable outcome. Its authors are focusing on three risks and are providing a reasonable way of pricing them, however, it is impossible to test whether or not there are some not-so-obvious risks that the authors are unconsciously omitting and therefore not mentioning.

The lack of testability of such a model is crucial and the testimony of the authors that MCPM provides more "reasonable" and "realistic" outcomes than other models is not forceful enough to make us throw away CAPM, on the contrary. This comes with a great contrast to CAPM, which has come during the last couple decades through serious testing, as we have seen in the previous sub-chapter.

Other critique falls on the actual calculation. Even though the bond and derivatives market witnessed unprecedented boom in the last decades it is still very difficult to gather enough data for calculating MCPM. Even for large multi-national companies it is difficult to find data on bonds and options that will cover a large enough time spectrum, which would then enable us to derive different rates according to the time structure of the projects. For small companies and start-ups it is merely impossible.

Take bonds first. Authors encourage the user of this formula, in case he or she is not able to find bond quotes on a given company, to use information from another company in the same field or better with equivalent credit rating. If we followed their advice I believe we would be committing bias.

Looking at the bond market, we see very clearly that the bond rates for companies with a same rating are not the same and thus such assumptions is somewhat dubious. Differences in rates of companies with the same credit rating are not awfully large, but they tend to rise with longer maturity, not to mention the fact that we are currently experiencing very low rates which also might cause small spread. However, if we do not follow their advice we are not able to complete the calculation at all.

It gets trickier with options. Even though MCPM is presented as a model that will enable users to come up with time specific rates, looking at the data available on the market we see that it is only a partial truth. Quotes can be found only for less than two years ahead and therefore we are not able to calculate rates for following periods.

This is not the end of the pitfalls. Option market is used in the calculation to derive expected volatility of the stock in a given period in the future. This lays on the assumption that options are priced based on the Black-Scholes Formula. But if that would hold, then we would get the same volatility from all quotes for given time period regardless of the fact that we take into account put or call options. This is sadly not the case, and the variability is, in this case, truly variable. Should we take the most traded one or rather the one that is closest to our forward break-even price? We might argue that not the most traded one, since it might be due to the fact that investors see it as badly priced and therefore trade it. Authors use the latter case and do not explain why.

Even though MCPM is very appealing in its nature, one has to bear these bottlenecks in mind. Until better theoretical framework is found or proper testing proves the usefulness of this model, its results should be taken with high caution.

3. Asset price estimations

The following section will focus on estimating of asset prices based on the empirical data obtained from the market.[16] As for the CAPM the ex post stock data will be analyzed in order to estimate beta of the stocks and subsequently its asset price. For MPCM spot quotes, bond quotes and quotes on options will be used to derive the asset prices.

In this section four stocks will be analyzed in order to provide data on which we will try to comment the suitability of the two models for asset pricing. These stocks include:

- Wal Mart Inc. – the world's biggest retailer with more than 250 billion dollars in sales in the past 12 months
- eBay Inc. – recently established leader in the e-commerce sector known for its online auction site
- Myriad Genetics Inc. - biopharmaceutical company focused on the development of novel therapeutic products and the development and marketing of predictive medicine products
- American Electric Power Company, Inc. (AEP) - registered public utility holding company. It is one of the biggest company in the American utility sector

These stocks have been chosen because they represent four very different industries as well as giving a possibility to compare the firms from "new" and "old" economy. Also they serve as good examples of bottlenecks connected with MCPM.

3.1. CAPM

3.1.1. Data

For the estimation of beta using Capital asset pricing model and more importantly for the linear regression analysis it is very important to decide what data will be used for the computation.

[16] Data used in this section are available on request at hajekf@mbox.fsv.cuni.cz

Rule of thumb suggests that more data will provide better and more reliable results, but for the sake of simplicity and manageability monthly data for the past 5 years are used. That is compatible with observation made by Berndt that: *"econometric studies based on such data*[17] *have found that in many studies (but with some notable exceptions),* β_i *has tended to be relatively stable over five-year (60 months) time span"* (Berndt 1991, pp. 35)

As for the data itself, monthly data ranging from May 1999 to May 2004 are computed as an average of high and low quotes.

3.1.2. Market portfolio

CAPM takes use of the market portfolio. As we have seen during our excurse in the preceding chapter selecting the right proxy for it is a very difficult exercise. Normally market portfolio is proxied by a market index such as Dow Jones Industrial Average, Standard & Poor's 500, or M-S World Stock Index. Berndt recommends the estimates of the Center for Research on Security Prices at the University of Chicago which uses estimate based on the value-weighted transactions of all stocks listed on the New York and American Stock Exchanges.

Unfortunately such data is not available and therefore S&P 500 will be used as a market proxy. It is important to stress once again that such proxy is not perfect and one has to take that into account when interpreting results.

Similarly as with the stocks, monthly data ranging from May 1999 to May 2004 and computed as an average of high and low quotes are used.

3.1.3. Risk free rate

As for the risk-free rate *"the typical measure of a risk-free rate is something like the 30-day U.S. Treasury bill rate"* (Berndt 1991, pp. 37). Even though U.S. Treasury bills are often referred to as a risk free rate, they are risk-free only if held until maturity and only in a nominal sense. The presence of inflation disables us to use them as a real risk free rate.

[17] NYSE data

However U.S. T-bills do not provide quotes for every month and therefore we have to use 1-month commercial paper rate (CPR) as a proxy. Fortunately they do not differ significantly from the T-bills rates and their rates are available for the time period.

Proxing risk free rate however does not posses too much danger as long as the Black's zero-beta assumption is fulfilled. In our case the correlation between risk free proxy - 1-month commercial paper rate and market proxy – S&P 500 is -0.085 which yields beta of -0.04393. This value is very close to zero and therefore we can quite safely use it as a risk free proxy.

3.1.4. Stocks

As mentioned in the introduction of this chapter stock of following companies will be analyzed: Wal Mart Inc., Ebay Inc., Myriad Genetics, Inc. and American Electric Power Company, Inc.

Generally returns on stocks are defined as in equation (2.18), however dividend gains are omitted in following calculations. Taxes are also omitted for the sake of simplicity.

3.1.5. Regression model

The following regression equation was employed for beta estimation

$$y_{i,t} = \alpha_i + \beta_i \cdot x_t + \varepsilon_{i,t} \tag{3.1}$$

$$rp_{i,t} = \alpha_i + \beta_i \cdot rp_{m,t} + \varepsilon_{i,t} \tag{3.2}$$

$$\left(r_{i,t} - r_{CPR,t}\right) = \alpha_i + \beta_i \cdot \left(r_{S\&P500,t} - r_{CPR,t}\right) + \varepsilon_{i,t} \tag{3.3}$$

where $rp_{i,t}$ is stock's risk premium in t, $rp_{m,t}$ is market risk premium in t, defined as commercial paper rate subtracted from stock's return and S&P 500 index return in t respectively, $\varepsilon_{i,t}$ is residuum in t.

While our attention is focused on β term in the previous three equations, these equations also include term α which does not appear in the Sharpe-Lintner CAPM equation (2.7). As we have already discussed in previous chapter, we expect α to be insignificantly different from zero. As we can see in table 5 α really is insignificant in all cases.

Estimation of beta is done using the ordinary least square (OLS) method. Such method provides the best linear unbiased estimate of beta given the fulfillment of the OLS assumptions. The following assumptions must be checked in order to use OLS method and most importantly to get meaningful results: (Gujarati 2003, pp. 66-75)

- *Regression model is linear in parameters*
- *Values of independent variables are non-random*
- *The residuals are normally distributed*
- *The expected value of residuals is equal to zero*
- *The residuals are independent from one another*
- *Homoskedasticity, i.e. equal variance of residuum*
- *Zero covariance between residuum and independent variables*
- *Number of observations is greater than number of estimated parameters*
- *Variance of independent variable is finite and positive number*
- *Non-existence of specific variance, or error in regression model*
- *Non-existence of multicolinearity among independent variables*

3.1.5.1. Normality

For the test of normality Jarque-Bera test[18] was used. This test pointed to the normality problem with MYGN as can be seen in Table 2. This might be happening due to the limited number of observations.

Under the null hypothesis of a normal distribution, the Jarque-Bera statistic is distributed as with 2 degrees of freedom. The reported Probability is the probability that a Jarque-Bera statistic exceeds (in absolute value) the observed value under the null - a small probability value leads to the rejection of the null hypothesis of a normal distribution.

[18] Jarque, C. M., Bera, A. K.: „A Test for Normality of Observations and Regression Residuals“, International Statistical Review 55, 1987, pp. 163-172.

Table 2.:Jargue-Bera statistic results

Company	Jarque-Bera statistic	P-value
American Electric Power	0,68987	0,708
eBay	2,06144	0,357
Myriad Genetics	12,53360*	0,002
Wal Mart	5,27808	0,071

Source: own calculations based on data

3.1.5.2. Autocorrelation

Durbin-Watson (DW) statistic[19] is a test for first-order serial correlation. More formally, the DW statistic measures the linear association between adjacent residuals from a regression model. If there is no serial correlation, the DW statistic will be around 2. The DW statistic will fall below 2 if there is positive serial correlation (in the worst case, it will be near zero). If there is negative correlation, the statistic will lie somewhere between 2 and 4.

Due to the fact that normality is assumed in Durbin-Watson statistic, MYGN results should be taken in view with consideration. As we see from the Table 3, Durbin Watson detected autocorrelation in the case of Myriad Genetics and eBay. Wal Mart lies on the edge and should also be taken with caution.

Table 3.:Durbin-Watson statistic results

Company	Durbin-Watson statistics
American Electric Power	1.958800
eBay	1.516365*
Myriad Genetics	1.290931*
Wal Mart	1.662800

Source: own calculations based on data

[19] Durbin, J., Watson, G. S.: „Testing for Serial Correlation in Least-Squares Regression", Biometrika 38, 1951, pp. 159-171.

3.1.5.3. Homoscedasticity

Test of the assumption that the variance around the regression line is equal for all values of the predictor variable i.e. homoscedasticity holds, was carried out with White's test.[20] Homoscedasticity was detected in all samples and since White's test does not assume normality these findings are valid for all four stocks.

Table 4.:White's statistic results

Company	White's statistic	P-value
American Electric Power	2.730493	0.255318
eBay	0.469386	0.790813
Myriad Genetics	1.682775	0.431112
Wal Mart	2.252119	0.324309

Source: own calculations based on data

3.1.5.4. OLS results

As we have found in previous sub-chapters some of the assumptions of OLS are not fulfilled. In order to receive more efficient results it would be correct to employ ARCH or GARCH models which would take into account founded heteroscedascity, but this thesis will not take data to this step, due to the increasing difficulty and knowledge needed to properly use them.

OLS method will be employed even though, but reader must take into account that not all assumptions are fulfilled and therefore estimates might not be as efficient as with above mentioned models. And since we are eventually trying to estimate the price of asset and for that purpose beta estimation is only a mean not an end, loss of efficiency in the variability of beta should not play major role.

Table 5 shows the results of both alpha and beta estimates.[21] In line with the theory alpha coefficient is statistically insignificant for all four companies. Beta is

[20] White, H.: „A Heteroscedasticity Consistent Covariance Matrix Estimator and a Direct Test of Heteroscedasticity", Econometrica 48, 1980, pp. 817-818.

[21] The star sign (*) by the P-value in the table indicates that the estimated coefficient is significant at 1 % level, the double star then on 5 %.

statistically significant at 5 % level of significance. Values seem to support the simple assumption that new and IT or biotechnology companies with short sales records should be more risky than companies with more stable cash flow. Both eBay and Myriad Genetics are aggressive stocks since their beta is higher than 1. AEP and Wal Mart are defensive as the very opposite is true.

Table 5. :OLS Results

Company	Alpha estimate	Standard Error	t-Statistic	Beta estimate	Standard Error	t-Statistic
AEP	-0.000743	0.009958	-0.074605	0.833920	0.300267	2.777261*
eBay	0.022515	0.015422	1.459969	1.427142	0.334956	4.260684*
Myriad Genetics	0.055271	0.030811	1.793850	2.025349	0.746953	2.711481*
Wal Mart	0.006435	0.007106	0.905549	0.700203	0.133397	5.249018*

Source: own calculations based on data

Table 6 shows the supplementary characteristics of respective models.

Table 6. :Supplementary regression characteristics

Company	RSS	SE reg.	R^2
American Electric Power	0.347469	0.077401	0.167655
eBay	0.823674	0.119169	0.199271
Myriad Genetics	3.256826	0.236965	0.112500
Wal Mart	0.178238	0.055435	0.216816

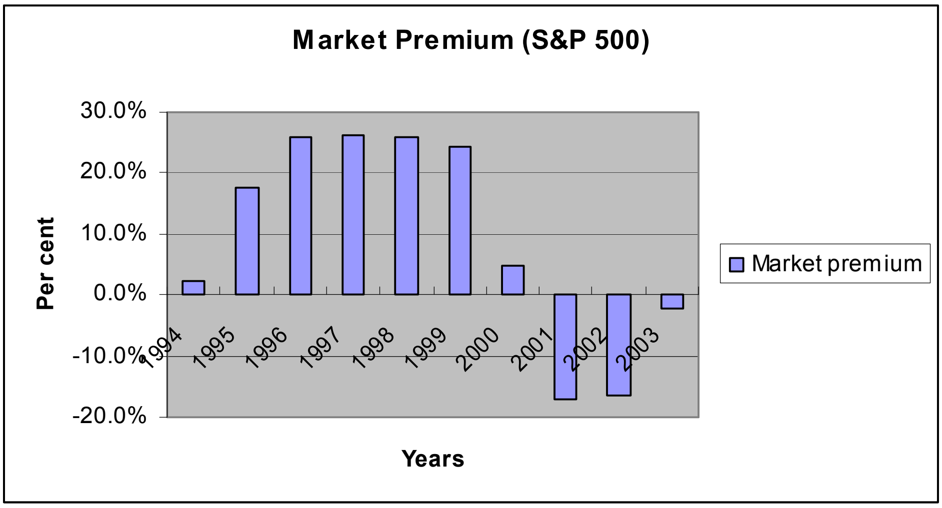
Source: own calculations based on data

3.1.6. Asset price estimate

Since we have estimated beta of the four companies we can now put them in equation (2.8). In order to come up with the result we have to decide on which values to use for market premium as well as for the risk free rate.

Berndt notes that "over the last 60 or so years in the United States the average market risk premium has been about 8.4% per year" (Berndt 1991, pp. 34).[22] Since market risk premium varies hugely in the last 10 years as Chart 2 uncovers, it is appropriate to use Berndt's figure. As for the risk free asset, average from 1-month commercial paper rate for last 12 months will be used.[23]

Chart 2. : Market Premium (1994 – 2003)



Source: own calculations based on data from http://moneycentral.msn.com

Based on the beta estimates in Table 5 and already mentioned data we employ the Black's CAPM equation

$$ER_i = (1 - \beta_i)ER_z + \beta_i ER_m \quad (3.4)$$

and come to the following results.

[22] Taken originally from Roger Ibbotson and Rex Sinquefield (1986) as noted in Berndt.
[23] In our case 1.029 % p.a.

Table 7: Results

Company	Expected return
American Electric Power	7.18%
eBay	11.55%
Myriad Genetics	15.96%
Wal Mart	6.19%

Source: own calculations based on data

3.2. MCPM

3.2.1. Data

MCPM is a forward looking model that gathers information from the bond and option market. Timeframe is given by the availability of data rather than by selecting past period that is evaluated. For the estimation of asset prices using MCPM the following data need to be used: company bonds, option quotes and risk free rate.

3.2.1.1. Bonds

For the purposes of the estimation it is necessary to obtain information on the cost of debt for given companies. As was mentioned in previous chapter it happens very often that such information is not available and we need to derive it from the bonds of same grade. Such is also the case here, where bond rates are obtained in this way for eBay and Myriad Genetics. Consequences of such selection might be, as was said before, severe and our estimates might be biased. This has to be taken into account when judging the results.

Data are taken from the bond market quotes available on 16th May 2004. [24]

3.2.1.2. Options

Even though MCPM is presented as a model that is able to predict asset prices for different time periods, in reality it is limited by the availability of data. Since option markets do not provide quotes for far future (in our case available dates are at furthest for January 2006). Due to the lack of data beyond this point, calculations are made only for

[24] Source: www.bondpage.com

January 2005 and January 2006 with the exception of Myriad Genetics. Market does not provide quotes for Myriad Genetics in the period beyond November 2004, and therefore rate only for this date is calculated and used.

Data as such are taken from on-line quotes and are based on the quotes from 16th May 2004.[25]

3.2.1.3. Risk free rate

Risk free rate is as in the previous case proxied by 1-month commercial paper rate (CPR). In this case however current rate is used.

3.2.2. Asset price estimate

Given the data the following expected returns were calculated for January 2005 and January 2006 period using the procedure described at length in sub-chapter 2.3. Results are in line with the expectations that authors of the model makes *"compensation for the risk of longer investments remain greater than that of shorter ones*[26] *... but the additional compensation for the risk that company takes on per year decreases over time"* (McNulty, Yeh, Schulze and Lubatkin 2002, pp.119)

Table 8: Results

Company	Expected return	
	January 2005	January 2006
American Electric Power	14.53 %	9.02 %
eBay	19.61 %	13.27 %
Myriad Genetics	22.17 %[27]	n.a.
Wal Mart	11.73 %	8.82 %

Source: own calculations based on data

[25] Source: http://moneycentral.msn.com
[26] 20 months of 9,02 is greater then 8 months of 14,53 % in case of AEP
[27] Quote for November 2004

3.3. Comparison

Since we have calculated the same outcomes – expected returns – using two different models we would naturally want to compare them. Unfortunately in the case of CAPM and MCPM is not straightforward. Authors of MCPM put as an undisputable advantage of their model the possibility to derive different rates according to the time period of the investment, compared to a "fit-for-all" beta, however this hinders our ability to make a comparison.

CAPM uses only the historical data to achieve a risk premium and changes slowly over the time. MCPM derives different a risk premium according to the length of the investment both of them come essentially to the same result – asset price. The fact that we do not have a chance to compare the models head-to-head does not make either one of them less helpful. In my opinion both models should be used and the managers' final decision should be based on the information that both models are giving. Since they both have their unique point of view and focus on slightly different aspects combining them seems to be reasonable.

In our case the only thing we might conclude from the comparison of the two results we have is the fact that MCPM gives higher expected rates and therefore higher cost of capital vis-à-vis CAPM. Other conclusions are unfortunately impossible.

4. Conclusion

The aim of this thesis has been to map the introduction of CAPM and to follow one line of the debate it started, as well as to introduce a challenging model based on information from derivative markets. We have explained thoroughly both models and pointed out the difficulties that may arise when interpreting the results as well as during the calculations themselves. Employing both models we have subsequently derived asset prices for four companies traded on the US market.

The results of this thesis are however not as straight forward as we would have wished them to be, due to the fact that even though both models are meant to give answer to the same question, their results are not comparable. We are missing a tool that would enable us to decide which model is providing us with more valuable and accurate predictions of capital assets prices.

Final judgment has to be given by the practitioners around the world, who use not only asset pricing models but also their shrewdness, intuition and experience. Without such insight we are not able to answer this question, but that does not mean that we cannot take sides. In my own view CAPM stands on firmer foundations and its predictions about the risk return relation seem to be valid in the long run.

As for the MCPM, the current boom of derivative markets around the globe makes me believe that it was worth revealing its framework. MCPM, despite the fact that has a lot of bottlenecks, as was pointed out in one sub-chapter, provides us with a very interesting alternative to the main stream asset pricing models. Further research in this field should be encouraged, with respect to the critique that was provided.

Amidst the critique of both models we should however realize that even though today we have quite enough empirical evidence to comfortably reject most of the works done during the last forty years, such rejection should not mislead us to discard the portfolio theory as such. Modern investment theory should be perceived as the indispensable building block for every academician and practitioner to start off, but not to be engrossed, with.

As Sharpe (1984) concludes, *"While the relative importance of various factors changes over time, as do the preferences of investors, we need not completely abandon a valuable framework within which we can approach investment decision methodically. We* *have developed a useful set of tools and should certainly continue to develop them. Meanwhile, we can use the tools we have, as long as we use them intelligently, cautiously, and humbly."*

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