Which Government Interventions Are Good in Alleviating Credit Market Failures?

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Which Government Interventions Are Good in Alleviating Credit Market Failures?

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Abstract:
Credit contracting between a lender with a market power and a small start-up entrepreneur may lead to a rejection of projects whose expected benefits are higher than their total costs when an adverse selection is present. This inefficiency may be eliminated by a government support in the form of credit guarantees or subsidies. The principal-agent model of this paper compares different forms of government support and concludes that a guarantee defined as a proportion of a gross interest rate is not a sufficiently robust policy instrument. Lump-sum guarantees and interest rate subsidies are evaluated as better instruments because they have a nonambiguous positive effect on a social efficiency since they enable funding of socially efficient projects which would not be financed otherwise.

Keywords: information asymmetry, credit, guarantees, subsidies

JEL: D82, G18, H25.
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1 Introduction

The topic of this paper is the public provision of subsidies and guarantees in the credit market characterized by an information asymmetry and a market power of a lender. In the following paragraphs we characterize the main results obtained from our principal-agent model of adverse selection.

The paper concentrates on the socially efficient projects which would be financed by a lender who would have available the same information as a borrower. The introduction of information asymmetry may lead to credit rationing under which some socially efficient projects would not be financed. The credit rationing by a lender with market power takes a specific form of redlining – i.e. rejection of credit to particular group of borrowers. Our binary result of credit being either provided or rejected is different from the well-know phenomenon of credit rationing obtained in a big family of related models, where the credit rationed borrower obtains credit with some probability \( \pi \in (0, 1) \). Our modeling result of redlining is easier to reconcile with the empirically observed lending practices than the random provision of credit predicted by the model of adverse selection at perfectly competitive credit markets. Another interesting new result of general importance is the finding that the choice of redlined borrower depends on the strength of adverse selection as captured in the relative difference between good and bad borrowers. As long as this relative difference is sufficiently strong, the entrepreneur with ex ante lower chance of success is the one who may be redlined. This empirically plausible result is reversed (the borrower with high chance of success may be redlined) if the relative difference in chances of success is too low.

The main part of the paper deals with three different forms of government interventions which may be used to decrease the extent of redlining socially efficient projects. We show that from the government budget point of view the desirability of interest rate subsidies, proportional guarantees and lump-sum guarantees is crucially dependent on the strength
of adverse selection. This result provides theoretical support to the empirically observed fact that governments sometimes prefer subsidies, sometimes guarantees. The only result of our model, which is sufficiently robust with respect to the strength of adverse selection, is the dominance of lump-sum guarantees and interest rate subsidies over proportional guarantees. Proportional guarantees are dominated by other intervention instruments both with respect to their budget cost and with respect to their feasibility to eliminate redlining. In our leading case of sufficiently strong adverse selection the cheapest way for government budget to eliminate redlining is to use lump-sum guarantees.

The analysis of the government interventions into credit markets is a subject of many theoretical or empirical articles in an academic literature. The most relevant theoretical ones with respect to our paper are the papers dealing with government support in the theoretical models of credit provision with financial informational frictions. DeMeza and Webb (1987) and Innes (1991) focus their attention on the efficiency of public interventions connected with variable size of investment projects. The model introduced by DeMeza and Webb (1987) is further developed by Hellmann and Stiglitz (2000). Similar topics are also covered by DeMeza (2002), Cressy (2002), and Lerner (2002). As opposed to our model of adverse selection, Williamson (1994) and Wang and Williamson (1998) investigate government interventions designed to overcome information asymmetries caused by costly state verification. The models closely related to our papers are used by Gale (1990) and Smith and Stutzer (1989). All these models are dealing with the perfectly competitive markets. The assumption of perfectly competitive markets is also shared in the models of government subsidies and guarantees provided by Lacker (1994) and Li (1998, 2002).

Arping, Loranth, and Morrison (2008) model the state sponsored credit guarantees and loan subsidies in a setting where entrepreneurs are capital constrained and subject to moral hazard. In their model, guarantees can raise social welfare because they reduce the cost of capital faced by entrepreneurs, and so potentially enhance entrepreneurial effort incentives. They also show that overly generous guarantees have perverse incentive effects since they
induce lenders to reduce lending standards and to lower their collateral requirements.

Government guarantees and subsidies for commercial credit are often targeted towards the small and medium size firms. The problem of insufficient financial resources for big investment projects is sometimes addressed by government guarantees and subsidies too, but usually other resources like the government support in the form of creating favorable conditions for foreign direct investment (Gersl (2008), Gersl and Hlavacek (2007)) are preferred.

An important example of the program targeted to small and medium size enterprises (SME) is US Small Business Administration (SBA). Since its creation in 1953, SBA engaged in provision of direct loans and bank loan guarantees. After recognizing that commercial banks are usually better than government institution in identifying which projects to support, SBA started to switch from direct loans towards loan guarantees in the mid-1980s. Currently, in 2007-2008, the SBA extends direct loans only under very special circumstances and specializes on the guaranteed loan program. The guarantees by SBA are provided to the commercial lenders who firstly structure their own loans according to underwriting requirements of SBA and then apply for and receive a guarantee from the SBA on a portion of the loan. The SBA usually guarantees about 50 to 85 percent of the amount of a loan. According to SBA (2008) maximum loan size is USD 2 million and the maximum guarantee on such a loan is USD 1.5 million. The activities of SBA are analyzed by Craig, Jackson, and Thomson (2008).

In the Europe, the credit guarantee institutions concerned with small and medium size firms formed a network AECM, which represents 34 guarantee organizations in 18 countries of the European Economic Area. The AECM does not serve as any EU counterpart to SBA in US. The task of AECM is to represent common interest of its member organization, to promote the harmonisation of the relevant legal framework, to provide information exchange among its members and to provide proposals and other ideas related to credit guarantees to economic policy makers, especially in the European Union. While the credit
support policies in different AECM member institutions are different, they often share some common programs and regulation because they usually serve as agents for distribution of EU support. The members of AECM usually provide not only guarantees but they also engage in provision of interest rate subsidies or in direct lending.

Credit guarantees are also extensively used by European financial institutions like European Bank for Reconstruction and Development (EBRD), European Investment bank (EIB) and European Investment Fund (EIF). These major international institutions usually do not provide guarantee for a specific loan extended by a commercial lender to a particular borrower. Their guarantees are in general given as a lump sum amount to a specific financial institution, which is able to use them for support of particular class of borrowers. The example of such guarantee may be provision of USD 1 million guarantee from EBRD to a leading Kyrgyz microfinance lender Bai Tushum in 2006 which was followed by additional USD 2 million in 2007 (Bai Tushum (2007)).

Significant part of EU support of loans for small businesses is currently channeled through the Competitiveness and Innovation Framework Programme (CIP). EUR 1.1 bn budgeted for CIP for the period 2007–2013 will be used to subsidize loans extended to SMEs by a range of financial institutions. The actual administration of CIP money is done by EIF. According to EIB (2008) the CIP SME Guarantee Facility comprises four main business lines: loan guarantees, micro-credit guarantees, equity guarantees and securitisation.

Credit guarantees are not the only form of credit support provided by these institutions. According to Patacchini and Rapisarda (2003) the EIB provided interest rate subsidies on its loans under its temporary lending facility in the 1990s. This subsidy was in the form of interest rebate of 2 base points and it was provided to the firms which already obtained EIB loans under easier terms than they would get in commercial credit market. Similarly as in the case of credit guarantees, international institutions like EIB keep providing indirect interest rate subsidies on commercial loans extended to SMEs. According to EIB (2008) the interest rate subsidy is technically realized through EIB lines of credit, by which the
EIB provides funds to commercial banks at low rates. These low rates are then passed on to SMEs through lower interest rates on the commercial loans approved by these banks.

Another often used tool of government support to commercial credit provision are interest rates subsidies. In addition to small and medium size firms promotion they are especially used for the agricultural credit or the home financing. The other classical area in which interest rate subsidies used to be widely utilized was the export promotion (Crane and Kohler (1985)). Nowadays, their use is significantly restricted by international agreements, especially by World Trade Organization (WTO). The WTO agreements explicitly forbid any use of subsidies promoting export. The exceptions to this general rule are covered in Arrangement on Officially Supported Export Credits (2005).


Out of these country studies, the Japanese ones are especially interesting since from 1998–2001, the Japanese government implemented a massive credit guarantee program that was unprecedented in both scale and scope. Using a new panel data set of Japanese firms Uesugi, Sakai, and Yamashiro (2006) empirically test whether government credit programs do more to stimulate small business investment or serve to worsen adverse selection problems in credit markets. They find evidence consistent with the former. Their empirical
study concludes that program participants significantly increase their leverage, particularly their use of long-term loans, and with the exception of high-risk firms, become more efficient. Overall, these findings suggest that government interventions in credit markets can be beneficial.

2 Model


Our model has two time periods which are referred to as ex ante and ex post. There are three classes of economic agents in this model. These are government, lenders, and borrowers. The government is modeled as a benevolent body whose only concern is an increase in social efficiency and whose only role is to distribute exogenously determined guarantees and subsidies. The role of lenders is to provide financial funds which are needed by borrowers in order to realize their projects. Risk neutral lenders are effectively colluded and act as a single principal with a market power. The existence of a lender’s market power is very characteristic for the banking sector in many countries, as documented in the empirical studies by Pruteanu-Podpiera, Weill, and Schobert (2008) and Bikker, Spierdijk, and Finnie (2007). The supply of funds facing lenders is perfectly elastic, so that the lenders have available any demanded amount of funds under the unit cost of $\rho$.

There are two types of risk neutral borrowers in this model, indexed as a type 1 and type 2. The two types are distinguished by their probability of successfully finishing their project, denoted as $0 < \delta_1 < \delta_2 < 1$, and by their reservation utilities from not participating in the project, denoted as $b_1 < b_2$. A type 1 borrower is labeled as a high risk borrower
and a type 2 borrower as a low risk borrower. The probability that the random borrower facing a lender is a type 1 borrower is \( \theta \), which is the proportion of type 1 borrowers in the total population of borrowers.

In the models of adverse selection, it is usual to consider the participation cost (alternative return) for the agent to be the same for all types of the agent. This simplifying assumptions may be questioned. Why the agents should be heterogeneous with respect to the project analyzed in the adverse selection model and to be homogeneous otherwise? In reality the probabilities of success in a analyzed project and the outside alternative returns may be independent or positively or negatively correlated. In our model we assume positive correlation, where type 2 borrower has both higher probability of success in the contemplated investment project and higher reservation utility. We have chosen to analyze the positive correlation case since we expect that better entrepreneurs (the ones with higher chance of success) are likely to obtain better outcomes (higher reservation utility) in the case they would abstain from the project considered in this model too.

Our model belongs to the family of adverse selection models of business credit provision, where the ex-ante unobservable different abilities of aspiring entrepreneurs create the selection problem for the lender. The bigger this unobservable difference in success probabilities among applicants for credit is, the stronger is the adverse selection problem for the lender. When we take the relative difference between the outside alternative returns of entrepreneurs as a fixed reference point, we may distinguish the cases when the heterogeneity in the terms of success probabilities in a contemplated investment project is higher or lower than a reference point chosen in this way. Therefore we define strong and weak adverse selection in the following sense. The strong adverse selection happens when the success probabilities \( \delta_1 \) and \( \delta_2 \) are sufficiently different such that \( \frac{\delta_2}{\delta_1} > \frac{b_2}{b_1} \). Weak adverse selection happens when \( \frac{\delta_2}{\delta_1} \leq \frac{b_2}{b_1} \). Notice that the assumption of the same alternative returns for agent, which is usually imposed in the literature, leads automatically to our strong adverse selection case. Therefore our weak adverse selection scenario may serve as
a robustness check on what happens when the usual assumption of uniform participation cost is relaxed and the differences in the success probabilities are relatively small.

The assumption of risk neutrality of borrowers serves to emphasize the adverse selection aspects of this model. In this way we do not confuse the exposition with the implications of the well known result that the optimal risk sharing arrangement between risk neutral and risk adverse agents is to have the risk neutral agent bear all the risk. On the intuitive level we can support the assumption of the risk neutrality of the borrower by pointing out that the borrower in this model does not ask for a consumer loan, but for a loan for production purposes. The production activity of the borrower is strictly separated from his personal life (we are using the framework of a limited liability company).

The borrower can either undertake one risky project, which yields $y$ in the case of success and $0$ in the case of failure, or he can become engaged in some other activity, which yields an expected return of $b_i$, $i \in \{1, 2\}$. In order to undertake the project, the borrower has to borrow a fixed amount of money from the lender. The size of this loan is normalized to 1.

The flow of funds from lenders to borrowers and the repayment of these funds is governed by contracts. Each lender offers two types of contract. Each contract is a pair $(\pi_i, R_i)$, $i \in \{1, 2\}$ where $\pi_i$ is the probability that the application of the borrower who chooses this contract will be satisfied and he will be really lent money and $R_i$ is the interest factor $(1 + \text{interest rate})$, which is equal to the required repayment because of our normalization of the loan size to 1. It is possible that both types of contracts will be the same, which would mean that there will only be one contract pooling all borrowers together.

While this type of interest rate/probability of rationing contract menu is a standard approach used in the theoretical literature mentioned at the beginning of this section, the empirical relevance of credit contracts which specify some nontrivial probability of rejection seems somehow doubtful. Nevertheless, the solution of our models shows that in the equilibrium, all credit applications are either accepted or rejected with probability
one. Therefore the equilibrium credit contracts in our model do not exhibit empirically disturbing stochastic character.

The expected utility of a borrower of type \( i \) who applies for a contract designed for a borrower of type \( j \) is given by:

\[
U_{ij} = \pi_j[\delta_i(y - R_j) - b_i].
\]  

The values of all parameters are known by borrowers, lenders and government. The only informational asymmetry in the model is that lenders or government do not know the type of borrower.

The expected profit to a lender on one loan provided to a borrower of type \( i \) under asymmetric information is given as:

\[
B_i = \pi_i[\delta_iR_i - \rho].
\]  

We assume that in the case that a lender is indifferent between lending and not-lending, he resolves this tie in the favor of lending.

The government can attempt to reduce the inefficiencies created by credit rationing through three types of interventions in this model. Under the proportional guarantees program, the government guarantees the payment of the fraction \( \alpha_i \) of the contracted loan repayment in the case of zero return from a project. The expected profit equation (2) is modified as:

\[
B_i = \pi_i[\delta_iR_i + (1 - \delta_i)\alpha_iR_i - \rho].
\]  

Under the lump-sum guarantees program the government guarantees the payment of an exogenously determined lump-sum \( g_i \) in the case of zero return from the project. The lender’s expected profit equation (2) is modified as:

\[
B_i = \pi_i[\delta_iR_i + (1 - \delta_i)g_i - \rho].
\]
The third considered type of an intervention is an interest rate subsidy $s_i$, which is paid only in the case of the project’s success, as opposed to guarantees, which are paid in the case of failure. While the subsidy reduces the interest rate paid by a borrower, we can treat it analytically just like an exogenous supplement repayment to a lender. The expected profit equation (2) is then modified as:

$$B_i = \pi_i\left[\delta_i(R_i + s_i) - \rho\right]. \quad (5)$$

The expected utility of a borrower, under all three types of interventions, is still given by equation (1) since the interventions influence the borrower’s utility only indirectly through their impact on the lender’s profit.

3 Economy without Government Intervention

As a benchmark against which inefficiencies caused by information asymmetries can be evaluated, we first consider the symmetric information case. Under this scenario the lender has exactly the same information as borrowers and he is able separate borrowers perfectly into two different markets. The optimal contract for the lender with a market power is the one in which he maximizes his expected profit subject to individual rationality (participation) constraint for entrepreneurs who wants to borrow money:

$$\max_{(\pi_i, R_i)} B = \pi_i[\delta_i R_i - \rho]$$

s.t.

$$\pi_i[\delta_i(y - R_i) - b_i] = 0, \quad (IRi)$$

$$0 \leq \pi_i \leq 1,$$

$$i \in \{1, 2\}.$$  

It is not difficult to show that the solution to this inequality constrained problem is given by:
As long as the lender has the same information as the borrower he is able to extract all surplus, and the individual rationality constraints (IRi) of a borrower $i$ is binding. There is no inefficiency in this case since the project is financed and undertaken if and only if the expected return of a project ($\delta_i y$) is equal or bigger than the social cost ($b_i + \rho$). The financing decision of the lender with a market power is the efficient one and consequently there is no efficiency reason for government intervention in this case. In the rest of the paper we will concentrate our attention on the projects satisfying the social efficiency criterion

$$\delta_i y > b_i + \rho.$$  \hfill (6)

We will investigate the cases when the introduction of information asymmetry between borrower and lender with a market power leads to the rejection of the project. When such a rejection happens we will suggest possible government interventions which would enable financing of the socially efficient projects which would not be undertaken because of information asymmetry.

Under asymmetric information, the lender does not know ex ante the type of entrepreneur who asks for a loan. There is a possibility that the entrepreneur will misrepresent his type. Consequently, the lender in his maximization problem has to take into account the borrower’s incentive compatibility constraints, which we denote (IC1) and (IC2) in the following formalization of the lender’s maximization problem:

$$\max_{(\pi_1, R_1, \pi_2, R_2)} B = \theta B_{11} + (1 - \theta) B_{22}$$
\[ = \theta \pi_1 [\delta_1 R_1 - \rho] + (1 - \theta) \pi_2 [\delta_2 R_2 - \rho] \]

s.t.
\[
\begin{align*}
\pi_i [\delta_i (y - R_i) - b_i] & \geq 0, \quad \text{(IRi)} \\
\pi_1 [\delta_1 (y - R_1) - b_1] & \geq \pi_2 [\delta_1 (y - R_2) - b_1], \quad \text{(IC1)} \\
\pi_2 [\delta_2 (y - R_2) - b_2] & \geq \pi_1 [\delta_2 (y - R_1) - b_2], \quad \text{(IC2)} \\
0 & \leq \pi_i \leq 1, \quad i \in \{1, 2\}.
\end{align*}
\]

In the case of strong adverse selection, the solution to this problem is given by:

\[
\begin{align*}
R_1^* & = \begin{cases} y - \frac{b_1}{\delta_1} & \text{if } \pi_1^* \neq 0, \\
\text{any value} & \text{if } \pi_1^* = 0, \end{cases} \\
R_2^* & = \begin{cases} y - \frac{b_1}{\delta_1} & \text{if } \pi_1^* = 1, \\
\frac{y - b_2}{\delta_2} & \text{if } \pi_1^* = 0, \end{cases} \\
\pi_1^* & = \begin{cases} 1 & \text{if } \delta_1 y - b_1 - \rho \geq \frac{1-\theta}{\theta} \delta_2 \left(\frac{b_1}{\delta_1} - \frac{b_2}{\delta_2}\right), \\
0 & \text{otherwise}, \end{cases} \\
\pi_2^* & = 1.
\end{align*}
\]

In the case of weak adverse selection the solution is:

\[
\begin{align*}
R_1^* & = \begin{cases} y - \frac{b_2}{\delta_2} & \text{if } \pi_2^* = 1, \\
\frac{y - b_1}{\delta_1} & \text{if } \pi_2^* = 0, \end{cases} \\
R_2^* & = \begin{cases} y - \frac{b_2}{\delta_2} & \text{if } \pi_2^* \neq 0, \\
\text{any value} & \text{if } \pi_2^* = 0, \end{cases} \\
\pi_1^* & = 1, \\
\pi_2^* & = \begin{cases} 1 & \text{if } \delta_2 y - b_2 - \rho \geq \frac{\theta}{1-\theta} \delta_1 \left(\frac{b_2}{\delta_2} - \frac{b_1}{\delta_1}\right), \\
0 & \text{otherwise}. \end{cases}
\end{align*}
\]

**Proof.** In Appendix.

As long as expected return of the project is sufficiently higher than the social cost, the credit contract is the pooling one, where all entrepreneurs obtain the financing under
the same conditions. When the difference between expected return and the social cost is still positive, but small, the credit contract leads to the separating outcome. In this separating outcome one of the borrowers obtains his symmetric information contract and the other one is rejected the credit (is redlined). The pooling outcome is socially efficient since all projects satisfying social efficiency criterion (6) which would be financed under symmetric information are still financed. The only difference between symmetric and asymmetric information situations is the distribution of social surplus among the lender and different types of borrower. This distributional effect is caused by the result that in pooling equilibrium one of the borrowers is charged lower price than under symmetric information.

In the separating outcome with strong adverse selection the outcome intuitively expected by a practitioner happens. The high risk borrower is redlined if the expected social surplus generated by his project is small. The separating outcome with weak adverse selection is characterized by redlining of low risk entrepreneurs, which is rather unintuitive result from the point of view of usual banking practice. Nevertheless this result does not reject the rationale for government support. From the point of view of keeping a high enough level of production in the sector targeted by government support, this weak adverse selection case actually presents higher pressure for government intervention than the strong adverse selection case.

4 Government Interventions

4.1 Economy with Strong Adverse Selection

The credit rationing of high risk borrowers caused by the informational asymmetry between lender and borrower could be eliminated by government intervention. We consider three different types of government intervention: proportional guarantees, lump-sum guarantees,
and interest rate subsidies. In the following subsections we analyze the credit market equilibrium and the government budget impact of these credit support programs.

### 4.1.1 Proportional Guarantees

The proportional guarantee intervention means that the government guarantees the payment of the fraction $\alpha$ of the loan in the case of zero return from the project. This leads to the lender’s expected profit function (3) which was derived in the Model section of this paper. The optimization problem for the lender with market power in the presence of proportional guarantees is therefore given as follows:

$$
\max \left( \pi_1, R_1, \pi_2, R_2 \right) \quad B = \theta B_{11} + (1 - \theta) B_{22} = \theta \pi_1 \left( \delta_1 R_1 + (1 - \delta_1) \alpha R_1 - \rho \right) + (1 - \theta) \pi_2 \left( \delta_2 R_2 + (1 - \delta_2) \alpha R_2 - \rho \right)
$$

subject to the same conditions as in the case without an intervention.

The solution to this problem is:

$$
R_1^* = \begin{cases} 
    y - \frac{b_1}{\delta_1} & \text{if } \pi_1^* \neq 0, \\
    \text{any value} & \text{if } \pi_1^* = 0 
\end{cases} \quad (8)
$$

$$
R_2^* = \begin{cases} 
    y - \frac{b_1}{\delta_1} & \text{if } \pi_1^* = 1, \\
    y - \frac{b_2}{\delta_2} & \text{if } \pi_1^* = 0 
\end{cases} \quad (9)
$$

$$
\pi_1^* = \begin{cases} 
    1 & \text{if } \delta_1 y - b_1 - \rho \geq \frac{1 - \theta}{\pi_1} \left( \frac{b_1}{\delta_1} - \frac{b_2}{\delta_2} \right) \delta_2 + (1 - \delta_2) \alpha \left( 1 - \delta_1 \right) \alpha \\
    0 & \text{otherwise} 
\end{cases} \quad (10)
$$

$$
\pi_2^* = 1. \quad (11)
$$

**Proof.** In Appendix.

For $\alpha = 0$, the critical expected social surplus for loan being granted to high risk entrepreneur is identical to the situation without any government support. The influence of any proportional guarantee $\alpha$ bigger than zero on the cut-off value of social surplus
leading to the credit rationing of high risk entrepreneur is not immediately obvious from the inequality (10) since the increase in $\alpha$ has both positive and negative effects on the right hand side of (10). The comparison of these effects shows that implementation of proportional guarantee decreases the required cut-off for credit provision if and only if the return in the case of the project’s success is sufficiently high:

\[
y > \frac{1 - \theta}{\theta} \left( \frac{b_1}{\delta_1} - \frac{b_2}{\delta_2} \right) \left( 1 - \delta_2 \right) + \frac{b_1}{\delta_1}.
\]

(12)

As long as this condition is satisfied, redlining of high risk entrepreneur may be eliminated. Obviously, the cheapest way to eliminate credit rationing by proportional guarantees is to provide guarantee $\alpha$ which solves inequality in (10) as an equation. This optimal value of $\alpha$ is

\[
\alpha = \frac{1 - \theta \left( \frac{b_1}{\delta_1} - \frac{b_2}{\delta_2} \right) \delta_2 - (\delta_1 y - b_1 - \rho)}{\delta_1 y - \frac{b_1}{\delta_1} \left( 1 - \delta_1 \right) - \frac{1 - \theta}{\theta} \left( \frac{b_1}{\delta_1} - \frac{b_2}{\delta_2} \right) \left( 1 - \delta_2 \right)}.
\]

(13)

Since government intervention is required only when the numerator of (13) is positive, positive intervention $\alpha$ is possible if and only if the denominator of (13) is also positive, which is guaranteed as long as condition (12) is satisfied.

4.1.2 Lump-sum Guarantees

Since the government guarantees the payment of an exogenously determined lump-sum $g$ in the case of zero return from a project, the maximization problem of the lender under this intervention is:

\[
\max_{(\pi_1, R_1, \pi_2, R_2)} B = \theta B_{11} + (1 - \theta) B_{22}
\]

\[
= \theta \pi_1 [\delta_1 R_1 + (1 - \delta_1)g - \rho] + (1 - \theta) \pi_2 [\delta_2 R_2 + (1 - \delta_2)g - \rho]
\]

(14)

subject to the same conditions as in the case without an intervention.

The solution to the lender’s maximization problem is:

\[
R_1^* = \begin{cases} 
    y - \frac{b_1}{\delta_1} & \text{if } \pi_1^* \neq 0, \\
    \text{any value} & \text{if } \pi_1^* = 0,
\end{cases}
\]

(15)
\[
R^*_2 = \begin{cases} 
  y - \frac{b_1}{\delta_1} & \text{if } \pi^*_1 = 1, \\
  y - \frac{b_2}{\delta_2} & \text{if } \pi^*_1 = 0,
\end{cases} 
\]

(16)

\[
\pi^*_1 = \begin{cases} 
  1 & \text{if } \delta_1 y - b_1 - \rho \geq \frac{1-\theta}{\theta} (\frac{b_1}{\delta_1} - \frac{b_2}{\delta_2}) \delta_2 - (1 - \delta_1) g, \\
  0 & \text{otherwise},
\end{cases} 
\]

(17)

\[
\pi^*_2 = 1. 
\]

(18)

**Proof.** In Appendix.

For \( g = 0 \), we obtain the same result as in the case without intervention. As opposed to the case of proportional guarantees, the effect of lump-sum guarantees on the cut-off value of social surplus determining the redlining of high risk borrower is nonambiguous and it is immediately obvious. Taking the derivative of a right hand side of (17) with respect to \( g \), which is equal to \((\delta_1 - 1)\), we see that an increase in a lump-sum guarantee increases the chance that a loan to a high risk borrower will be granted with a probability \( \pi^*_1 = 1 \). Solving the inequality in (17) as an equation provides the smallest value \( g \) for which loans to a high risk borrower will be always granted with a probability of \( \pi^*_1 = 1 \):

\[
g = \frac{1-\theta}{\theta} \delta_2 (\frac{b_1}{\delta_1} - \frac{b_2}{\delta_2}) - (\delta_1 y - b_1 - \rho)}{1 - \delta_1}. 
\]

(19)

**4.1.3 Interest Rate Subsidies**

The maximization problem of the lender under the interest rate subsidies is given by:

\[
\max \left( \pi_1, R_1, \pi_2, R_2 \right) B = \theta B_{11} + (1 - \theta) B_{22} \\
= \theta \pi_1 [\delta_1 (R_1 + s) - \rho] + (1 - \theta) \pi_2 [\delta_2 (R_2 + s) - \rho] 
\]

(20)

s.t. the same conditions as in the case without an intervention.

The subsidy is paid only in the case of the project’s success, as opposed to guarantees which are paid in the case of failure. The subsidy is just an exogenous supplement to
a repayment to a lender and it does not enter into the (IC) and (IR) constraints of a borrower.

The solution of the lender’s maximization problem is:

\[
R_1^* = \begin{cases} 
    y - \frac{b_1}{\delta_1} & \text{if } \pi_1^* \neq 0, \\
    \text{any value} & \text{if } \pi_1^* = 0,
\end{cases} \tag{21}
\]

\[
R_2^* = \begin{cases} 
    y - \frac{b_1}{\delta_1} & \text{if } \pi_1^* = 1, \\
    y - \frac{b_2}{\delta_2} & \text{if } \pi_1^* = 0,
\end{cases} \tag{22}
\]

\[
\pi_1^* = \begin{cases} 
    1 & \text{if } \delta_1 y - b_1 - \rho \geq \frac{1-\theta}{\theta} \delta_2 \left( \frac{b_1}{\delta_1} - \frac{b_2}{\delta_2} \right) - \delta_1 s, \\
    0 & \text{otherwise},
\end{cases} \tag{23}
\]

\[
\pi_2^* = 1. \tag{24}
\]

**Proof.** In Appendix.

In the same way as in the cases of guarantees, we obtain the same result as in the credit market without intervention if \( s = 0 \). Taking the derivative of the right hand side of (23) with respect to \( s \), which is equal to \((-\delta_1)\), we immediately see that an increase in interest payment subsidies increases the chance that the loan to a high risk borrower will be granted with a probability \( \pi_1^* = 1 \). Solving inequality in (23) as an equation provides the smallest value \( s \) for which credit rationing of a high risk borrower will be eliminated:

\[
s = \frac{1-\theta}{\theta} \delta_2 \left( \frac{b_1}{\delta_1} - \frac{b_2}{\delta_2} \right) - (\delta_1 y - b_1 - \rho) \frac{1}{\delta_1}. \tag{25}
\]

### 4.1.4 Government Budget Impact of Interventions

In order to compare government budget impact of various types of interventions we consider such values \( G_s, G_g, G_\alpha \) of subsidies \( s \), lump-sum guarantees \( g \), and proportional guarantees \( \alpha \) which make sure that a loan to a type 1 borrower will be always granted with a probability \( \pi_1^* = 1 \). From (20) we get the expected budget cost of government subsidies:

\[
G_s = \theta \delta_1 s + (1 - \theta) \delta_2 s = s[\theta \delta_1 + (1 - \theta) \delta_2]. \tag{26}
\]
From (14) we get the expected budget cost of lump-sum guarantees:

\[ G_g = \theta (1 - \delta_1)g + (1 - \theta)(1 - \delta_2)g = g \{ 1 - [\theta \delta_1 + (1 - \theta)\delta_2] \}. \] (27)

From (7) we get the expected budget cost of proportional guarantees:

\[ G_\alpha = \alpha(y - \frac{b_1}{\delta_1})[\theta (1 - \delta_1) + (1 - \theta)(1 - \delta_2)] \]
\[ = \alpha(y - \frac{b_1}{\delta_1})\{ 1 - [\theta \delta_1 + (1 - \theta)\delta_2] \}. \] (28)

In order to compare the costs of subsidies and lump-sum guarantees we substitute \( s \) from (25) into (26) and \( g \) from (19) into (27) and we compare \( G_s \) and \( G_g \):

\[ G_g - G_s = \frac{1 - \theta}{\delta_1} \frac{\delta_2(b_1 - b_2)}{\delta_1 - \delta_2} - (\delta_1 y - b_1 - \rho) \{ 1 - [\theta \delta_1 + (1 - \theta)\delta_2] \} - \frac{1 - \theta}{\delta_1} \frac{\delta_2(b_1 - b_2)}{\delta_1 - \delta_2} - (\delta_1 y - b_1 - \rho)[\theta \delta_1 + (1 - \theta)\delta_2]. \]

This can be simplified as:

\[ G_g - G_s = \frac{1 - \theta}{\delta_1} \frac{\delta_2(b_1 - b_2)}{\delta_1 - \delta_2} - (\delta_1 y - b_1 - \rho) \left( \frac{1 - \theta}{\delta_1} \frac{\delta_1 y - b_1 - \rho - (\delta_1 y - b_1 - \rho)}{\delta_1} \right) < 0. \] (29)

This shows that in order to eliminate credit rationing of high risk borrowers, it is cheaper for the government to use lump-sum guarantees than subsidies.

To compare the costs of proportional guarantees \( G_\alpha \) and lump-sum guarantees \( G_g \), we substitute \( \alpha \) from (13) into (28):

\[ G_g - G_\alpha = \frac{\delta_1 y - b_1}{\delta_1} \left( 1 - \delta_1 \right) - \frac{\delta_1 y - b_1}{\delta_1} \left( 1 - \delta_1 \right) - \frac{1 - \theta}{\delta_1} \frac{\delta_2(b_1 - b_2)}{\delta_1 - \delta_2} - (\delta_1 y - b_1 - \rho) \left[ \frac{1 - \theta}{\delta_1} \frac{\delta_1 y - b_1 - \rho - (\delta_1 y - b_1 - \rho)}{\delta_1} \right] < 0. \] (30)
The inequality (30) is satisfied if and only if inequality (12) is satisfied.

Since the positive proportional guarantee $\alpha$ can be used only if (12) is satisfied, the inequality (30) implies that the lump-sum guarantees are cheaper for the government than proportional guarantees.

When comparing $G_s$ and $G_\alpha$ we get:

$$G_s - G_\alpha = \left[\frac{1 - \theta}{\theta} (\frac{b_1}{\delta_1} - \frac{b_2}{\delta_2}) \delta_2 - (\delta_1 y - b_1 - \rho)\right],$$

which is positive iff

$$y > \frac{[\theta \delta_1 + (1 - \theta) \delta_2] \frac{1 - \theta}{\delta_1} (\frac{b_1}{\delta_1} - \frac{b_2}{\delta_2})(1 - \delta_2)}{(1 - \theta)(\delta_2 - \delta_1)} + \frac{b_1}{\delta_1}. \quad (31)$$

The result of the comparison of budget cost of different intervention programs is that lump-sum guarantees are the cheapest form of intervention. When the yield from a successful project $y$ is very high so that inequalities (31) and (12) are satisfied, it is possible to use proportional guarantees which are cheaper for the government than subsidies (but they are still more expensive than lump-sum guarantees). If the size of a return in a successful state of nature is intermediate, so that (12) is satisfied and (31) is not, then it is still possible to use the proportional guarantees, but they are in this case the most costly intervention program for a government budget.

Since the subsidies are paid in the case of success and guarantees are paid in the case of failure, the result that guarantees are cheaper for the government is quite intuitive for high probabilities of success $\delta_i$. What is not so obvious is that according to (29), guarantees are cheaper even in the case of low probabilities of a success.

From the point of view of a lender, the ordering of the desirability of different forms of government interventions is exactly reversed since the lender prefers the highest possible transfers from the government.
4.2 Economy with Weak Adverse Selection

The structure of maximization problems facing lenders and the approach to their solution is the same as in the analysis of government interventions in the strong adverse selection economy case (as presented in the section 4.1) with modifications along the lines of the analysis of the weak adverse selection economy (as presented in the proof of credit contract under asymmetric information).

The solution under all three types of interventions considered in this paper is:

\[
R^*_1 = \begin{cases} 
  y - \frac{b_2}{\delta_2} & \text{if } \pi^*_2 = 1, \\
  y - \frac{b_1}{\delta_1} & \text{if } \pi^*_2 = 0,
\end{cases}
\]

\[
R^*_2 = \begin{cases} 
  y - \frac{b_2}{\delta_2} & \text{if } \pi^*_2 \neq 0, \\
  \text{any value} & \text{if } \pi^*_2 = 0,
\end{cases}
\]

\[
\pi^*_1 = 1,
\]

\[
\pi^*_2 = \begin{cases} 
  1 & \text{if } \delta_2 y - b_2 - \rho \geq A, \\
  0 & \text{otherwise}.
\end{cases}
\]

The expression A in the solution for \(\pi^*_2\) takes the following form for the different intervention programs:

Proportional guarantees:

\[
A = \frac{\theta}{1 - \theta} \left( \frac{b_2}{\delta_2} - \frac{b_1}{\delta_1} \right) \left[ \delta_1 + (1 - \delta_1)\alpha \right] - \frac{\delta_2 y - b_2}{\delta_2} (1 - \delta_2)\alpha.
\]

Lump-sum guarantees:

\[
A = \frac{\theta}{1 - \theta} \left( \frac{b_2}{\delta_2} - \frac{b_1}{\delta_1} \right) \delta_1 - (1 - \delta_2)g.
\]

Interest rate subsidies:

\[
A = \frac{\theta}{1 - \theta} \left( \frac{b_2}{\delta_2} - \frac{b_1}{\delta_1} \right) \delta_1 - \delta_2 s.
\]

If the government decides to remove the redlining of low risk borrowers, it is always possible to do it in the economy with weak adverse selection by using lump-sum guarantees or interest rate subsidies.

20
In the case of proportional guarantees, it is possible to remove the redlining of low risk borrowers only if the projects’ yield in the successful state of nature is high enough so that:

\[ y > \theta \left( \frac{b_2}{\delta_2} - \frac{b_1}{\delta_1} \right) \frac{1 - \delta_1}{1 - \delta_2} + \frac{b_2}{\delta_2}. \]

The comparison of budget impacts of lump-sum guarantees and subsidies, where budget cost \( G_g, G_s \) and the sizes of government interventions \( s, g \) are defined analogically as in the strong adverse selection case, shows that:

\[ G_g - G_s = \left[ \frac{\theta}{1 - \theta} \left( \frac{b_2}{\delta_2} - \frac{b_1}{\delta_1} \right) \delta_1 - (\delta_2 y - b_2 - \rho) \right] \frac{\theta(\delta_2 - \delta_1)}{\delta_2(1 - \delta_2)} > 0. \]

This means that it is cheaper for the government to use subsidies than lump-sum guarantees. This result is completely opposite to the finding in the case of a strong adverse selection.

The comparison of budget impacts of lump-sum guarantees and proportional guarantees shows that:

\[ G_g - G_\alpha = \left[ \theta(1 - \delta_1) + (1 - \theta)(1 - \delta_2) \right] \left[ \frac{\theta}{1 - \theta} \left( \frac{b_2}{\delta_2} - \frac{b_1}{\delta_1} \right) \delta_1 - (\delta_2 y - b_2 - \rho) \right] \]

\[ \frac{-\frac{\theta}{1 - \theta} \left( \frac{b_2}{\delta_2} - \frac{b_1}{\delta_1} \right)(1 - \delta_1)}{(1 - \delta_2) \left[ \frac{\delta_2 y - b_2}{\delta_2} (1 - \delta_2) - \frac{\theta}{1 - \theta} \left( \frac{b_2}{\delta_2} - \frac{b_1}{\delta_1} \right)(1 - \delta_1) \right]} < 0. \]

So, in a weak adverse selection case the ordering of budget costs required to remove the redlining of low risk borrowers is \( G_\alpha > G_g > G_s \).

Possible recommendation on the use of different forms of government support are usually based on the assumption that the government is choosing the forms of support such that the government monetary outlays are minimized. If we admit the possibility that the political influence of lenders is strong enough to ensure that the government intervention programs are biased toward providing high transfers to banks, then the situation is reversed. Under this different political economy scenario we should expect credit guarantees to be prevalent in a weak adverse selection and credit subsidies to be prevalent in a strong adverse selection case.
5 Conclusions

This paper presents a policy-relevant model of government interventions in a credit market. The credit guarantees and subsidies provided essentially automatically by government to all applicants, who passed the credit screening process by the commercial bank, are potentially very strong policy instruments. Our model shows that they are also efficient instruments, as long as the forms of credit support are chosen in a right way. The policy relevance of our model is obvious from the fact, that the model is based on basic features of a number of successful credit support programs all over the world. The introduction of the credit support program is especially beneficial in the time of credit crunch on sectoral level, as happened in agriculture and other restructured industries in many transition economies in nineties, or on economy-wide level, which was the case in Japan during 1998–2001.

The public support of commercially granted credit does not exhibit the squeeze out effect on commercial loans which may be caused by direct governmental provision of soft loans. Nevertheless there are still two contradictory effects of this type of public intervention. The positive effect is the alleviating of the credit crunch and enabling the banks to finance potentially profitable business projects which would not be financed otherwise. The negative effect could be connected with adverse selection and moral hazard problems associated with subsidized lending, which we did not consider in this paper. There could be an adverse selection where primarily companies with low profitability and socially inefficient projects would use the public support program. Or there could be a significant moral hazard on the side of banks which would not exercise due screening of the loan applicants and would not provide proper monitoring of the approved loans. The experience of both transition economies with sectoral credit support programs which was analyzed by Janda (2008) and the Japanese economy-wide program open to all small and medium enterprises (SME) which was analyzed by Fukanuma, Nemoto, and Watanabe (2006) and Uesugi, Sakai, and Yamashiro (2006) show that the positive effects prevailed and credit
support programs had a positive impact on the economy.

The experience of US SBA credit support program and the practice of many European credit support institutions participating in AECM network shows that the public policy role for government credit guarantees and interest rate subsidies is viable not only in the economies troubled with credit crunches or restructuring, but it is appropriate in a highly developed economies in any phase of the business cycle too. The related problems of cyclical increases of credit risk which leads to higher demand for government credit support are analyzed from the macroeconomic perspective by Jakubik (2007).

The model of this paper starts with the benchmark situation of an information symmetry between lender and borrower. As long as there is a symmetric information on the characteristics of the participants in the credit market, the lender with market power is perfectly able to discriminate and to fully extract all surplus from borrowers. Each borrower is offered a different rate of interest. In the conditions of an asymmetric information, there are two possible outcomes of the lender’s decision process. If the social surplus from the realization of the project of the borrower who has a lower interest payment under perfect information is sufficiently high, than both types of borrowers are pooled together. The pooled borrowers are offered the same contract. Otherwise the borrowers are separated.

In a strong (weak) adverse selection economy, the following happens. Under a pooling equilibrium, a high risk (low risk) borrower will expect to just break even and a low risk (high risk) borrower will expect a positive surplus. Under a separating equilibrium, a low risk (high risk) borrower will expect just to break even and a high risk (low risk) borrower will be rejected credit. The separating equilibrium is socially inefficient under both regimes since the borrowers’ projects, which generate a positive social surplus, are not undertaken. This seems especially wasteful in the weak adverse selection case since the borrowers who get financed are not the borrowers with the highest chance of success.

The social inefficiency created by the rejection of socially efficient projects presents a clear case for possible government interventions. This paper analyzes three types of gov-
ernment intervention: proportional credit guarantees, lump-sum credit guarantees, and interest rate subsidies. Credit guarantees and subsidies are the main intervention instruments used by many governments in credit market support. The government intervention in our model is realized only under such combination of projects’ and borrowers’ characteristics, which lead to redlining of one type of borrower. The government has no information advantage over lenders or borrowers required to conduct credit market interventions.

The main idea behind government interventions is to decrease the critical level of the expected return required by lenders in order to provide loans to borrowers with a lower interest repayment in the case of full information. The optimal level of government support equates this critical level with the symmetric information state so that all socially efficient projects are undertaken.

The model of this paper shows that the guarantee defined as a proportion of a gross interest rate is not a sufficiently robust political instrument. The reason is that the size of the government support is not in this case sufficiently endogenous. It is determined by the interest rate chosen by a lender who takes a government guarantee into account in his optimization problem. For some values of the exogenous parameters, an increase in a government proportional guarantee decreases the critical level of a required return on a high risk borrower’s loans, and for some values this critical level is increased. Moreover, for some combinations of exogenous parameters (proportions of high risk types in population, reservation utilities, probabilities of successful projects) there does not exist a positive proportional guarantee which would ensure the realization of all socially efficient projects.

The other two instruments (government lump-sum guarantees and interest rate subsidies) have nonambiguous effects on social efficiency. Both enable the government to ensure that all socially efficient projects will be undertaken. The principal difference between these two instruments is in their budgetary implications, which are quite different for the economies with a strong and a weak adverse selection. The expected size of monetary transfer from the government to lenders is lowest for lump-sum guarantees in a strong
adverse selection. It is lowest for interest rate subsidies in a weak adverse selection.

This means that as long as the participation cost of low risk entrepreneurs are sufficiently close to the participation cost of high risk entrepreneurs, the budget-cost-minimizing government should prefer guarantees over interest rate subsidies as an intervention instrument for elimination of credit rationing in a targeted credit market segment. Our results show that a relaxation of usually maintained modeling assumption of uniform participation cost for all borrowers does not eliminate the theoretical argument for the desirability of government support. But it has an effect on the choice of the most cost-efficient form of this support in the case that the difference in the participation costs is sufficiently high.

Government intervention is always favorable both for redlined and financed types of entrepreneurs. The entrepreneur, who would be credit rationed in the absence of government support, will be able to run his project and the other type of entrepreneur will receive better contract conditions than would be the case in the absence of government intervention. A low risk type of borrower is made better off by government intervention in the strong adverse selection case since the induced pooling means that a low risk borrower gets to keep a positive surplus as compared to just breaking even under a separating equilibrium. The same is true for a high risk borrower in the weak adverse selection case.

As our model shows, the public support of commercial credit provision is beneficial for all borrowers and lenders. This may explain the widespread use of these programs and their favorable treatment by policymakers, financiers and businessmen. The lenders appreciate not only the possibility to extend the guaranteed credit, but they benefit from the positive effect of government guarantees on their regulatory capital (Teply (2007)). Channeling the government funds through the commercial lending instead of direct provision of subsidies to firms is also considered to be a generally accepted practice. The firm owners may prefer to receive lump-sum payments in a form of direct government subsidies, but they realize that tying the government support with commercial loans may actually bring more funds available for the firm. Additionally the entrepreneurs realize, that provision of indirect
support through credit guarantees and subsidies to commercially extended credit is easier to accept for general public and for policymakers than asking for direct support from public funds.

The danger of government support channeled through the lender with a market power could be that the lender may adjust the terms of lending such that all benefits would accrue to him and the borrower would not be better off after the intervention. Our model shows that this situation will not happen with the credit guarantees and interest rate subsidies. Since all the borrowers will be strictly better off, this type of intervention is universally acceptable for politicians, voters and civil servants. The widespread acceptance of this type of support also means that it is difficult to remove it unless a different form of support is offered instead. As an example of successful downsizing we could mention the importance of commercial credit guarantees and interest rates subsidies provided to farmers in the Czech Republic by the Supporting and Guarantee Agricultural and Forestry Fund since 1994. This very successful program was responsible for a significant part of Czech government expenditures on agriculture policy in the second half of nineties, but its funding significantly diminished with the gradual incorporation of the Common Agricultural Policy (CAP) of EU. Czech farmers and agricultural policymakers were willing to sacrifice public funding for commercial credit support in return for higher payments from EU and Czech public funds in the framework of CAP.

The result of our model that in some situation credit guarantees are better support instrument and in some cases interest rate subsidies perform better is consistent with the fact, that policymakers across the world are using both subsidies and guarantees and that the relative weights of these two instruments change over the time. The finding that the lump-sum guarantees are better instrument than proportional guarantees may serve as an interesting recommendation to the economic policymakers. According to our model the provision of guarantees which are not directly related to the interest rate payments is more robust policy instrument. This finding supports the practice of international institutions
like EBRD, EIB, or EIF which provide lump-sum guarantee support to financial intermediaries without ties to the details of the loans provided to the final beneficiaries of their programs.

6 Appendix - The Proofs

6.1 Economy without Government Intervention

We first prove the part of the solution dealing with strong adverse selection. We start with assumption that the constraints (IC1) and (IR2) will not be violated in the solution, that is, we first ignore (IC1) and (IR2). This means that a high risk borrower will always non-strictly prefer his contract to a low risk borrower’s contract, and that a low risk borrower will obtain a positive surplus.

In this “less-constrained” problem we hold $\pi_i$ as parameters and we optimize with respect to $R_i$. We, then, substitute the solutions $R_i(\pi_1, \pi_2)$ into the original maximization problem and we optimize with respect to $\pi_i$. Having solved the “less-constrained” problem, we show that this solution indeed satisfies (IC1) and (IR2).

The Lagrangian for the “less-constrained” problem is:

$$
\max_{(R_1, R_2)} L = \theta \pi_1 [\delta_1 R_1 - \rho] + (1 - \theta) \pi_2 [\delta_2 R_2 - \rho] - \\
\mu \{ \pi_1 [\delta_2 (y - R_1) - b_2] - \pi_2 [\delta_2 (y - R_2) - b_2] \} + \lambda [\delta_1 (y - R_1) - b_1].
$$

The Kuhn-Tucker conditions for this problem are given by FOC:

$$
\frac{\partial L}{\partial R_1} = \theta \pi_1 \delta_1 + \mu \pi_1 \delta_2 - \lambda \pi_1 \delta_1 = 0,
$$

$$
\frac{\partial L}{\partial R_2} = (1 - \theta) \pi_2 \delta_2 - \mu \pi_2 \delta_2 = 0,
$$

and by (IC2), (IR1), complementary slackness conditions, and non-negativity of multipliers.
When \( \pi_i = 0 \), the corresponding values of \( R^*_i \) can be set equal to any value since the loan is not granted anyway. If \( \pi^*_i = 0 \) only for one type of a borrower, the other type is treated as in the full information case.

We solve the FOC for the values of multipliers:

\[
\mu = 1 - \theta, \\
\lambda = \frac{\theta \delta_1 + (1 - \theta) \delta_2}{\delta_1}.
\]

A positive value of a multiplier \( \lambda \) implies through a complementary slackness that (IR1) is binding, which means:

\[
R^*_1 = y - \frac{b_1}{\delta_1} \text{ if } \pi^*_1 > 0.
\]

A positive value of a multiplier \( \mu \) implies through a complementary slackness that (IC2) is binding, which means:

\[
R^*_2 = y - \frac{b_2}{\delta_2} + \frac{\pi_1}{\pi_2} \left( \frac{b_2}{\delta_2} - \frac{b_1}{\delta_1} \right). \tag{32}
\]

Substituting these optimal values of \( R^*_i \) into the lender’s objective function gives the lender’s utility as a function of \( \pi_1 \) and \( \pi_2 \).

In order to solve for probabilities \( \pi_1 \) and \( \pi_2 \) of being granted a credit, we solve the problem:

\[
\max_{(\pi_1, \pi_2)} Z = \pi_1 \left[ \theta (\delta_1 y - b_1 - \rho) + (1 - \theta) \delta_2 \left( \frac{b_2}{\delta_2} - \frac{b_1}{\delta_1} \right) \right] + \pi_2 (1 - \theta) (\delta_2 y - b_2 - \rho).
\]

The FOC for this problem are:

\[
\frac{\partial Z}{\partial \pi_2} = (1 - \theta) (\delta_2 y - b_2 - \rho) > 0 \quad \Rightarrow \pi^*_2 = 1,
\]

\[
\frac{\partial Z}{\partial \pi_1} = \theta (\delta_1 y - b_1 - \rho) + (1 - \theta) \delta_2 \left( \frac{b_2}{\delta_2} - \frac{b_1}{\delta_1} \right) \quad \Rightarrow
\]

\[
\Rightarrow \pi^*_1 = \begin{cases} 
1 & \text{if } \delta_1 y - b_1 - \rho \geq \frac{1 - \theta}{\theta} \delta_2 \left( \frac{b_2}{\delta_2} - \frac{b_1}{\delta_1} \right), \\
0 & \text{otherwise}.
\end{cases}
\]
Substituting $\pi_2^* = 1$ into the equation (32) yields the interest factor $R_2^*$, which solves the "less-constrained" problem:

$$R_2^* = \begin{cases} y - \frac{b_1}{\delta_1} & \text{if } \pi_1^* = 1, \\ y - \frac{b_2}{\delta_2} & \text{if } \pi_1^* = 0. \end{cases}$$

To check that the solution to the "less-constrained" problem satisfies (IC1), we first substitute into (IC1) the values of $R_i^*$ for $\pi_i^* = 1$. This simplifies as $0 \geq 0$, which means that (IC1) will be satisfied by the solution of the "less-constrained" problem. Then we substitute into (IC1) the values of $R_i^*$ for $\pi_1^* = 0, \pi_2^* = 1$. In that case, (IC1) simplifies as $\frac{b_1}{\delta_1} \geq \frac{b_2}{\delta_2}$, which is by definition always true in the strong adverse selection case.

To verify that the solution to the "less-constrained" problem satisfies (IR2), we first substitute into (IR2) the value of $C_2^*$ for $\pi_i^* = 1$, which simplifies as:

$$\frac{b_1}{\delta_1} \geq \frac{b_2}{\delta_2}. \quad (33)$$

Rewriting (33) as $\frac{\delta_2}{\delta_1} \geq \frac{b_2}{b_1}$, we see that it is satisfied when the relative disparities in opportunities costs are smaller than the relative differences between success probabilities. This is the condition which defines our strong adverse selection case.

In the case $\pi_1^* = 0, \pi_2^* = 1$, we obtain, after a substitution of appropriate value of $R_2^*$ into (IR2), a trivial truism $0 \geq 0$ which means that (IR2) is satisfied.

By this we proved the solution for the case of strong adverse selection. Now we finish the proof with the case of weak adverse selection. We first assume that the constraints (IC2) and (IR1) will not be violated in the solution, that is, we first ignore (IC2) and (IR1). Going through the same steps as in the strong adverse selection case, we find that:

$$R_2^* = y - \frac{b_2}{\delta_2} \quad \text{if } \pi_2^* > 0,$$

$$R_1^* = y - \frac{b_1}{\delta_1} + \frac{\pi_2}{\pi_1} \left( \frac{b_1}{\delta_1} - \frac{b_2}{\delta_2} \right).$$

Solving an optimization problem in variables $\pi_1, \pi_2$, we get a solution:
\[ \pi_1^* = 1, \]
\[ \pi_2^* = \begin{cases} 
1 & \text{if } \delta_2 y - b_2 - \rho \geq \frac{\theta}{1-\theta} \delta_1 (\frac{b_2}{\delta_2} - \frac{b_1}{\delta_1}), \\
0 & \text{otherwise}.
\end{cases} \]

Q.E.D.

6.2 Government Interventions with Strong Adverse Selection

6.2.1 Proportional Guarantees

We follow the strategy used in the case without an intervention.

The Lagrangian for the “less-constrained” problem is:

\[ \max_{(R_1, R_2)} L = \theta \pi_1 \delta_1 R_1 + (1 - \delta_1) \alpha R_1 - \rho + (1 - \theta) \pi_2 \delta_2 R_2 + (1 - \delta_2) \alpha R_2 - \rho - \mu \{ \pi_1 [\delta_2 (y - R_1) - b_2] - \pi_2 [\delta_2 (y - R_2) - b_2] \} + \lambda \pi_1 [\delta_1 (y - R_1) - b_1]. \]

Kuhn-Tucker conditions are FOC:

\[ \frac{\partial L}{\partial R_1} = \theta \pi_1 \delta_1 + (1 - \delta_1) \alpha + \mu \pi_1 \delta_2 - \lambda \pi_1 \delta_1 = 0, \]
\[ \frac{\partial L}{\partial R_2} = (1 - \theta) \pi_2 \delta_2 + (1 - \delta_2) \alpha - \mu \pi_2 \delta_2 = 0, \]

and (IC2), (IR1), complementary slackness conditions, and non-negativity of multipliers.

We solve the FOC for the values of multipliers:

\[ \mu = (1 - \theta) \frac{\delta_2 + (1 - \delta_2) \alpha}{\delta_2}, \]
\[ \lambda = \frac{\theta [\delta_1 + (1 - \delta_1) \alpha] + (1 - \theta) \delta_2 \frac{\delta_2 + (1 - \delta_2) \alpha}{\delta_2}}{\delta_1}. \]

Since (IR1) and (IC2) are the same as in the case without intervention, and multipliers \( \lambda \) and \( \mu \) are again positive, we get the same optimal values of \( R_i^* \) as in the case without intervention.
Substituting these optimal values of \( R^*_i \) into the lender’s objective function gives the lender’s utility as a function of \( \pi_1 \) and \( \pi_2 \).

To get the values of probabilities \( \pi_1, \pi_2 \), we solve the problem:

\[
\max_{(\pi_1, \pi_2)} Z = \pi_1 \left\{ \theta \left[ \delta_1 y - b_1 - \rho + \frac{\delta_1 y - b_1}{\delta_1} (1 - \delta_1) \alpha \right] + (1 - \theta) \left[ \frac{b_2}{\delta_2} - \frac{b_1}{\delta_1} \right] \left[ \delta_2 + (1 - \delta_2) \alpha \right] \right\} + \\
\pi_2 \left( 1 - \theta \right) \left[ \delta_2 y - b_2 - \rho + \frac{\delta_2 y - b_2}{\delta_2} (1 - \delta_2) \alpha \right].
\]

The FOC are:

\[
\frac{\partial Z}{\partial \pi_2} = (1 - \theta) \left[ \delta_2 y - b_2 - \rho + \frac{\delta_2 y - b_2}{\delta_2} (1 - \delta_2) \alpha \right] > 0 \\
\Rightarrow \pi_2^* = 1,
\]

\[
\frac{\partial Z}{\partial \pi_1} = \theta \left[ \delta_1 y - b_1 - \rho + \frac{\delta_1 y - b_1}{\delta_1} (1 - \delta_1) \alpha \right] + (1 - \theta) \left[ \frac{b_2}{\delta_2} - \frac{b_1}{\delta_1} \right] \left[ \delta_2 + (1 - \delta_2) \alpha \right] \\
\Rightarrow \pi_1^* = \begin{cases} 
1 & \text{if } \delta_1 y - b_1 - \rho \geq \frac{1 - \theta}{\theta} \left( \frac{b_1}{\delta_1} - \frac{b_2}{\delta_2} \right) \left[ \delta_2 + (1 - \delta_2) \alpha \right] - \frac{\delta_1 y - b_1}{\delta_1} (1 - \delta_1) \alpha, \\
0 & \text{otherwise.} 
\end{cases}
\]

The rest of the solution (checking our assumptions about (IC1) and (IR2)) is identical to the case without intervention.

Q.E.D.

6.2.2 Lump-sum Guarantees

The strategy employed is similar to the case with a proportional guarantee.

Kuhn-Tucker conditions are FOC:

\[
\frac{\partial L}{\partial R_1} = \theta \pi_1 \delta_1 + \mu \pi_1 \delta_2 - \lambda \pi_1 \delta_1 = 0,
\]

\[
\frac{\partial L}{\partial R_2} = (1 - \theta) \pi_2 \delta_2 - \mu \pi_2 \delta_2 = 0,
\]

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and (IC2), (IR1), complementary slackness conditions, and non-negativity of multipliers.

Similarly like in the case without intervention multipliers \( \lambda \) and \( \mu \) are again found to be positive and optimal values of \( R_i^* \) are the same as in the case without intervention.

The lender’s objective function is as follows:

\[
\max_{(\pi_1, \pi_2)} Z = \pi_1 [\theta (\delta_1 y - b_1 - \rho + (1 - \delta_1)g) + (1 - \theta) \delta_2 (\frac{b_2}{\delta_2} - \frac{b_1}{\delta_1})] + \\
\pi_2 (1 - \theta) (\delta_2 y - b_2 - \rho + (1 - \delta_2)g).
\]

The FOC are:

\[
\frac{\partial Z}{\partial \pi_2} = (1 - \theta) (\delta_2 y - b_2 - \rho + (1 - \delta_2)g) > 0 \\
\Rightarrow \pi_2^* = 1,
\]

\[
\frac{\partial Z}{\partial \pi_1} = \theta (\delta_1 y - b_1 - \rho + (1 - \delta_1)g) + (1 - \theta) \delta_2 (\frac{b_2}{\delta_2} - \frac{b_1}{\delta_1})
\]

\[
\Rightarrow \pi_1^* = \begin{cases} 
1 & \text{if } \delta_1 y - b_1 - \rho \geq \frac{1 - \theta}{\theta} \delta_2 (\frac{b_2}{\delta_2} - \frac{b_1}{\delta_1}) - (1 - \delta_1)g, \\
0 & \text{otherwise}.
\end{cases}
\]

Q.E.D.

### 6.2.3 Interest Rate Subsidies

The problem is identical to the problem without intervention up to a formulation of the lender’s maximization of utility function \( Z \) with respect to \( \pi_1, \pi_2 \) :

\[
\max_{(\pi_1, \pi_2)} Z = \pi_1 [\theta (\delta_1 y - b_1 - \rho + \delta_1 s) + (1 - \theta) \delta_2 (\frac{b_2}{\delta_2} - \frac{b_1}{\delta_1})] + \\
\pi_2 (1 - \theta) (\delta_2 y - b_2 - \rho + \delta_2 s).
\]

The FOC are:

\[
\frac{\partial Z}{\partial \pi_2} = (1 - \theta) (\delta_2 y - b_2 - \rho + \delta_2 s) > 0 \\
\Rightarrow \pi_2^* = 1,
\]

\[
\frac{\partial Z}{\partial \pi_1} = \pi_1^* = \begin{cases} 
1 & \text{if } \delta_1 y - b_1 - \rho \geq \frac{1 - \theta}{\theta} \delta_2 (\frac{b_1}{\delta_1} - \frac{b_2}{\delta_2}) - \delta_1 s, \\
0 & \text{otherwise}.
\end{cases}
\]

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References


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