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Charles University in Prague

# **Operational Risk Management and Implications for Bank's Economic Capital – A Case Study**

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# Operational Risk Management and Implications for Bank's Economic Capital – A Case Study

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**Abstract:**

In this paper we review the actual operational data of an anonymous Central European Bank, using two approaches described in the literature: the loss distribution approach and the extreme value theory ("EVT"). Within the EVT analysis, two estimation methods were applied; the standard maximum likelihood estimation method and the probability weighted method ("PWM"). Our results proved a heavy-tailed pattern of operational risk data consistent with the results documented by other researchers in this field. Additionally, our research demonstrates that the PWM is quite consistent even when the data is limited since our results provide reasonable and consistent capital estimates. From a policy perspective, it should be noted that banks from emerging markets such as Central Europe are exposed to these operational risk events and that successful estimates of the likely distribution of these risk events can be derived from more mature markets.

**Keywords:** operational risk, economic capital, Basel II, extreme value theory, probability weighted method

**JEL:** G18, G21, G32.

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## Abstract

In this paper we review the actual operational data of an anonymous Central European Bank, using two approaches described in the literature: the loss distribution approach and the extreme value theory (“EVT”). Within the EVT analysis, two estimation methods were applied; the standard maximum likelihood estimation method and the probability weighted method (“PWM”). Our results proved a heavy-tailed pattern of operational risk data consistent with the results documented by other researchers in this field. Additionally, our research demonstrates that the PWM is quite consistent even when the data is limited since our results provide reasonable and consistent capital estimates. From a policy perspective, it should be noted that banks from emerging markets such as Central Europe are exposed to these operational risk events and that successful estimates of the likely distribution of these risk events can be derived from more mature markets.

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## 1. Introduction

Operational risk has become one of the most discussed topics by both academics and practitioners in the financial industry in the recent years. The reasons for this attention can be attributed to higher investments in information systems and technology, the increasing wave of mergers and acquisitions, emergence of new financial instruments, and the growth of electronic dealing (Sironi and Resti, 2007). In addition, the New Basel Capital Accord (effective since 2007) demands a capital requirement for operational risk and further motivates financial institutions to more precisely measure and manage this type of risk.

According to de Fontouville et al. (2003), financial institutions have faced more than 100 operational loss events exceeding \$100 million since the end of 1980s. The highest losses stemming from operational risk have been recorded in Societe Generalé in 2008 (\$7.3 billion), Sumitomo Corporation in 1996 (\$2.9 billion), Orange County in 1994 (\$1.7 billion), Daiwa Bank in 1995 (\$1.1 billion), Barings Bank in 1995 (\$1 billion) and Allied Irish Bank in 2002 (\$700 million)<sup>1</sup>. Operational risk also materialised during the US subprime mortgage crisis in 2007, when mortgage frauds became a serious issue<sup>2</sup>. As noted by Dille (2008), “*mortgage applicants with weak financial standing or poor credit history have an obvious temptation to exaggerate their income or assets in order to secure a loan*”. However, the fraud entailed in issuing these mortgages is that although it could be alleged the lenders knew better, they deliberately denied the risks<sup>3</sup> and benefited themselves by collecting fees and

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<sup>1</sup> See Chernobai et al. (2007) or Peters and Terauds (2006) for an overview of examples of operational risk events.

<sup>2</sup> Naturally, mortgage frauds occurred also before the crisis. However, the number of cheating applicants was not as high as the mortgages were not provided to so many applicants.

<sup>3</sup> We should note that some loans were provided intentionally to applicants with a low creditworthiness – such as NINJA loans (No Income, No Job, No Assets).

commissions from borrowers who could only rely on rising real-estate prices to support their mortgage payments<sup>4</sup>.

Moreover, there have been several instances of operational risk in Central Europe too. In 2000 for example, a trader and his supervisor at one of the biggest Czech banks exceeded their trading limits when selling US Treasury bonds causing a US\$53 million loss to the bank. And, in the late 1990s, another Central European bank suffered a US\$180 million loss as a result of providing financing based on forged documents. More typical examples of operational risk experienced by both Central European and other global banks include cash theft, fee rounding errors in IT systems or internet crashes.

Although large operational losses are extreme events occurring very rarely, a bank — or a financial institution in general — has to consider the probability of their occurrence when identifying and managing future risks. In order to have reasonable estimates of possible future risks a bank needs an in-depth understanding of its past operational loss experience. As a result, a bank may create provisions for expected losses and set aside capital for unexpected ones. In this paper we focus on modelling of the economic capital that should be set aside to cover unexpected losses resulting from operational risk failures.

The contribution of this study is threefold. The first contribution is the presentation of a complete methodology for operational risk management. Banks in Central Europe generally do not possess a methodology to model operational risk since they rely on the competence of their parent companies to calculate operational risk requirement on the consolidated basis of the whole group. Therefore, our study that proposes the complete methodology might be beneficial for banks willing to model their operational risk but not selected a sophisticated methodology yet.

Secondly, our study is an empirical study which uses real operational risk data from an anonymous Central European bank (the “Bank”). We are going to test various approaches and methods that are being used to model operational risk and calculate capital requirements based on the results. The final outcome of our study is to propose the model of operational risk that could be implemented by the Bank. Our estimates ought to be consistent with the real capital requirement of this bank.

Lastly, our analysis provides important results and conclusions. We have found out that even a general class distribution is not able to fit the whole distribution of operational losses. On the other hand, extreme value theory (EVT) appears more suitable to model extreme events. Additionally, we have discovered that traditional estimation using maximum likelihood does not provide consistent results while estimation based on probability weighted moments proved to be more coherent. We attribute it to limited dataset and conclude that probability weighted moments estimation that assign more weight to observations further in the tail of a distribution might be more appropriate to model operational loss events.

This paper is organised as follows; the second part provides a literature review; the third part discusses the modelling issues of operational risk and implications for economic capital, while the fourth part describes the data used and exploratory data analysis. The methodology is described in the fifth and sixth chapter and in the seventh part we discuss the results of our research and compare them with the findings of other studies. Finally, the eighth part concludes the paper and state final remarks.

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<sup>4</sup> As real-estate prices fell, many home owners were forced into foreclosure or into maintaining “upside-down” loans where they owed more to the bank in floating-rate mortgages than the house was worth; simultaneously, the lenders were required to take multi-billion dollar write-downs.

## 2. Literature Overview

“Operational risk is not a new risk... However, the idea that operational risk management is a discipline with its own management structure, tools and processes... is new.” This quotation from British Bankers Association in Power (2005) well describes the development of operational risk management in the last years. Until Basel II requirements in the mid 1990s<sup>5</sup>, operational risk was largely a residual category for risks and uncertainties that were difficult to quantify, insure and manage in traditional ways. For this reasons one cannot find many studies focused primarily on operational risk until the late 1990s, although the term ‘operations risk’ already existed in 1991 as a generic concept of Committee of Sponsoring Organizations of the Treadway Commission.

Operational risk management methods differ from those of credit and market risk management. The reason is that operational risk management focuses mainly on low severity/high impact events (tail events) rather than central projections or tendencies. As a result, the operational risk modelling should also reflect these tail events which are harder to model (Jobst, 2007b). Operational risk can build ideas from insurance mathematics in the methodological development (Cruz (2002), Panjer (2006) or Peters and Terauds (2006)). Hence one of the first studies on operational risk management was done by Embrechts et al. (1997) who did the modelling of extreme events for insurance and finance. Later, Embrechts conducted further research in the field of operational risk (e.g. Embrechts et al. (2003), Embrechts et al. (2005) and Embrechts et al. (2006)) and his work has become classic in the operational risk literature.

Cruz et al. (1998), Coleman and Cruz (1999) and King (2001) provided other early studies on operational risk management. Subsequently, other researchers such as van den Brink J. (2002), Hiwatshi and Ashida (2002), de Fontnouvelle et al. (2003), Moscadelli (2004), de Fontnouvelle et al. (2005), Nešlehová (2006) or Dutta and Perry (2007) experimented with operational loss data over the past few years.

To this date Moscadelli (2004) is probably the most important operational risk study. He performed a detailed Extreme Value Theory (EVT) analysis of the full QIS data set<sup>6</sup> of more than 47,000 operational losses and concluded that the loss distribution functions are well fitted by generalised Pareto distributions in the upper-tail area. The estimated tail parameters ( $\zeta$ ) for different business lines ranged from 0.85 for asset management to 1.39 for commercial banking. Six of the business lines have an estimate of  $\zeta$  greater than one, corresponding to an infinite mean model. Based on these QIS data, the estimated capital requirements ( $\beta$ , as defined in the *Section 3.3*) ranged from 8.3% for retail banking to 33.3% for payment & settlement, with an overall  $\alpha$  of 13.3%, slightly below the Basel II value of 15% used in the Basic Indicator Approach<sup>7</sup>.

Operational risk modelling helps the risk managers to better anticipate operational risk and hence it supports more efficient risk management. There are several techniques and methodological tools developed to fit frequency and severity models including the already-mentioned EVT (Cruz (2002), Embrechts et al. (2005) or Chernobai et al. (2007)), Bayesian inference (Schevchenko and Wuthrich

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<sup>5</sup> For more details see the Annex 1 – The Evolution of the Regulatory Treatment of Operational Risk.

<sup>6</sup> QIS – Quantitative Impact Study by the Basel Committee on Banking Supervision's, another important collection of data is the exercise of the Federal Reserve Bank of Boston (see e.g. de Fontnouvelle et al. (2004))

<sup>7</sup> For more details see the *Section 3.3*.

(2006) or Cruz (2002)), dynamic Bayesian networks (Ramamurthy et al., 2005) and expectation maximisation algorithms (Bee, 2006).

When modelling operational risk, other methods that change the number of researched data of operational risk events are used. The first one are the robust statistic methods used Chernobai and Ratchev (2006) that exclude outliers from a data sample (e.g. 5% or 10% the highest operational risk events). On the other hand, a stress-testing method adds more data to a data sample and is widely used by financial institutions (Arai (2006), Rosengren (2006) or Rippel (2008)).

More recently, Peters and Terauds (2006), van Leyveld et al. (2007), Chernobai et al. (2007) or Jobst (2007c) summarise an up-to-date development of operational risk management from both views of academics and practitioners.

### 3. An Overview of Operational Risk and Economic Capital

#### 3.1 Basics of Operational Risk

There are many definitions of operational risk such as “*the risk arising from human and technical errors and accidents*” (Jorion, 2000) or “*a measure of the link between a firm’s business activities and the variation in its business results*” (King, 2001). The Basel Committee offers a more accurate definition of operational risk as “*the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events failures*” (BCBS, 2006, p.144). This definition encompasses a relatively broad area of risks, with the inclusion of for instance, strategic, transaction or legal risk (see *Table 1*).

**Table 1:** Operational risk and main factors

People	Systems	Processes	External Events
Fraud, collusion and other criminal activities	IT problems (hardware or software failures, computer hacking or viruses etc.)	Execution, registration, settlement and documentation errors ( <i>transaction risk</i> )	Criminal activities (theft, terrorism or vandalism)
Violation of internal or external rules (unauthorized trading, insider dealing etc.)	Unauthorized access to information and systems security	Errors in models, methodologies and mark to market ( <i>model risk</i> )	Political and military events (wars or international sanctions)
Errors related to management incompetence or negligence	Unavailability and questionable integrity of data	Accounting and taxation errors Inadequate formalization of internal procedures	Change in the political, regulatory and tax environment ( <i>strategic risk</i> )
Loss of important employees (illness, injury, problems in retaining staff etc.)	Telecommunications failure	Compliance issues Breach of mandate	Change in the legal environment ( <i>legal risk</i> ) Natural events (fire, earthquake, flood etc.)
Violations of systems security	Utility outages	Inadequate definition and attribution of responsibilities	Operational failure at suppliers or outsourced operations

**Source:** Based on Sironi and Resti (2007)

However, the reputation risk (damage to an organisation through loss of its reputational or standing) and strategic risk (the risk of a loss arising from a poor strategic business decision) are excluded from

the Basel II definition. The reason is that the term “loss” under this definition includes only those losses that have a discrete and measurable financial impact on the firm. Hence strategic and reputational risks are excluded, as they would not typically result in a discrete financial loss (Fontnouvelle et al., 2003). Other significant risks such as market risk<sup>8</sup> and credit risk<sup>9</sup> are treated separately in the Basel II.

Some peculiarities of operational risk exist compared to market and credit risks. The main difference is the fact that operational risk is not taken on a voluntary basis but is a natural consequence of the activities performed by a financial institution (Sironi and Resti, 2007). In addition, from a view of risk management it is important that operational risk suffers from a lack of hedging instruments. For other peculiarities see *Table 2*.

**Table 2:** *Operational risk peculiarities*

<b>Market and Credit Risks</b>	<b>Operational Risks</b>
Consciously and willingly face	Unavoidable
“Speculative” risk, implying losses and profits	Pure risks, implying losses only*
Consistent with an increasing relationship between risk and expected return	Not consistent with an increasing relationship between risk and expected return
Easy to identify and understand	Difficult to identify and understand
Comparatively easy to measure and identify	Difficult to measure and identify
Large availability of hedging instruments	Lack of effective hedging instruments
Comparatively easy to price and transfer	Difficult to price and transfer

\* with few exceptions

**Source:** Based on Sironi and Resti (2007)

### **3.2 Modelling operational risk**

There are two main ways to assess operational risk – the top-down approach and the bottom-up approach. Under the top-down approach, operational losses are quantified on a macro level only, without attempting to identify the events or causes of losses (Chernobai et al., 2007). The main advantage of these models is their relative simplicity and no requirement for collecting data. Top-down models include multifactor equity price models, capital asset pricing model, income-based

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<sup>8</sup> The risk of losses (in and on- and off-balance sheet positions) arising from movements in market prices, including interest rates, exchange rates, and equity values (Chernobai et al., 2007).

<sup>9</sup> The potential that a bank borrower or counterparty fails to meet its obligations in accordance with agreed terms (Chernobai et al., 2007).

models, expense-based models, operating leverage models, scenario analysis and stress testing and risk indicator models.

On the other hand, bottom-up models quantify operational risk on a micro level and are based on the identification of internal events. Their advantages lie in a profound understanding of operational risk events (the way how and why are these events formed). Bottom-up models encompass three main subcategories: process-based models (causal models and Bayesian belief networks, reliability models, multifactor causal factors), actuarial models (empirical loss distribution based models, parametric loss distribution based models, models based on extreme value theory) and proprietary models.<sup>10</sup>

As recommended by many authors such as Chernobai et al. (2007) or van Leyveld (2007), the best way for operational risk management is a combination of both approaches. In the paper we follow this best practice and employ bottom-up approaches for operational risk modelling (LDA and EVT methods as described below) and compare the results.

### 3.3 Top-down approach of modelling operational risk

Basel II provides an operational risk framework for banks and financial institutions. The framework includes identification, measurement, monitoring, reporting, control and mitigation of operational risk. Stated differently, it requires procedures for proper measurement of operational risk losses (i.e. ex-post activities such as reporting and monitoring) as well as for active management of operational risk (i.e. ex-ante activities such as planning and controlling). The Basel Committee distinguishes seven main categories of operational risk and eight business lines for operational risk measurement as depicted in the following table (Table 3).

**Table 3:** Business lines and event types according to Basel II

Business line	Event type
1. Corporate Finance	1. Internal Fraud
2. Trading & Sales	2. External Fraud
3. Retail Banking	3. Employment Practices and Workplace Safety
4. Commercial Banking	4. Clients, Products and Business Practices
5. Payment & Settlement	5. Damage to Physical Assets
6. Agency Services	6. Business Disruption and System Failure
7. Asset Management	7. Execution, Delivery and Process Management
8. Retail Brokerage	

Source: BCBS (2006)

Basel II is based on three main pillars. Pillar I of Basel II provides guidelines for measurement of operational risk, Pillar II requires adequate procedures for managing operational risk and Pillar III sets up requirements on information disclosure of the risk.

Basel II distinguishes three main approaches to operational risk measurement:

- 1) Basic indicator approach (BIA)

<sup>10</sup> For more detailed description of these models see Chernobai et al. (2007), pages 67–75.

- 2) Standardised approach (SA)
- 3) Advanced measurement approach (AMA)

Under the BIA, the simplest approach, gross income<sup>11</sup> serves as a proxy for the scale of operational risk of the bank. Hence the bank must hold capital for operational risk equal to the average over the previous three years of a fixed percentage (denoted as alpha,  $\alpha$ ) of positive annual gross income<sup>12</sup>. Alpha was set at 15 %.

The capital charge ( $K_{BIA}$ ) can be expressed as follows:

$$K_{BIA} = \frac{\left[ \alpha \cdot \sum_{t=1}^n GI_t \right]}{n} \quad (1)$$

$GI_t$  - gross income at time t

n - the number of the previous three years for which gross income was positive

$\alpha$  - the fixed percentage of gross income (15%)

The SA<sup>13</sup> is very similar to the BIA, only the activities of banks are dividend into eight business lines. Within each business line, gross income is a broad indicator of operational risk exposure. Capital requirement ranges from 12 to 18 % (denoted as beta,  $\beta$ ) of gross income in the respective business line (see *Table 4*).

The total capital charge ( $K_{SA}$ ) can be rewritten as follows:

$$K_{SA} = \frac{\sum_{t=1}^3 \max \left\{ \sum_{k=1}^8 GI_{tk} \cdot \beta_k, 0 \right\}}{3} \quad (2)$$

$GI_{tk}$  - gross income at time i for business line k

$\beta_k$  - a fixed percentage of GI for each of eight business lines

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<sup>11</sup> Gross income = interest income + non-interest income.

<sup>12</sup> When gross income is negative, the figure is excluded from both numerator and denominator.

<sup>13</sup> An alternative to the SA exists – the Alternative Standardised Approach, which uses for the Retail Banking and the Commercial Banking total loans and advances as a proxy for the scale of operational risk of the bank (instead of gross income).

**Table 4:** Beta Factors under the Standardised Approach

Business Lines	Beta Factors
Corporate finance	18%
Trading and sales	18%
Retail banking	12%
Commercial Banking	15%
Payment and settlement	18%
Agency services	15%
Asset management	12%
Retail brokerage	12%

Source: BCBS (2006)

### 3.4 Bottom-up approaches of modelling operational risk

Under the Advanced Measurement Approach (AMA), the regulatory capital requirement shall equal the risk measure generated by the bank's internal operational risk measurement system. The bank must meet certain qualitative (e.g. quality and independence of operational risk management, documentation of loss events, regular audit) and quantitative (internal and external data collection, scenario analysis) standards to qualify for using the AMA. For instance, a bank must demonstrate that its operational risk measure is evaluated for one-year holding period and a high confidence level (99.9% under Basel II<sup>14</sup>). The use of the AMA is subject to supervisory approval.

The above-mentioned description of three approaches indicates that the BIA is the simplest while the AMA is the most advanced. The idea behind Basel II requirements lies in the assumption that

$$K_{BIA} > K_{SA} > K_{AMA} \quad (3)$$

In other words, *equation 3* implies that the AMA capital charge ( $K_{AMA}$ ) should be lower than  $K_{BIA}$  and  $K_{SA}$ . Therefore banks should be motivated to use the most advanced approach – AMA<sup>15</sup>. At present most banks use a combination of two AMA approaches to measure operational risk:

- The loss distribution approach (LDA), which is a quantitative statistical method analysing historical loss data.
- The scorecard approach, which focuses on qualitative risk management in a financial institution (this approach was developed and implemented at the Australian New Zealand Bank (Lawrence, 2000)).

The above-mentioned approaches complement each other. As a historical data analysis is backward-looking and quantitative, the scorecard approach encompasses forward-looking and qualitative indicators. In our analysis we concentrate on the first approach because of the data availability.

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<sup>14</sup> BCBS (2006) p.151.

<sup>15</sup> The lower capital charge hold by a bank should result in its higher profitability.

However, we would like to point out that a combination of both approaches is necessary for successful operational risk management.

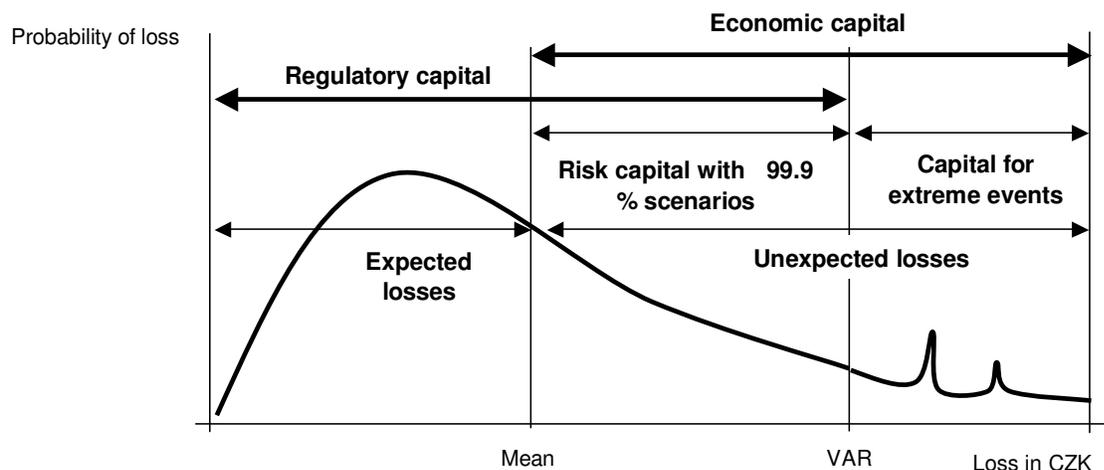
Once operational risks have been assessed both qualitatively and quantitatively, the next step is to manage them, the following ways are suggested (Fitch Ratings, 2007):

- avoidance of certain risks;
- acceptance of others, but an effort to mitigate their consequences;
- or simply acceptance some risks as a part of doing business.

### 3.5 Economic capital

A concept of economic capital is used for modelling operational risk through the AMA. However, no unique definition of economic capital exists. For instance, Mejstřík, Pečená and Teplý (2007) state “*economic capital is a buffer against future, unexpected losses brought about by credit, market, and operational risks inherent in the business of lending money*”. Alternatively, van Leyveld (2007) offers the following definition: “*economic capital can be defined as the amount of capital that a transaction or business unit requires in order to support the economic risk it originates, as perceived by the institution itself*”. Alternatively, Chorofas (2006) defines economical capital as “*the amount necessary to be in business – at a 99% or better level of confidence – in regard to assume risks*”. We should distinguish economic capital from regulatory capital that can be defined as capital used for the computation of capital adequacy set by the Basel II requirements (Mejstřík, Pečená and Teplý, 2008) or as the minimum amount needed to have a license (Chorofas, 2006). *Figure 1* presents the difference between economic and regulatory capital.

**Figure 1** – Classification of bank’s capital requirements according to risk



**Source:** Authors based on Chorofas (2006) and BCBS (2006)

As the figure shows, regulatory capital should cover (e.g. in the form of provisions) both expected losses and unexpected losses (but excluding extreme events) while economic capital should cover unexpected losses. In addition, economic capital should cover both risk capital with 99.9% scenarios and capital for extreme events. The latter is important for modelling operational risk as “low

frequency/high severity” losses often occur, what is supported by many researchers such as Chernobai (2006), Dutta and Perry (2006) or as it will be shown later, by our results. As the examples of extreme events, we can list 9/11 events in 2001, flooding in the Czech Republic in 2002 or Hurricane Katrina in 2005.

## 4. Data analysis

### 4.1 Data used

In this study we have used data from the Bank. Altogether the dataset consists of more than six hundred operational losses over the period 2001-2007. However, there are disproportionately fewer observations in the beginning of the sample (January 2001-November 2003) signalling lower quality of data when the process of collecting operational losses data was just starting. In order to remove possible bias, we have left out 14 observations of this period.

Moreover, the threshold for collecting the data in the Bank (about \$1,000) is set quite low compared to other studies, the threshold is typically of the order of \$10,000, hence we further cut some of the observations from the beginning as we describe in the section dealing with LDA. By setting the threshold up to \$10,000 we have left out many small losses, hence the number of observation in our dataset further decreased up to 236<sup>16</sup>.

Observations across years starting from December 2004 are by simple graphical inspection quite stationary and hence can be considered to be collected by consistent methodology. However, there is a significant variation across months; particularly losses in December are significantly more frequent. This can be explained by the end of fiscal year when all possible unrecorded losses up to a date finally appear on the books. This is not a problem when losses are treated on annual basis or independent of time, however, it hinders the possibility to take into account monthly information.

Generally, our dataset is not very big, but it is satisfactory enough for operational risk analysis at the level of the whole bank. For analysis focusing on particular business lines and/or particular type of loss events we would need more observations.

### 4.2 Exploratory data analysis

To get a better understanding of the structure and characteristics of the data we have firstly performed Exploratory Data Analysis as suggested by Tukey (1977). Operational risk data are skewed and heavy-tailed; hence skewness and kurtosis are the most important characteristics. We have utilised some of the measures proposed by Hoaglin (1985) and Tukey (1977) used in Dutta and Perry (2007) to analyse skewness and kurtosis. Regarding skewness, if the data are symmetric then

$$X_{0.5} - X_p = X_{1-p} - X_{0.5},$$

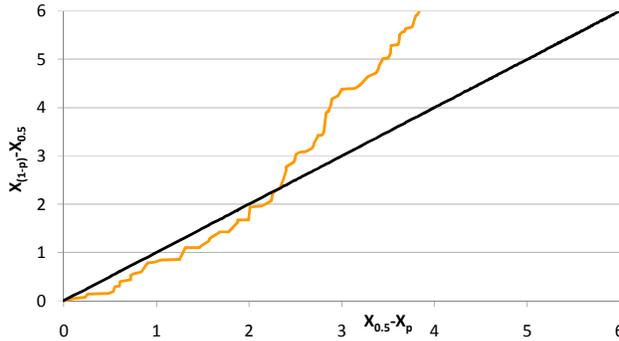
where  $X_p$ ,  $X_{1-p}$ , and  $X_{0.5}$  are the  $100p^{th}$  percentile,  $100(1-p)^{th}$  percentile, and the median of the data respectively. If the data are from a symmetric distribution such as the normal distribution, the plot  $X_{0.5}$

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<sup>16</sup> Although the number of observations left out is high, they account only for about 2.5% of the sum of total operational losses in the sample. A \$ 10,000 threshold is commonly used in operational risk modelling (see Duta, Perry (2007) or Chernobai (2007)).

–  $X_p$  against  $X_{1-p} - X_{0.5}$  would be a straight line with unit slope. As *Figure 2*<sup>17</sup> clearly reveals, our data are far from symmetric, near the median they are negatively skewed owing to the low threshold for operational losses in the Bank, then it turns to positive as a consequence of high losses in the right tail of the distribution.

**Figure 2** – Data skewness relative to a symmetric distribution



Another measure of skewness is provided by a mid-summary plot in which the mid-summary of the data is defined as

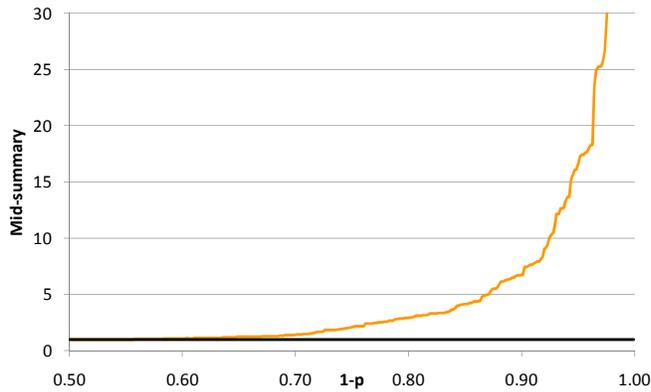
$$mid_p = \frac{1}{2}(X_p - X_{1-p}).$$

For symmetric data, the mid-summary must be equal to the median for all percentiles  $p$ . For a dataset exhibiting systematic skewness, the mid-summary plot against  $1 - p$  exhibits gradual diversion from the median, for unsystematic skewness, the plot changes sharply with varying quantiles driven by extreme observations. *Figure 3* displays both, the systematic skewness for the lower quantiles and unsystematic skewness for the highest 10% of data, which is even stronger for the last 5%. Both indicators of skewness thus confirm that operational losses are highly skewed driven by extreme observations.

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<sup>17</sup> Please note that in this figure and later in our analysis we have followed the common practice and have replaced actual numbers on the axes by normalised numbers as to preserve the confidentiality of the data.

**Figure 3** – *Mid-summary plot to detect unsystematic skewness*

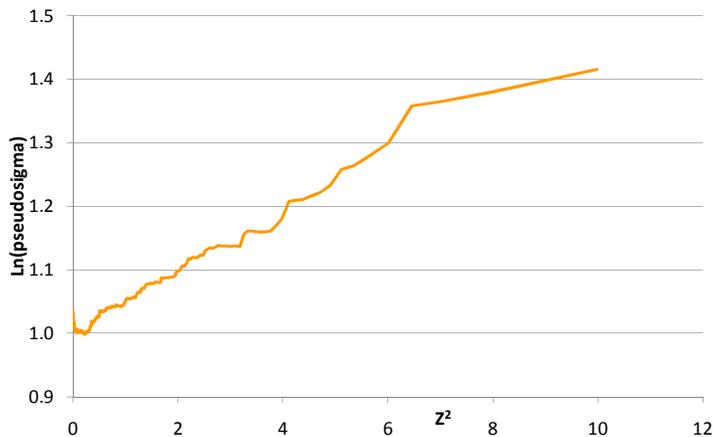


To measure excess kurtosis of operational losses we have utilised pseudo sigma defined as

$$\frac{X_p - X_{1-p}}{2Z_p},$$

where  $Z_p$  is the quantile of the standard normal distribution. For the normal distribution the pseudo sigma is constant and equal to a standard deviation  $\sigma$ . On the other hand, increasing or decreasing pseudo sigma with  $p$  is a signal of leptokurtotic (heavy-tailed) or platokurtotic (light-tailed) distribution, respectively. As Hoaglin (1985) suggested, we have plot natural logarithm of pseudo sigma against  $Z^2$  (Figure 4). Steadily increasing plot confirms the hypothesis that our data are heavy-tailed.

**Figure 4** – *Mid-summary plot to identify excess kurtosis*



## 5. Methodology

### 5.1 Concept of VAR, modelling frequency and aggregation of losses

Before describing individual approaches to model operational risk, we would like to define Value at Risk (VAR), a risk informative indicator recognised by Basel II requirements.<sup>18</sup> Jorion (2007) defines VAR as “the maximum loss over a target horizon such that there is a low, prespecified probability that the actual loss will be higher”. Usually VAR is expressed as a corresponding value (in currency units) of  $p\%$  quantile of a distribution<sup>19</sup> where  $p$  is the prespecified low probability and  $f(x)$  is a density function of operational losses:

$$p = \int_{VAR}^{\infty} f(x)dx$$

Alternatively, VAR is a cut-off point of the distribution beyond which the probability of the loss occurrence is less than  $p$ . For operational risk losses the quantile defined in Basel II is 99.9% (see *Figure 1*), thus we will report VAR<sub>99.9</sub> for each modelling method used. The target horizon is one year, so a 99.9% VAR requirement can be interpreted as the maximum loss incurred over 1,000 years.

There is one complication associated with the above definition of VAR and the requirement of Basel II. The above density function  $f(x)$  has to combine both the severity and frequency of losses for a period of one year which is analytically difficult in specific cases (Embrechts et al., 2005). One of the approaches suggested (e.g. Cruz, (2002), Embrechts et al. (2005) or Dutta and Perry (2007)) is the Monte Carlo (MC) simulation where for a simulation of a given year a number of losses is drawn from a frequency distribution and each loss in the year is simulated by a random quantile of a severity distribution. All losses in each of the simulated years are then summed to arrive at the estimation of the combined distribution function. The 99.9% quantile is then taken from these simulated annual losses as the estimator of the 99.9% VAR. We have simulated 10,000 years, however, as argued by Embrechts et al. (2005) for rare events, the convergence of the MC estimator to the true values may not be particularly fast, so in real applications either using more iterations or refining the standard MC by importance sampling technique is suggested<sup>20</sup>.

To model frequency we have used Poisson distribution, which is typically employed, having the density function

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!},$$

having a single parameter  $\lambda$ . We have estimated it using three complete years 2004-2006 and for each year of the simulation we generated a random number of losses based on this parameter.

For EVT we have not modelled the whole distribution but rather the tail by applying either the generalised extreme value (GEV) or the generalised Pareto distribution (GPD). In these cases

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<sup>18</sup> For more details on the VAR methodology see the traditional risk management books such as Jorion (2007), Saunders and Cornett (2006) or Sironi and Resti (2007).

<sup>19</sup> Although it is sometimes also defined as the difference between the mean and the quantile.

<sup>20</sup> Furthermore, the outlined aggregation of losses assumes that individual losses and the density function for severity and frequency are independent; in the context of operational losses this is a reasonable assumption.

(following Dutta et al., 2007) we have used empirical sampling<sup>21</sup> for the body of the distribution. Hence, the VAR has been calculated by a MC simulation in which a part of losses was drawn from the actual past losses and the other part was modelled by an EVT model. The proportion of losses in the tail for the calculation of VAR was set to 2% as this percentage of the highest losses appears to be the best to fit the data. The frequencies were again modelled using the Poisson distribution.

## 5.2 Loss distribution approach

In the loss distribution approach (LDA) we have made use of a few parametric distributions to try to model the whole distribution of the operational losses. As we have seen in the exploratory data analysis, the empirical distribution of our data is highly skewed and leptokurtotic, hence the distribution we have chosen allows for this. As the benchmark, exponential distribution with only one parameter is utilised, secondly, three two-parameter distributions (standard gamma, lognormal, and log-logistic) and the five-parameter generalised hyperbolic (GH) distribution. GH distribution belongs into general class of distributions and entails a wide range of other distributions and hence is more flexible for modelling.

Adequacy of each of the distributions is verified graphically by QQ-plots (Embrechts et al., 1997) and by the Kolmogorov-Smirnov statistics  $D^+$ ,  $D^-$  and  $D$  and the Kuiper statistic  $V$ . The statistics are defined as following

$$D^+ = \max_i \left[ \frac{i}{n} - F(x_i) \right] \quad D^- = \max_i \left[ F(x_i) - \frac{i-1}{n} \right] \quad D = \max(D^+, D^-) \quad V = D^+ + D^-$$

To calculate critical values for the statistics for different distributions we have followed procedure in D'Agostino and Stephens (1986). Based on the sample parameters we have drawn 10,000 simulations of the size  $n$  where  $n$  is the number of our observations. For each simulation we have reestimated the parameters, calculated the test statistics based on these parameters and used 10%, 5%, and 1% of the highest values of the statistics as the critical values.

As we have already mentioned, the threshold for the operational losses in the Bank is set quite low, so in order to improve the fit as low losses might be differently distributed we have increased the threshold to \$3,000, \$6,000, and \$10,000. Since, the last figure provided the best results and is in line with other studies we report only outcomes using this threshold.

To estimate the parameters for the four simple distributions maximum likelihood estimation (MLE) has been employed, whereas for the estimation of the GH distribution we have utilised quantile-based method given in Hoaglin (1985). As argued in Duta and Perry (2007), quantile-based methods can potentially be more accurate for fitting the tails of distribution compared to MLE.

The random variable  $X$  has an exponential distribution, if its density is

$$f(x) = \lambda \exp(-\lambda x), \quad x > 0, \quad \lambda > 0,$$

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21 Empirical sampling – randomly drawing actual losses from the dataset.

where  $\lambda$  is the only parameter referred to as rate or as scale if expressed as  $1/\lambda$ .

The random variable  $X$  has a standard 2-parameter gamma distribution, if its density is

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} \exp(-\beta x), \quad x > 0, \quad \alpha > 0, \quad \beta > 0,$$

where  $\alpha$  is the shape parameter,  $\beta$  is the scale parameter and  $\Gamma(\alpha)$  is the gamma function defined as

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} \exp(-x) dx, \quad \alpha > 0.$$

The random variable  $X$  has a 2-parameter lognormal distribution, if  $\ln(X)$  is distributed as normal distribution  $N(\mu, \sigma^2)$  defined as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], \quad x > 0, \quad \alpha > 0, \quad \beta > 0,$$

where  $\mu$  is the location and  $\sigma$  the scale parameter.

The random variable  $X$  has a log-logistic distribution (also known as the Fisk distribution), if its density is

$$f(x) = \frac{ax^{a-1}}{b^a \left[1 + \left(\frac{x}{b}\right)^a\right]^2}, \quad a > 0, \quad b > 0, \quad x > 0,$$

where  $a$  is the shape and  $b$  is the scale parameter.

The GH family of distributions introduced by Tukey (1977) is a transformation of the standard normal variable  $Z$  to

$$X_{g,h}(Z) = A + B(e^{gZ} - 1) \frac{e^{\frac{hZ^2}{2}}}{g}$$

where  $A$ ,  $B$ ,  $g$ , and  $h$  are the location, the scale, the shape parameter responsible for skewness, and the shape parameter responsible for kurtosis<sup>22</sup>, respectively. Martinez and Iglewicz (1984) have shown that GH distribution can approximate a wide variety of distributions by choosing appropriate values of  $A$ ,  $B$ ,  $g$ , and  $h$ . The following summarises estimation of parameters of the distributions based on Dutta and Perry (2007), the details can be found in Hoaglin (1985).

Defining  $X_p$  and  $Z_p$  as the  $100p^{\text{th}}$  percentiles of the  $g$ -distribution and standard distribution respectively, then

$$g_p = -\left(\frac{1}{Z_p}\right) \ln\left(\frac{X_{1-p} - X_{0.5}}{X_{0.5} - X_p}\right)$$

where  $X_{0.5}$ , the median of the data, is equal to  $A$ . Because there are many different  $g_p$  depending on the percentile  $p$ , Hoaglin (1985) suggests choosing  $g$  equal to the median of  $g_p$ . It can be shown that

$$\ln\left[\frac{g(X_p - X_{1-p})}{e^{gZ_p} - e^{-gZ_p}}\right] = \ln(B) + h\frac{Z_p^2}{2}.$$

Given that operational risk data are positively skewed and heavy-tailed to the right, it is more appropriate to express the left-hand side of this expression using the upper half spread (UHS) as defined in Hoaglin (1985):

$$UHS = \frac{g(X_{1-p} - X_{0.5})}{e^{-gZ_p} - 1}.$$

So once  $A$  and  $g$  are determined, the values of  $B$  and  $h$  can be found from OLS regression of  $\ln(UHS)$  on  $Z_p^2/2$ . The exponential value of the intercept is the estimate of  $B$ , and the coefficient of the regression is an estimate of  $h$ .

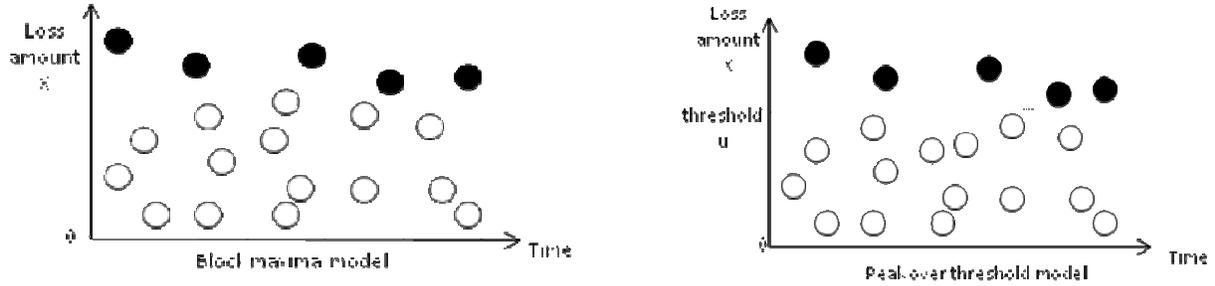
## 6. Extreme value theory

Extreme value theory (EVT) is a promising class of approaches to modelling of operational risk. Although originally utilised in other fields such as hydrology or non-life insurance, EVT is capable of modelling low frequency, high severity instances of operational losses. There are two main kinds of models in EVT. More traditional models are *block maxima* models which are for the largest observations collected from large samples of identically distributed observations. The whole sample is divided into equal non-overlapping time intervals and the biggest loss from each interval is used for modelling (*Figure 5*, left pane). In the *peak over threshold* (POT) model (or the threshold exceedances model), a more-modern approach, the large enough threshold is determined and the observations above are considered. For both block maxima and POT there is a theorem regarding limiting distribution.

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<sup>22</sup> The parameters  $g$ , and  $h$  can possibly be polynomial functions of  $Z^2$ , we considered only constant  $g$  and  $h$  in the estimation.

**Figure 5** – Block maxima model vs. Peak over threshold model



### 6.1 Block maxima models

Using the Fisher-Tippett and Gnedenko theorem the limiting distribution for normalised maxima is the GEV distribution (for more details see e.g. Embrechts et al., 2005). The distribution function of the (standard) GEV distribution is given by

$$F(x) = \begin{cases} \exp\left\{-\left(1 + \xi \frac{x - \mu}{\sigma}\right)^{\frac{1}{\xi}}\right\} & \text{if } \xi \neq 0 \\ \exp\left\{-e^{-\frac{x - \mu}{\sigma}}\right\} & \text{if } \xi = 0 \end{cases}$$

where (following Chernobai et al., 2007)

$$1 + \xi \frac{x - \mu}{\sigma} > 0 \quad x > \mu - \frac{\sigma}{\xi} \quad \text{if } \xi > 0 \quad x < \mu - \frac{\sigma}{\xi} \quad \text{if } \xi < 0 \quad x \in \mathbb{R} \quad \text{if } \xi > 0;$$

$x$  refers to the maxima,  $\mu \in \mathbb{R}$ , and  $\sigma > 0$ ,  $\mu$  is the location parameter,  $\sigma$  is the scale parameter, and  $\xi$  is the shape parameter.

The GEV distribution can be divided into three cases based on the value of the shape parameter. For  $\xi > 0$ , the GEV is of the *Fréchet* case which is particularly suitable for operational losses as the tail of the distribution is slowly varying (power decay), hence it is able to account for high operational losses. It may be further shown that  $E(X^k) = \infty$  for  $k > 1/\xi$ , thus for instance if  $\xi \geq 1/2$  a distribution has infinite variance and higher moments (Embrechts et al., 1997).

The *Gumbel* case ( $\xi = 0$ ) is also plausible for operational losses, although a tail is decreasing faster (exponential decay), it has a heavier tail than the normal distribution. The moments are always finite ( $E(X^k) < \infty$  for  $k > 0$ ). The *Weibull* case ( $\xi < 0$ ) is of the least importance as the right endpoint is finite, hence unable to model heavy tails of operational losses. The GEV distribution can be fitted using various methods, we are going to describe and use the two most commonly used, maximum likelihood and probability-weighted moments. Denoting  $f_{\xi, \mu, \sigma}$  the density of the GEV distribution, and  $M_1, \dots, M_m$  being the block maxima, the log-likelihood is calculated to be

$$l(\xi, \mu, \sigma; M_1, \dots, M_m) = \sum_{i=1}^m \ln f_{\xi, \mu, \sigma}(M_i) = -m \ln \sigma - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^m \ln \left(1 + \xi \frac{M_i - \mu}{\sigma}\right) - \sum_{i=1}^m \ln \left(1 + \xi \frac{M_i - \mu}{\sigma}\right)^{-\frac{1}{\xi}},$$

which must be maximised subject to the parameter constraints that  $\sigma > 0$  and  $1 + \xi(M_i - \mu)/\sigma > 0$  for all  $i$ . (for more details see Embrechts et al., 2005).

Probability weighted moments (PWM), the second used approach to estimate parameters of GEV, has better applicability to small samples than maximum likelihood (ML) method (Landwehr et al., 1979). Following Hosking et al. (1985), although probability weighted estimators are asymptotically inefficient compared to ML estimators, no deficiency is detectable in samples of 100 or less. As the number of extreme observations is typically limited, this property of PWM makes it very valuable in operational risk modelling. The probability-weighted moments of the GEV distribution for  $\xi \neq 0$  are given by<sup>23</sup>

$$\beta_r = \frac{1}{1+r} \left\{ \mu - \frac{\sigma}{\xi} \left[ 1 - \left( \frac{1}{1+r} \right)^{-\xi} \Gamma(1-\xi) \right] \right\} \quad \xi < 1$$

From this we have

$$\begin{aligned} \beta_0 &= \mu - \frac{\sigma}{\xi} [1 - \Gamma(1-\xi)] \\ 2\beta_1 - \beta_0 &= -\frac{\sigma}{\xi} \Gamma(1-\xi) (1 - 2^\xi) \\ \frac{3\beta_2 - \beta_0}{2\beta_1 - \beta_0} &= \frac{1 - 3^\xi}{1 - 2^\xi} \end{aligned}$$

From this, the PWM estimators  $\hat{\mu}$ ,  $\hat{\sigma}$ ,  $\hat{\xi}$  are obtained when  $\beta_r$  are replaced by their estimators. Given a random sample of size  $n$  from the distribution  $F$ , estimation of  $\hat{\beta}_r$  can be based on the ordered sample  $x_1 \leq x_2 \leq \dots \leq x_n$ . The statistic

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<sup>23</sup> In the following four expressions, we changed the sign of  $\xi$  as in the original paper the distribution function was defined with the inverse sign of  $\xi$  compared to the definition we use.

$$b_r = \frac{1}{n} \sum_{j=1+r}^n \frac{(j-1)(j-2)\dots(j-r)}{(n-1)(n-2)\dots(n-r)} x_j$$

is an unbiased estimator of  $\hat{\beta}_r$  (Landwehr et al., 1979).

Adequacy of the GEV model is verified similarly to LDA by QQ-plots (Embrechts et al., 1997) and the Kolmogorov-Smirnov statistics  $D^+$ ,  $D^-$  and  $D$  and the Kuiper statistic  $V$  based on Chandra et al. (1981). The statistics are defined as in the *Section 5.2*.

## 6.2 Points over threshold models

As argued by Embrechts et al. (2005) block maxima models are very wasteful of data as they consider only the highest losses in large blocks. Consequently, methods based on threshold exceedances are used more frequently in practice. These methods utilize all data that exceed a particular designated high level. Based on the Pickands-Balkema-de Haan theorem, the limiting distribution of such points over thresholds (POT) is the GPD. The distribution function of the generalised (two-parameter) GDP distribution is given by

$$F(x) = \begin{cases} 1 - \left(1 + \frac{\xi}{\sigma} x\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - e^{-\frac{x}{\sigma}} & \text{if } \xi = 0 \end{cases}$$

where  $\sigma > 0$ , and  $x \geq 0$ , when  $\xi \geq 0$  and  $0 \leq x \leq -\sigma/\xi$  when  $\xi < 0$ ;

$x$  refers to the extreme observations above the threshold,  $\beta$  is the scale parameter, and  $\xi$  is the shape parameter.

Similarly to the GEV distribution, the generalised GDP contains a number of special cases: when  $\xi > 0$  the distribution is of an ordinary Pareto distribution; when  $\xi = 0$  there is an exponential distribution,  $\xi < 0$  leads to a short-tailed, Pareto type II distribution. The condition for existence of moments in the heavy-tailed case ( $\xi > 0$ ) is  $E(X^k) = \infty$  for  $k \geq 1/\xi$ .

The critical issue in this approach is to determine the threshold  $u$ . A simple approach using an excess plot is typically employed. For positive-valued loss data  $X_1, \dots, X_n$  the sample mean excess function is defined as an empirical estimator of the mean excess function

$$e_n(v) = \frac{\sum_{i=1}^n (X_i - v) I_{\{X_i > v\}}}{\sum_{i=1}^n I_{\{X_i > v\}}}$$

where  $v$  is the value above threshold ( $v \geq u$ ). Threshold values against mean excess values provide the mean excess plot. If the data support a GPD model, this plot should become increasingly “linear” for higher values of  $v$ .

Maximum likelihood (ML) and probability weighted moments (PWM) are again the primary methods used for parameters estimation. The log-likelihood for excess losses  $Y_i$  ( $X_i - u$ , where  $u$  is the given threshold) given the density function  $f_{\xi, \sigma}$  can be calculated to be (e.g. Embrechts et al., 2005)

$$\begin{aligned} l(\xi, \sigma; Y_1, \dots, Y_{N_u}) \\ = \sum_{i=1}^{N_u} \ln f_{\xi, \sigma}(Y_i) = -N_u \ln \sigma - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^{N_u} \ln \left(1 + \xi \frac{Y_i}{\sigma}\right) \end{aligned}$$

which must be maximised subject to  $\sigma > 0$  and  $1 + \xi Y_i / \sigma > 0$  for all  $i$ .

The parameters using PWM can be calculated (provided  $\xi < 1$ ) by (Hosking et al. 1997)<sup>24</sup>

$$\begin{aligned} \sigma &= \frac{2\alpha_0\alpha_1}{\alpha_0 - 2\alpha_1}, \\ \xi &= 2 - \frac{\alpha_0}{\alpha_0 - 2\alpha_1}. \end{aligned}$$

The PWM estimators  $\alpha$  and  $\xi$  are obtained by replacing  $\alpha_0$  and  $\alpha_1$  by estimators based on an observed sample of size  $n$ . The unbiased and consistent possibility is

$$a_r = n^{-1} \sum_{j=1}^{n-r+1} \frac{(n-j)(n-j-1)\dots(n-j-r+1)}{(n-1)(n-2)\dots(n-r)} x_j,$$

where  $x_1 \leq x_2 \leq \dots \leq x_n$  is the ordered sample.

Again, the adequacy of the model is verified by QQ-plots and the Kolmogorov-Smirnov statistics  $D^+$ ,  $D^-$  and  $D$  and the Kuiper statistic. As critical values for the GPD have not been found, we have estimated them using the simulation approach described in the section devoted to LDA.

## 7. Empirical results

### 7.1 Loss distribution approach

As would be expected, the simple parametric distributions with one or 2-parameters are far too simple to model operational loss data. Although moving from exponential to a gamma distribution and from a gamma to a lognormal or a log-logistic somewhat improves the fit, both QQ plots and the test

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<sup>24</sup> In the following two expressions, the sign of  $\xi$  is again changed as the distribution function was defined with the inverse sign.

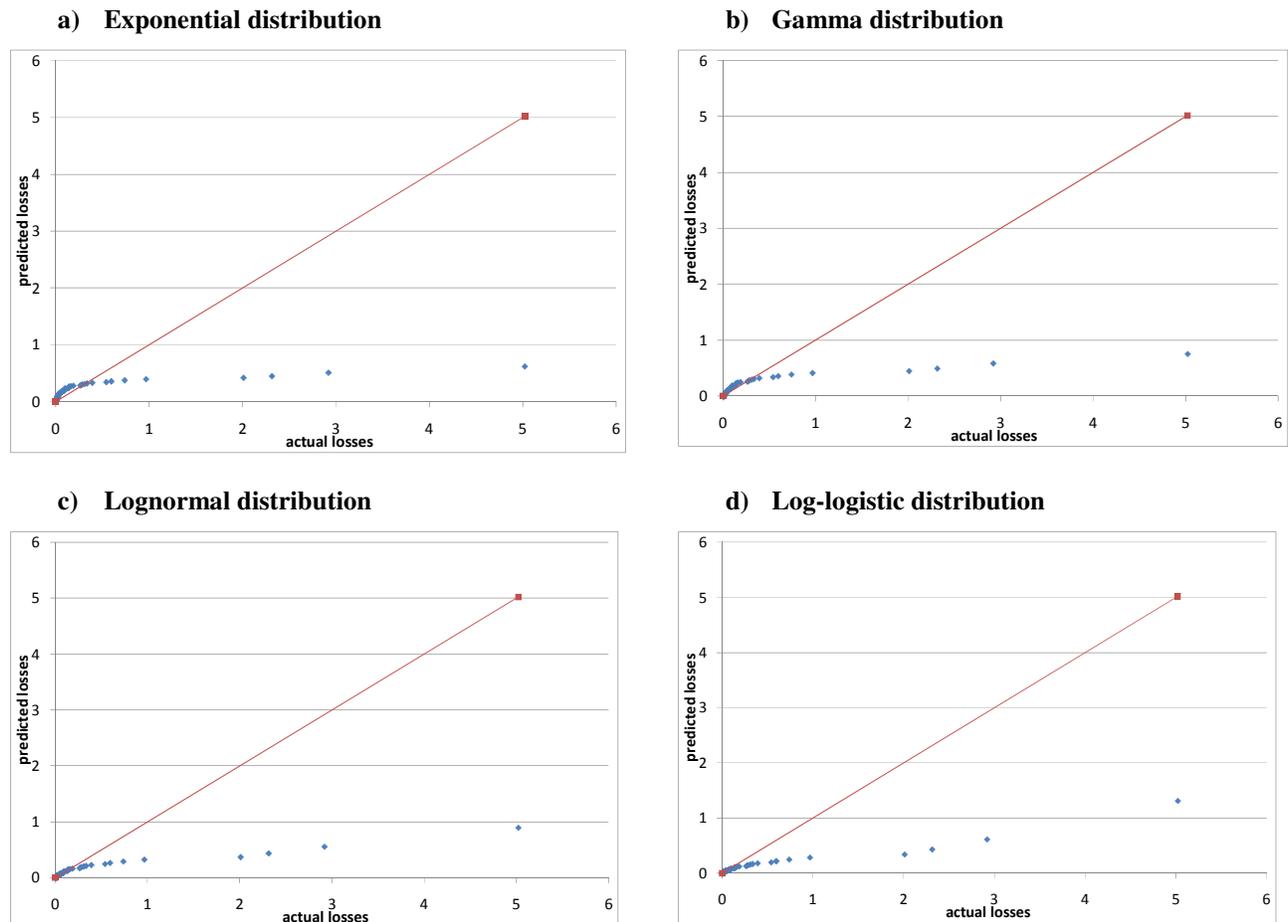
statistics (*Table 5*) reject the hypothesis that the data follow any of these distributions. The reason is that the losses in the end of the tail of the distribution are significantly underpredicted as can be seen in *Figure 6*.

**Table 5** – Simple parametric distributions - the goodness-of-fit statistics (*p-values*)

	MLE	
	$\sqrt{n}D$	$\sqrt{n}V$
Exponential	<0.01	<0.01
Gamma	<0.01	<0.01
Lognormal	<0.01	<0.01
Log-logistic	<0.01	<0.01

**Note:**  $\sqrt{n}D$  stands for the Kolmogorov-Smirnov and ( $\sqrt{n}V$ ) the Kuiper statistic

**Figure 6** – QQ plots for the exponential (panel a), gamma (b), lognormal (c) and the log-logistic distribution (d)



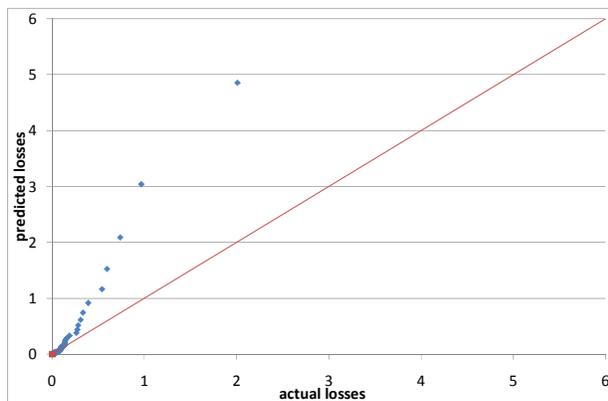
The results for the GH distribution are not much better (*Table 6, Figure 7*). Although this distribution is flexible enough to model extremely high losses, the highest loss in the dataset that is almost twice

the second largest loss causes the estimated GH distribution parameter for kurtosis to be very high and hence the distribution overpredicts the high losses, while underpredicting the lower losses. We can conclude that the whole distribution pattern of operational losses with rather limited observations is not possible to be captured even with a general class of distributions such as the GH distribution.

**Table 6** – GH distribution (Quantile Estimation)- the goodness-of-fit statistics (p-values)

	QE	
	$\nu nD$	$\nu nV$
GH	<0.01	<0.01

**Figure 7** – QQ plots for the GH distribution



Although none of the parametric distributions got close to a reasonable fit, we have still calculated VAR for these models (Table 7) to have at least an idea of the calculated VAR. From the table we can draw similar conclusion as from the Q-Q plots. The first three distributions provide relatively low capital requirements in the range (2.0-2.7%). Based on the Fisk distribution the calculated capital requirement is much higher as this distribution allow for higher losses. Finally, the GH distribution provides unreasonably high capital requirement owing to the high shape parameter and overprediction of the highest losses.

**Table 7** – Summary of calculated VAR – Parametric distributions

Distribution	VAR (99.9%) - Monte-Carlo	
	MLE	QE
Exponential	2.7%	
Gamma	2.1%	
Lognormal	2.0%	
Log-logistic	9.5%	
GH distribution		>100%

## 7.2 Block maxima models

Two different scenarios have been employed when applying the block maxima model, the highest losses in each month and the highest dozen (twelve) of losses<sup>25</sup>. For each scenario the parameters were estimated by MLE and PWM.

Table 8 shows the resulting estimate of the shape parameter<sup>26</sup>.

**Table 8** – Block maxima models – the shape parameter

	MLE	PWM
Max. each month	1.22	0.78
Max. dozen	1.95	0.45

Although both estimation methods indicate a heavy tail of the distribution, MLE and PWM yield quite different results for both block maxima models. While for PWM the parameters are less than one, (even less than 0.5 for the second model indicating finite variance) the parameters derived from MLE are well above one (infinite mean), indicating extremely heavy tailed data.

Table 9 depicts the goodness-of-fit statistics, the Kolmogorov-Smirnov ( $\sqrt{n}D$ ) and the Kuiper statistic ( $\sqrt{n}V$ ), if the p-value is below 1%, the hypothesis of a good fit of the model is rejected on the 1% significance level. On the contrary, if it is above 10%, the model appears as very appropriate to model the data. The other cases are in-between these two boundary cases.

**Table 9** – Block maxima models - the goodness-of-fit statistics (p-values)

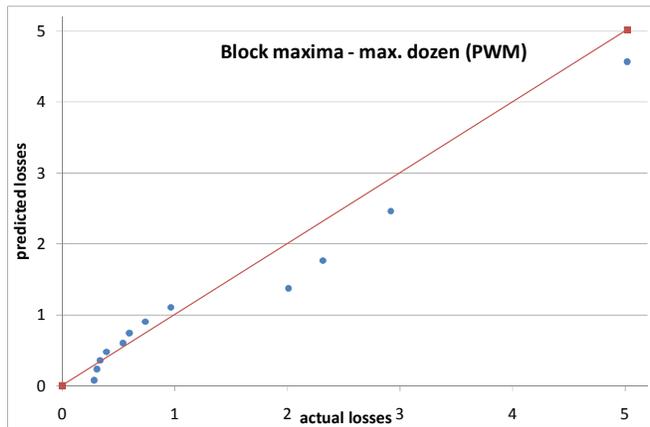
	MLE		PWM	
	$\sqrt{n}D$	$\sqrt{n}V$	$\sqrt{n}D$	$\sqrt{n}V$
Max. each month	<0.01	<0.01	>0.01	<0.01
Max. dozen	<0.01	>0.10	>0.10	>0.10

From the above table we can conclude that the second model (the maximum dozen model) fitted by PWM produces the best results, while the use of MLE for the first model can be rejected. The other two cases deliver mixed results.

<sup>25</sup> As the twelve losses are not the maximas as defined in the theorem for the limiting distribution, there is no assurance that this scenario will even in the limit follow the GEV distribution. However, the GEV can still be a good model that fits the data well.

<sup>26</sup> We again follow the current practice not to show the location and the scale parameter for the confidentiality reasons and we just show the shape parameter which is of the highest importance from the modelling perspective.

**Figure 8** – Block maxima model – QQ-plot for max. dozen model fitted by PWM

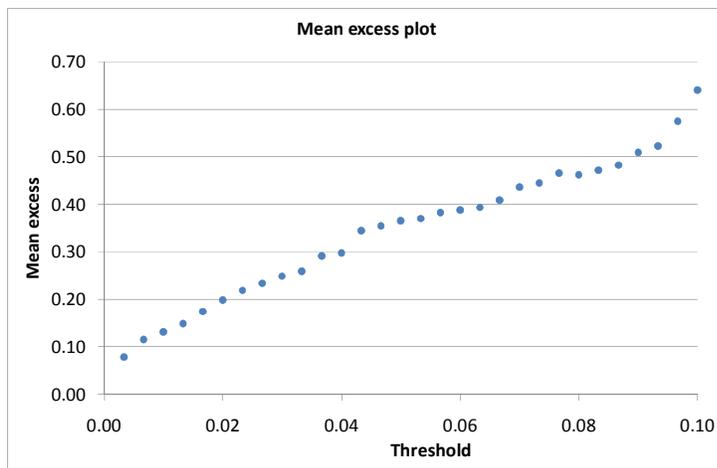


The QQ-plot above shows that although the maximum dozen model estimated by PWM slightly underpredicts the highest losses, the fit of the data is very good, supporting the adequacy of this model.

### 7.3 Points over threshold models

We have chosen four different models. Firstly, using the excess plot we have identified a threshold (*Figure 9*). The plot is reasonably linear over the given range; the threshold is set at the level of a small “kink” where the slope decreases slightly<sup>27</sup>. This threshold is slightly higher than 10% of all losses in the data set. Additionally, we have used 2%, 5% and 10% of the highest losses.

**Figure 9** – POT model – Mean excess plot



<sup>27</sup> Slightly above 0.04 on the virtual horizontal axis.

Again, the shape parameter obtained from different methods differs significantly (

*Table 10*). However, we can trace some consistency at least from the PWM results. As noted by Embrechts (2005) the shape parameter of the limiting GPD for the excesses is the same as the shape parameter of the limiting GEV distribution for the maxima. Indeed, for our data, the block maxima model of maximum dozen losses (approximately 2% of losses) is close to the threshold of 2% highest losses from the POT model. Additionally, the other three POT models have the shape estimates close to each other.

**Table 10** – Threshold exceedances models - the shape parameter

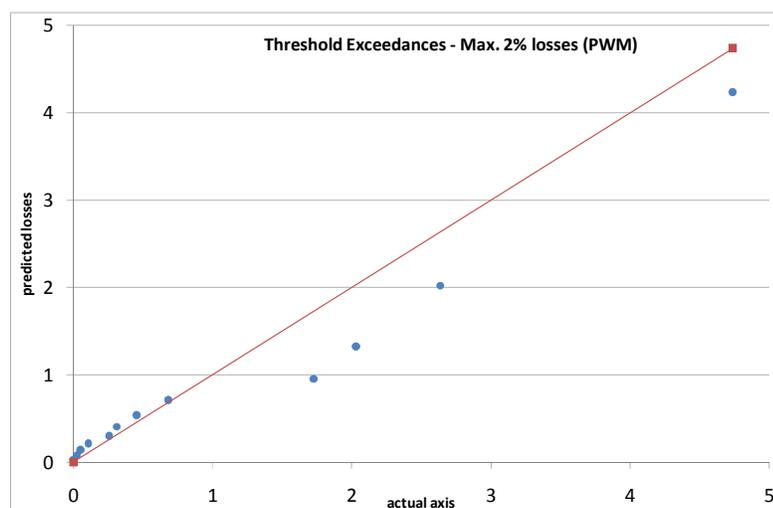
	MLE	PWM
Losses > a threshold	1.02	0.77
Max. 10% losses	1.08	0.77
Max. 5% losses	1.55	0.73
Max. 2 % losses	0.93	0.48

Regarding the goodness-of-fit, the outcomes (*Table 11*) are generally plausible for both estimation methods. Therefore, we can conclude, that the models appear reasonable from the statistical point of view. QQ-plot is produced (*Figure 10*) for the maximum 2% model estimated by PWM, which exhibits the best visual fit and at the same time displays consistency with the block maxima model.

**Table 11** – Threshold exceedances models - the goodness-of-fit statistics (p-values)

	MLE		PWM	
	$\sqrt{n}D$	$\sqrt{n}V$	$\sqrt{n}D$	$\sqrt{n}V$
Losses > a threshold	>0.10	>0.05	>0.01	>0.05
Max. 10% losses	>0.10	>0.10	>0.01	>0.10
Max. 5% losses	>0.10	>0.10	<0.01	>0.025
Max. 2 % losses	>0.10	>0.10	>0.10	>0.10

**Figure 10** – POT model – QQ-plot for maximum 2% model fitted by PWM



The *Table 12* summarises the result for EVT. The high shape parameters for some of the models estimated by MLE result in unreasonable high capital estimates, higher than 100% of the corresponding bank income<sup>28</sup>. On the other hand, capital estimates by PWM are quite consistent from a practical point of view, ranging from 6.9%–10.0%, indicating alongside with the arguments already mentioned that this method might be more suitable in the estimation of operational risk when the data are limited.

As discussed above, Central European banks usually do not possess a methodology to model operational risk since they rely on the competence of their parent companies to calculate operational risk requirement on the consolidated basis of the whole group. The issue worth investigating is if there is any benefit from shifting the calculation of operational risk capital requirement to the subsidiary level, especially taking into account the translation and transaction risks necessary for consolidated reporting. Although the PWM methodology might give reasonable results for a subsidiary, parent companies need to consolidate capital requirements of their subsidiaries (not only operational risk but also for credit, market and other risks). Therefore the parent companies use their models and the subsidiaries usually provide these models only with some modifications (e.g. more data or scenario analysis). As documented both in the theory (OWC, 2001) and practice (Deutsche Bank (2007) or BBVA (2007)), this portfolio approach brings a diversification effect resulting in a lower capital requirement. For instance, Deutsche Bank recorded a 20% positive diversification effect of an overall economic capital requirement in the year 2007. Similarly, Banco Bilbao Vizcaya Argentaria estimated a 45–58% positive diversification effect for operational risk capital requirement in 2007.

**Table 12** – Summary of results - Extreme value theory

Model	Description	Shape ( $\xi$ )		VAR (99.9%) - Monte-Carlo	
		MLE	PWM	MLE	PWM
1	GEV - monthly maxima	1.22	0.78	83.4%	8.1%
2	GEV - max. dozen	1.95	0.45	>100%	7.2%
3	GPD - losses > a threshold	1.02	0.77	33.7%	7.7%
4	GPD - max. 10% losses	1.08	0.77	39.9%	6.9%
5	GPD - max. 5% losses	1.55	0.73	>100%	10.0%
6	GPD - max. 2% losses	0.93	0.48	>100%	9.2%

*Table 13* presents a summary of our research. As we indicated earlier, EVT shows the best statistical fit when estimating capital of the Bank on a 99.9% confidence level.

<sup>28</sup> For a comparison, Basel II requires banks to hold a capital requirement for operational risk at 15% of banking income in case of using the Basic Indicator Approach (see the *Section 3.3*).

**Table 13** – Summary of results – LDA & selected EVT models

Body	Tail	Statistical fit	Capital estimate (99.9%)
Exponential	Exponential	very poor	2.7%
Gamma	Gamma	very poor	2.1%
Lognormal	Lognormal	poor	2.0%
Log-logistic	Log-logistic	poor	9.5%
GH distribution	GH distribution	poor	>100%
Empirical sampling	EVT (block maxima, max. dozen, PWM)	excellent	7.2%
Empirical sampling	EVT (block maxima, max. 2%, PWM)	excellent	9.2%

## 8. Conclusion

In this paper we have attempted to analyse and model real operational data of a Central European Bank. We have utilized two approaches currently described in the literature. The LDA, in which parametric distributions are fitted to the whole data sample, was not able to capture the pattern of the data and was rejected based on the goodness-of-fit statistics. Hence we conclude that the parametric distributions like exponential, gamma, log-normal, log-logistic and GH do not fit well the data. This result proves an unusual (heavy-tailed) pattern of operational risk data as documented by many researchers such as Muller (2002), Cruz (2002), Moscadelli (2004), de Fontnouvelle et al. (2005) or Duta, Perry (2007).

The EVT, on the other hand, for both block maxima and POT proved to fit the data in the tail of the distribution. We have used two estimation methods in the EVT approach, the standard MLE in which all the observation have the same weight and the PWM in which the observations higher in the tail have a higher weight.

When applying the block maxima model we have found out that the maximum dozen model fitted by PWM produces the best results. Cruz (2002) used PWM to analyse fraud loss data on an undisclosed source for the 1992–1996 period and deduced that the data in 1994 and 1996 recorded a heavy-tailed GEV distribution. In addition, the Kuiper statistics for PWM showed the best results in all four years, which confirms our findings.

POT models are frequently used for application of EVT to operational loss data. We observed that the high shape parameters for some of the MLE models bring unreasonable high capital estimates, what is consistent with Moscadelli (2004), de Fontnouvelle et al. (2005) or Chavez-Demoulin et al. (2005). These authors also mention the estimates are highly sensitive to the chosen threshold, what again underpins our conclusions. Unlike the others, our research showed that PWM are quite consistent from a practical point of view and they might be suitable in the estimation of operational risk when data is limited. This result might be useful for the banks that have limited data series of operational risk events, what is typical for many Central European banks.

From a policy perspective, it should be noted that emerging market banks, such as Central European banks, face increasing exposure to operational risk events. Data from the Bank evidenced an improvement over time, attributed to managements' devoting more attention to recording and mitigating operational risk events. Moreover, as demonstrated in this analysis, successful estimates of the distribution of these risk events can be estimated based on data derived from more mature markets.

Despite the conclusions cited above, there are still several ways in which our research can be improved. Firstly, a similar study can be done on a larger sample of data (we used the data from one

Central European bank). Secondly, the research provided on all eight business lines recognised by Basel II may reveal interesting facts about different operational risk features among various business lines. Finally, other research might include other results derives from modelling operational risk using such techniques as robust statistics, stress-testing, Bayesian inference, dynamic Bayesian networks and expectation maximisation algorithms.

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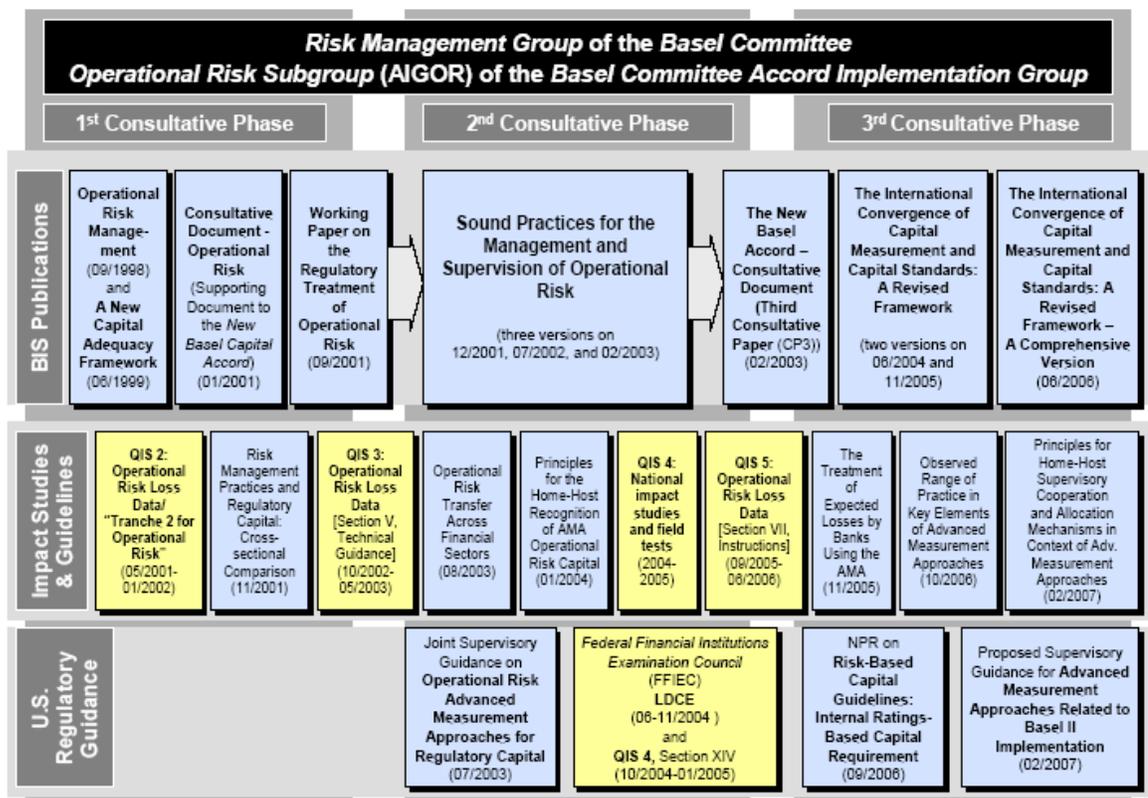
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## 10. Annex 1 - The Evolution of the Regulatory Treatment of Operational Risk



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