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Implied Market Loss Given Default: structural-model approach

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Abstract:
This paper focuses on the key credit risk parameter–Loss Given Default (LGD). We describe its general properties and determinants with respect to seniority of debt, characteristics of debtors or macroeconomic conditions. Further, we illustrate how the LGD can be extracted from market observable information with help of the adjusted Mertonian structural approach. We present a derivation of the formula for expected LGD and show its sensitivity analysis with respect to other structural parameters of the company. Finally, we estimate the 5-year expected LGDs for companies listed on Prague Stock Exchange and find out, that the average LGD for this analyzed sample is around 20%. To the author's best knowledge, those are the first implied market estimates of LGD in the Czech Republic.

Keywords: loss given default, credit risk, structural models.

JEL: C02, G13, G33.

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Introduction

The awareness of the credit risk has largely enlarged in last decades due to an increase in the volatility in the underlying real economy, integration of financial markets and development of new financial instruments. The increased uncertainty has lead to development of new procedures and mechanisms how to determine the causality between the attributes of the borrowing entity and its potential bankruptcy. The credit risk techniques have therefore experienced a significant development of new refined methods concerning the estimation of risks and other parameters specifying possible losses.

One of those parameters is also Loss Given Default (LGD), expressing percentage of exposure, which will be not recovered after counterparty's default.[1] While estimation of probability of default (PD) has received considerable attention over the past 20 years, LGD has obtained a greater acceptance only in recent years as the New Basel Accord identified it as one of the key risk parameters. Yet, loss given default modeling is still a quite new open problem in the credit risk management and its estimation is not straightforward, because it depends on many driving factors, such as the seniority of the claim, quality of collateral or state of the economy. Moreover, the insufficient database with experienced LGDs makes it more difficult to develop accurate LGD estimates based on the historical data. Hence, the extraction of LGD for credit-sensitive securities based on the market observable information is an important issue in the current credit risk area and may bring other improvements into present credit risk management.

This paper therefore discusses this key risk parameter for single corporate exposures and deals with the possibility of LGD's extractions from market information. This type of LGD is denoted as ***implied market LGD***. We utilize so called structural models, which are based on the initial Merton's framework, and present the derivation of closed-form formula for LGD and its sensitivity analysis with respect to other structural parameters of the company. Further, we

[1] Before Basel II formalized the use of LGD, this concept was also called ***Severity*** (see Stephanou and Mendoza 2005).

empirically implement this contingent claim approach for the set of companies in the Czech Republic. Since the application of structural models requires a value of firm's asset and its volatility as input parameters, which are non-observable variables, we also present the methods for their estimation using equity prices and balance sheet data. As the result, we estimate 5-year expected LGD for almost thirty companies listed on Prague Stock Exchange in period 2000–2008. Those are to the author's knowledge the first estimates of LGD from market information in Czech Republic.

♦ Basic characteristics of LGD

LGD is usually defined as the loss rate experienced by a lender on a credit exposure if the counterparty defaults.[1] Thus, despite default the lender still recovered $1-\text{LGD}$ percent of the exposure. One minus LGD is therefore called recovery rate (RR). In principle, LGD comprises also other costs related to default of the debtor, and the correct formula should rather be

$$\text{LGD} = 1 - \text{RR} + \text{Costs} \tag{1.1}$$

Nevertheless, costs are relevant only in a specific type of LGD and are not usually so high to influence losses markedly in comparison with recovery rate. Therefore we use recovery rate as the complement of LGD in the following text and take these two parameters as conceptually the same.

Usually three basic types of LGD for defaulted facilities are used. *Market LGD* employs the price of bond after default as a proxy of the recovered amount. "*The market value is the best estimate for the expected recovery and the market price reflects the expected recovery suitably discounted*" (Bhatia 2006, p. 285). However, post-default price is available only for the fraction of the debt that is traded and for which after-default market exists – very often they are available only for corporate bonds issued by large companies.[2] Market LGD is therefore highly limited for defaulted bank loans that are traditionally not traded. For them one must turn to the other approach.

Workout LGD considers all relevant facts that may influence the final economic value of the recovered part of the exposure arising in the long-running workout process. LGD is then determined by (i) loss of principal, (ii) carrying costs of non-performing assets, e.g. interest income lost or foregone, and (iii) recovery and workout expenses, for example direct and indirect administrative costs. However, bankruptcy claims are often not settled only in cash but with securities (equity, options, warrants, etc.) with no secondary market, which means that their value will be unclear for years. Another problem is that appropriate discount rate (which should reflect the risk of holding defaulted asset) is not known. Computation of workout LGD there-

[1] In principle we should mark the loss rate given default as LGDR and LGD use for the absolute amount of loss. However, LGD is used to indicate the loss rate by many practitioners including the Basel II, while the absolute loss is indicating as LGD.EAD (see BCBS 2005).

[2] What's more, outside the USA the market for defaulted bonds either non-exist or does not have the required depth and liquidity.

fore depends on an unknown and variable discount rate which is difficult to estimate for particular situation.[1] Therefore we must speak also about estimate of LGD even if we are trying to measure it from ex-post data. The last method of measuring of LGD is concept of ***Implied Market LGD***, which is estimated ex-ante from market prices of non-defaulted loans, bonds, or credit default instruments by structural or reduced-form models. The idea is that prices of risky instruments reflect market's expectation of the loss and may be broken down into PD and LGD. Implied market LGD estimation does not rely on historical data and can be especially used for low default facilities, for which is an insufficient historical database with experienced LGD. Thus, the chosen method of LGD estimation depends on data availability and all mentioned concepts have the indispensable importance in an area of recovery estimation.

As can we see, the stress should be admittedly put on distinguishing between measuring LGD ex-post and estimating it ex-ante. However, any study of LGD has to exactly specify, which criterion should be used to define default event because a different classification will lead to diverse results in LGD's both ex-ante and ex-post estimates. A widely chosen definition of default leads to a lower estimate of PD but higher estimate of LGD because fewer exposures will be classified as "in default", but those will have relatively lower quality with a low recovery outlook. Conversely, from a narrower and more severe definition stems higher default rates and also higher recoveries.

A firm can default on the debt obligations and still not declare bankruptcy. It depends on the negotiations with its creditors. One can observe a certain pattern of typical developments, which we call after default scenarios – cure, restructuring, liquidation (see Christian 2006). When firm goes to the bankruptcy[2] and there is no other possibility than liquidation, the capital structure of the firm and absolute priority rule (APR)[3] is an important determinant of recovery rate. This states that the value of the bankrupted firm must be distributed to suppliers of capital so that *"...senior creditors are fully satisfied before any distributions are made to more junior creditors, and paid in full before common shareholders*" (Schuermann 2004, p. 11). The rate of recovery of the defaulted bond depends on where the claims in the firm's capital structure are. Bonds are frequently classified in terms of seniority and allocated collateral. Seniority is capturing the order mentioned above of the claimants'priority over the assets of the defaulted company and collateralization measures the allocation of specific assets as guaranties for the facility in the case of default. The bank loans are on the top of the debt waterfall and are often highly collateralized.

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[1] Sometimes the discount rate based on historical values is used. For example, on average market rates observed between the default and the end of workout process, this would lead according to Resti and Sironi (2007, p. 349) to backward-looking measure, because in estimation of LGD on future bad loans we are interested in interest rate that might be on the market after a new default. The use of past interest rate is not appropriate for the present and future market conditions. What discount factor should be used is dealt in e.g. Maclachlan (2005).

[2] The bankruptcy has a form of reorganization or liquidation.

[3] Eberhart and Weiss (1998) are confirming that APR is routinely violated because of speed of resolution. Creditors agree to violate APR to resolve bankruptcies faster.

Empirical evidence on recovery rates is usually based on defaulted bonds because the LGDs data is simply available through after-default market information. Loans are usually expected (ceteris paribus) to perform better because they are typically more senior in capital structure, have tighter covenants and banks can more actively monitor the evolving financial health of the obligor (see Amihud et al. 2000).[1]

The results of several empirical studies have confirmed that the RR increases with the seniority and security of the defaulted bonds and decrease with its degree of subordination. Results tend to be also rather similar in term of average recovery rates – for bank loans (70–84%), for bonds: senior secured (53–66%), senior unsecured (48–50%), senior subordinated (34–38%), and subordinated (26–33%). All studies also reported high standard deviation that characterizes recovery rate across all bond debt-classes, regularly overrunning 20%.[2] This implies a high degree of uncertainty concerning the expected RR and observed ex-post results may significantly differ from ex-ante estimates (see Altman, Kishore 1996, Castle, Keisman 1999, Keenan et al. 2000 or Hu, Perraudin 2002). Also, there is a general understanding that collateral can help to reduce LGD radically, which is also empirically confirmed (Dermine and Neto 2005).

Recovery rates are ultimately determined by the value of assets that can be seized in case of default. Because many asset types differ between industries,[3] it is therefore intuitive to assume that the debtor's industry characteristics can influence LGD. Also firm-specific characteristics, mostly financial, which contribute towards reducing leverage help to improve RR. Leverage indicates the extent of claimants for assets in case of default; therefore its lower value improves the enforcement of claim. The firm-specific quality of assets has also its importance, as their values are the source of repayment after default. Assets, whose quality have lower likelihood to deteriorate over time and are less likely to "disappear", provide better guarantee.

Although industry-type seems like a straightforward determinant of RR, the literature does not give wholly unified answers (see Altman and Kishore 1996, Grossman et al. 2001 or Acharya et al. 2003). Those studies have broken out LGD of corporate bonds by industry and have found evidence that some industries such as *public utilities* and *chemicals*[4] do evidently better than the others. Nonetheless, they also showed that the standard deviation of RR per industry and within a given industry is still very large. An opposite view of the industry influence is presented by Gupton et al. (2000) or Araten et al. (2004) which has on the contrary found no

[1] Banks are sometimes able to change their lending relationship to better position in capital structure of the firm with anticipation of forthcoming debtor's bankruptcy and thereby raise expected recovery. The dispersed nature of bond ownership makes it impractical for bondholders to renegotiate the terms of contract as debtor's conditions changes (see Schuermann 2004).

[2] It is interesting to note that if the recovery rate probability distribution were uniform, which means that the probability of occurrence of values from 0 to 100% is the same, then its standard deviation would be approximately equal 29%. This clearly shows the big variance in RRs.

[3] Firm in some sectors have a large amount of asset that can be easily sold on the market in case of default, while other sectors can be more e.g. labor-intensive.

[4] The exact name of the group is "Chemicals, petroleum, rubber, and plastic products" (see Altman, Kishore, 1996).

evidence of different LGDs across industries. They state, that the use of recovery averages broken out by industry does not capture the industry variability in recovery rates across time. Some sectors may enjoy periods of high recoveries, but can fall later below average recoveries at other times, it means that industry recovery distributions change over time and therefore cannot be expected to hold in the future. As a result they concluded that industry type is not so appropriate factor for future RR predictions.

These unambiguous results of different studies might be attributed to LGD cyclicality in relation with economic environment. Every industry has specific traits and can be in different stage of economic cycle, which can more influence LGD than the industry-type itself because LGD is not stable in time and is underlying cyclical variability, which can be taken in relation with macroeconomic conditions. Acharya et al. (2003) showed that when the industry is in distress, mean LGD is on average 10–20% higher than otherwise.

Behind the cyclical variation is the fact that as the economy enters into recession, default rates increase. Recoveries from collateral will depend on the possibility of selling the respective assets. We can generally suppose that greater supply of collateral-assets will lead to their lower prices, of course, depending on the market size and structure observed for a certain asset.[1] Also banks have to accept discounts for distressed sale. Moreover, the demand for these assets declines because non-defaulted companies are not able to invest the same amount of money in recession as during an expansion. The result is that macroeconomic situation can significantly influence the recovery rate, which was as well demonstrated by several authors (see Araten et al. 2004 or Altman et al. 2005a).

As it was shown, LGD is influenced by many factors as facility's seniority and presence of collateral, borrower's industry characteristics or more general factors as macroeconomic conditions. However, previous research gives ambiguous results concerning some LGD's properties. The relatively rare occurrence of default events for some facilities could cause that the research was based on relatively small empirical samples. Also a non-homogenous methodology was used (e.g. for extracting LGD in workout process), which could also influence some conclusions. It is clear that further research is needed and hopefully with the acceptance of Basel II accord, setting rules for LGD's data gathering and its estimates, this research will be based on better data sample offering more exact outcomes.

As we could see, LGDs differ with respect to the types of borrowers, seniority of debt, or development of macroeconomic conditions. Although we could notice different relations among those variables, a major difficulty of such information is their complete dependence on historical data. The LGD predictions based on their past data are not thus necessarily coherent with the evolution of fundamentals across time and can result in inaccurate estimates being not able to capture the real trend in economy.

[1] For instance, a substantial number of defaulted firms in the telecom industry in 2001 in US. The very large inflow of specific telecom assets being liquidated increased the imbalance between supply and demand and depressed the value of these assets in the market.

♦ Loss Given Default modeling

In this part we focus on analytical tools which enable forward-looking estimates of LGD from market observable information. We employ asset pricing models which are aiming at determining the equilibrium arbitrage-free price of risky assets. Each risky asset should offer an expected return corresponding to its degree of risk; therefore all risky parameters must be evaluated by market in order to get the equilibrium price. This assumption, that prices include all information is then used by credit risk pricing models which utilize market information (e.g. share or bond price) to measure credit risk and are trying to extract the key risk parameters such as PD or LGD from the prices. Those models are forward-looking, estimating the risk parameters which are expected by the market in the future and not those that occurred in the past. From the nature of this method such estimate of LGD is called *implied market LGD*.

Those credit risk pricing models can be further classified as *structural* and *reduced-form* models. The category of structural-form models are based on the framework developed by Merton in 1974 using the theory of option pricing presented by Black and Scholes (1973). The intuition behind is quite straightforward, a company defaults, when the value of its assets becomes lower at the time of debt's maturity than that of its liabilities. For that reason, the default process is driven by the value of the company's assets and the risk of default is explicitly related to the assets variability. The term *structural* comes from the fact, that these models focus on the structural characteristics of the company such as asset volatility or leverage that determine relevant credit risk elements. Default and RR are a function of those variables.

In contrast, *reduced-form* models generally ignore the structural parameters as the cause of the default and simply assume that default is possible and is driven by some exogenous random variable. The result is that default and recovery is modeled independently from the firm's structural features, which lacks the clear economic intuition behind the default event. The basic input parameters for extracting LGD in reduced-form approach are the prices of risky corporate bonds. However, the companies in the Czech Republic are still using more traditional bank loans as the source of finance than issuing bonds (see Dvořáková 2003). It results in the fact that the domestic market with corporate debt is rather illiquid and incomplete and can hence barely reflect market expectations about default and recovery risk of particular company or its security. The result is that the reduced-form models are nowadays hardly applicable for LGD estimation in the Czech Republic.

The stock market provides an alternative source of information assuming that the share prices incorporate all available information including future prospects of the company as well as its creditworthiness.[1] Structural models for extracting company's default risk typically utilize the observed stock prices, stock volatility and specifics about the company's capital

[1] This is true only if an efficiency hypothesis holds, which was doubted by some studies (see e.g. Sloan 1996). There is also a question, whether the volatility of stock price is caused solely by incorporation of new information about future stocks' returns, or if it is caused largely by trading itself (see French 1980 or French and Roll 1986).

structure. Even if the number of quoted companies in the Czech Republic is also limited, for some of them seems to be sufficiently liquid to apply structural models and estimate demanded credit risk parameters. As the result, we will utilize the Merton's structural approach to derive formula for implied market LGD for particular company.

The seminal structural Merton's (1974) model relies on many hypotheses which are mostly coming from the Black–Scholes option pricing theory. Some of them became source of criticism and have been later relaxed.[1] The original framework in which is the valuing process of firm's assets embedded requires for the application of standard corporate credit risk pricing many assumptions. There are no transactions costs, taxes, or short-selling restrictions. The term structure of risk-free interest rate is flat and known with certainty. The price of riskless bond paying \$1 at time T is hence $B_0(T) = \exp[-rT]$, where r is the instantaneous riskless interest rate. Total value of firm V is financed by equity E and one zero-coupon non-callable debt contract D, maturing at time T with face value F. Also holds $V_t = D_t + E_t$, with no-taxes assumption this implies that the value of the firm and the values of assets are identical and do not depend on the capital structure itself. This corresponds with Modigliani–Miller theorem.

The dynamics of the firm's value through time can be described by the stochastic differential equation called geometric Brownian motion

$$dV_t = m_V V_t dt + s_V V_t dW_t^V \tag{1.2}$$

where m_V is the assets drift (i.e. the instantaneous expected rate of return on the firm's value V per unit time), s_V is the standard deviation of its return, and dW_t^V is a standard Gauss–Wiener process.

In such framework based on those assumptions, credit risk concerns the possibility that the value of the company evolves stochastically, will be on the maturity day T less than the repayment value of the loan F. The debtholders receive at T neither the value F (if $V_T > F$), or they receive the entire value of the firm and the owners of the firm receive nothing (if $V_T < F$). The risk of default is therefore explicitly linked to the volatility on the firm's asset value. The Merton's contingent claim analysis shows, how this risk should be priced. Merton derived the fundamental differential equation, which determines the value of the debt as at any time t as a function of the value of a firm. We use the famous Merton's conclusion that the value of equity identical to the formula for pricing "*...an European call option on a non-dividend paying com-*

[1] The alternative approaches, which try to remove one or more of those problematic drawbacks of the seminal model, have been developed. Black and Cox (1976) introduced the possibility of more complex capital structure of the company's liabilities, Geske (1977) presented the interest paying debt, or Vasicek (1984) established the distinction between the short and long-term debt. All previous authors also enhanced the model by treating default as an event that can occur any time before debt's maturity. More recent improvements such as works by Longstaff and Schwartz (1995), Hull and White (1995) reject the constant risk-free interest rate and considered interest rate as stochastic variable instead of that. For detailed development of later structural models see e.g. Altman et al. (2005a) and the references therein.

mon stock where firm value corresponds to a stock price and F corresponds to the exercise price" (Merton 1974, p. 10). This is given as

$$E(V,0) = \max\left[0; V - F\right] \tag{1.3}$$

Indeed, at maturity time T, the equity holders will exercise option and pay to debtholders face value of liabilities if $V_T \geq F$, otherwise they let this option expire. By applying the Black–Scholes option pricing formula it is straightforward to get solution for equation (1.3) as

$$E(V,t) = V\Phi\left(d_1\right) - Fe^{-rt}\,\Phi\left(d_2\right) \tag{1.4}$$

where $d_1 = \dfrac{\ln\dfrac{V}{F} + \left(r + \dfrac{1}{2}s_V^2\right)t}{s_V\sqrt{t}}$, $d_2 = d_1 - s_V\sqrt{t} = \dfrac{\ln\dfrac{V}{F} + \left(r - \dfrac{1}{2}s_V^2\right)t}{s_V\sqrt{t}}$ and Φ (.) is cumulative standard normal distribution. And since $V = D(V,t) + E(V,t)$, where $\tau = T - t$ is the length of time until maturity, we can express the value of debt at time τ as

$$D(V,t) = V\Phi\left(-d_1\right) + Fe^{-rt}\,\Phi\left(d_2\right) \tag{\textbf{1.5}}$$

Now we can already look how credit risk parameters as PD and RR can be extracted. The default occurs, when firm's value drops below some ***default barrier*** (DB), which is in the seminal Merton's model represented by face value of debt F at its maturity. The probability of default is therefore simply expressed as

$$PD = \Pr(V_T \leq F) \tag{1.6}$$

To get this probability, the more information about probability distribution of V has to be known. However, we can use the assumption that the value of the firm V is log-normally distributed, what is according to Crouhy et al. (2000) quite robust hypothesis confirmed by actual data, and we can get information about probability distribution of $\ln V_T$,[1] what is

$$\ln V_T \sim \Phi\left[\ln V_0 + \left(m_V - 0{,}5 s_V^2\right)T, s_V^2 T\right] \tag{1.7}$$

from properties of natural logarithm can be the probability (1.6) expressed as

$$PD = \Pr(\ln V_T \leq \ln F) \tag{1.8}$$

and from that by using (1.7) we can get

$$PD = \Phi\left(-\frac{\ln\dfrac{V_0}{F} + \left(m_V - \dfrac{1}{2}s_V^2\right)T}{s_V\sqrt{T}}\right) = \Phi(-d_2^*) \tag{1.9}$$

[1] The Itô's Lemma can be again used to get dynamics for $d\ln V_t$ and from that can be determined parameters of normal distribution for $\ln V_t$.

which is the PD of a company at the time of maturity T expected at time $t = 0$, $(t = T)$, when the value of the firm V_0 is known with certainty.[1] Nearer look at values d_2^* and d_2 discloses that probability of default occurs also in final equation for pricing risky debt (1.5). This comes from the fact that $\Phi(d_2)$ is the probability that the European call option will be exercised by equity holders, and company will not default. The term $1-\Phi(d_2) = \Phi(-d_2)$ then characterize default probability. However, while $\Phi(-d_2^*)$ in (1.9) gives the real-world (physical) probability of default, $\Phi(-d_2)$ presents the default probability in the risk-neutral world. This is caused by using riskless interest rate r instead of expected rate of return m_V in the formula for d_2. In the real world, investors are demanding more than risk-free rate of return and therefore $d_2^* > d_2$ what implies $\Phi(-d_2^*) < \Phi(-d_2)$ and the fact that risk-neutral PD overstates its physical measure. Similarly it has to be distinguished between physical and risk-neutral RR.[2]

The recovery rate, when assuming no liquidation costs after default, will by given by the ratio of firm's value at T to the debt F, (V_T/F). More formally expressed as

$$RR = E\left(\frac{V_T}{F}\middle| V_T < F\right) = \frac{1}{F} E\left(V_T \middle| V_T < F\right) \tag{1.10}$$

as was already mentioned, V is log-normal variable, therefore to get an explicit formula for RR we can use for the method presented in Liu et al. (1997), that derives conditional mean for log-normal distributed variable, what is exactly the case of equation (1.10).

Let's suppose that variable Y is log-normal and $\ln Y$ is normally distributed with mean μ and variance σ^2. Then variable $Z = (\ln Y - \mu)/\sigma$ has a standard normal distribution. The conditional mean of Y, giving $Y < c$, can be then expressed as follows

$$E\left(Y \middle| Y < c\right) = E\left(\exp[s\,Z + m] \middle| \exp[s\,Z + m] < c\right)$$

$$= E\left(\exp[s\,Z + m] \middle| Z < (\ln c - m)/s\right) \tag{1.11}$$

to simplify following expression, let's define

$$g = (\ln c - m)/s \quad \text{and} \quad h = \Phi(g) \tag{1.13}$$

where $\Phi(.)$ is again normal c.d.f. with these notations, the equation (1.11) becomes

$$E\left(Y \middle| Y < c\right) = h^{-1} \int_{-\infty}^{g} \exp[s\,Z + m] (2p)^{-1/2} \exp[-z^2/2] dz$$

$$= \exp[m + s^2/2] h^{-1} \int_{-\infty}^{g} (2p)^{-1/2} \exp[-(z - s)^2/2] dz$$

[1] From (1.9) it can be seen that PD is the function of the distance between current V_0 and the face value of debt F, adjusted by the expected growth of asset m_V relative to its volatility s_V^2. The d_2^* is hence called distance-to-default (DD) and the higher its value is, the lower is PD.

[2] As e.g. Deliandes and Geske (2003) state, risk-neutral default probabilities can serve as an upper bound to physical default probabilities. For recoveries hold reverse relation – the risk-neutral expected recovery rate is less than its physical (real-world) counterpart (see Madan et al. 2006, p. 5).

(1.14) $$= \exp[m + s^2/2] \frac{\Phi\left(\left(\ln c - m\right)/s - s\right)}{\Phi\left(\left(\ln c - m\right)/s\right)}$$

considering the parameters of normal distribution of ln$V$ stated in (1.7), we can write conditional mean of V_T, giving $V_T < F$ as

(1.15) $$E\left(V_T \middle| V_T < F\right) = \exp[m_v^* + s_V^{*2}/2] \frac{\Phi\left(\left(\ln F - m_v^*\right)/s_V^* - s_V^*\right)}{\Phi\left(\left(\ln F - m_v^*\right)/s_V^*\right)}$$

where $m_v^* = \ln V_0 + \left(m_V - 0{,}5 s_V^2\right)T$ and $s_V^{*2} = s_V^2 T$, after substituting and rearranging we get

$$E\left(V_T \middle| V_T < F\right) = \exp\left[\ln V_0 + m_V T\right] \frac{\Phi\left(-\dfrac{\ln\left(V_0/F\right) + \left(m_V + 0{,}5 s_V^2\right)T}{s_V \sqrt{T}}\right)}{\Phi\left(-\dfrac{\ln\left(V_0/F\right) + \left(m_V - 0{,}5 s_V^2\right)T}{s_V \sqrt{T}}\right)}$$

(1.16) $$= V_0 \exp[m_V T] \frac{\Phi(-d_1^*)}{\Phi(-d_2^*)}$$

using this term in equation (1.10) we get final expression for the expected recovery rate at time $t = 0$ in the form

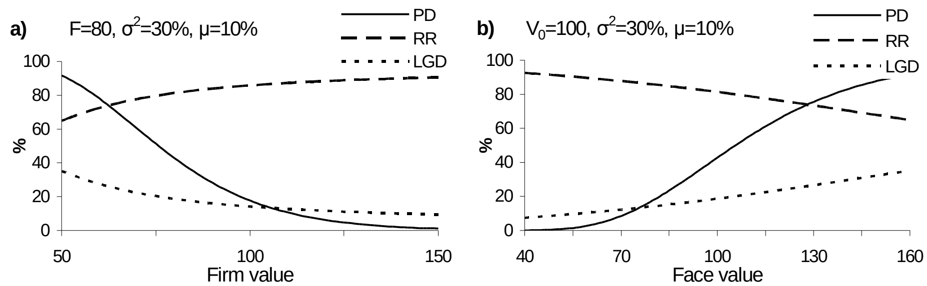
(1.17) $$RR = \frac{1}{F} E\left(V_T \middle| V_T < F\right) = \frac{V_0}{F} \exp[m_V T] \frac{\Phi(-d_1^*)}{\Phi(-d_2^*)}$$

which is the physical recovery rate and the risk-neutral RR would be obtained by replacement m_V with r. RR function is homogenous of degree zero in V_0 and F, which means that proportional change in those variables does not influence its value (ceteris paribus). Moreover, RR is dependent, as the PD, on the uncertain development of firm's value and therefore is not constant through the time but stochastic.

As we could see from the above derived model's dynamics, both PD and RR are simultaneously given from arbitrage-free equilibrium conditions. Using the presented expression for PD and RR, the sensitivity analyses with respect to other company's structural parameters can be made. Consider the firm with given $F = 80$, $V_0 = 100$, $\sigma^2 = 30\%$, $\mu = 10\%$ and T=1. The variables will be shocked to see how PD and RR change.

Figure 1

The sensitivity analysis for PD and RR (LGD) – part 1

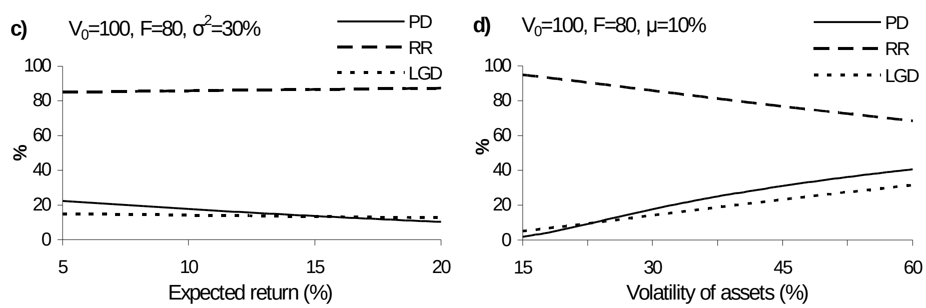


Source: computed from eq. (1.9) and (1.17)

The figure presents results for RR and PD in physical measure. It could be supposed, that the higher is the firm's value at the time of risk parameters prediction, the lower is the expected LGD and lower PD – part a), the linkage is reverse with the value of debt *F*. An increase in firm's leverage brings about higher both PD and LGD. The similar impact also has an increase in assets' volatility (leaving leverage unchanged) which causes higher uncertainty of future firm's value at the maturity *T* and therefore fall in RR.

Figure 2

The sensitivity analysis for PD and RR (LGD) – part 2



Source: computed from eq. (1.9) and (1.17)

In summary, Merton's approach evidently generates the negative correlation between PD and RR because both variables depend on the same firm's structural characteristics. The RR is significantly determined by the value of firm's assets at the maturity time *T*.

However, the original Merton's model does not include any payouts to security holders. Since the interest payouts occur over the life of the debt and they are considerably lower than the principal amount, they represent lower default risk. Their neglecting should not hence bring important bias into our analysis. However, to disregard dividend stream, as Hillegeist et al. (2004) state, could introduce significant errors in estimation of current market value of the firm and its volatility and influence resulting LGD estimate. Therefore is necessary to modify the seminal Merton approach and incorporate into model payout of dividends.

If we denote dividend rate δ as the ratio between the sum of the prior year's common and preferred dividends and the market value of the firm's asset, then the equation for the equity value reflecting the dividend stream paid by the firm accrues to equity holders would change as proposed by Hillegeist et al. (2004) in

$$E(V,T) = V\exp[-dT]\Phi\left(d_1\right) - Fe^{-rT}\Phi\left(d_2\right) + (1-\exp[-dT])V \tag{1.18}$$

where the additional $\exp[-dT]$ in the first term accounts for the reduction in the assets' value due to dividends distributed before maturity T. The last expression $(1-\exp[-dT])V$ does not appear in the traditional equation for call option on a dividend paying stock since dividends do not accrue to option holders. Equation (1.18) is derived under risk-neutral measure, therefore risk-free rate is taken as the expected rate of return on the firm's value. This rate is however lowered by the dividend rate and hence the terms d_1, d_2 have to be modified as

$$d_1 = \frac{\ln(V_0/F) + \left(r - d + 0{,}5s_V^2\right)T}{s_V\sqrt{T}},\ d_2 = d_1 - s_V\sqrt{T}$$

where all parameters were defined above.

♦ Model's implementation

The empirical use of any structural model is based on variables, which are not directly observable. Similarly in our case, the market value of assets V and also asset volatility σ_V, must be estimated in order to compute expected LGD.[1] The procedure for estimation of those variables was firstly proposed by Jones et al. (1984) for publicly listed companies exploiting the prices of their shares. Their approach is based on simultaneous solving two equations, which are matching the value of equity E and its volatility σ_E with two unknown variables V and σ_V. The equity data is generally used since actual daily prices are observable and equity is the firm's most liquid security. Jones et al. (1984) used as the first equation the relation (1.4). Nonetheless, this equation does not consider dividends' payout and we will hence utilize modified equation (1.18). The second equation linking the observable and unknown values is in the form

$$s_E E = s_V \exp[-dT]V\Phi(d_1) \tag{2.1}$$

and its derivation uses the Itô's lemma and is presented in Appendix. This system of two equations has to be solved to arrive at unobservable market value of firm's asset and its volatility. Due to the non-linearity of those equations it is necessary to solve the system iteratively.[2]

The accuracy of the expected LGD estimate is therefore dependent on the estimates of parameters in equation (1.17). Although some of them as the debt's face value [3] or its matur-

[1] The market value of the firm is the sum of the equity and debt's market value. However, the market value of the debt is not usually available since companies are not financed entirely by traded debt.

[2] To solve two non-linear equations of the form $F(x,y)=0$ and $G(x,y)=0$ can be minimized the function $[F(x,y)]^2 + [G(x,y)]^2$ (see Kulkarni et al. 2005).

[3] This holds only if the debt is traded.

ity are observable, some assumptions about them must be made to be able to implement Merton's simplifying approach. For example, the model requires reducing firm's capital structure into one single liability. Since the large share of the firm's debt is not very often traded, we have to use the book values as a proxy. As a result, the book value of total liabilities reported in firms' balance sheets is used as the notional face value of the zero coupon bond. This approach is often used because equity holders earn the residual value of the firm once all debt is paid off (see e.g. Helwege et al. 2004 or Hillegeist et al. 2004).[1]

To determine the maturity time of zero coupon bond representing all firm's liabilities, we could compute the weighted maturity of the individual claims' maturities.[2] However, our intention is to provide LGD comparable across the sample of analyzed companies, which would be hardly practicable in case of different maturities. Therefore we will assume five years debt's maturity for all companies, which should be an assumption considering the length of both short-term and long-term debt's maturity. By setting the longer time horizon we should also avoid inaccuracies coming from the fact that we use for firm's asset value dynamics poor diffusion process without possible jumps.

From our previous discussion is obvious that V and σ_V estimates are highly dependent through the system of two equations on the value of equity and its volatility. While the market value of equity E is simply obtained as the shares' closing price at the end of the fiscal year multiplied by outstanding number of stocks, the value of equity's volatility depends on chosen method of estimation. For that reason it is desirable to use different types of estimation techniques for mutual comparison.

The standard approaches of estimating σ_E can be based on the historical data of stock prices or can exploit bond prices for getting so called implied volatility. Bond implied volatility is acquired when one chooses the asset volatility such that the price generated by our model fits to actual bond's market value.[3] Nevertheless, since this volatility's estimates incorporates all possible errors of used model and also considering our discussion about illiquid and insufficient bond market, we will use only historical approach using the development of stocks' returns.

Let P_i denotes the day i closing price of the stock. Then the continuously compounded one day return r_i is defined as $r_i = lnP_i - lnP_{i-1}$ and the unbiased estimate of the one day volatility using the m observations of the r_i is

[1] Moody's KMV model specifies the notional default point as the book value of short-term liabilities plus half of the value of long-term liabilities (see Crosbie and Bohn 2003). They put a greater weight on short-term obligations because debts due in the near term are more likely to cause a default. However, this approach is probably more convenient in the first-passage time models than in seminal Merton, where the default may occur only at debt's maturity.

[2] Another method widely used among academics is to group the short-term and long-term obligations and find out the maturity by weighting the maturities of those two groups. For example Dalianedis and Geske (2001) made assumption of 1 year maturity for short-term and 10 years for long-term debt. The weights would be the book values of claims.

[3] Similarly, one could get the option-implied volatility for the companies with options written on their stock by using standard Black–Scholes formula for pricing option (see Hull 2002).

(2.2) $$s_E = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}\left(r_i - \bar{r}\right)^2}$$

where $\bar{r}$ denotes the mean of r_i's (see Hull 2003). The appropriate observation interval depends on the time horizon, which we are dealing with. Since we set the maturity time to 5 years, we should also use the long-term volatility for our predictions. From that reason we used volatility of 5-trading years.[1] In addition, to take into account the possible changes in volatility in the shorter time period, we also estimate last 250 trading days' volatility similarly as did e.g. Kulkarni et al. (2005).

The improvement over those traditional methods of volatility estimate that give equal weigh to each observation, is the estimate using the exponentially weighted moving average (EWMA), where more recent observations carry higher weights. This method, capturing better the volatility dynamics, is recommended in RiskMetrics™ (1996) and for a given set of m observations can be exponentially weighted volatility computed as

(2.3) $$s_E = \sqrt{(1-l)\sum_{i=1}^{m} l^{i-1}\left(r_i - \bar{r}\right)^2} \quad 0 < l < 1$$

where λ is referred as the ***decay factor*** that determines the relative weights for particular observation. For our sample of companies we used monthly observations over the five years with decay factor equal to 0,97. This value is based on the analysis relating to optimal λ that was provided in RiskMetrics™ (1996).

The fourth and the last method that we used is GARCH(1,1) from class of ARCH models that consider the fact that variance of time series returns tends over time to revert to its long-run average (see Bollerslev 1986). We estimate GARCH(1,1) model for daily data over the five year interval in the form

(2.4) $$s_t^2 = b + a_1 r_{t-1}^2 + a_2 s_{t-1}^2 \quad a_0 > 0, a_1 \geq 0, a_2 \geq 0$$

where $b = a_0 s_{LR}^2$, s_{LR}^2 represents the long-run unconditional variance of daily returns r and a_0, a_1, a_2 are the weights whose sum is equal to 1. Since we are concentrating on the long-run volatility, we use only the long-run average variance σ^2_{LR} to which the process will convert in the future. The long-run volatility is therefore computed from estimated parameters as

(2.5) $$s_E = \sqrt{\frac{b}{1 - a_0 - a_1}}$$

However, for some companies was not the GARCH's long run volatility estimated as their return's time series was not weakly stationary. The GARCH is also unstable, when fitted parameters $\hat{a}_1 + \hat{a}_2$ are close to 1. This leads to integrated IGARCH(1,1) model with additional constraint $a_1 + a_2 = 1$. However, the unconditional variance σ^2_{LR} is not in this case defined.

[1] In the case of insufficient long time series, we use the longest available one. This holds also for other 5-year estimates computed later in this section.

Nonetheless, as it can be found in Tsay (2005), this special IGARCH(1,1) model can be rewritten as EWMA formula, with that we have already σ_E estimated.

For the most of the companies in our sample we estimated by aforementioned methods four types of daily equity's volatility. Those must be still scaled to obtain the annualized volatility used in later computations.

All estimates are enclosed in Appendix, Table 2. Since the higher volatility of equity results in higher volatility of firm's value and higher default risk, the choice of estimated σ_E can significantly influence further results. However, we consider it more desirable to provide as the rule of prudence rather overstated values of LGD than vice versa. Therefore we use the average of the two highest σ_E estimates, σ_E^* as a parameter entering the system of two equations.

The derived system for obtaining unobservable values of V and σ_V exploits as the firm's expected rate of return the risk-free rate r_f, for which we used the yield of 5-years government bond. Therefore, the last parameter that must be estimated, in order to solve the equations, is dividend rate δ. Nonetheless, for acquiring δ, one needs to get the market value of the firm V. Hence we use the approximate market value V' as the sum of equity's market value E and book value of debt.[1] Since we are estimating 5-year horizon, we will use in computations the adjusted rate δ^* capturing dividend stream in the last five years, instead of one year dividend rate δ.[2]

We solved the two equations simultaneously by the iterative Newton search algorithm. As the starting values for V and σ_V the approximate value V' and volatility of equity were used, respectively. In almost all of the cases, the process converges within ten iterations. Note that the equation linking equity and asset volatility given by (2.1) holds only instantaneously, what causes the bias in V and σ_V estimates when the leverage changes. Crosbie and Bohn (2003) assert that a quick decrease in the leverage would lead to overestimation of asset volatility and vice versa, if the leverage increases. The impact of the change in firm's leverage on ELGD is presented later in the sensitivity analysis section.

Note that dynamics of estimated σ_V follows the equity's volatility σ_E^*, nevertheless, σ_V is always lower than σ_E^*. This is caused by presence of leverage, since the debt is considered as non-traded. With the increase of leverage, the equity occupies a lower share in the overall value of the firm and therefore V is less volatile than E.

For estimate of expected LGD in risk-neutral measure we already know all necessary parameters, however, as the risk-free rate can significantly differ from the real firm's value rate of return, we estimate also the expected market return on assets, μ_V, as the return on assets during previous year. We can easily utilize estimated values of firm's market value V and one-year return μ_V get as

[1] This approach, as Wong and Li (2004) show, overestimates the true market value of the firm.
[2] We used exponentially weighted average with decay factor $\lambda = 0{,}9$.

(2.6) $$m_V(t) = \frac{V(t) + Div(t) - V(t-1)}{V(t-1)}$$

where $V(t)$ is the firm's market value at the end of year t and $Div(t)$ denotes the sum of the common and preferred dividends declared during this year. Since the 5-year expected return will not be solely based on a one year observation only, we use in our calculations adjusted μ_V^* again as the five-year weighted average, in which recent years carry more weight to react faster to current information.

♦ Estimate of LGD in the Czech Republic

We will implement the aforementioned methods on a sample of firms, which are listed on Prague Stock Exchange (PSE) and present dynamics of 5-year expected LGD for each company between 2000 and 2008. We restrict our sample to non-financial firms, so that the leverage ratios could be comparable across them. In addition, we exclude enterprises that become listed after 2007 to obtain at least one year time series of share prices necessary to estimate asset volatility. The list of 27 analyzed companies can be found in Appendix, Table1.

Income statements and the balance-sheet items for our set of PSE corporations were obtained from Magnus (2008) database and for some of them were completed from company's annual reports. Share prices, dividend yields and the number of shares outstanding are available on the web of Prague Stock Exchange.[1] We used time series of share prices from the beginning of 1999 to the end of 2007 and accounting information reported at the end of fiscal year. The series of five year risk-free interest rates comes from ARAD database of Czech National Bank (CNB).

The non-existence of dividends' payouts in the seminal Merton's model was modified in the last section. Still, one should also incorporate the costs of bankruptcy which result that debt holders in the case of default receive less than the total firm value. Additional default costs also arise from deviations in APR where equity holders gain at the expense of bondholders. While Betker (1997) estimated the direct administration costs relating to bankruptcy around 5% of firm value, study by Andrade and Kaplan (1998) indicates higher costs of financial distress in the range of 15–20%. Based on those empirical studies we consider exogenous common bankruptcy costs $(1-\varphi)$ equal to 10%.

The final formula for 5-year expected LGD at the beginning of year t in physical measure, including both dividends payout and bankruptcy cost, is then

(2.7) $$ELGD_t = 1 - j\,\frac{V_t}{F_t}\exp[(m_{V,t}^* - d_t^*)T]\,\frac{\Phi(-d_1^*)}{\Phi(-d_2^*)}$$

[1] The information is also available for the Czech companies in Magnus (2008) database.

$$d_1^* = \frac{\ln(V_t / F_t) + \left((\mu_{V,t}^* - \delta_t^*) + 0,5\sigma_{V,t}^2\right)T}{\sigma_{V,t}\sqrt{T}}, \text{ and } d_2^* = d_1^* - \sigma_{V,t}\sqrt{T}$$

where time indexes represent particular values at the beginning of year t (end of the previous year), and $\mu_{V,t}^*$, δ_t^* denotes adjusted rates considering 5-year historical observations. One can get the expected LGD in risk-neutral measure by replacing $\mu_{V,t}^*$ by r_f.

The results are given in the Figure 3 which presents the expected LGD for each company estimated at the beginning of every year during the period 2000–2008 in both risk-neutral and physical measure. The estimates in physical measure begin from year 2001 since we lost one observation for acquiring firm's growth rate. All parameter used for computations are given in Appendix, Table 2.

In the theoretical framework the risk-neutral LGD is always an upper bound to its physical counterpart. Nevertheless, this holds only if assets drift μ_V is greater than the risk-free rate. In the conventional analysis the r_f rate is supposed to be always lesser than drift μ_V. For example, Hillegeist et al. (2004) compute μ_V for PD estimates and use r_f as a minimal bound for μ_V, since their claim that lower expected growth rates than r_f are inconsistent with asset pricing theory. However, this approach can result in highly underestimated values of LGD if the real growth rate is lower than r_f. This can be demonstrated from given results.

Figure 3

The 5-year expected LGD in the period 2000–2008.

	Expected LGD (%) - risk neutral measure									Expected LGD (%) - physical measure							
Company	2000	2001	2002	2003	2004	2005	2006	2007	2008	2001	2002	2003	2004	2005	2006	2007	2008
CETV	-	-	-	-	-	-	18,0	22,5	21,4	-	-	-	-	-	-	23,1	19,1
Č. NÁM. PLAVBA	28,7	26,1	23,1	24,0	22,2	34,8	16,2	13,7	12,5	23,5	28,6	24,6	22,8	34,7	16,0	12,9	12,6
ČEZ	24,1	27,7	34,4	35,7	35,3	30,7	29,3	29,2	24,1	32,7	47,1	39,3	29,6	21,2	18,1	18,7	16,7
ECM	-	-	-	-	-	-	-	13,8	18,4	-	-	-	-	-	-	-	13,3
ENERGOAQUA	13,0	24,4	37,7	35,7	33,4	22,4	17,8	14,0	13,6	17,2	22,7	20,8	19,6	17,4	14,6	12,4	12,2
JČ PAPÍRNY VĚTŘNÍ	29,2	23,7	26,3	26,5	21,3	32,4	23,1	23,0	33,6	30,3	52,6	33,2	57,9	33,2	13,0	14,1	36,2
JM PLYNÁRENSKÁ	44,7	38,3	34,4	45,6	32,2	23,8	19,4	14,9	11,6	48,2	21,5	27,7	19,5	14,8	17,4	13,4	11,2
LÁZNĚ TEPLICE	17,7	16,4	16,9	15,5	17,0	17,2	16,9	15,9	14,7	49,5	12,6	11,5	12,9	13,7	14,1	14,7	14,9
LEČ. L. JÁCHYMOV	30,1	18,0	25,9	20,4	19,1	16,8	14,0	15,6	13,3	85,8	49,5	19,7	19,0	17,1	13,7	14,9	13,4
ORCO	-	-	-	-	-	-	21,3	22,5	29,5	-	-	-	-	-	-	13,2	16,7
PARAMO	30,4	17,6	16,2	20,5	19,5	23,8	25,0	21,4	22,5	78,4	65,4	44,3	16,5	20,6	19,1	18,7	19,6
PEGAS	-	-	-	-	-	-	-	28,4	19,0	-	-	-	-	-	-	-	20,4
PHILIP MORRIS	-	17,0	25,4	36,9	32,1	31,1	28,9	32,5	41,2	-	15,8	21,7	18,8	20,8	21,0	29,5	41,2
PR. ENERGETIKA	51,5	40,8	42,5	44,0	35,9	28,8	25,1	22,9	20,6	52,7	53,5	40,4	28,5	22,0	18,5	17,4	16,2
PR. PLYNÁREN.	30,2	33,0	34,6	40,3	38,5	36,8	30,9	29,1	26,4	66,7	33,2	36,3	36,4	26,6	19,4	19,2	18,9
PR. SLUŽBY	18,3	25,6	22,2	22,0	20,1	14,4	12,1	11,2	10,7	17,0	22,1	21,9	17,1	13,3	11,7	11,0	10,6
RM-S HOLDING	29,2	34,6	32,7	27,1	34,5	35,4	24,6	12,5	11,4	58,8	50,4	33,4	29,1	31,2	24,5	12,6	11,5
SETUZA	30,0	30,6	28,2	28,0	28,7	29,8	29,8	28,4	27,4	13,3	14,3	17,4	21,3	23,5	31,0	30,7	22,7
SLEZAN FM	26,4	34,4	34,4	32,4	29,4	30,2	25,9	23,1	18,5	88,3	70,4	27,8	23,1	27,9	25,3	23,2	19,8
SM PLYNÁREN.	31,5	25,1	40,6	29,9	33,7	33,7	23,9	21,7	19,2	25,2	36,5	21,1	29,6	33,5	21,5	19,6	17,4
SPOL. CH.H..VÝR.	20,0	16,2	23,0	23,4	24,9	22,4	25,5	22,0	20,0	70,1	37,8	28,1	23,9	15,8	14,5	13,7	13,2
SPOLANA	33,3	33,5	36,1	34,2	35,0	34,9	27,8	27,5	26,6	42,9	76,6	58,5	44,3	45,0	28,9	27,1	30,0
TELEFÓNICA	23,9	32,5	36,7	36,0	33,4	33,3	26,3	22,9	43,4	40,2	49,5	51,7	35,4	32,7	23,0	20,9	37,1
TOMA	29,9	29,1	23,0	23,5	21,0	19,7	23,5	21,4	18,7	67,5	24,2	29,6	18,4	15,6	16,5	15,8	13,4
UNIPETROL	36,1	30,1	26,5	24,8	26,4	29,8	35,0	36,3	31,3	24,0	25,3	23,4	22,1	27,0	18,8	22,3	21,6
VČ PLYNÁRENSKÁ	42,8	34,9	48,6	63,5	56,9	55,3	48,4	49,1	30,7	33,6	33,1	41,1	39,7	42,5	39,0	41,0	28,9
ZENTIVA	-	-	-	-	-	18,6	22,6	22,9	24,0	-	-	-	-	-	15,3	18,7	19,2
Mean (%)	29,6	27,7	30,4	31,4	29,6	28,5	24,5	22,9	22,4	46,0	38,3	30,6	26,6	25,0	19,8	19,2	19,5
Std. Dev. (%)	9,1	7,4	8,4	10,8	9,0	8,9	7,4	8,3	8,7	23,4	18,5	11,7	10,7	9,2	6,5	7,1	8,3

Source: computed from eq. (2.7)

Company Paramo ended year 2000 with a loss counting more than 430 mil. CZK and almost 24% drop in the market firm's value. This negative result has no impact on expected risk-neutral LGD at the beginning of 2001 and its value is even below-average for given year.

However, the physical estimate captures the huge deterioration in firm's asset value which leads to more than four times higher expected LGD. Also Spolana recorded as a result of negative development in the market with plastics in 2001 looses about 700 mil. CZK. Subsequent year was negatively affected by floods which lead to other losses. While risk-neutral LGDs in these years do not incorporate any problem comparing to other years estimates, the physical measure counterparts indicate company's poor performance quite well. The same situation can be also found in the case of Lázně Jáchymov in 2001, Slezan FM in 2001, 2002 or e.g. Papírny Větřní in 2002 and 2004. Contrary to that, when the growth rate of firm's assets μ_V is higher than r_f, risk-neutral estimates overstate ELGD.

The relatively high ELGD in both measures for ČEZ in 2002 might seem contradictory, since ČEZ ended year 2001 successfully with increase in net profit over 26% to more than 9 bill. CZK. However, the share price drops from initial 101 CZK at the end of 2000 to 77,5 CZK at the end of 2001 what lead to more than 23% decrease in the market value of equity. This development together with high dividend rate was reflected in almost 14% deterioration of assets value and lead to significant increase in ELGD. Similarly, high decrease in market value of equity caused the worsening of predictions for Telefónica in 2002 and 2003. Nonetheless, the sharp rise of ELGD in 2008 is solely incurred by rash increase in assets volatility.

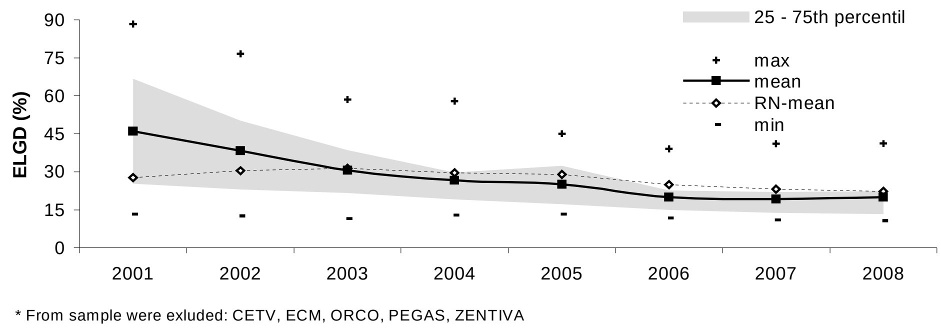
The Figure 4 displays the average ELGD over the period from 2001 to 2008. To provide comparable estimates across time, we excluded companies which were not quoted on PSE during the whole period. The shaded strip covers the quartile range with extending from the 25^{th} to the 75^{th} percentile, which illustrates the variability of ELGD in particular year. From the figure, decreasing trend both in the average physical ELGD and in its variability is also evident. The expected downswing of economic activity due to global and domestic factors (see CNB 2008) was not incorporated enough in the share prices at the end of 2007. Therefore the average ELGD at the beginning of 2008 is relatively small, still capturing good economic development in the recent years. However, expected slowdown in economic growth resulted for some of the analyzed companies drop in the market prices of equity. As a result, the rough average ELGD estimate [1] at the beginning of May 2008 has raised to 24%, which indicates the increase in the credit risk in non-financial corporations sector. A slight increase in the corporate sector's credit risk in 2008 is also indicated by the creditworthiness indicator reported in CNB (2008).[2]

[1] The estimate is using all other parameters constant except market value of equity.

[2] This indicator calculates the outlook for the corporate sector's risk at the one-year forecast horizon based on financial indicators of solvency, profitability, liquidity and activity. More details can be found in Jakubík and Teplý (2008).

Figure 4

Development of average 5-year ELGD in the period 2001–2008



Source: computed from eq. (2.7)

Risk-neutral estimates are based on the same company's structural values relating to its credit risk, as do physical estimates, except different assumptions about expected growth of company's assets. Kulkarni et al. (2005) even state that since risk-neutral estimates can be calculated without estimating the firm's expected return, they may provide more accurate information. Nevertheless, as it was demonstrated, risk-neutral estimates are not properly characterizing the actual company's riskiness. The more μ_V differs from r_f, the more inaccurate results they provide compared to its physical counterpart. Therefore, creditor trying to appraise its possible recovered amount in the case of obligor's default should consider the real future growth rate of firm's assets μ_V, as the main determinant of the future LGD,[1] even if the average values of physical and risk-neutral measures can be almost identical (Figure 4). From this point hence it is more desirable to use the real physical estimates also for the credit management in the Basel II framework.

♦ Sensitivity analysis

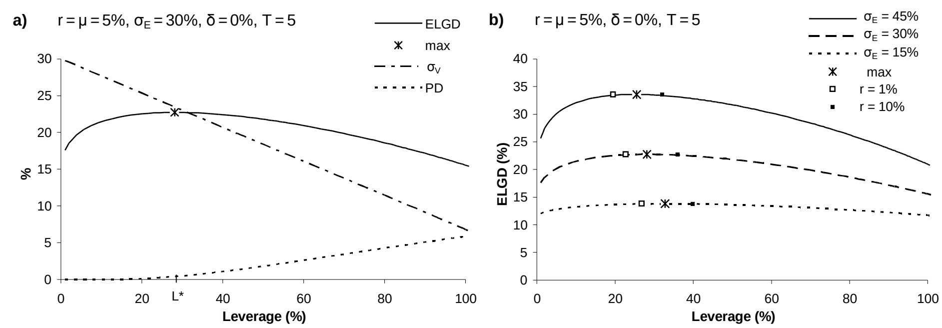
The sensitivity analysis relating to initial Merton's model discussed in the theoretical section assumed that all necessary structural variables are known. However, as was already said, the value of firm's assets and its volatility are not directly observable and they have to be estimated through the system of two equations, which hold only in the given time. Therefore, the following analysis concentrates on sensitivity of ELGD coming out from potential changes in structural variables of the company influencing also the estimates of σ_V and V. The stress is put especially on the leverage, defined as the ratio between total liabilities and market value of all assets (F/V), since this belongs to the mostly watched indicator affecting the company's creditworthiness.

[1] Also the risk-neutral estimates consider changes in markets value of company's asset through the leverage ratio. Still, as we could see, it does not seem to be sufficient.

Before we present the ELGD's sensitivity for individual companies in analyzed sample, we provide a general theoretical discussion based on different scenarios of input parameters. The main difference between the current analysis and the previous one illustrated in Figure 2 is caused by the fact that the change in the leverage influences the estimate of firm's assets volatility σ_V. Thus, by leverage's increase the weight of equity in the firm value declines and the volatility is decreasing. The rate of declining is for a given set of parameters presented in the first part of Figure 5.

Figure 5

The sensitivity analysis for ELGD – part 1



Source: computed from eq. (2.7) and system (1.18), (2.1)

This figure also illustrates the impact of increase in the firm's leverage on the PD and ELGD. However, while the leverage's growth has positive unambiguous effect on PD, the ELGD reaches its maximum value for a particular leverage ratio and then starts to decrease.

The negative relation between ELGD and leverage may look contra-intuitive; nevertheless, this development is caused by the decreasing assets' volatility σ_V.[1] Although the PD is increasing in leverage, the expected value of firm's assets at the maturity T, conditioned by default ($V_T < F$) has increased with respect to given leverage. In other words, due to lower volatility σ_V is less likely that the expected firm's value will be excessively below the value of default barrier F at time T and therefore the expected recovery ratio (V_T/F) in the case of default has increased.

The result is, that by leaving the initial volatility of equity as a constant,[2] the increase in leverage causes the decline in assets volatility, which from particular leverage ratio (L* – breakpoint) generates a negative correlation between PD and ELGD. Nevertheless, for all presented scenarios the increase in PD outweighs the LGD's decline and expected loss for unit of exposure (PD.ELGD) is hence strictly increasing with leverage.

[1] The previous analysis reported in the Figure 2 shows the strictly positive correlation between ELGD and leverage. However, the σ_V was taken as a constant and did not change with leverage.

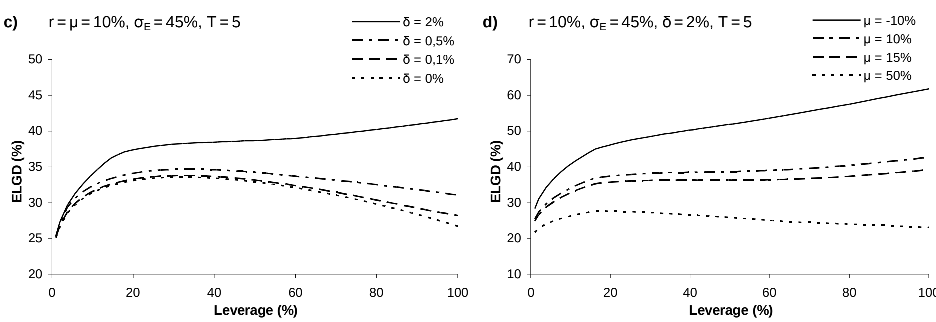
[2] The change in leverage will also affect the equity's volatility. However, since we use the long-run volatility σ_E^* for computation, in which does not the sudden short-term changes take effect; the assumption of constant σ_E in the sensitivity analysis is maintainable.

Pursuing the issue further, we analyze the changes in breakpoints with respect to other parameters. The maximum ELGD points are presented for 3 different values of r_f rate and σ_E. As it can be seen, the decline in the risk-free interest rate shifts the max ELGD points to the left, similarly as the increase in the equity's volatility (Figure 5, b). It is evident that any increase in σ_E will lead (because of higher uncertainty) ceteris paribus to higher values of ELGD. However, the figure also presents the variability of potential ELGDs along the whole range of leverages. While for $\sigma_E = 45\%$ ELGDs vary from 22 to 33 percent, the volatility for $\sigma_E = 30\%$ is only 7 percentage points, and in the case of $\sigma_E = 15\%$ is the variability of possible ELGDs minimal. This further highlights the importance of volatility as a crucial variable for LGD predictions and indicates that the companies with identical leverage ratios can have substantially different ELGD's sensitivity.

The existence of dividend rate in the system of equations lowers the estimated market value of the company *V*, since the part of its value is paid out to the equity holders. Supposing the same value of equity, the presence of dividends increases the estimated assets volatility, compared to the state with zero dividends rate. Thus, dividends offset the initial lowering of σ_V given by increase in leverage, which results in higher ELGD and consequently lower ELGD decrease behind the breakpoint. Moreover, the increase in assets volatility given by sufficiently high dividend rate outweighs the volatility's after breakpoint decline and the overall effect with increase in leverage on ELGD is positive (see Figure 6, c).

Figure 6

The sensitivity analysis for ELGD – part 2



Source: computed from eq. (2.7) and system (1.18), (2.1)

Till now we did not consider any differences between physical and risk-neutral measure in the analysis of ELGD's sensitivity to leverage. Since the real asset growth μ_V does not figure in *V* and σ_V estimation, it may seem that physical ELGD will differ for given set of parameters only in the absolute terms, keeping the same rate of change with respect to leverage. The right-hand side of the Figure 6 displays evolution of ELGD for various growth rates relating to the increasing ELGD's sensitivity curve from previous figure (2% dividend rate). As we can see, the μ_V affects also the slope of ELGD's curve, not only its parallel shift. Bad company's performance represented by small and negative μ_V will raise the rate of growth of ELGD, while good development will offset the presence of the dividend payout and the curve will become decreasing from the breakpoint again. The result is that the ELGD in the physical measure has lower growth rate in the leverage for the $\mu_V > r_f$ and for sufficiently high values of μ_V may by even the initial growth rate from some point inverted from increasing to decreasing (see part d, $\mu_V = 50\%$). This holds also vice versa for low and negative values of μ_V.

The empirical results for the analyzed sample are reported in the following table that shows the leverage elasticity of ELGD in both measures at the beginning of 2008.

As it can be seen, the most of the analyzed companies have inelastic ELGD with respect to leverage. Only Spolek pro chem. a hut. výrobu has negative elasticity slightly exceeding 1. The lowest elasticity in absolute terms belongs to Pražské služby and the highest positive sensitivity of ELGD to 1% increase in leverage has Pražská plynárenská, both in martingale and physical measure.

Based on our previous discussion we can analyze differences in risk-neutral (ε^Q) and physical (ε^P) elasticity with respect to other parameters. For example, CET or Pr. Služby, companies with zero dividend rate and low leverage at the beginning of 2008, are located on the increasing part of their ELGD's sensitivity curve. However, because μ_V lowers ELGD's rate of growth and the expected assets' rate μ_V is for both companies higher than r_f, their "physical" elasticity is lower than ε^Q.

Figure 7

The elasticity of ELGD with respect to Leverage

Company	$e^{ELGD^Q}_{Leverage}$	$e^{ELGD}_{Leverage}$	Company	$e^{ELGD^Q}_{Leverage}$	$e^{ELGD}_{Leverage}$	Company	$e^{ELGD^Q}_{Leverage}$	$e^{ELGD}_{Leverage}$
CETV	0,071	0,022	ORCO	0,344	-0,128	SLEZAN FM	0,432	0,493
Č. NÁM. PLAVBA	0,042	0,045	PARAMO	-0,393	-0,498	SM PLYNÁREN.	0,308	0,228
ČEZ	0,078	-0,034	PEGAS	0,341	0,405	SPOL. CH.HUT.VÝR.	-1,072	-1,095
ECM	-0,607	-0,643	PHILIP MORRIS	0,403	0,403	SPOLANA	-0,647	-0,477
ENERGOAQUA	0,188	0,080	PR. ENERGETIKA	0,268	0,128	TELEFÓNICA	0,175	0,150
JČ PAPÍRNY VĚTŘNÍ	0,116	0,129	PR. PLYNÁREN.	0,856	0,423	TOMA	-0,093	-0,179
JM PLYNÁRENSKÁ	0,198	0,092	PR. SLUŽBY	0,011	0,004	UNIPETROL	-0,025	-0,148
LÁZNĚ TEPLICE	-0,055	-0,047	RM-S HOLDING	0,022	0,024	VČ PLYNÁRENSKÁ	0,271	0,244
LEČ. L. JÁCHYMOV	0,026	0,028	SETUZA	-0,867	-0,890	ZENTIVA	0,012	-0,109

Source: computed from eq. (2.7) and system (1.18), (2.1)

On the contrary, Č. Nám. Plavba or JČ Papírny indicate inverse inequality between ε^P and ε^Q since their $\mu_V < r_f$. Further, if the company's position is at the decreasing part of the sensitivity curve, the high values of expected μ_V will raise the rate of the curve's decline and contrariwise for $\mu_V < r_f$ (ECM, Lázně Teplice, Paramo or Spolana). The dividend payout causes the positive sensitivity behind the breakpoint in the case of JM Plynárenské or Philip Morris, however, ε^P for JM Plynárenská is lowered by $\mu_V > r_f$.[1]

The sensitivity analysis further illustrates already pointed differences between risk-neutral and physical measure. However, the more important finding seems to be that ELGD is quite inelastic in leverage and its sudden changes do not incur significantly high turns in expected LGD. The possible inaccuracies is estimation V and σ_V, mentioned by Crosbie and Bohn

[1] The values of leverage and expected assets' growth are reported in the Table 2 in the Appendix .

(2003), caused by change in leverage might be more relevant for PD estimate, but should not bring important changes to predictions of ELGD. However, the discussion about other limits and shortcomings of presented estimates should be accomplished in more details.

♦ Criticism and limitations

The first implementation of Merton's model applied by Jones et al. (1984), Ogden (1987) or Franks and Torous (1989) suggested that the model generates lower credit spreads than those ones observed on the market do. Similarly more recent studies by Lyden and Saraniti or Helwege et al. (2004) showed that basic Mertonian contingent claim model under predicts actual bond's spread especially for low-leveraged and low-volatility companies. Based on those findings, our ELGD estimates would be undervalued. However, considering the fact that bonds' spreads reflects also market risk, tax or liquidity effects the mentioned studies only confirmed Merton's inability to capture other components of debt's spread, saying nothing about model's ability to reveal default and recovery risk.

This issue can further be confirmed by Longstaff (2000) who has argued that corporate bond markets are much more illiquid than government bonds and stock market and therefore it seems likely that credit spread is only partly attributed to default risk. In spite of these well known complications and imperfections, majority of the literature empirically testing structural models has presumed that the credit spread is primarily attributed to default risk, since the other components are hardly tractable.[1] Sarig and Warga (1989) did not compare absolute values of theoretical corporate bond spreads, but only their rates of change with respect to change in actual bond's default riskiness and approved good predictive power of Merton's model. Further, Dalianedis and Geske (2001) termed the difference between observed and modeled spread the residual spread and empirically confirmed that the spreads estimated with Merton approach correctly evaluates default risk and residual spread is driven by liquidity, tax and other effects.[2] These conclusions move towards the correctness of our LGD estimates, since accuracy of ELGDs is based on the capturing the company's default risk.

If we assume that share prices reflect all relevant information considering future development of the company as well as the expected conditions for given industry or economy, this expectations are also incorporated in our ELGD's, since they are dependent on the development of the stock market. Thus, ELGDs based on market value of equity are forward-looking estimates which may be used to instantaneous watching company's riskiness and may serve as indicator of early-warning. Nevertheless, ELGD's stock market dependence can also embody excessive movements in the share prices caused by market bubbles. Also, the stock market may

[1] This idea stems from the theoretical assumption that markets for corporate bonds are perfect, complete and trading takes place continuously (see Dalianedis and Geske 2001).

[2] Structural models may also understate spreads in short-run, since the pure diffusion process is not able to capture unpredicted extreme changes in firm's asset value given by shock. Therefore is also possible to add jump process to Brownian motion or to model asset value as a discontinuous Lévy process (see Bhatia 2006, p. 126 and references therein).

not efficiently incorporate all publicly-available information about default probability and especially in a case of a young market, such as that one of the Czech Republic, limitations of information given by share prices and particularly by companies, which shares are not so frequently traded, should be considered.[1]

For purposes of Basel II framework, the ELGD's based on equity development are procyclical and due to increase in the minimum required capital in the recession would lead to the credit crunch and contrariwise to the overlending in the time of strong economic growth. The definition of default used in the model corresponds more to the state of bankruptcy than to the obligor's ninety days past due obligation defined under Basel II. Thus, model's definition of default leads to overstated ELGD; however, the analyzed companies should have high capabilities to raise funds. So if the company is past due more than 90 days on its obligation, it has probably exhausted all means to raise the funds and the bankruptcy will follow. The different default definitions hence should not bring significant inaccuracies.

The computations also do not consider any debt's priority, therefore ELGDs for secured and more senior claims should be lower than presented estimates and vice versa higher for subordinated debt, however, the distribution of the value of the bankrupted firm depends also on the violation of APR, what is difficult to predict for single cases. The bankruptcy costs were determined by using other empirical studies, nonetheless, bankruptcy laws and other procedures differ substantially by country, and may therefore differ in the Czech Republic. The calibration on the empirical sample would be needed to obtain more accurate estimates, but the appropriate data sample is not available due to low number of defaults of comparable companies.

The computed ELGDs suffer also by others shortcomings, like the assumption of constant interest rate, no tax shield, and other simplifications coming out from the seminal Mertonian approach. On the other side, more sophisticated models demand higher number of parameters, which have to be estimated. This increases the computation complexity and might therefore produce higher errors. Also, some introduced amendments relating e.g. to stochastic interest rate have unambiguous effects, and sometimes have only little impact on the results (Lyden and Saraniti 2000). Nevertheless, the empirical application of more complex models will be the goal of the further research.

In spite of all mentioned limitations, the presented results are the first estimates of expected LGD based on the market information for single companies listed on Prague Stock Exchange and should therefore serve as the stepping stone for their further improvement. The estimates should not deputize the estimated values of LGD based on historical data, as is requested in Basel II, however, they may serve as the early warning signal and improve thereby the current credit management.

[1] Č. nám. plavba, Energoaqua, Jihomoravská plynárenská, Pražské služby, RM-S HOLDING, SLEZAN FM, or Východočeská plynárenská. Nonetheless, we estimate LGD also for these less liquid companies because our estimates are based on 5-year volatility and we can still acquire some information even if liquidity in one year is low.

♦ Conclusion

Among intensively studied topics in quantitative finance currently belongs also the concept of Loss Given Default, which is a rather unexplored territory in credit risk area. Especially with the implementation of the New Capital Accord, LGD has obtained increased attention and became a frequent object of empirical and theoretical research. The goal of this paper was to present the basic pieces of knowledge concerning this key input parameter of credit risk analysis and primarily to introduce modeling technique which enables estimation of forward-looking LGDs from market observable data.

We exploited the information embedded in the stock market and utilized Mertonian structural approach based on the contingent claim analysis, which as the recovered amount in the case of default considers the rest value of firm's assets. This demonstrates that LGD is even in the initial Merton's framework stochastic since it depends on uncertain development of assets' value. We also pointed out the joint dependence between PD and LGD which implies that those parameters should not be in the credit risk modeling treated as independent.

We analyzed companies listed on Prague Stock Exchange in the 2000–2008 period and computed expected LGD for every single company at given year. The average LGD of the sample across the time was estimated in the range around 20–45%. We also described the estimation procedures exploiting prices of equity and its volatility and showed that LGD is relatively inelastic in leverage of the company. We also demonstrated that LGD in the physical measure is a more reliable indicator than its risk-neutral counterpart.

The concept of implied market LGD may serve as early warning system in the current credit management and especially provides information about possible recovery rate of low-default facilities with insufficient historical data sample concerning experienced LGD.

The paper dealt with LGD's properties, possible modeling technique, and its estimates from market data, respectively. As the main value added of this work are the unique estimates of LGD for the Czech corporate sector. This altogether should bring a perspective on LGD and provide better understanding of difficulties related to this credit risk parameter.

Appendix

Companies listed on PSE

Table 1

Companies listed on Prague Stock Exchange (PSE) at the beginning of June 2008

Name of the company	Abbreviation	ISIN	Observed years
AAA Auto Group N.V. [1)]	AAA	NL0006033375	-
CENTRAL EUROPEAN MEDIA ENTERPRISES LTD.	CETV	BMG200452024	2005 - 2007
Česká námořní plavba, a.s.	Č. NÁM. PLAVBA	CZ0008413556	1999 - 2007
ČEZ, a.s.	ČEZ	CZ0005112300	1999 - 2007
ECM REAL ESTATE INVESTMENTS A.G.	ECM	LU0259919230	2006 - 2007
Energoaqua, a.s.	ENERGOAQUA	CS0008419750	1999 - 2007
Erste Bank der oesterreichischen Sparkassen AG [2)]	ERSTE BANK	AT0000652011	-
Jihočeské papírny, a.s. Větřní	JČ PAPÍRNY VĚTŘNÍ	CZ0005005850	1999 - 2007
Jihomoravská plynárenská, a.s.	JM PLYNÁRENSKÁ	CZ0005078956	1999 - 2007
Komerční banka, a.s. [2)]	KOMERČNÍ BANKA	CZ0008019106	-
Lázně Teplice v Čechách, a.s.	LÁZNĚ TEPLICE	CS0008422853	1999 - 2007
Léčebné Lázně Jáchymov, a.s	LEČ. L. JÁCHYMOV	CS0008446753	1999 - 2007
New World Resources N.V. [1)]	NWR	NL0006282204	-
ORCO PROPERTY GROUP S.A.	ORCO	LU0122624777	2005 - 2007
PARAMO, a.s.	PARAMO	CZ0005091355	1999 - 2007
PEGAS NONWOVENS SA	PEGAS	LU0275164910	2005 - 2007
Philip Morris ČR a.s.	PHILIP MORRIS	CS0008418869	2000 - 2007
Pražská energetika, a.s.	PR. ENERGETIKA	CZ0005078154	1999 - 2007
Pražská plynárenská, a.s.	PR. PLYNÁREN.	CZ0005084350	1999 - 2007
Pražské služby, a.s.	PR. SLUŽBY	CZ0009055158	1999 - 2007
RM-S HOLDING, a.s.	RM-S HOLDING	CS0008416251	1999 - 2007
SETUZA, a.s.	SETUZA	CZ0008460052	1999 - 2007
SLEZAN Frýdek-Místek, a.s.	SLEZAN FM	CZ0005018259	1999 - 2007
Severomoravská plynárenská, a.s.	SM PLYNÁREN.	CZ0005084459	1999 - 2007
SPOLANA, a.s.	SPOLANA	CS0008424958	1999 - 2007
SPOLEK PRO CHEM.A HUT.VÝR.,a.s	SPOL. CH.HUT.VÝR.	CZ0005092858	1999 - 2007
Telefónica O2 Czech Republic,a.s.	TELEFÓNICA	CZ0009093209	1999 - 2007
TOMA, a.s.	TOMA	CZ0005088559	1999 - 2007
UNIPETROL, a.s.	UNIPETROL	CZ0009091500	1999 - 2007
Východočeská plynárenská,a.s.	VČ PLYNÁRENSKÁ	CZ0005092551	1999 - 2007
VGP NV [1)]	VGP	BE0003878957	-
VIENNA INSURANCE GROUP [1)]	VIG	AT0000908504	-
ZENTIVA N.V.	ZENTIVA	NL0000405173	2004 - 2007

[1)] Firm was excluded - insufficient long time series (issud after 2. 1. 2007)

[2)] Firm was excluded - financial institution

Source: Prague Stock Exchange (www.pse.cz)

The derivation of equation (2.1)

Let's assume that the dynamics of equity value can be described by the stochastic differential equation $dE_t = (m_E - d\ )E_t dt + s_E E_t dW_t^E$, where m_E is the equity drift, s_E is the standard deviation of equity's return, and dW_t^E is a standard Gauss–Wiener process. Dynamics of security, whose market value is at any time t a function of the value of the firm and time, i.e. $E_t = f(V_t, t)$, can be expressed by using Itô's Lemma in terms of V_t as follows

$$dE_t = \left(\frac{\partial E_t}{\partial t} + (m_V - d_V) V_t \frac{\partial E_t}{\partial V_t} + \frac{1}{2} s_V^2 V_t^2 \frac{\partial^2 E_t}{\partial V_t^2} \right) dt + \frac{\partial E_t}{\partial V_t} s_V V_t dW_t^V$$

Comparing corresponding variables in two equations above, we get $s_E E_t = \frac{\partial E_t}{\partial V_t} s_V V_t$,

and since $\frac{\partial E_t}{\partial V_t} = \Phi(d_1)$ [1] (see Helwege et al. 2004), then holds $s_E E_t = \Phi(d_1) s_V V_t$. After considering our discussion about dividend rate that lowers the value of *V*, we get the relation presented in equation (2.1) $s_E E = s_V \exp[-dT] V \Phi(d_1)$ .Estimated parameters

Table 2

All relevant parameters for the sample of analyzed companies

(see the next page)

[1] This relation is also called option (equity) delta (see Hull 2002).

Table 2 - part 1

		Estimates of equity's volatility					Parameters used for ELGD computation						Other used parameters				
Company	End of year	σ_E MA(5y) (%)	σ_E MA(1y) (%)	σ_E EWMA (%)	σ_E GARCH (%)	σ^*_E (%)	r_f (%)	μ^* (%)	δ^* (%)	σ_V (%)	V (bill. CZK)	F (bill. CZK)	δ (%)	μ (%)	Leverage (%)	V´ (bill. CZK)	Equity (bill. CZK)
CETV	2005	22,7	22,7	21,9	22,8	22,7	3,1	-	0,0	17,5	62,94	16,99	0,0	-	27,0	65,35	48,36
	2006	28,2	30,7	27,5	28,7	29,7	3,3	1,2	0,0	23,4	63,69	15,91	0,0	1,2	25,0	66,09	50,18
	2007	28,4	28,6	27,5	28,7	28,7	4,0	17,6	0,0	25,0	82,73	12,74	0,0	29,9	15,4	85,02	72,28
Č. NÁM. PLAVBA	1999	37,8	37,8	33,2	39,3	38,6	6,7	-	0,0	32,3	0,09	0,02	0,0	-	22,9	0,10	0,08
	2000	35,4	32,8	28,4	38,4	36,9	6,8	20,7	0,0	34,2	0,11	0,01	0,0	20,7	10,5	0,12	0,10
	2001	32,2	24,4	23,7	36,7	34,4	4,8	-19,6	0,0	33,0	0,06	0,00	0,0	-49,9	5,5	0,06	0,05
	2002	29,7	20,0	19,2	36,4	33,0	3,2	0,7	0,0	29,7	0,07	0,01	0,0	27,4	11,9	0,07	0,06
	2003	26,6	4,9	12,7	38,3	32,5	3,8	0,2	0,0	30,8	0,07	0,00	0,0	-0,6	6,2	0,07	0,07
	2004	20,5	0,0	6,9	73,5	47,0	3,4	3,6	0,0	40,5	0,08	0,01	0,0	10,6	16,6	0,08	0,07
	2005	16,4	18,0	6,8	21,5	19,7	3,1	4,4	0,0	15,4	0,09	0,02	0,0	9,9	25,9	0,09	0,07
	2006	13,1	9,6	4,7	16,8	15,0	3,3	13,0	0,0	12,0	0,10	0,02	0,0	17,8	23,3	0,11	0,08
	2007	10,3	9,5	4,5	16,2	13,2	4,0	2,3	0,0	12,2	0,09	0,01	0,0	-16,0	9,6	0,09	0,08
ČEZ	1999	35,6	35,6	31,2	39,0	37,3	6,7	-	0,0	17,9	113,76	84,34	0,0	-	74,1	136,83	52,49
	2000	36,0	36,4	33,1	36,7	36,5	6,8	-0,2	0,5	21,1	112,36	81,61	0,8	-0,2	72,6	141,49	59,87
	2001	38,1	41,7	35,2	41,9	41,8	4,8	-8,0	0,8	24,6	95,30	76,45	1,2	-13,9	80,2	122,37	45,92
	2002	36,9	33,3	34,0	39,9	38,4	3,2	-1,3	1,3	26,2	99,83	69,54	2,1	7,5	69,7	124,30	54,76
	2003	34,2	20,9	29,1	41,1	37,7	3,8	14,6	1,8	28,4	137,20	78,22	2,9	42,2	57,0	164,51	86,29
	2004	32,8	27,6	28,1	38,3	35,5	3,4	41,5	2,0	30,2	264,57	79,22	1,9	96,7	29,9	280,99	201,77
	2005	31,9	32,1	28,4	36,4	34,2	3,1	67,1	1,9	30,3	546,00	132,92	1,6	109,7	24,3	568,96	436,04
	2006	29,0	29,6	27,4	39,6	34,6	3,3	61,5	1,9	30,8	701,40	161,00	1,6	30,6	23,0	729,52	568,52
	2007	27,7	27,0	26,7	29,4	28,6	4,0	57,9	2,0	26,9	942,99	169,56	2,4	37,8	18,0	976,15	806,59
ECM	2006	16,0	16,0	16,0	15,0	16,0	3,3	-	0,0	8,7	11,16	5,98	0,0	-	53,6	12,07	6,09
	2007	25,8	26,3	23,6	33,5	29,9	4,0	26,6	0,0	11,1	14,13	10,91	0,0	26,6	77,3	16,03	5,12
ENERGOAQUA	1999	21,5	21,5	15,1	19,6	21,5	6,7	-	0,0	4,4	0,24	0,26	0,0	-	110,9	0,31	0,05
	2000	31,0	37,9	23,3	31,4	34,7	6,8	35,1	0,0	20,2	0,32	0,19	0,0	35,1	59,2	0,37	0,18
	2001	27,2	17,1	16,6	26,5	26,8	4,8	39,1	14,1	26,8	0,33	0,15	32,6	42,2	43,6	0,38	0,23
	2002	25,9	21,5	15,2	25,6	25,8	3,2	41,8	11,3	25,8	0,45	0,20	6,4	45,2	43,6	0,55	0,35
	2003	23,9	12,9	11,9	23,4	23,6	3,8	40,5	8,2	23,6	0,61	0,32	1,8	38,2	52,6	0,76	0,45
	2004	22,2	9,7	8,4	21,6	21,9	3,4	29,1	7,9	21,9	0,61	0,13	5,2	5,8	20,9	0,67	0,54
	2005	15,0	10,8	5,8	20,2	17,6	3,1	29,5	7,3	17,6	0,76	0,13	4,2	31,7	17,3	0,83	0,70
	2006	14,2	13,2	4,9	13,5	13,9	3,3	34,3	3,9	13,9	1,09	0,14	3,0	47,2	12,9	1,16	1,02
	2007	10,8	5,6	3,3	11,7	11,2	4,0	23,3	2,4	11,1	1,12	0,29	0,0	3,5	25,5	1,27	0,98
JČ PAPÍRNY VĚTŘNÍ	1999	44,8	44,8	41,6	45,2	45,0	6,7	-	0,0	22,9	0,43	0,30	0,0	-	71,3	0,51	0,20
	2000	41,1	37,0	35,3	40,7	40,9	6,8	-0,9	0,0	15,7	0,42	0,38	0,0	-0,9	89,1	0,53	0,15
	2001	43,4	47,8	39,0	44,5	46,1	4,8	-15,6	0,0	17,7	0,31	0,25	0,0	-26,7	81,9	0,36	0,11
	2002	40,0	27,2	31,4	40,9	40,4	3,2	-6,1	0,0	20,4	0,33	0,20	0,0	6,4	59,6	0,35	0,16
	2003	39,8	38,6	30,2	42,0	40,9	3,8	-18,0	0,0	12,3	0,20	0,18	0,0	-38,6	86,8	0,23	0,06
	2004	42,0	53,5	35,7	45,7	49,6	3,4	2,2	0,0	26,1	0,29	0,17	0,0	43,5	59,1	0,31	0,14
	2005	45,6	54,2	41,7	49,3	51,8	3,1	43,0	0,0	12,5	0,65	0,61	0,0	126,1	92,5	0,74	0,13
	2006	45,8	48,7	42,9	51,8	50,2	3,3	30,6	0,0	12,7	0,58	0,53	0,0	-11,5	92,2	0,66	0,12
	2007	50,9	56,7	47,6	51,3	54,0	4,0	-8,0	0,0	53,1	0,04	0,00	0,0	-92,8	2,0	0,04	0,04
JM PLYNÁRENSKÁ	1999	32,9	32,9	26,2	32,2	32,9	6,7	-	1,4	25,7	5,41	7,55	1,4	-	139,7	9,78	2,23
	2000	24,1	8,5	13,4	n.a.	18,8	6,8	11,0	1,3	18,8	5,29	7,78	1,2	11,0	146,9	10,43	2,65
	2001	20,5	9,6	9,3	n.a.	15,0	4,8	16,5	1,9	15,0	6,68	8,69	2,8	20,7	130,1	11,90	3,21
	2002	21,4	23,9	11,2	n.a.	22,7	3,2	22,0	2,6	22,7	8,14	10,58	3,6	29,2	130,0	14,52	3,94
	2003	19,1	0,2	7,1	n.a.	13,1	3,8	14,1	3,0	13,1	8,19	9,60	3,9	0,3	117,2	13,53	3,93
	2004	12,1	0,0	3,4	n.a.	7,8	3,4	9,6	3,8	7,8	8,05	8,41	5,0	0,3	104,6	12,35	3,93
	2005	13,7	16,4	5,1	n.a.	15,0	3,1	10,7	5,5	15,0	7,08	2,74	8,4	13,5	38,7	9,79	7,05
	2006	13,4	6,9	3,9	n.a.	10,1	3,3	13,4	6,9	10,1	7,77	2,92	9,1	21,2	37,5	10,69	7,77
	2007	8,1	0,0	1,9	n.a.	5,0	4,0	11,3	6,8	5,0	7,77	3,55	6,0	10,8	45,7	11,33	7,77
LÁZNĚ TEPLICE	1999	27,0	27,0	23,1	29,7	28,3	6,7	-	0,0	10,5	0,13	0,12	0,0	-	88,3	0,17	0,05
	2000	24,3	21,3	18,2	24,4	24,3	6,8	-15,0	0,0	9,8	0,11	0,10	0,0	-15,0	84,0	0,14	0,05
	2001	25,8	28,7	19,8	25,7	27,3	4,8	26,7	0,0	9,5	0,18	0,15	0,0	58,0	83,2	0,21	0,06
	2002	23,5	14,3	15,7	23,3	23,4	3,2	36,1	0,0	8,4	0,27	0,20	0,0	48,5	75,0	0,30	0,10
	2003	23,5	23,3	16,4	23,4	23,5	3,8	30,6	0,0	11,7	0,32	0,20	0,0	21,2	60,7	0,36	0,16
	2004	21,6	17,8	14,1	23,4	22,5	3,4	24,7	0,0	13,0	0,36	0,18	0,0	12,5	50,0	0,39	0,21
	2005	20,6	15,5	11,2	22,6	21,6	3,1	19,7	0,0	13,3	0,36	0,16	0,0	0,0	44,8	0,39	0,22
	2006	17,0	11,9	8,0	21,5	19,2	3,3	10,8	0,0	13,4	0,37	0,13	0,0	1,6	35,8	0,39	0,26
	2007	16,0	6,2	5,9	18,1	17,1	4,0	3,1	0,0	11,2	0,36	0,15	0,0	-3,8	41,8	0,38	0,23
LEČ. L. JÁCHYMOV	1999	42,8	42,8	35,3	42,0	42,8	6,7	-	0,0	26,3	0,58	0,32	0,0	-	55,4	0,67	0,35
	2000	33,9	20,5	18,7	n.a.	27,2	6,8	-41,3	0,0	11,7	0,34	0,28	0,0	-41,3	80,6	0,42	0,15
	2001	30,3	21,3	17,7	48,2	39,2	4,8	-18,0	0,0	20,1	0,34	0,22	0,0	-0,5	63,7	0,38	0,17
	2002	28,0	19,5	15,0	28,1	28,1	3,2	5,3	0,0	16,6	0,46	0,22	0,0	35,8	48,2	0,49	0,27
	2003	25,8	13,3	12,2	25,6	25,7	3,8	4,2	0,0	15,4	0,47	0,23	0,0	2,4	48,3	0,51	0,28
	2004	17,8	9,9	7,4	23,9	20,9	3,4	2,1	0,0	14,3	0,46	0,17	0,0	-2,3	37,4	0,49	0,32
	2005	14,9	0,0	4,7	16,3	15,6	3,1	5,0	0,0	10,6	0,47	0,17	0,0	0,8	37,5	0,49	0,32
	2006	13,4	15,7	4,1	21,4	18,6	3,3	7,9	0,0	13,9	0,52	0,16	0,0	12,4	29,8	0,55	0,39
	2007	11,1	9,8	3,8	17,1	14,1	4,0	3,4	0,0	11,5	0,53	0,12	0,0	0,9	22,7	0,55	0,43
ORCO	2005	21,0	21,0	19,6	35,9	28,5	3,1	-	0,0	18,9	29,57	11,58	0,0	-	39,2	31,18	19,60
	2006	25,8	29,6	24,6	31,7	30,7	3,3	76,4	0,2	18,1	51,91	26,72	0,4	76,4	51,5	56,57	29,86
	2007	26,7	28,1	25,6	28,8	28,4	4,0	34,4	0,4	17,0	53,07	52,69	0,5	3,0	99,3	76,15	23,46
PARAMO	1999	50,1	50,1	48,8	49,9	50,1	6,7	-	0,0	22,4	2,68	2,18	0,0	-	81,5	3,27	1,09
	2000	46,1	40,9	40,3	51,3	48,7	6,8	-24,3	0,0	7,0	2,03	2,51	0,0	-24,3	123,7	2,74	0,24
	2001	40,9	27,4	32,7	42,7	41,8	4,8	-17,1	0,0	6,1	1,79	1,98	0,0	-11,6	110,7	2,21	0,23
	2002	39,1	33,1	28,3	44,0	41,5	3,2	-11,7	0,0	11,1	1,71	1,51	0,0	-4,7	88,2	1,91	0,41
	2003	37,1	27,0	24,6	39,7	38,4	3,8	9,6	0,0	10,5	2,50	2,24	0,0	46,5	89,6	2,87	0,63
	2004	32,2	28,2	22,8	39,3	35,8	3,4	11,3	0,0	18,2	2,87	1,69	0,0	15,0	58,9	3,12	1,42
	2005	33,0	45,5	29,6	39,2	42,4	3,1	16,7	0,0	17,0	3,42	2,47	0,0	19,1	72,0	3,74	1,28
	2006	34,2	33,8	30,6	35,8	35,0	3,3	9,5	0,0	14,2	3,01	2,14	0,0	-12,0	71,1	3,32	1,18
	2007	33,2	27,6	28,8	34,8	34,0	4,0	11,2	0,0	16,6	3,35	2,12	0,0	11,2	63,2	3,71	1,60
PEGAS	2006	28,6	28,6	28,6	25,3	28,6	3,3	-	1,7	23,3	9,63	4,78	1,7	-	49,6	11,73	6,95
	2007	20,9	20,6	20,2	20,6	20,7	4,0	0,3	0,7	15,4	9,66	4,47	0,0	0,3	46,3	11,40	6,93
PHILIP MORRIS	2000	13,8	13,8	10,5	12,5	13,8	6,8	-	12,4	13,8	12,74	3,46	12,4	-	27,2	14,47	11,01
	2001	23,3	24,0	20,3	n.a.	23,7	4,8	63,8	15,5	23,7	17,46	3,21	17,9	63,8	18,4	19,06	15,85
	2002	29,2	34,5	26,8	33,3	33,9	3,2	60,7	15,4	33,9	23,69	4,69	15,3	58,5	19,8	26,03	21,34
	2003	28,3	26,2	25,8	29,5	28,9	3,8	61,0	14,0	28,9	33,90	7,61	11,5	61,4	22,4	37,70	30,10
	2004	28,9	31,0	26,9	29,4	30,2	3,4	44,9	13,2	30,2	35,24	6,27	11,5	16,9	17,8	38,37	32,10
	2005	29,0	28,4	26,9	29,1	29,1	3,1	35,7	11,3	29,1	38,14	6,42	7,4	16,9	16,8	41,34	34,93
	2006	30,6	32,0	27,6	29,9	31,3	3,3	10,6	9,2	31,3	23,39	5,28	6,3	-34,4	22,6	26,03	20,74
	2007	28,6	25,0	26,0	29,9	29,3	4,0	3,9	8,6	29,3	21,37	12,38	8,8	1,7	57,9	27,56	15,18
PR. ENERGETIKA	1999	50,1	50,1	44,0	48,3	50,1	6,7	-	4,7	47,8	6,77	3,24	4,7	-	47,9	7,88	4,64
	2000	38,1	19,5	26,5	28,5	33,3	6,8	-4,4	4,7	33,3	5,86	3,52	4,7	-4,4	60,0	7,86	4,35
	2001	33,0	19,0	20,9	26,7	29,9	4,8	-0,3	5,0	29,9	5,12	3,87	5,5	2,9	75,6	7,79	3,92
	2002	33,5	35,0	21,0	31,0	34,3	3,2	7,0	6,2	34,3	5,92	3,22	8,3	16,6	54,4	7,79	4,57
	2003	30,6	14,3	17,2	25,3	28,0	3,8	21,6	6,4	28,0	7,27	3,56	6,8	46,8	49,0	10,16	6,60
	2004	21,8	14,7	12,7	22,9	22,4	3,4	22,5	6,5	22,4	8,23	3,64	6,3	24,5	44,2	11,52	7,88
	2005	21,8	19,3	13,8	24,7	23,2	3,1	30,5	6,5	23,2	11,31	3,08	5,9	40,4	27,3	14,32	11,24
	2006	21,1	14,7	12,6	18,9	20,0	3,3	25,4	7,6	20,0	10,91	3,20	9,8	8,3	29,4	14,06	10,86
	2007	16,7	19,8	13,5	20,1	20,0	4,0	29,3	5,1	20,0	15,05	3,67	0,0	35,4	24,4	18,71	15,04
PR. PLYNÁREN.	1999	28,0	28,0	19,5	28,1	28,1	6,7	-	1,1	21,2	3,63	3,06	1,1	-	84,1	5,36	2,30
	2000	22,0	13,4	12,2	20,0	21,0	6,8	-4,7	1,0	19,2	3,01	3,48	0,9	-4,7	115,5	5,34	1,86
	2001	21,0	18,9	12,1	19,6	20,3	4,8	7,4	0,9	18,9	3,64	4,21	0,8	16,4	115,7	6,24	2,03
	2002	22,4	26,1	12,1	21,0	24,3	3,2	5,7	1,8	23,7	3,78	3,90	3,2	3,6	103,2	6,04	2,14
	2003	20,8	12,3	9,5	19,4	20,1	3,8	6,5	2,8	20,1	3,57	3,99	4,8	8,0	111,5	6,12	2,13
	2004	16,8	6,8	6,5	17,8	17,3	3,4	11,9	3,2	17,3	4,34	4,88	3,8	22,9	112,5	7,23	2,35
	2005	17,0	14,3	6,2	14,6	15,8	3,1	20,2	3,0	15,8	5,38	5,14	2,0	33,3	95,5	8,77	3,64
	2006	16,2	15,0	5,4	16,1	16,1	3,3	21,9	3,1	16,1	6,21	5,37	2,7	24,6	86,5	10,14	4,77
	2007	11,7	7,2	3,8	13,9	12,8	4,0	16,0	2,1	12,8	6,16	6,24	0,0	-0,4	101,3	10,54	4,31

Table 2 - part 2

Company	End of year	Estimates of equity's volatility					Parameters used for ELGD computation						Other used parameters				
		σ_E MA(5y) (%)	σ_E MA(1y) (%)	σ_E EWMA (%)	σ_E GARCH (%)	σ^*_E (%)	r_f (%)	μ^* (%)	δ^* (%)	σ_V (%)	V (bill. CZK)	F (bill. CZK)	δ (%)	μ (%)	Leverage (%)	V´ (bill. CZK)	Equity (bill. CZK)
PR. SLUŽBY	1999	27,2	27,2	24,1	28,4	27,8	6,7	-	0,0	11,9	0,72	0,58	0,0	-	80,4	0,88	0,30
	2000	33,2	38,3	28,6	33,8	36,1	6,8	44,4	0,0	21,9	1,04	0,58	0,0	44,4	56,0	1,20	0,62
	2001	30,0	22,2	22,3	29,5	29,8	4,8	5,4	0,0	25,8	0,79	0,13	0,0	-23,8	17,0	0,82	0,69
	2002	28,4	22,5	19,5	30,2	29,3	3,2	3,5	0,0	25,1	0,80	0,13	0,0	0,9	16,9	0,82	0,68
	2003	27,3	22,2	16,9	28,7	28,0	3,8	30,4	0,0	25,8	1,42	0,13	0,0	77,1	9,3	1,44	1,31
	2004	24,4	0,0	10,3	n.a.	17,3	3,4	20,8	0,0	15,8	1,43	0,15	0,0	1,3	10,5	1,46	1,31
	2005	17,4	0,0	5,3	n.a.	11,4	3,1	14,8	0,0	9,6	1,55	0,28	0,0	7,9	18,1	1,59	1,31
	2006	14,2	0,0	2,8	n.a.	8,5	3,3	13,1	0,0	7,1	1,56	0,29	0,0	0,6	18,9	1,60	1,31
	2007	10,1	3,0	1,4	n.a.	6,6	4,0	10,6	0,0	5,5	1,59	0,31	0,0	2,4	19,6	1,65	1,34
RM-S HOLDING	1999	38,9	38,9	34,0	39,5	39,2	6,7	-	0,5	36,5	3,47	0,46	0,5	-	13,3	3,60	3,14
	2000	41,7	44,4	31,7	47,4	45,9	6,8	-33,6	0,2	39,1	2,30	0,52	0,0	-33,6	22,5	2,45	1,93
	2001	36,9	24,6	25,8	38,8	37,8	4,8	-26,5	2,4	36,2	1,72	0,35	5,2	-21,2	20,2	1,80	1,45
	2002	33,0	16,7	19,8	35,2	34,1	3,2	-15,6	1,5	33,2	1,70	0,20	0,0	-1,2	11,7	1,73	1,53
	2003	30,2	14,6	15,3	32,9	31,6	3,8	16,7	22,2	31,6	1,73	0,25	64,6	72,6	14,5	1,86	1,61
	2004	26,2	19,7	13,9	30,7	28,4	3,4	10,9	16,6	28,4	1,72	0,46	0,0	-1,0	26,7	1,95	1,49
	2005	18,2	13,1	11,0	25,9	22,0	3,1	3,2	12,4	22,0	1,32	0,29	0,0	-23,1	21,8	1,46	1,18
	2006	14,5	0,0	6,5	16,8	15,6	3,3	0,6	8,9	15,6	1,18	0,01	0,0	-10,4	1,0	1,19	1,18
	2007	12,6	5,1	4,5	13,8	13,2	4,0	-1,1	6,7	13,2	1,12	0,00	0,0	-5,2	0,4	1,12	1,12
SETUZA	1999	37,7	37,7	29,3	37,7	37,7	6,7	-	0,0	18,7	1,41	1,49	0,0	-	105,6	1,82	0,33
	2000	37,0	36,2	25,7	36,5	36,7	6,8	87,4	0,0	18,0	2,64	3,06	0,0	87,4	115,6	3,51	0,46
	2001	33,2	23,7	23,2	33,7	33,4	4,8	44,4	0,0	14,7	2,96	3,38	0,0	12,1	114,2	3,68	0,30
	2002	33,2	33,4	23,9	39,2	36,3	3,2	21,3	0,0	14,0	2,70	2,92	0,0	-9,0	108,4	3,12	0,20
	2003	32,2	27,7	21,5	40,0	36,1	3,8	13,8	0,0	15,1	2,72	2,93	0,0	0,8	107,9	3,21	0,28
	2004	28,4	15,5	17,5	41,2	34,8	3,4	12,1	0,0	17,2	2,95	2,91	0,0	8,6	98,5	3,39	0,48
	2005	24,4	16,5	14,5	38,9	31,6	3,1	1,7	0,0	18,0	2,90	2,66	0,0	-1,6	91,7	3,27	0,61
	2006	23,8	20,7	13,8	27,4	25,6	3,3	0,7	0,0	17,8	2,94	2,56	0,0	1,2	87,2	3,32	0,76
	2007	19,0	9,5	9,7	28,0	23,5	4,0	10,7	0,0	16,0	3,78	3,63	0,0	28,8	95,9	4,43	0,80
SLEZAN FM	1999	21,4	21,4	14,9	22,1	21,7	6,7	-	0,0	13,0	0,72	0,92	0,0	-	127,2	0,98	0,06
	2000	16,9	10,6	8,4	18,7	17,8	6,8	-32,0	0,2	10,6	0,49	0,87	0,4	-32,0	179,0	0,92	0,05
	2001	19,7	23,5	10,4	19,9	21,7	4,8	-16,1	0,4	16,2	0,46	0,62	0,6	-4,1	133,4	0,77	0,15
	2002	22,5	29,5	14,7	30,8	30,2	3,2	8,5	0,5	18,5	0,65	0,65	0,5	40,8	100,0	0,92	0,27
	2003	20,9	12,7	10,9	30,8	25,9	3,8	12,8	0,5	17,3	0,77	0,73	0,5	20,3	94,8	1,12	0,39
	2004	19,6	12,8	9,2	28,1	23,9	3,4	5,7	0,4	16,3	0,70	0,73	0,1	-8,8	103,9	1,05	0,32
	2005	20,8	19,6	10,9	n.a.	20,2	3,1	3,7	0,3	14,0	0,64	0,61	0,1	-9,4	96,1	0,94	0,33
	2006	17,8	0,0	6,8	n.a.	12,3	3,3	3,2	0,2	12,0	0,64	0,61	0,1	0,4	96,4	0,94	0,33
	2007	12,0	0,0	3,5	n.a.	7,7	4,0	2,7	0,1	7,7	0,71	0,75	0,0	10,6	106,3	1,08	0,33
SM PLYNÁREN.	1999	28,1	28,1	18,3	28,4	28,3	6,7	-	0,8	21,2	4,96	4,60	0,8	-	92,6	7,26	2,66
	2000	22,2	13,9	12,3	21,0	21,6	6,8	6,7	0,3	15,8	5,30	5,12	0,0	6,7	96,6	7,86	2,74
	2001	25,8	32,0	17,1	35,3	33,6	4,8	9,3	1,2	24,8	5,69	6,30	2,3	11,2	110,7	8,84	2,54
	2002	26,4	28,0	17,0	36,2	32,1	3,2	22,1	0,7	20,8	7,90	5,75	0,0	38,8	72,7	10,09	4,34
	2003	23,7	3,5	11,1	45,6	34,6	3,8	10,5	1,4	25,4	6,89	4,51	2,9	-9,6	65,4	8,70	4,20
	2004	21,0	14,2	8,8	27,2	24,1	3,4	3,5	2,4	24,1	5,82	3,95	4,2	-10,8	67,8	7,80	3,85
	2005	21,2	15,1	8,6	26,4	23,8	3,1	12,0	4,2	23,8	7,02	1,67	7,3	30,3	23,8	7,85	6,18
	2006	17,5	17,6	8,2	24,8	21,2	3,3	11,6	5,8	21,2	7,08	1,62	8,7	10,6	22,9	7,89	6,27
	2007	12,5	4,9	5,2	22,8	17,6	4,0	12,0	8,1	17,6	7,48	1,76	11,5	19,2	23,6	8,36	6,60
SPOL. CH.HUT.VÝR.	1999	47,3	47,3	44,4	46,6	47,3	6,7	-	0,0	9,7	1,43	1,63	0,0	-	114,6	1,88	0,25
	2000	41,3	34,2	36,9	41,0	41,2	6,8	-18,1	0,0	6,1	1,17	1,42	0,0	-18,1	121,7	1,57	0,15
	2001	41,5	41,5	38,0	41,4	41,5	4,8	-8,0	0,0	14,5	1,16	0,99	0,0	-0,4	85,5	1,37	0,37
	2002	41,7	42,3	37,2	42,5	42,4	3,2	-2,4	0,0	14,8	1,22	0,96	0,0	5,0	78,8	1,35	0,39
	2003	39,3	28,0	31,5	40,4	39,9	3,8	5,5	0,0	17,9	1,45	0,99	0,0	19,1	68,2	1,61	0,62
	2004	35,7	30,1	28,3	40,9	38,3	3,4	23,6	0,0	14,5	2,34	1,76	0,0	60,8	75,1	2,59	0,83
	2005	39,8	52,3	35,4	43,8	48,1	3,1	45,1	0,0	16,2	4,19	3,39	0,0	79,3	80,9	4,64	1,25
	2006	39,1	37,8	35,5	46,4	42,7	3,3	34,8	0,0	12,8	4,31	3,67	0,0	2,8	85,2	4,84	1,16
	2007	36,7	30,1	32,9	40,4	38,5	4,0	30,4	0,0	11,1	4,92	4,38	0,0	14,2	88,9	5,68	1,30
SPOLANA	1999	44,0	44,0	39,4	44,4	44,2	6,7	-	0,0	18,4	5,29	6,79	0,0	-	128,5	7,20	0,40
	2000	39,4	34,2	31,8	40,1	39,8	6,8	-0,7	0,0	19,0	5,25	6,59	0,0	-0,7	125,5	7,13	0,54
	2001	40,2	41,7	34,2	40,5	41,1	4,8	-26,3	0,0	23,0	2,86	2,94	0,0	-45,5	102,6	3,48	0,54
	2002	40,5	41,7	32,9	43,3	42,5	3,2	-13,8	0,0	19,8	2,94	3,05	0,0	2,7	103,7	3,37	0,33
	2003	37,3	19,2	24,8	39,9	38,6	3,8	-4,5	0,0	21,7	3,28	3,26	0,0	11,4	99,4	3,82	0,56
	2004	33,7	25,9	21,7	38,9	36,3	3,4	-5,8	0,0	22,2	3,00	2,82	0,0	-8,4	94,0	3,42	0,60
	2005	37,8	50,9	29,7	51,7	51,3	3,1	1,6	0,0	18,3	3,54	2,80	0,0	18,0	79,2	3,91	1,11
	2006	36,2	34,4	28,9	62,2	49,2	3,3	3,9	0,0	18,6	3,45	2,67	0,0	-2,5	77,3	3,83	1,16
	2007	32,5	21,9	24,9	57,0	44,8	4,0	-0,5	0,0	18,7	3,11	2,29	0,0	-10,0	73,8	3,49	1,20
TELEFÓNICA	1999	31,9	31,9	28,7	32,0	32,0	6,7	-	0,0	26,8	221,55	49,96	0,0	-	22,6	235,65	185,68
	2000	38,2	43,8	36,4	39,9	41,9	6,8	-10,4	0,7	36,2	196,08	45,58	1,2	-10,4	23,2	208,95	163,36
	2001	43,2	51,8	42,9	44,4	48,1	4,8	-18,5	0,4	39,1	147,99	38,95	0,0	-24,5	26,3	155,71	116,76
	2002	42,8	41,7	41,6	43,3	43,1	3,2	-18,0	6,2	32,5	103,75	34,19	16,4	-17,4	33,0	113,01	78,82
	2003	41,8	37,6	39,1	42,3	42,0	3,8	0,5	5,4	28,1	132,06	55,46	3,7	32,6	42,0	149,28	93,82
	2004	40,7	24,0	34,0	40,3	40,5	3,4	4,8	4,0	33,3	149,91	38,74	0,0	13,5	25,8	157,65	118,92
	2005	36,6	17,8	26,6	n.a.	31,6	3,1	16,4	5,3	29,1	190,22	29,24	7,3	36,6	15,4	198,17	168,94
	2006	30,1	22,0	22,8	n.a.	26,4	3,3	13,1	6,9	23,5	168,01	29,40	8,8	-3,2	17,5	182,71	153,31
	2007	25,0	18,2	19,9	80,4	52,7	4,0	20,8	6,5	48,6	200,88	30,76	7,8	29,1	15,3	206,23	175,47
TOMA	1999	28,1	28,1	22,7	27,6	28,1	6,7	-	0,0	20,1	0,22	0,20	0,0	-	94,9	0,27	0,07
	2000	26,3	24,4	21,7	25,9	26,1	6,8	-21,1	0,0	19,5	0,17	0,16	0,0	-21,1	95,0	0,22	0,05
	2001	32,1	41,4	28,9	31,7	36,8	4,8	2,9	0,0	16,2	0,21	0,15	0,0	20,9	72,8	0,24	0,09
	2002	30,9	27,0	24,9	31,6	31,3	3,2	-13,3	0,0	25,8	0,13	0,03	0,0	-34,5	20,6	0,14	0,11
	2003	29,1	20,0	20,5	29,9	29,5	3,8	24,4	0,0	27,3	0,26	0,02	0,0	89,7	8,9	0,26	0,24
	2004	29,3	28,8	21,6	31,2	30,3	3,4	64,0	0,0	29,4	0,63	0,02	0,0	145,3	3,3	0,63	0,61
	2005	29,6	26,4	20,8	32,8	31,2	3,1	54,4	0,0	25,8	0,72	0,15	0,0	14,4	20,3	0,74	0,59
	2006	24,6	18,4	16,9	31,4	28,0	3,3	41,4	0,0	21,7	0,76	0,20	0,0	6,8	26,7	0,80	0,59
	2007	22,9	18,4	16,3	25,8	24,4	4,0	48,6	0,0	16,1	1,11	0,46	0,0	45,5	41,6	1,20	0,73
UNIPETROL	1999	47,5	47,5	42,7	55,0	51,3	6,7	-	0,0	32,7	15,23	8,09	0,0	-	53,1	17,36	9,27
	2000	41,4	34,1	36,6	44,1	42,7	6,8	23,2	0,0	26,5	18,76	10,32	0,0	23,2	55,0	21,59	11,27
	2001	40,2	37,7	36,2	42,4	41,3	4,8	7,0	0,0	20,0	17,80	12,06	0,0	-5,1	67,8	20,23	8,17
	2002	41,1	44,0	37,9	43,5	43,8	3,2	5,4	0,0	16,4	18,38	13,99	0,0	3,3	76,1	20,26	6,27
	2003	38,5	25,2	32,9	40,7	39,6	3,8	14,1	0,0	20,9	23,73	13,88	0,0	29,1	58,5	25,93	12,05
	2004	33,8	23,2	27,1	47,0	40,4	3,4	11,0	0,0	29,3	24,88	8,23	0,0	4,8	33,1	26,04	17,81
	2005	38,6	53,7	33,8	47,9	50,8	3,1	71,9	0,0	30,4	74,49	36,75	0,0	199,4	49,3	78,91	42,16
	2006	37,8	33,8	31,6	64,8	51,3	3,3	52,0	0,0	33,0	69,26	30,75	0,0	-7,0	44,4	73,23	42,49
	2007	34,2	24,8	28,2	50,0	42,1	4,0	44,4	0,0	32,0	81,23	24,00	0,0	17,3	29,5	85,22	61,22
VČ PLYNÁRENSKÁ	1999	25,3	25,3	21,5	23,8	25,3	6,7	-	2,7	25,3	1,97	2,42	2,7	-	122,4	3,61	1,20
	2000	18,4	5,6	10,7	16,4	17,4	6,8	9,4	1,9	17,4	2,07	2,73	1,4	9,4	131,8	3,94	1,21
	2001	20,9	25,1	11,8	19,2	23,0	4,8	15,6	4,0	18,0	2,35	3,44	6,7	20,1	146,1	4,51	1,08
	2002	48,8	90,7	23,9	51,4	71,1	3,2	37,7	4,2	54,5	4,42	3,28	4,5	66,8	74,3	5,45	2,17
	2003	44,2	15,7	18,1	48,4	46,3	3,8	23,4	4,9	42,7	4,00	3,44	6,3	-1,4	85,9	5,52	2,08
	2004	42,8	6,9	12,2	43,2	43,0	3,4	18,1	5,6	43,0	3,96	2,84	6,5	7,2	71,8	5,13	2,29
	2005	43,1	13,5	11,8	49,8	46,4	3,1	19,5	5,7	46,4	4,52	1,45	5,0	20,4	32,0	4,83	3,38
	2006	42,0	11,7	9,1	59,1	50,5	3,3	20,1	5,5	50,5	5,20	1,30	5,3	21,4	24,9	5,42	4,13
	2007	11,1	0,0	3,9	49,7	30,4	4,0	9,3	6,6	30,4	4,53	1,08	8,7	-1,6	23,8	5,20	4,13
ZENTIVA	2004	24,1	24,1	23,2	25,2	24,7	3,4	-	1,0	24,4	30,70	2,14	1,0	-	7,0	31,03	28,89
	2005	27,7	29,4	25,6	28,0	28,7	3,1	68,2	0,8	25,2	51,28	9,29	0,7	68,2	18,1	52,61	43,32
	2006	28,3	29,2	25,4	30,8	30,0	3,3	32,3	0,8	28,2	53,59	6,17	0,8	5,4	11,5	54,53	48,36
	2007	29,6	32,4	9,7	29,8	31,1	4,0	21,0	0,5	21,0	56,84	24,97	0,0	6,1	43,9	62,04	37,07

Source: author's computation, Magnus (2008), Prague Stock Exchange

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