On a priori evaluation of power of veto

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Introduction

Existing constitutional design of any democratic country must favour a consensus among the society it addresses. As it is pointed out by John Rawls (Rawls, 1987):

> "In a constitutional democracy one of its most important aims is presenting a political conception of justice that can not only provide a shared public basis for the justification of political and social institutions but also helps ensure stability from one generation to the next. … such a basis must be I think, even when moderated by skilful constitutional design, a mere modus vivendi, dependent on a fortuitous conjunction of contingencies. What is needed … thereby specifying the aims the constitution is to achieve and the limits it must respect. In addition, this political conception needs to be such that there is some hope of its gaining the support of an overlapping consensus, that is a consensus in which it is affirmed by the opposing religious, philosophical and moral doctrines likely to thrive over generation in a more or less just constitutional democracy, where the criterion of justice is that political conception itself."

Therefore, relations between all constitutional and government organs must be moderated and evaluated depending on their way of decision making. Montesquieu's tripartite of power may be realized via different attributes of involved sides. Among those attributes one may find the right to veto. We think, that priori veto is rather strengthening the position of beholder. So, any considerations about consensus process must include evaluation of veto attribute as well, for to preserve the balance between sides.

The main goal of the paper is the evaluation of power connected with veto attribute of the decision maker. In certain cases, it is possible to calculate a value of power of veto attributed to the decision maker and to give the exact value of the power index as well. In other cases, it is only possible to compare the situation with and without veto attribute. Actually, significant numbers of power indices are in use for evaluation of power of player. The main differences between these indices are the ways in which coalition members share the final outcome of their cooperation and the kind of coalition players choose to form. The latter started from negation of equiprobability (Laplace criterion) of possible coalition transformation into a winning coalition (as it was done in Owen, 1977) and consequently it leads to different assumptions and results. A special kind of action attributed to some players is the right to veto, i.e. to stop the action of others permanently or temporarily. In this paper we would like to analyse the power of a player with a right to veto, expecting that the difference between the power of player with veto and his power without veto allows us to evaluate directly or indirectly the power of veto itself.

Veto

The meaning of veto can be explained by the following artificial example: $\{2; 1_a, 1_b, 1_c\}$ where the voting is a majority voting (voting quota equals 2) and weights of all voters *a, b, c* (*N=3*) are equal and fixed at 1. As it can be seen, there are four winning coalitions: {a,b}, {a,c}, {b,c}, {a,b,c}. The first three coalitions are vulnerable and the veto (called the veto of the first degree) of any coalition's members transforms it from winning into non-winning one.

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The classical example of such a veto is a possible veto of permanent members of the Security Council of United Nations.

The last coalition, {a,b,c}, is different: a single member's veto can be overruled by two other members. This type of veto is called the second degree veto. A very typical example of such a veto is a presidential veto, which under certain circumstances can be overruled.

We also believe, that any real life decision process of legislation can be framed with the above simple games and, for example, associations between "a" and "house of representatives", "b" and "senate" and "c" and "president" are well-founded. In that case, "a" can represent a sub-game, for which weight 1 means that a bill has passed through the House of Representatives, and so on. It is also noticed, that "president" is a single player game, as 1_c in our example. Then, when analysing a bill passing through a legislative way, we are in fact analysing a supra-game where players are games on different levels. This lets us to use standard power indices which must be modified for certain conditions and assumptions.

Power index of veto

Before we start the evaluation of power of veto, we need to define a standard power concept and a power index consequently. Let $N = \{1, 2, ..., n\}$ be the set of players (individuals, parties) and ω_i $(i = 1, ..., n)$ be the (real, non-negative) weight of the *i-th* agent and τ be the total sum of weights of all players. Let γ be a real number such as $0 < \gamma < \tau$ (minimal sum of weights necessary to approve a proposal). The $(n+1)$-tuple $[\gamma,\boldsymbol{\omega}] = [\gamma; \omega_1, \omega_2, ..., \omega_n]$ such that

$$\sum_{i=1}^{n} \omega_i = \tau, \omega_i \geq 0,\ 0 < \gamma \leq \tau\ ,$$

we call a weighted voting body of the size $n = card\ \{N\}$ with quota γ, total weight τ and allocation of weights $\boldsymbol{\omega} = (\omega_1, \omega_2, ..., \omega_n)$. We assume that each player i casts all his votes either as "yes" votes, or as "no" votes. Any non-empty subset of players $S \subseteq N$ we shall call a voting coalition. Given an allocation ω and a quota γ, we shall say that $S \subseteq N$ is a winning voting coalition, if $\sum_{i \in S} \omega_i \geq \gamma$ and a losing voting coalition, if $\sum_{i \in S} \omega_i < \gamma$. Let

$$T = \left[(\gamma,\boldsymbol{\omega}) \in \mathrm{R}_{n+1} : \sum_{i=1}^{n} \omega_i = \tau, \omega_i \geq 0, 0 \leq \gamma \leq \tau \right]$$

be the space of all coalitions of the size n, total weight τ and quota γ.

Most of measures of power are designed to evaluate a priori power of players in a setting that is not structured by any rules except a voting rule. A power index (as it is formulated in Turnovec et al. 2008) is a vector valued function $\boldsymbol{\Pi} : T \rightarrow R_n^+$ that maps the space T of all coalitions of the size n into a non-negative quadrant of R_n. A power index for each of the coalition players represents a "reasonable expectation" that it will be "decisive" in the sense that its vote (YES or NO) will determine the final outcome of voting.

Generally, there are two properties, related to the positions in voting of the coalition players, which are being used as a starting point for quantification of an a priori voting power: swing position and pivotal position of a coalition player.

Let $(i_1, i_2, ..., i_n)$ be a permutation of players of the committee, and let player k be in position r in this permutation, i.e. $k = i_r$. We shall say that a player k of the coalition is in a pivotal situation (has a pivot) with respect to a permutation $(i_1, i_2, ..., i_n)$, if

$$\sum_{j=1}^{r}\omega_{i_j} \geq \gamma \quad and \quad \sum_{j=1}^{r}\omega_{i_j} - \omega_{i_r} < \gamma$$

The most known pivotal position based on the a priori power measure was introduced by Shapley (1953) and Shapley and Shubik (1954), and is called SS-power (SS-index). The Shapley-Shubik index is therefore defined as follows:

$$SS(\gamma, \boldsymbol{\omega}) = \frac{p_i}{n!}$$

where p_i is the number of pivotal positions of the coalition player *i* and *n!* is the number of permutations of all coalition players (number of different strict orderings).

The most known swing position based on the a priori power measure is called Penrose-Banzhaf power index (Penrose, 1946; Banzhaf, 1965, 1968). Let *S* be a winning configuration in a coalition [γ, **ω**] and $i \in S$. We say that a member *i* has a swing in configuration *S* if

$$\sum_{k\varepsilon S}\omega_k \geq \gamma \quad and \quad \sum_{k\varepsilon S\backslash\{i\}}\omega_k < \gamma$$

Let s_i denote the total number of swings of the member *i* in the committee [γ, **ω**]. Then PB-power index of player *i* is defined as

$$PB_i(\gamma, \boldsymbol{\omega}) = \frac{s_i}{\sum_{k\in N} s_k}$$

This form is usually called a relative PB-index. Original Penrose definition of power of the member *i* was

$$PB_i^{abs}(\gamma, \omega) = \frac{s_i}{2^{n-1}},$$

which is nothing else but the probability that a given member will be decisive (probability to have a swing). This form is usually called an absolute PB-index.

It is clear that both swing and pivotal attempts are connected with veto. If we assume that coalition is already formed, and one of its member defeats (swings), then this is equivalent to veto (the first or the second degree). So, possible power of veto should be rather measured somehow with participation of power index based on swings. The PB power index is the first one which can be used for it, but as far as we do not know which power index is better fitted for veto measuring, we still keep in minds other power indices.

Johnston power index

Other a priori indices are also in use depending on a situation when voting is taken. Among them there are: the Coleman coalition prevent index (1971) – *CP(i)*; the Coleman action initiation index (1971) – *CI(i)*; the Coleman group capacity index (power to act) (1971) – *C(A)*; the Rae index (1969) – *R(i)*; the Zipke index (Nevison, 1979) – *Z(i)*; the Brams-Lake index (1978) – *BL(i)*; the Deegan-Packel index (1979) – *DP(i)*; the Holler index (1982) – *H(i)*; and the Johnston index (1978) – *J(i)*. The last one is suggested by Brams (Brams, 1990) as the best suited (as swing type power index) for veto analysis and it will be presented more detailed below.

Definition: a winning coalition is vulnerable if, among its members, there is at least one in swing position, whose swing would cause the coalition to lose. Such a member is called critical. If only one player is critical, then this player is uniquely powerful in the coalition.

Defining Johnston power index, first we count number of players being in swing position in vulnerable coalition *c*. Reciprocal of number of swings in coalition *c* is the share of *i*-th member swings in critical coalition *c* in total number of swings in critical coalition *c*, *f(c)*. For example, if there are only two such players in the coalition *c*, thus *f(c)=1/2*.

The Johnston power of player *i* is the sum of the reciprocal of number of swings in vulnerable coalition *c* in which *i* is critical, divided by the total number of reciprocal of number of swings in vulnerable coalition *c* of all players, or *i*'s proportion of reciprocal of number of swings in coalition *c*.

Formally, if $V \subseteq S$ is a coalition of players, for each coalition $c \in V$, we define the set $f_i(c)$

$$f_i(c) = \begin{cases} f(c) & \textbf{\textit{i is swing in c}} \\ 0 & \textbf{\textit{otherwise}} \end{cases} ,$$

And Johnston power index:

$$J(i) = \frac{\sum_{c \in V} f_i(c)}{\sum_{j=1}^{n} \sum_{c \in V} f_j(c)}$$

Let us consider the following example (Mercik, 2009): game [4; 3, 2, 1], i.e. voting where there are three voters with 3, 2 and 1 votes each. Majority for the decision is 4 (quota). There are the following vulnerable coalitions in this game: (3, 2), (3, 1) and (3, 2, 1) (vulnerable coalitions must be winning coalitions).

Vulnerable coalitions	Number of vulnerable coalitions	Critical swings			Reciprocal of number of critical swings		
		3 votes player	2 votes player	1 vote player	3 votes player	2 votes player	1 vote player
(3, 2)	1	1	1	0	½	½	0
(3, 1)	1	1	0	1	½	0	½
(3, 2, 1)	1	1	0	0	1	0	0
Total	3	3	1	1	2	½	½
J(i)					4/6	1/6	1/6

Tab. 1 the example of Johnston power index for the game [4; 3, 2, 1] (Source: Mercik, 2009).

It can be noticed that value (4/6, 1/6, 1/6) of Johnston power index in this example differs from Penrose-Banzhaf power index (3/5, 1/5, 1/5) and it is equal to Shapley-Shubik power index (4/6, 1/6, 1/6). Probabilistic interpretation of Johnston power index (a probability that a certain player is in swing position) shows that Johnston power index belongs to the family of "swing" indices. Shapley-Shubik index describes probability that a certain player is in pivotal position, so SS index belongs to the "pivotal" family of indices.

Evaluation of power of veto of the first degree

Let us remind that veto of the first degree means that the veto can not be overruled. It means that any winning coalition must include a veto empowered player or such a player should stop to use his/her veto. Consequently, it means that probabilities that coalitions will occur, are not equal. In the literature, such attempt is commonly recognizable as games with the pre-coalition structure (Owen, 1977) and priori power indices must be modified respectively.

As it was previously mentioned, a good example of decisive body where some (at least) members are empowered with veto attribute is the UN Security Council. Veto of one of the permanent members of the UN Security Council (i.e. China, France, Russia, the United

Kingdom and the United States) can not be overruled, so this is a typical example of the veto of the first degree. The rest, ten non-permanent members of the Council have no right to veto resolutions of the Council. In table 2, there are shown the results of calculation of different power indices for the UN Security Council. It can be noticed that veto attributed to a permanent member of the UN Security Council makes them from 10 (based on swing type index) to 103 times more powerful (based on pivotal type index) – this is an indirect estimation of power of veto of permanent members.

Power index	Value	Power ratio P_j/N_j
SS power index		
$SS(P_j)$	0.1963	
$SS(N_j)$	0.0019	103.85
Penrose-Banzhaf power index		
$PB(P_j)$	0.1669	
$PB(N_j)$	0.0165	10.09
Coleman Power to prevent action index CP(i)		
$CP(P_j)$	1.0000	
$CP(N_j)$	0.0990	10.10
Coleman Power to initiate action index CI(i)		
$CI(P_j)$	0.0266	
$CI(N_j)$	0.0026	10.23

Tab. 2 Values of different power indices for the UN Security Council (P_j stands for permanent members, N_j stands for non-permanent members).

Analysing legislative way, it can be noticed that some decisions are made sequentially. Traditional power measures use the notion of a composite voting game introduced in order to deal with inter-body decision-making (see for example Felsenthal and Machover, 1998). At each stage players make their decisions and final result is a consequence of all of them. At fig. 1 and 2 we symbolically illustrate this process.

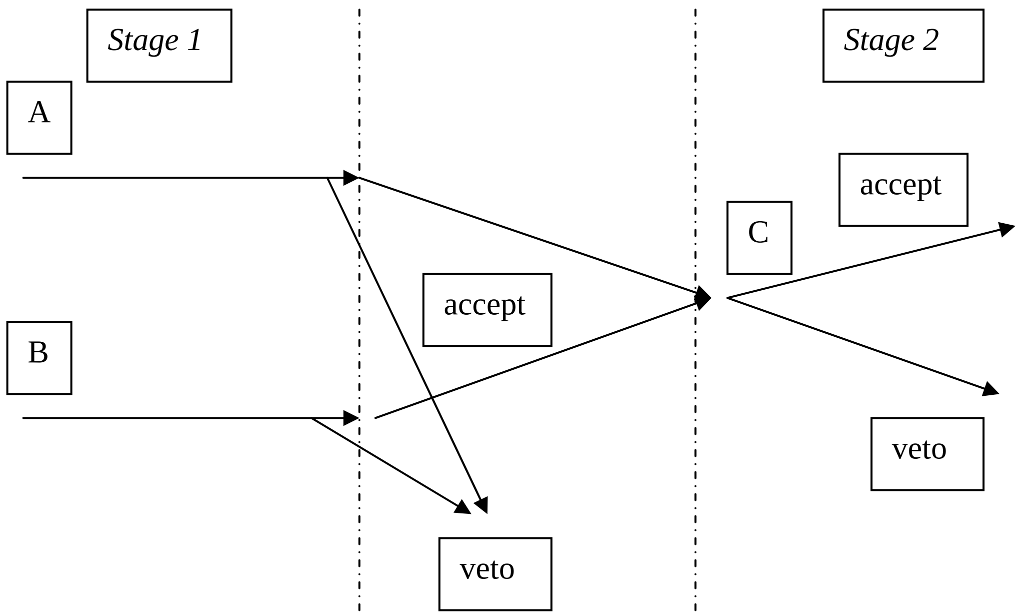

Fig. 1 Two-stage process of decision making with two parallel decision-makers (A and B) and one final decision-maker.

If A, B and C are single players (not bodies) as presented in fig. 1, estimation of a priori power of veto depends on logic of parallel part of the process: when both A and B must accept before C accepts, then value of power of veto is equal for A, B and C each. When only one of A and B must accept before C accepts, all veto power belongs to C. Whichever power index is in use, it must fulfil already mentioned conditions.

It is also clear, that when at least one of the A, B and C players is a decisive body, the veto power has to be distributed among members of the decisive body respectively. In that case, we may use one of the “swing” type indices and evaluate veto part of the power by comparing power of a certain player with and without veto attribute.

If A, B and C are single players as presented in fig. 2, each of the decision-makers has the same veto power. For decisive bodies, marked with A, B and C, the standard “swing” type index can be used and all veto power has to be distributed among members of a decisive body.

Any other veto of the first degree cases is the combination of those two already analysed examples and veto power index can be evaluated in the same way, i.e. by comparing power with and without veto.

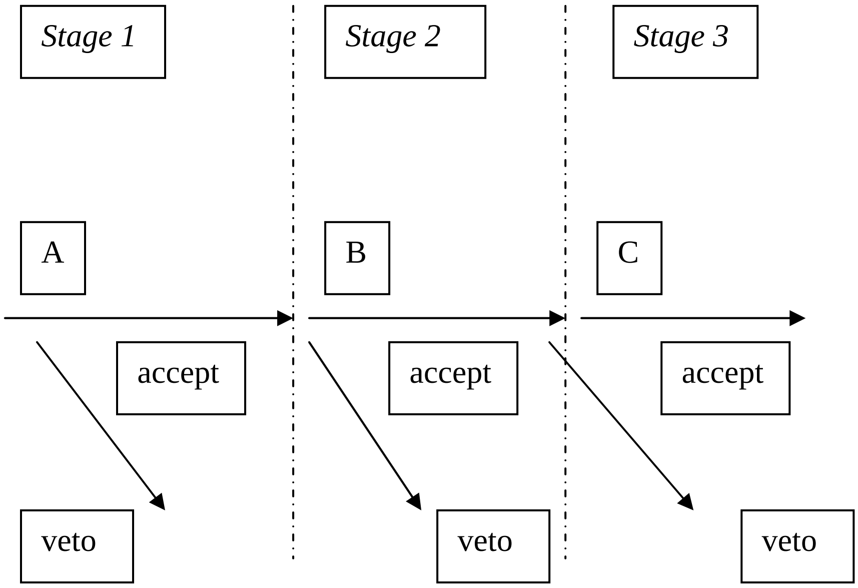

Fig. 2 Three-stages process of decision making with three serial decision-makers (A, B and C).

Evaluation of power of veto of the second degree

Possibility of overruling the veto (veto of the second degree) changes the situation drastically: the same decision maker may accept and/or veto at least more than once. It can be seen that inter-body decision is made and positions of decision-makers are different, depending on their ability to overrule a veto.

Let us have a look at the following example. In Poland, in the process of legislation, any bill accepted by the *Sejm* (lower chamber of the Polish parliament – 460 representatives) is considered by the *Senate* (100 senators), which may accept, amend or reject a bill. If a bill is amended or rejected by the *Senate*, then it comes back to the *Sejm*. The *Sejm* may, by absolute majority, reject the Senate's objection. After that, a bill accepted by the *Sejm* comes to the President of Poland who can accept and sign a bill within 21 days or may declare veto and send a bill back to the *Sejm*.

The presidential veto is considered as a cognizable attribute of the president regarding any bill resolved by the parliament. According to the Constitutional Act, the president signs and declares a bill in the official monitor (gazette). In the case of important state interests or poor quality of constituted law, the president may reject a bill. Presidential rejection of a bill (veto) has a conditional character: the *Sejm* may accept a bill once more by a majority of 3/5 of votes in the presence of at least half of the members of the *Sejm* (representatives). In this case, the president has to sign a bill within seven days and publish the bill in the official monitor. The real effectiveness of the president's veto is therefore strongly subordinated to the present structure of parties in the *Sejm*.

Therefore, we have coalitions: $\left(z, p_{j_1}, p_{j_2}, ..., p_{j_{460}}\right)$ where z means the president and p means a representative. Notice, that as far as the *Senate* has no right to veto a bill (any *Senate*'s amendment or veto may be overruled by the *Sejm* by simple majority), the *Senate* does not participate in the game. Assuming that the parliament has no party structure, the winning coalition is as follows:

- $\left(z, p_{j_1}, p_{j_2}, ..., p_{j_n}\right)$, where $n \geq 231$ (we also assume that all representatives participate in every voting).

Among the winning coalitions there are the following vulnerable coalitions:

- for $n = 231$ all players, i.e. the president and 231 representatives, are critical, i.e. each of them can swing,
- for $232 \leq n < 276$ only the president can swing.

Note, that for $n \geq 276$ all coalitions are winning but no member of such a coalition is in a swing position.

Using Johnston's power index one may find (Mercik, 2009) that those values of index of power for the *Sejm* and the president are:

- for the president: 0.92342978817
- for a representative: 0.0001664570
- for the *Sejm* as a whole: 0.0765702118

It means that the President of Poland is more than 12 times stronger than the House of Representatives, when no party structure is assumed. The position of the president changes radically when the party structure matters. In the tab. 2 one may find adequate calculations. In that case, party structure matters and it changes power of the president, decreasing it deeply, even when the president is attributed with a veto. To some extent, we may say that power of the president in this case consists of power of veto, and in this situation Johnston power index for the president measures only the power of veto directly.

	Johnston power index		
	No party structure	With party structure	The USA (no party structure)
The President	0.9234	0.0067	0.7700
The House of representatives	0.0766	0.9933	0.0736
The Senate	0	0	0.1560

Tab. 2 Summary of calculations of Johnston power index for different assumptions about the Polish Parliament (Mercik, 2009) compared to the USA (values for the USA are from the book of Brams, 1990).

The immanent characteristics of the veto of the second degree is the inter-body decision making and the necessity of strategic thinking. Any decision-maker has to look ahead and reason back. Strategy of a certain legislator must include evaluation and sometimes it is not possible at all without strategies of other decision bodies (players). In this sense, also veto becomes conditional and depending on other decision-makers. In fig. 3, one may see the legislative way a bill goes through the Polish parliamentary system. The House of Representatives (the *Sejm*) may be in action from 1 to 3 times during the legislation:

- one time when a bill is accepted by the Senate and the president without restrictions,
- two times when the Senate has restrictions and the president has none,
- three times when the Senate and the president have restrictions.

Each of these situations may result in different estimation of power of every player (i.e. the *Sejm*, the Senate and the president) and only the president has the right to veto the bill. Therefore, the estimation of veto power is connected with the president only but depending on the power of other players.

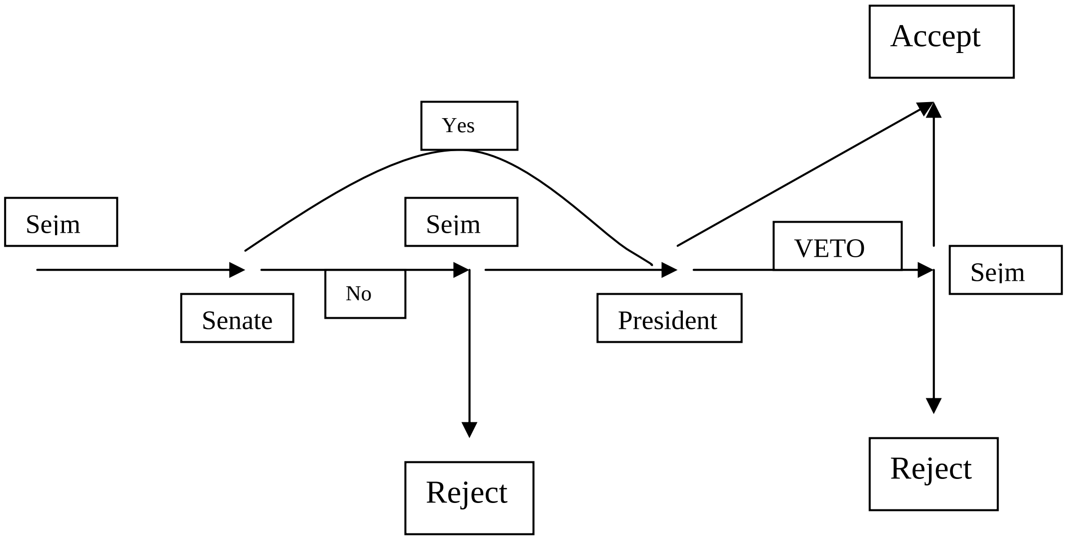

Fig. 3 Legislative way through the Polish parliamentary system (The *Sejm* stands for the House of Representatives).

The system of presidential vetoes may differ from country to country[2], but the general idea is the same: players must look ahead and reason back. It seems that the strategic type power index may be in use for the power evaluation. Among them probably the most adequate could be the index proposed by Steunenberg, Schmidtchen and Koboldt (1999). In their paper one may find also the example of evaluation of power for the European Union. In the paper, (Schmidtchen, Steunenberg 2002) they argue that this type of power index can be also used for a priori and a posteriori analysis of power.

Final remarks

In most cases, the power of the right to veto cannot be measured directly, because this right is only the part of the characteristics of players. Therefore, evaluation of chances for consensus can be done indirectly too. However, we can indirectly estimate the influence of the right to veto on the power of a player by comparing its power both with and without this

[2] The Polish example is rather untypical because the Senate is not participating in the president's veto overruling. For example, in the USA overruling of presidential veto requires the cooperation of the House of Representatives and the Senate (3/5 of members from both chambers)

right. Quite intuitively, the right of veto will increase the power of a player in most cases. It is not so obvious how large this increase will be and in some cases power is associated only with the right to veto (as is the case of the President of Poland and the parliament with a party structure). This last example shows measure of veto power in an absolute term. In most cases it is not possible to do it so directly.

Constitutional analyses try to identify the power of the players, including the power of veto, anticipated from behind a veil of ignorance. In fact, strategies of all players need to fix a right of veto as a permanent element of the situation. The veto right of the first degree can be modelled using a standard "swing" type power index. The veto right of the second degree can be estimated by the modified Banzhaf or Johnston power in simple cases, but a new index remains to be shown for complicated games (compound games), with more general structure.

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