Voting experiments: Measuring susceptibility of voting procedures to manipulation - responsiveness of strategic voting to information

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Abstract The paper computationally simulates 10 different voting procedures for small numbers of voters and small numbers of competing alternatives so as to study the vulnerability of these procedures to strategic voting. This is followed by a study of vulnerability of strategic voting to the variation in the amount of information that individual strategic agents possess. The susceptibility to strategic voting manipulation was found to be a diminishing function of the number of participants and an increasing function of the number of alternatives. Least susceptible voting procedures were the Condorcet-consistent procedures: Black's, Copeland's and Condorcet's procedure; relatively more susceptible procedures involved three elimination procedures: Coombs', Hare's and Plurality with runoff; most manipulable were the simple plurality, approval, max-min and Borda's procedures. We confirm vulnerability of strategic voting both to an absolute and relative reduction in the amount of information.

Keywords strategic voting, information, voting behaviour, distance in preferences, computation-based simulations

JEL classification C72, D72, D81

1. Introduction

This paper is devoted to computation-based simulations of voting. We use computation-based simulations to randomly generate a collective preference profile of a set of voters. All but one of these voters cast their votes sincerely in order to come at a collective decision, which the last voter attempts to manipulate through her strategic vote. We evaluate susceptibility to manipulation of 10 different voting procedures. We also target to estimate the change in the success rate of strategic voter's manipulations, given that her information about other voters' preferences shrinks. The information that the agent possesses shrinks because we assume away her full knowledge of other voters` preference profiles. The strategic voter instead expects other voters to vote according to some previously specified probability distribution.

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In my work I find that the susceptibility to strategic voting manipulation is a diminishing function of the number of voting participants, an increasing function of the number of competing alternatives, function of the currently chosen voting procedure and prominently of the amount of information that the individual voter holds. Once I strip the agent from the full knowledge of the collective preference profile, I confirm the vulnerability of strategic voting both to an absolute and relative reduction in the amount of owned information. A minimal reduction in her holding of information severely threatens agent's ability of voting manipulation. The precision in selecting the correct best manipulating voting pattern is also decreasing in the relative amount of information withheld. Consistently, the agent more often ends up with worse payoffs than sincere voting would yield, when a relatively larger share of information is withheld from her.

2. Literature survey

In modern social choice theory two impossibility theorems step up most prominently: famous Arrow's impossibility theorem (Arrow, 1963), and further interpreted Gibbard- Satterthwaithe theorem (Gibbard, 1973, Satterhwaite, 1975), unified e.g. by Reny (2000). Gibbard-Satterthwaite theorem states that there exists no voting system with three or more alternatives designed to select a single winner, which would be unrestricted in domain, would not be dictatorial, and which would not provide an agent, who has a full knowledge of other voters' preference profiles, with an incentive to strategically misrepresent her voting preference so as to swing the election outcome into her favour.

Approach of these and numerous following authors focuses on the properties of social choice functions and correspondences realised by different voting procedures and on theoretical consequences of their use. These authors forgo the individual view of voters involved in voting. We provide just a few from a long list of examples dealing with manipulability of SCFs and SCCs without intentions of actual reviewing them: Bandyopadhyay (1983), Barbera (1977), Feldman (1979), Gärdenfors (1979), Pattanaik (1973), or more recently Barbera, Dutta, Sen (2001), or Rodriguez-Alvarez (2007).

Contrary, numerous empirical studies focus on econometric detection and measurement of strategic voting in multiparty systems or in systems with numerous candidates, such are Alvarez, Nagler (2000), Blais et al. (2001), Blais, Bodet (2007), Fisher (2001a, 2001b), Schmitt (2001) and numerous other.

Many studies provide microeconomic rationales underlying either **voting turnout** or voting patterns including strategic voting. Much of this literature descends from **rational voter model** stemming from rational choice theory. For instance, a notion of **rational ignorance** is present in literature due to the contribution of Downs (1957). One of potential solutions has been proposed e.g. by Aldrich (1997), who incorporates social welfare into individual's utility to explain the voting turnout. General rational voter models are well described in the literature by Myerson, Weber (1993), Feldman, Serrano (2006); or Edlin, Gelman, Kaplan (2007).

Last branch of economics related to voting relies on simulations of voters' preferences, e.g. Laslier (2009). Economic experiments on strategic voting show up occasionally, see for illustration Harrison (2007).

There are a few approaches to specifying strategy-proofness. Umezawa (2009) provides an overview of the literature, which either makes explicit references to expected utilities, where probability measures over alternatives are given (e.g. Feldman, 1980; Barbera et al., 2001; Rodriguez-Alvarez, 2007; and numerous other.) or the other approach defines strategy-proofness in a non-probabilistic framework, where individuals evaluate the sets of alternatives based on their preference orders over alternatives by focusing on the best and/or the worst alternative in the sets (see, e.g. Bandyopadhyay, 1983; Barbera, 1977; Pattanaik, 1973).

A midway between these two approaches is specification of strategy-proofness based on probabilistically measured expected utilities, which are however not over specific alternatives but over voting distances between individual's preference ordering and ordered voting outcomes. Bossert, Storcken (1992) use Kemeny's distance to evaluate the voting distances between preference orderings. Duddy, Perote-Peña and Piggins (2009) investigate the problem of constructing a social welfare function that is non-manipulable in a context, where individuals attempt to manipulate a social ordering as opposed to a social choice. It is this middle way that we take in this paper, with an important distinction of using Euclidian rather than Kemeny's distance to construct the ranking of particular social orderings.

3. Methodology

3.1 Preference generating cultures

The random generation of a collective preference profile is in literature commonly called **a culture**. An overview of different preference generating cultures has been provided in the exposition of Laslier (2009). From among different specified Rousseauist, distributive or spatial cultures, we choose the '*impartial culture*' for our simulations.

The impartial culture attributes to each individual sincere voter a preference ordering from among m! strict total preference orderings, where m is the number of competing alternatives. Let's assume n individual voters with complete, transitive and anti-symmetric preference relations R_i on the set of alternatives A, where i is an index for an individual voter and separate alternatives are denoted by [a b c ... m]. The assumptions on voter's preference relations are equivalent for any R_i to be characterised as a total preference ordering.

An essential characteristic of the impartial preference-generation culture is that the preference orderings attributed to individuals are **i.i.d. according to uniform distribution** over m! logical strict preference orderings. The culture treats all the alternatives symmetrically and learning something about preference orderings of some voters yields no information about the preferences of the rest of voters.

Due to the symmetrical treatment of alternatives, we may fix the preference profile of the single strategic voter by attributing to her an alphabetical ordering of alternatives [a b c d… m]. The uniformity and i.i.d. properties of the preference generating culture allow us to proceed this way without the loss of generality.

3.2 Distance function specifications

We want to cardinalise individual's utility derived not from a particular alternative but from a complete social preference ordering. For that purpose we use a function that reflects both original individual's preference and the aggregated social preference order. Utilities reflect to what degree these two orders agree or how close is the generated voting outcome from the original voter's preference. For these reasons we compare the two orderings by the means of a distance function. We consider a distance function, which resembles the mathematical Euclidian distance. Minimization of a distance between individual's preference ordering and a social preference ordering then corresponds to maximization of utility of the strategic voter.

Definition: (Distance function) Let r_j and s_j constitute two systems of non-negative weights attached to all alternatives of an individual and social preference order, respectively. The two systems of weights are intertwined in the following manner: if particular alternative is located at the j^{th} position in the individual preference order $\Re_i$, then it bears individual weight r_j. An equal weight s_x will be attached to such position x in the social order S, at which the alternative was placed by the voting aggregation rule. The distance function D_{iS} between i^{th} individual preference ordering $\Re_i = [r_1\ r_2\ \ldots\ r_m]$ and social preference ordering $S = [s_1\ s_2\ \ldots\ s_m]$ is subsequently given by:

$$D_{iS} = \sqrt{\sum_{j=1}^{m}(r_j - s_j)^2}\ .$$

Distance function D_{iS} maps the systems of weights into one real number.

The weights that we attach to alternatives shall be unified since now on for the rest of our study. We assume them to correspond to scores, by which an individual would evaluate alternatives during **Borda's voting.** Individual's first and best alternative is associated with a weight of (m-1) points. The weights attached to consecutive options are consecutively falling by 1 point, with the last option being weighted by 0.

In case the voting outcome involves tie(s) between alternatives, we resolve the issue by calculating an average distance between all potential social outcomes and the individual preference ordering.

Definition: (Average distance) Let $L = (p_1, ..., p_K)$ be a lottery, which assigns equal probabilities to all potential social orders k that may occur after a random breaking of tie(s) involved in a voting outcome. Let D_{iSk} represent k-th potential distance of social ordering S_k from individual ordering $\Re_i$ for all k = (1,…, K). Then

$$\overline{D_i} = p_1 D_{iS1} + \ldots + p_K D_{iSK}$$

is the average distance between the individual preference ordering $\Re_i$ and all potential social preference orders.

3.3 Voting procedures

We compare the susceptibility to voting manipulation for a list of 10 voting procedures. For details on these procedures see Nurmi (1987) or Turnovec (2001).

1. **Simple plurality voting**

Each voter needs to decide, to which single alternative to assign a score of 1, while assigning 0 to all other alternatives. Plurality winner is such an alternative that collects the highest number of votes.

2. **Condorcet's voting procedure**

In standard understanding, a winning alternative is chosen by this procedure if and only if it is not defeated by a strict majority by any other alternative in a pair-wise vote. In this paper we order the alternatives according to the number of wins of an alternative over other alternatives in a pair-wise vote.

3. **Plurality voting with runoff**

Plurality voting with runoff involves two rounds. The first round proceeds just like simple plurality voting. The second round involves a vote between two alternatives with the highest scores obtained in the first round. The purpose of the first round, so-called runoff is to eliminate the least preferred options.

4. **Borda's voting procedure**

Given m alternatives each voter's first ranked alternative obtains (m-1) points, second ranked alternative obtains (m-2) points, the third one gets (m-3) points, and so forth, down to a minimum of 0 points for the last alternative. The scores are added up and the option with the highest score becomes the **Borda's winner.**

5. **Approval voting**

Individual voter may assign a score of 1 to as many alternatives as she wishes and assign 0 to all other. The winning alternative is the one, which gathers the most votes.

6. **Black's voting procedure**

Black's procedure simply chooses the Condorcet's winner if one exists. Otherwise it chooses the winner and ranks the alternatives according to Borda's voting.

7. **Hare's single transferable vote system**

If some alternative in Hare's voting procedure is ranked first by more than 50% of voters, it wins the election. If none such alternative exists, the alternative with fewest first ranks is eliminated from the count and the rest of alternatives is being pushed upwards in the preference lists of the voters. We again determine if any alternative ranks first by more than 50% of the voters. If so, it becomes a winner. If not, another round of eliminations proceeds. Eventually, after a number of rounds of eliminations one alternative must become **Hare's winner** or a tie is established in the final round.

8. **Coombs' technique**

Coombs suggested a slight modification to Hare's voting procedure and that was to eliminate during the rounds of elimination such an alternative that is ranked last by the largest number of voters. The qualification criterion for victory stayed the absolute majority of the first ranks in voters' preference profiles.

9. **Max-min voting technique**

Max-min procedure counts how many voters rank an alternative above each of other alternatives. For every alternative the procedure finds the lowest of these numbers. The procedure then ranks the alternatives according to the retrieved minima.

10. **Copeland's voting procedure**

The procedure attributes a number of wins and a number of losses to each alternative. The alternative wins over other alternative, if it gains a majority of votes in a pair-wise vote. Otherwise it loses. The social ordering consists of an ordered list of differences between a sum of wins and sum of losses of each alternative. Copeland's procedure obviously selects the Condorcet's winner if it exists.

3.4 Voter's behaviour given the level of information

Knowledge of the full collective preference profile

The role of a fully informed strategic voter is straightforward. The agent calculates all possible distances that could occur between her individual preference ordering and the aggregated social preference orderings and then selects such voting pattern so as to minimise the voting distance.

The role is straightforward, because voter's choice finalises the aggregation of the social preference ordering, and because the agent is the sole strategic voter, which means she faces no uncertainty about voting patterns of other voters.

To evaluate how successful is the voter in her endeavours, we need to decompose the question into two parts. How many times **did the voter have** and how many times **did she use** the opportunity to strategically manipulate the voting result? The success rate may be calculated either as a number of cases when the strategic voter succeeded to lower the relevant distance relatively to distance associated with her sincere voting or we may calculate the success rate as a number of cases when the voter succeeded to manipulate the voting result so as to make it copy her own individual preference ordering. I shall evaluate the former statistic. Under full information, the number of opportunities that the strategic voter **had** to manipulate and the number of opportunities that the voter **used** fully match**.** The difference between the two statistics emerges under voter's restrained information.

We simulate the preference profile of (n-1) voters and m competing alternatives using 100 000 independent draws for all listed voting procedures. We simulated the voting processes for n = {2, 3, 5, 7 and 11}. We use these values since we focus on voting manipulation in small groups or committees, where the informational assumption that a particular voter might know all or majority of other voters' preference profiles is feasible. All simulation codes are provided in Appendix F.

Information about full rankings of a subset of voters

The manipulating ability of a voter may be hampered by a lack of knowledge about voting patterns of a subset of the voters. This may happen, for instance, when some part of the electorate does not meet all sufficient conditions for their preference rankings to be classified as preference orderings. Alternative interpretation says that a part of the electorate may from various reasons behave **non-rationally** in their decision-making. May it be due to their bounded rationality, inadequate cognitive abilities, indifference, laziness, should they be constrained by time pressure, lack of appropriate incentives, or by any other feasible constraint. We shall avoid such explanations and will simply assume away strategic voter's full knowledge of the collective preference profile. The reduction of information consists in letting the strategic voter know about the full collective preference profile except for a preference ordering of one sincere voter.

The strategic voter can nevertheless determine some partial scores that the alternatives have gathered from voters, about which she has information. We will assume that the strategic voter knows that the voting patterns of all voters are i.i.d. from the uniform distribution.

Now, we may think of some simple heuristic rules that the strategic voter could use given her limited information. For instance, we may think her to attempt to manipulate the partially aggregated social ordering as if it were the fully aggregated social ordering. Nevertheless such heuristic rule could often lead into situations, where the strategic voter would end up with even worse payoffs than she would receive under sincere voting.

Another heuristic option is to make the strategic voter calculate all possible social orderings, into which the partially aggregated social ordering could lead and make her vote according to a min-max principle. That means to make her select such pattern, which would lead to potential social orderings, from among which the furthest one from the individual order is the closest one across different voting patterns.

Alternatively, the voter could stick to a minimalistic approach to strategic voting. She would opt for strategic voting only in cases, where the payoffs from her insincere voting strategy would never be dominated by payoffs accruing to her sincere voting.

Nevertheless to be consistent with previous specifications, we make the voter decide for a concrete voting pattern according to a minimisation of a weighted distance between her individual preference ordering and all plausibly aggregated social orderings associated with that voting pattern. The weights would be the probabilities of a particular combination of voting patterns to take place.

The questions we ask under limited information look for identical answers as the question raised under full information. Given a number of voters about which the strategic voter owns information, how many times **did the agent use the opportunity** to manipulate the voting result? How many times **was she successful** in her manipulation, in the sense that the resulting social ordering was closer to her own preference ordering than a social ordering associated with sincere voting would be? How many times did the agent manipulate with **adverse consequences**, in the sense that the resulting social ordering was further from her own preference ordering than would be a social ordering associated with sincere voting? How do these answers change, if the strategic agent **knows of fewer** voters' profiles? We contrast all these figures to figures obtained from cases of agent's full information to obtain relative measures of "successful manipulation". We again answer these questions for all 10 specified voting procedures.

4. Results

4.1 Full knowledge of the collective preference profile

Appendix A provides the complete tabulated overview of the opportunities for strategic manipulation of a sole strategic voter under full information. As we have already suggested, the number of opportunities for manipulation under full information mirrors the number of actual successful manipulations. Fully informed strategic voter moreover cannot end up with worse payoff by voting strategically than by voting sincerely. Table 1 and Table 2 present the summary statistics on the probability of manipulation under full information by number of players and number of competing alternatives. Figure 1 and Figure 2 graphically outline the evolution of room for strategic manipulation for all considered voting procedures, related to the number of players.

We observe 3 results:

1. strategic manipulation opportunity levels vary substantially across the used voting procedures,

2. strategic manipulation opportunity levels for 4 alternatives surpass those of 3 alternatives in every simulated procedure for all numbers of voters,

3. the number of sincere voters does not affect the manipulation opportunities, if we allow for wider confidence intervals, the opportunity for strategic manipulation is diminishing in the number of voters.

1. Levels of strategic voting vary substantially across the used voting procedures

Consider Figure 1 and Figure 2 in this regard. In both figures we can discern a distinct arrangement of layers.

Table 1 - Summary statistics for probability of manipulation, full information, m=3

Full information, m=3	n = 2	n = 3	n = 5	n = 7	n = 11
Average	0.142	0.144	0.156	0.127	0.129
Min	0	0	0	0	0
Max	0.333	0.361	0.432	0.325	0.289
Variance	0.019	0.016	0.019	0.010	0.010

Table 2 – Summary statistics for probability of manipulation, full information, m=4

Full information, m=4	n = 2	n = 3	n = 5	n = 7	n = 11
Average	0.404	0.361	0.358	0.365	0.346
Min	0.167	0.147	0.124	0.107	0.082
Max	0.794	0.684	0.704	0.592	0.562
Variance	0.052	0.032	0.036	0.032	0.031

The lowest probability of manipulation can be attributed to Copeland’s, Condorcet’s and Black’s voting procedures. This comes at no surprise, as these procedures are exactly the Condorcet- consistent procedures, in other words they always select the Condorcet’s winner if it exists. The second layer of manipulability of voting procedures involves three elimination procedures: Coombs’, Hare’s and Plurality with runoff voting procedures. Although these procedures are not Condorcet- consistent, the probability of manipulation is only slightly higher than in the preceding group. The reason is the difficult process of consecutive rounds of eliminations, where it is not only necessary for the strategic voter to find a situation where her vote is pivotal, moreover she has to find voting pattern, which does not harm her in later rounds of eliminations.

Figure 1 – Probabilities of manipulation by number of players and voting procedure, full information, m=3

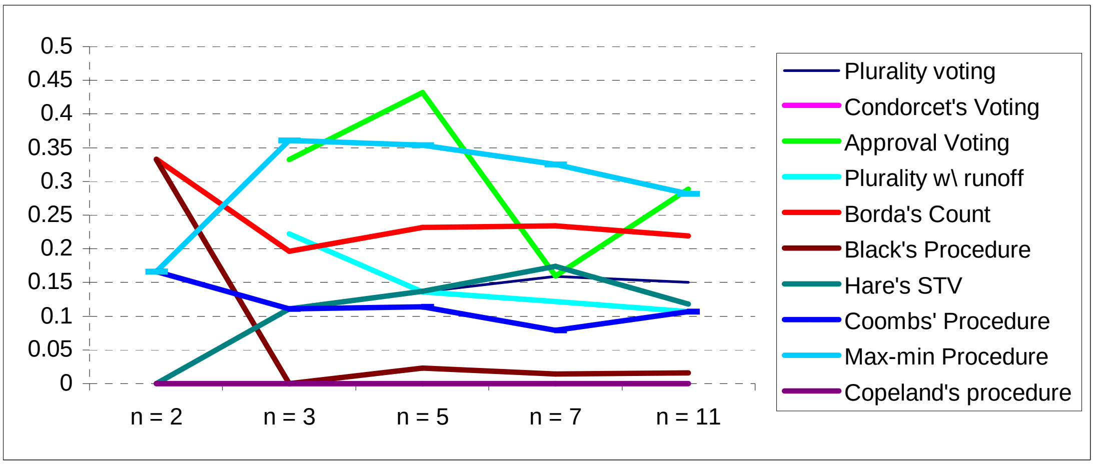
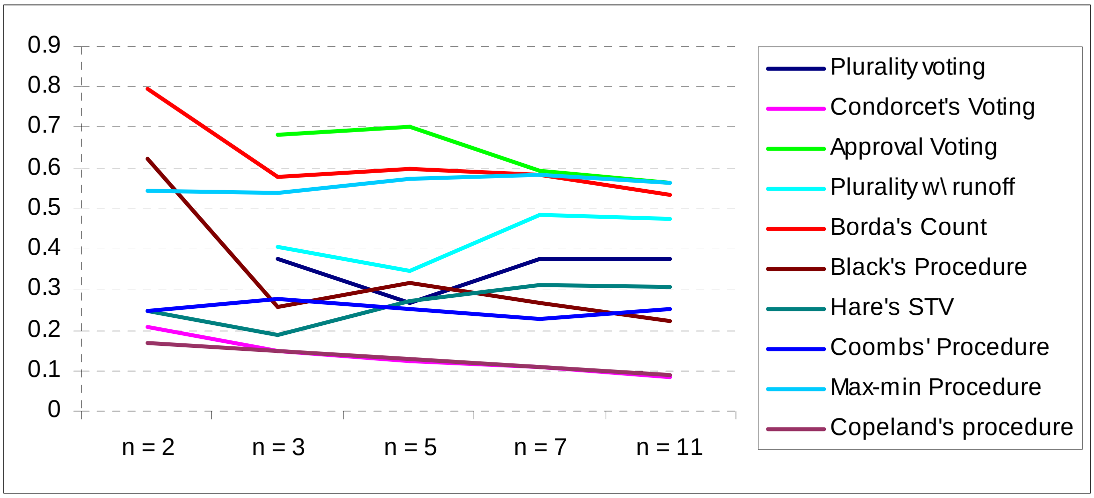


Figure 2 – Probabilities of manipulation by number of players and voting procedure, full information, m=4



The last most manipulable layer groups together the remaining procedures: approval voting, max-min voting and Borda's count. The common feature of these three procedures is that they allow the strategic voter to allocate wide ranges of scores to individual alternatives. This property gives to the strategic voter power to swing with scores more flexibly.

Noteworthy, some level of susceptibility to manipulation needs to be attributed also to the use of the impartial preference generating culture. Had we been using some other culture, where fewer ties would occur during the preference aggregation, the strategic voter would face fewer opportunities for gainful manipulation. That applies most apparently for procedures, where the range of points that determine the social ordering is the narrowest, e.g. max-min procedure.

The considerably higher susceptibility to strategic manipulation of the last three voting procedures (Borda, max-min, approval) manifests itself visibly on the histogram of the probabilities of manipulation. The three procedures contribute to the second peak in the probability distribution, just below the 60% mark in Figure 2.

In Table 3 we use simple ordinary least squares (OLS) regression to explain the variability in the susceptibility to strategic manipulation. The susceptibility is captured in the explained variable "Prob". Regarding the explanatory variables n_i captures the number of players, while $m4_i$ is a dummy signifying that we choose from 4 voting alternatives rather than from 3 alternatives. The rest of the variables Plurality to Copeland are dummy variables corresponding to the 10 voting aggregation rules. They are included in the (10x1) vector "$proced_i$", to which correspond 10 coefficients contained in the (10x1) vector δ. The formal model can be expressed as follows:

$$Prob_i = \beta\, n_i + \gamma\, m4_i + \delta'\, proced_i + \varepsilon_i\,,$$

index i does not stand here for the individual voter, but for a particular observation of the susceptibility to strategic manipulation.

Regression table 3 – Probability of manipulation on predictors, full information

Source	SS	df	MS
Model	9.06521324	12	.755434436
Residual	.482688736	72	.00670401
Total	9.54790197	84	.1136655

Number of obs =	84
F(12, 72) =	112.68
Prob > F =	0.0000
R-squared =	0.9494
Adj R-squared =	0.9410
Root MSE =	.08188

Prob	Coef.	Std. Err.	t	P>|t|		[95% Conf.	Interval]
n	-.005	.002	-1.99	0.051	*	-.011	.000
m4	.250	.019	13.14	0.000	***	.212	.288
Plurality	.155	.035	4.36	0.000	***	.084	.226
Condor	-.084	.044	-1.91	0.060	*	-.172	.003
Approval	.381	.035	10.68	0.000	***	.309	.452
Runoff	.198	.035	5.58	0.000	***	.127	.270
Borda	.337	.031	10.58	0.000	***	.273	.400
Black	.114	.031	3.59	0.001	***	.050	.177
Hare	.093	.031	2.93	0.004	***	.029	.157
Coombs	.089	.031	2.82	0.006	***	.026	.153
Max-min	.335	.031	10.53	0.000	***	.272	.399
Copeland	-.090	.044	-2.05	0.044	**	-.179	-.002

*** significant at 1%, ** significant at 5%, * significant at 10%

The column of coefficients allows us to rank the particular procedures according to their susceptibility to manipulation, consistently with previously described manner.

2. Susceptibility to manipulation in cases with 4 alternatives surpasses that of cases with 3 alternatives in every procedure for all considered numbers of voters

Regression table 3 reads that if we are choosing from among 4 alternatives rather than from 3, there is a 25% higher chance that the strategic voter comes to a situation where it is beneficial for her to manipulate her vote. Nonetheless, it is rather sound not to generalise this result with respect to the higher numbers of competing alternatives. The pattern does not have to be increasing in the number of alternatives in the least. A sound expectation for this pattern would be to be non-linear and rather depend on the difference (n-m) if not on a ratio of the number of voters and number of competing alternatives (n/m).

3. The number of sincere voters does not affect the manipulation opportunities under full information

Table 3 shows a very slight and rather marginal negative trend of susceptibility to voting manipulation in the number of voters. We cannot reject the H_0 hypothesis of no impact of this variable at 5% confidence level, and we have to allow for wider confidence intervals to be able to reject the H_0. The logic of our expectations for the coefficient to be negative is nevertheless straightforward: the more voters are involved in a voting situation, the lesser relative weight of one vote should become, in the sense that the strategic voter becomes less often pivotal.

If we evaluate the “n” coefficient for different voting procedures separately, we could conclude that it is only in Condorcet’s, Copeland’s and Plurality voting with runoff voting procedures with 3 alternatives that the susceptibility to manipulation decreases monotonically in the number of voters. Such conclusions are rather weak.

In Figure 3 we provide a scatter plot of actual values of the susceptibility to manipulation versus the values predicted by the linear regression. The grey area represents the 95% confidence interval of the linear prediction.

Figure 3 – Probability of manipulation vs. fitted values, full information

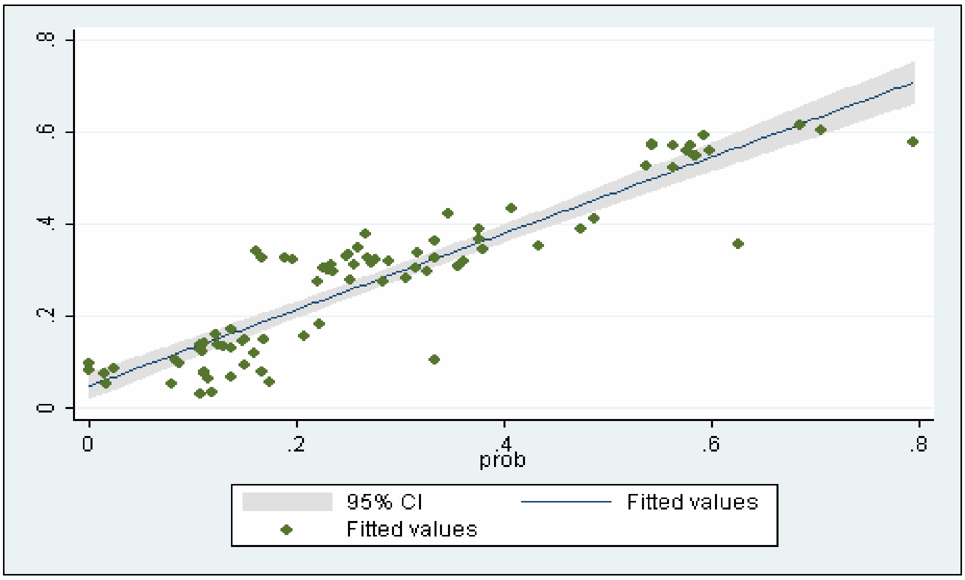


Figure 3 discloses slight differences in the variance between the group with smaller susceptibility to manipulation and group with higher susceptibility. This observation is not uncommon, since there are more observations on lower susceptibilities to manipulation, which is usually associated with larger variances.

Table 4 – Shapiro-Wilk test for the normality of residuals

Shapiro-Wilk W test for normal data					
Variable	Obs	W	V	z	Prob>z
resid	84	0.86043	9.972	5.053	0.00000

In Table 4 we perform a Shapiro-Wilk test for the normality of residuals, as a statistical test about the necessary assumptions for OLS. We fail to find enough supportive evidence to confirm the normality, which would break one of the OLS assumptions. We perceive the result of this test only as a supportive statistic.

4.2 Information about full rankings of a subset of voters

First, we have simulated a probability that the strategic voter **attempts for strategic manipulation**. The strategic voter does not have the full information, so she is coerced to decide on the basis of the weighted distances whether to attempt for a manipulation or not. The number of attempts may therefore be both higher and lower than the number of cases when the strategic voting was actually optimal. The uncertainty of the voter whether to manipulate or not propagates the other three kinds of results.

We provide the number of cases, when the strategic voter decides to manipulate and then acquires the same voting distance as she would acquire had she had the full information. This is captured in the variable of '**Maintained best manipulation**'. The variable does not include the cases when it was optimal for the strategic voter to vote sincerely and she correctly chose to do so. As a consequence the variable 'Maintained best manipulation' can be only equal or lower than the number of successful manipulations in the settings with full information.

Thirdly, an alternative measure of successful manipulation was produced. It is a probability that the strategic voter on the grounds of a weighted distance chose such voting pattern, which yielded not necessarily the best voting distance, but nonetheless **better** distance **than sincere voting** would yield. Appendix B provides the results.

Last, we provide a variable of the number of cases, when the attempt for voting manipulation has lead to a **worse** voting distance **than sincere** voting would lead to. Even this variable can be considered as an alternative measure of successful manipulation. The residual number of cases, i.e. (100 000 simulations – 'Worse than sincere') captures the number of cases when the strategic voter either correctly decided to manipulate or decided incorrectly but the voting distance was not worse than if she had voted sincerely, or the voter decided correctly not to manipulate.

Table 5 displays the summary statistics on the listed four measures of individual manipulation success. Table 6 measures correlations between these variables and the probability of manipulation under full information. All observations on manipulability for n=2 were dropped together with the observations of non-manipulable Condorcet's and Copeland's procedures for m=3.

Table 5 –Summary statistics for measures of individual manipulation success

Variable	Obs.	Mean	Std. Dev.	Min	Max
Attempts	72	.261	.220	0	.749
Maint. Best	72	.114	.099	0	.359
Better	72	.129	.119	0	.496
Worse	72	.044	.057	0	.336

Table 6 – Correlation table for measures of individual manipulation success

	Prob	Attempts	Worse	Better	Maint. best
Prob	1.0000				
Attempts	0.7899	1.0000			
Worse	0.4820	0.6781	1.0000		
Better	0.8219	0.9348	0.5989	1.0000	
Maint. Best	0.7501	0.8903	0.6259	0.9627	1.0000

We do not intend to comment on the statistic of ‘Better than sincere’, whereas here the results are tightly correlated with those of ‘Maintained best manipulation.’

Maintained best manipulation

The results, provided in Appendix C can be summarised in 5 points:

1. the levels of susceptibility to manipulation of various procedures differ less significantly under reduced information than under full information;

2. we observe a rapid drop in the susceptibility to manipulation of all considered procedures under limited information, i.e. we confirm the severe vulnerability of strategic voting to an absolute reduction in information owned;

3. the order of the most susceptible to the least susceptible voting procedure remains unchanged when compared to the order of procedures under full information;

4. the susceptibility to strategic manipulation grows in the number of voters under reduced information, i.e. we confirm the vulnerability of strategic voting to a relative reduction in possessed information;

5. the levels of manipulability are higher for cases with more alternatives.

1. The lower variability in the susceptibility across different voting procedures is well observable from both Figures 4 and 5, and could be also documented on a lower dispersion in the coefficients from Regression table 7. Numerous coefficients are moreover found not being significantly different from zero.

Figure 4 - Probabilities of manipulation by number of players and voting procedure – reduced information, m=3

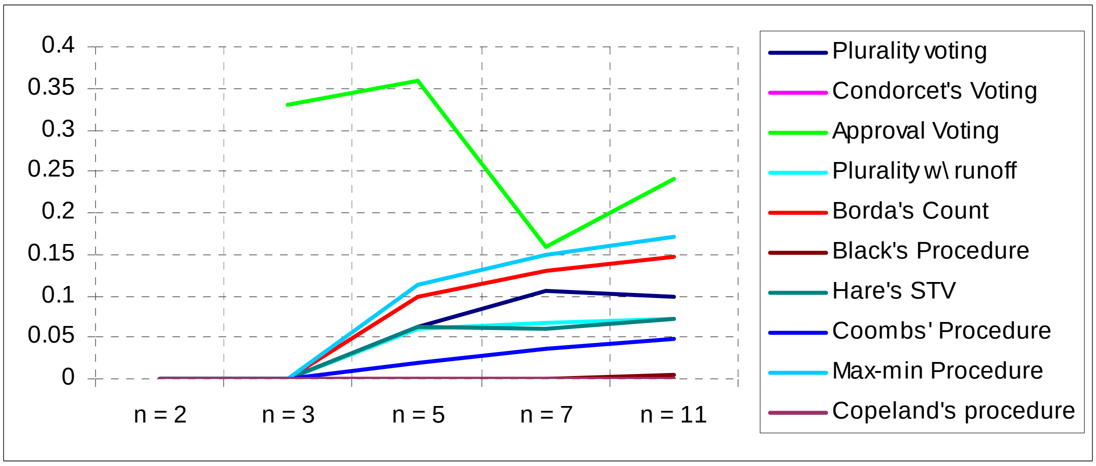
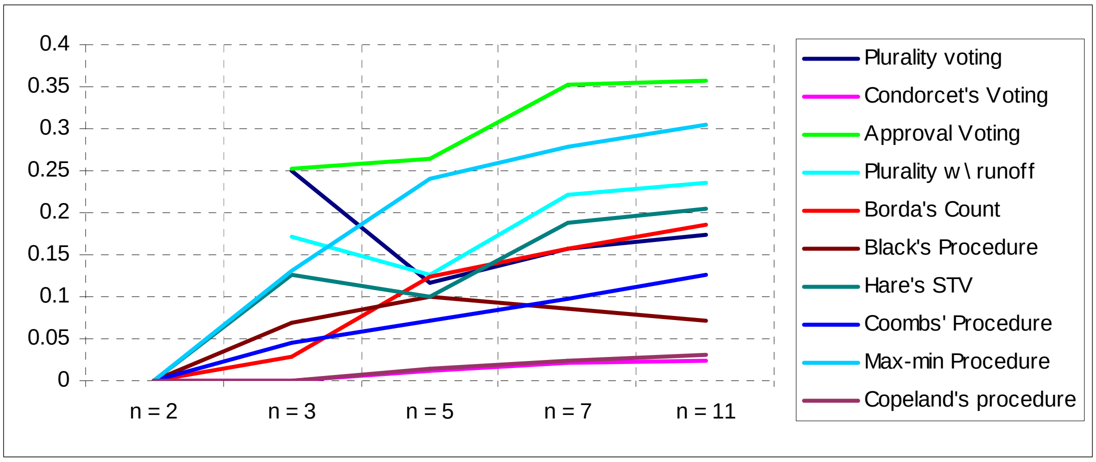


Figure 5 - Probabilities of manipulation by number of players and voting procedure – reduced information, m=4



Noteworthy, having 3 alternatives and 3 voters all voting procedures except for approval voting became immune to strategic manipulation. Having 4 alternatives and 3 voters the levels of manipulability remained significantly positive.

Regression table 7 - Probability of manipulation on predictors, reduced information

Source	SS	df	MS		
Model	1.51674706	12	.126395588	Number of obs =	72
Residual	.126827921	60	.002113799	F(12, 60) =	59.80
				Prob > F =	0.0000
				R-squared =	0.9228
				Adj R-squared =	0.9074
Total	1.64357498	72	.02282743	Root MSE =	.04598

Maint. Best	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
n	.007	.001	4.22	0.000***	.004	.011
m4	.085	.011	7.44	0.000***	.062	.108
Plurality	.027	.020	1.32	0.190	-.014	.069
Condorcet	-.121	.028	-4.28	0.000***	-.177	-.064
Approval	.196	.020	9.39	0.000***	.154	.238
Runoff	.026	.020	1.26	0.213	-.015	.068
Borda	.016	.020	0.76	0.448	-.025	.057
Black	-.051	.020	-2.48	0.016**	-.093	-.009
Hare	.008	.020	0.41	0.686	-.033	.050
Coombs	-.037	.020	-1.78	0.081*	-.079	.004
Max-min	.080	.020	3.86	0.000***	.038	.122
Copeland	-.118	.028	-4.18	0.000***	-.174	-.061

*** significant at 1%, ** significant at 5%, * significant at 10%

2. The rapid drop in the susceptibility to voting manipulation is best observable from the regression Table 8. Here the formal model resembles the previous model, apart from the facts that the explained variable $Prob_i$ includes both probabilities from full and reduced informational settings, and an additional explanatory dummy variable $(\text{Reduced Info})_i$ controls for this difference. The formal model and the results captured in Table 8 follow:

$$Prob_i = \beta\, n_i + \gamma\, m4_i + \delta'\, proced_i + \lambda\, (\text{Reduced Info})_i + \varepsilon_i\,,$$

Regression table 8 - Probability of manipulation on predictors, merged informational groups

Source	SS	df	MS		
Model	9.87427496	13	.759559613	Number of obs =	164
Residual	1.31720199	151	.008723192	F(13, 151) =	87.07
				Prob > F =	0.0000
				R-squared =	0.8823
				Adj R-squared =	0.8722
Total	11.191477	164	.068240713	Root MSE =	.0934

Prob	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
n	.003	.002	1.33	0.186	-.001	.007
m4	.162	.015	10.56	0.000***	.131	.192
Reduced Info	**-.185**	.014	-12.70	0.000***	-.214	-.156
Plurality	.173	.029	5.87	0.000***	.115	.232
Condorcet	-.021	.039	-0.53	0.593	-.099	.057
Approval	.371	.029	12.53	0.000***	.312	.429
Runoff	.194	.029	6.58	0.000***	.136	.253
Borda	.253	.026	9.50	0.000***	.200	.306
Black	.114	.026	4.31	0.000***	.062	.167
Hare	.128	.026	4.82	0.000***	.075	.181
Coombs	.108	.026	4.07	0.000***	.055	.161
Max-min	.278	.026	10.45	0.000***	.225	.331
Copeland	-.000	.039	-0.01	0.991	-.079	.078

*** significant at 1%, ** significant at 5%, * significant at 10%

The regression suggests the reduction in the knowledge of the strategic voter about the preference profile of one sincere voter reduces the probability of maintaining the best voting manipulation by 18%. That is moreover only a partial reduction since we need to take into regard also decline in coefficients of particular voting procedures. Lower amount of information depresses all of these coefficients simultaneously.

Speaking in absolute terms, under limited information none of the voting procedures is susceptible to manipulation in more than 35% of cases. Under 3 alternatives, the level of 35% is only approached by the approval voting procedure. Disregarding approval voting, the level of susceptibility would not overcome 18%.

3. The order of manipulability of individual voting procedures stayed unchanged

We again find the Condorcet-consistent procedures to be least manipulable, followed by the elimination-based procedures, placing the approval, Borda's and max-min procedures at the highest ranks in the order of the most to the least manipulable voting procedures.

Next, we order the procedures according to their **vulnerability to information reduction**, i.e. according to their **information intensity**. We construct a ratio of 'Maintained best manipulation' over 'Probability of successful manipulation' from under full information. The lower is the ratio, the more vulnerable to information reduction the procedure is. We present the ratios in Table 9.

Table 9 – Vulnerability of voting procedures to reduction in information

Variable*	Obs	Mean	Std. Dev.	Min	Max
Plurality	8	.471	.21	0	.66
Condorcet	4	.149	.13	0	.30
Approval	8	.705	.25	.36	1
Runoff	8	.426	.19	0	.67
Borda	8	.316	.23	0	.67
Black	7	.219	.12	0	.31
Hare	8	.463	.22	0	.67
Coombs	8	.307	.18	0	.50
Max-min	8	.384	.19	0	.60
Copeland	4	.173	.15	0	.35

* (Maintained best manipulation / probability of successful manipulation under full information)

The least information intensive are the approval, plurality, Hare's and plurality with runoff procedures. The most vulnerable are Condorcet's, Copeland's and Black's procedures.

We observe that the order of manipulability of voting procedures is co-determined by the information intensity of the procedures. The least manipulable procedures are in the largest extent vulnerable to the reduction in the amount of information and the most susceptible procedures to manipulation do not suffer from information reduction that much.

4. Under reduced information the susceptibility to manipulation grows in the number of voters

The reason for the increasing manipulation in the number of voters can be attributed to the relatively lower share of withheld information from the strategic voters at higher numbers of voters. Knowing less of 1 sincere voter's profile when there are 11 voters is less important for the agent's ability of strategic manipulation than knowing less of 1 sincere voter's profile when there are just 3 voters. Hereby we confirm the vulnerability of strategic manipulation not only to an absolute reduction in the individual information, but also to a relative reduction.

5. The levels of manipulability are higher for cases with more alternatives.

The discussion is analogous to the one in the section of full information

Attempts for voting manipulation

Table of ‘Attempts for voting manipulation’ is provided in Appendix D. Regression table 10 explains the number of attempts for manipulation by an OLS regression.

Noteworthy, the reader must not draw direct inference from absolute figures in the appendix, since these figures ignore the correlation between the number of attempts and the actual probability of manipulation. It is natural to expect that the weighted distances bid the strategic voter to attempt for strategic manipulation more often in those procedures, which are more susceptible to manipulation. On the other hand, regressing the number of attempts on the probability of voting manipulation would induce endogeneity issues, since both variables are caused by third factors, such as by the number of voters, by the relative amount of withheld information, etc.

The formal model used hence puts on the left side of the regression the ratio of the ‘number of attempts’ over the ‘probability of strategic manipulation under full information’. This ratio is captured in the variable (Rel. Attempts)$_i$. The formal model and the results follow:

$$(\text{Rel. Atempts})_i = \frac{Attempts_i}{\Pr ob_i} = \beta\, n_i + \gamma\, m4_i + \delta'\, proced_i + \varepsilon_i\,,$$

Regression table 10 – Number of attempts for manipulation on predictors, reduced information

Source	SS	df	MS
Model	505.076542	12	42.0897118
Residual	12.9608789	51	.254134881
Total	518.037421	63	8.22281621

Number of obs	63
F(12, 51)	165.62
Prob > F	0.0000
R-squared	0.9750
Adj R-squared	0.9691
Root MSE	.50412

Rel.Attempts	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
n	-.058	.022	-2.58	0.013**	-.104	-.013
m4	-.008	.139	-0.06	0.949	-.288	.270
Plurality	2.33	.270	8.65	0.000***	1.794	2.878
Condorcet	3.04	.379	8.02	0.000***	2.280	3.803
Approval	2.48	.250	9.95	0.000***	1.985	2.989
Runoff	2.11	.270	7.84	0.000***	1.575	2.660
Borda	3.69	.270	13.69	0.000***	3.155	4.239
Black	6.26	.291	21.51	0.000***	5.679	6.849
Hare	2.20	.270	8.17	0.000***	1.664	2.749
Coombs	2.75	.270	10.19	0.000***	2.209	3.294
Max-min	2.77	.270	10.28	0.000***	2.233	3.318
Copeland	2.94	.361	8.13	0.000***	2.214	3.665

From the Regression table 10 we can say that there are only few voting procedures where the relative number of attempts significantly differs from other procedures. In other words, the strategic agent attempts relatively for strategic manipulation in majority of procedures to a comparable extent. Majority of coefficients accruing to individual voting procedures fall into the 95% confidence intervals of the coefficients of other voting procedures. Only Black's and Borda's procedures differ from the other procedures in this respect, and their relative number of attempts for manipulation is higher. Nevertheless, we will see that in the case of Black's procedure this increased number of attempts leads eventually to an increased number of adverse outcomes.

As a positive result we view the independence of the number of attempts on the number of alternatives. The agent opts for attempts for manipulation irrespectively of the number of alternatives, which makes her decision making consistent.

Thirdly, a decrease in the relative number of attempts in the number of voters can be interpreted as getting more exact in attempting for manipulation, which we perceive just as well positively.

Overall, we can see that the number of attempts exceeds the number of cases when voting manipulation was optimal by twofold or even more. Luckily for the strategic voter, in cases when she attempts for a voting manipulation and she does not succeed she brings about either a result that is equally good as sincere voting or is better than sincere voting although it is the best manipulating option.

Adverse outcomes of attempting for manipulation

The results for outcomes, which are 'Worse than sincere' voting would deliver are provided in Appendix E. Table 11 explains the results through an OLS regression.

Absolutely speaking, the voting outcome is worse than sincere voting would yield on average in 5% of simulated situations when we are voting over 3 alternatives or in 15% of situations when we are voting over 4 alternatives. This percentage appears as a relatively small price to be paid for attempting for manipulation, given how many times the strategic agent succeeded in misrepresentation of her preferences. Moreover, since the strategic agent decides on a basis of a weighted distance, this distance is most probably not that much worse than the distance associated with sincere voting.

Speaking of relative figures, we relate the number of 'Worse than sincere outcomes' to the number of actual cases when voting manipulation was optimal. We capture the ratio of these two variables in a variable $(\text{Rel. Worse})_i$. The formal model follows:

$$(\text{Rel. Worse})_i = \frac{Worse_i}{\Pr ob_i} = \beta\ n_i + \gamma\ m4_i + \delta'\ proced_i + \varepsilon_i\ ,$$

Regression table 11 - Probability of manipulation into a worse than sincere outcome on predictors, reduced information

Source	SS	df	MS	
Model	1.92406925	12	.160339104	
Residual	.462447355	51	.009067595	
Total	2.3865166	63	.037881216	

Number of obs	= 63
F(12, 51)	= 17.68
Prob > F	= 0.0000
R-squared	= 0.8062
Adj R-squared	= 0.7606
Root MSE	= .09522

Rel. worse	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
n	-.017	.004	-4.14	0.000***	-.026	-.009
m4	.084	.026	3.22	0.002***	.031	.137
Plurality	.348	.051	6.84	0.000***	.246	.451
Condorcet	.106	.071	1.48	0.145	-.037	.249
Approval	.219	.047	4.64	0.000***	.124	.314
Runoff	.224	.051	4.40	0.000***	.122	.327
Borda	.210	.051	4.12	0.000***	.107	.312
Black	.226	.055	4.12	0.000***	.116	.337
Hare	.197	.051	3.88	0.000***	.095	.300
Coombs	.261	.051	5.12	0.000***	.158	.363
Max-min	.200	.051	3.92	0.000***	.097	.302
Copeland	.189	.068	2.77	0.008***	.052	.326

We see that the voting procedures are not statistically distinguishable between each other in the regard of how many 'Relative worse than sincere' outcomes they deliver. The strategic agent selects on average the unsatisfactory voting pattern in a similar extent across all voting procedures.

The number of relatively worse outcomes is diminishing in the number of voters. A careful reader has noticed, that to an increased number of voters we have previously attributed an increasing exactness of attempting for strategic manipulation. Now we discover, that the increase in exactness extends also on the ability of attempting for such voting patterns, which do not harm the individual strategic voter relatively to her sincere voting. This increase may originate in the lowest relative share of withheld information at higher numbers of voters.

The selection of unsatisfactory voting patterns is higher when selecting from 4 competing alternatives. We nevertheless find no motivation for this result.

5. Conclusions

Strategic voting is not only an act predicted by the economic theoretical models but also empirically manifested and widely observed pattern of the voting behavior. The sophisticated voters, who out of their short-term instrumental motivations want to best influence the election result, misrepresent in the elections their individual voting preferences in the expectation of manipulating the aggregated social preference order into an order, which would reflect their own sincere wishes as closely as possible. On the other hand, the strategic voters in their effort for best influencing the voting outcome stumble upon different impediments of the voting situation.

This study has computationally simulated 10 different voting procedures for small numbers of voters and small numbers of competing alternatives so as to study the susceptibility of these procedures to strategic voting. This was followed by a study of vulnerability of strategic voting to the variation in the amount of information that an individual strategic agent possessed.

The susceptibility to strategic voting manipulation was found to be a diminishing function of the number of election participants and an increasing function of the number of voting alternatives. All procedures could be characterised by their own specific extent to which they were susceptible to manipulation. The procedure-specific extent of manipulation was in turn dependent on the amount of information that the procedure typically requires from a participating agent to disclose, in combination with the strictness of the voting procedure, which is the amount of points that the procedure allows the agent to manipulate with. Least susceptible voting procedures were the Condorcet-consistent procedures: Black's, Copeland's and Condorcet's procedure itself. The second group of relatively more susceptible voting procedures involved three elimination procedures: Coombs', Hare's and Plurality with runoff voting procedures. Most manipulable procedures were the plurality voting procedure, approval voting procedure, max-min voting and Borda's count.

If the strategic agent had a full access to information about other voters' voting patterns, the opportunity for a strategic manipulation has occurred in up to 80% of cases for some procedures, although the average percentage of opportunities moved around 15% for 3 alternatives and 40% for 4 alternatives. Once we have stripped the agent from the full knowledge of the collective preference profile, we have confirmed the vulnerability of strategic voting to both an absolute and relative reduction in the amount of information. Having withheld information from the strategic agent about just one sincerely voting agent has reduced the number of cases, when the strategic agent was able to correctly choose the best manipulating voting pattern, by approx. 15-30 %. We found that the most manipulable procedures were least information intensive with regard to strategic manipulation. That is strategic voting was least vulnerable to a reduction in possessed information in approval, plurality, Hare's and plurality with runoff procedures and most vulnerable in Condorcet's, Copeland's and Black's procedures. The precision of selection of the best manipulating voting pattern was decreasing in the relative amount of information withheld from the strategic agent. Consistently, the agent has more often ended up with worse payoff than sincere voting would yield, when a relatively larger share of information was withheld from her.

There is much work left undone in the field, which is mostly related to the alternative specifications of the preference generating cultures or to the means of withholding information from the strategic voter, not speaking of the cases with numerous strategic voters. The future research should aim at these tasks.

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Appendix A Optimal number of voting manipulations, full info

Full information		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality Voting	M = 3	*	0.111	0.136	0.159	0.150
	M = 4	*	0.376	0.265	0.376	0.378
Condorcet's voting	M = 3	0	0	0	0	0
	M = 4	0.207	0.150	0.124	0.107	0.082
Approval Voting	M = 3	*	0.332	0.432	0.160	0.289
	M = 4	*	0.684	0.704	0.592	0.562
Plurality w\ runoff	M = 3	*	0.222	0.136	0.122	0.107
	M = 4	*	0.406	0.345	0.486	0.474
Borda' s Count	M = 3	0.333	0.196	0.232	0.234	0.219
	M = 4	0.794	0.578	0.598	0.585	0.536
Black' s Procedure	M = 3	0.332	0	0.023	0.014	0.016
	M = 4	0.625	0.259	0.316	0.267	0.225
Hare' s STV	M = 3	0	0.111	0.137	0.174	0.118
	M = 4	0.249	0.189	0.272	0.314	0.305
Coombs' Procedure	M = 3	0.166	0.111	0.114	0.079	0.107
	M = 4	0.247	0.275	0.254	0.228	0.251
Max - min Procedure	M = 3	0.166	0.361	0.354	0.325	0.282
	M = 4	0.542	0.541	0.576	0.582	0.562
Copeland's Procedure	M = 3	0	0	0	0	0
	M = 4	0.167	0.147	0.128	0.109	0.087

* For plurality, Condorcet's and Approval voting procedures, the results are trivial for n=2

Appendix B Alternative measure of manipulation, reduced info

Better than sincere		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality Voting	m = 3	*	0	0.062	0.106	0.099
	m = 4	*	0.251	0.117	0.156	0.175
Condorcet's voting	m = 3	0	0	0	0	0
	m = 4	0	0	0.012	0.021	0.026
Approval Voting	m = 3	*	0.331	0.359	0.160	0.242
	m = 4	*	0.496	0.315	0.439	0.407
Plurality w\ runoff	m = 3	*	0	0.061	0.067	0.072
	m = 4	*	0.171	0.130	0.239	0.247
Borda' s Count	m = 3	0	0	0.099	0.131	0.148
	m = 4	0	0.040	0.175	0.211	0.238
Black' s Procedure	m = 3	0	0	0	0.001	0.004
	m = 4	0	0.091	0.144	0.121	0.099
Hare' s STV	m = 3	0	0	0.062	0.060	0.072
	m = 4	0	0.126	0.103	0.187	0.208
Coombs' Procedure	m = 3	0	0	0.020	0.036	0.048
	m = 4	0	0.046	0.073	0.105	0.135
Max - min Procedure	m = 3	0	0	0.114	0.150	0.172
	m = 4	0	0.235	0.314	0.337	0.353
Copeland's Procedure	m = 3	0	0	0	0	0
	m = 4	0	0	0.015	0.025	0.032

Appendix C Probability of agent to maintain the optimal outcome under reduced information

Maintained best manipulation		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality Voting	m = 3	*	0	0.062	0.106	0.099
	m = 4	*	0.251	0.117	0.156	0.175
Condorcet's voting	m = 3	0	0	0	0	0
	m = 4	0	0	0.012	0.021	0.025
Approval Voting	m = 3	*	0.331	0.359	0.160	0.242
	m = 4	*	0.253	0.264	0.352	0.357
Plurality w\ runoff	m = 3	*	0	0.061	0.067	0.072
	m = 4	*	0.171	0.127	0.222	0.235
Borda' s Count	m = 3	0	0	0.099	0.131	0.148
	m = 4	0	0.029	0.124	0.156	0.185
Black' s Procedure	m = 3	0	0	0	0.001	0.004
	m = 4	0	0.068	0.100	0.085	0.071
Hare' s STV	m = 3	0	0	0.062	0.060	0.072
	m = 4	0	0.126	0.100	0.187	0.205
Coombs' Procedure	m = 3	0	0	0.020	0.036	0.048
	m = 4	0	0.046	0.072	0.098	0.126
Max – min Procedure	m = 3	0	0	0.114	0.150	0.172
	m = 4	0	0.132	0.241	0.278	0.304
Copeland’s Procedure	m = 3	0	0	0	0	0
	m = 4	0	0	0.014	0.025	0.031

Appendix D Number of attempts for manipulation, reduced info

Attempts for manipulation		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality Voting	m = 3	*	0	0.149	0.209	0.205
	m = 4	*	0.498	0.203	0.251	0.290
Condorcet's voting	m = 3	0	0	0	0	0
	m = 4	0	0	0.032	0.052	0.065
Approval Voting	m = 3	*	0.667	0.592	0.374	0.419
	m = 4	*	0.749	0.542	0.729	0.707
Plurality w\ runoff	m = 3	*	0	0.109	0.132	0.146
	m = 4	*	0.248	0.185	0.338	0.397
Borda' s Count	m = 3	0	0	0.291	0.375	0.422
	m = 4	0	0.120	0.429	0.527	0.616
Black' s Procedure	m = 3	0	0	0	0.007	0.019
	m = 4	0	0.449	0.530	0.470	0.410
Hare' s STV	m = 3	0	0	0.111	0.134	0.151
	m = 4	0	0.257	0.130	0.265	0.338
Coombs' Procedure	m = 3	0	0	0.057	0.083	0.110
	m = 4	0	0.088	0.194	0.234	0.239
Max - min Procedure	m = 3	0	0	0.184	0.277	0.347
	m = 4	0	0.505	0.627	0.666	0.671
Copeland’s Procedure	m = 3	0	0	0	0	0
	m = 4	0	0	0.049	0.066	0.080

Appendix E   Cases with worse outcomes than sincere voting would yield, reduced info

Worse than sincere		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality Voting	m = 3	*	0	0.037	0.040	0.022
	m = 4	*	0.247	0.058	0.085	0.069
Condorcet's voting	m = 3	0	0	0	0	0
	m = 4	0	0	0.0006	0.001	0.008
Approval Voting	m = 3	*	0.336	0.111	0	0.047
	m = 4	*	0.061	0.050	0.066	0.067
Plurality w\ runoff	m = 3	*	0	0	0.007	0.002
	m = 4	*	0.077	0.043	0.079	0.076
Borda' s Count	m = 3	0	0	0.041	0.042	0.033
	m = 4	0	0.027	0.063	0.072	0.058
Black' s Procedure	m = 3	0	0	0	0	0.001
	m = 4	0	0.156	0.108	0.080	0.057
Hare' s STV	m = 3	0	0	0	0	0
	m = 4	0	0.130	0.017	0.034	0.028
Coombs' Procedure	m = 3	0	0	0.018	0.002	0.016
	m = 4	0	0.021	0.052	0.030	0.041
Max - min Procedure	m = 3	0	0	0.009	0.011	0.011
	m = 4	0	0.133	0.122	0.108	0.082
Copeland's Procedure	m = 3	0	0	0	0	0
	m = 4	0	0	0.011	0.010	0.009

Appendix F   CD carrier with simulation codes, Stata data files, and regression code

The CD carrier carries beside a copy of the paper also two folders, in which we include the Matlab code for our voting simulations. The first folder includes the simulation code for all considered voting procedures under full informational settings. The name of a particular Matlab .m files indicates the used voting procedure, the used number of alternatives and currently used number of voters as follows:

Name: simulation_Procedure_Voters_x_Alternatives.m

The second folder includes the simulation codes for the environment of incomplete information. The generic name of a particular Matlab .m file follows:

Name: incomplete_Procedure_Voters_x_Alternatives.m

Apart from the simulation codes we include on the CD carrier also manually gathered results from the voting simulations contained in three Stata data files. They correspond to the two voting environments of full or incomplete information. To each of the three Stata data files corresponds a Stata code, which performs the presented regressions and inference.