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Heterogeneous Agent Model with Memory and Asset Price Behaviour

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Abstract:

The Efficient Markets Hypothesis provides a theoretical basis on which technical trading rules are rejected as a viable trading strategy. Technical trading rules, providing a signal of when to buy or sell asset based on such price patterns to the user, should not be useful for generating excess returns.

Technical traders and chartists tend to put little faith in strict efficient markets. Fundamentalists rely on their model employing fundamental information basis to forecasting of the next price period. The traders determine whether current conditions call for the acquisition of fundamental information in a forward looking manners, rather than relying on post performance. This approach relies on heterogeneity in the agent information and subsequent decisions either as fundamentalists or as chartists. Changing of the chartist's profitability and fundamentalist's positions is the basis of cycles behaviour. A more detail analysis is introduced in the Brock and Hommes model. We analyze this model in a memory case. This branch consists of a behaviour analysis among fundamentalists, technical traders, chartists, and contrarians. It is possible to show that in the case without contrarians but with a length memory adding, and increasing of the intensity choice parameter, fundamentalist's strategy is preferable.

Next, the case with fundamentalists and contrarians including different values of coefficients is observed. This case is sensitive on the structure of memory weights and the memory lengths. It is shown that different values of these memory coefficients can significantly change the preferences of trader strategies.
The last case, where fundamentalists, trend chasers, and contrarians are presented, shows high profitability of contrarians strategy at the prescribed memory lengths. The implementation of memory in the system with heterogeneous agents model is, in all these simulations, very important. In this case, these results have had very different values in comparison with the case without memory.

Keywords:

efficient markets hypothesis, technical trading rules, fundamentalists, technical traders, chartists, and contrarians, heterogeneous agent model with memory, asset price behaviour

JEL *classifications:* **C610; G140; D840**

1 Introduction

Assumptions about rational behaviour of agents, homogeneous models, and efficient market hypothesis were paradigm of economic and finance theory for the last year. After empirical data analysis on financial markets and economic and finance progress these paradigm are gotten over. There are phenomena observed in real data collected from financial markets that cannot be explained by the recent economic and finance theories. One paradigm of recent economic and finance theory asserts that sources of risk and economic fluctuations are exogenous. Therefore the economic system would converge to a steady-state path, which is determined by fundamentals and there are no opportunities for speculative profits in the absence of external shocks prices. It means that the other factors play important role in construction of real market forces as heterogeneous expectations. Since agents no have sufficient knowledge of the structure of the economy to form correct mathematical expectations, it is impossible for any formal theory to postulate unique expectations that would be held by all agents (Gaunersdorfer (2000)). Prices are partly determined by fundamentals and partly by the observed fluctuations endogenously caused by non-linear market forces. This implies that technical trading rules need not be systematically bad and

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may help in predicting future price changes. Developments in the theory of non-linear dynamic systems have contributed to new approaches in economics and finance (Brock (2001)). Introducing non-linearity in the models may improve research of a mechanism generating the observed movements in the real financial data. Financial markets are considered as systems of the interacting agents processing new information immediately. A heterogeneity in expectations can lead to market instability and complicated dynamics.

Our approach assumes that agents are intelligent having no full knowledge about the underlying model in sense of the rational expectation theory and no having the computational equipment can interpret the same information by different way. Therefore prices are driven by endogenously market forces. The approach Adaptive Belief Approach by Brock and Hommes (1997) is employed in this paper. Agents adapt their predictions by choosing among a finite number of predictors. Each predictor has a performance measure. Based on this performance agents make a rational choice between the predictors. Brock and Hommes shown that the adaptive rational equilibrium dynamics incorporates a general mechanism which may generate local instability of the equilibrium steady state and complicated global equilibrium dynamics.

We focus on a version of the model with two types of trades, i.e., fundamentalists, and technical traders. Technical traders tend to put little faith in strict efficient markets. Fundamentalists rely on their model employing fundamental information basis to forecasting of the next price period. The traders determine whether current conditions call for the acquisition of fundamental information in a forward looking manners, rather than relying on post performance. This approach relies on heterogeneity in the agent information and subsequent decisions either as fundamentalists or as chartists. Changing of the chartist's profitability and fundamentalist's positions is a basis of the cycles behaviour. A more detailed analysis is introduced in the Brock and Hommes model. We analyse this model in a memory case.

This paper is organized as follows. In Section 2 we briefly introduce the Brock-Hommes model without memory. This branch consists of a behaviour analysis among fundamentalists, technical traders, chartists, and contrarians. It is possible to show that in the case without contrarians but with a length memory adding, and increasing of the intensity choice parameter, fundamentalist's strategy is preferable.

In Section 3 this model is studied with memory in the performance measure. This case with fundamentalists and contrarians including different values of coefficients is observed. This case is sensitive on the structure of memory weights and the memory lengths. It is shown that different values of these memory coefficients can significantly change the preferences of trader strategies.

In Section 4, fundamentalists, trend chasers, and contrarians are presented again. This case shows high profitability of contrarians strategy at the prescribed memory lengths. The implementation of memory in the system with heterogeneous agents model is, in all these simulations, very important. In this case, these results have had very different values in comparison with the case without memory. Results of numerical analysis are introduced.

2 Model

The analysed model presents a form of evolutionary dynamics, which is called **Adaptive Belief System**, in a simple present discounted value (PDV) pricing model. Such model without memory (and only one period) was presented by Brock and Hommes (1998)[1].

Let us consider an asset-pricing model with one risky asset and one risk-free asset. Let p_t be the share price (ex dividend) of the risky asset at time t, and let $\{y_t\}$ be

[1] The model was inspired by the model of Lucas (1978)

i.i.d. the stochastic dividend process of the risky asset. The risk free asset is perfectly elastically supplied at gross return $R > 1$. The dynamics of wealth can be written as

$$W_{t+1} = R \cdot W_t + \left(p_{t+1} + y_{t+1} - R \cdot p_t \right) \cdot z_t , \tag{2.1}$$

where z_t denotes the number of shares of the asset purchased at time t. E_t and V_t are the conditional expectation and conditional variance operators, based on the public available information set consisting of past prices and dividends, i.e., on the information set $\mathcal{F}_t$ = $\{p_t,\ p_{t-1},\dots;y_t,\ y_{t-1}\ ,\dots\}$. Let $E_{h,t}$, $V_{h,t}$ denote **beliefs** of investor of type h about the conditional expectation and conditional variance. The conditional variance of wealth is

$$V_{h,t}\left(W_{t+1}\right) = z_t^2 \cdot V_{h,t}\left(p_{t+1} + y_{t+1} - R \cdot p_t\right). \tag{2.2}$$

We assume that beliefs about the conditional variance of excess returns are constant for all investor types h

$$V_{h,t}\left(p_{t+1} + y_{t+1} - R \cdot p_t\right) \equiv \sigma_h{}^2 = \sigma^2 . \tag{2.3}$$

Assume each investor type is a **myopic mean variance maximizer.** So for type h, the demand for shares z_{ht} is solved as follows

$$\max_z \left\{ E_{h,t} W_{t+1} - \frac{a}{2} V_{h,t}\left(W_{t+1}\right) \right\}, \tag{2.4}$$

i.e.,

$$E_{h,t}\left(p_{t+1} + y_{t+1} - Rp_t\right) - a\sigma^2 z_{s,t} = 0 , \tag{2.5}$$

$$z_{h,t} = \frac{E_{h,t}\left(p_{t+1} + y_{t+1} - R \cdot p_t\right)}{a \cdot \sigma^2} . \tag{2.6}$$

The risk aversion a is here assumed to be the same for all traders. Let $z_{s,t}$ be a supply of shares per investor and $n_{h,t}$ the fractions of investors of type h at date t. The equilibrium among demand and supply is expressed in the following form

$$\sum_h n_{h,t} \left\{ E_{h,t}\left(p_{t+1} + y_{t+1} - R \cdot p_t\right) / a \cdot \sigma^2 \right\} = z_{s,t} . \tag{2.7}$$

If there is only one type h, the market equilibrium yields the pricing equation

$$R \cdot p_t = E_{h,t}\left(p_{t+1} + y_{t+1}\right) - a \cdot \sigma^2 \cdot z_{s,t} . \tag{2.8}$$

For the special case of zero supply, i.e., $z_{st} = 0$, for all t, a benchmark notion of the rational expectation fundamental solution p_t^* is obtained. Then the expression (2.8) can be written in the following form

$$R \cdot p_t^* = E_t\left\{p_{t+1}^* + y_{t+1}\right\} . \tag{2.9}$$

If the dividend process $\{y_t\}$ is i.i.d., the expectation $E_t\{y_{t+1}\} = \overline{y}$, and a standard notion of **fundamental** is obtained. Let us put $p_t^* = \overline{p}$, where $\overline{p}$ is solution of

$$R \cdot \overline{p} = \overline{p} + \overline{y} . \tag{2.10}$$

The equation (2.9) has infinitely many solutions but only the constant solution $\overline{p} = \overline{y}/(R-1)$ of the equation (2.10) satisfies no the bubbles condition, i.e. $\lim\limits_{t\to\infty} E\left(p_t\right)/R^t = 0$. For our purpose, it is better to work with the deviation x_t from the benchmark fundamental p_t^*, i.e.,

$$x_t = p_t - p_t^* . \tag{2.11}$$

Heterogeneous beliefs will be now introduced and we shall study their influences on equilibrium of the dynamical systems. In the case of zero supply of outside shares we get from the equation (2.7)

$$R \cdot p_t = \sum_h n_{h,t} \cdot E_{h,t}\left(p_{t+1} + y_{t+1}\right). \tag{2.12}$$

The class of beliefs for every trader type h must be specified. Therefore the following assumption is introduced:

All beliefs are of the form

$$E_{h,t}\left(p_{t+1}+y_{t+1}\right)=E_t\left(p^*_{t+1}+y_{t+1}\right)+f_h\left(x_{t-1},...,x_{t-L}\right). \tag{2.13}$$

where p^*_{t+1} denotes the fundamental, $E_{h,t}\left(p^*_{t+1}+y_{t+1}\right)$ is the conditional expectation of the fundamental on the information set $\mathcal{F}_t$, $x_t = p_t - p^*_t$ is the deviation from the fundamental, and f_h is some deterministic function which can differ across trader types h, i.e. **we restrict beliefs to deterministic functions of past deviations from the fundamental**. As a special case, the assumption includes the case of an i.i.d. dividend process, with $E_t y_{t+1} = \bar{y}$ and the corresponding constant fundamental $p^*_t = \bar{p} = \bar{y}/(R-1)$. We can rewrite equation (2.12) in the deviations form

$$R\cdot p_t = R\cdot x_t + R\cdot p^*_t. \tag{2.14}$$

i.e.,

$$R\cdot x_t = \sum_h n_{h,t}\cdot E_{h,t}\left(p_{t+1}+y_{t+1}\right)-E_t\left(p^*_{t+1}+y_{t+1}\right). \tag{2.15}$$

Now we use equation (2.13), and the fact that $\sum n_{h,t}=1$ for all t, and we obtain

$$R\cdot x_t = \sum_h n_{h,t}\cdot\left[E_t\left(p^*_{t+1}+y_{t+1}\right)+f_h\left(x_{t-1},...,x_{t-L}\right)\right]-E_t\left(p^*_{t+1}+y_{t+1}\right)\frac{n!}{r!(n-r)!}. \tag{2.16}$$

$$R\cdot x_t = \sum_h n_{h,t}\cdot f_h\left(x_{t-1},...,x_{t-L}\right)\equiv\sum_h n_{h,t}\cdot f_{h,t}. \tag{2.17}$$

Denote the excess returns by expression $R_{t+1}=p_{t+1}+y_{t+1}-R\cdot p_t$. Let $\rho_{h,t}=E_{h,t}\left(R_{t+1}\right)$ be the conditional expectation of R_{t+1}. Let us consider the **goal function**

$$\max_z\left\{E_{h,t}R_{t+1}\cdot z-\left(\frac{a}{2}\right)\cdot z^2\cdot V_{h,t}\left(R_{t+1}\right)\right\}=\max_z\left\{\rho_{h,t}\cdot z-\left(\frac{a}{2}\right)\cdot z^2\cdot\sigma^2\right\}. \tag{2.18}$$

The equation (2.18) is equivalent to the equation (2.4) up to a constant, so the optimum choice of shares of the risky asset is the same. So the optimum solution of the equation (2.18) is denoted by $z\left(\rho_{h,t}\right)$.

3 The Dynamics of Fractions

Let us concentrate on the adoption of beliefs, i.e., on dynamics of the fractions $n_{h,t}$ of different trader types. Next, let us change slightly the timing of updating beliefs, i.e.,

$$R\cdot x_t = \sum_h n_{h,t-1}\cdot f_h\left(x_{t-1},...,x_{t-L}\right)\equiv\sum_h n_{h,t-1}\cdot f_{h,t}, \tag{3.1}$$

where $n_{h,t-1}$ denotes the fraction of trader type h at the beginning of period t, before than the equilibrium price x_t has been observed. Now the **realized excess return** over period t to the period $t+1$ is computed,

$$R_{t+1}=p_{t+1}+y_{t+1}-R\cdot p_t. \tag{3.2}$$

$$R_{t+1}=x_{t+1}+p^*_{t+1}+y_{t+1}-R\cdot x_t-R\cdot p^*_t. \tag{3.3}$$

$$R_{t+1}=x_{t+1}-R\cdot x_t+p^*_{t+1}+y_{t+1}-E_t\left(p^*_{t+1}+y_{t+1}\right)+E_t\left(p^*_{t+1}+y_{t+1}\right)-R\cdot p^*_t. \tag{3.4}$$

From the equation (2.9) we get

$$E_t\left(p^*_{t+1}+y_{t+1}\right)-R\cdot p^*_t=0,\text{ and }\delta_{t+1}=p^*_{t+1}+y_{t+1}-E_t\left(p^*_{t+1}+y_{t+1}\right),$$

which is a martingale difference sequence with respect to $\mathcal{F}_t$ i.e., $E_t\left(\delta_{t+1}|\mathcal{F}_t\right)=0$ for all t. So the expression (3.4) can be written as follows

$$R_{t+1}=x_{t+1}-R\cdot x_t+\delta_{t+1}. \tag{3.5}$$

The decomposition of the equation (3.5) as separating the 'explanation' of realized excess returns R_{t+1} into the contribution $x_{t+1} - R \cdot x_t$ of the theory is investigated here and the conventional Efficient Markets Theory term δ_{t+1} is shown.

Let the fitness measure (or the performance measure) $\pi\left(R_{t+1}, \rho_{h,t}\right)$ be defined by

$$\pi_{h,t} = \pi\left(R_{t+1}, \rho_{h,t}\right) = R_{t+1} \cdot z\left(\rho_{h,t}\right) = \left(x_{t+1} - R \cdot x_t + \delta_{t+1}\right) \cdot z\left(\rho_{h,t}\right), \tag{3.6}$$

so the fitness is given by the realized profits for trader h. In the following paragraphs, numerical simulations with a stochastic dividend process $y_t = \bar{y} + \varepsilon_t$, where ε_t is i.i.d.[2], with a uniform distribution on an interval $\langle -\varepsilon, +\varepsilon \rangle$ will be used.

Now write type h beliefs $\rho_{h,t} = E_{h,t}\left(R_{t+1}\right) = f_{h,t} - R \cdot x_t$ in the deviations form. Let the updated fractions $n_{h,t}$ be given by the discrete choice probability

$$n_{h,t} = \exp\left(\beta \cdot \pi_{h,t-1}\right) / Z_t, \text{ where } Z_t = \sum_h \exp\left(\beta \cdot \pi_{h,t-1}\right). \tag{3.7}$$

The parameter β is the **intensity of choice** measuring how fast agents switch between different prediction strategies. The parameter β is a measure of trader's rationality. The variable Z_t is just a normalization so that fractions $n_{h,t}$ sum up to 1. If the intensity of choice is infinite $(\beta = +\infty)$, the entire mass of traders uses the strategy that has the highest fitness. If the intensity of choice is zero, the mass of traders distributes itself evenly across the set of available strategies.

The timing of predictor selection is important. The fractions $n_{h,t}$ depend upon fitness π and return R at the time $t-1$ in order to ensure that depends only upon observable deviations x_t at time t. The timing ensures that past realized profits are observable quantities that can be used in predictor selection.

4 Memory in the performance measure

For the case with memory in the performance measure the fitness is not given by the most recent past (last period), but by summation of more values of fitness measure in the past with different weights for these values. The weights sum up to one.

$$n_{h,t} = \exp\left(\beta \cdot \sum_{p=1}^{m} \eta_{h,t} \cdot \pi_{h,t-p}\right) / Z_t, \; Z_t = \sum_h \exp\left(\beta \cdot \sum_{p=1}^{m} \eta_{h,t} \cdot \pi_{h,t-p}\right). \tag{4.1}$$

where m denotes the memory length, η is the vector of memory weights.

All beliefs will be of the simple form

$$f_{h,t} = g \cdot x_{t-1} + b_h \tag{4.2}$$

where g_h denotes the trend and b_h the bias of trader type h.

If $b_h = 0$, the agent h is called a **pure trend chaser** if $g > 0$ (strong trend chaser if $g > R$) and a **contrarian** if $g < 0$ (strong contrarian if $g < -R$).

If $g_h = 0$, type h trader is said to be **purely biased**. He is upward (downward) biased if $b_h > 0$ $(b_h < 0)$.

In the special case $g_h = b_h = 0$, type h trader is called **fundamentalist** i.e., the trader is believing that prices return to their fundamental value. Fundamentalists do have all past prices and dividends in their information set, but they do not know the fractions $n_{h,t}$ of the other belief types.

Now we derive the fitness measure for the simple belief type (2.2). Rewriting the equation (2.6) in deviations form yields the demand for shares by type h (by the assumption (2.13))

[2] In this case we have $\delta_{t+1} = \varepsilon_{t+1}$

$$z_{h,t-1} = \frac{E_{h,t-1}\left(p_t + y_t - R \cdot x_{t-1}\right)}{a \cdot \sigma^2} = \frac{f_{h,t-1} - R \cdot x_{t-1}}{a \cdot \sigma^2}. \tag{4.3}$$

Now the fitness function (3.6) can be rewritten, hence the realized profit is

$$\pi_{j,t-1} = R_t \cdot z_{h,t-1} = \frac{\left(x_t - R \cdot x_{t-1} + \delta_{t+1}\right)\left(g_h \cdot x_{t-2} + b_h - R \cdot x_{t-1}\right)}{a \cdot \sigma^2}. \tag{4.4}$$

The most common trader type in our numerical analysis is fundamentalist with parameters $g_h = b_h = 0$. Hence for fundamentalists we can write

$$\pi_{j,t-1} = \frac{\left(x_t - R \cdot x_{t-1} + \delta_{t+1}\right)\left(-R \cdot x_{t-1}\right)}{a \cdot \sigma^2}. \tag{4.5}$$

5 Numerical Analysis of the model under different memory structures

This section demonstrates numerically the importance of the memory for a behaviour of this model. We show that there are significant differences in profitability of trader's strategies as memory length is changed. In the second and third case we also use different memory structures (memory weights), which also influence the traders profitability, i.e., the trader's participation on the market.

Numerical analysis is focused only on the model with four types of traders, each with different beliefs. We examine three different cases, where types one are fundamentalists that interact with other trader's types such as trend chasers, contrarians, or with both of them.

For all three cases in this section we add noise to a dividend process. The noise has a uniform distribution on the interval $\langle -0.005, +0.005 \rangle$. The equation (5.1) is used for the memoryless system (one period), and the equation (5.2) for the system with memory where m denotes the memory length. Memory weights $\eta_{j,p}$ have the same value for different memory positions in case 1, but in case 2 and 3 they are given experiments with different values for these positions. The sum of $\eta_{j,p}$'s must add up to one.

Memoryless system

$$x_t = \frac{1}{R}\sum_{j=1}^{4} \exp\left(\beta \cdot \pi_{j,t-1}\right). \tag{5.1}$$

System with memory, where m denotes memory length and η memory weights.

$$x_t = \frac{1}{R}\sum_{j=1}^{4} \exp\left(\beta \cdot \sum_{p=1}^{m} \eta_{j,p} \cdot \pi_{j,t-p}\right). \tag{5.2}$$

Case 1: Fundamentalists, Trend chasers

First simulation is without memory, i.e. agents make decisions according to the last period of performance measure. For values of beta larger than 90 there arise chaotic price fluctuations and trading strategy labelled N2 becomes dominant on the market, figure (5.1). In this paper we do not want to explore dynamic features in the sense of chaotic behaviour but mainly the presence of traders on the market.

Type	Parameters		
N1	$g_1 = 0$	$b_1 = 0$	Fundamentalists
N2	$g_2 = 1.1$	$b_2 = 0.2$	Trend with upward bias
N3	$g_3 = 0.9$	$b_3 = -0.2$	Trend with downward bias
N4	$g_4 = 1$	$b_4 = 0$	Trend chasers

Table 5.1 Parameters of the system for case 1.

The effect of different memory lengths (for all types) is displayed in figures (5.3), (5.4). There is a dramatic change at m = 2 where fundamentalists becomes dominant strategy to m = 18 where no price fluctuations occur and strategies are equally represented on the market. This example shows stabilizing effect of memory for the system.

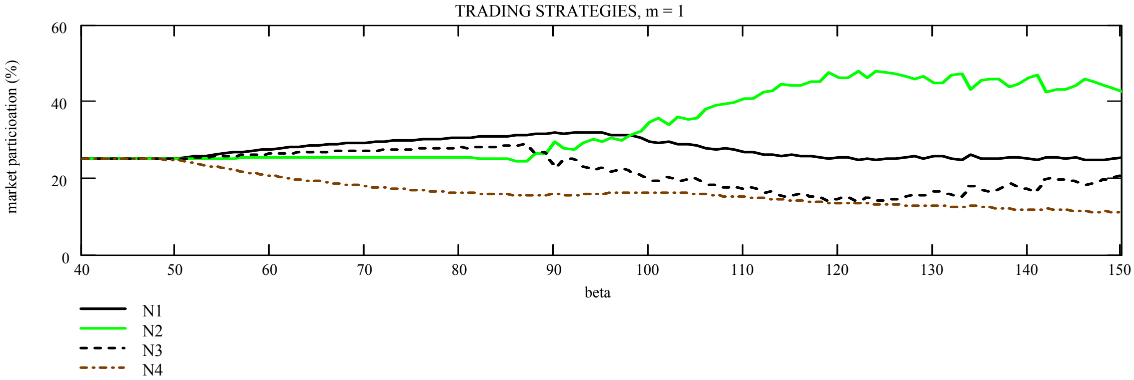

Figure 5.1 Participation of trading strategies on the market

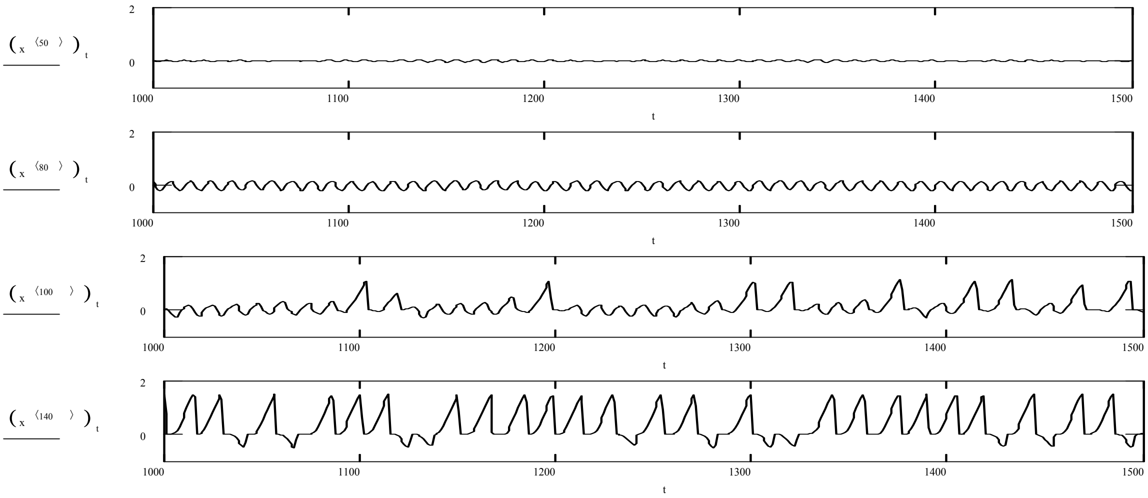

Figure 5.2 Time series of x_t with different values of β: 50, 80, 100, 140

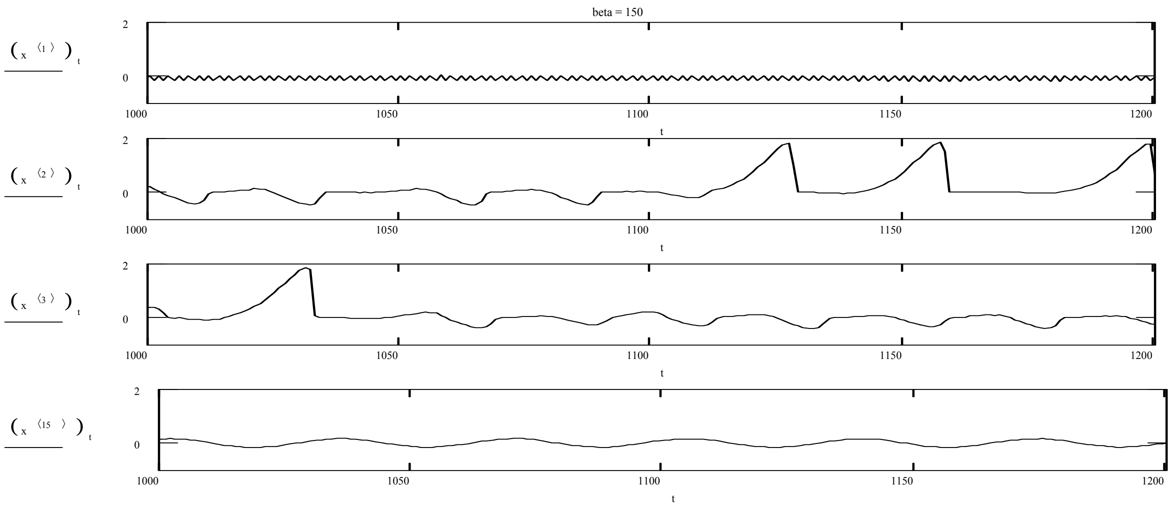

Figure 5.3 Time series of x_t with different memory lengths m: 1, 2, 3, and 15.

Figure (5.5) displays simulations with equal memory length (m=20), but with different values of beta. It confirms the result – higher profitability of fundamentalists N1 on the market also with rising beta values.

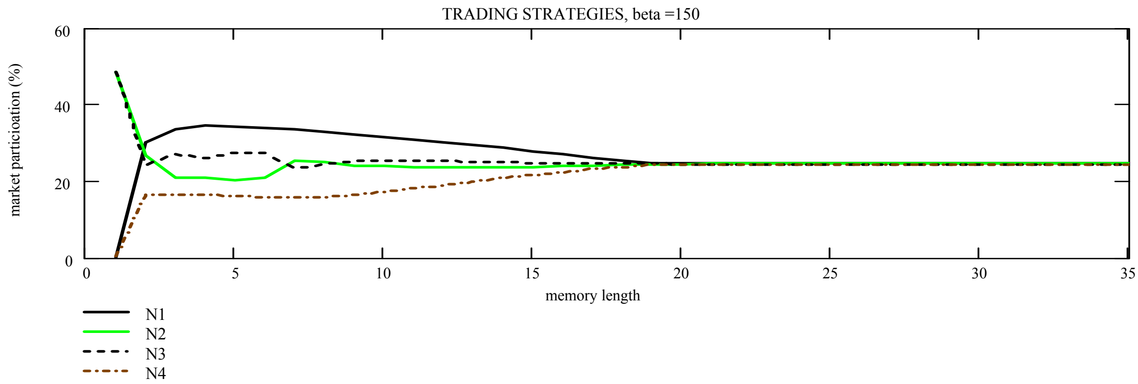

Figure 5.4 Participation of trading strategies on the market

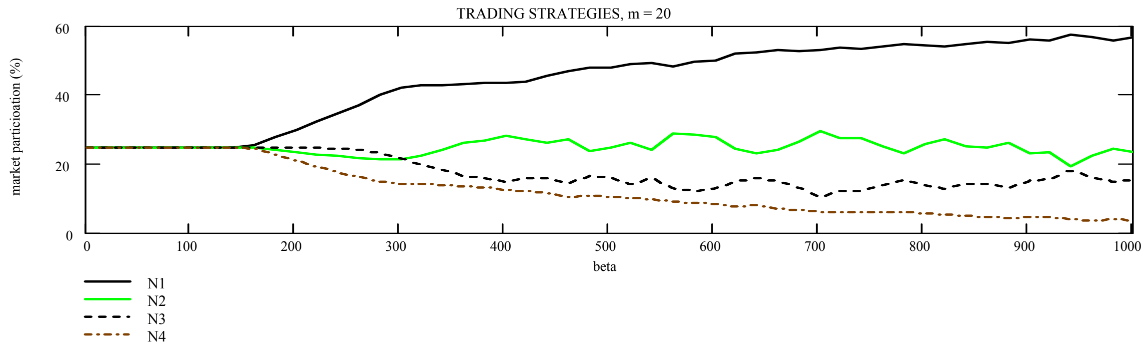

Figure 5. 5 Participation of trading strategies on the market

Case 2: Fundamentalists, Contrarians

Next, we consider an example with four different belief types with parameters:

Type	Parameters		
N1	$g_1 = 0$	$b_1 = 0$	Fundamentalists
N2	$g_2 = -1.1$	$b_2 = 0.2$	Trend with upward bias
N3	$g_3 = -0.3$	$b_3 = -0.2$	Trend with downward bias
N4	$g_4 = -0.5$	$b_4 = 0$	Contrarians

Table 5.2 Parameters of the system for case 2.

Without memory the system has complicated dynamics with maximum values of x within the interval $\langle -0.5, +0.7 \rangle$. The role of fundamentalists (N1) and contrarians without bias is with rising β is negligible (figure 5.6). With longer memory (20 periods, constant memory structure) the system is more stable, the price is less volatile and the amplitude is smaller (figure 5.9).

With higher β (>4300*)* the strategy of fundamentalists (labelled N1) becomes the most profitable strategy on the market. From the beginning contrarians without bias (labelled N4) lose their positions and almost diminish from the market. With such memory structure is visible the importance of bias for contrarians (N3 versus N4).

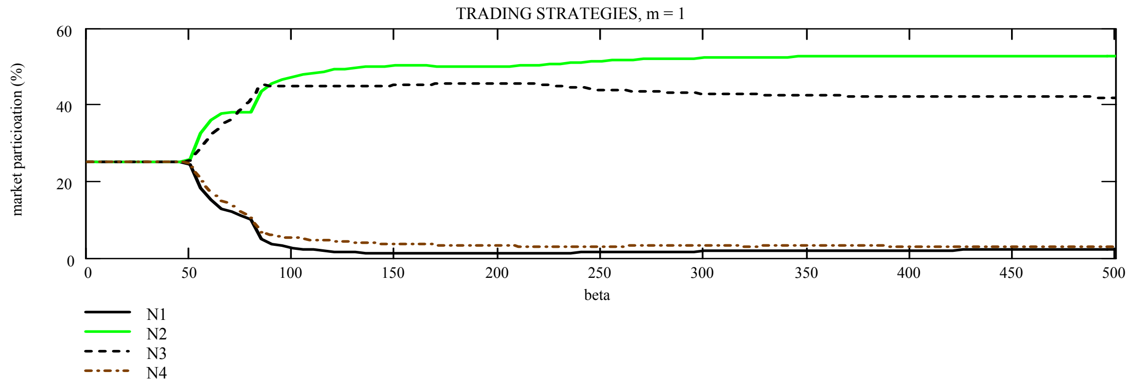

Figure 5.6 Participation of trading strategies on the market.

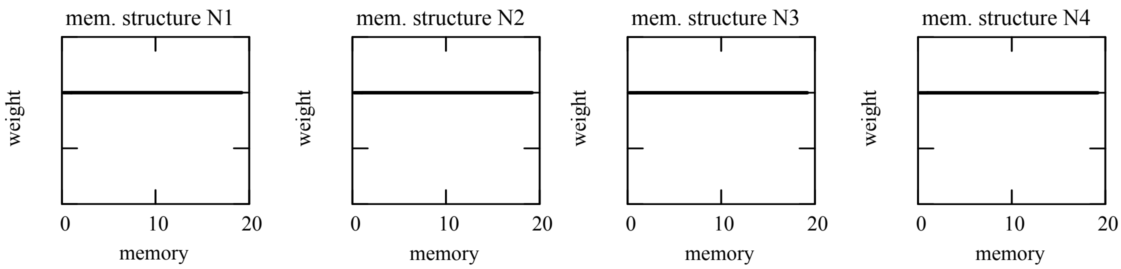

Figure 5.7 Memory length and structure for trading strategies.

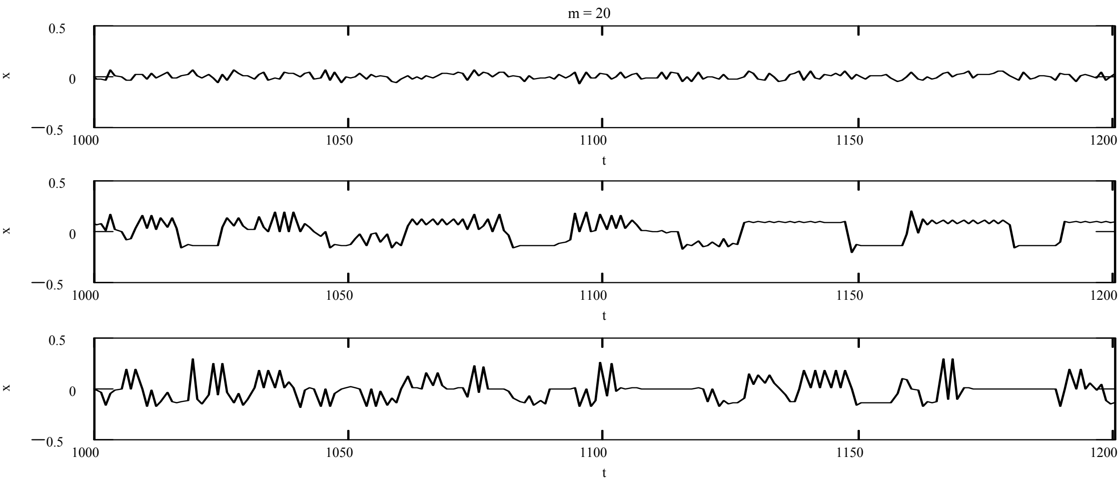

Figure 5.8 Time series of x_t with different values of β: 500, 2000, and 6000.

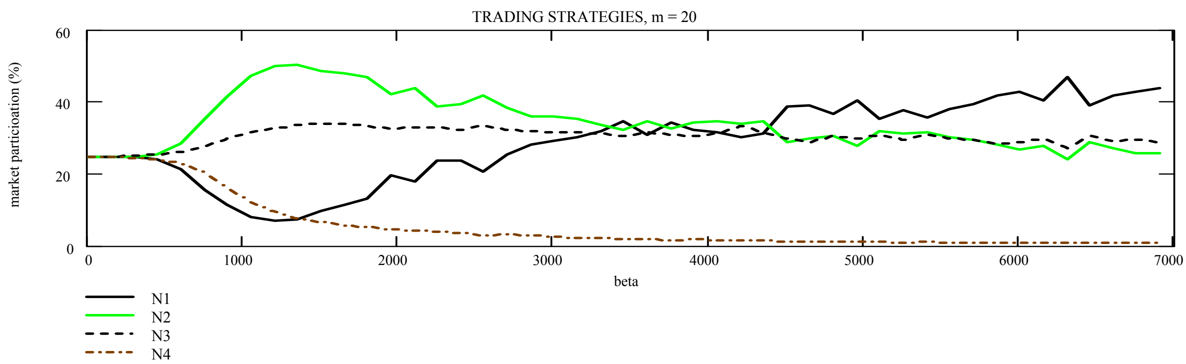

Figure 5.9 Participation of trading strategies on the market

Further analysis has shown system sensitivity on the memory length and structure (memory weights).

The following example has the same coefficients for trading strategies but with shorter memory length for pure contrarians (N4), figure (5.10). In this example is evident the increase of profitability of the strategy number four and also for $\beta > 2300$ it becomes the most profitable one, figure (5.12).

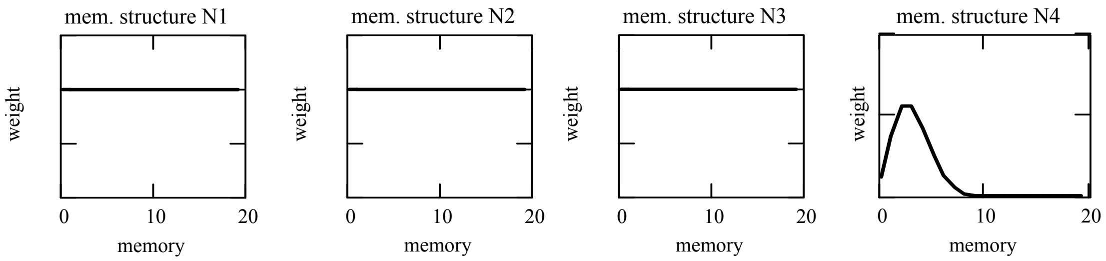

Figure 5.10 Memory length and structure for trading strategies.

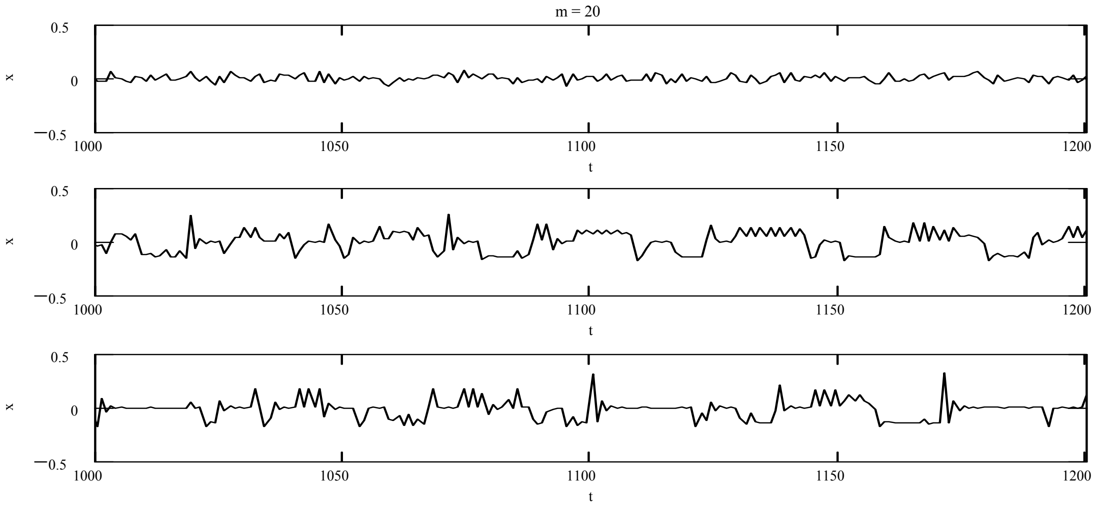

Figure 5.11 Time series of x_t with different values of β: 500, 2000, and 6000.

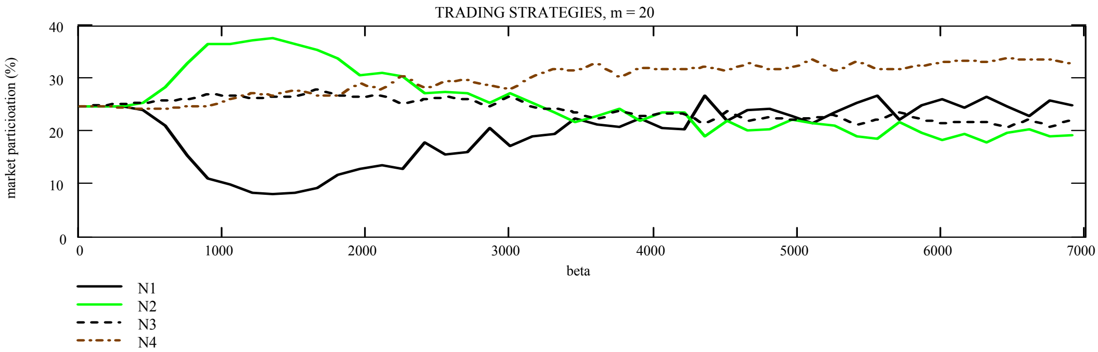

Figure 5.12 Participation of trading strategies on the market

Case 3: Fundamentalists, Trend chasers and Contrarians

For the last case consider system with parameters:

Type	Parameters		
N1	$g_1 = 0$	$b_1 = 0$	Fundamentalists
N2	$g_2 = 1.0$	$b_2 = 0.2$	Trend with upward bias
N3	$g_3 = 0.6$	$b_3 = -0.2$	Trend with downward bias
N4	$g_4 = -0.5$	$b_4 = 0$	Contrarians

Table 5.3 Parameters of the system for case 3.

For memoryless model the leading strategy on the market are trend chasers with downward bias (N3), figure (5.13). With memory (m=20, constant memory structure) contrarians (N4) are becoming the leading strategy on the market. (N1) and (N3) are almost exiting the market, figure (5.15). In this case there also exists significant sensitivity on memory length and structure. Changing the memory structure for fundamentalists N1, figure (5.16) they are becoming significant part of the market, figure (5.17). When using the memory reduction for N3 (the least profitable strategy), figure (5.18) then we get similar result as in preceding case with fundamentalists but the dominance of contrarians N4 is still high, figure (5.19).

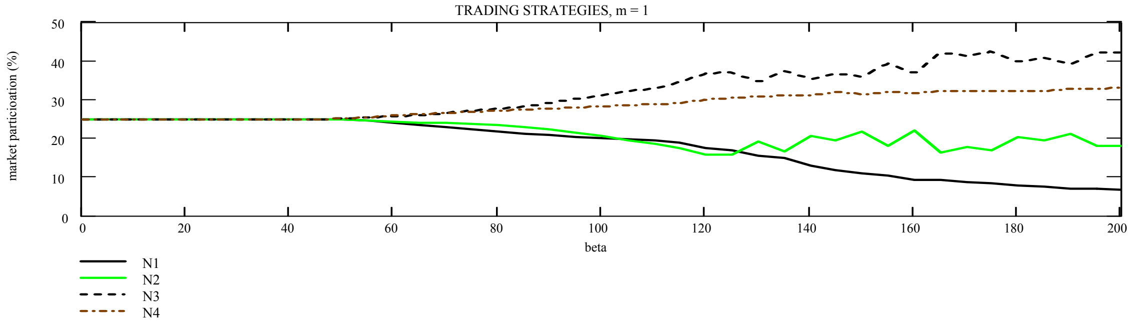

Figure 5.13 Participation of trading strategies on the market

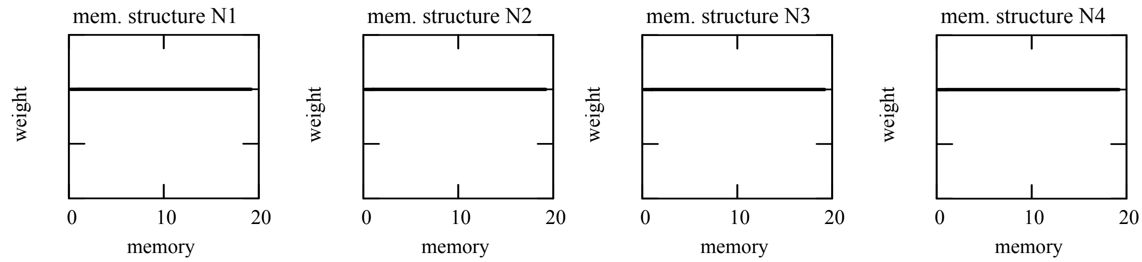

Figure 5.14 Memory length and structure for trading strategies.

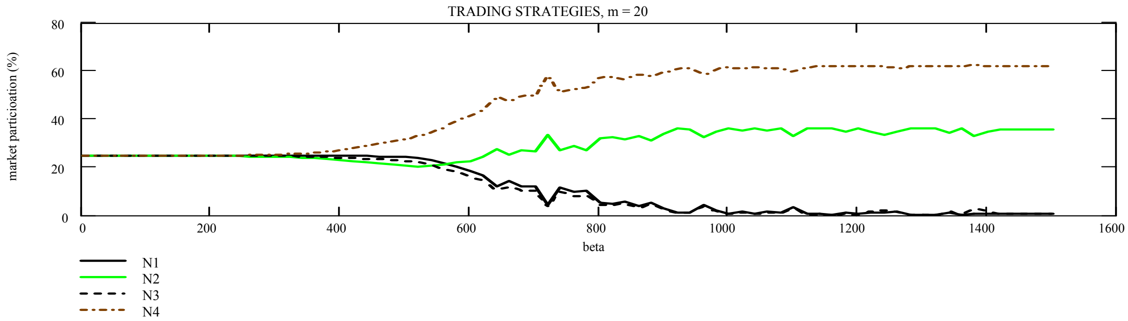

Figure 5.15 Participation of trading strategies on the market

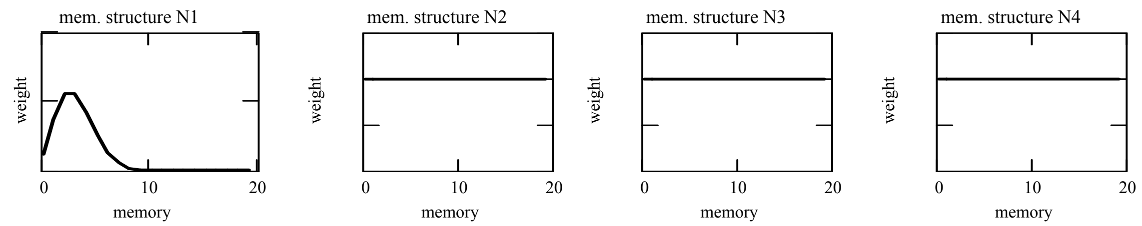

Figure 5.16 Memory length and structure for trading strategies.

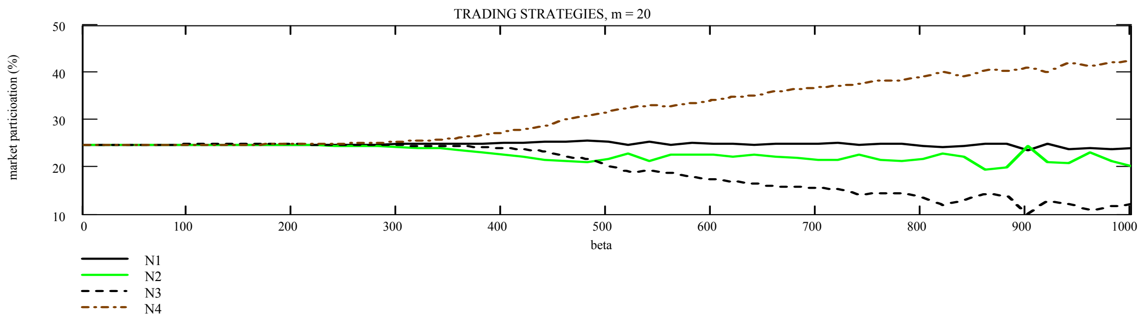

Figure 5.17 Participation of trading strategies on the market

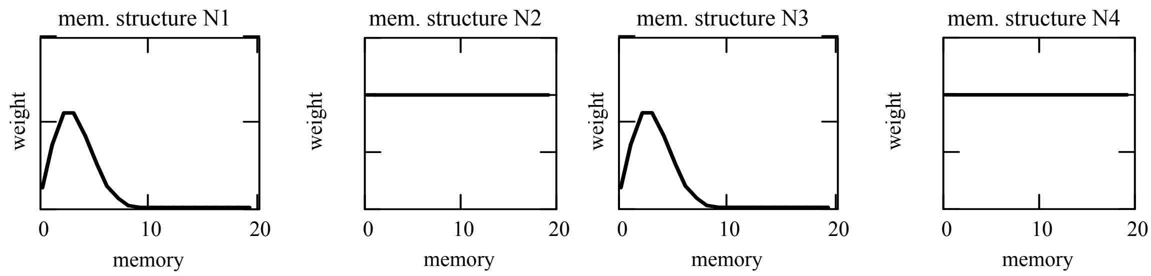

Figure 5.18 Memory length and structure for trading strategies.

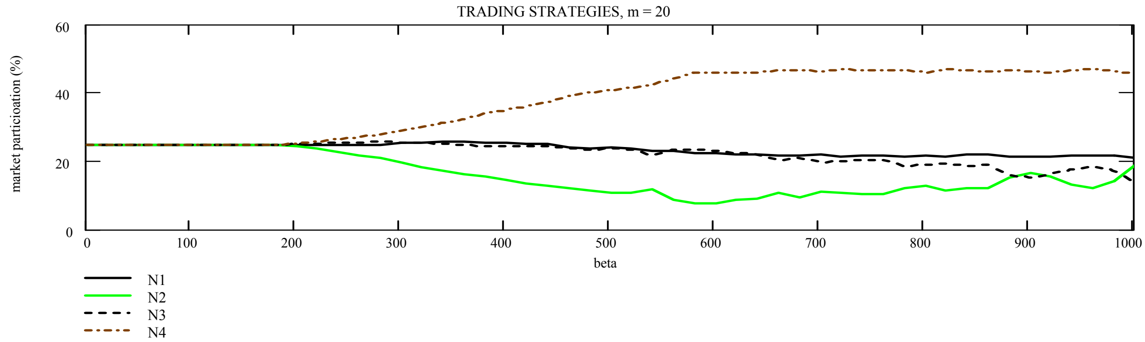

Figure 5.19 Participation of trading strategies on the market

Alternative way how to study this system is changing the memory length with fixed b, figure (5.20). For shorter memory lengths (m < 10) interesting dynamics of traders strategies participation arises and price volatility is higher in that region. For memory lengths higher than 10 contrarians N4 starting to dominate the market.

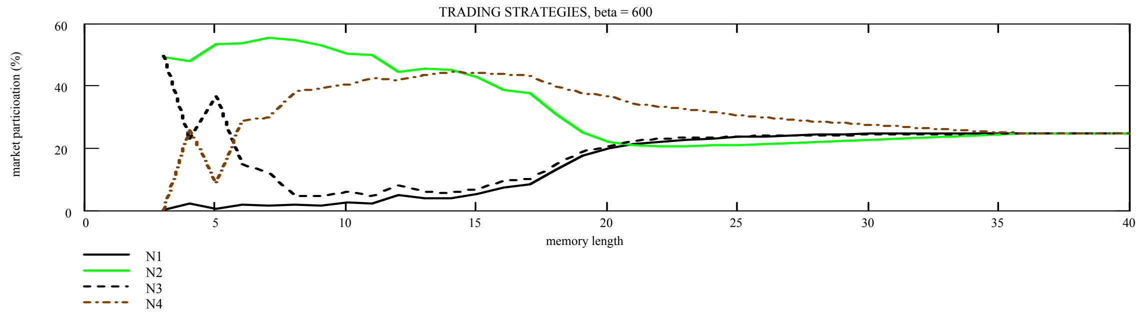

Figure 5.20 Participation of trading strategies on the market

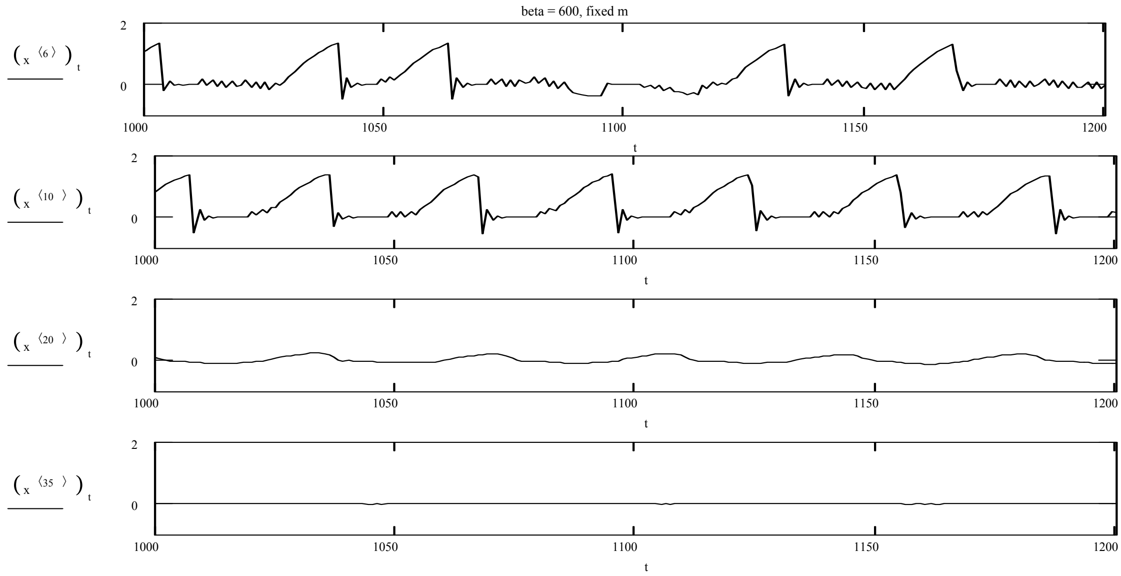

Figure 5.21 Time series of x_t with different memory lengths m: 6, 10, 20, and 35.

6 Conclusions

- The system with memory is more stable than the memoryless system. Higher values of β are needed to generate chaotic behaviour.
- In all cases, memory adding helps fundamentalists to increase profit, i.e., to increase a participation on the market. Especially in the first case and the second case they even become the most profitable strategy as β increases. As was demonstrated in numerical analysis (the first case and the second case) fundamentalists become the most profitable one as β increases. That is a remarkable difference with comparison to the memoryless system.
- A fact of shorter memory length for pure contrarians changes a profitability of this strategy significantly. From the marginal participation, this strategy is becoming leading strategy as β increases.
- It is shown that increased memory helps contrarians outperform other strategies on the market.

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