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Estimating Elasticity of Substitution in CES Production Function: Examining Different Nesting Structures and EU Regions

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Abstract:
Elasticity of factor substitution is one of the key parameters of any computational general equilibrium model. Despite a wide use of this model in a policy analysis, there are a few estimates of the elasticity, with almost none for transition economies in Europe. To fill this gap, we estimate the elasticity of substitution between Capital, Labour, Energy and Material in the constant elasticity of substitution (CES) production function. We use a non-linear estimation technique to derive these elasticities for the whole economy and for five different sectors, for the EU as a whole and for its two sub-regions. We find that Cobb–Douglas and the Leontief production functions do not fit the data better than more flexible CES specification, and after evaluating several multiple KLEM nesting structures of the CES production function we conclude that KL-E and KL-EM nesting structures fit the data best in both EU regions and for the most economic sectors. The economy-wide factor substitution elasticity complies to the one reported in the literature, however, its magnitude varies across sectors, and it is much larger for the energy-intensive sectors. The elasticities also differ between the EU economies in the West and in the East, although their magnitude is converging in more recent years. We recommend a set of the specific elasticities to be used in the impact modelling and conclude that the estimates based on more recent data and that are region-specific should be used in CGE-based policy applications.

JEL: C51, D24
Keywords: Elasticity of substitution; Constant elasticity of substitution (CES); nesting structure; KLEM production function; Central and Eastern European Countries

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1 Introduction

In modern applied economics and especially in the field of environmental and climate policy, Computable General Equilibrium (CGE) models have become one of the leading tools to evaluate policy measures and scenarios (Böhringer et al., 2003). CGE, macro-econometric, input-output or linear programming models use different types of nested production function with Constant Elasticity of Substitution (CES) to describe the production of an economy (Kemfert, 1998). Elasticities of substitution (EoS) are the key parameters in these models as they measure the ease or difficulty of substitution between the inputs in economic production.

Jacoby et al. (2006) perform a sensitivity analysis of structural parameters of their MIT-EPPA CGE model and find that elasticities of substitution between energy and value added (the capital-labour composite) are the main drivers of model results. Similarly, Antimiani et al. (2015) confirms the importance of substitution elasticities using a dynamic CGE model based on the GTAP framework with sector specific values for capital-energy and inter-fuel elasticities. A change in their values generates a different distribution of impacts. A lower flexibility of energy substitution possibilities induces more expensive abatement efforts. The criticism of elasticity of substitution estimates for CGE models has several dimensions. First, the CES function in CGE model should have the same structure as the CES function used for elasticity estimation, since the elasticity of substitution differs across different kinds of nesting structures of CES function. Second, as van der Werf (2008) points out, many models use different values of EoS, even when the models use the same nesting structure. Furthermore, many research papers use elasticity estimates taken from literature, but empirical validations for the nesting structures and values chosen are missing (van der Werf, 2008). Sorrell (2014) adds other difficulties in using empirical to infer values of elasticity substitution specifically for CGE models originating from the fact that CGE models typically: 1) differ from empirical studies in the manner in which individual inputs are aggregated and in the level of sectoral aggregation; 2) require estimates of the elasticity of substitution between nests of inputs, while the parameters estimated by most empirical studies relate to individual pairs of inputs; and 3) define production function by means of Hicks/Direct Elasticity of substitution (HES), while the most empirical studies estimate other types of elasticity of substitution. Furthermore, only a small number of papers have applied nonlinear estimation techniques (Gechert et al., 2019) even though they perform significantly better in comparison to the standard linear estimations using Kmenta approximation (Koesler and Schymura, 2015). Although the regional transferability of substitution elasticity estimates is limited, many papers apply EoS values estimated for different regions. To our knowledge, there is no study focused primarily on Central and Eastern European (CEE) countries so far. As a consequence, the lack of proper estimates of elasticity of substitution specific for CEE countries is higher than in other regions. In this paper, we seek to fill this gap by providing consistent sectoral estimates of elasticity of production factors' substitution in 10 CEE countries, 17 other EU Member States (WEST) and the whole EU to compare them with the CEE region. We report the accompanying elasticity of substitution for both three and four-input CES production functions in five different nesting structures.

The remainder of the paper is organized as follows. First, we review the existing literature. Next, we specify our models and describe the data and our estimation approach. In section 4, we present our results and verify whether often used CES production functions in Cobb-Douglas and Leontief form fit our data. We test our estimated EoS for regional and time difference. The last section concludes.

2 Literature review

Lack of empirical validations for the EoS values either taken from literature or expert-based mentioned by van der Werf (2008) can be detected in a variety of models.Often, the incohorence between nesting structure or data in the calibrated model versus the source for its calibration is present.

An applied general equilibrium model SAGE (Marten et al., 2019) of the US economy is calibrated using the sector specific elasticity estimates by Koesler and Schymura (2015) for a pool of 40 countries using the WIOD database. The sectors in the SAGE model adhere to the sectors' structure used by Koesler and Schymura (2015) only partly. The elasticities of substitution between capital, labour and energy in the CGE model ICES (Intertemporal Computable Equilibrium System) (Parrado and De Cian, 2014) are calibrated based on Carraro and Cian (2012) who use a non-nested CES production function with an elasticity of substitution estimated as 0.38. Parrado and De Cian (2014) use this same elasticity value for eight world regions. Another example of regional data incoherence is a CGE model for Korea by Oh et al. (2020) with a three-level CES nesting structure ((KL)E)M. The Korean model is calibrated based on Okagawa and Ban (2008) who estimate substitution elasticities for a pool of 14 major world economies. The well known Emissions Prediction and Policy Analysis (EPPA) recursive-dynamic CGE model for the whole world economy (Paltsev, 2005) uses for its elasticity calibration values proposed by Cossa (2004) who conducted a literature review and a expert elicitation. The EPPA model uses the same elasticity values across sectors and countries. The I3E model (de Bruin and Mert Yakut, 2020) is a country-specific intertemporal CGE model focused on assessment of climate policies' economic and environmental impacts specifically for Ireland. The elasticities of the CES production function in the I3E model are based on expert judgement without any further specification. The same complication applies for the calibration of the JRC-GEM-E3 model (Vandyck et al., 2016) which uses expert based values of substitution elasticity in a two-level (KL)(EM) CES production function. Kiuila et al. (2019) propose for both the substitution between capital and labor and capital and electricity a value of 0.2 without further specifying the source for the calibration. A capital-energy substitution elasticity of 0.5 is assumed across sectors in the GTAP-E model (Burniaux and Truong, 2002) based on a literature review.

The substitution between capital-labor-energy composite and materials is assumed to be in the Leontief form without empirical verification of its appropriateness for the GTAP-E model (Burniaux and Truong, 2002) model. Similarly, authors assume a Leontief specification in the energy-materials nest and a Cobb-Douglas specification in the capital-labour nest in the case of a Japanese CGE model (Huang and Kim, 2019). An empirically not validated Cobb-Douglas structure for capital-labor substitution is used also by the model World Induced Technical Change Hybrid (WITCH) integrated assessment model (Emmerling, 2016). Other elasticities used to calibrate the WITCH model are based on a review of literature from the 90's. Moreover, the model uses the same elasticities across sectors and countries. Analogously, the CGE model NEWAGE (National European World Applied General Equilibrium) developed within the project REEM Pathways (REEEM Project, 2019) assumes a Cobb-Douglas EoS between capital and labor and Leontief EoS between KLE composite and materials, the KL-E EoS is calibrated at 0.5. Values are calibrated based on Beestermoeller (2016).

Regarding the estimation methods for the CES production function parameters, Gechert et al. (2019) collected 121 studies estimating substitution elasticities. A vast majority of estimates come from single-level production functions with capital and labor as inputs. Almost 70 % of considered studies estimated EoS via either single first order conditions (FOCs) for capital or labor or their systems. Another large part of studies used Kmenta (Kmenta, 1967) linear approximation of production function. Henningsen and Henningsen (2011) cites main complications of this approach as a method reliable only for the Cobb-Douglas production function, e.g. when $\sigma \to 1$, and that Kmenta approximation is by itself a truncated Taylor series with the remainder term being an omitted variable.

Nonlinear estimation techniques have been applied by only a limited number of research papers (Gechert et al., 2019). Kemfert (1998) estimates the elasticity of substitution for tree two-level nested CES production functions for the entire German industry and individual industrial sectors and comes to a conclusion that the specification with capital and energy in one nest (KE)L fits best the entire German industry, but a nest of capital and labour (KL)E might be closer to reality for several industrial sectors. The appropriateness of the (KL)E nesting structure was confirmed for industrial level data from 12 developed OECD countries for the 1978-1996 time horizon by van der Werf (2008). Contrarily to both Kemfert (1998) and van der Werf (2008), a nesting structure where substitution between labor-energy composite and capital is allowed, fits best the Canadian economy as Dissou et al. (2015) found out by fitting a production function with three inputs of capital, labor and energy to Canadian data. Including materials as a fourth production input, Okagawa and Ban (2008) estimated a nested CES function using more disaggregated OECD dataset with 19 sectors compared to van der Werf (2008) defining 7 sectors. The elasticity of substitution for the German industry provided by Kemfert (1998) was re-estimated by Henningsen and Henningsen (2011) using the same data and a non-linear least squares estimation method. They apply several estimation approaches that yield robust results significantly different from those obtained by Kemfert (1998). Koesler and Schymura (2015) use the World-Input-Output Database (WIOD) to estimate elasticity of substitution for a three-level four-input nested CES ((KL)E)M production function via non-linear least squares estimation method developed by Henningsen and Henningsen (2011). They estimate the substitution elasticity for 36 sectors pooled across all 40 countries included in WIOD over a period of 12 years (1995-2006). This dataset has the advantage of higher number of observation, but since the WIOD includes not only European countries but also 13 other major world countries, Koesler and Schymura (2015) lose the geographic consistency. Lecca et al. (2011) investigate the issue of correct nesting of the KLEM production function with focus on energy based on a Macro-micro model Of Scotland (AMOSENVI) CGE model parameterised on Scottish data and criticize arbitrary choice of a nesting structure of production function.

The different conclusions of mentioned studies serve as an example of limited regional transferability of substitution elasticity estimates. All of them focus on well developed countries but to our knowledge, there is no study focused primarily on Central and Eastern European (CEE) countries so far. As a consequence, the lack of proper estimates of elasticity of substitution specific for CEE countries is higher than in other regions.

3 Methodology

3.1 Nesting structure specifications

The CES production function as a general form of the Cobb-Douglas (CD) was introduced by Solow (1956) and later popularized by Arrow et al. (1961). In contrast to CD, CES allows for non-unity elasticities of substitution between production factors. As Zha and Zhou (2014) mentioned, the nesting of production factors allows for different elasticities since factors on the same level are substituted with the same elasticity.

In our analysis, we benefit from the flexibility of the CES production function (Böhringer et al., 2003) and employ three different ways of specification with a total of five nesting structures.

First, in a two-level CES production function, the three inputs of capital (K), labor (L) and energy (E) can be combined as follows:

$$y_t = \gamma e^{t\lambda} \left[\alpha \left(\alpha_1 K_t^{-\rho_1} + (1-\alpha_1) E_t^{-\rho_1} \right)^{\frac{\rho}{\rho_1}} + (1-\alpha) L_t^{-\rho} \right]^{\frac{-\nu}{\rho}}, \quad (1)$$

$$y_t = \gamma e^{t\lambda} \left[\alpha \left(\alpha_1 K_t^{-\rho_1} + (1-\alpha_1) L_t^{-\rho_1} \right)^{\frac{\rho}{\rho_1}} + (1-\alpha) E_t^{-\rho} \right]^{\frac{-\nu}{\rho}}, \quad (2)$$

$$y_t = \gamma e^{t\lambda} \left[\alpha \left(\alpha_1 L_t^{-\rho_1} + (1-\alpha_1) E_t^{-\rho_1} \right)^{\frac{\rho}{\rho_1}} + (1-\alpha) K_t^{-\rho} \right]^{\frac{-\nu}{\rho}}, \quad (3)$$

where y is the output, $\gamma \in (0, \infty)$ is an efficiency parameter, $\lambda \geq 0$ is the rate of technological change, t is time variable, α and $\alpha_1 \in (0, \infty)$ set the optimal distribution of inputs, ρ and $\rho_1 \in (-1, 0) \cup (0, \infty)$ determine the (constant) elasticity of substitution, and $\nu \in (0, \infty)$ is equal to 1 in the case of constant returns to scale.

Second, we consider also intermediate inputs (M) and estimate the four-input two level nested CES production function ((KL)(EM)) introduced by Sato (1967) and estimated by Lecca et al.

(2011) as follows:

$$y_t = \gamma e^{t\lambda} \left[\alpha \left(\alpha_1 K_t^{-\rho_1} + (1-\alpha_1) L_t^{-\rho_1} \right)^{\frac{\rho}{\rho_1}} + (1-\alpha) \left(\alpha_2 E_t^{-\rho_2} + (1-\alpha_2) M_t^{-\rho_2} \right)^{\frac{\rho}{\rho_2}} \right]^{\frac{-\nu}{\rho}}, \quad (4)$$

Lastly, we estimate a three-level CES nesting structure ((KL)E)M as in Koesler and Schymura (2015) based on Sato (1967) and Henningsen and Henningsen (2011). The production function has the following form:

$$y_t = \gamma e^{t\lambda} \left[\alpha_2 M_t^{-\rho_2} + (1-\alpha_2) \left((\alpha_1 E_t^{-\rho_1} + (1-\alpha_1) VA_t^{-\rho_1})^{\frac{1}{-\rho_1}} \right)^{-\rho_2} \right]^{\frac{1}{-\rho_2}} \quad (5)$$

with

$$VA_t = \left(\alpha K_t^{-\rho} + (1-\alpha) L_t^{-\rho} \right)^{\frac{1}{-\rho}} \quad (6)$$

where VA is a value-added compound of K and L. As Koesler and Schymura (2015) state, the separability implied by the CES framework allows us to divide the three-level nesting structure into two equations 5 and 6 and overcome the limitation of the software that we are using for the estimation. The micEconCES package proposed by Henningsen and Henningsen (2011) allows for two-level nesting structures only.

For 1 - 5, the elasticities of substitution σ, σ_1 and σ_2 are defined as:

$$\sigma = \frac{1}{1+\rho}, \quad (7)$$

$$\sigma_1 = \frac{1}{1+\rho_1}, \quad (8)$$

and

$$\sigma_2 = \frac{1}{1+\rho_2}. \quad (9)$$

We estimate the Hicks-McFadden (direct) elasticity of substitution (HES) between the inputs in the lower nest and the Allen-Uzawa (partial) elasticity of substitution (AES) between the nests. HES elasticity of substitution describes the input substitutability of two inputs i and j along an isoquant given that all other inputs are constant. The AES describes the input substitutability of two inputs when all other input quantities are allowed to adjust[1]. Two inputs within an individual nest are necessary HES substitutes, they may at the same time be AES

[1]For details on the HES and AES specification, please see Henningsen and Henningsen (2011).

complements (Sorrell, 2014).

3.2 Data and estimation procedure

For our analysis we benefit from the World-Input-Output database (WIOD) (Timmer et al., 2012) as a consistent source of data. Since we take into account the limited transferability of the substitution elasticity, we focus only on the EU Member States and CEE countries as a subsample of the EU. We combine the Gross output (Y), Intermediate inputs (M), Gross Value added (VA), Labour compensation (L), and Gross capital stock (K) from the Socio-Economic Accounts in the WIOD database with the Gross energy use (E) from the Environmental Accounts. All prices (Y, K, L, VA, M) are converted to USD 2010 using the sector and variable specific price indices. Data covers the 15-year period (2000-2014) in 27 EU Member States including 10 CEE countries listed in Table 1.

Table 1: Countries included in the analysis

Country	Code	Region	Country	Code	Region
Austria	AUT	WEST	Ireland	IRL	WEST
Belgium	BEL	WEST	Italy	ITA	WEST
Bulgaria	BGR	CEE	Lithuania	LTU	CEE
Cyprus	CYP	WEST	Luxembourg	LUX	WEST
Czech Republic	CZE	CEE	Latvia	LVA	CEE
Germany	DEU	WEST	Malta	MLT	WEST
Denmark	DNK	WEST	Netherlands	NLD	WEST
Spain	ESP	WEST	Poland	POL	CEE
Estonia	EST	CEE	Portugal	PRT	WEST
Finland	FIN	WEST	Romania	ROU	CEE
France	FRA	WEST	Slovakia	SVK	CEE
United Kingdom	GBR	WEST	Slovenia	SVN	CEE
Greece	GRC	WEST	Sweden	SWE	WEST
Hungary	HUN	CEE			

The 34 WIOD sectors are aggregated according to Baccianti (2013) in two ways. First, we use the classical division on primary, secondary and tertiary sectors. Second, we divide the economy into energy intensive and energy efficient industries. A sector is classified as energy intensive in case of average energy share on total costs surpasses 5%. Among such sectors, we can find agriculture, mining, energy generation, transport, and manufacture of selected goods. The full overview of sector aggregation is provided in Table 4 in Appendix. After dropping observation with missing or zero values, first and last 4 percentiles with outlier values of capital/output ratio and extreme values of energy/output ratio, we obtain 21 883 observations for the EU, 7 486 for CEE and 14 397 for WEST countries. Table 2a describes data summary statistics across regions

and Table 2b describes data summary statistics across aggregated sectors.

Table 2a: Summary statistics of our data sample across regions

Region	Variable	Unit	Obs.	Mean	Std. dev.	Min	Max
EU	Y	mil.$	21 883	18 628	41 152	1.5	514 835
	VA	mil.$	21 883	8 779	21 520	0.4	236 439
	K	mil.$	21 883	16 807	53 678	1	994 282
	L	mil.$	21 883	5 989	15 922	0.4	190 768
	E	TJ	21 883	59 390	328 625	0	6 466 591
	M	mil.$	21 883	9 883	22 251	0.4	363 322
CEE	Y	mil.$	7 486	3 911	7 025	2.4	112 486
	VA	mil.$	7 486	1 650	3 130	0.5	37 114
	K	mil.$	7 486	3 636	6 648	1.2	71 276
	L	mil.$	7 486	918	1 882	0.7	27 593
	E	TJ	7 486	24 554	124 088	0.1	1 891 246
	M	mil.$	7 486	2 271	4 481	1.7	98 352
WEST	Y	mil.$	14 397	26 280	48 758	1.5	514 835
	VA	mil.$	14 397	12 486	25 665	0.4	236 439
	K	mil.$	14 397	23 656	64959	1	994 282
	L	mil.$	14 397	8 487	19 076	0.3	190 768
	E	TJ	14 397	77 503	393 939	0	6 466 592
	M	mil.$	14 397	13 841	26 389	0.4	363 322

Figure 1 shows the average K/Y ratio in CEE countries which is approximately 11 percentage points (pp) higher than in WEST countries almost during the whole period 2000-2014. This indicates a higher average efficiency of capital in WEST countries. After 2008, the ratio increased by about 10 percentage points (pp) and remains fluctuating around 1.1. The ratio increased slightly after 2009 also in the case of WEST by about 7 pp. The L/Y ratio remains approximately the same over the whole period for both regions. On average, the ratio rests by about 3 pp lower in CEE countries than in WEST countries. This reflects the lower average wage in CEE countries in comparison to Western Europe (Goraus-Tanska and Lewandowski, 2016). The labour/capital ratio shows a slightly decreasing trend in both regions presumably arising from productivity improvements obtained thru capital investments and automatizing of production procedures.

Table 2b: Summary statistics of our data sample across aggregated sectors

Aggregated sector	Variable	Unit	Obs.	Mean	Std. dev.	Min	Max
Primary	Y	mil.$	1 669	7 808	18 066	2	119 619
	VA	mil.$	1 669	4 118	11 477	1	99 119
	K	mil.$	1 669	13 528	33 956	2	262 653
	L	mil.$	1 669	1 808	3 698	1	31 260
	E	TJ	1 669	36 510	183 386	0.2	2 770 169
	M	mil.$	1 669	3 724	8 206	1	65 962
Secondary	Y	mil.$	9 519	16 599	39 312	1	514 836
	VA	mil.$	9 519	5 351	13 432	0.1	177 714
	K	mil.$	9 519	11 195	35 404	1	644 308
	L	mil.$	9 519	3 325	9 440	0.1	145 183
	E	TJ	9 519	103 326	484 357	0.3	6 466 591
	M	mil.$	9 519	11 273	26 597	0.2	363 323
Tertiary	Y	mil.$	10695	22 123	44 774	6	380 111
	VA	mil.$	10 695	12 558	27 174	1	236 439
	K	mil.$	10 695	22 314	67 383	9	994 282
	L	mil.$	10 695	8 825	20 499	3	190 768
	E	TJ	10 695	23 855	62 183	1	786 806
	M	mil.$	10 695	9 608	19 113	1	195 627
Energy Intensive	Y	mil.$	7 237	11 899	23 019	2	181 723
	VA	mil.$	7 237	4 271	9 396	1	99 119
	K	mil.$	7 237	14 241	33 982	2	323 185
	L	mil.$	7 237	2 277	4 867	0.3	57 027
	E	TJ	7 237	151 804	558 157	0.2	6 466 591
	M	mil.$	7 237	7 659	15 113	0.1	128 815
Energy non-intensive	Y	mil.$	14 646	21 953	47 277	1	514 836
	VA	mil.$	14 646	11 007	25 167	0.3	236 439
	K	mil.$	14 646	18 075	61 072	1	994 282
	L	mil.$	14 646	7 687	18 905	0.1	190 768
	E	TJ	14 646	13 725	33 563	0.4	786 806
	M	mil.$	14 646	10 983	24 967	1	363 323

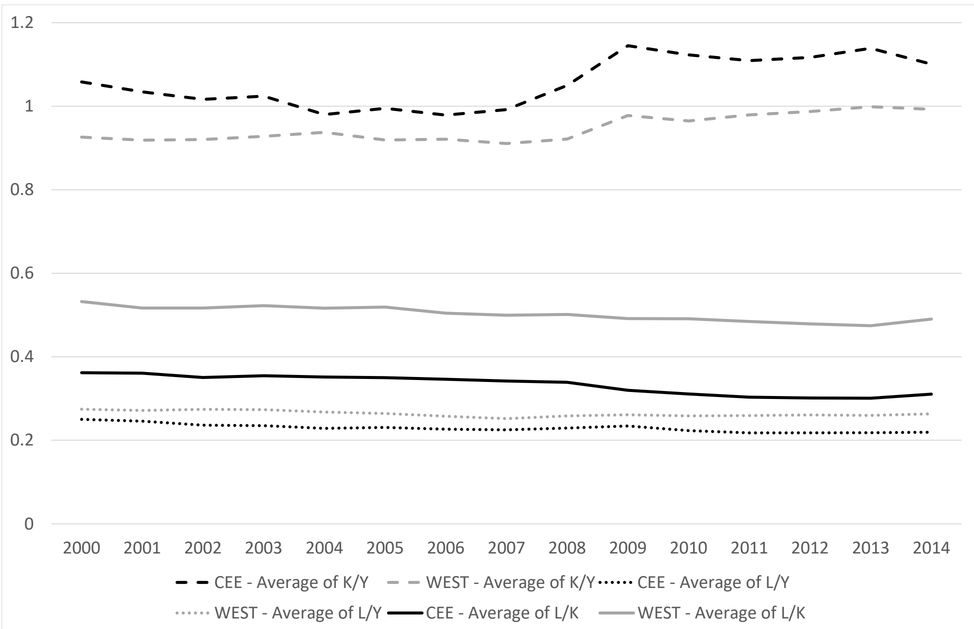

Figure 1: Capital/Output ratio (K/Y) Labour/Output(L/Y) and Labor/Capital (L/K) ratio

The biggest difference between the CEE region and the rest of Europe is in the energy intensity (Figure 2). In the period 2000 - 2008 the difference between the production energy intensity in CEE countries and the rest of Europe reached almost 50 % or even higher in some years. After 2009, the energy intensity in CEE countries has been slowly converging to WEST countries' level. This could be a consequence of a change in the economic structure of CEE countries, ie. reorienting towards less energy-intensive industries such as services. The energy/capital ratios for both regions are steadily decreasing over the observed period and the gap between them is slowly closing and ending with a 19 % difference in 2014. This could indicate the rising energy efficiency of machinery and equipment and an overall technological progress as well as modernisation of equipment in CEE countries.

For the estimation of the substitution elasticity, we use a micEconCES package in R developed by Henningsen and Henningsen (2011). The micEconCES package provides a robust estimation tool for elasticity of substitution (Koesler and Schymura, 2015).

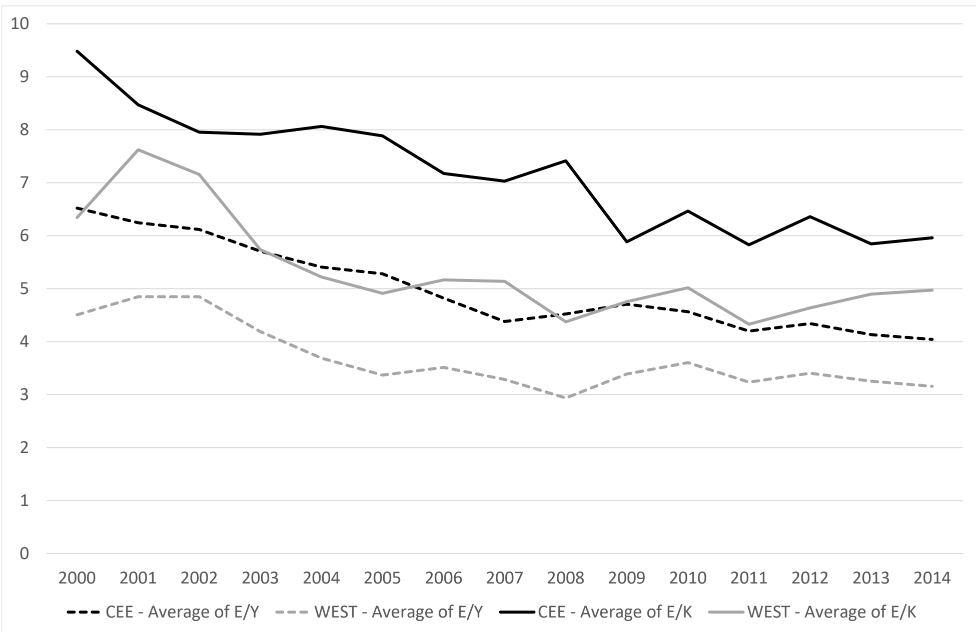

Figure 2: Energy/Output and Energy/Capital ratio [TJ / mil. €million] in CEE and WEST coutries

Besides the Kmenta approximation critique, Henningsen and Henningsen (2011) further mention the Levenberg-Marquardt algorithm (LM) (Marquardt, 1963) as a method with tendencies for biasing the elasticity estimates towards zero and the Conjugate Gradients (CG) method based on Fletcher and Reeves (1964) as less suitable for CES estimation due to its focus on well-behaved approximately quadratic objective functions and large and sparse Hessian matrix. For the above mentioned reasons, we decided to leave Kmenta approximation, CG and LM optimisation algorithms out of our analysis.

However, we employed multiple optimisation algorithms for our estimation of the CES function parameters, namely the Newton algorithm (Dennis and Schnabel, 1983), Nelder-Mead routines (NM) (Nelder and Mead, 1965), the Simulated Annealing algorithm (SANN) (Kirkpatrick et al., 1983), both the Broyden-Fletcher-Goldfarb-Shanno (BFGS) and the restricted BFGS (L-BFGS-B) (Broyden, 1970; Fletcher, 1970; Goldfarb, 1970; Shanno, 1970) routines, Differential Evolution (DE) (Storn and Price, 1997), PORT routines (Gay, 1990) and a two-dimensional grid search for ρ_1 and ρ_1 using PORT algorithm and this algorithm using starting values equals to the estimates from the grid search (Henningsen and Henningsen, 2011).

The SANN algorithm yields results with the best fit to our data in all nesting structures, most significant estimates and the least residual sum of squares in most cases. Henningsen and Henningsen (2011) describes SANN algorithm as a "robust global optimiser" with a possibility to apply to "a large search space, where it provides fast and reliable solutions" and Feng and Zhang (2018) apply SANN as a well performing method to find global optima for not well-behaved objective function. Detailed description of the SANN algorithm is provided by Kirkpatrick et al. (1983) and Cerny (1985). For clarity and brevity, in the remainder of the paper we only present results based on the SANN optimisation algorithm.

4 Results

4.1 Choice of nesting structure

Estimates of substitution elasticity for the two-level three-input nesting structures of the CES function (KE)L, (KL)E and (LE)K given by equations 1 to 3, respectively, are displayed in Table 5 together with their standard errors s in parentheses. Estimates are provided separately for the aggregated sectors and for CEE and WEST regions as well as the whole EU. Table 5 also provides the goodness of fit R^2 for each estimation.

Kemfert (1998) and van der Werf (2008) suggest the goodness of fit R^2 as a criterion for identifying the most fitting nesting structure. However, Feng and Zhang (2018) points out the compliance of estimated CES parameters to economic meaning and convergence to assumed elasticities as important points in the decision process.

Table 5 shows that all three nesting specifications provide very similar goodness of fit to the data across the regions and sectors. Only the average R^2 for the (LE)K specification is by about 1 pp lower than the other two nesting structures. A very similar result is achieved when comparing the residual sum of squares (RSS) where all three specifications appear to be equivalently suitable[2].

(LE)K specification According to the review by Lagomarsino (2020), the (LE)K specification has been chosen only very sporadically. Few examples include CES elasticity estimates by Turner et al. (2012) for the UK, Dissou et al. (2015) for Canada, and Su et al. (2012) and Shen and Whalley (2013) for China. While in our case, the elasticity estimates σ and σ_1 are consistent across regions, some of the parameters' estimates do not comply with their economic meaning. Specifically, Feng and Zhang (2018) propose to verify that α, α_1, ν and λ all rest in acceptable ranges. In five cases, $\alpha > 1$ and in some cases α_1 is negative. For energy non-intensive (EnI) and tertiary (III.) sector, as well as for the whole economy, $\sigma_{L,E}$ estimates are very inconsistent across regions, see Table 5. $\sigma_{L,E}$ for EnI in CEE region is 0.56 while for WEST countries reaches 1.61. Similarly for III. sector, $\sigma_{L,E} = 0.39$ for CEE and $\sigma_{L,E} = 1.91$ for WEST. On the level of the whole economy, the difference between regional estimates reaches 0.73. On the upper nest, four estimates of $\sigma_{LE,K}$ are insignificant and the regional inconsistency is present for the EnI sector with a difference of 0.92.

(KE)L specification Feng and Zhang (2018) propose $\sigma_{K,E} = 0.5$ and $\sigma_{KE,L} = 1$ as initial points taken from the GTAP-E model (Burniaux and Truong, 2002). Our estimates of $\sigma_{K,E}$ are approaching these values on the level of the whole economy. However, on sectoral level, estimates differ and range between 0.6 and 4.13 for CEE, and 0.4 and 0.89 for WEST countries. The vastest inconsistency in estimates for CEE and WEST regions is present in energy intensive industries. All estimates $\sigma_{K,E}$ are significant on 0.1% level. Even more regionally inconsistent are $\sigma_{KE,L}$ estimates especially in case of primary (difference 2.6), tertiary (difference 3.2) and energy non-intensive (difference 2.1) sectors. The average $\sigma_{KE,L}$ for CEE region is 0.79 with average standard error of 0.8 while for WEST countries those numbers are 2.15 and 0.87, respectively. Three of $\sigma_{KE,L}$ estimates are insignificant, see Table 5.

[2]RSS are not provided due to space limitation. The full set of results is available upon request

(KL)E specification According to Feng and Zhang (2018) and the GTAP-E model (Burniaux and Truong, 2002), initial points for $\sigma_{K,L}$ and $\sigma_{KL,E}$ are 0.59 and 0.94, respectively. In four cases, estimates of α and α_1 do not lie within acceptable ranges. Five of our $\sigma_{K,L}$ estimates are insignificant. All estimates of $\sigma_{KL,E}$ are significant on at least 5% level. The average estimated $\sigma_{K,L}$ for CEE region is 1.0 (with average standard error (s.e.) 0.2) and 0.95 (average s.e. is 2.84) for WEST countries. For the upper nest, the average $\sigma_{KL,E}$ is 0.77 (avg. s.e. 0.12) for CEE and 1.5 (avg. s.e. 0.32) for WEST region. The difference between $\sigma_{K,L}$ and $\sigma_{KL,E}$ estimates for the two regions is much smaller than for the (KE)L structure except for the upper nest in the energy non-intensive industries with $\sigma_{KL,E}$ estimated as 0.55 for CEE and 5.36 for WEST countries.

Overall, the production function in the CES form seems as a reliable and reasonable assumption. The (KL)E and (KE)L nesting specifications provide similar fit to our data but the (KL)E structure shows considerably lower elasticity inconsistencies across regions. Thus, our results support the (KL)E nesting structure as a superior choice in case of European data.

((KL)E)M specification In the three-level four-input ((KL)E)M CES nesting structure, four estimates on the bottom, two on the middle and four on the upper nest are not significant, see Table 6. The residual sum of squares is by one order lower compared to the three-input specification. The R^2 of the ((KL)E)M specification is 0.98. However, in eight cases in total, λ is negative, which violates the assumption of non-negative rate of technological change. If we do not take insignificant estimates into account, the average estimate of $\sigma_{K,L}$ is 2.04 with avg. s.e. 0.45 for CEE countries and 1.03 (avg. s.e. 0.12) for WEST region. Average regional estimates of $\sigma_{KL,E}$ are similar, 0.56 (avg. s.e. 0.11) and 0.66 (avg. s.e. 0.08) for CEE and WEST, respectively. On the upper level, average estimate of $\sigma_{KLE,M}$ is equal to 1.47 (avg. s.e. 0.16) for CEE and 2.18 (avg. s.e. 0.13) for WEST region.

(KL)(EM) specification The goodness of fit and the residual sum of squares of the (KL)(EM) model is similar to the three-input CES function, see Table 7. On the bottom nest, two estimates of $\sigma_{K,L}$ for the CEE region and three estimates of $\sigma_{E,M}$ are not significant on 10 % level. On the upper nest, all estimated $\sigma_{KL,EM}$ are significant on 5 % level. The estimates on the lower nest are consistent across regions. The average estimates of: $\sigma_{K,L}$ are 0.7 (avg. s.e. 0.5) and 0.74 (avg. s.e. 0.16), $\sigma_{E,M}$ are 1 (avg. s.e. 0.52) and 0.84 (avg. s.e. 0.11), and $\sigma_{KL,EM}$ are 1.21 (avg. s.e. 0.11) and 1.62 (avg. s.e. 0.05) for CEE and WEST, respectively. Consequently, the (KL)(EM) model appears as the best fit for our data.

4.2 Elasticity of factor substitution: central estimate

Gechert et al. (2019) prove the irrelevancy of the Cobb-Douglas production function and find that the most representative elasticity of substitution between capital and labor found in the literature is 0.5 when considering aggregated economy level data, and 0.3 under the restriction of industry-level country-level data.

When examining closely the results of the (KL)(EM) specification given in Table 7, we find that the estimates of substitution elasticity between capital and labor approach the value found by Gechert et al. (2019) on the aggregate level both for the two regions and the EU27. Our estimate of $\sigma_{K,L}$ are 0.41 for CEE, 0.49 for WEST and 0.56 for EU27. However, the substitution elasticities differ on the level of sectors. Both the divisions into three sectors and according to the energy intensity show that industries with a necessity of machine use report an easier substitution between capital and labor. For energy-intensive industries, $\sigma_{K,L}$ rounds at 0.98 for both regions and for the primary sector, the elasticity even exceeds one and reaches 1.16 for CEE and 1.29 for WEST countries. Conversely, the substitution is more complicated in the tertiary sector where the human capital is important and often cannot be replaced by automatisation. Thus, $\sigma_{K,L}$ is equal to 0.45 and 0.41, for CEE and WEST countries, respectively. The secondary sector lies in between the primary and tertiary with elasticity of substitution of 0.48 for CEE and 0.65 for WEST countries. Since the energy non-intensive industries overlap with the secondary and tertiary sector, the estimates are consequently also lower than in the energy intensive case, 0.7 for CEE and 0.61 for WEST countries.

While the energy and materials substitution is more difficult in energy intensive industries in comparison to capital-labor substitution, the opposite is true for energy efficient, secondary and tertiary sectors as well as on the economy-wide level, where $\sigma_{E,M}$ equals to 1.48 for CEE, 0.79 for WEST region, and 0.62 for EU27. In the primary sector, the values of substitution elasticities for CEE and EU27 are inconclusive due to their statistical insignificance. While the estimation of $\sigma_{KL,EM}$ across regions and sectors is inconclusive regarding the difficulty of substitution, the two composites can be, with an exception in primary sector, substituted with each other more easily. Average $\sigma_{KL,EM}$ being 1,21 (avg. s.e. 0.11) and 1,62 (avg. s.e. 0.05) in WEST and CEE region, respectively. On the EU level, the composites can be substituted straightforwardly as well, with average $\sigma_{KL,EM}$ equal to 2.37 (avg. s.e. 0.09).

See Table 3 for the summary of substitution elasticity estimates recommended as the best fitting for European countries based on our analysis.

Table 3: Central estimates of elasticities of substitution

	EU			WEST			CEE		
	K-L	E-M	KL-EM	K-L	E-M	KL-EM	K-L	E-M	KL-EM
Whole economy	0.56	0.62	3.36	0.49	0.79	2.98	0.41	1.48	1.79
Energy Intensive	1.59	0.47	1.77	0.98	0.66	1.47	0.98	0.79	0.63
Energy non-intensive	0.75	0.72	6.24	0.61	0.95	2.72	0.70	0.79	1.79
Primary	0.96	0.57	0.51	1.29	0.78	0.3	1.16	1.2	0.77
Secondary	0.61	0.96	0.82	0.65	1.06	0.76	0.48	0.58	1.36
Tertiary	0.49	0.63	1.51	0.41	0.8	1.5	0.45	1.13	0.93

Note: Standard errors and statistical significance of estimates of elasticity of substitution are displayed in Table 7.

4.3 Cobb-Douglas and Leontief production function

The Cobb-Douglas and Leontief forms, often found in the literature, are both a special case of the CES production function. The former occurs when $\rho \to 0$ and consequently $\sigma \to 1$ while for the later $\rho \to \infty$ and $\sigma \to 0$. Gechert et al. (2019) reject the Cobb-Douglas specification, as a special case of CES production function based on an extensive meta-analysis

Using the two-sided Wald test, we test the hypotheses of both Cobb-Douglas and Leontief specifications' suitability for all the nests within the (KL)(EM) nesting structure.

The assumption of Leontief function ($\sigma_{KL} = 0, \sigma_{E,M} = 0$ or $\sigma_{KL,EM} = 0$) can be rejected on a 5 % significance level for all the estimates . The same result is true for the hypothesis of the Cobb-Douglas simplification of the production function which assumes $\sigma_{KL} = 1, \sigma_{E,M} = 1$ or $\sigma_{KL,EM} = 1$. The hypothesis is rejected for all elasticity estimates on all levels within the (KL)(EM) nesting structure. Our results support the findings by van der Werf (2008), Gechert et al. (2019) and Koesler and Schymura (2015) that a production function specified in a Leontief or Cobb-Douglas form may lead to inaccurate conclusions of an analysis.

4.4 Are elasticities different between CEE and WEST?

By means of the double-sided Welch's t-test we examine whether there is a statistically significant regional difference in substitution elasticities of production factors between the CEE and WEST countries, as well as the EU. Table 8 presents the results of the Welch's t-test with the null hypothesis H0 of equal elasticity estimates. Zeros in the Table 8 suggest that H0 cannot be rejected at a level of 0.1 % significance (p-value > 0.001). The elasticity equivalence is tested for all nests within the (KL)(EM) nesting structure.

With the exception of capital-labor substitution elasticity in energy intensive industries, the hypothesis of elasticity equivalence in CEE countries and Western Europe can be rejected for all sectors. There is a statistically significant difference of substitution elasticities between EU and CEE and also between EU and WEST in all nests for all sectors. When comparing WEST countries and EU, the null hypothesis cannot be rejected for half of nests within Transport and for one case within both Manufacturing and Energy sector. Should we compare CEE region with the EU, we come to a similar conclusion on only a very few nests where we cannot reject H0. Hence, the difference between CEE results and WEST and EU result supports the need of special estimates for the CEE region.

4.5 Are elasticities different before and after 2008?

We conduct an analysis of change in input substitutability over time in two periods divided by the financial crisis: 2000-2008 and 2009-2014. Equation 4 is reestimated for the two time-restricted subsamples and the accompanying elasticity estimates are compared using the double-sided Welch's test. Table 9 summarises the results. Similar to Table 8, null values indicate that the null hypothesis of equal substitution elasticites in both time periods cannot be rejected at 0.1 % level. The convergence was not achieved for one estimate, the concerned sector is indicated with a NA value. Some of the time restricted elasticity estimates are not significant on at least 10 % level. Test performed on such estimates are reported in Table 9 with an asterisk and are to be interpreted with caution. .

On the aggregate level, the hypothesis of equal substitution elasticities H0 over time can be rejected for all nests and for both regions. Over time, elasticities of substitution between capital and labor, energy and material, and value added composite and energy-materials composite for CEE and WEST region converge and the difference between regions diminishes.

On the upper KL-EM nest, the null hypothesis of equal elasticities can be rejected for all sectors, both for CEE and WEST. A closer look on the development of elasticity values shows their convergence in all sectors with an average difference in elasticities of 0.84 between regions in the first time period, and 0.11 in the second one. The zero hypothesis is rejected for energy intensive, second and third sector for both bottom nests K-L and E-M, and for both CEE and WEST. Moreover, elasticities of substitution between capital and labor, energy and material converge over time for II. and III. sector. The statistical insignificance of time-restricted elasticity estimates for the primary sector stems from a low number of observations in each period (below 650). The only case where we reliably cannot reject H0 is for E-M nest in CEE region.

Thus, our analysis suggests that the elasticity of substitution of production factors changes over time. This result suggest that future research on production factors' elasticity of substitution should take into consideration the time aspect and investigate it more rigorously.

5 Conclusion

We apply non-linear estimation techniques to estimate substitution elasticity directly from the CES production function using the World Input Output Database (WIOD) as a data source. We focus on CEE countries and their differences from the rest of the European Union due to a general lack of specific empirical evidence for this region.

We estimate five different nesting specifications of a CES production function both with and without materials as a fourth input besides capital, labor and energy. Estimation is done on a economy-wide level, as well as on the level of five aggregated sectors - primary, secondary, tertiary, energy intensive and energy non-intensive. In the three-input two-level nesting without material, we confirmed findings by van der Werf (2008) of value added composite substitution with energy as a best fit to our data. However, based on the performance of the models, adding materials as a fourth production input seems as a reasonable choice. Out of the two four-input nesting structures, (KL)(EM) performs significantly better and is thus the preferred nesting structure of a CES production function. Based on our estimation, we conclude that while the values of substitution elasticities on the economy-wide level conform to the literature they differ on the level of sectors. Generally, human labor oriented sectors (tertiary, energy non-intensive, any partly secondary) show a more difficult substitution between capital and labor than machine oriented industries (primary, energy intensive). The substitution between energy and materials is more difficult in energy intensive industries in comparison to capital-labor substitution. The opposite is true for energy efficient, secondary and tertiary sectors as well as on the economy-wide level. With a few exceptions, the KL and EM composites can be substituted more easily compared to the bottom nests.

In line with Gechert et al. (2019), van der Werf (2008) and Koesler and Schymura (2015) we reject the suitability of the production function specified in a Leontief or Cobb-Douglas form often found in the literature. The hypothesis is rejected across sectors for both regions as well as the EU as a whole.

Based on the significant differences in capital efficiency, energy intensity, energy/capital and capital/labor ratios between the CEE countries and the rest of the European Union we test the hypothesis that the elasticity of substitution between production factors is different in the Central and Eastern Europe and in the rest of European Union. We reject the hypothesis of equal elasticity estimates for the vast majority of cases. Thus we confirm the need for special estimates for the CEE region.

Lastly, we find that the elasticites in two time periods 2000-2008 and 2009-2014 differ for the majority of sectors in both regions suggesting that the elasticity of substitution of production factors changes over time. Hence, CGE models should take into consideration not only the temporal but also regional aspect and choose values for its calibration carefully.

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Appendix

Tables

Table 4: Aggregation of WIOD sectors

Sector	NACE Code	Group
Agriculture, hunting, forestry and fishing	AtB	I. & EI
Mining and quarrying	C	I. & EI
Food, beverages and tobacco	15t16	II. & EnI
Textiles and textile	17t18	II. & EI
Leather, leather and footwear	19	II. & EI
Wood and products of wood and cork	20	II. & EI
Pulp, paper, paper, printing and publishing	21t22	II. & EI
Coke, refined petroleum and nuclear fuel	23	II. & EI
Chemicals and chemical	24	II. & EI
Rubber and plastics	24	II. & EnI
Other non-metallic mineral	25	II. & EI
Basic metals and fabricated metal	27t28	II. & EI
Machinery, nec	29	II. & EnI
Electrical and optical equipment	30t33	II. & EnI
Transport equipment	34t35	II. & EnI
Manufacturing nec; recycling	36t37	II. & EI
Electricity, gas and water supply	E	II. & EI
Construction	F	II. & EnI
Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of fuel	50	III. & EnI
Wholesale trade and commission trade, except of motor vehicles and motorcycles	51	III. & EnI
Retail trade, except of motor vehicles and motorcycles; repair of household goods	52	III. & EnI
Hotels and restaurants	H	III. & EnI
Inland transport	60	III. & EI
Water transport	61	III. & EI
Air transport	62	III. & EI
Supporting and auxiliary transport activities; activities of travel agencies	63	III. & EI
Post and telecommunication	64	III. & EnI
Financial intermediation	J	III. & EnI
Real estate activities	70	III. & EnI
Renting of m&eq and other business activities	71t74	III. & EnI
Public admin and defence; compulsory social security	L	III. & EnI
Education	M	III. & EnI
Health and social work	N	III. & EnI
Other community, social and personal services	O	III. & EnI

Note: I. = Primary sector, II. = Secondary sector, III. = Tertiary sector, EI = Energy intensive sectors, EnI = Energy non-intensive sectors

Table 5: Results of the two-level three-input K, L, E estimation

Region	Sector	$\sigma_{K,E}$			$\sigma_{KE,L}$			R^2	$\sigma_{K,L}$			$\sigma_{KL,E}$			R^2	$\sigma_{L,E}$			$\sigma_{LE,K}$			R^2
CEE	all	0.76	(0.05)	***	0.87	(0.03)	***	0.71	1.33	(0.11)	***	0.5	(0.02)	***	0.72	0.35	(0.02)	***	1.18	(0.12)	***	0.72
WEST	all	0.52	(0.03)	***	1.33	(0.03)	***	0.84	0.68	(0.26)	***	1.12	(0.01)	***	0.84	1.08	(0.01)	***	0.76	(0.23)	***	0.84
EU	all	0.52	(0.02)	***	1.33	(0.03)	***	0.85	0.78	(0.15)	***	0.97	(0.01)	***	0.85	1.27	(0.02)	***	0.6	(0.1)	***	0.84
CEE	EI	4.13	(1.83)	**	1.57	(0.14)	***	0.8	0.77	(0.12)	***	1.59	(0.11)	***	0.79	2	(0.23)	***	0.81	(0.14)	***	0.79
WEST	EI	0.72	(0.04)	***	1.19	(0.05)	***	0.85	0.92	(0.12)	***	1.24	(0.04)	***	0.86	0.77	(0.02)	***	0.9	(0.17)	***	0.85
EU	EI	2.64	(0.55)	***	0.81	(0.02)	***	0.86	0.84	(0.09)	***	1.12	(0.03)	***	0.86	0.81	(0.02)	***	1.1	(0.13)	***	0.86
CEE	EnI	1.21	(0.42)	***	0.6	(0.04)	***	0.83	1.2	(0.47)	**	0.55	(0.03)	***	0.83	0.56	(0.03)	***	0.57	(0.23)	**	0.82
WEST	EnI	0.9	(0.22)	***	2.7	(0.47)	***	0.84	0.95	(1.75)		5.36	(1.63)	***	0.84	1.61	(0.11)	***	1.49	(1.33)		0.84
EU	EnI	0.51	(0.13)	***	16.59	(14.75)		0.85	1.02	(0.9)		8.68	(3.49)	**	0.85	1.61	(0.09)	***	1.49	(1.08)		0.85
CEE	I.	0.86	(0.28)	***	0.65	(0.24)	***	0.94	0.86	(0.29)	***	0.92	(0.5)	*	0.94	0.66	(0.31)	**	0.78	(0.23)	***	0.94
WEST	I.	0.4	(0.03)	***	3.27	(3.92)		0.93	0.84	(0.37)	**	0.35	(0.03)	***	0.93	0.83	(1.03)		0.45	(0.03)	***	0.93
EU	I.	0.43	(0.02)	***	2.08	(1.51)		0.93	3.61	(2.51)		0.4	(0.02)	***	0.94	0.76	(5.5)		0.43	(0.03)	***	0.93
CEE	II.	0.6	(0.1)	***	0.49	(0.03)	***	0.78	1.33	(0.23)	***	0.58	(0.06)	***	0.78	0.4	(0.05)	***	0.45	(0.06)	***	0.78
WEST	II.	0.48	(0.02)	***	0.63	(0.02)	***	0.92	1.5	(0.12)	***	0.51	(0.02)	***	0.91	0.6	(0.01)	***	0.43	(0.02)	***	0.92
EU	II.	0.48	(0.02)	***	0.63	(0.01)	***	0.92	0.76	(0.03)	***	0.45	(0.01)	***	0.92	0.49	(0.01)	***	0.63	(0.03)	***	0.92
CEE	III.	0.94	(0.27)	***	0.55	(0.02)	***	0.82	0.52	(0.07)	***	0.5	(0.02)	***	0.83	0.39	(0.02)	***	0.85	(0.32)	***	0.83
WEST	III.	0.68	(0.1)	***	3.79	(0.78)	***	0.89	0.82	(14.46)		0.44	(0.2)	**	0.88	1.91	(0.12)	***	0.68	(0.43)		0.89
EU	III.	0.58	(0.09)	***	2.1	(0.2)	***	0.9	0.73	(0.78)		0.51	(0.03)	***	0.89	1.96	(0.14)	***	0.83	(0.62)		0.9

*Note: I. = Primary sector, II. = Secondary sector, III. = Tertiary sector, EI = Energy intensive sectors, EnI = Energy non-intensive sectors; */**/*** indicates that coefficient differs from zero at 10/5/1% level of significance*

Table 6: Results of the three-level four-input K, L, E, M estimation

Region	Sector	$\sigma_{K,L}$			$\sigma_{KL,E}$			$\sigma_{KLE,M}$			R^2
CEE	all	4.2	(1.04)	***	0.77	(0.02)	***	1.39	(0.02)	***	0.98
WEST	all	0.56	(0.03)	***	0.68	(0.05)	***	83.39	(70.59)		0.99
EU	all	0.56	(0.03)	***	1.02	(0.02)	***	2.22	(0.04)	***	0.99
CEE	EI	1.02	(0.06)	***	0.49	(0.07)	***	1.68	(0.12)	***	0.99
WEST	EI	n.a.	(n.a.)		0.51	(0.07)	***	2.81	(0.33)	***	0.97
EU	EI	n.a.	(n.a.)		0.48	(0.02)	***	n.a.	(n.a.)		0.97
CEE	EnI	6.08	(6.54)		0.66	(5.05)		1.48	(0.04)	***	0.98
WEST	EnI	0.41	(0.03)	***	0.95	(0.05)	***	2.31	(0.05)	***	0.99
EU	EnI	0.56	(0.03)	***	0.82	(0.18)	***	2.52	(0.05)	***	0.99
CEE	I.	7.8	(33.05)		0.55	(0.92)		1.07	(0.64)	*	0.97
WEST	I.	0.93	(0.08)	***	0.76	(0.15)	***	n.a.	(n.a.)		0.93
EU	I.	0.85	(0.05)	***	0.42	(0.04)	***	n.a.	(n.a.)		0.93
CEE	II.	1.51	(0.09)	***	0.44	(0.24)	*	1.68	(0.07)	***	0.99
WEST	II.	2.77	(0.42)	***	0.49	(0.04)	***	0.88	(0.02)	***	1
EU	II.	3.27	(0.48)	***	0.49	(0.02)	***	1.12	(0.02)	***	1
CEE	III.	1.47	(0.63)	**	0.55	(0.13)	***	1.52	(0.08)	***	0.97
WEST	III.	0.47	(0.05)	***	0.53	(0.05)	***	2.72	(0.11)	***	0.99
EU	III.	0.58	(0.09)	***	1.5	(0.33)	***	2	(0.05)	***	0.99

Note: I. = Primary sector, II. = Secondary sector, III. = Tertiary sector, EI = Energy intensive sectors, EnI = Energy non-intensive sectors; $*/{*}{*}/{*}{*}{*}$ indicates that coefficient differs from zero at 10/5/1% level of significance

Table 7: Results of the two-level four-input K, L, E, M estimation

Region	Sector	$\sigma_{K,L}$			$\sigma_{E,M}$			$\sigma_{KL,EM}$			R^2
CEE	all	0.41	(0.02)	***	1.48	(0.12)	***	1.79	(0.03)	***	0.97
WEST	all	0.49	(0.05)	***	0.79	(0.09)	***	2.99	(0.09)	***	0.99
EU	all	0.56	(0.02)	***	0.62	(0.06)	***	3.36	(0.09)	***	0.99
CEE	EI	0.98	(0.24)	***	0.79	(0.08)	***	0.63	(0.03)	***	0.98
WEST	EI	0.98	(0.09)	***	0.66	(0.04)	***	1.47	(0.08)	***	0.96
EU	EI	1.59	(0.18)	***	0.47	(0.04)	***	1.77	(0.1)	***	0.97
CEE	EnI	0.7	(0.13)	***	0.79	(1.36)		1.79	(0.05)	***	0.98
WEST	EnI	0.61	(0.04)	***	0.95	(0.11)	***	2.72	(0.08)	***	0.99
EU	EnI	0.75	(0.05)	***	0.72	(0.09)	***	6.24	(0.31)	***	0.99
CEE	I.	1.16	(0.94)		1.2	(1.4)		0.77	(0.42)	*	0.97
WEST	I.	1.29	(0.63)	**	0.78	(0.3)	***	0.3	(0.03)	***	0.94
EU	I.	0.96	(0.25)	***	0.57	(0.54)		0.51	(0.03)	***	0.94
CEE	II.	0.48	(1.52)		0.58	(0.03)	***	1.36	(0.07)	***	0.99
WEST	II.	0.65	(0.12)	***	1.06	(0.09)	***	0.76	(0.01)	***	0.99
EU	II.	0.61	(0.04)	***	0.96	(0.09)	***	0.82	(0.01)	***	0.98
CEE	III.	0.45	(0.12)	***	1.13	(0.11)	***	0.93	(0.02)	***	0.97
WEST	III.	0.4	(0.02)	***	0.8	(0.04)	***	1.5	(0.04)	***	0.99
EU	III.	0.49	(0.02)	***	0.63	(0.02)	***	1.51	(0.03)	***	0.99

Note: I. = Primary sector, II. = Secondary sector, III. = Tertiary sector, EI = Energy intensive sectors, EnI = Energy non-intensive sectors; $*/{*}{*}/{*}{*}{*}$ indicates that coefficient differs from zero at 10/5/1% level of significance

Table 8: Test for regional difference in substitution elasticity

Region	Sector	$\sigma_{K,L}$	$\sigma_{E,M}$	$\sigma_{KL,EM}$
CEE-WEST	all	1	1	1
CEE-EU	all	1	1	1
WEST-EU	all	1	1	1
CEE-WEST	EI	1	1	1
CEE-EU	EIs	1	1	1
WEST-EU	EI	1	1	1
CEE-WEST	EnI	0	1	1
CEE-EU	EnI	1	1	1
WEST-EU	EnI	1	1	1
CEE-WEST	I.	1	1	1
CEE-EU	I.	1	1	1
WEST-EU	I.	1	1	1
CEE-WEST	II.	1	1	1
CEE-EU	II.	1	1	1
WEST-EU	II.	1	1	1
CEE-WEST	III.	1	1	1
CEE-EU	III.	1	1	1
WEST-EU	III.	1	1	1

Note: 1 = H0 rejected on at least 0.1 % significance level

Table 9: Test for time difference in substitution elasticity

Region	Sector	$\sigma_{K,L}$	$\sigma_{E,M}$	$\sigma_{KL,EM}$
CEE	all	1	1	1
WEST	all	1	1	1
CEE	EI	1	1	1
WEST	EI	1	1	1
CEE	EnI	1	0*	1
WEST	EnI	0*	1*	1
CEE	I.	NA	0	1
WEST	I.	1*	0*	1
CEE	II.	1*	1*	1
WEST	II.	1	1	1
CEE	III.	1	1	1
WEST	III.	1	1	1

Note: 1 = H0 rejected on at least 0.1 % significance level; ∗ indicates that the test was perfomed on statistically insignificant estimates

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