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Price Level Targeting with Imperfect Rationality: A Heuristic Approach

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Abstract:
The paper compares price level targeting and inflation targeting regimes in a New Keynesian model with bounded rationality. Economic agents form their expectations using heuristics—they choose between a few simple rules based on their past forecasting performance. Two main specifications of the price level targeting model are examined—the agents form expectations either about price level or about inflation, which is ex ante not equivalent because of sequential nature of the model. In addition, several formulations of the forecasting rules are considered. Both regimes are assessed by loss function comparison. According to the results, price level targeting is preferred in the case with expectations created about price level under the baseline calibration; but it is sensitive to some model parameters. Furthermore, when expectations are created about inflation, price level targeting over time loses credibility and leads to divergence of the economy. On the other hand, inflation targeting model functions stably. Therefore, while potential benefits of price level targeting have been confirmed under certain assumptions, the results suggest that inflation targeting constitutes more robust choice for monetary policy.

JEL: E31, E37, E52, E58, E70
Keywords: Price level targeting, Inflation targeting, Monetary policy, Bounded rationality, Heuristics

1 Introduction

Inflation targeting (IT) has become state-of-the-art in monetary policymaking during last three decades and it has been officialy adopted by 41 central banks by now (IMF, 2020). In addition, other central banks including US Fed and ECB are using many aspects of IT despite not being formal inflation targeters. Nevertheless, there has been an ongoing debate and research of alternative policy frameworks, reinforced especially by the Great Recession and subsequent era of nominal interest rates close to their zero lower bound—problem, which has recently gained in topicality even more since outbreak of global pandemic, as central banks across world lowered their policy rates and returned back to the lower bound (or in cases such as ECB they did not even abandon it before).

One significant stream of the research of alternative frameworks is focused on price level targeting (PLT), under which the targeted path of price level can still increase over time (consistently with small but positive inflation rate), but contrary to the inflation targeting, the price level targeting regime compensates for past deviations of inflation from its steady state rate. The difference seems small, but economic literature in general suggests that it could have significant positive impact on the economy due to stabilizing role of inflation expectations.

The crucial aspect of the PLT framework is that lower inflation now leads to higher inflation expectations for the future and vice versa. As a result, dynamics of inflation expectations mitigate shocks into inflation and decrease macroeconomic volatility. This mechanism works well in theoretical models with rational expectations and fully credible PLT regime—which might be, however, too strong assumptions to hold in reality. If people did not understand the regime correctly or did not trust the central bank sufficiently, the inflation expectations would not adjust in the desired way and the key mechanism of the regime would cease to hold.

Moreover, there is very limited historical experience with price level targeting, so it is not possible to compare it with inflation targeting empirically. In fact, lack of belief in the strong assumptions among policymakers is probably the main reason why the discussion remains only theoretical and no central bank uses pure PLT in practice. Therefore, probably the only option is to try to relax the assumptions within the framework of the theoretical models and to compare IT and PLT in a different setting.

This paper represents one attempt to do so by replacing rational expectations hypothesis with heuristic approach as proposed for example by De Grauwe (2012). Economic agents form expectations by choosing between a few simple rules based on their past forecasting performance; in other aspects, the model corresponds to a standard 3-equation New Keynesian DSGE model. The model of De Grauwe (2012) is here extended for price level targeting regime and its corresponding rules of expectations formation.

Such modelling of expectations formation inherently contains endogenous credibility of monetary policy framework and therefore it helps to examine the crucial aspects of PLT—credibility and understandability by public. Furthermore, the heuristic approach comprises larger departure from rational expectations than other models of bounded rationality, such as adaptive learning models. The heuristic methodology thus enables to assess robustness of performance of price level targeting in a framework more distant from perfect rationality, so the monetary policy regime is subject to a greater scrutiny. Furthermore, the methodology is supported by some empiricial evidence about expectations formation (see Section 3 for details).

To author's best knowledge, heuristic modelling has been applied on PLT only by Ho *et al.* (2019), who however examined only very specific case, while present article considers much broader set of forecasting rules, which in the end leads to quite different results than Ho *et al.* (2019). The analysis should contribute to the discussion of whether the theoretical benefits of price level targeting might hold also in reality, even without fully rational economic agents.

Overall, the analysis suggests that inflation targeting is more robust and thus safer choice for monetary policy regime than price level targeting in presence of model uncertainty. Theoretical advantages of price level targeting are confirmed in certain model specifications, but they are sensitive to particular assumptions about expectations formation—the PLT regime is unstable for other specifications. Conversely, inflation targeting performs reasonably well in all examined cases.

The paper is structured as follows: Section 2 presents key characteristics of price level targeting and related literature review. Section 3 discusses the macroeconomic model with expectations created using heuristics. Section 4 presents results of the analysis, while Section 5 assesses their implications. Section 6 concludes. Appendices then contain some supplementary mathematical derivations and calibration together with robustness analysis, respectively.

2 Price Level Targeting

In theory, price level targeting could offer several benefits as compared with inflation targeting. As Cournede & Moccero (2011) or Popescu (2012) discuss, if the regime is credible and well understood by public, then any shock to prices in one direction leads to movement of inflation expectations in the opposite direction. Real interest rate then adjusts in desired direction even for stable nominal interest rate. This mechanism leads not only to less volatile nominal interest rate, but more importantly, it can also anchor inflation closer to its long-term desired rate and decrease output volatility (see more detailed discussion about the volatility under PLT in subsection 2.1). In addition, there is also lower probability of hitting the lower bound on nominal interest rates.

Both Cournede & Moccero (2011) and Popescu (2012) also mention that price level targeting could decrease probability or severity of asset price bubbles. Nominal rates below the neutral level for a sustained period of time can contribute to a creation of asset price bubbles—and as just discussed, PLT requires less aggressive adjustment of the rates. In addition, PLT leads to lower uncertainty about price level in the long-term, which makes intertemporal planning easier. On the contrary, under IT, price level is not stationary and its variance increases up to the infinity in the very long term (Minford & Peel, 2003).

The benefits of PLT hinge on the anchoring expectations about future prices. The mechanism functions very well when all economic agents are forward-looking, rational, and when the monetary policy regime is fully credible. But these assumptions are hardly realistic. In practice, central bank's communication would be difficult, as short-term targeted inflation rate changes over time in reaction to past shocks to the price level, while communicating targeted price level (e.g. level of consumer price index) would be cumbersome as this is much more abstract variable. Moreover, with imperfect credibility of central bank, people could no longer form inflation expectations in the direction opposite to the shock and the key advantage of price level targeting stops working. Finally, PLT is prone to dynamic inconsistency problem (Bohm *et al.*, 2012).

These challenges show why the regime remains rather theoretical concept and central banks are reluctant to use it in practice. The only historical experience widely considered as price level targeting was Swedish monetary policy in 1930s (see e.g. Berg & Jonung (1999) or Straumann & Woitek (2009)). Bohm *et al.* (2012) suggest that deflationary policy as a part of currency reform in Czechoslovakia after WWI could be also considered as PLT. Nevertheless, neither of the experiences says much about effectiveness of the PLT framework in general, as both were short, adopted during turbulent times and with very specific characteristics.[1]

On the other hand, Fed after its monetary policy framework revision in August 2020 effectively adopted average inflation targeting, which allows temporary overshooting of inflation target after a period below the target. The policy can be considered as a compromise between IT and PLT.[2] While PLT has not been adopted in the pure sense, such shift in Fed's policy confirms that it is at the frontier of current monetary policy discussions and that there is a need for its further research. In addition, average inflation targeting is also discussed in relation with monetary policy review by ECB conducted in 2020-2021.

2.1 Literature Review

Much of the research regarding price level targeting is influenced by seminal paper of Svensson (1999b). Model presented in his paper contains neoclassical Phillips curve and assumes that central bank has complete control over inflation rate and discretionary chooses the inflation in each period in order to minimize loss function. The loss function has two main specifications—one with deviation of inflation rate from its target (corresponding to IT) and one with deviation of price level from its targeted path (corresponding to PLT); both versions also contain deviation of output gap from its (non-negative) target. If output gap is at least moderately persistent, PLT brings lower inflation variability than IT. In particular, if the desired output gap is zero, level of output gap persistence above 0.5 is sufficient for such result. Moreover, even if society prefers inflation stabilization as opposed to price level stabilization (i.e. loss function of the society is specified in terms of inflation deviation from the target), assigning goal of price level stabilization to the central bank leads to results closer to optimal monetary policy than assigning inflation target.

[1] Price level targeting element is also part of "foolproof way" for escaping from liquidity trap presented by Svensson (2000), which served as a theoretical underpinning of exchange rate commitment in the Czech Republic in 2013-2017. As Franta *et al.* (2014) discuss, the Czech National Bank in fact foresaw that the commitment would lead to an increase in inflation above 2% target, followed by convergence to the target from above. This should compensate for previous period of low inflation and lead to average inflation close to the target; the policy therefore implicitly included price level targeting characteristics. On the other hand, the policy was based on nominal exchange rate peg without any commitment towards future price level and without intention to abandon inflation targeting framework. Such approach hence represented both the potential benefits of PLT as well as the reluctance of policymakers to adopt the regime fully due to perceived cognitive limitations of people.

[2] Technically, average inflation targeting with the average computed over infinite time horizon corresponds to price level targeting.

While Svensson uses backward-looking model, Vestin (2006) confirms his key result in a standard forward-looking New Keynesian model. He assumes that central bank can operate only in discretionary environment, but optimal commitment solution can be replicated under PLT in scenario with no inflation persistence, which is rather strong result. In addition, PLT leads to better inflation-output volatility trade-off than IT even with inflation persistence. Cover & Pecorino (2005) also complements Svensson's analysis and shows that his results are valid under broader set of assumptions (even without output gap persistence).

Some further support for PLT is provided e.g. by Berentsen & Waller (2011) in real business cycle setting; by Ball *et al.* (2005) in inattention framework; by Cateau (2017) in presence of model uncertainty; or by Eggertsson & Woodford (2003) in presence of the zero lower bound.

While support for price level targeting in rational expectations models is clearly substantial, there are several papers relaxing some of strong assumptions of such models. For example, Masson & Shukayev (2011) introduce an escape clause allowing for reset of the price level target after large shocks. It turns out that this possibility endangers credibility of the policy regime, it may lead to multiple equilibria, and the result can be worse performance of PLT than of IT.

Honkapohja & Mitra (2019) replace rational expectations with adaptive learning; full credibility of PLT is replaced by only partial credibility, which endogenously evolves over time. Their results are mixed—as long as there is at least some initial credibility of PLT, the regime outperforms IT during a liquidity trap; on the other hand, IT is superior when zero lower bound constraint is not binding. Implications of imperfect knowledge and adaptive learning are also explored by Eusepi & Preston (2018) and Gaspar *et al.* (2007) who find that price level targeting can be appropriate policy regime even without rational expectations.

Transition from IT to PLT is examined by Cateau *et al.* (2009), who use large scale macroeconomic model used in Bank of Canada (ToTEM). They model transition between regimes as a Markov switching process with endogenous state of either low or high credibility of the policy regime. Low credibility corresponds to positive probability of switching back to IT, while high credibility represents rational expectations consistent with PLT. It turns out that potential welfare gain from the switch to PLT can be quite substantial, and it would become negative only if the economy was in the low credibility state for at least 13 years.

Yetman (2005) introduces consumers using rule-of-thumb forecasting into otherwise standard macroeconomic model with rational expectations and shows that even a small portion of rule-of-thumb consumers reverses optimality of price level targeting. While performance of both IT and PLT diminishes with increasing number of these consumers, PLT is less robust and deteriorates faster. Such conclusion is actually in line with results of this paper to be presented in chapter 4, even though particular details of both methodology and results differ between Yetman's paper and this work.

More detailed surveys of literature on price level targeting are provided by Ambler (2009), who in general confirms superiority of PLT in rational expectations models; and by Hatcher & Minford (2016), who focus on development since Ambler's survey and discuss papers on optimal monetary policy, zero lower bound, transition from IT to PLT, or financial frictions. Overall, the authors favour PLT as well, although the conclusion is not unequivocal.

To summarize, models with rational expectations provide strong support for price level targeting in various model settings. There have been a few attempts to model expectations differently, which have led to mixed results—PLT is still superior to IT in some cases (but at least certain level of its credibility is crucial for such result), while it ceases to perform well without rational expectations according to other studies.

3 Model

While rational expectations models generally favour price level targeting, they are usually considered as too unrealistic in this context. This motivates using approach with bounded rationality. In particular, rational expectations will be replaced by heuristics, where economic agents form their expectations based on a few simple rules, as proposed by De Grauwe (2012). The method is also motivated by some empirical evidence (e.g. by Branch (2004) from survey data and by Hommes (2011) based on a laboratory experiment) suggesting that agents actually form expectations by switching between a few simple rules. Furthermore, significant stream of literature including Mankiw *et al.* (2003) or Andrade *et al.* (2016) shows significant and time-varying heterogeneity in inflation expectations (and in case of the latter paper also in expectations of other macroeconomic variables) across economic agents, which the heuristic approach is able to capture.

The heuristic approach is not the only possible way of modelling imperfect rationality. One common branch of bounded rationality models uses adaptive learning. Although details depend on a particular setup, the departure from the rational expectations is in general still quite small in adaptive learning framework, as economic agents are assumed to know full structure of the economic model—they just do not know values of model parameters. They instead estimate them using simple econometric techniques, usually least squares. They re-estimate the parameters in each time period and learn about their true values over time. Such approach generally converges to rational expectations, but transition dynamics may have consequences for business cycle. Contrarily, heuristic expectations represent larger departure from the rational expectations with no convergence towards them. The methodology of this paper thus scrutinizes performance of price level targeting to a greater extent than adaptive learning models.[3]

Apart from expectations formation, the applied model corresponds to a standard small New Keynesian DSGE model. In particular, dynamic IS curve (or aggregate demand equation) based on utility maximization of individual households looks as follows:

$$y_t = a_1 \widetilde{E}_t y_{t+1} + (1 - a_1) y_{t-1} + a_2 (i_t - \widetilde{E}_t \pi_{t+1} - \overline{r}) + \epsilon_t \tag{1}$$

where y_t is the output gap in period t, i_t is the nominal interest rate, $\overline{r}$ is the steady state real interest rate (assumed to be constant over time for simplicity), π_t is the inflation rate, and ϵ_t is a white noise disturbance term (no autocorrelation in the error term is used in the baseline model). The tilde above the expectations operator E captures that expectations are not rational. Lagged output gap accounts for habit formation in consumption. The parameter a_1 is inversely related to the degree of habit formation ($0 < a_1 < 1$) and $a_2 < 0$.

New Keynesian Phillips curve (aggregate supply equation) based on profit maximization of individual firms operating in monopolistic competition and Calvo price setting environment then has the form

$$\pi_t = b_1 \widetilde{E}_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 y_t + \eta_t \tag{2}$$

[3] Adaptive learning models are summarized in a comprehensive way by Evans & Honkapohja (2001) and more recently surveyed by Milani (2012) and Eusepi & Preston (2018)). Other option of modelling bounded rationality is e.g. behavioural New Keynesian model with cognitive discounting parameter directly in IS curve and Phillips curve introduced by Gabaix (2020). Further overview of the topic can be also found in Woodford (2013).

where η_t is a white noise disturbance term. The equation includes also lagged inflation, which captures indexation of prices. Parameter b_1 then captures relative weight of forward and backward looking terms and it is a function of the underlying Calvo parameter ($0 < b_1 < 1$).

The model is closed by central bank interest rate rule. Central bank targeting inflation uses Taylor rule with interest rate smoothing:

$$i_t = c_3 \left[\pi^* + \overline{r} + c_1^{IT}(\pi_t - \pi^*) + c_2 y_t\right] + (1 - c_3)i_{t-1} + u_t \tag{3}$$

and rule corresponding to price level targeting has the form[4]

$$i_t = c_3 \left[\pi^* + \overline{r} + c_1^{PLT}(p_t - \overline{p}_t) + c_2 y_t\right] + (1 - c_3)i_{t-1} + u_t \tag{4}$$

where p_t is log of price level in period t, while $\overline{p}_t$ is targeted price level in that period. π^* is inflation target in case of IT and increase in targeted price level under PLT (it is assumed to be same in both policy regimes). Price level target then develops according to[5]

$$\overline{p}_t = \overline{p}_{t-1} + \pi^{*q} \tag{5}$$

See Appendix A.1 for an explicit solution of the two versions of the model for given expectations about output gap and inflation (to be defined below).

Furthermore, quadratic loss function of standard form (in line with e.g. Clarida *et al.* (1999) or Walsh (2003)) will be defined to compare the two monetary policy regimes:

$$L = \sum_{t=1}^{T} \left[(\pi_t - \pi^*)^2 + \lambda y_t^2\right] \tag{6}$$

The function thus punishes deviations of inflation from the target and of output from its potential, with larger deviations having higher importance (given the quadratic nature of the function). Parameter λ captures relative weight of output gap stability and T is number of periods over which the comparison is performed. Note that the loss function is the same for both policy regimes—even loss under PLT is defined in terms of inflation volatility, which is the

[4]This type of rule is called Wicksellian (see Woodford (2003) or Giannoni (2014) for discussion).

[5]Superscript q distinguishes that the relationship holds for quarterly inflation, while model equations contain annualized inflation. Log-linearised relationship is simply $\pi = 4 * \pi^q$.

ultimate goal; PLT is approached as just a tool to possibly deliver more stable inflation (and output gap).

Let us turn into description of expectations formation, which is based on De Grauwe (2012). Note however, that De Grauwe does not consider price level targeting at all—this paper hereby extends his model for this monetary policy framework.

In particular, economic agents are assumed to choose between two simple rules for forecasts of inflation and separately of output gap. For both variables, there is a fundamentalist rule (corresponding to variable at its equilibrium level) and an extrapolative rule (when the agents use adaptive expectations). Let us focus on output gap forecasts at first. Formally, fundamentalist and extrapolative rules, respectively, for output gap under both policy regimes have the form

$$\widetilde{E}_t^f y_{t+1} = 0 \tag{7}$$

$$\widetilde{E}_t^e y_{t+1} = y_{t-1} \tag{8}$$

While economic agents are not assumed to have rational expectations, they are not choosing forecasting rule completely randomly—they take into account past performance of the rules; we are speaking about bounded rationality here. Subsequent formalization of switching between the rules is based on discrete choice theory as described e.g. by Anderson *et al.* (1992) or Brock & Hommes (1997). The agents evaluate past forecasting performances of both rules by calculation of utilities (defined as negative weighted mean squared forecast error of given rule) as follows:

$$U_{f,t}^y = -\sum_{k=0}^{\infty} \omega_k \left[y_{t-k-1} - \widetilde{E}_{t-k-2}^f y_{t-k-1} \right]^2 \tag{9}$$

$$U_{e,t}^y = -\sum_{k=0}^{\infty} \omega_k \left[y_{t-k-1} - \widetilde{E}_{t-k-2}^e y_{t-k-1} \right]^2 \tag{10}$$

where $U_{f,t}^y$ and $U_{e,t}^y$ are the utilities from fundamentalist and extrapolative rules in output gap forecasting, respectively (calculated in period t); and ω_k are geometrically declining weights—errors made in distant past have lower weight than recent errors—which captures tendency to forget. When we define $\omega_k = (1-\rho)\rho^k$ with $0 < \rho < 1$, we can rewrite equations 9 and 10 as follows (Appendix A.2 shows this explicitly):

$$U^y_{f,t} = \rho U^y_{f,t-1} - (1-\rho)\left[y_{t-1} - \widetilde{E}^f_{t-2} y_{t-1}\right]^2 \tag{11}$$

$$U^y_{e,t} = \rho U^y_{e,t-1} - (1-\rho)\left[y_{t-1} - \widetilde{E}^e_{t-2} y_{t-1}\right]^2 \tag{12}$$

The coefficient ρ then captures memory of people. $\rho = 0$ means no memory, while with increasing ρ (up to 1), the importance of more distant errors grows as well.

The utilities represent deterministic components in the choice between the two rules, but there are also stochastic elements $\epsilon_{f,t}$ and $\epsilon_{e,t}$. Resulting probability of choosing the fundamentalist rule is

$$\alpha_{f,t} = P\left[U^y_{f,t} + \epsilon_{f,t} > U^y_{e,t} + \epsilon_{e,t}\right] \tag{13}$$

Such specification is based on an idea that the choice between the rules is influenced by both actual performance of the rules as well as by current mood of individual decision makers, which is captured by the stochastic components. Furthermore, $\epsilon_{f,t}$ and $\epsilon_{e,t}$ are assumed to be logistically distributed, which leads to probability of selecting the fundamentalist rule as follows:

$$\alpha_{f,t} = \frac{\exp(\gamma U^y_{f,t})}{\exp(\gamma U^y_{f,t}) + \exp(\gamma U^y_{e,t})} \tag{14}$$

and to probability of choosing the extrapolative rule:

$$\alpha_{e,t} = P\left[U^y_{e,t} + \epsilon_{e,t} > U^y_{f,t} + \epsilon_{f,t}\right] = \frac{\exp(\gamma U^y_{e,t})}{\exp(\gamma U^y_{f,t}) + \exp(\gamma U^y_{e,t})} = 1 - \alpha_{f,t} \tag{15}$$

where parameter γ measures intensity of choice or in other words willingness to learn from past errors; and it is given by variance of $\epsilon_{f,t}$ and $\epsilon_{e,t}$. When the variance is zero, $\gamma = \infty$ and the choice purely deterministic. When the variance goes to infinity, $\gamma = 0$ and the choice becomes random—probability of choice of each rule is 0.5.

The market expectations are then given by

$$\widetilde{E}_t y_{t+1} = \alpha_{f,t}\widetilde{E}_t^f y_{t+1} + \alpha_{e,t}\widetilde{E}_t^e y_{t+1} = \alpha_{f,t}0 + \alpha_{e,t}y_{t-1} \tag{16}$$

The same approach is applied for inflation forecasting. The extrapolative rule under IT is defined by

$$\widetilde{E}_t^{e,IT}\pi_{t+1} = \pi_{t-1} \tag{17}$$

and the fundamentalist rule by

$$\widetilde{E}_t^{f,IT}\pi_{t+1} = \pi^* \tag{18}$$

The situation is, however, more complicated for PLT. The model is constructed in a way that an agent at time t forms expectations about $t+1$ based on data from $t-1$. As $t-1$ and $t+1$ are not adjacent periods, it actually matters whether the agent makes expectations about future prices or inflation. In the baseline model, let us assume that expectations are formed about price level; the other case will be described and examined in section 4.2. The model will be solved for p_t using relationship $\widetilde{E}_t\pi_{t+1}^q = \widetilde{E}_t p_{t+1} - p_t$. Agents therefore form expectations $\widetilde{E}_t p_{t+1}$, while inflation expectations $\widetilde{E}_t\pi_{t+1}^q$ are created only ex post, after p_t is determined. Fundamentalist rule in this case is then

$$\widetilde{E}_t^{f,PLT}p_{t+1} = \overline{p}_{t+1} \tag{19}$$

while for the extrapolative rule there are more possibilities how to form $\widetilde{E}_t^{e,PLT}p_{t+1}$. For clarity, the options will be specified in chapter 4 when they will be examined one by one.

Forecast performances of both rules are represented by their respective utilities computed from weighted mean squared forecast errors just as for output gap. This leads to probabilities of choosing fundamentalist and extrapolative rule in inflation/price level forecasting (for both regimes) as follows

$$\beta_{f,t} = \frac{\exp(\gamma U_{f,t}^{\{\pi,p\}})}{\exp(\gamma U_{f,t}^{\{\pi,p\}}) + \exp(\gamma U_{e,t}^{\{\pi,p\}})} \tag{20}$$

$$\beta_{e,t} = \frac{\exp(\gamma U_{e,t}^{\{\pi,p\}})}{\exp(\gamma U_{f,t}^{\{\pi,p\}}) + \exp(\gamma U_{e,t}^{\{\pi,p\}})} \tag{21}$$

Finally, market forecast of inflation under IT is given by

$$\widetilde{E}_t^{IT}\pi_{t+1} = \beta_{f,t}\widetilde{E}_t^{f,IT}\pi_{t+1} + \beta_{e,t}\widetilde{E}_t^{e,IT}\pi_{t+1} = \beta_{f,t}\pi^* + \beta_{e,t}\pi_{t-1} \tag{22}$$

and under PLT by

$$\widetilde{E}_t^{PLT}p_{t+1} = \beta_{f,t}\widetilde{E}_t^{f,PLT}p_{t+1} + \beta_{e,t}\widetilde{E}_t^{e,PLT}p_{t+1} = \beta_{f,t}\overline{p}_{t+1} + \beta_{e,t}\widetilde{E}_t^{e,PLT}p_{t+1} \tag{23}$$

where $\widetilde{E}_t^{e,PLT}p_{t+1}$ will be specified later. Furthermore, parameter $\beta_{f,t}$ can be also interpreted as credibility of given monetary policy regime.

The rules described above have one drawback—it would be preferred to derive heuristic rules at micro-level with individuals having cognitive limitations. Nevertheless, there is no recognized approach how to do this because of our limited knowledge of information processing by human brains. Therefore, the rules are not micro-founded, but only imposed ex post into model equations. On the other hand, while the exact formulation of forecasting rules is a bit arbitrary, some empirical support for this way of inflation expectations formation by Branch (2004) or Hommes (2011) has already been mentioned.

Finally, let us define supplementary variable AS_t, which stands for *animal spirits*:

$$AS_t = \begin{cases} \alpha_{e,t} - \alpha_{f,t} & \text{if } y_{t-1} > 0, \\ -\alpha_{e,t} + \alpha_{f,t} & \text{if } y_{t-1} < 0. \end{cases} \tag{24}$$

The variable AS_t hence ranges between -1 and 1 and is equal to 0 when fraction of agents using each rule is the same, i.e. 0.5. When $y_{t-1} > 0$, extrapolative rule is optimistic, while fundamentalist rule is pessimistic. The more agents use extrapolative rule (the higher $\alpha_{e,t}$), the higher is the fraction of optimists and the higher is AS_t. The opposite situation holds for $y_{t-1} < 0$. The variable AS_t hence captures degree of optimism and pessimism. Values close to 1 indicate substantial wave of optimism, values close to -1 then mean strong pessimism. Values around 0 suggest neutral state of the economy.

For calibration of the model parameters, see appendix B. Now let us briefly discuss main characteristics of heuristic models as described by De Grauwe (2012) and subsequent research.

In particular, the model leads to self-fulfilling waves of optimism and pessimism, which arise when extrapolative rules after a shock prevail for some time. De Grauwe (2012) calls the waves *animal spirits* (and equation 24 is a formalization of this term), which refers to famous concept of Keynes (1936), more recently emphasized also by Akerlof & Shiller (2009). Note however, that Keynes used the term in quite positive sense of spontaneous optimism, while Akerlof & Shiller (2009) and De Grauwe (2012) consider *animal spirits* to be behavioural tendencies which drive the economy out of the equilibrium.

Furthermore, the model generates fat tails in output gap distribution. Periods of large bubbles and recessions are hence more probable than in the mainstream models.

To author's best knowledge, the only previous work using the behavioural model with heuristics to the issue of price level targeting is paper of Ho *et al.* (2019). The authors use an open-economy version of the model to examine different monetary policy regimes, and they conclude that PLT leads to better economic stability and social welfare than IT. While the general approach applied in that paper is similar to methodology used here, these works differ in several ways. Ho *et al.* (2019) examine only one very specific case, where agents under PLT forecast future price level (not future inflation), and at the same time targeted price level is constant. The analysis conducted here will start from the same basis, but after that it will examine implications of different assumptions—case with people making forecasts about future inflation will be explored as well and more versions of forecasting rules (not present elsewhere in the literature) will be considered. The model used here also slightly differs in exact formulation of policy rules, it allows for positive inflation target and positive natural real interest rate. Most importantly, it turns out that general implications of the analysis here will be quite different from the results of Ho and his co-authors.

It should be noted that overall superiority of the heuristic model to the mainstream models has not been established, as few studies attempted to fit data and compare the heuristic and rational expectations models. One such analysis has been conducted by Liu & Minford (2014), who actually reject the behavioural model on US data, while the rational expectations model passes the test. On the other hand, Jang & Sacht (2016) prefer heuristic approach for the euro area using method of moments, while they admit that this model has problem to deal with structural break in US data at the beginning of Great Moderation. Furthermore, Kukacka *et al.* (2018) use simulated maximum likelihood estimation for the euro area and the US and they also favour the behavioural model, even though not unequivocally.

There is hence no clear consensus regarding comparison of the models. The heuristic approach surely has certain relevance and it can serve as a tool to look into PLT issues from different perspective, at the same it should not be considered as clearly superior to the mainstream models and the results should thus be treated with some caution.

4 Results

This chapter presents results obtained from simulating the model described in chapter 3. Agents in price level targeting regime are assumed to form expectations about price level in the first section and about inflation in the second one. Agents in inflation targeting regime always form expectations about inflation.

In each case, simulations will be conducted 1000 times for 1000 time periods for both monetary policy regimes, and loss function described by equation 6 will be computed for each simulation of each regime. Models of both regimes are simulated under the same set of random disturbances. Presented graphs depict one random draw from the 1000 simulations.

Wilcoxon signed rank test will be applied to compare loss functions formally. Under the null hypothesis, difference between two paired sets of measurements come from a distribution with zero median. In present context, each loss value under IT has a corresponding value under PLT based on the same set of model shocks. Rejection of the null hypothesis would hence suggest that difference between two regimes is statistically significant.

4.1 Expectations about price level

Zero target Let us start with a case of zero inflation target under IT and constant targeted price level under PLT. The extrapolative rule under PLT then can be defined as

$$\widetilde{E}_t^{e,PLT} p_{t+1} = p_{t-1} \tag{25}$$

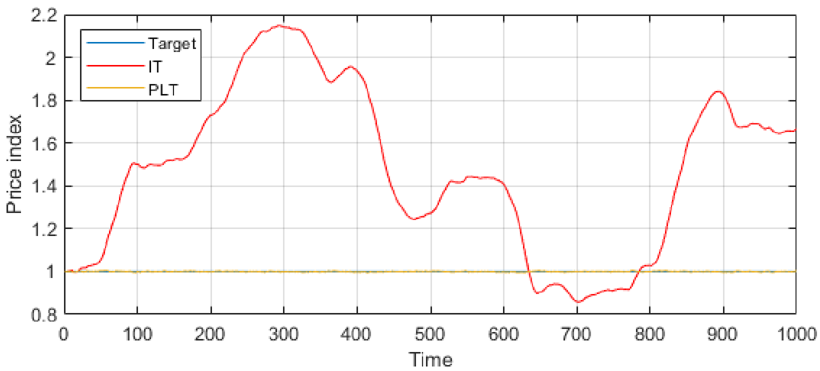
Economic agents hence choose between forecasting price level in the next period to be equal either to targeted price level in that period (fundamentalist rule 19) or to price level in the previous period.

Table 1 presents basic descriptive statistics of loss function under both regimes. We can see that price level targeting here clearly outperforms inflation targeting, as its loss function is lower by a substantial margin in all simulations, which is formally confirmed by virtually zero p-value of Wilcoxon test.

Table 1: Loss functions under constant targeted prices

	IT Loss	PLT Loss
Mean	0.4	0.03
Standard deviation	0.136	0.003
Range	0.12-1.07	0.025-0.048
Higher loss	1000	0
Wilcoxon test p-value	<0.001	

Figure 1: Price level



In addition, there is one more benefit of price level targeting not captured by the loss function—as figure 1 shows, development of price level under PLT is very stable around targeted path, while it is fluctuating significantly in the IT regime.

Figures 2 and 3 depict several key variables for both regimes. *Animal spirits* variable around extreme values of -1 and 1 in case of IT suggests strong waves of optimism and pessimism, while the PLT regime leads to much lower number of extreme AS values. Notice also difference in scales for other variables, which are better anchored around steady state values for PLT. Persistence of the variables is higher under IT than under PLT, especially in case of inflation persistence (first lag autocorrelation of 0.93 versus 0.24). Figure 4 depicts autocorrelation function for more lags and confirms that PLT substantially decreases persistence in inflation due to its compensation for past inflation shocks. Finally, figure 5 shows fraction of agents using the extrapolative rules for both output gap and inflation and for both policy regimes.

The results are not surprising as assumptions used in this scenario are very similar to those of Ho *et al.* (2019), who also concluded that PLT outperforms IT in such setting, as mentioned in the previous chapter. However, there is a question regarding reliability of such results. While PLT model is in the baseline version calibrated with $c_1^{PLT} = 0.3$, one startling feature is the fact that loss function under PLT is lowest for values of parameter c_1^{PLT} in range of about $0 - 0.5$ (not shown in detail for brevity), i.e. the regime leads to favourable results even for $c_1^{PLT} = 0$. On the other hand, $c_1^{IT} > 1$ is (approximately) needed for convergence under IT. But for $c_1 = 0$, both interest rate rules 3 and 4 are identical and the models differ only in expectations formation. The thing is that both forecasting rules under PLT have embedded certain level of regime credibility. Even extrapolative rule 25 implicitly assumes zero inflation, which is itself quite stabilizing (and it actually corresponds to the fundamentalist rule under IT). Better anchor for inflation expectations is the key theoretical advantage of price level targeting, but it is hard to believe that if central bank did not react to development of inflation and price level at all, the expectations would remain anchored in this way. Therefore, the next section will introduce increasing targeted inflation; an extrapolative agent under PLT will then have to make an implicit assumption about inflation at time t, which influences the dynamics.

Figure 2: IT with zero inflation target

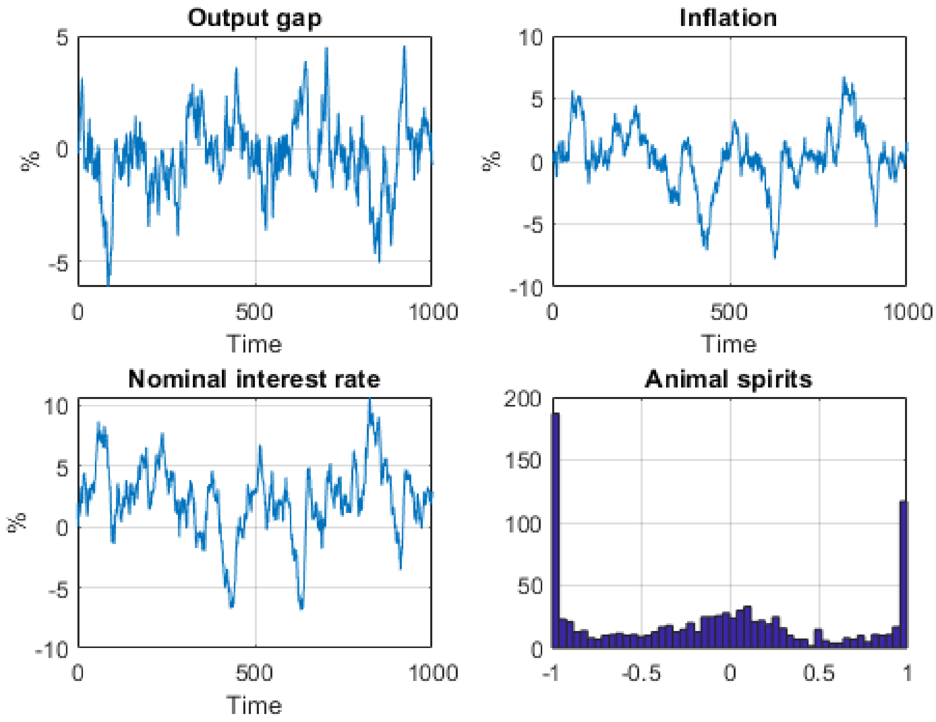


Figure 3: PLT with constant price level target

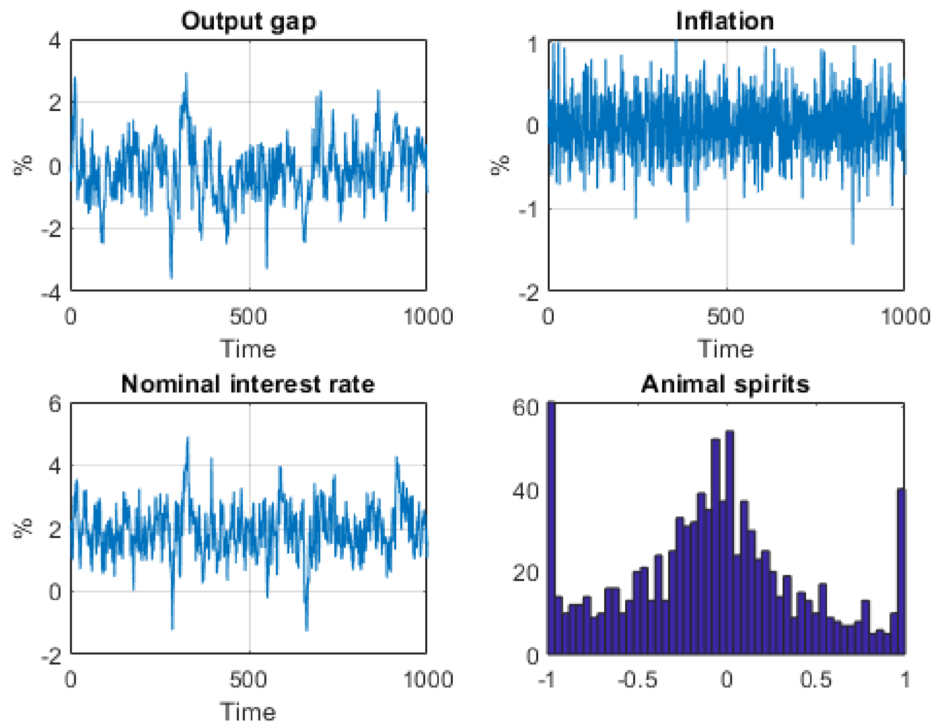


Figure 4: Inflation autocorrelation function for zero target

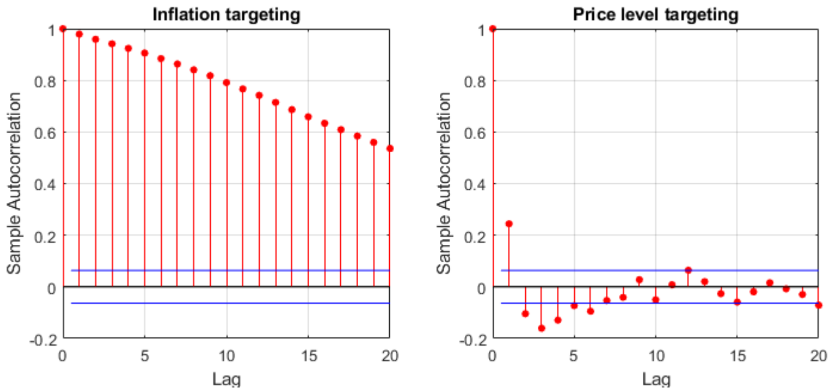
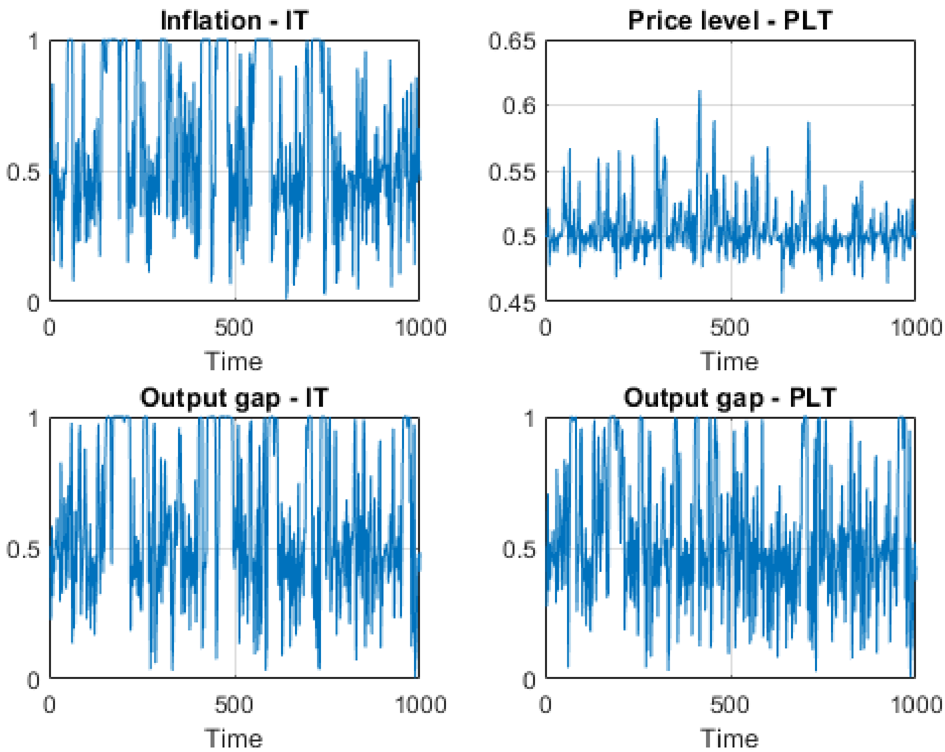


Figure 5: Fraction of extrapolatives with zero target



Positive target Now suppose that $\pi^* = 2\%$. Forecasting future price level to be equal to past price level no longer makes sense; even extrapolative agents have to address increasing trend in prices. Two main options arise. First, the extrapolative rule can be defined as

$$\widetilde{E}_t^{e,PLT} p_{t+1} = p_{t-1} + 2 * \pi^{*q} \tag{26}$$

This version still contains substantial credibility of the central bank, as inflation at times t and $t+1$ is assumed to be at its long-term target. Rule 25 is actually a special case of 26 with target set to 0. The PLT extrapolative rule is again quite similar to the fundamentalist rule under IT and hence it is itself quite stabilizing; the model with this rule is thus prone to similar criticism as the one in previous section. Results are also very similar to those in previous section with PLT leading to lower loss in all simulations, so details are not presented here to save space.

Alternative version of the rule could be

$$\widetilde{E}_t^{e,PLT} p_{t+1} = p_{t-1} + 2 * \pi_{t-1}^{q} \tag{27}$$

This rule is of purely extrapolative form; it abandons partial credibility of rules 25 and 26, and it is hence not subject to the aforementioned criticism. Agents simply assume that the economy will develop in the same way as in the past. The rationale of such rule corresponds better to the logic behind the IT extrapolative rule in equation 17.

Inflation targeting model is in all cases considered so far almost the same (it differs only in value of the target), so it leads to very similar results as before. On the other hand, price level targeting is now susceptible to model divergence. At the same time, in cases where the model does not diverge, PLT still outperforms IT. To analyse probability of divergence with higher precision, the model has been now simulated 100.000 times.

The divergence of PLT model occurs in 18.9% of cases. This number captures simulations in which the software actually provided *not available* result. Nevertheless, table 2 reveals that while most of the remaining simulations leads to value of loss function closely distributed around median, there are several extremely large observations. Mean value is actually higher than 0.97 quantile; and the steep increase in loss value occurs around this quantile. Clearly, the extreme observations also represent model divergence—the divergence is not that substantial that the software would provide *not available* result, but economically it still captures very large instability. Therefore, all simulations with loss higher than 0.97 quantile will be considered as divergent (these account for additional 2.4% of original 100.000 simulations).

With this adjustment in mind, let us look at actual results. Table 3 summarizes that price level targeting leads to better results with 78.7% probability, i.e. in all cases where it behaves stably. But with 21.3% probability, the PLT model diverges, while IT still performs relatively well.

Table 2: PLT loss functions—descriptive statistics

Statistic	Value
0.05 quantile	0.036
Median	0.042
0.95 quantile	0.055
0.96 quantile	0.068
0.97 quantile	0.201
0.98 quantile	1.2
0.99 quantile	7.44
Maximum	186.6

Notes: The numbers are based on 81.1% stable simulations.

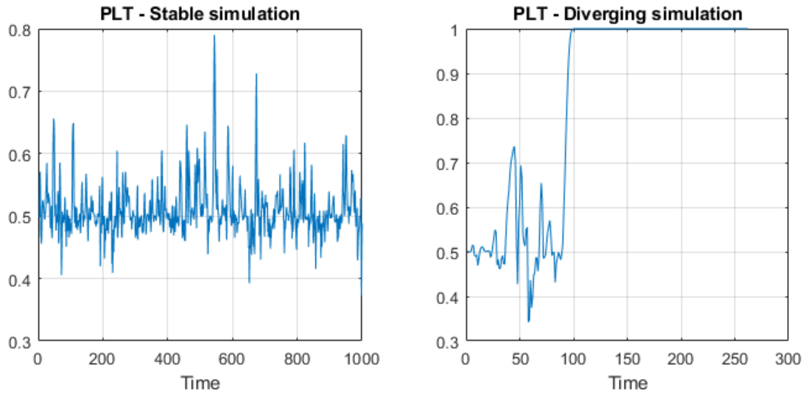
Table 3: Loss functions with increasing targeted prices

	IT Loss	PLT Loss
Divergence	0	21.3%
Mean (all simulations)	0.41	n/a
Mean (stable simulations)	0.39	0.04
Higher loss	78.7%	0
Wilcoxon test p-value	<0.001	

Notes: Wilcoxon signed rank test has been conducted only for stable simulations.

The mechanism of divergence is such that if a sufficiently large shock causes the economy to deviate from steady state by a lot, the fundamentalist rule becomes very imprecise and the extrapolative rule prevails. Once this happen, the situation is much more difficult to reverse under PLT than under IT. In addition, PLT performs very poorly under the extrapolative rule. Chapter 5 discusses whole process in more detail. Figure 6 shows a typical development of fraction of extrapolatives under PLT in a stable simulation and in a diverging simulation. The

Figure 6: Fraction of extrapolatives with positive target



main result is not sensitive to value of parameter c_1^{PLT}—the model diverges with probability in range 15-25% for all reasonable values of c_1^{PLT}.

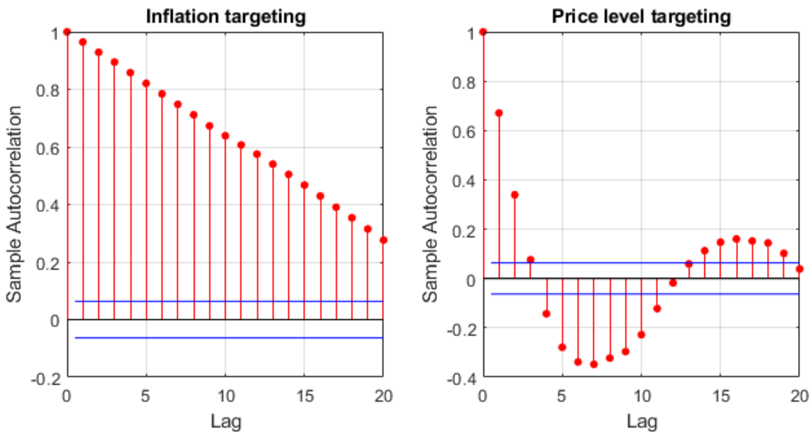
Note that 1000 periods means 250 years, so probability of divergence 21.3% within this time span is not that high. For example, in the time range of 50 years, PLT outperforms IT with about 96% probability, while it leads to very bad outcome in the remaining cases. The risk of divergence is therefore neither very high nor completely negligible.

When PLT simulation is stable, results are similar as before (not shown in detail for brevity)—*animal spirits* are concentrated mostly around 0 for PLT and more at extremes for IT; inflation persistence under PLT is now higher than before (0.66), but still lower than for IT; and price level is again very predictable under PLT.

Finally, figure 7 depicts that autocorrelation in inflation under PLT is now statistically significant even for more lags than the first one; and it turns negative in the fourth period. The shape of the autocorrelation function nicely illustrates the key mechanism of price level targeting—higher inflation now causing lower inflation in the future, and vice versa.

So far, the analysis compared models with just two forecasting rules in order to contrast characteristics of the individual rules. It is, however, easily possible to extend the model for more rules by combination of previous cases. As with one version of the rule PLT clearly outperforms IT, while with the other it may diverge, such approach lets the model decide which version will prevail; and the results can be thus considered as more conclusive than individual analyses before.

Figure 7: Inflation autocorrelation function (rule 27)



Inflation targeting will be modelled in the same way as before with two rules only; the focus will be on price level targeting. In particular, let us consider one fundamentalist (rule 19) and two extrapolative (26 and 27) rules. All remaining aspects of the model are the same as before. Computation of utilities from the rules, probability of using each of them, and resulting market expectations are computed by straightforward extension of approach described in section 3.

When the model is simulated, it turns out that the PLT regime is now stable and superior to IT, as table 4 shows—divergence is no longer a problem. PLT works only slightly worse than in case without the purely extrapolative rule, and it outperforms IT by a large margin.

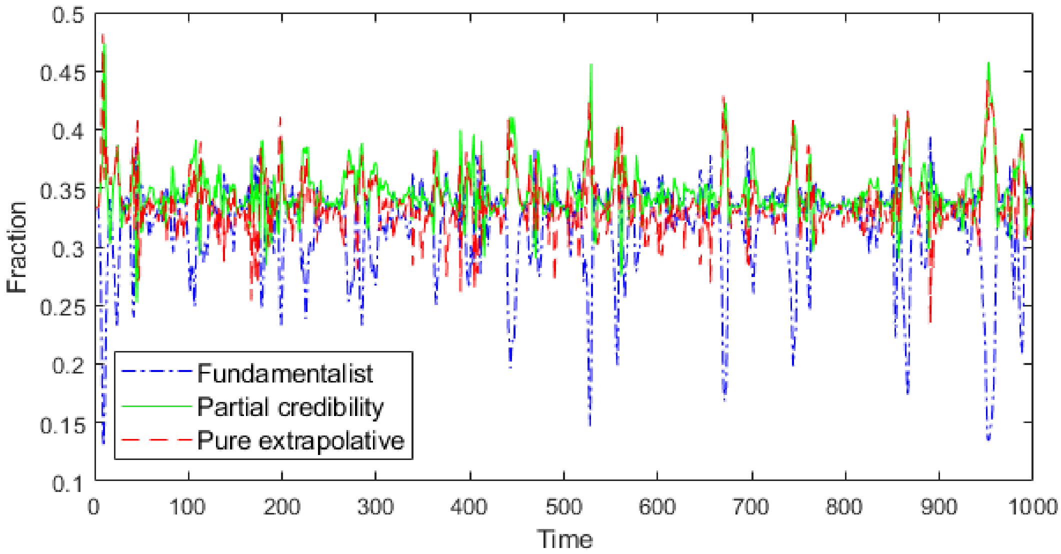
Figure 8 shows that share of economic agents using each rule fluctuates around one third. Overall, as opposed to case with zero target, where the results could have seem caused by construction due to stabilizing nature of even the extrapolative PLT rule, results of the analysis with three rules actually provide stronger support for price level targeting, as the regime performs well even in presence of the purely extrapolative rule.

Table 4: Loss functions with three forecasting rules

	IT Loss	PLT Loss
Mean	0.4	0.04
Standard deviation	0.128	0.005
Range	0.15-0.88	0.027-0.077
Higher loss	1000	0
Wilcoxon test p-value	<0.001	

Notes: IT model is the same as before, but the simulations have been conducted again, so values sensitive to individual observations (such as range) slightly differ.

Figure 8: Probabilities of using each of three rules



4.2 Expectations about inflation

Now let us assume that agents even under price level targeting still create expectations about inflation at $t+1$; while price level expectations are now determined only ex post. The reasoning behind such approach may be such that agents still care about inflation rather than price level (which is captured also by loss function 6, which contains deviations of inflation from target, not of price level). PLT regime serves just as a tool to deliver possibly more stable inflation, which is still the ultimate goal. The agents hence still think about how large inflation rate will be in the next period, not what will be the price level.

Conversely to the previous case, extrapolative rule is now the one which is straightforward:

$$\widetilde{E}_t^{e,PLT}\pi_{t+1} = \pi_{t-1} \tag{28}$$

This is the same as the IT extrapolative rule. Extrapolative agents now really do not distinguish between policy regimes, they simply use adaptive expectations; and only fundamentalists actually take into account the monetary policy regime. This is quite favourable feature of this approach as opposed to that with expectations formed about price level.

On the other hand, there are now more potential specifications of the fundamentalist rule. The goal is to have inflation rate such that targeted price level in the next period is met, but this is not directly possible as the agents know only past values of variables. Therefore, an assumption about current inflation and current price level needs to be made. Two reasonable specifications are to assume either that current inflation is equal to the long-term target ($\widetilde{E}_t^f \pi_t = \pi^*$) and deviation of price level will be corrected for in next period only, or that deviation is distributed over current and next period with the same inflation rate. This leads to following rules[6]

$$\widetilde{E}_t^{f,PLT} \pi_{t+1}^q = \overline{p}_{t+1} - \widetilde{E}_t^f p_t = \overline{p}_{t+1} - p_{t-1} - \widetilde{E}_t^f \pi_t^q = \overline{p}_{t+1} - p_{t-1} - \pi^{*q} = \overline{p}_{t-1} - p_{t-1} + \pi^{*q} \quad (29)$$

$$\widetilde{E}_t^{f,PLT} \pi_{t+1}^q = \frac{\overline{p}_{t+1} - p_{t-1}}{2} \quad (30)$$

Thus, we have two different specifications of the fundamentalist rule. Nevertheless, it turns out that regardless of which one is used, PLT leads to divergence of the model in all 1000 simulations; the divergence occurs within 102-367 periods (without any significant difference between the two rules). The extrapolative rule always gains dominance over time, which is then very difficult to reverse. On the contrary, IT model remains reasonably stable as before.

It is again very straightforward to extend the analysis by combining both versions into one model with three rules. But since extrapolative rule prevails in both individual scenarios, it can be expected that such model will lead again to divergence; and actual simulations confirm the intuition. When people create expectations about inflation regardless of monetary policy regime, price level targeting leads to divergence of the economy regardless of particular model specification.

5 Discussion

The results presented in chapter 4 crucially depend on assumption about the expectations formation, but they are fairly robust to particular values of model parameters. The only exceptions

[6]Note that for some initial price level p_1 (leading to first considered inflation rate being π_2), price levels can be expressed by repeated substitution as $\overline{p}_{t-1} = p_1 + (t-2)\pi^{*q}$ and $p_{t-1} = p_1 + \sum_{j=2}^{t-1} \pi_j^q$. The rule in equation 29 can then be rewritten as

$$\widetilde{E}_t^{f,PLT} \pi_{t+1}^q = \overline{p}_{t-1} - p_{t-1} + \pi^{*q} = \pi^{*q} + \sum_{j=2}^{t-1} (\pi^{*q} - \pi_j^q)$$

This formulation nicely illustrates how price level targeting compensates for past deviations of inflation rate from the long-term target.

are shock of NKPC equation 2 (with increasing standard deviation of the shock and with its increasing autoregressive component, both monetary policy regimes deteriorate, but susceptibility of PLT to instability grows substantially, while IT is still stable) and intensity of choice parameter γ (with decreasing γ, PLT becomes more stable, but it still performs worse than IT in cases when it was previously unstable). Sensitivity analysis is thus in accordance with overall message of the results. Details of the sensitivity analysis are presented in appendix B.

In total, the results of whole analysis are mixed. When economic agents form expectations about price level, price level targeting is generally superior to inflation targeting. This holds whether the target is zero or positive. Such conclusion confirms results of Ho *et al.* (2019) that PLT in heuristic model leads to higher social welfare than IT in the specific setting with zero target. PLT may diverge with probability of 21.3% in one particular specification with positive target, but this is, first, not as high as it might seem since it captures very long time span (the number is based on 1000 time periods, corresponding to 250 years), and second, this feature disappears when the model is expanded to contain one additional forecasting rule.

On the other hand, the outcomes of section 4.1 are based on the particular assumption about expectations. Under alternative case with expectations being created about inflation, it turns out that price level targeting is very prone to losing credibility and it diverges in all simulations. Furthermore, it has been discussed that even in the first case, PLT starts to behave poorly with high standard deviation of shock in NK Phillips curve or with autoregressive specification of that shock. Therefore, while potential benefits of price level targeting have been confirmed, complete results are rather in favour of the inflation targeting regime, which is significantly more robust choice. The overall message of the analysis is hence different to the one by Ho *et al.* (2019), who provide stronger support for PLT.

One crucial question obviously arises—what in fact causes so contradictory results of the model depending on whether the agents form expectations about price level or inflation? For a potential explanation, let us firstly clearly state actual difference between the two approaches. When it is assumed that expectations are created about price level, people simply forecast future price level based on the past with no regard to present. Fundamentalists just assume that price level will be at the target and they do not care whether the compensation for a past inflation deviation will occur in the current time period, in the next one, or gradually in both. There is hence more flexibility and the rule remains viable and stabilizes the economy.

On the other hand, when people form expectations about inflation, the fundamentalist rule requires an implicit assumption about current inflation rate in order to make forecast about the future one. Even when price level targeting is itself working well and it delivers targeted price level, if the core of the compensation for past inflation deviations occurs in the current period, while people assumed it would occur in the next period (or vice versa), the fundamentalist rule becomes imprecise. Moreover, for some particular realization of current inflation rate, the fundamentalist rule can in fact become inconsistent with the policy regime and thus not stabilizing for the economy. Therefore, good performance of the fundamentalist rule needs sufficiently precise forecast of both present and future state of the economy, which poses stronger requirements on the economic agents. The extrapolative rule with its simple nature then may become more attractive and it predominates over time. This part of the answer hence explains why price level targeting in case of expectations created about inflation leads to higher prevalence of the extrapolative rule than under the assumption of expectations formed about price level.

Second part of the answer is related to actual performance of the policy regimes under the extrapolative rules. To examine this issue, the model has been generated without the fundamentalist rules in inflation forecasting. In other words, all economic agents use adaptive expectations and the two regimes therefore differ only in central bank rule; monetary policy has no credibility. The fundamentalist rule comprises a stabilizing element in the original model, so the performance of inflation targeting is worse without it; but the regime still leads to relatively stable development of the economy. In fact, some of classic inflation targeting studies by Svensson (1997, 1999a) were conducted under adaptive expectations. On the other hand, price level targeting is unstable under adaptive expectations. Such conclusion holds regardless of whether the adaptive expectations are about price level or inflation; and it does not depend on value of c_1^{PLT} parameter in the policy rule either. Furthermore, the outcome is the same whether it is assumed that output gap forecasts are created according to the heuristic switching mechanism just as before, or whether adaptive expectations are assumed even in case of output gap.

To describe actual dynamics of the model under adaptive expectations, let us assume that the economy is in a boom due to some shocks; inflation is above target and output gap is positive; the central bank then reacts by increasing nominal interest rate. As a result, inflation and output gap fall and return to steady state values; but it takes a few time periods because of inflation persistence, so deviation of price level from its targeted path accumulates. When inflation and output gap are at the steady state, central bank targeting inflation returns also the nominal rate to its steady state value and the economy remains stable. But central bank targeting price level needs to depress inflation further in order to meet its price level target, so the nominal rate remains high; and output gap declines as a by-product. The nominal rate now does not smooth the cycle caused by shocks, but on the contrary, it enhances it. When the price level finally meets the target and the nominal rate declines, inflation is below the steady state; and because of the inflation persistence, it remains there for some time, generating negative deviation of price level from its target. The central bank now decreases the nominal rate below the steady state level and the mechanism is repeated. Every time, correcting for deviation in price level requires larger deviation of inflation from its long-term target, which then generates even larger deviation in price level in the opposite direction, and so on.

In addition, even though inflation persistence is key part of the mechanism, the final outcome does not depend on parameter b_1 in NKPC (equation 2). As $\widetilde{E}_t \pi_{t+1} = \pi_{t-1}$ under adaptive expectations, term $b_1 \widetilde{E}_t \pi_{t+1} + (1 - b_1)\pi_{t-1}$ in that equation is independent on value of b_1. The inflation persistence is thus inherent in the adaptive expectations framework.

To summarize the arguments, price level targeting performs well as long as it retains certain credibility, but it is, first, more vulnerable to losing the credibility than inflation targeting (and the vulnerability is higher in the case of expectations formed about inflation), and second, it behaves much worse when this occurs and adaptive expectations prevail among people. Response to price level in the policy rule needs to be combined with regime-consistent expectations for PLT to function well.

On the other hand, the analysis here might be a bit unjust towards price level targeting, which constitutes certain limitation of the applied method. Firstly, higher vulnerability of PLT under the assumption of expectations created about inflation may be partly caused artificially by sequential nature of the model. If people knew current state of the economy, expectations about future inflation and price level would be equivalent; and in reality, people presumably have more information about present than the model suggests. (On the other hand, we can expect that the information is imperfect and official macroeconomic data usually come with a lag.) Secondly, regarding the behaviour under adaptive expectations, the PLT rule contains only current values of economic variables; forward-looking central bank could take the inflation persistence into account and normalize the interest rate sooner than when price level is at the target.

Therefore (and considering that inflation targeting faces substantial difficulties in the form of the zero lower bound), disregarding price level targeting completely based on its low stability could be a mistake, as it offers a potential solution to at least partially deal with the zero lower bound problem. At the same time, the model does reveal sensitivity of the PLT regime to particular model specification and to certain model parameters; and it implies that PLT leads to substantially higher requirements on forward-looking behaviour of the central bank. Thus, while completely disregarding price level targeting might be a mistake, the results at the very least call for high caution before potential implementation of PLT in practice, and they also corroborate the paramount importance of credibility for proper conduct of the regime.

The analysis focused on comparison of pure price level and inflation targeting, note however, that these are not the only options. For example, already mentioned average inflation targeting could be a potential way to utilize advantages of price level targeting without endangering credibility of monetary policy; under certain assumptions, it can in fact outperform both IT and PLT (see Nessen & Vestin (2005)). Its treatment is beyond scope of this text, but this framework surely comprises an area for further economic research.

6 Concluding Remarks

The paper has analysed price level targeting, which has been previously found to outperform inflation targeting in various theoretical studies; and the difference between the two regimes is even more pronounced when zero lower bound on nominal interest rates is considered. Such results are, however, to a large extent given by strong assumption of rational expectations, which seems to be unrealistic in practice. Therefore, a simple New Keynesian 3-equation model which abandons rational expectations hypothesis has been applied here. Economic agents instead form expectations about future inflation (or price level) and output gap using heuristics—they choose between a few simple forecasting rules based on their past performance. Two rules have been used in the baseline setting—fundamentalist rule consistent with given monetary policy regime, and extrapolative rule corresponding to adaptive expectations. Furthermore, two main approaches to PLT have been discussed—one with assumption that people form expectations about future price level, and the other assuming expectations about inflation (which is ex ante not equivalent due to sequential nature of the model).

It has been shown that price level targeting outperforms inflation targeting under assumption that expectations are formed about price level. Nevertheless, the case with expectations about inflation causes PLT to diverge regardless of particular specification of the fundamentalist forecasting rule. The regime is (as compared with IT) more prone to losing credibility, so the extrapolative rule begins to prevail. And if that happens, the PLT regime behaves very poorly, self-fulfilling wave arises and gets the economy on an explosive path, while IT remains relatively stable. Moreover, even in models in which PLT is stable and superior under the baseline calibration, it is very sensitive to characteristics of shock in NK Phillips curve. Inflation targeting is hence under model uncertainty safer choice and, according to results of the analysis, it is therefore overall preferred to price level targeting (even though the preference is not unequivocal). Results are in general robust to remaining model parameters and hinge especially on the actual assumptions about expectations formation.

The results are thus rather contradictory to much of existing research, which favours price level targeting—but the research is largely determined by the assumption of rational expectations. On the other hand, the paper is in accordance with approach common among policymakers, who admit theoretical advantages of PLT, but who are also aware that the rational expectations hypothesis might be too strong assumption and that the regime might not gain and retain sufficient credibility. This is what has been shown here.

At the same time, it needs to be said that the model used here is very simplified. It is a model of closed economy, it takes into account neither length of monetary policy horizon nor many other characteristics or sectors of the economy, and central bank policy rules are backward-looking. Dealing with all of these issues can be a path for future research. Therefore, it would be premature to claim based on results of this paper that price level targeting is definitely not a good option for monetary policy, which would lead to high risk of explosive path of the economy in practice. The policy regime surely offers substantial potential benefits, which might be especially useful in prevalence of the zero lower bound on nominal interest rates over the world. But at the very least, the results suggest that policymakers should be very cautious before actually adopting PLT in practice, and that introducing only its certain elements, e.g. in a form of average inflation targeting, might be a better option than pure price level targeting itself.

References

Akerlof, G. A. & R. J. Shiller (2009): *Animal Spirits.* Princeton University Press.

Ambler, S. (2009): "Price-Level Targeting and Stabilisation Policy: A Survey." *Journal of Economic Surveys* **23(5)**: pp. 974–997.

Anderson, S. P., A. De Palma, & J. F. Thisse (1992): *Discrete Choice Theory of Product Differentiation.* The MIT Press.

Andrade, P., R. K. Crump, S. Eusepi, & E. Moench (2016): "Fundamental disagreement." *Journal of Monetary Economics* **83**: pp. 106–128.

Ball, L., N. G. Mankiw, & R. Reis (2005): "Monetary policy for inattentive economies." *Journal of Monetary Economics* **52**: pp. 703–725.

Berentsen, A. & C. Waller (2011): "Price-Level Targeting and Stabilization Policy." *Journal of Money, Credit and Banking* **43(2)**: pp. 559–580.

Berg, C. & L. Jonung (1999): "Pioneering price level targeting: The Swedish experience 1931-1937." *Journal of Monetary Economics* **43(3)**: pp. 525–551.

Bohm, J., J. Filacek, I. Kubicova, & R. Zamazalova (2012): "Price-Level Targeting – A Real Alternative to Inflation Targeting?" *Czech Journal of Economics and Finance* **62(1)**: pp. 2–26.

Branch, W. A. (2004): "The Theory of Rational Heterogeneous Expectations: Evidence from Survey Data on Inflation Expectations." *The Economic Journal* **114**: pp. 592–621.

Brock, W. A. & C. H. Hommes (1997): "A Rational Route to Randomness." *Econometrica* **65(5)**: pp. 1059–1095.

Cateau, G. (2017): "Price-level versus inflation targeting under model uncertainty." *Canadian Journal of Economics* **50(2)**: pp. 522–540.

Cateau, G., O. Kryvtsov, M. Shukayev, & A. Ueberfeldt (2009): "Adopting Price-level Targeting under Imperfect Credibility in ToTEM." *Working paper*, Bank of Canada.

Clarida, R., J. Gali, & M. Gertler (1999): "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature* **37(4)**: pp. 1661–1707.

Cournede, B. & D. Moccero (2011): "Is there a case for price-level targeting?)." In L. Odor, D. P. Cobham, & M. Beblavy (editors), "The Euro Area and the Financial Crisis," pp. 292–319. Cambridge University Press.

Cover, J. P. & P. Pecorino (2005): "Price and Output Stability under Price-Level Targeting." *Southern Economic Journal* **72(1)**: pp. 152–166.

De Grauwe, P. (2012): *Lectures on Behavioral Macroeconomics.* Princeton University Press.

Eggertsson, G. B. & M. Woodford (2003): "The Zero Bound on Interest Rates and Optimal Monetary Policy." *Brookings Papers on Economic Activity* **(1)**: pp. 139–211.

Eusepi, S. & B. Preston (2018): "The Science of Monetary Policy: Imperfect Knowledge Perspective." *Journal of Economic Literature* **56(1)**: pp. 3–59.

Evans, G. W. & S. Honkapohja (2001): *Learning and Expectations in Macroeconomics.* Princeton University Press, Frontiers of Economic Research edition.

Franta, M., T. Holub, P. Kral, I. Kubicova, K. Smidkova, & B. Vasicek (2014): "The Exchange Rate as an Instrument at Zero Interest Rates: The Case of the Czech Republic." *Research and policy notes 3*, Czech National Bank.

Gabaix, X. (2020): "A Behavioral New Keynesian Model." *The American Economic Review* **110(8)**: pp. 2271–2327.

Gaspar, V., F. Smets, & D. Vestin (2007): "Is Time Ripe for Price Level Path Stability?" *Working Paper Series 818*, European Central Bank.

Giannoni, M. P. (2014): "Optimal interest-rate rules and inflation stabilization versus price-level stabilization." *Journal of Economic Dynamics* & *Control* **41**: pp. 110–129.

Hatcher, M. & P. Minford (2016): "Stabilisation Policy, Rational Expectations and Price-Level versus Inflation Targeting: A Survey." *Journal of Economic Surveys* **30(2)**: pp. 327–355.

Ho, T.-K., Y.-C. Lin, & Y. Kuo-Chun (2019): "Alternative Monetary Policies under Keynesian Animal Spirits." *Macroeconomic Dynamics* pp. 1–27.

Hommes, C. (2011): "The heterogeneous expectations hypothesis: Some evidence from the lab." *Journal of Economic Dynamics & Control* **35**: pp. 1–24.

Honkapohja, S. & K. Mitra (2019): "Price level targeting with evolving credibility." *Journal of Monetary Economics.* **In press**.

IMF (2020): "Annual Report on Exchange Arrangements and Exchange Restrictions 2019." *Annual report*, International Monetary Fund, Washington D.C.

Jang, T.-S. & S. Sacht (2016): "Animal Spirits and the Business Cycle: Empirical Evidence from Moment Matching." *Metroeconomica* **67(1)**: pp. 76–113.

Keynes, J. M. (1936): *The General Theory of Employment, Interest, and Money.* Palgrave Macmillan.

Kukacka, J., T.-S. Jang, & S. Sacht (2018): "On the Estimation of Behavioral Macroeconomic Models via Simulated Maximum Likelihood." *Kiel University Economic Working Paper* **(2018-11)**.

Liu, C. & P. Minford (2014): "Comparing behavioural and rational expectations for the US post-war economy." *Economic Modelling* **43**: pp. 407–415.

Mankiw, G. N., R. Reis, & J. Wolfers (2003): "Disagreement About Inflation Expectations." *NBER Macroeconomics Annual (MIT Press)* **(18)**: pp. 209–248.

Masson, P. R. & M. D. Shukayev (2011): "Are bygones not bygones? Modeling price-level targeting with an escape clause and lessons from the gold standard." *Journal of Macroeconomics* **33**: pp. 162–175.

Milani, F. (2012): "The Modeling of Expectations in Empirical DSGE Models: a Survey." *Working Papers, University of California-Irvine* .

Minford, P. & D. Peel (2003): "Optimal Monetary Policy: Is Price-Level Targeting the Next Step?" *Scottish Journal of Political Economy* **50(50)**: pp. 650–667.

Nessen, M. & D. Vestin (2005): "Average Inflation Targeting." *Journal of Money, Credit and Banking* **37(5)**: pp. 837–863.

Popescu, I. V. (2012): "Price-Level Targeting—A Viable Alternative to Inflation Targeting?" *CES Working Papers* **4(4)**: pp. 809–823.

Straumann, T. & U. Woitek (2009): "A pioneer of a new monetary policy? Sweden's price-level targeting of the 1930s revisited." *European Review of Economics History* **13(2)**: pp. 251–282.

Svensson, L. E. (1997): "Inflation forecast targeting: Implementing and monitoring inflation targets." *European Economic Review* **41**: pp. 1111–1146.

Svensson, L. E. (1999a): "Inflation Targeting: Some Extensions." *The Scandinavian Journal of Economics* **101(3)**: pp. 337–361.

Svensson, L. E. (1999b): "Price-Level Targeting versus Inflation Targeting: A Free Lunch?" *Journal of Money, Credit and Banking* **31(3)**: pp. 277–295.

Svensson, L. E. (2000): "The Zero Bound in an Open Economy: A Foolproof Way of Escaping from a Liquidity Trap." *CEPR Discussion Papers* **(2566)**.

Taylor, J. (1993): "Discretion versus policy rules in practice." *Carnegie-Rochester Conference Series on Public policy* **39**: pp. 195–214.

Vestin, D. (2006): "Price-level versus inflation targeting." *Journal of Monetary Economics* **53**: pp. 1361–1376.

Walsh, C. E. (2003): "Speed Limit Policies: The Output Gap and Optimal Monetary Policy." *The American Economic Review* **93(1)**: pp. 265–278.

Woodford, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press.

Woodford, M. (2013): "Macroeconomic Analysis Without the Rational Expectations Hypothesis." *Annual Review of Economics* **5**: pp. 303–346.

Yetman, J. (2005): "The credibility of the monetary policy "free lunch"." *Journal of Macroeconomics* **27**: pp. 434–451.

Appendices

A Mathematical Derivations

This appendix shows several explicit derivations, which were skipped in the main body of the paper.

A.1 Model Solution

The applied model consists of three key equations, as presented in chapter 3. These are for inflation targeting regime

$$y_t = a_1 \widetilde{E}_t y_{t+1} + (1 - a_1) y_{t-1} + a_2 (i_t - \widetilde{E}_t \pi_{t+1} - \overline{r}) + \epsilon_t \tag{31}$$

$$\pi_t = b_1 \widetilde{E}_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 y_t + \eta_t \tag{32}$$

$$i_t = c_3 \left[\pi^* + \overline{r} + c_1 (\pi_t - \pi^*) + c_2 y_t\right] + (1 - c_3) i_{t-1} + u_t \tag{33}$$

For given expectations of output gap and inflation, the model can be easily solved by plugging equation 33 into 31, which leads to

$$\begin{aligned} y_t = a_1 \widetilde{E}_t y_{t+1} + (1 - a_1) y_{t-1} + a_2 c_3 \pi^* + a_2 c_3 \overline{r} + a_2 c_1 c_3 (\pi_t - \pi^*) + a_2 c_2 c_3 y_t + \\ a_2 (1 - c_3) i_{t-1} + a_2 u_t - a_2 \widetilde{E}_t \pi_{t+1} - a_2 \overline{r} + \epsilon_t \end{aligned} \tag{34}$$

This equation together with equation 32 can be further rearranged:

$$\pi_t - b_2 y_t = b_1 \widetilde{E}_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + \eta_t \tag{35}$$

$$\begin{aligned} - a_2 c_1 c_3 \pi_t + (1 - a_2 c_2 c_3) y_t = -a_2 \widetilde{E}_t \pi_{t+1} + a_1 \widetilde{E}_t y_{t+1} + (1 - a_1) y_{t-1} + a_2 (1 - c_1) c_3 \pi^* + \\ a_2 (1 - c_3)(i_{t-1} - \overline{r}) + a_2 u_t + \epsilon_t \end{aligned} \tag{36}$$

Which can be rewritten in matrix notation as

$$\begin{pmatrix} 1 & -b_2 \\ -a_2c_1c_3 & 1-a_2c_2c_3 \end{pmatrix} \begin{pmatrix} \pi_t \\ y_t \end{pmatrix} = \begin{pmatrix} b_1 & 0 \\ -a_2 & a_1 \end{pmatrix} \begin{pmatrix} \widetilde{E}_t\pi_{t+1} \\ \widetilde{E}_t y_{t+1} \end{pmatrix} + \begin{pmatrix} 1-b_1 & 0 \\ 0 & 1-a_1 \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ y_{t-1} \end{pmatrix}$$
$$+ \begin{pmatrix} 0 \\ a_2(1-c_1)c_3 \end{pmatrix} \pi^* + \begin{pmatrix} 0 \\ a_2(1-c_3) \end{pmatrix} (i_{t-1}-\overline{r}) + \begin{pmatrix} \eta_t \\ a_2u_t+\epsilon_t \end{pmatrix} \tag{37}$$

or simply

$$\mathbf{A}\mathbf{Z}_t = \mathbf{B}\widetilde{E}_t\mathbf{Z}_{t+1} + \mathbf{C}\mathbf{Z}_{t-1} + \mathbf{b}\pi^* + \mathbf{c}(i_{t-1}-\overline{r}) + \upsilon_t \tag{38}$$

The solution is then given by

$$\mathbf{Z}_t = \mathbf{A}^{-1}\left[\mathbf{B}\widetilde{E}_t\mathbf{Z}_{t+1} + \mathbf{C}\mathbf{Z}_{t-1} + \mathbf{b}\pi^* + \mathbf{c}(i_{t-1}-\overline{r}) + \upsilon_t\right] \tag{39}$$

The solution exists if $\mathbf{A}$ is not singular, i.e. if $1 - a_2c_2c_3 - a_2b_2c_1c_3 \neq 0$, which can be rewritten as $1 \neq a_2c_3(c_2 + b_2c_1)$. The solution yields y_t and π_t, which can be plugged into interest rate rule 33 to generate i_t.

Under assumption that agents forecast inflation even under PLT (as examined in section 4.2), the same approach can be applied to price level targeting, which differs only in the interest rate rule:

$$i_t = c_3\left[\pi^* + \overline{r} + c_1(p_t - \overline{p}_t) + c_2y_t\right] + (1-c_3)i_{t-1} + u_t \tag{40}$$

As equation 32 determines current inflation rate, we need to put inflation also into the interest rate rule, which can however be done easily as $p_t = p_{t-1} + \pi_t^q$. But as the model equations contain annualized inflation, we need to use the form $p_t = p_{t-1} + \frac{1}{4}\pi_t$. The interest rate rule can be therefore written in form

$$i_t = c_3\left[\pi^* + \overline{r} + c_1(\frac{\pi_t}{4} + p_{t-1} - \overline{p}_t) + c_2y_t\right] + (1-c_3)i_{t-1} + u_t \tag{41}$$

where p_{t-1} and $\overline{p}_t$ are known values. This rule can be then plugged into equation 31 and rearranged just as in case of inflation targeting above. This yields

$$\begin{pmatrix} 1 & -b_2 \\ -\frac{a_2 c_1 c_3}{4} & 1 - a_2 c_2 c_3 \end{pmatrix} \begin{pmatrix} \pi_t \\ y_t \end{pmatrix} = \begin{pmatrix} b_1 & 0 \\ -a_2 & a_1 \end{pmatrix} \begin{pmatrix} \widetilde{E}_t \pi_{t+1} \\ \widetilde{E}_t y_{t+1} \end{pmatrix} + \begin{pmatrix} 1 - b_1 & 0 \\ 0 & 1 - a_1 \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ y_{t-1} \end{pmatrix}$$
$$+ \begin{pmatrix} 0 \\ a_2 c_3 \end{pmatrix} \pi^* + \begin{pmatrix} 0 \\ a_2(1 - c_3) \end{pmatrix} (i_{t-1} - \overline{r}) + \begin{pmatrix} 0 \\ a_2 c_1 c_3 \end{pmatrix} (p_{t-1} - \overline{p}_t) + \begin{pmatrix} \eta_t \\ a_2 u_t + \epsilon_t \end{pmatrix} \tag{42}$$

or

$$\widetilde{\mathbf{A}}\mathbf{Z}_t = \mathbf{B}\widetilde{E}_t\mathbf{Z}_{t+1} + \mathbf{C}\mathbf{Z}_{t-1} + \widetilde{\mathbf{b}}\pi^* + \mathbf{c}(i_{t-1} - \overline{r}) + \mathbf{d}(p_{t-1} - \overline{p}_t) + \upsilon_t \tag{43}$$

which differs from the case of inflation targeting by adjustments in matrix $\widetilde{\mathbf{A}}$, vector $\widetilde{\mathbf{b}}$ (the tilde captures that it is slightly different than matrix $\mathbf{A}$ and vector $\mathbf{b}$ in equation 38), and by the term $\mathbf{d}(p_{t-1} - \overline{p}_t)$. The solution is given by

$$\mathbf{Z}_t = \widetilde{\mathbf{A}}^{-1} \left[\mathbf{B}\widetilde{E}_t\mathbf{Z}_{t+1} + \mathbf{C}\mathbf{Z}_{t-1} + \widetilde{\mathbf{b}}\pi^* + \mathbf{c}(i_{t-1} - \overline{r}) + \mathbf{d}(p_{t-1} - \overline{p}_t) + \upsilon_t \right] \tag{44}$$

with condition $1 \neq a_2 c_3 (c_2 + \frac{b_2 c_1}{4})$.

Derivation for PLT model is slightly different when it is assumed that people form expectations about price level (as in section 4.1). Again, we need to explicitly account for difference between annualized and quarterly inflation and to rewrite model equations using $\widetilde{E}_t \pi_{t+1} = 4(\widetilde{E}_t p_{t+1} - p_t)$ and $\pi_t = 4(p_t - p_{t-1})$. This leads to

$$y_t = a_1 \widetilde{E}_t y_{t+1} + (1 - a_1) y_{t-1} + a_2 i_t - 4a_2 \widetilde{E}_t p_{t+1} + 4a_2 p_t - a_2 \overline{r} + \epsilon_t \tag{45}$$

$$4(p_t - p_{t-1}) = 4b_1 \widetilde{E}_t p_{t+1} - 4b_1 p_t + 4(1 - b_1)(p_{t-1} - p_{t-2}) + b_2 y_t + \eta_t \tag{46}$$

$$i_t = c_3 \left[\pi^* + \overline{r} + c_1 (p_t - \overline{p}_t) + c_2 y_t \right] + (1 - c_3) i_{t-1} + u_t \tag{47}$$

Plugging nominal interest rate from 47 to 45 just as before yields

$$y_t = a_1\widetilde{E}_t y_{t+1} + (1-a_1)y_{t-1} + a_2c_3\pi^* + a_2(c_3-1)\overline{r} + a_2c_1c_3p_t - a_2c_1c_3\overline{p}_t + a_2c_2c_3y_t$$
$$+ a_2(1-c_3)i_{t-1} + a_2u_t - 4a_2\widetilde{E}_tp_{t+1} + 4a_2p_t + \epsilon_t \quad (48)$$

Then let us put all terms with y_t and p_t in equations 46 and 48 on the left hand side and all remaining terms on the right hand side. This results in

$$\begin{pmatrix} 4(1+b_1) & -b_2 \\ -a_2(4+c_1c_3) & 1-a_2c_2c_3 \end{pmatrix} \begin{pmatrix} p_t \\ y_t \end{pmatrix} = \begin{pmatrix} 4b_1 & 0 \\ -4a_2 & a_1 \end{pmatrix} \begin{pmatrix} \widetilde{E}_tp_{t+1} \\ \widetilde{E}_ty_{t+1} \end{pmatrix} +$$
$$\begin{pmatrix} 4(2-b_1) & 0 \\ 0 & 1-a_1 \end{pmatrix} \begin{pmatrix} p_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} -4(1-b_1) & 0 \\ 0 & -a_2c_1c_3 \end{pmatrix} \begin{pmatrix} p_{t-2} \\ \overline{p}_t \end{pmatrix} +$$
$$\begin{pmatrix} 0 \\ a_2(1-c_3) \end{pmatrix} (i_{t-1}-\overline{r}) + \begin{pmatrix} 0 \\ a_2c_3 \end{pmatrix} \pi^* + \begin{pmatrix} \eta_t \\ a_2u_t+\epsilon_t \end{pmatrix} \quad (49)$$

or

$$\overline{\mathbf{A}}\widetilde{\mathbf{Z}}_t = \overline{\mathbf{B}}\widetilde{E}_t\widetilde{\mathbf{Z}}_{t+1} + \overline{\mathbf{C}}\widetilde{\mathbf{Z}}_{t-1} + \widetilde{\mathbf{b}}\pi^* + \mathbf{c}(i_{t-1}-\overline{r}) + \widetilde{\mathbf{d}}\mathbf{P}_t + \upsilon_t \quad (50)$$

with $\mathbf{P}_t = \begin{pmatrix} p_{t-2} \\ \overline{p}_t \end{pmatrix}$. This can be solved again as

$$\widetilde{\mathbf{Z}}_t = \overline{\mathbf{A}}^{-1}\left[\overline{\mathbf{B}}\widetilde{E}_t\widetilde{\mathbf{Z}}_{t+1} + \overline{\mathbf{C}}\widetilde{\mathbf{Z}}_{t-1} + \widetilde{\mathbf{b}}\pi^* + \mathbf{c}(i_{t-1}-\overline{r}) + \widetilde{\mathbf{d}}\mathbf{P}_t + \upsilon_t\right] \quad (51)$$

with condition $4(1+b_1)(1-a_2c_2c_3) \neq a_2b_2(4+c_1c_3)$.

A.2 Forgetfulness

This part shows equality between equations 9 and 11, which was claimed to hold in chapter 3. Let us start with the latter equation and substitute for $U^y_{f,t-1}$, $U^y_{f,t-2}$ etc.

$$
\begin{aligned}
U^y_{f,t} = \rho U^y_{f,t-1} - (1-\rho)\left[y_{t-1} - \widetilde{E}^f_{t-2} y_{t-1}\right]^2 = \\
\rho\left[\rho U^y_{f,t-2} - (1-\rho)\left[y_{t-2} - \widetilde{E}^f_{t-3} y_{t-2}\right]^2\right] - (1-\rho)\left[y_{t-1} - \widetilde{E}^f_{t-2} y_{t-1}\right]^2 = \\
\rho\left\{\rho\left[\rho U^y_{f,t-3} - (1-\rho)\left[y_{t-3} - \widetilde{E}^f_{t-4} y_{t-3}\right]^2\right] - (1-\rho)\left[y_{t-2} - \widetilde{E}^f_{t-3} y_{t-2}\right]^2\right\} \\
- (1-\rho)\left[y_{t-1} - \widetilde{E}^f_{t-2} y_{t-1}\right]^2 \quad (52)
\end{aligned}
$$

Further repeating the substitution $j-1$ times then leads to

$$
\begin{aligned}
\rho^j U^y_{f,t-j} - \rho^{j-1}(1-\rho)\left[y_{t-j} - \widetilde{E}^f_{t-j-1} y_{t-j}\right]^2 - \rho^{j-2}(1-\rho)\left[y_{t-j+1} - \widetilde{E}^f_{t-j} y_{t-j+1}\right]^2 - \\
\ldots - \rho^0(1-\rho)\left[y_{t-1} - \widetilde{E}^f_{t-2} y_{t-1}\right]^2 = \rho^j U^y_{f,t-j} - \sum_{k=0}^{j-1} \rho^k (1-\rho)\left[y_{t-k-1} - \widetilde{E}^f_{t-k-2} y_{t-k-1}\right]^2 \quad (53)
\end{aligned}
$$

For j going to infinity, with $0 < \rho < 1$, and after defining $\omega_k = \rho^k(1-\rho)$, this expression yields

$$
U^y_{f,t} = -\sum_{k=0}^{\infty} \rho^k (1-\rho)\left[y_{t-k-1} - \widetilde{E}^f_{t-k-2} y_{t-k-1}\right]^2 = -\sum_{k=0}^{\infty} \omega_k \left[y_{t-k-1} - \widetilde{E}^f_{t-k-2} y_{t-k-1}\right]^2 \quad (54)
$$

which is identical to equation 9. Equivalence of equations 10 and 12 can be shown in completely same way.

B Calibration and Robustness Analysis

Table A1 contains calibrated values of parameters in the baseline version. The calibration is such that one time period in the model corresponds to one quarter, but variables are presented in annualized form to be consistent with usual convention. Most of the parameters are the same as in De Grauwe (2012) or Ho *et al.* (2019) and correspond to standard DSGE models. Memory parameter ρ with value 0.5 means that the last forecast error has weight of one half, while all the previous forecast errors have joint weight of the remaining one half. Intensity of choice parameter γ corresponds to De Grauwe (2012) as well, although it has different numerical value for technical reasons.[7]

[7] De Grauwe (2012) sets the model in a way that inflation of, say, 2 corresponds to 2% inflation rate. But as price level is an explicit part of the model here, 2% inflation rate is captured by value 0.02 to make the log-

Neutral real interest rate $\overline{r}$ is based on Taylor (1993) and weight of output gap in the loss function λ on Walsh (2003). Parameters c_2 and c_3 are based on own simulation and are the same for both monetary policy regimes (the model has been simulated for a sequence of reasonable values of these parameters; the loss function generally decreases with higher value of these parameters, but the rate of decrease starts to be negligible at values presented in the table under both regimes). Parameter c_1 has been calibrated in the same manner (for values approximately below 1 the IT model diverges—the model is in fact stable even for slightly lower values, about 0.95; but overall logic is in the spirit of Taylor principle), but it differs by regime. These values led to results presented in the main body of the paper.

Table A1: Calibration

Parameter	Value
a_1	0.5
a_2	-0.2
b_1	0.5
b_2	0.05
ρ	0.5
γ	20000
σ_ϵ	0.005
σ_η	0.005
σ_u	0.005
$\overline{r}$	0.02 (2%)
π^*	0 or 0.02 (2%)
λ	0.25
c_1^{IT}	1.5
c_1^{PLT}	0.3
c_2	0.7
c_3	0.3

Notes: σ_ϵ, σ_η, and σ_u are standard deviations of random disturbances in model equations, i.e. in dynamic IS curve, NKPC and interest rate rule, respectively. Shock u_t relates to both IT and PLT rules 3 and 4.

To assess robustness, the model has been simulated repeatedly with always one departure from the baseline calibration—all coefficients were one by one increased and decreased by a reasonable amount as compared with their baseline value, and the effect on the results has been examined. This holds not only for parameters in model equations, but also memory parameter

linearised relationship between price level and inflation work. To compensate for this in calculation of utilities (and given their quadratic nature), parameter γ, originally equal to 2, has to be multiplied by 10000. Similarly for standard deviations of model shocks.

ρ. Furthermore, autocorrelation into error terms in the model equations has been introduced (with autoregressive coefficient of various sizes). Most of these changes did not influence the implications of the results under the baseline calibration. PLT still outperforms IT in the version with zero target and with extrapolative rule 26 by substantial margin; probability of PLT divergence with rule 27 does not fluctuate much from the value for the baseline calibration (21.3%) and PLT is superior in cases where it is stable; and PLT model always diverges in the model used in section 4.2.

The exceptions which actually influence results are the following: size of standard deviation σ_η of shock η in New Keynesian Phillips curve 2; adding AR(1) process to the same shock (let us label AR coefficient as ξ_η); and intensity of choice parameter γ. Moreover, these parameters influence only results of two models with positive target in section 4.1 containing purely extrapolative rule 27 (either separately or together with partially extrapolative rule 26); and γ parameter also influences divergence of model in section 4.2. Therefore, let us focus sensitivity analysis on these cases only and let us start with σ_η and ξ_η.

Table A2: Sensitivity to characteristics of shock η

	$\sigma_\eta = 0.001$	$\sigma_\eta = 0.01$	$\xi_\eta = 0.1$	$\xi_\eta = 0.2$	$\xi_\eta = 0.5$	$\xi_\eta = 0.9$
PLT divergence (27)	0	1000	410	745	1000	1000
PLT divergence (3 rules)	0	717	0	9	768	1000
Median loss (PLT 27)	0.019	n/a	0.047	0.05	n/a	n/a
Median loss (PLT 3 rules)	0.019	0.14	0.041	0.047	0.139	n/a
Median loss (IT)	0.023	1.61	0.5	0.65	1.76	35.7
Loss(IT) $<$ Loss(PLT 27)	0	n/a	33	34	n/a	n/a
Loss(IT) $<$ Loss(PLT 3 rules)	0	32	0	0	35	n/a

Notes: The table analyses two versions of PLT model, one using extrapolative rule 27 and one using this rule together with 26, i.e. the model with three rules. A reminder: standard deviation $\sigma_\eta = 0.005$ and autoregressive coefficient $\xi_\eta = 0$ in the baseline calibration. Each case has been simulated 1000 times.

Results from table A2 suggests that with increasing standard deviation of NKPC equation shock and with increasing autoregressive component in the same shock, both monetary policy regimes deteriorate. Most importantly, susceptibility of price level targeting to instability grows substantially; and while it still generally outperforms inflation targeting in cases where it is stable, PLT is clearly much less robust to the characteristics of η shock. With only small autoregressive coefficient in the shock, it already starts to perform poorly (especially in the specification using rule 27 only), and with a moderate size of 0.5 it diverges for most of the time regardless of particular model specification.

Now let us turn to the intensity of choice parameter γ. As a brief reminder—lower value of the parameter means higher role of randomness in choice between forecasting rules (as discussed in chapter 3, $\gamma = 0$ leads to purely stochastic choice between the rules with no regard to their performance). Periods with all agents using just one rule hence cease to exist with decreasing value of the parameter, and probabilities of using one or the other rule are closer to 0.5 (with the extrapolative rule being slightly preferred for most of the time unless $\gamma = 0$). In this case, the sensitivity analysis includes again the model with the extrapolative rule 27, and now also the model with expectations about inflation from section 4.2 with the fundamentalist rule 30. Model with rule 29 would lead to similar results as one with rule 30, while remaining model specifications were stable even for initial $\gamma = 20000$.

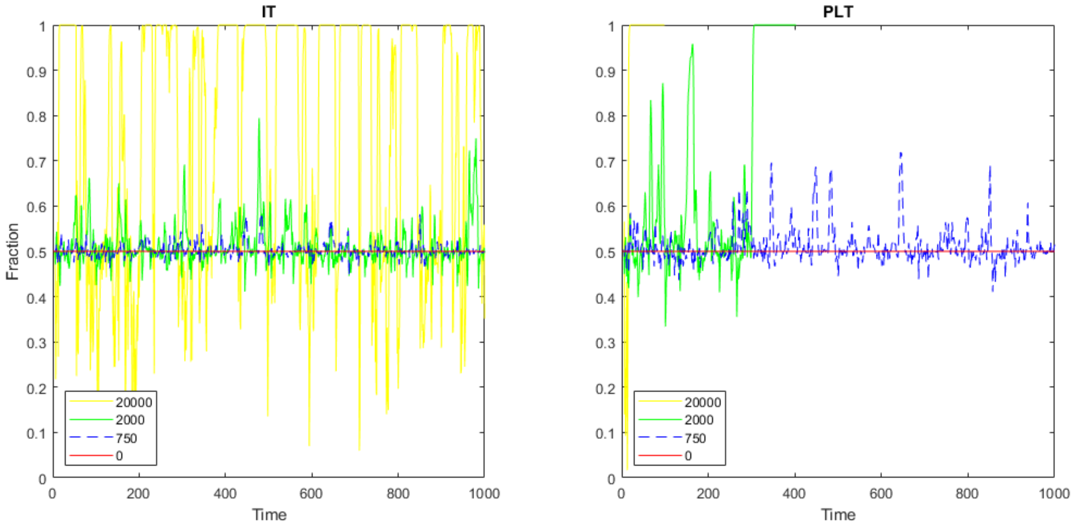
Table A3 compares simulations for different values of γ. Stability of PLT increases with decreasing γ, but notice that lower parameter also improves performance of IT, as it eliminates *animal spirits* and enhances viability of the fundamentalist rule. The model under expectations created about inflation, which previously diverged in all cases, starts to be stable with high level of randomness in choice of the forecasting rule; but even when it is stable, it still performs worse than IT in all simulations. Conversely, the other model using rule 27 dominates IT as long as it is stable—just as before. To gain more intuition about what different values of γ mean, see figure A1, which depicts fraction of extrapolatives in forecasting inflation under several values of the parameter.

Table A3: Sensitivity to γ

γ	0	500	1000	5000	10000	20000
PLT divergence (27)	0	0	0	0	2	194
PLT divergence (30)	0	0	43	1000	1000	1000
Median Loss (PLT 27)	0.034	0.034	0.035	0.036	0.039	0.043
Median Loss (PLT 30)	0.078	0.078	0.078	n/a	n/a	n/a
Median Loss (IT)	0.069	0.07	0.07	0.11	0.26	0.4
Loss(IT)<Loss (PLT 27)	0	0	0	0	0	28
Loss(IT)<Loss (PLT 30)	959	942	885	n/a	n/a	n/a

Notes: Last column with $\gamma = 20000$ corresponds to the baseline calibration. Particular numbers presented here may slightly differ from those presented in chapter 4, as the simulations for the sensitivity analysis has been conducted separately; however, the difference is negligible.

Figure A1: Fraction of extrapolatives for different values of γ



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