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SCENARIO GENERATION FOR IFRS9 PURPOSES USING A BAYESIAN MS-VAR MODEL

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IES Working Paper 10/2021

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Bibliographic information:
Kuchta M. (2021): "Scenario Generation for IFRS9 Purposes using a Bayesian MS-VAR Model" IES Working Papers 10/2021. IES FSV. Charles University.

Scenario Generation for IFRS9 Purposes using a Bayesian MS-VAR Model

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April 2021

Abstract:
The industry consensus on the implementation of the International Financial and Reporting Standard 9 - Financial Instruments (IFRS9) in the field of credit risk is that the estimation of credit risk parameters should be conditioned in the baseline, upside and downside macroeconomic scenarios presumed to be representative of the respective state of the economy. The existing approaches to scenario generation and probability weights assignment suffer from arbitrary inputs, e.g. expert judgment, quantiles selection, severity metric, the specification of a conditioned path. We present a pioneering forecasting approach using a Bayesian MS-VAR which is net of these arbitrary components. This method allows for the consistent contemporaneous formulation of the baseline and alternative scenarios and endogenously ties them to their respective probability weights. We propose to generate representative scenarios as unconditional regime-specific forecasts and to calculate the probability weights associated with representative scenarios as unconditional lifetime transition probabilities. We illustrate the method on artificial as well a real data and conduct an empirical backtest, in which generated scenarios are compared to the actual development during the financial crisis. The method is challenged with the DSGE model and conditional forecasting.

JEL: C11, C32, C53, G17, G38
Keywords: scenario generation, IFRS9, Markov-switching VAR, Bayesian

Acknowledgements: This research has been supported by the Czech Science Foundation Grant 18-05244S "Innovation Approaches to Credit Risk Management“.

1 Introduction

Macroeconomic scenario generation attracted attention after the introduction of the new (i) accounting/provisioning International Financial and Reporting Standard 9 (IFRS9) and (ii) regular stress-testing exercises conducted either Europe-wide under the supervision of the European Banking Authority (EBA) or in-house under the Internal Capital Adequacy Assessment Process (ICAAP). The new regulatory requirements were a response to the 2008-2009 financial crisis and the empirical evidence of the macroeconomic environment being the main driver of the losses suffered by the financial and banking sector. The IFRS9 introduced the new concept of Expected Credit Losses (ECL), which reflects the need for forward-looking point-in-time estimates, responsive and sensitive to the current macroeconomic environment and desirable from both regulatory and business perspective. The inclusion of the forward-looking indicators partially answers the wide critique of the pro-cyclical behaviour of the BASEL II regulation for the calculation of Tier 1 capital. IFRS9 methodology imposes countercyclical behaviour, as it intends to forecast future losses and create additional capital buffers with respect to the reporting date ahead of a potential future downturn.

ECL is an unbiased, forward-looking and probability-weighted best estimate of credit losses discounted to the reporting date and considered over multiple outcomes (IFRS 9.5.5.17). The forward-looking component is usually included through macroeconomic scenarios, which also complies with the multiple outcomes requirement. However, since it is virtually impossible to calculate credit losses over a large set of alternative scenarios, the industry consensus is to condition the estimation of credit risk parameters (PD, LGD etc.) to the baseline, upside and downside macroeconomic scenarios. This dimension reduction requires the definition of a representative scenario which approximates to the given state of the economy, represents the full set of scenarios in that state and is not biased towards extreme events (GPPC 2.8.2.3). The concept of a representative scenario is essential also for the scenario probability weights: since the occurrence probability of any distinct scenario path is inherently (close to) zero, the probability weights associated with the representative scenario relates rather to the state of the economy than to the distinct scenario path.

Popular methods of scenario generation include conditional forecasting (Banbura et al., 2015; Baumeister and Kilian, 2014), Monte Carlo simulations (Jacobs, 2016) or simpler heuristic methods. Conditional forecasts are projections of variables of interest on the future paths of one or multiple variables. The scenario analysis conducted in a conditional forecasting exercise in principle answers, for example, the question "what would happen if interest rate was fixed at one percentage point for certain period" and imposes elicit a priori restrictions on the unobserved structural shocks (Wagonner and Zha, 1999). On the other hand, Antolin-Diaz et al. (2020) argue that only economically meaningful shocks should be used to construct scenarios. They prefer to ask the question "what would happen if external shock fixed the interest at one percentage point". They specify the conditions on observable variables and select which shocks drive the forecast, while others are restricted to their unconditional distributions.

However, conditional forecasting is more suitable for scenario analysis in terms of policy evaluation than for the generation of alternative scenarios. E.g. Banbura et al. (2015) examines the effect of a 0.1% increase in GDP growth above the baseline (unconditional) forecast. Baumeister and Kilian (2014) feed in a sequence of historical shocks, or calibrate a single structural shock to evaluate alternative scenarios and their impact on oil prices. Antolin-Diaz et al. (2020), among others, study the effect of the shock corresponding to the financial crisis using the FED's adverse scenario for GDP and the unemployment rate. Although the last example resembles scenario generation as we understand it, the stress testing is conducted on the asset prices and on the aggregate level. The main drawback of conditional forecasting as such is that it requires the addition of an external scenario for at least one variable.

Monte Carlo simulations include either Bayesian or frequency models enhanced by bootstrapping applied on univariate (ARMA)[1] or multivariate (VAR) models. With unconditional forecast being considered as the baseline scenario, the (p^{th}) and (1 −

[1] e.g. Breeden and Ingram (2010)

p^{th}) percentiles represent alternative scenarios. Jacobs (2016) employs an MS-VAR model to generate stress testing scenarios. He shows that the MS-VAR model provides more conservative scenarios than the ordinary VAR model and it also exhibits greater accuracy in model testing, better capturing extreme historical events. He identifies a severe scenario as an average of all paths for which all included variables crossed the 99th percentile in at least a single period. However, this method still requires analytical and arbitrary input for the quantile selection.

Notice that Monte Carlo methods in general consist of scenario generation and scenario selection. The latter part is especially arbitrary and often requires a severity function for the selection of an appropriate scenario, e.g. Mokinski (2017) . However, the severity function is heavily dependent on the selected metric and does not deal either with the selection of "representative" quantile[2] or with the assignment of probabilities. With the severity distribution available, the (p^{th}) severity quantile represents the share of all scenarios with an equal or worse outcome. E.g. Franta et al. (2014) approximates the probability that the forecasted variable is below the stress-test scenario path, implicitly treating the Bayesian confidence intervals (fan charts) as a severity distribution. However, the probability weights as we understand them represents rather the probability mass around the representative scenarios.

We propose a pioneering approach to scenario generation using an MS-VAR model which is net of arbitrary components, such as quantile selection, conditioning path specification or a severity function. Instead, we define representative scenarios as unconditional, regime-specific forecasts endogenously associated with regime probability. The analytical input to scenario generation and the assignment of probability weights is limited to the calibration of model hyper parameters. The choice of a Markov-switching type of model is essential and especially useful for IFRS9 purposes, since it allows for the contemporaneous formulation of the baseline and alternative scenarios and naturally

[2]The selection of the 99th quantile is natural for stress testing which is expected to capture extreme events. On the other hand, the definition of representativeness is vague and results in an inherently arbitrary selection of quantiles.

ties them to their respective probability weights as required by the Standard.

The philosophical foundation of the approach lies in the interpretation of observable history. At first, let us assume the existence of an unknown but finite number of M regimes unique in their macroeconomic properties associated with a strictly positive probability of occurrence. Then, we interpret the historical time series as a sequence formed by these regimes. We limit the number of regimes to $M = 3$, and interpret them as baseline, upside and downside states of the economy. Due to the specific nature of the threshold model family (Tong, 1980), such MS-VAR reduces into three plain vanilla VAR models once the regimes are known. Then, with three sets of parameters in hand, the representative scenarios are obtained through three separate unconditional forecasting exercises. Such regime-specific models also capture non-linearities in the business cycle. This means that the representative scenarios are conditioned purely to data-driven regimes rather than an external scenario path or quantile selection, and are not biased towards extreme events.

We propose to derive the probability weights of representative scenarios from the limiting distribution of the estimated transition probability matrix, i.e. to associate the average occupancy time for a given state of the economy with the representative scenario. This endogeneity between the representative scenarios and the probability weights is believed to provide results superior to those of separate models.

The main goal of the benchmarking analysis is to compare the proposed regime-switching to "established" scenario generation methods, i.e. the primary focus is on the scenario generation method and the secondary focus is on the model type[3]. We employ conditional forecasting as in, for example, Baumiester and Kilian (2014) to generate alternative scenarios through the small open economy DSGE model of Schmitt-Grohe and Uribe (2003). We select the most extreme historical developments of GDP growth as alternative scenario paths and a pattern for the other variables. For the model evaluation and testing, we leverage the only closed period of economic stress during the financial

[3]To challenge the performance of MS-BVAR to another regime-switching model is outside the scope of this paper.

crisis in 2007-2008. Such a set-up of the backtest allows the direct comparison of the examined scenario generation methods in terms of their ability to capture and forecast possible economic events.

The paper is organized as follows: Section 2 covers the set-up of the MS-VAR model, describes the estimation algorithm together with the setting of the prior distributions, and presents the approach to scenario generation and probability weight assignment. Section 3 is dedicated to the benchmark DSGE model and the data preparation process, due to its naturally rigid structure, which imposes relatively strict requirements on data. The fourth section provides details on the estimation of both models and illustrates the methods employed in the empirical examples. Firstly we demonstrate the suitability of the MS-VAR model in terms of regime identification performance using artificial data, and then we compare the scenarios of MS-VAR and benchmark conditional forecasting through a DSGE model.

2 Bayesian Markov-switching VAR model

2.1 Model Setting

Consider the following three-regimes MS-VAR model according to Hamilton (1994) or Geweke (2005):

$$\boldsymbol{Y}_t = c_{S_t} + \sum_{l=1}^{L} \boldsymbol{B}_{S_t} \boldsymbol{Y}_{t-l} + \varepsilon_t, \ \varepsilon_t \sim N(0, \boldsymbol{\Omega}_{S_t}), \tag{1}$$

where $S_t = 1, 2, 3$ is an unobserved, latent discrete variable indicating structural shifts and lag degree $L = 2$. It is generated by a discrete-state, homogeneous, irreducible and ergodic first order Markov chain:

$$PR[S_t = i | S_{t-1} = j] = p_{ij} \tag{2}$$

The probability measure $p_{ij} \in (0, 1)$ refers to the probability of transition to regime i at time t, given that the regime at the previous time $(t-1)$ was j and also determine the persistence of each regime. The transition probabilities for the specific case of $M = 3$

regimes are summarized in a transition probability matrix $\boldsymbol{P}$:

$$\boldsymbol{P} = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} \tag{3}$$

The likelihood function of the MS-VAR model is a weighted average of the likelihoods of each considered regime conditional on information I_{t-1} known at time $(t-1)$, with weights given by their respective probability of materialization $f(S_t = i|I_{t-1})$:

$$f(\boldsymbol{Y}_t|I_{t-1}) = \sum_{i=0}^{t} f(\boldsymbol{Y}_t|s_t = i, I_{t-1}) \times f(S_t = i|I_{t-1}) \tag{4}$$

The model is estimated using extended Gibbs sampling suitable for MS-VAR models, e.g. Blake and Mumztaz (2017) or Osmundsen et. al. (2019). Before it is initiated, (i) a Hamilton (1994) filter is applied to calculate the probability terms $Pr[S_t = i|I_t]$, (ii) the latent state variable $\tilde{S}_t = \{S_1, S_2, S_3\}$ is sampled by backward recursion, and (iii) the transition probability matrix $\boldsymbol{P}$ is sampled from its density. The Gibbs sampler then samples from these conditional posterior distributions:

1. $\boldsymbol{b}_{S_t}$ from $H(\boldsymbol{b}_{S_t}|\boldsymbol{\Omega}_{S_t}, \tilde{S}_t, \boldsymbol{P}, \boldsymbol{Y}_t)$
2. $\boldsymbol{\Omega}_{S_t}$ from $H(\boldsymbol{\Omega}_{S_t}|\boldsymbol{b}_{S_t}, \tilde{S}_t, \boldsymbol{P}, \boldsymbol{Y}_t)$
3. $\boldsymbol{P}$ from $H(P|\boldsymbol{b}_{S_t}, \boldsymbol{\Omega}_{S_t}, \tilde{S}_t, \boldsymbol{Y}_t)$
4. $\tilde{S}_t$ from $H(\tilde{S}_t|\boldsymbol{b}_{S_t}, \boldsymbol{\Omega}_{S_t}, \boldsymbol{P}, \boldsymbol{Y}_t)$,

with $\boldsymbol{b}_{S_t} = \{c_{S_t}; \boldsymbol{B}_{S_t}\}$ containing intercepts and a matrix of coefficients. In more detail:

1. Set priors. We apply the natural conjugate Normal inverse Wishart prior for the matrix of parameters $\boldsymbol{b}_{S_t}$ and covariance matrix $\boldsymbol{\Omega}_{S_t}$, $S = \{1, 2, 3\}$. Following Banbura et al. (2007), Banbura et al. (2015), Blake and Mumtaz (2017) or Chiu et al. (2016), the prior is implemented via artificial dummy observations $\boldsymbol{Y}_D$ and $\boldsymbol{X}_D$ appended to the regular sample such that $\boldsymbol{Y}^* = [\boldsymbol{Y}, \boldsymbol{Y}_D]$ and $\boldsymbol{X}^* = [\boldsymbol{X}, \boldsymbol{X}_D]$. Regressing $\boldsymbol{X}_D$ on $\boldsymbol{Y}_D$ yields the prior mean and the prior scale matrix:

$$b_0 = (\boldsymbol{X}'_D \boldsymbol{X}_D)^{-1}(\boldsymbol{X}_D \boldsymbol{Y}_D) \tag{5}$$

$$\boldsymbol{S} = (\boldsymbol{Y}_D - \boldsymbol{X}_D b_0)'(\boldsymbol{Y}_D - \boldsymbol{X}_D b_0). \tag{6}$$

Denoting T_D as the length of these artificial data, K as the number of regressors in each equation and $\tilde{b}_0 = vec(b_0)$, the prior is set such that:

$$p(\boldsymbol{b}|\boldsymbol{\Omega}) \sim N(\tilde{b}_0, \boldsymbol{\Omega} \otimes (\boldsymbol{X}'_D \boldsymbol{X}_D)^{-1}) \tag{7}$$

$$p(\boldsymbol{\Omega}) \sim IW(\boldsymbol{S}, T_D - K). \tag{8}$$

If we factorize the transition probability matrix into columns such that $\boldsymbol{P} = \{P_1, P_2, P_3\}$, the conjugate prior for each column with probabilities summing to one is the Dirichlet distribution with M parameters α_i and PDF:

$$f(x_1, ..., x_{M-1}, \alpha_1, ..., \alpha_M) = \prod_{i=1}^{M} x_i^{\alpha_i - 1}, \tag{9}$$

where $x_M = 1 - \sum_{i=1}^{M-1} x_i$. Mean μ is given by $\mu = \frac{\alpha_i}{\alpha}$ and variance $\sigma = \frac{\alpha_i(\tilde{\alpha}-\alpha_i)}{\tilde{\alpha}^2(\tilde{\alpha}-\alpha_i)}$, where $\tilde{\alpha} = \sum_{i=1}^{M} \alpha_i$. The parametrization of the Dirichlet distribution ensures that the transition from each regime separately sums to one.

2. Run the Hamilton (1994) Filter.

3. Draw the state variable $\tilde{S}_t$ by backward recursion. In the three-regimes MS-VAR, the probability-based algorithm for the transition from the state $S_{t-1} = j$ to the state $S_t = \{1, 2, 3\}$ is:

$$u < Pr[S_{t+1} = 1|S_t] = \frac{Pr[S_t=1|S_{t+1}=j]}{\sum_{i=1}^{3} Pr[S_t=1|S_{t+1}=j]} \longrightarrow \text{Regime 1}$$

$$u < Pr[S_{t+1} = 2|S_t] = \frac{Pr[S_t=2|S_{t+1}=j]}{\sum_{i=2}^{3} Pr[S_t=2|S_{t+1}=j]} \longrightarrow \text{Regime 2}$$

$$u < Pr[S_{t+1} = 3|S_t] = \frac{Pr[S_t=3|S_{t+1}=j]}{Pr[S_t=3|S_{t+1}=j]} = 1 \longrightarrow \text{Regime 3},$$

where u is a random draw from uniform distribution $u \sim U(0, 1)$.

4. Draw the transition probability matrix $\boldsymbol{P}$ from its conditional posterior density

$$H(\boldsymbol{P}_j|S_t) \sim D(\alpha_{j1} + \eta_{j1}, \alpha_{j2} + \eta_{j2}, \alpha_{j3} + \eta_{j3}),$$

where η_{ij} refers to the number of empirical (actual) transitions from j to the state i, i.e. empirical transition matrix $\boldsymbol{\eta}$:

$$\boldsymbol{\eta} = \begin{pmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ \eta_{31} & \eta_{32} & \eta_{33} \end{pmatrix} \tag{10}$$

The matrix η is constructed using the current draw of the state variable $\tilde{S}_t = \{S_1, S_2, S_3\}$ from its conditional distribution $H(\tilde{S}_t|b_{S_t}, \boldsymbol{\Omega}_{S_t}, \boldsymbol{P}, \boldsymbol{Y}_t)$ and thus encompasses all the information currently available in the data and other parameters. Therefore, the conditional posterior distribution $H(\boldsymbol{P}_j|S_t)$ for the j^{th} column $j = \{1, 2, 3\}$, depends only on the state variable S_t.

5. The data are partitioned conditional on the current draw of $\tilde{S}_t$ into the respective regimes $i = 1, 2, 3$. The meaningful model estimation is ensured by retaining only such draws of $\tilde{S}_t$ satisfying $n_i \geq n_{crit}$ for $\forall i = 1, 2, 3$, with n_{crit} denoting the minimum acceptable number of observations for each regime i.

6. Initiate the Gibbs sampler in each regime to draw the VAR parameters from their conditional posterior densities:

$$H(\boldsymbol{b}_{S_t}|\boldsymbol{\Omega}_{S_t}, \tilde{S}_t, \boldsymbol{P}, \boldsymbol{Y}_t) \sim N(vec(\boldsymbol{B}^*_{S_t}), \boldsymbol{\Omega}_{S_t} \otimes (\boldsymbol{X}^{*'}_{S_t}\boldsymbol{X}^*_{S_t})^{-1}) \tag{11}$$

$$H(\boldsymbol{\Omega}_{S_t}|\boldsymbol{b}_{S_t}, \tilde{S}_t, \boldsymbol{P}, \boldsymbol{Y}_t) \sim IW(\boldsymbol{S}^*_{S_t}, T^*), \tag{12}$$

where

$$\boldsymbol{B}^*_{S_t} = (\boldsymbol{X}^{*'}_{S_t}\boldsymbol{X}^*_{S_t})^{-1}(\boldsymbol{X}^{*'}_{S_t}\boldsymbol{Y}^*_{S_t}) \tag{13}$$

$$\boldsymbol{S^*}_{S_t} = (\boldsymbol{Y}^{*'}_{S_t} - \boldsymbol{X}^*_{S_t}\boldsymbol{b}_{S_t})(\boldsymbol{Y}^{*'}_{S_t} - \boldsymbol{X}^*_{S_t}\boldsymbol{b}_{S_t}) \tag{14}$$

and T^* is the length of $\boldsymbol{Y^*}$. Note that (i) the draws of the VAR parameters are conditioned on the already drawn matrix $\boldsymbol{P}$ and vector $\tilde{S}_t$ and that (ii) at this point, the regimes are known, and thus this problem reduces to the three conventional VAR models applied to the respective observations $\boldsymbol{X}^*_{S_t}$ and $\boldsymbol{Y}^*_{S_t}$.

7. Repeat x-times to obtain the empirical marginal posterior distributions.

Each iteration of the Gibbs sampler triggers the stability check consisting of the eigenvalue decomposition of matrix $\boldsymbol{b}_{S_t}$. The draw is considered stable only when all eigenvalues of $\boldsymbol{b}_{S_t}$ are non-explosive roots within the unit circle. Such a draw is saved and the algorithm proceeds into the next iteration. To ensure consistent model properties, rejection sampling is employed such that only draws satisfying $det(\boldsymbol{\Omega}_3) >= det(\boldsymbol{\Omega}_2)$ are retained. This condition ensures that the third regime exhibits higher overall volatility, and is intended to impose identification restriction on the third regime as a downside scenario.

2.2 Prior Distributions

The prior for each row (state) of the transition probability matrix is the Dirichlet distribution $p(\boldsymbol{P}) \sim D(3, 3, 3)$, implying the prior transition probability matrix and the starting value to initiate the Gibbs sampler:

$$\boldsymbol{P} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \tag{15}$$

with prior variance of 0.022.

The artificial dummy observations $\boldsymbol{Y}_D$ and $\boldsymbol{X}_D$ appended to the regular sample such that $\boldsymbol{Y}^* = [\boldsymbol{Y}, \boldsymbol{Y}_D]$ and $\boldsymbol{X}^* = [\boldsymbol{X}, \boldsymbol{X}_D]$ are generated satisfying:

$$\boldsymbol{Y}_D = \begin{pmatrix} \frac{diag(\chi\sigma)}{\tau} \\ 0_{N\times N} \\ 0_{N\times N} \\ diag(\sigma) \\ \delta\mu \end{pmatrix}, \boldsymbol{X}_D = \begin{pmatrix} 0_{N\times 1} \quad \frac{diag(\sigma)}{\tau} \quad 0_{N\times N(L-1)} \\ 0_{N\times(N+1)} \quad \frac{diag(\sigma 2^d)}{\tau} \\ (\frac{1}{\lambda_c})_{N\times 1} \quad 0_{N\times NL} \\ 0_{N\times(NL+1)} \\ \delta \quad (\mu\delta)_{1\times L} \end{pmatrix} \tag{16}$$

The prior is parametrized such that λ_c=10000, the decay parameter $d = 1$, hyperparameter $\tau = 0.6$, lag degree $L = 2$ and vectors $\chi = \{\chi_1, \chi_2, ..\chi_N\}$ and $\sigma = \{\sigma_1, \sigma_2, ...\sigma_N\}$ for the variables indexed over $i = 1, 2, ...N$ are obtained from the AR(1) process:

$$vec(\boldsymbol{Y}_t) = (I_N \otimes \boldsymbol{Y}_{t-1})vec(\boldsymbol{b}) + vec(u_t)\ , \quad u_t \sim N(0, \sigma_i) \tag{17}$$

such that $\boldsymbol{b}$ is factorized into $\boldsymbol{b} = \{b_0, \chi\}$. The last row of 16 expresses the prior belief about the common stochastic trend, i.e. co-integration. Denoting μ as a $[1 \times N]$ vector of sample means, the artificial dummy observations imply that the sample mean of each variable i is the linear combination of every other variable at all lags, including itself:

$$\delta\mu = [\delta \quad \delta\mu \quad \delta\mu] \tag{18}$$

The choice of the co-integration in this MS-VAR model is crucial as (i) the data consist mainly of the GDP and its components, due to the (ii) "mild" mean-reversion property caused by the application of the one-sided HP filter and (iii) the presence of four co-integration relationships indicated by the Johansen test[4]. The purpose of the co-integration prior is to tie the explanatory variables together[5] and to reduce the potential explosion of forecasts. The value of δ is set to $\delta = 1.5$, and note that the co-integration prior is implemented more tightly with $\delta \to \infty$. Finally, the $n_{crit} = 20$.

2.3 Generating Scenarios

Let us postulate that at each point in time $t = 1, 2, ...T$ there exists an unknown, but finite number of M unique regimes associated with the strictly positive probability of materialization p, and typified by their imposition of unique macroeconomic conditions. At each point in time t, only one of the M regimes can materialize, on account of its associated probability. Next, imagine that the data generating process behind the historically observed macroeconomic time series is an unknown sequence formed by these regimes uniquely materialized at each point in time $t = 1, 2, ...T$.

[4]For the data preparation process please refer to section 3.

[5]Imposing relative tightness of the model similar to the decision rules in DSGE models (Dib et al., 2008)

Such an interpretation of history and problem formulation allows us to endogenously generate regime-specific unconditional, i.e. mean, forecasts that could be interpreted as representative scenarios for a given regime or state of the economy. This is due to the reduction of M-regimes MS-VAR model into M plain vanilla VAR models once the regimes are known, which allows for the calculation of regime-specific unconditional forecasts or scenarios which are, by definition, the most representative for a given state. By the representative scenario, we understand a distinct scenario path which approximates a given state of the economy, is representative of the full set of scenarios in a given regime and is not biased towards extreme events.

Compared to other approaches, the scenarios are conditioned purely on data-driven macroeconomics regimes rather than on an arbitrary external scenario path, and the problem of scenario or quantile selection is thus mitigated even without a severity function. Moreover, the sets of parameters unique for each regime take account of non-linearities in the different stages of the business cycle. For the purposes of IFRS9, we select $M = 3$, mimicking the baseline, upside and downside state of the economy. The representative scenario path for regime $i = 1, 2, 3$ and with lag degree $L = 2$ up to forecasting horizon k is obtained as an unconditional forecast:

$$\hat{\boldsymbol{Y}}_{t+k}|(S_t = i) = \hat{c}_{S_t} + \sum_{l=1}^{L} \hat{\boldsymbol{B}}_{S_t}\boldsymbol{Y}_{t+k-l} \tag{19}$$

Regime identification in the MS-VAR model can be imperfect, as there still exists estimation uncertainty due to (i) the unknown number of the true regimes, (ii) the unknown macroeconomic properties of the true regimes and (iii) the imperfect granularity of the time periods t. Consequently, (i) regimes, i.e. the latent state variable S_t, are identified only with the associated probability, as demonstrated in Figure 2, (ii) and the state-dependent matrix of VAR coefficients $\boldsymbol{B}_{S_t}$ and covariance matrix $\boldsymbol{\Omega}_{S_t}$ exhibit estimation uncertainty, as (iii) the transition probability matrix $\boldsymbol{P}$.

Therefore, unique regimes, as estimated by the MSVAR model, can actually be composed of multiple true regimes or be "under-identified", when the properties of one true regime are assigned to multiple estimated ones. This might results in the absence of clearly separated, stable, internally consistent and easily interpretable scenario paths. That is to say, one scenario may represent higher volatility and fit both an upside and a downside peak, while another may be a volatility scenario evolving around the zero mean. Consequently, the model does not ensure distinctly separated scenarios at each point in time.

On the other hand, the demand for distinctly separated scenario paths is also arbitrary and lacks rigorous statistical foundations. Therefore, this is not perceived as model misbehaviour, but rather as a feature due to the data generating process being composed of an unknown number of regimes with unknown properties. However, this is consistent with our interpretation of the history.

2.4 Probability weights

The concept of a representative scenario is essential also for the calculation of probability weights, since the probability of any distinct scenario path is inherently (close to) zero and thus unusable for IFRS9 purposes. Instead, the probability weights associated with the representative scenario relate rather to the respective state of the economy, i.e. the regime, than to the distinct scenario path. Such probability weights can be derived from the model transition matrix $\boldsymbol{P}$ as unconditional transition probabilities, i.e. limiting (or steady state) distribution. This approach provides fixed and strictly positive lifetime transition probabilities that are independent of the initial state (Geweke, 2005), and thus the scenario probability weights in the ECL calculations are time and state independent.

Rigorously, the transition probability matrix $\boldsymbol{P}$ is associated with the latent state variable $S_t = 1, 2, 3$. A necessary and sufficient condition for the existence of single uniquely determined limiting distribution with strictly positive probability weights is that the S_t is generated by a discrete-state, homogeneous, irreducible and ergodic first order Markov chain:

$$PR[S_t = i | S_{t-1} = j] = p_{ij} \tag{20}$$

Homogeneity means the existence of the single Markovian transition probability matrix, which application is valid at any point in time and which probabilities from any initial state $i = 1, 2, ..s$ to state $j = 1, 2, ..s$ sum to one: $\sum_{j=1}^{s} p_{ij} = 1$. Irreducibility implies $p_{ij} < 1$ for $i = j$ or equivalently $diag(\boldsymbol{P}) < 1$, thus the non-existence of an absorbing state defined as:

$$p_{ij} = \begin{cases} 1, & \text{for } i = j, \\ 0, & \text{for } i \neq j. \end{cases} \tag{21}$$

This lifetime accessibility condition also restricts the peculiar structure of transition matrices such as cycles among states, separations or non-accessible states (Geweke, 2005). Note that irreducibility is indeed a necessary condition, as the existence of the absorbing state does not violate ergodicity, because aperiodicity means the existence of at least one recurrence path within two transition periods, not every possible path.

Ergodicity, i.e. a finite recurrence time and period of one, states that at any point in time $t = 1, 2...\infty$, the initial state i is reachable from any other state j (including state i) within two consecutive transition periods.

An irreducible Markov chain has a stationary distribution iff the Markov chain is ergodic. If the Markov chain is ergodic, the stationary distribution is unique. Thus, when all listed conditions are satisfied, there exists a uniquely determined limiting stationary distribution with strictly positive probabilities for each state embodied in a stationary $[M \times 1]$ vector π satisfying:

$$\boldsymbol{P}\pi = \pi \tag{22}$$

Denoting δ_i as an $[M \times 1]$ zero-one indicator vector for state i, the limiting distribution is obtained such that (Hamilton, 1994):

$$\lim_{n \to \infty} \boldsymbol{P}^n \delta_i = \pi \quad \text{for} \quad \forall \ i \tag{23}$$

That is to say, irrespective of the initial state, an irreducible and ergodic Markov chain always converges to its stationary limiting distribution in the infinite time horizon (Geweke, 2005). It is sufficiently clear that (22) is the characteristic equation of the transition matrix $\boldsymbol{P}$. Let us rewrite it in the standard form with λ denoting eigenvalues:

$$\boldsymbol{P}\pi = \lambda\pi \tag{24}$$

The basic algebraic property of Markovian matrices is that their first eigenvalue is always $\lambda_1 = 1$. Then, the right eigenvector π associated with the eigenvalue $\lambda_1 = 1$ is a steady-state vector π, i.e. limiting distribution, if it is normalized to $e\pi = 1$, where e is an $[1 \times M]$ vector of ones, i.e. the elements of π sum exactly to one (Olver and Shakiban, 2006). Vector π also represents the unconditional lifetime probabilities of each considered state and be can thought of as an average occupancy time (Guidolin, 2011). Therefore, it is straightforward to apply these probabilities as scenario weights and combine them with forecasts and the scenario-dependent values of risk parameters in the calculation of ECL.

3 DSGE Model

Model set up

The core of this DSGE model and solution method is the replication of Schmitt-Grohe and Uribe (2003), to be precise, the model with a stochastic discount factor and with minor adjustments is parameterized as in Mendoza (1991). Specifically, we use te period utility function

$$U(C_t, N_t) = \frac{\left(\mathrm{e}^{\epsilon_t^c} C_t - \frac{\mathrm{e}^{\epsilon_n^l} N_t^{-\phi}}{\phi}\right)^{1-\gamma} - 1}{1-\gamma} \tag{25}$$

extended with the consumption preference and the labour supply shock ϵ_t^c and ϵ_t^n respectively defined as AR(1) processes:

$$\epsilon_t^c = \rho_c \epsilon_{t-1}^c + \varepsilon_t^c, \;\; \varepsilon_t^c \sim N(0, \sigma_c^2) \tag{26}$$

$$\epsilon_t^n = \rho_n \epsilon_{t-1}^n + \varepsilon_t^n, \;\; \varepsilon_t^n \sim N(0, \sigma_n^2). \tag{27}$$

The endogenous discount factor is

$$\beta(C_t, N_t) = \left(1 + C_t - \frac{N_t^{\phi}}{\phi}\right)^{-\psi_n} \tag{28}$$

an the production technology has the standard Cobb-Douglas form with the constant returns to scale

$$Y_t = \mathrm{e}^{a_t} K_t^{\alpha} N_t^{1-\alpha}, \tag{29}$$

where

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a,\ \varepsilon_t^a \sim N(0, \sigma_a^2) \tag{30}$$

is the TFP shock process defined as AR(1) included in levels, i.e. the aggregate level of technology. The law of motion of capital is extended with investment-specific shock such that

$$K_{t+1} = \mathrm{e}^{b_t} I_t + (1-\delta)K_t \tag{31}$$

$$b_t = \rho_b b_{t-1} + \varepsilon_t^b,\ \varepsilon_t^b \sim N(0, \sigma_b^2). \tag{32}$$

The introduction of additional shocks to the model is required for the purposes of the Bayesian estimation and due to the relative tightness of the model behaviour caused by the decision rules in the DSGE model (Dib at al., 2008), which can provide a good fit to data only when enough shocks are specified (Smets and Wouters, 2007). Therefore, additional sampling variability is introduced to the core model through measurement errors in the observation equations discussed below.

The model is solved on levels providing an analytical solution for the deterministic steady state and allowing for higher order approximation and thus greater precision when compared to log-linearization. For a detailed derivation and analytical solution of the model, please refer to Appendix A.

Linking the model to the data

Following mainly Adjemian et al. (2011), Griffoli (2008), Ireland (1999) and Pfeifer (2013), the data transformations and the specification of the observation equations necessary for linking the model to the data corresponds to the non-linear model for log-linearization. Examining the properties of the general model shock process s_t such that

$$s_t = \rho_s s_{t-1} + \varepsilon_t^s \text{ , where } \varepsilon_t^s \sim N(0, \sigma_s^2),$$

we find that its level form $S_t = \mathrm{e}^{s_t}$ is the fluctuation around an unspecified trend. The model is therefore stationary and describes the behaviour of the economy along its balanced growth path. Consequently, the general model variable Z_t corresponds to the stationary and per capita values. Z_t is the intensive form of the empirically observed variable Z_t^{data} obtained as $Z_t = \frac{Z_t^{data}}{L_t}$, where L_t is the two-sided HP-filtered overall labour force L_t^{data}. The filtration corrects for undesired fluctuations due to measurement errors or statistical revisions able to create a spurious economic cycle. Z_t^{data} represents the level variable of the seasonally adjusted time series at constant prices. Seasonal adjustment is the necessary condition for the successful estimation of any macroeconomic model preventing the existence of an undesired spurious economic cycle in the data. Current prices, i.e. real terms, account for the absence of the deflator in the model.

The model relevant empirical variable z_t^{obs} is obtained by the application of the causal one-sided HP-filter, as in Stock and Watson (1999), to the logarithm of Z_t, $z_t = log(Z_t)$. The logarithmic transformation ensures scale invariant deviations from the steady state, and the causal one-sided, i.e. backward-looking HP-filter does not contradict the backward-looking state space representation of DSGE models and forecasting exercises unlike its commonly used two-sided counterpart (Pfeifer, 2013). The filtration provides empirical data for the model variables $\boldsymbol{z}_t^{obs} = \{y_t^{obs}, c_t^{obs}, i_t^{obs}, n_t^{obs}, tb_t^{obs}, ca_t^{obs}\}$. i.e observed output, consumption, investments, hours worked, trade balance and current account respectively. The observed variables in $\boldsymbol{z}_t^{obs}$ represent deviation from their respective steady state Z_t^{SS} and possess an asymptotically zero mean. For the exact matching of the model market clearing equation with its empirical counterpart, the model private consumption variable contains both private and government consumption. Note that the national accounting methodology adopted in the Czech Republic distinguishes between government consumption and investments, which allows for modification devoid of the introduction of bias. Additional transformation is applied to the ratio variables:

$$ca\_y^{obs} = \left(\frac{ca_t^{obs}}{y_t^{obs}}\right) - \overline{\left(\frac{ca_t^{obs}}{y_t^{obs}}\right)} \tag{33}$$

and

$$tb\_y^{obs} = \left(\frac{tb_t^{obs}}{y_t^{obs}}\right) - \overline{\left(\frac{tb_t^{obs}}{y_t^{obs}}\right)}. \tag{34}$$

Then, defining model variable $\hat{z}_t = z_{obs}$ as a deviation z_t from its respective steady state z_t^{SS}, the observation equation with the additional measurement error $\varepsilon_t^z \sim N(0, \sigma_z^2)$

is:

$$\hat{z}_t = z_t - z_t^{ss} + \varepsilon_t^z \tag{35}$$

Prior distributions

The setting of the prior distributions follows the original calibration of Mendoza (1991) used as the prior means, partially adjusted for monthly data frequency in the case of the interest rate and capital depreciation parameter. A relatively loose setting with high standard deviations allows for variation across the subspace of potential values and the extraction of the maximum information contained in the data. The selection of the priors follows the conventions of the DSGE literature, e.g. Buriel et al. (2007), Griffoli (2008), Hristov (2016), Pfeifer (2013) and Smets and Wouters (2007). The inverse Gamma distribution with infinite standard deviation is applied for the shocks, the Beta distribution is used for the parameters restricted to the interval $[0, 1]$, the Gamma distribution is used as a more informative prior to bind the posterior mean closer to the prior mean, and the Normal distribution is considered to be a non-informative prior to be applied when no prior information about the parameters is known.

Tables 2 to 4 in Appendix B summarize the estimation results, listing the actual setting of the prior for each estimated parameter, shock, and pairwise correlations, together with its posterior counterpart including Highest Probability Density Intervals, the Bayesian equivalent of the confidence intervals.

4 Scenario Generation: A case Study

4.1 Data and model estimation

For the estimation of both models we use the Czech headline macroeconomic time series with a monthly frequency on GDP, consumption, investments, total hours worked, the trade balance and current account from the *Moody's Analytics'* branch *Economy.com*. The data spans the period from 01-1996 to 12-2006, providing 132 observations in total. The choice of the in-sample period for model estimation stems from the desire to perform an empirical backtest of a forecasted alternative scenario, and the last period of macroeconomic stress was experienced during the financial crisis in 2007-2008[6]. The comprehensive expanding window backtest is not feasible due to the extreme computational demands mainly of the DSGE model.

The data preparation process in ruled primarily by the rigid nature of the DSGE model and its extensive but precise requirements on data structure. For comparability in the benchmarking analysis, the data preparation process and applied transformations are the same for both models with the exception of the current account to GDP ratio, which is excluded from the MS-VAR model estimation due to undesired volatility causing over-fitting of the MS-VAR model to the volatility-driven regimes[7]. Table 1 shows the descriptive statistics of the included time series, and Figure 1 depicts their historical development.

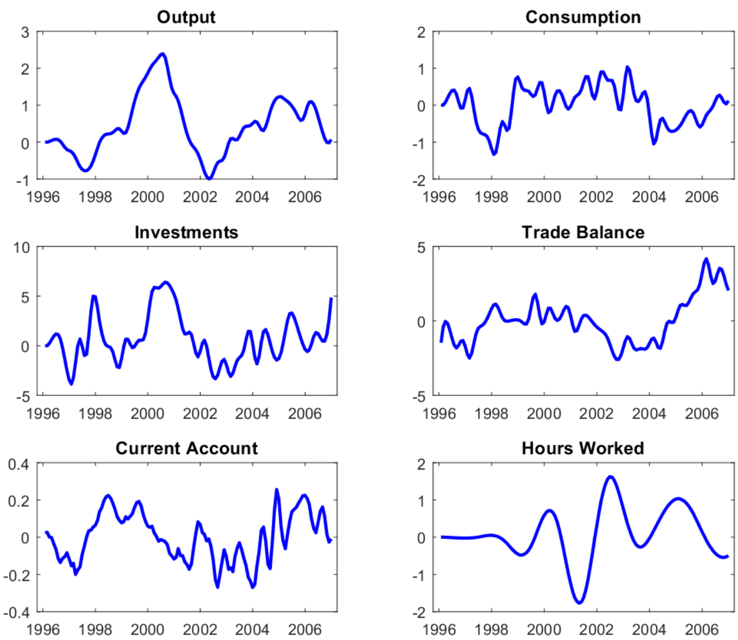
Table 1: Descriptive Statistics (deviations from SS)

	Mean	Std. dev.	5^{th} perc.	95^{th} perc.	min	max
Output	0.43	0.8	-0.77	2.07	-1	2.39
Consumption	0	0.52	-0.93	0.77	-1.33	1.03
Investments	0.68	2.39	-2.96	5.84	-3.86	6.4
Trade Balance	0	1.52	-2.17	3.14	-2.59	4.16
Current Account	0	0.13	-0.2	0.21	-0.27	0.26
Hours Worked	0.08	0.7	-1.4	1.22	-1.77	1.62

[6]The data for the Covid-19 economic downturn are not fully available yet.

[7]For details on the data preparation, please refer to the section obs:eq.

Figure 1: Final data for the estimation

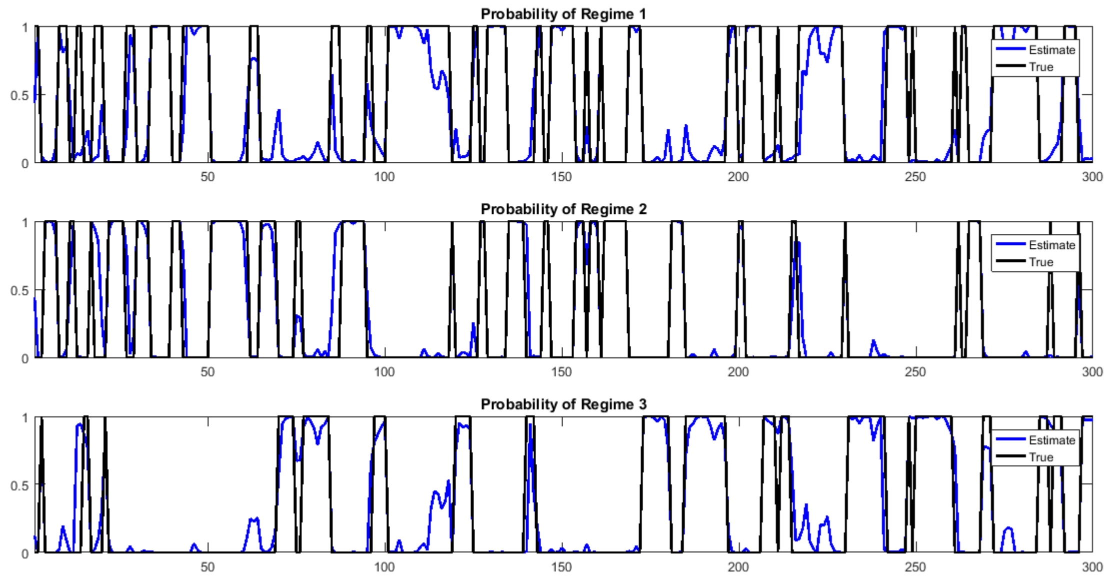


The DSGE model is estimated with an RWMH algorithm using 6 MH blocks with 500 000 draws each and a burn ratio of 0.9. Multiple MH chains are initiated to increase the robustness of the results, randomly picking the starting values around the posterior mode rather than the mode itself. For the setting of the prior, please refer to the 3. The MS-VAR model is estimated with the algorithm described in 2.1 using 100 000 draws with a burn ratio of 0.8 and the very same data as for the DSGE model. For the detailed estimation results for both models, please refer to Appendix B.

4.2 Example 1: Regime Identification Performance on Artificial Data with the MS-VAR model

Due to the nontrivial model definition and specification, the performance of the MSVAR model is tested before the forecasting exercise in an empirical simulation using artificial data. Following Blake and Mumtaz (2017), the artificial data of 300 observations in total are generated using the regime-specific pre-specified VAR parameters $\boldsymbol{B}_{S_t}$ and the covariance matrices $\boldsymbol{\Sigma}_{S_t}$ for each regime $\tilde{S} = 1, 2, 3$, and the probability transition matrix $\boldsymbol{P}^{true}$. Vector $\tilde{S}^{true}$ containing true regimes is generated only with the transition probability matrix $\boldsymbol{P}^{true}$ by the comparison of the transition probability p_{ij}^{true} with the random draw $u \sim U(0,1)$. Figure 2 depicts the model performance in terms of the correct regime-identification of each regime $i = \{1, 2, 3\}$. Note that thanks to the artificial data, the true regime is always known and the probability of each regime as identified by the model is compared to the true value. This shows that the model is quite responsive even to less persistent regimes and predicts false changes only in a limited number of cases.

Figure 2: MSVAR - Regimes Probability on Artificial Data

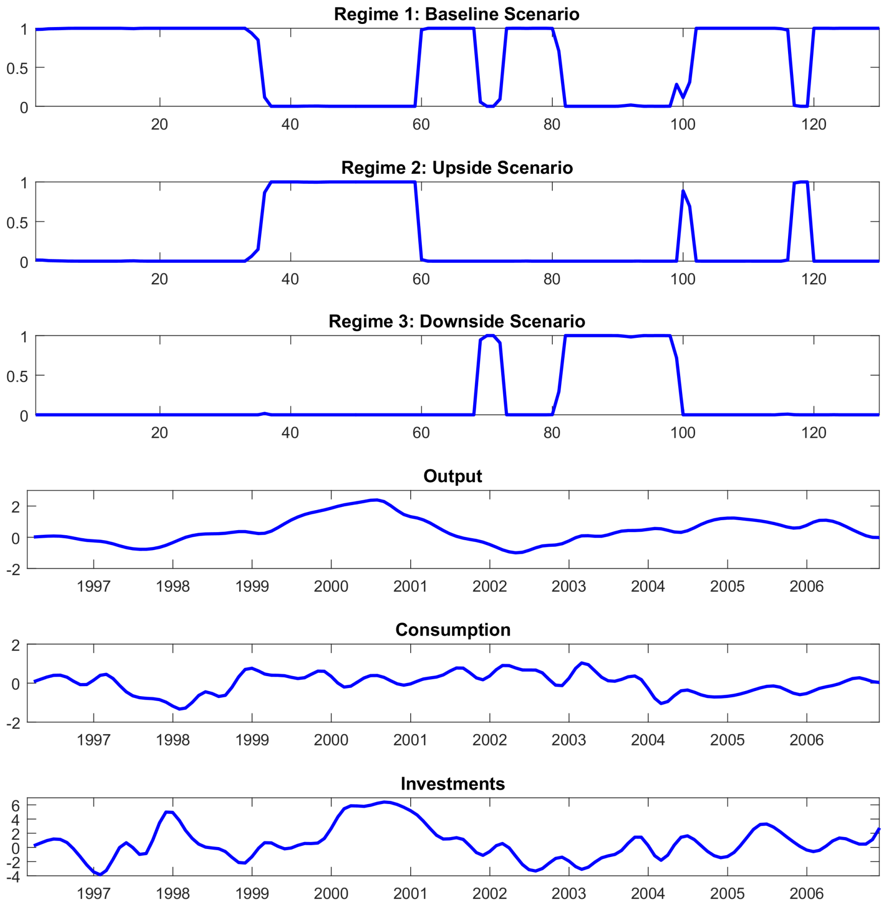


4.3 Example 2: Scenario generation with the MSVAR model

Figure 3 depicts historical regimes between 1996 and 2007 as identified by the MS-VAR model. The assignment of regimes to baseline, upside and downside scenarios is twofold. The technical aspect comes from the estimation algorithm and the requirement of the third regime being associated with the increased volatility typical for downturn periods. The expert assignment is based on the persistence, average occupancy time, volatility and the historical development of GDP, consumption and investments. In other words, it is not only the direction, but also the volatility and magnitude that govern the regime identification.

The baseline regime contains 80 periods, i.e. 60% of all available observations and the alternative scenarios amount to 28 and 22 (21% and 17%) historical periods for the upside and downside regime respectively. The upside regime covers primarily the period around the dot-com bubble in early 00's and a few periods in the late 2006. The downside regime covers the downturn after the dot-com bubble as well as the period between 2003 and 2005 characterised by increased volatility and the change in foreign country position.

Figure 3: MSVAR - Regimes Probability on Historical Data

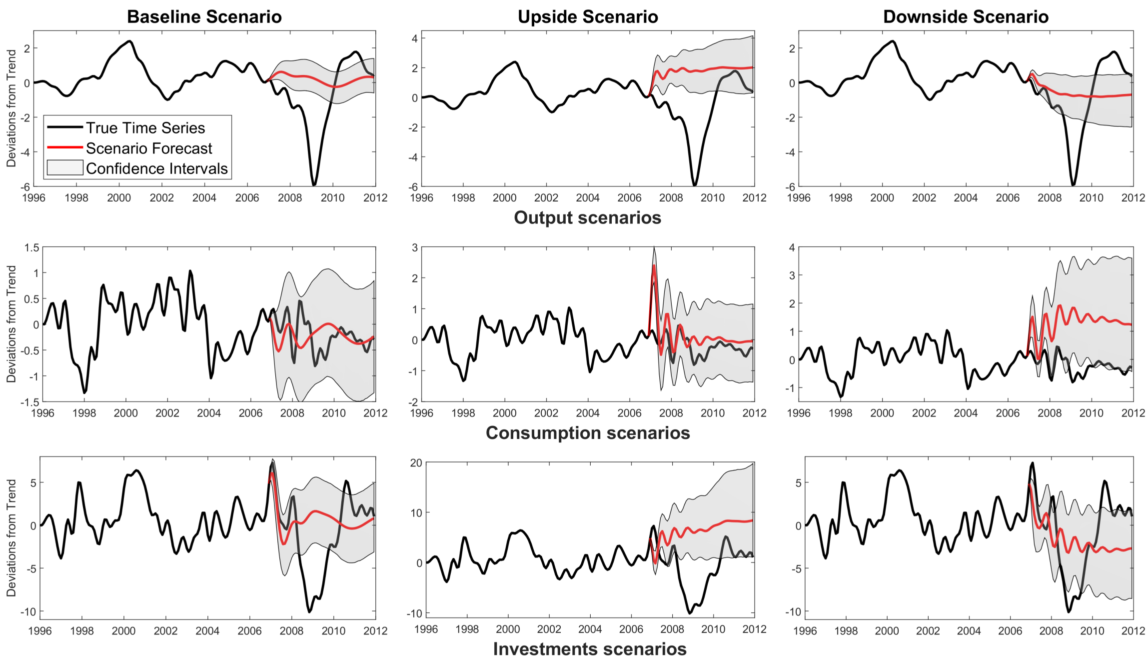


The set of Figures in 4 depicts scenarios generated by the MS-VAR model as unconditional, regime-specific forecasts with forecasting horizon $k = 60$ months and $L = 2$:

$$\hat{\boldsymbol{Y}}_{t+k}|(S_t = i) = \hat{c}_{S_t} + \sum_{l=1}^{L} \hat{\boldsymbol{B}}_{S_t} \boldsymbol{Y}_{t+k-l} \tag{36}$$

The separation of the scenarios in output and investments and the inherently highest volatility in the third regime support the identification of the regimes as baseline, upside and downside scenarios. However, the forecasts of consumption lack direct interpretation: while the first regime si truly the baseline, as it mimics the historical realization, and the second can be considered upside thanks to the initial peak, the third scenario deviates from conventions. After an initial period of high volatility, the forecast leaves the subspace of historical data and fails to converge. However, it is still interpretable as a stabilization policy, since the consumption model variable encompasses government expenditures and the deviation is less than 1 percentage point. On the other hand, despite the absence a precedence for the crisis, investments manage to capture partially the crisis downturn 24 periods ahead. Relatively mild scenarios for output are not considered to be a weakness, since the model is not expected or even supposed to capture the extreme macroeconomic stress induced by the financial crisis, but, rather, to provide representative scenarios.

Figure 4: MSVAR - Scenario Forecasts



Finally, we present the estimated transition probability matrix $\hat{\boldsymbol{P}}$ and its limiting distribution π, i.e. the probability weights of the baseline, upside and downside scenario respectively. Notice that the scenario weights roughly correspond to the relative share of historical observations in the individual regimes. The estimated transition matrix is consistent with macroeconomic theory and empirical observations. The baseline regime is naturally the most persistent, and migrations to alternative states are almost perfectly symmetric. The same applies to the persistence of alternative states, implying that (extreme) upturn and downturn periods are of the same length. The higher migration probability from the downside straight to the upside rather than the baseline regime is also consistent with the empirically observed sharp recovery following crises. In the reverse case, the baseline regime is more likely to follow the upside before the crisis hits, i.e. there is a cool-down period.

$$\hat{\boldsymbol{P}} = \begin{pmatrix} 0.883 & 0.059 & 0.058 \\ 0.157 & 0.764 & 0.079 \\ 0.140 & 0.121 & 0.738 \end{pmatrix}, \quad \hat{\pi} = \begin{pmatrix} 0.5604 \\ 0.2413 \\ 0.1982 \end{pmatrix}. \tag{37}$$

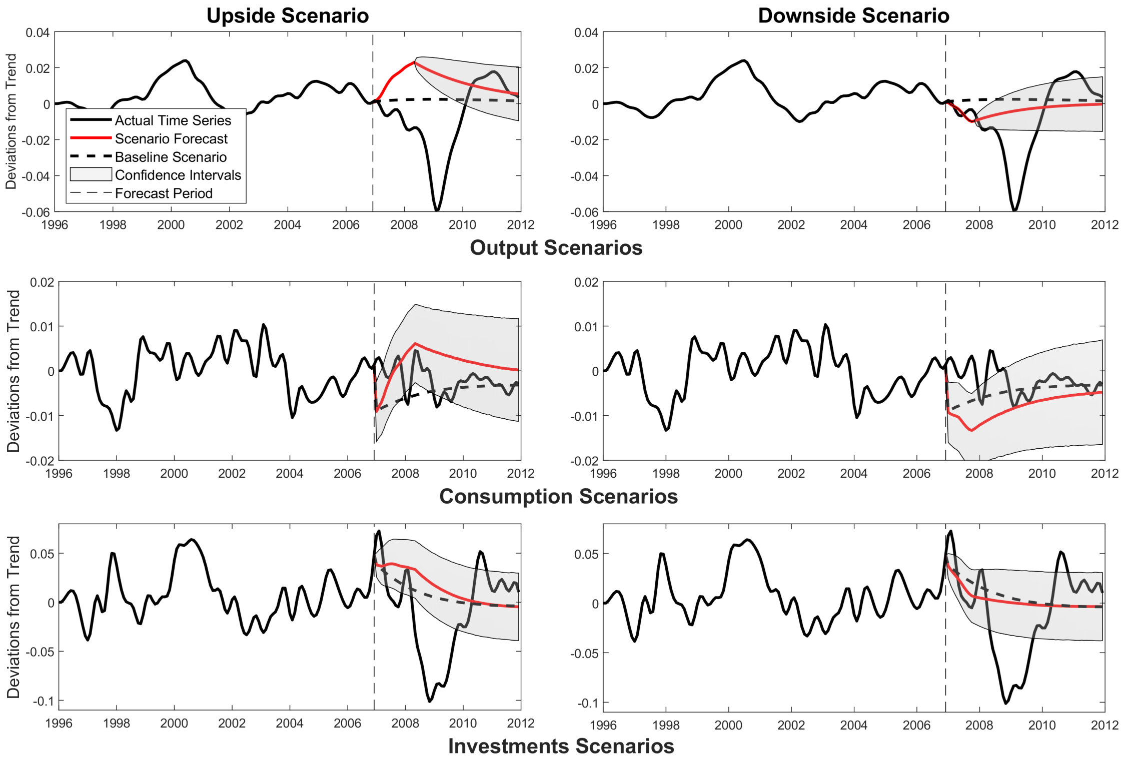
Overall, the method provides relatively well-behaved scenarios with respect to the desired properties such as scenario separation, representativeness, relatively smooth scenario paths and, especially, economic interpretability. Data-driven regimes identification and scenario generation comes at the expense of less control over the scenario paths.

4.4 Example 3: Scenario generation with the DSGE model

In line with the conventions of scenario analysis, a baseline scenario is represented by an unconditional forecast, while upside and downside scenarios are constructed as conditional forecasts fitting a prescribed future path, as in Banbura et al. (2015) or Baumeister and Kilian (2014). The initial paths of stressed variables are specified according to the maximum historical upside and downside peaks in GDP, as in the FED CCAR 2018 stress testing methodology, and later convergence is determined endogenously by the model. Also note that this method, unlike the MS-VAR model, does not require regime identification, and only a single set of parameters is available. The consequences are twofold: A researcher has better control over the alternative scenarios, but the model does not take account of the non-linearities in the business cycle, i.e. the response is the same in all scenarios.

The set of figures in Figure 5 depicts the baseline and alternative scenarios for output, consumption and investments. The baseline forecast for all variables quickly converges to the unconditional mean due to the rigid structure of the model, although for consumption it is characterized by the initial steep fall present also in the alternative scenarios. Since the alternative scenario paths are generated according to the peak and bottom of the historical GDP, the downside scenario in particular tends to capture better the initial cool-down before the financial crisis. However, from the perspective of 2007 such a development could be considered extreme, not representative. On the other hand, note that unlike the MSVAR model, the DSGE failed to generate investment scenarios based on the empirical evidence taht it had volatility of output three times greater and the scenarios are very mild in both alternative regimes instead. In favour of this approach is the general gradual convergence of alternative paths towards the baseline. Overall, the method seems viable, but compared to MS-VAR it bears additional costs: (i) a cumbersome model solution and estimation, (ii) a rigid structure, (iii) extreme computational demands, (iv) excessive analytical expertise and (v) the determination of an alternative path of at least one variable which still does not guarantee the desired scenario properties. Moreover, the method does not mitigate the assignment of scenario probabilities.

Figure 5: DSGE - Scenario Forecasts



Technically, conditional forecasts depend on (i) the order of the shocks, (ii) the selection of the prescribed variable and (iii) controlled shocks. Here, a conditional forecasting exercise is conducted by exogenizing the path of GDP, while controlling the TFP shock. The order of shocks is consistent with the original model. Forecasts paths are generated using 50 000 MCMC draws.

5 Conclusion

Scenario generation is an integral part of both the academic and business environment and has become a necessary everyday activity of financial institutions, corporates, government organization and other stakeholders for planning, provisioning or assessing the health of a financial system. The literature on this subject is still relatively scarce and consists primarily of stress testing exercises and the coverage of extreme events, while the generation of representative scenarios for IFRS9 purposes is omitted. At the same time, the existing scenario generation approaches are either inappropriate for representative scenarios or suffer from excessive expert judgment. There is also a desire to associate a scenario with probability.

Our pioneering approach leverages the well-established framework of the Markov-switching model, but sets up the a philosophical foundation to scenario generation. We specify a representative scenario as an unconditional regime-specific forecast which, thanks to the separate sets of parameters, accounts for the non-linearities in the business cycle. The representative scenarios are naturally associated with the probability weights related to the respective state of the economy and derived from the model transition matrix. Both scenarios and probabilities are purely data driven, and the analytical input is limited solely to the setting up of priors and the calibration of model hyper parameters. The method also allows for the incorporation of prior beliefs or macroeconomic theory either through regime-sampling restrictions, priors, or the a priori specification of baseline and alternative regimes in historical data without compromising data-driven scenarios. Further improvements could be achieved using a structural VAR or different regime-switching underlying model, e.g. a regime-switching DSGE. Further research is also needed into the calculation of probability weights: the natural extension is to consider a time-varying transition probability matrix and the conditioning of the probabilities to the current state of the economy.

In the empirical examples we demonstrate outstanding model performance in terms of regime identification on artificial as well as real data. We show that the generated scenarios are economically interpretable and possess most of the desired properties, such as scenario separation, representativeness, or a volatility driven crisis regime. The generated scenarios are, indeed, not extreme but rather representative given the historical data. Once the model is calibrated, its relative simplicity compared to other robust methods, the intuitive interpretation of the set-up and scenarios, and its possible adjustments to te requirements research questions make it perfectly suitable for academic and business purposes.

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7 Appendix A: DSGE Model

7.1 Model

Following the Schmitt-Grohe and Uribe (2003) and Mendoza (1991), the domestic economy is populated by an infinite number of identical and eternal households. Preferences are unitary and non-separable in both labour and consumption:

$$U(C_t, N_t) = \frac{\left(e^{\epsilon_t^c} C_t - \frac{e^{\epsilon_n^l} N_t^{-\phi}}{\phi}\right)^{1-\gamma} - 1}{1-\gamma}. \tag{38}$$

C_t is the per capita composite consumption index, γ is the intertemporal elasticity of substitution between consumption and labour, N_t is the per capita labour supply and ϕ is the corresponding Frisch labour supply elasticity. Preferences are extended with the consumption preference and the labour supply shock, ϵ_t^c and ϵ_t^n respectively, defined as AR(1) processes:

$$\epsilon_t^c = \rho_c \epsilon_{t-1}^c + \varepsilon_t^c \quad \text{, where } \varepsilon_t^c \sim N(0, \sigma_c^2) \tag{39}$$

$$\epsilon_t^n = \rho_n \epsilon_{t-1}^n + \varepsilon_t^n \quad \text{, where } \varepsilon_t^n \sim N(0, \sigma_n^2) \tag{40}$$

Households maximize their utility with respect to their budget constraint:

$$C_t + I_t + TB_t \leq Y_t - \frac{\Phi}{2}(\Delta K_{t+1})^2, \tag{41}$$

where I_t are gross investments, b_t is the investment specific shock, Y_t is output or GDP, TB_t is the trade balance and $\Phi(\Delta K_{t+1})$ is the function capturing investment adjustment costs parametrized such that $\Phi(0) = \Phi'(0) = 0$, and K_{t+1} is one-period-ahead predetermined capital stock. There exists a market with foreign assets A_t available to households of the domestic economy, paying (or receiving) a foreign interest rate r_t^*. One can think of this asset as foreign bonds and its holding as an international lending position or foreign debt:

$$A_{t+1} = TB_t + A_t(1 + r_t). \tag{42}$$

Production technology in this economy is represented by the standard Cobb-Douglas production technology with constant returns to scale and composed of the capital K_t

and labour N_t services:

$$Y_t = \mathrm{e}^{a_t} K_t^{\alpha} N_t^{1-\alpha}, \tag{43}$$

where

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \quad \text{, where } \varepsilon_t^a \sim N(0, \sigma_a^2) \tag{44}$$

is the TFP shock process defined as AR(1) included in the levels, i.e. the aggregate level of technology. The law of motion of capital is also in standard form:

$$K_{t+1} = \mathrm{e}^{b_t} I_t + (1-\delta) K_t, \tag{45}$$

with added investment-specific shock:

$$b_t = \rho_b b_{t-1} + \varepsilon_t^b \quad \text{, where } \varepsilon_t^b \sim N(0, \sigma_b^2). \tag{46}$$

The evolution of foreign debt is obtained by the substitution of (42) into the budget constraint (41):

$$A_t = (1 + r_{t-1}) A_{t-1} - Y_t + C_t + I_t + \Phi(\Delta K_{t+1})^2. \tag{47}$$

Note that foreign debt is decreasing in domestic output and increasing in consumption, investments, interest costs and capital adjustment costs. Substituting (43) and (45) into (47) leads to the final constraint for households optimization:

$$A_t = \Big((1+r_{t-1})A_{t-1} - A_t K_t^{\alpha} N_t^{1-\alpha} + C_t + \mathrm{e}^{-b_t} K_{t+1} - (1-\delta)\mathrm{e}^{-b_t} K_t + \frac{\Phi}{2}(\Delta K_{t+1})^2\Big). \tag{48}$$

At last, stochastic endogenous discount factor (EDF) θ is defined:

$$\theta_{t+1} = \beta(C_t, N_t)\theta_t \tag{49}$$

with

$$\beta(C_t, N_t) = \Big(1 + C_t - \frac{N_t^{\phi}}{\phi}\Big)^{-\psi_n}. \tag{50}$$

When they are all combined, the stochastic utility maximization problem is:

$$\underset{C_t, N_t, A_t, \theta_{t+1}, K_{t+1}}{max} : \; U(C_t, N_t) \text{ s.t. } BC \text{ and } EDF.$$

This optimization problem is solved by solving the Lagrangian function

$$\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \theta_t \Bigg[\frac{\Big(\mathrm{e}^{\epsilon_t^c} C_t - \frac{\mathrm{e}^{\epsilon_t^n} N_t^{\phi}}{\phi}\Big)^{1-\gamma} - 1}{1-\gamma} - \eta_t \Big(\theta_{t+1} - \Big(1 + C_t - \frac{N_t^{\phi}}{\phi}\Big)^{-\psi_n} \theta_t \Big)$$

$$-\lambda_t \Big(-A_t + (1+r_{t-1}) A_{t-1} - \mathrm{e}^{a_t} K_t^{\alpha} N_t^{1-\alpha} + C_t + \mathrm{e}^{-b_t} K_{t+1} - (1-\delta) \mathrm{e}^{-b_t} K_t + \frac{\Phi}{2} \Big(\Delta K_{t+1}\Big)^2 \Big) \Bigg]$$

The first order conditions for this optimization problem are:

$\frac{\partial \mathcal{L}}{\partial C_t}:\ \theta_t \Big[\Big(\mathrm{e}^{\epsilon_t^c} C_t - \frac{\mathrm{e}^{\epsilon_t^n} N_t^{\phi}}{\phi}\Big)^{-\gamma} - \lambda_t - \eta_t(-\psi_n)\Big(1 + C_t - \frac{N_t^{\phi}}{\phi}\Big)^{-\psi_n - 1} \Big] = 0$

$\frac{\partial \mathcal{L}}{\partial N_t}:\ \theta_t \Big[\Big(\mathrm{e}^{\epsilon_t^c} C_t - \frac{\mathrm{e}^{\epsilon_t^n} N_t^{\phi}}{\phi}\Big)^{-\gamma} (-N_t^{\phi-1}) + \lambda_t (1-\alpha) e^{a_t} \Big(\frac{K_t}{N_t}\Big)^{\alpha} - \eta_t(-\psi_n)\Big(1 + C_t - \frac{N_t^{\phi}}{\phi}\Big)^{-\psi_n - 1} (-N_t^{\phi-1}) \Big] = 0$

$\frac{\partial \mathcal{L}}{\partial A_t}:\ \theta_t \lambda_t = \theta_{t+1} \lambda_{t+1} (1+r)$

$\frac{\partial \mathcal{L}}{\partial \theta_{t+1}}:\ \frac{\Big(\mathrm{e}^{\epsilon_{t+1}^c} C_{t+1} - \frac{\mathrm{e}^{\epsilon_{t+1}^n} N_{t+1}^{\phi}}{\phi}\Big)^{1-\gamma} - 1}{1-\gamma} - \eta_{t+1}\Big(1 + C_t - \frac{N_t^{\phi}}{\phi}\Big)^{-\psi_n} - \eta_t$

$\frac{\partial \mathcal{L}}{\partial K_{t+1}}:\ -\theta_t \lambda_t \Big(\mathrm{e}^{-b_t} + \Phi(\Delta K_{t+1})\Big) - \theta_{t+1} \lambda_{t+1} \Big(-(1-\delta) \mathrm{e}^{-b_{t+1}} - \mathrm{e}^{a_{t+1}} \alpha \Big(\frac{N_{t+1}}{K_{t+1}}\Big)^{1-\alpha} + \Phi(\Delta K_{t+2}) \Big)$

By evaluating the first order conditions at $t = 0$ and substituting (49) for θ_{t+1}, we can rewrite them as:

$$C_t:\ \lambda_t = \Big(\mathrm{e}^{\epsilon_t^c} C_t - \frac{\mathrm{e}^{\epsilon_t^n} N_t^{\phi}}{\phi}\Big)^{-\gamma} - \eta_t(-\psi_n)\Big(1 + C_t - \frac{N_t^{\phi}}{\phi}\Big)^{-\psi_n - 1} \tag{51}$$

$$N_t:\ \lambda_t (1-\alpha) e^{a_t} \Big(\frac{K_t}{N_t}\Big)^{\alpha} = (-\mathrm{e}^{\epsilon_t^n} N_t^{\phi-1}) \Big[-\Big(\mathrm{e}^{\epsilon_t^c} C_t - \frac{\mathrm{e}^{\epsilon_t^n} N_t^{\phi}}{\phi}\Big)^{-\gamma} + \eta_t(-\psi_n)\Big(1 + C_t - \frac{N_t^{\phi}}{\phi}\Big)^{-\psi_n - 1} \Big] \tag{52}$$

$$A_t:\ \lambda_t = \frac{\theta_{t+1}}{\theta_t} \lambda_{t+1}(1+r) = \lambda_{t+1}(1+r)\Big(1 + C_t - \frac{N^{\phi}}{\phi}\Big)^{-\psi_n} \tag{53}$$

$$\theta_{t+1}:\ \eta_t = -\frac{\Big(\mathrm{e}^{\epsilon_{t+1}^n} C_{t+1} - \frac{\mathrm{e}^{\epsilon_{t+1}^n} N_{t+1}^{\phi}}{\phi}\Big)^{1-\gamma} - 1}{1-\gamma} + \eta_{t+1}\Big(1 + C_t - \frac{N_t^{\phi}}{\phi}\Big)^{-\psi_n} \tag{54}$$

$$K_{t+1}: \ \lambda_t\Big(1+\mathrm{e}^{b_t}\Phi(\Delta K_{t+1})\Big) = \lambda_{t+1}\Big(1+C_t-\frac{N_t^{\phi}}{\phi}\Big)^{\psi_n}\frac{\mathrm{e}^{b_t}}{\mathrm{e}^{b_{t+1}}}\Bigg((1-\delta)+...$$

$$...+\mathrm{e}^{a_{t+1}+b_{t+1}}\alpha\left(\frac{N_{t+1}}{K_{t+1}}\right)^{1-\alpha}+\mathrm{e}^{b_{t+1}}\Phi(\Delta K_{t+2})\Bigg). \quad (55)$$

Lagrange multipliers λ and η represent ... To close up the model, the foreign sector in this small open economy model is represented by the balance of trade (TB) and the evolution of the current account (CA). Dividing the household budget constraints outlined in (41) by the domestic output Y_t and rearranging the formula, the definition of the trade balance to output ratio is given by:

$$\frac{TB_t}{Y_t} = 1 - \frac{C_t + I_t + \frac{\Phi}{2}(\Delta K_{t+1})^2}{Y_t}. \quad (56)$$

The current account is by definition given by the change in the foreign debt A_t, and thus the current account to output ratio is:

$$\frac{CA_t}{Y_t} = \frac{A_{t-1} - A_t}{Y_t}. \quad (57)$$

The purpose of defining the trade balance and the current account as ratios is to induce stationarity properties in the model. Finally, domestic agents face an interest rate which is identical to the world interest rate:

$$r_t = \bar{r}_t \quad (58)$$

7.2 Steady State and Analytical solution

To find the analytical solution of the model, the deterministic steady state is calculated from the model equations. Firstly, (i) the variables uniquely determined only by the model parameters are calculated, then (ii) the capital-labour ratio is found, allowing us to express (iii) the remaining variables directly dependent on those inputs. Firstly, debt evolution has to be pinned down to its equilibrium value:

$$A = \bar{A}. \quad (59)$$

From (58) it is clear that:

$$r = \bar{r}. \tag{60}$$

The above derived FOC provides an optimality condition for the factors of production and marginal utility variables λ and η. It is useful to derive the capital-labour ratio, by equalizing (53) and (54) through the definition of the endogenous discount factor $\beta(C_t, N_t)$:

$$1 + r = \alpha\left(\frac{N}{K}\right)^{1-\alpha} + 1 - \delta$$

$$r + \delta = \alpha\left(\frac{N}{K}\right)^{1-\alpha}$$

$$\frac{K}{N} = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}}. \tag{61}$$

Then, by equalizing consumption FOC (51) and labour FOC (52) through the marginal utility λ and simultaneously pre-multiplying (52) by the factor of (-1), both right-hand-sides can be eliminated, except $(-N^{\phi-1})$, to obtain:

$$N^{\phi-1} = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha}.$$

With the capital-labour ratio (61) in hand, it is straightforward that:

$$N = \left[(1-\alpha)\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{\phi-1}} \tag{62}$$

$$K = N\left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} \tag{63}$$

and the steady state output is obtained from the production function (43):

$$Y = K^{\alpha}N^{1-\alpha}. \tag{64}$$

In the steady state, equilibrium investments have to cover the depreciation of the capital stock exactly. From the law of motion of capital (45), it is:

$$I = \delta K. \tag{65}$$

From (42) it follows that in the steady state $TB = rA$, and thus consumption can be derived from the market clearing condition (41) reduced to:

$$C = Y - I - rA. \tag{66}$$

From (54) comes the relationship for η:

$$\eta = \frac{-\frac{\left(C_t - \frac{N_t^{-\phi}}{\phi}\right)^{1-\gamma} - 1}{1-\gamma}}{1 - \left(1 + C_t - \frac{N_t^{\phi}}{\phi}\right)^{-\psi_n}}. \tag{67}$$

Now we have all necessary components to pin down the steady state value for λ, directly from (51):

$$\lambda = \left(C - \tfrac{N^{\phi}}{\phi}\right)^{-\gamma} - \eta(-\psi_n)\left(1 + C - \tfrac{N^{\phi}}{\phi}\right)^{-\psi-1}. \tag{68}$$

The steady state trade balance to output ratio is:

$$\frac{TB}{Y} = 1 - \frac{C+I}{Y}. \tag{69}$$

The last remaining things are the current account and the trade balance to output ratio and the shock processes:

$$\frac{CA}{Y} = \frac{TB}{Y} = a_t = b_t = \mathrm{e}^{\epsilon_t^c} = \mathrm{e}^{\epsilon_t^n} = 0. \tag{70}$$

8 Appendix B

The following set of Tables 2 to 4 summarizes the prior and posterior distribution of DSGE model parameters.

Table 2: Prior and Posterior distributions of Parameters

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	5% HPD	95% HPD
γ	Gamma	2	0.2	2.261	0.1514	2.0837	2.5920
ϕ	Gamma	2	0.1	1.985	0.0536	1.9114	2.0850
ρ_a	Beta	0.8	0.1	0.955	0.0071	0.9438	0.9667
ρ_b	Beta	0.8	0.1	0.972	0.0111	0.9546	0.9897
ρ_c	Beta	0.7	0.2	0.960	0.0110	0.9414	0.9778
ρ_n	Beta	0.7	0.2	0.979	0.0077	0.9679	0.9908
δ	Beta	0.1	0.08	0.017	0.0030	0.0121	0.0216
ψ_l	Normal	0.4	0.2	0.765	0.0848	0.6132	0.8670
α	Beta	0.3	0.1	0.180	0.0107	0.1629	0.1978
Φ	Normal	0.4	0.4	0.548	0.1084	0.3622	0.7174

Table 3: Prior and Posterior Distributions of Standard Deviations of Structural Shocks

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	5% HPD	95% HPD
σ_a	Inverse Gamma	0.005	Inf	0.002	0.0001	0.0016	0.0020
σ_b	Inverse Gamma	0.005	Inf	0.003	0.0007	0.0018	0.0039
σ_c	Inverse Gamma	0.005	Inf	0.002	0.0002	0.0014	0.0020
σ_n	Inverse Gamma	0.005	Inf	0.003	0.0002	0.0030	0.0038
σ_y	Inverse Gamma	0.005	Inf	0.001	0.0001	0.0007	0.0010
σ_{tb}	Inverse Gamma	0.005	Inf	0.015	0.0009	0.0131	0.0161
σ_h	Inverse Gamma	0.005	Inf	0.001	0.0001	0.0006	0.0008

(Continued on next page)

Table 3: (continued)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	5% HPD	95% HPD
σ_{cay}	Inverse Gamma	0.005	Inf	0.001	0.0000	0.0006	0.0007

Table 4: Prior and Posterior distributions for Correlation of Structural Shocks

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	5% HPD	95% HPD
$corr(\varepsilon_a, \varepsilon_b)$	Beta	0.5	0.3	0.240	0.0673	0.1283	0.3444
$corr(\varepsilon_a, \varepsilon_c)$	Beta	0.0	0.3	-0.193	0.0612	-0.2859	-0.0737
$corr(\varepsilon_a, \varepsilon_n)$	Beta	0.0	0.3	0.895	0.0191	0.8644	0.9264
$corr(\varepsilon_b, \varepsilon_c)$	Beta	0.0	0.3	0.812	0.0416	0.7488	0.8830
$corr(\varepsilon_b, \varepsilon_n)$	Beta	0.0	0.3	0.269	0.0574	0.1747	0.3580
$corr(\varepsilon_c, \varepsilon_n)$	Beta	0.0	0.3	-0.061	0.0642	-0.1768	0.0410

The following set of Tables 5 to 7 summarizes the estimated coefficients of the MS-VAR model for each regime.

Table 5: MS-VAR Coefficients for Baseline Regime

	Output	Consumption	Investments	Hours Worked	Trade balance
Constant	-0.001	-0.019	0.075	0.001	-0.045

(Continued on next page)

Table 5: MS-VAR Coefficients for Baseline Regime

	Output	Consumption	Investments	Hours Worked	Trade balance
	(0.004)	(0.012)	(0.057)	(0.001)	(0.022)
Output (-1)	1.770	-0.153	-0.106	-0.001	-0.681
	(0.044)	(0.139)	(0.652)	(0.016)	(0.253)
Consumption (-1)	-0.025	1.627	0.277	-0.008	-0.159
	(0.027)	(0.07)8	(0.367)	(0.009)	(0.140)
Investments (-1)	0.005	-0.044	1.619	0.003	0.070
	(0.005)	(0.014)	(0.064)	(0.002)	(0.025)
Hours Worked (-1)	0.119	-0.127	-0.798	1.970	-0.126
	(0.045)	(0.129)	(0.588)	(0.013)	(0.219)
Trade Balance (-1)	0.012	-0.002	-0.052	0.005	1.498
	(0.011)	(0.034)	(0.159)	(0.004)	(0.061)
Output (-2)	-0.809	0.146	0.065	0.014	0.782
	(0.043)	(0.135)	(0.636)	(0.016)	(0.248)
Consumption (-2)	0.013	-0.744	-0.439	0.009	0.045
	(0.028)	(0.078)	(0.365)	(0.009)	(0.139)
Investments (-2)	-0.002	0.037	-0.826	-0.002	-0.054
	(0.004)	(0.013)	(0.061)	(0.002)	(0.023)
Hours Worked (-2)	-0.108	0.117	0.685	-1.005	0.141
	(0.045)	(0.131)	(0.592)	(0.013)	(0.222)
Trade Balance (-2)	-0.008	0.006	0.146	-0.007	-0.587
	(0.010)	(0.033)	(0.156)	(0.004)	(0.060)

Table 6: MS-VAR Coefficients for Upside Regime

	Output	Consumption	Investments	Hours Worked	Trade balance
Constant	0.108	0.007	-0.342	0.011	0.095
	(0.018)	(0.019)	(0.148)	(0.008)	(0.070)
Output (-1)	1.900	0.015	0.459	-0.007	1.993
	(0.101)	(0.101)	(0.739)	(0.040)	(0.376)
Consumption (-1)	-0.144	1.749	-1.399	-0.014	-1.629
	(0.087)	(0.089)	(0.674)	(0.034)	(0.335)
Investments (-1)	0.017	0.090	0.952	-0.006	-0.281
	(0.014)	(0.014)	(0.105)	(0.005)	(0.052)
Hours Worked (-1)	0.072	0.777	-0.666	1.948	-3.652
	(0.120)	(0.120)	(0.839)	(0.047)	(0.448)
Trade Balance (-1)	-0.075	0.121	-0.461	0.005	1.068
	(0.031)	(0.031)	(0.218)	(0.012)	(0.114)
Output (-2)	-0.986	-0.263	0.626	0.011	-1.344
	(0.097)	(0.095)	(0.697)	(0.038)	(0.357)
Consumption (-2)	0.067	-0.725	1.181	0.000	0.968
	(0.069)	(0.067)	(0.505)	(0.027)	(0.258)
Investments (-2)	-0.006	0.013	-0.287	-0.001	-0.028
	(0.013)	(0.013)	(0.094)	(0.005)	(0.048)
Hours Worked (-2)	-0.087	-0.848	0.763	-0.992	3.882
	(0.118)	(0.119)	(0.83)0	(0.046)	(0.446)
Trade Balance (-2)	0.090	0.019	0.147	-0.006	-0.499
	(0.026)	(0.028)	(0.206)	(0.011)	(0.101)

Table 7: MS-VAR Coefficients for Downside Regime

	Output	Consumption	Investments	Hours Worked	Trade balance
Constant	0.063	0.327	0.049	0.024	0.051
	(0.018)	(0.051)	(0.285)	(0.011)	(0.059)
Output (-1)	1.278	-0.357	-3.236	-0.117	1.318
	(0.165)	(0.447)	(2.72)	(0.107)	(0.522)
Consumption (-1)	-0.064	1.354	1.172	0.01	-0.155
	(0.025)	(0.063)	(0.402)	(0.015)	(0.075)
Investments (-1)	0.022	0.018	0.912	-0.004	0.088
	(0.014)	(0.014)	(0.105)	(0.005)	(0.052)
Hours Worked (-1)	-0.562	-0.83	-1.227	1.788	0.075
	(0.106)	(0.278)	(1.699)	(0.066)	(0.343)
Trade Balance (-1)	-0.017	0.118	-0.465	0.012	1.062
	(0.042)	(0.108)	(0.646)	(0.025)	(0.129)
Output (-2)	-0.466	-0.35	3.558	0.097	-1.671
	(0.153)	(0.412)	(2.516)	(0.098)	(0.48)
Consumption (-2)	-0.005	-0.871	-1.56	-0.026	0.09
	(0.029)	(0.071)	(0.469)	(0.017)	(0.089)
Investments (-2)	0.014	-0.028	-0.272	0.002	0.053
	(0.008)	(0.022)	(0.126)	(0.005)	(0.024)
Hours Worked (-2)	0.48	0.499	0.316	-0.867	0.099
	(0.074)	(0.201)	(1.185)	(0.047)	(0.24)
Trade Balance (-2)	-0.003	-0.047	0.44	-0.017	-0.119
	(0.041)	(0.107)	(0.642)	(0.025)	(0.126)

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