INSTITUTE
OF ECONOMIC STUDIES
Faculty of Social Sciences
Charles University

FREQUENCY-DEPENDENT HIGHER MOMENT RISKS

Jozef Baruník
Josef Kurka

IES Working Paper 11/2021

Institute of Economic Studies,
Faculty of Social Sciences,
Charles University in Prague

[UK FSV – IES]

Opletalova 26
CZ-110 00, Prague
E-mail : ies@fsv.cuni.cz
http://ies.fsv.cuni.cz

Institut ekonomických studií
Fakulta sociálních věd
Univerzita Karlova v Praze

Opletalova 26
110 00 Praha 1

E-mail : ies@fsv.cuni.cz
http://ies.fsv.cuni.cz

Bibliographic information:
Baruník J. and Kurka J. (2021): "Frequency-Dependent Higher Moment Risks" IES Working Papers 11/2021. IES FSV. Charles University.

This paper can be downloaded at: http://ies.fsv.cuni.cz

Frequency-Dependent Higher Moment Risks

Jozef Baruník[a]
Josef Kurka[b]

[a]Institute of Economic Studies, Charles University, Prague, Czech Republic
Institute of Information Theory and Automation, Academy of Sciences of the Czech Republic
E-mail: barunik@fsv.cuni.cz
[b]Institute of Economic Studies, Charles University, Prague, Czech Republic
Institute of Information Theory and Automation, Academy of Sciences of the Czech Republic
E-mail: josef.kurka@fsv.cuni.cz

April 2021

Abstract:
Based on intraday data for a large cross-section of individual stocks and Exchange traded funds, we show that short-term as well as long-term fluctuations of realized market and average idiosyncratic higher moments risks are priced in the cross-sectionof asset returns. Specifically, we find that market and average idiosyncratic volatility and kurtosis are significantly priced by investors mainly in the long-run even if controlled by market moments and other factors, while skewness is mostly short-run phenomenon. A conditional pricing model capturing the time-variation of moments confirms downward-sloping term structure of skewness risk and upward-sloping term structure of kurtosis risk, moreover the term structures connected to market skewness risk and average idiosyncratic skewness risk exhibit different dymanics.

JEL: C14, C22, G11, G12
Keywords: Higher Moments, frequency, Spectral Analysis, Cross-sectional

Acknowledgements: We are grateful to Wolfgang Hardle, Antonio Galvao, Lukas Vacha, Martin Hronec, and the participants at the CFE 2019 for many useful comments, suggestions, and discussions. We gratefully acknowledge the support from the Czech Science Foundation under the EXPRO GX19-28231X project, support from the Grant Agency of Charles University under project No. 1188119, and from Charles University Research Centre program No. UNCE/HUM/035.

1 Introduction

Higher moments capturing non-normalities of return distributions have been recognized as an important source of risk in pricing securities for long time (Fama, 1965). More recent papers suggest that additional features of individual securities' payoff distribution may be relevant for understanding differences in assets' returns. For example Amaya et al. (2015); Neuberger and Payne (2021) argue that time variation of moments is an important aspect that induces changes in investment opportunity set by changing the expectation of future market returns, or by changing the risk-return trade-off.[1] In addition, risk premium associated with idiosyncratic and systemic counterparts are documented to impact the pricing kernel of an investor unequally (Langlois, 2020). These risks related to higher moments of return distributions are however exclusively being modelled as constant across frequencies which imposes strong restrictions on risk measurement across horizons (Bandi et al., 2021). In contrast to this assumption, recent literature documents theoretically as well as empirically[2] that investor's preferences are frequency-specific (Dew-Becker and Giglio, 2016; Neuhierl and Varneskov, 2021; Bandi et al., 2021). It remains an open question how different sources of risk to an investor are high-frequency (low-frequency) fluctuations of higher moments such as skewness or kurtosis, associated with transitory (persistent) risks.

The main goal of this paper is to provide a systematic investigation of how short-term and long-term fluctuations of important higher moments measures are priced in the cross-section of expected stock returns. Since these moment based risks are highly time-varying and have transitory (short-term) as well as persistent (long-term) components, we want to determine the role of these components using recent advances in financial econometrics coupled with newly available high-frequency intraday data enabling accurate

[1] See also Kelly and Jiang (2014); Harvey and Siddique (2000); Ang et al. (2006). Amaya et al. (2015) show that realized skew measures computed from intraday return data on individual stocks can be used to sort stocks into portfolios that have significantly different excess returns, while Boyer et al. (2009) and Conrad et al. (2013) show that high idiosyncratic skewness in individual stocks is also correlated with positive returns. Ghysels et al. (2016) present similar results for emerging market indices.

[2] Importance of horizon-specific decision making of investors has been recognized by literature for decades. Choice of horizons significantly affects model outcomes in terms of asset pricing (Levhari and Levy, 1977), portfolio selection (Tobin, 1965), and portfolio performance (Levy, 1972). Such discoveries stressed the importance of capturing heterogeneous preferences of investors across investment horizons. Models incorporating such assumption started emerging shortly (Gressis et al., 1976; Lee et al., 1990), however the increased attention to modelling the horizon-specific risks is a very recent phenomenon (Dew-Becker and Giglio, 2016; Neuhierl and Varneskov, 2021; Bandi et al., 2021).

measurement of the time-varying higher moments.

Why should we model investor preferences over higher moment risks as horizon-specific? Risk changes across investment styles as well as frequencies (Bandi et al., 2021) hence models that assume constant risk across investment horizons generally fail to describe a range of key characteristics including pricing of cross-sections when confronted with data. While long-run risk models (Bansal and Yaron, 2004) suggest that persistent components of risk are those of importance, empirical evidence is mixed suggesting that they do not capture the dynamics in returns fully. In contrast, Neuhierl and Varneskov (2021) argue that key feature of an asset pricing model should be ability to decompose the risk into frequency-specific components. Higher moments of return distribution exhibit strong time dynamics (Amaya et al., 2015) which implies that they will carry important transitory and permanent sources of risk. For example skewness risk often perceived as a manifestation of tail risk or crash risk may have transitory as well as persistent components that can be well connected to the transitory and permanent shocks in the economy creating horizon-specific risk. Our work is closely related to Neuberger and Payne (2021) who suggest how to compute higher moments of long-horizon returns from daily returns. In contrast, we use cyclical decomposition of fluctuations from intraday data that offers full decomposition of information to any frequency band of interest, and we exploit both transitory and persistent components of the higher moments. We view this decomposition as a natural way to explicitly model heterogeneous investment horizons and describe their dynamics fully.

More generally, returns and risk can be decomposed to elements with various levels of persistence (Adrian and Rosenberg, 2008). In their seminal work, Bansal and Yaron (2004) suggest frequency decomposition of consumption and dividend growth processes as a key to explaining various asset markets puzzles. Shocks to consumption at different frequencies have different implications for model outcomes; they enter the pricing kernel with different weights (Dew-Becker and Giglio, 2016), have varying effects on asset returns (Ortu et al., 2013; Yu, 2012) and lifetime utility Bidder and Dew-Becker (2016), moreover exposure of firms' cash flow to shocks with different persistence varies (Li and Zhang, 2016). Bandi et al. (2021) decompose the betas in consumption CAPM model, thus disentangle the effect of exposure to market risk for various horizons. Kamara et al. (2016) identify the sources of transitory and persistent risks in five cornerstone factors (MKT, SMB, HML, MOM, LIQ) by observing their power to explain the cross-section of expected returns over different horizons.

Varying preferences of investors across horizons justify certain degree of horizon-dependence in their risk attitudes. Several theoretical concepts explain such behaviour of investors. E.g. myopic loss aversion connects willingness of an individual to participate on an investment (alternatively on a bet, game, etc.) with the horizon of evaluation (for details see Benartzi and Thaler, 1995) thus perceives decision making of investors as horizon-specific. Standardly used preferences, e.g. Epstein-Zin (Epstein and Zin, 2013), are described by a discount factor and risk aversion parameter. Under horizon-dependent risk aversion, the representation of investor preferences needs to be adjusted by adding a patience coefficient (Gonzalo and Olmo, 2016). In order to prevent the outcomes of our model to be driven by selection of specific utility function (Dittmar, 2002), we approximate the stochastic discount factor using the model-free approach (e.g., Dittmar, 2002; Chabi-Yo, 2012). The empirical model we propose disentangles the short-run and long-run characteristics of investors' risk attitudes connected to various sources of transitory and persistent risk through empirical decomposition of higher moment risks into different horizons.

Assuming the idiosyncratic risk can be fully diversified, literature for long considered that only market risk enters the decision making of the investors. However, it has been documented both theoretically and empirically (e.g., Amaya et al., 2015; Jondeau et al., 2019) that idiosyncratic risk is also priced in the asset returns. The reason for this may be well grounded in the fact that idiosyncratic risk can be diversified away only in an unconnected system of stocks. As argued by Elliott et al. (2014); Barunik and Ellington (2020b) investors require risk premia for idiosyncratic risk in case the stocks form a connected network. Another important reason for the relevance of idiosyncratic risk originates from deliberate under-diversification of investors who, for example, do not hold fully diversified portfolios because they want to take advantage of the extreme positive returns stemming from holding positively skewed assets. In turn, these investors are exposed to the average idiosyncratic risk due to the network structures of stock markets, and their documented behaviour which violates rational decision making in the traditional sense.

While considerable research has examined the time-series relation between the idiosyncratic moments and the cross-section of returns, the question of how the aggregate moments affect the cross-section of expected returns has received less attention. Literature documents that while idiosyncratic skewness risk is important, systematic return skewness offers defensive returns during bad times as well (Langlois, 2020), and average market skewness is priced in the cross-section of returns (Jondeau et al., 2019). Our work is related to this recent debate and we further explore how the two sources of risk are being priced by investors with horizon-specific preferences.

Our key contribution is that we document how the higher moments are priced by investors at different investment horizons. We establish our main empirical results for a cross-section of U.S. firms. While empirical literature is predominantly build on evidence from the U.S. financial markets, we like to contrast this phenomenon of "academic home bias puzzle" with assets that are one of the most important financial innovations in decades, exchange-traded funds (ETFs). Using the ETFs dataset allows us to control the robustness of our empirical results by including assets representing securities from non-U.S. developed and emerging countries as well as small cap stocks. As such, ETFs are of considerable interest to economists, but the literature is still in its early stage. They have grown substantially in recent years and have potential to dramatically reshape the broader investment landscape (Lettau and Madhavan, 2018). To our knowledge, we are the first to look at higher moments risks in ETFs using high-frequency data.

The sample collected over the period January 2010 to November 2018 uses high frequency data to calculate realized moments using one minute sampled prices filtered to 5-minute prices, and we build daily and weekly returns and moment factors database to test the conditional asset pricing models. We find that both realized market and idiosyncratic higher moment risks are priced in the cross-section of asset returns with heterogeneous persistence. In particular, on sorting the individual stocks into portfolios based on their weekly short-run component of skewness, we document a significant equally-weighted weekly return differential between stocks in the lowest quintile and the stocks in the highest quintile. Sorts on the market skewness are also driven mainly by its transitory short-run component as we document returns with similar significance. These results remain also after controlling for all effects using Fama-MacBeth type cross-sectional predictability regressions, both conditionally and unconditionally. This means that market and average idiosyncratic skewness are both predominantly short-run phenomena, however their term structures exhibit different dynamics.

2 Pricing of Frequency-Dependent Higher Moments

The empirical search for explanation of why different assets earn different average returns centers around risk factor models arising from the Euler Equation. Whereas literature documents large number of factors, their overall poor performance supports the focus on risk factors capturing the properties of asset returns such as moments of distribution. Higher moments are highly informative once data depart from normality, that is otherwise convenient to assume. Non-normalities of return distribution have been recognized in the literature empirically (Fama, 1976, 1996; Bakshi et al., 2003), and problems documented by standard asset pricing and portfolio selection models, e.g. equity premium puzzle[3] or deliberate underdiversification[4], may be explainable through tails of distribution expressed by the higher moments. Barberis and Huang (2008) observe that investors deciding under risk often depart from the expected utility framework, hence the decision-making would be better modelled under cumulative prospect theory overweighting the tails. Higher weight given to information in the tails of returns distribution better coincides with the documented preferences of investors towards positively skewed assets.[5] Desire to exploit extreme positive returns stemming from holding "lottery-like" assets is among the explanations of the documented deliberate underdiversification of investors (Simkowitz and Beedles, 1978; Mitton and Vorkink, 2007).[6]

Harvey and Siddique (2000) note that failures of CAPM are most significant for assets in the lowest deciles of market-cap, i.e. the most significantly skewed assets. In their model based on pricing kernel quadratic in market returns[7], excess returns are dependent on covariance and coskewness with the market. Chabi-Yo (2012) derives a pricing kernel with

[3] Mehra and Prescott (1985) noted that class of general equilibrium models is not able to explain large average equity risk premia and low risk-free rate observed on US markets.

[4] Investors deliberately hold insufficiently diversified portfolios, although they would be capable of obtaining a sufficient number of assets to fully diversify away the idiosyncratic risk.

[5] Barberis and Huang (2008) claim their model could explain e.g. poor performance of IPOs or success of momentum strategies.

[6] Investors forego the opportunity to exploit returns of positively skewed assets by becoming completely diversified (Simkowitz and Beedles, 1978; Mitton and Vorkink, 2007).

[7] Such pricing kernel is derived by Taylor expansion cut-off before the fourth derivative.

stochastic volatility, skewness and kurtosis risk using an alternative model-free approach; small-noise expansion. Maheu et al. (2013) build on this result and derive an autoregressive asset pricing model containing jumps stemming from the higher moment risk. One of the intermediate steps in the derivation relates excess returns to second, third and fourth centralized moment of returns on aggregate wealth.

2.1 Frequency Decomposition

Higher moments are exclusively assumed as a risk that is constant across frequencies in the literature. This is too restrictive for the data, instead the risk factors should be modelled as having heterogeneous impact across frequencies. Our main aim is to investigate how the short-term as well as long-term fluctuations of higher moments matter in the cross-section of returns. This endeavor stems from the recent discussion which points to changing risk attitudes across investment styles as well as frequencies (Bandi et al., 2021; Neuhierl and Varneskov, 2021) and suggests that key feature of an asset pricing model should be ability to decompose the risk into frequency-specific components. To our knowledge, the ability of the transitory and persistent fluctuations in higher moments to price stock returns has not been evaluated. Similarly to the notion of spectral factor models (Bandi et al., 2021) that decompose CAPM beta into several frequencies, we decompose the higher moments to their horizon-specific components.

Assume that a higher moment M_t has two orthogonal components capturing economic cycles shorter than 2^j periods and longer than 2^j periods (for example months) for $j \geq 1$. These represent the short-run and long-run components capturing transitory and permanent information respectively. Bandi et al. (2021) show formally that it is always possible to decompose covariance-stationary time-series in such a way that these two components are orthogonal, they are non anticipative, and hence suitable for out of sample applications. These are key for the purpose of using such factors in asset pricing models.

Assuming the higher moment is a covariance-stationary time-series, we can decompose higher moment risk factor into components operating over the short and the long horizons as

$$M_t = M_t^{(s)} + M_t^{(l)} \tag{1}$$

$$= M_t^{<2^j} + M_t^{>2^j}, \tag{2}$$

where $M_t^{(s)}$ captures the short-run component of the moment computed as a sum of the corresponding elements up to j, and $M_t^{(l)}$ captures the long-run component of the moment consisting of the elements larger than j.

Based on this notion, we formalize an asset pricing model that connects horizon-specific components of higher moments to a risk premium. While this decomposition is general and allows for user specified choices of horizons, in the empirical application on the daily and weekly data we use $j = 7$ and $j = 5$ respectively. This choice corresponds to approximately less than half year for the short-term component and more than half year for the long-term component, and we focus on this decomposition specifically to disentangle the persistent risks from the transitory. Hence from now on, we will refer to short-run and long-run components of higher moment risk within this definition. The two distinct sources of risk will have transitory and permanent nature, and our aim is to find how these risks are priced in the cross-section of stocks as well as exchange traded funds.

2.2 Parameterization of the equity premium

Asset pricing models rely on approximating the stochastic discount factor (SDF) either by assuming a particular form of utility function, or in a model-free manner using Taylor expansion. Dittmar (2002) argues that results of many multi-factor or nonlinear pricing kernel[8] models stem from arbitrary assumptions about utility function. Preference for skewness and aversion to kurtosis can be motivated by already existing concepts while employing the model-free approach preventing the model outcomes to be affected by restrictive assumptions. Arditti (1967) motivates preference for positive skewness via decreasing risk premia in wealth, Kraus and Litzenberger (1976) formulate the three-moment CAPM based on properties of theoretically feasible utility functions[9], and Fang and Lai (1997) add cokurtosis to formulate four-moment CAPM. Dittmar (2002) derives set of conditions to eliminate counterintuitive risk taking by investors consisting of risk aversion, decreasing absolute risk aversion and decreasing absolute prudence. Pricing kernel satisfying these assumptions consists of elements representing moments of returns distribution

[8] Terms stochastic discount factor and pricing kernel both refer to M_{t+1} from the Euler equation (see Equation (3)), and we treat them as interchangeable.

[9] Desirable properties of utility function according to Arrow (1970): i) positive marginal utility with respect to wealth, ii) decreasing marginal utility in wealth, i.e. risk aversion, iii) non-increasing absolute risk aversion (Kraus and Litzenberger, 1976).

up to the fourth order and implies that investors have preferences over both skewness and kurtosis.

To formalize the discussion, we build on Harvey and Siddique (2000); Dittmar (2002); Maheu et al. (2013); Chabi-Yo (2012), and we assume a general utility function $U(W_{t+1})$[10] depending on wealth W_t can be accurately approximated by taking a Taylor expansion up to the fourth order (Dittmar, 2002). Defining R^w_{t+1} as the simple net return on wealth, we expand $U(W_{t+1})$ around $W_t(1+C_t)$ where C_t is an arbitrary return $C_t = E_t(R^w_{t+1})$. The pricing kernel is defined as $M_{t+1} = U'(W_{t+1})/U'(W_t)$ and it can be approximated by (Maheu, 2005)

$$\begin{aligned} M_{t+1} &\approx \sum_{n=0}^{3} \frac{U^{(n+1)}(1+C_t)}{U'(1)n!}(R^w_{t+1}-C_t)^n \\ &= g_{0,t} + g_{1,t}(R^w_{t+1}-C_t) + g_{2,t}(R^w_{t+1}-C_t)^2 + g_{3,t}(R^w_{t+1}-C_t)^3, \end{aligned} \tag{3}$$

where $g_{n,t} = [U^{(n+1)}(1+C_t)/U'(1)][1/n!] = [U^{(n+1)}(1+C_t)/U'(1+C_t)n!][U'(1+C_t)/U'(1)]$.

Further, we assume that both excess return of ith asset R^i_{t+1} as well as simple net return on wealth R^w_{t+1} can be decomposed to elements consisting of short-term and long-term fluctuations as

$$R_{t+1} \equiv \sum_{j=1}^{N} R^{(j)}_{t+1} + R^{(\infty)}_{t+1} = R^{(short)}_{t+1} + R^{(long)}_{t+1}, \tag{4}$$

where $R^{(short)}_{t+1} = \sum_{j=1}^{J} R^{(j)}_{t+1}$, and $R^{(long)}_{t+1} = R^{(\infty)}_{t+1} = R^{(j>J)}_{t+1}$, and choice of J depends on the economic meaning of short-term and long-term fluctuations.[11]

The following proposition establishes how the prices are linked with frequency dependent higher moments.

Proposition 1 (Pricing of Frequency-Dependent Higher Moments)**.** *Assume an ith asset return R^i_{t+1} and return on wealth R^w_{t+1} can be decomposed to their short-term and long-term fluctuations that satisfy Eq. (4), and the pricing kernel M_{t+1} that satisfies Eq. (3). Further assume orthogonality of components* $\left(\epsilon^{(s)}_{t+1}\right)^n \left(\epsilon^{(l)}_{t+1}\right)^n = \left(\epsilon^{(s)}_{t+1}\right)^n R^{(l)}_{t+1} = \left(\epsilon^{(l)}_{t+1}\right)^n R^{(s)}_{t+1} = 0$, *for* $n \in \{1,2,3\}$.

[10] The restriction we place on the utility function $U(W_{t+1})$ is that its derivatives, $U^{(n)}(W_{t+1})$ for $n \in \{1,2,3,4\}$, exist, and are finite.

[11] Note that due to the equivalence in Equation (4), the decomposition is not restricted to two horizons. In fact, we are able to construct components from arbitrary number of horizons by splitting the sum in intermediate points.

Excess returns of asset i then conform

$$E_t\left(R^e_{t+1,i}\right) = \sum_{h\in\{s,l\}} \theta^{(h)}_{1,t,i} E_t\left[\left(\epsilon^{(h)}_{t+1}\right)^2\right] + \sum_{h\in\{s,l\}} \theta^{(h)}_{2,t,i} E_t\left[\left(\epsilon^{(h)}_{t+1}\right)^3\right] + \sum_{h\in\{s,l\}} \theta^{(h)}_{3,t,i} E_t\left[\left(\epsilon^{(h)}_{t+1}\right)^4\right], \tag{5}$$

where $E_t\left[(\epsilon_{t+1})^n\right]$ *denotes n-th centralized moment of returns on aggregate wealth,* $\epsilon^{(s)}_{t+1}$ *consists of the short-run components of returns on aggregate wealth, and* $\epsilon^{(l)}_{t+1}$ *consists of the long-run components of returns on aggregate wealth.*

Proof. See Appendix A. □

Proposition 1 allows us to build a model valid for the whole set of applicable utility functions while maintaining the interpretation of individual coefficients in the model, i.e. it allows us to disentangle and quantify the horizon-specific components of investors' risk tastes (risk aversion, absolute risk aversion, absolute prudence). Heterogeneity of risk tastes across different horizons may be a key step towards explaining the cross-section of stock (and other assets) returns.

Finally, the Equation (5) can be expressed as (see Appendix A for detailed discussion)

$$E_t(R^e_{t+1,i}) = \sum_{h\in\{s,l\}} \beta^{(h)}_{t,i} \sqrt{v^{(h)}_t} + \sum_{h\in\{s,l\}} \delta^{(h)}_{t,i} s^{(h)}_t + \sum_{h\in\{s,l\}} \mathcal{K}^{(h)}_{t,i} k^{(h)}_t, \tag{6}$$

where $\sqrt{v^{(h)}_t}$, $s^{(h)}_t$ and $k^{(h)}_t$ are short- and long-run components of volatility, skewness and kurtosis of returns on aggregate wealth, respectivelly. The connection of the respective coefficients is detailed in the Appendix A.

2.3 Computing the Horizon-Specific Higher Moments from High Frequency Data

The discussion assumes that higher moments evolve dynamically, but at the same time we need to realize that higher moments are generally hard to measure. In this subsection, we provide brief summary of high-frequency based estimation of higher moment risk measures that we plug into the model. We rely on recent advances in high-frequency econometrics to measure the realized volatility, realized skewness and realized kurtosis and then decompose their fluctuations to transitory and persistent parts so we define horizon-specific higher moments. Here, we also briefly discuss the high-frequency data that we use in our main empirical investigations.

2.4 Realized moments

The daily realized variance (RDV), realized skewness (RDS), and realized kurtosis (RDK) representing the second, third, and fourth moment of daily returns distribution can be computed from 5-minute prices. Using already well-known arguments of Andersen et al. (2001, 2003) realized variance can be constructed as sum of the squared high-frequency intraday returns as

$$RDV_t = \sum_{j=1}^{K} r_{t,j}^2, \tag{7}$$

where $r_{t,j} = p_{t,j/K} - p_{t,(j-1)/K}$ with $p_{t,j/K}$ denoting a natural logarithm of j-th intraday price at day t. We use five-minute returns so that in 6.5 trading hours we have $K = 78$ intraday returns. Realized Volatility (RDVOL) is computed as $RDVOL_t = \sqrt{RDV_t}$.

Since we are mainly interested in measuring asymmetry and higher order moments of the daily return's distribution, we construct a measure of ex-post realized skewness based on intraday returns standardized by the realized variance following Amaya et al. (2015) as

$$RDS_t = \frac{\sqrt{N} \sum_{j=1}^{K} r_{t,j}^3}{RDV_t^{3/2}}. \tag{8}$$

The negative values of the realized skewness indicate that stock's return distribution has a left tail that is fatter than the right tail, and positive values indicate the opposite. In addition, extremes of the return distribution can be captured by realized kurtosis

$$RDK_t = \frac{N \sum_{j=1}^{K} r_{t,j}^4}{RV_t^2}. \tag{9}$$

Note that as discussed by Amaya et al. (2015), with increasing sampling frequency K realized skewness in the limit separates the jump contribution from the continuous contribution to cubic variation and it captures mainly jump part. This feature is important to note since the measure does not capture leverage effect arising from correlation between return and variance innovations. Hence assets with positive jumps on average will have a positive realized third moment and vice versa. Hence higher moments measured by high frequency data are likely to contain different information from those computed from daily data or options (see Amaya et al. (2015) for rigorous discussion).

2.4.1 Decomposition of the Realized Moments

The realized higher-order moments exhibit strong time series dynamics Amaya et al. (2015) and may thus have unexplored transitory and persistent components that create heterogeneous types of risks matching our theoretical expectation. To explore such risk, we decompose the realized measures to their horizon-specific components.

Assume that a realized moment $RDM_t \in \{RDVOL_t, RDS_t, RDK_t\}$ has two orthogonal components capturing economic cycles shorter than 2^j periods (for example months) and longer than 2^j periods for $j \geq 1$. These represent the short-run and long-run components capturing transitory and permanent information contained in the higher moments respectively. As discussed earlier, we can decompose our risk factors into components operating over the short and the long horizons as

$$RDM_t = RDM_t^{(s)} + RDM_t^{(l)}, \quad (10)$$

$$= RDM_t^{<2^j} + RDM_t^{>2^j},, \quad (11)$$

where $RDM_t^{(s)}$ denotes short-run realized moment computed as a sum of the corresponding elements up to j, and $RDM_t^{(l)}$ denotes long-run realized moment consisting of the elements larger than j.

2.5 Idiosyncratic and Market Moments

Another important aspect of the discussion is the type of moment based risk we use in the analysis. A traditional view in the literature is that idiosyncratic moment risks can be diversified away, and only systematic components of moments should be rewarded (Harvey and Siddique, 2000). However, enormous literature emphasizes the ability of idiosyncratic risks to predict subsequent returns. Recently, Jondeau et al. (2019) document that average of monthly skewness across firms predicts future market returns, and they argue that systematic market skewness is not the main channel by which investor's preferences for skewness affect future market return. In addition, Langlois (2020) documents that systematic and idiosyncratic skewness are connected with different expected returns across stocks. While stocks with higher systematic skewness are appealing because they offer defensive returns during bad times, stocks with positive idiosyncratic skewness attract investors seeking high returns regardless of broad market movements, and are connected to a lottery-like payoff.

Empirical ability of idiosyncratic skewness risk to predict the cross-section of returns has been recognized across many different measures of skewness, specifically option implied measures (Conrad et al., 2013), realized measures computed from high-frequency data (Amaya et al., 2015), or idiosyncratic skewness forecasted by a time series model (Boyer et al., 2009). These findings indicate that investors are willing to accept low returns and high volatility if they are compensated by positive skewness. Such phenomenon is closely connected to deliberate underdiversification (Simkowitz and Beedles, 1978; Mitton and Vorkink, 2007) that is driven by "lotto investors" demanding assets with high upside potential. Moreover, preference of investors over "lottery-like" assets is connected to strong predictive power of maximum past returns (Bali et al., 2011), and it plays a central role in explaining the idiosyncratic volatility puzzle (Hou and Loh, 2016).[12]

Generally, there is evidence that idiosyncratic higher moments can help explaining multiple financial market puzzles. We contribute to this debate by assessing the role of both market and average idiosyncratic moments with respect to their short-term as well as long-term fluctuations. In other words, we investigate how the short-term and long-term fluctuations of average idiosyncratic and market higher moments are priced in the cross-section of assets.

This discussion leads to the final model where both market as well as average idiosyncratic moments are combined. Using the results of Jondeau et al. (2019), both types of the risks can directly enter our pricing model based on the assumption that a stock return is a linear combination of market return and the idiosyncratic (unexplained) part. We add the

[12]Idiosyncratic volatility puzzle is a phenomenon observed by Ang et al. (2006), who document a negative relationship between idiosyncratic volatility and returns. This is very puzzling as investors should require positive risk premia, if any, for idiosyncratic volatility. However, high idiosyncratic volatility indicates possible high future exposure to idiosyncratic skewness (Boyer et al., 2009). Preference for right skewed assets along with market frictions holds a prominent place amongst explanations of idiosyncratic volatility puzzle (Hou and Loh, 2016).

idiosyncratic source of risk to our model in Equation (6) as[13]

$$E_t(R^e_{t+1,i}) = \sum_{h\in\{s,l\}} \beta^{(h,m)}_{t,i}\sqrt{v^{(h,m)}_t} + \sum_{h\in\{s,l\}} \delta^{(h,m)}_{t,i} s^{(h,m)}_t + \sum_{h\in\{s,l\}} \mathcal{K}^{(h,m)}_{t,i} k^{(h,m)}_t + \sum_{h\in\{s,l\}} \beta^{(h,I)}_{t,i}\sqrt{v^{(h,I)}_t} + \sum_{h\in\{s,l\}} \delta^{(h,I)}_{t,i} s^{(h,I)}_t + \sum_{h\in\{s,l\}} \mathcal{K}^{(h,I)}_{t,i} k^{(h,I)}_t, \tag{12}$$

where the upper case I indicates idiosyncratic risk, and the lower case m indicates market risk. Thus $M^{(h,I)}_t \in \{\sqrt{v^{(h,I)}_t}, s^{(h,I)}_t, k^{(h,I)}_t\}$, $h \in \{s,l\}$, stands for the horizon-specific component of the corresponding idiosyncratic moment, and $M^{(h,m)}_t \in \{\sqrt{v^{(h,m)}_t}, s^{(h,m)}_t, k^{(h,m)}_t\}$, $h \in \{s,l\}$, stands for the horizon-specific component of the corresponding market moment. Analogously, $\phi^{(h,I)}_{t,i} \in \{\beta^{(h,I)}_{t,i}, \delta^{(h,I)}_{t,i}, \mathcal{K}^{(h,I)}_{t,i}\}$, $h \in \{s,l\}$, denotes the coefficient associated with the corresponding horizon-specific idiosyncratic risk factor, and $\phi^{(h,m)}_{t,i} \in \{\beta^{(h,m)}_{t,i}, \delta^{(h,m)}_{t,i}, \mathcal{K}^{(h,m)}_{t,i}\}$, $h \in \{s,l\}$, denotes the coefficient associated with the corresponding horizon-specific market risk factor.

Estimation of such model is possible mainly due to use of high-frequency data allowing computation of the realized moments. Specifically, we proxy the aggregate representations of the three moments $\{\sqrt{v^{(I)}_t}, s^{(I)}_t, k^{(I)}_t\}$ with an average idiosyncratic higher moment at time

$$RDM^{(I)}_t = \frac{1}{N}\sum_{i=1}^{N} RDM_{t,i}, \tag{13}$$

where $RDM_{t,i} \in \{RDVOL_{t,i}, RDS_{t,i}, RDK_{t,i}\}$. The short-term and long-term components of the three moments $\{\sqrt{v^{(h,I)}_t}, s^{(h,I)}_t, k^{(h,I)}_t\}$, $h \in \{s,l\}$, are proxied with an average horizon-specific idiosyncratic higher moment at time t

$$RDM^{(h,I)}_t = \frac{1}{N}\sum_{i=1}^{N} RDM^{(h)}_{t,i}, \tag{14}$$

where $h \in \{s,l\}$ and $RDM^{(h)}_{t,i} \in \{RDVOL^{(s)}_{t,i}, RDVOL^{(l)}_{t,i}, RDS^{(s)}_{t,i}, RDS^{(l)}_{t,i}, RDK^{(s)}_{t,i}, RDK^{(l)}_{t,i}\}$. In addition to average idiosyncratic moments, we use the S&P 500 index returns for measurement of market higher moments $\{\sqrt{v^{(m)}_t}, s^{(m)}_t, k^{(m)}_t, \sqrt{v^{(h,m)}_t}, s^{(h,m)}_t, k^{(h,m)}_t\}$, $h \in \{s,l\}$. In

[13] Note that derivation of this model follows directly from the arguments in Jondeau et al. (2019) who assume simply an asset return to have market as well as purely idiosyncratic component creating a linear combination of these two sources of risk.

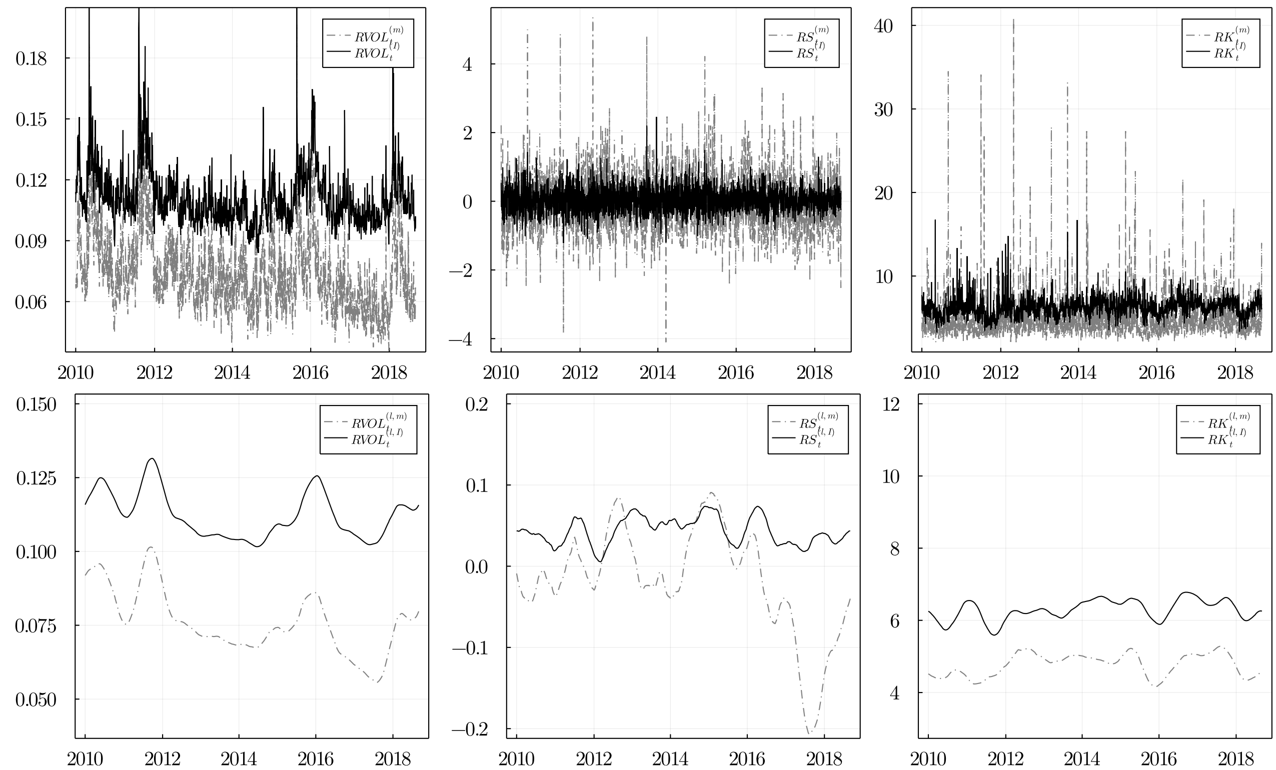

Figure 1. Daily realized measures for cross-section of stocks

The top row depicts market (dashed) and average idiosyncratic (black line) realized measures of the cross-section of daily stock returns. The bottom row depicts the long-term components that capture fluctuations of all measures longer than half a year.

our cross-sectional analysis, we also use weekly returns and we construct corresponding weekly realized measures by summing the corresponding daily realized variance, skewness and kurtosis over the week. Specifically, assuming t is Monday, we compute the weekly realized measures as $RVOL_t = \frac{252}{5}\left(\sum_{i=0}^{4} RDV_{t-i}\right)^{1/2}$, $RS_t = \frac{1}{5}\sum_{i=0}^{4} RDS_{t-i}$, $RK_t = \frac{1}{5}\sum_{i=0}^{4} RDK_{t-i}$, where we annualize realized volatility as standard in the literature to facilitate the interpretation of the results.

2.6 Data

Our empirical analysis focuses on two main datasets. Firstly the U.S. stocks, and secondly the exchange-traded funds (ETFs) that recently grew in popularity since they provide an easy and cheap form of diversification. The main reason we chose the ETF data in addition to the usually studied U.S. equities is that they cover variety of different securities in terms of asset classes, industrial sectors, market cap, and most importantly country of origin. Including the ETFs ensure robustness of our estimation to the "academic home bias

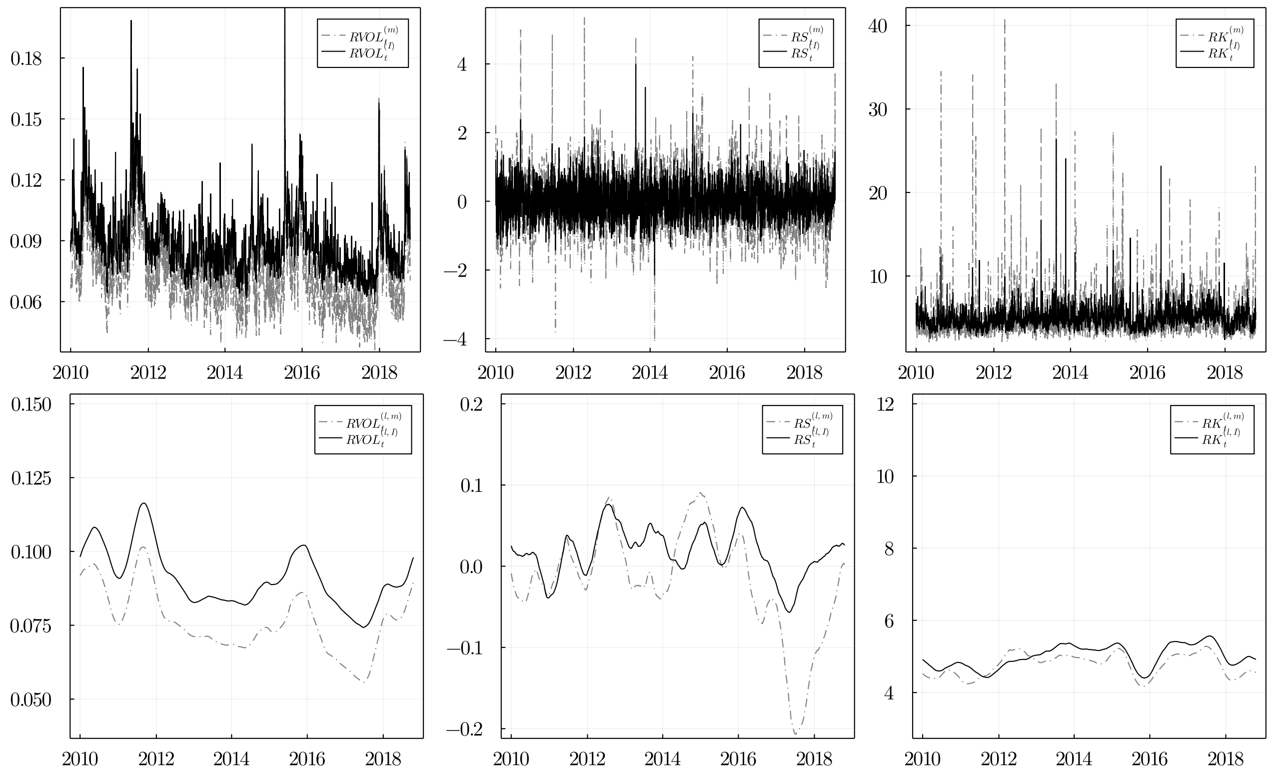

Figure 2. Daily realized measures for cross-section of ETFs

The top row depicts market (dashed) and average idiosyncratic (black line) realized measures of the cross-section of daily ETF returns. The bottom row depicts the long-term components that capture fluctuations of all measures longer than half a year.

puzzle".

We analyze high-frequency intraday data about all companies listed on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and NASDAQ that have been constituents of the S&P 500 index over the period from January 1998 until August 2018. We also analyze Exchange Traded Funds traded on NYSE ARCA during the same period.[14] We record prices every five minutes starting 9:30 EST and construct five-minute log-returns for the period 9:30 EST to 16:00 EST for a total of 78 intraday returns. We construct the five- minute grid by using the last recorded price within the preceding five-minute period, and we consider excess returns as required by our empirical model outlined later in the text. A "coarse" five-minute sampling scheme aims to balance the bias induced by market microstructure effects and mirrors common practice in the literature.

After data cleaning, we are left with the cross-section of 367 stocks and 100 ETFs cover-

[14]The data were obtained from Tick Data and Quant Quote databases.

ing the period of 01/2010 to 11/2018.[15] The daily (weekly) return of an asset is constructed from dividends and splits adjusted prices as an excess return of a logarithmic difference between opening and closing price for a given period. We use the 3-month Treasury Bill rate as the risk-free rate.

Note that our sample starts after the year 2010 which since we are using intraday data to compute the measures, data after year 2010 are especially interesting after the introduction of Globex platform and millisecond time stamps recording. Moreover, there is a recent discussion in the literature pointing to substantial time-variation in the risk premia connected to various risk factors (Boons et al., 2020; Barroso et al., 2021). Selected sample period affects the magnitude of effects displayed by the individual risk factors, and the sign of these effects may change in different subsamples as well. Most papers studying the effects of moment based risks using high-frequency data use the sample period 1993-2013 (Amaya et al., 2015; Bollerslev et al., 2020). In light of the documented time-variation in the state risk variables and the highly time-conditional nature of the risk-return relationships on financial markets, we believe that it is important to use contemporary high-frequency data to explain the recent fluctuations in asset returns.

Figures 1 and 2 show all the computed realized moments for stocks and ETFs respectively. Specifically, the first row of Figure 1 contrasts market and average idiosyncratic realized volatility, realized skewness and realized kurtosis of stock returns. It is visible that the measures of average idiosyncratic realized volatility and realized kurtosis are larger in magnitude than the market ones, and that market realized skewness shows larger fluctuations than average idiosyncratic realized skewness. The bottom row comparing long-term fluctuations of the market and average idiosyncratic measures shows similar dynamics of these two sources of volatility and kurtosis risk, while interestingly different dynamics of the skewness risks. While market skewness fluctuates around zero with a large negative spike during 2017, average idiosyncratic skewness is positive for the whole period.

In contrast to the stock returns' moments, ETFs returns' moments depicted in Figure 2 show less significant discrepancies between average idiosyncratic and market moments.[16]

[15] The dataset ends after 08/2018 in case of stocks.

[16] Note that the market returns are approximated by the returns of the S&P 500 index for both stocks and ETFs data. Hence, the lines representing the market measures in Figure 1 and Figure 2 are identical.

Specifically, average idiosyncratic volatility and average idiosyncratic kurtosis are only slightly larger in comparison to corresponding market measures. While skewness shows similar overall dynamics, its long-run component is more connected to the market long-run component, although again stays in positive values most of the time. Table 1 further reveals

Table 1. Correlations: Stocks

This table provides the correlation matrix for the realized market and average idiosyncratic realized moments in the Panel A, correlation matrix for the frequency-decomposed realized market and average idiosyncratic moments in the Panel B, and Panel C.

Panel A	$RVOL_t^{(m)}$	$RS_t^{(m)}$	$RK_t^{(m)}$	$RVOL_t^{(I)}$	$RS_t^{(I)}$	$RK_t^{(I)}$
$RVOL_t^{(m)}$						
$RS_t^{(m)}$	-0.045					
$RK_t^{(m)}$	-0.037	0.168				
$RVOL_t^{(I)}$	0.921	-0.023	-0.084			
$RS_t^{(I)}$	-0.035	0.618	0.123	-0.009		
$RK_t^{(I)}$	-0.389	0.188	0.293	-0.25	0.199	

Panel B	$RVOL_t^{(s,m)}$	$RVOL_t^{(l,m)}$	$RS_t^{(s,m)}$	$RS_t^{(l,m)}$	$RK_t^{(s,m)}$	$RK_t^{(l,m)}$
$RVOL_t^{(s,m)}$						
$RVOL_t^{(l,m)}$	0.11					
$RS_t^{(s,m)}$	-0.071	-0.003				
$RS_t^{(l,m)}$	0.023	0.314	0.026			
$RK_t^{(s,m)}$	0.02	-0.015	0.171	0.006		
$RK_t^{(l,m)}$	-0.062	-0.703	0.004	-0.048	0.045	

Panel C	$RVOL_t^{(s,m)}$	$RVOL_t^{(l,m)}$	$RS_t^{(s,m)}$	$RS_t^{(l,m)}$	$RK_t^{(s,m)}$	$RK_t^{(l,m)}$	$RVOL_t^{(s,I)}$	$RVOL_t^{(l,I)}$	$RS_t^{(s,I)}$	$RS_t^{(l,I)}$	$RK_t^{(s,I)}$
$RVOL_t^{(s,I)}$	0.905	0.148	-0.036	0.021	-0.032	-0.091					
$RVOL_t^{(l,I)}$	0.125	0.92	-0.002	0.143	-0.018	-0.748	0.156				
$RS_t^{(s,I)}$	-0.049	0.004	0.62	0.014	0.124	0.002	-0.015	0.005			
$RS_t^{(l,I)}$	0.053	0.023	0.022	0.493	0.007	0.046	0.073	-0.083	0.048		
$RK_t^{(s,I)}$	-0.289	-0.079	0.205	-0.016	0.285	0.055	-0.116	-0.079	0.216	-0.027	
$RK_t^{(l,I)}$	-0.178	-0.829	-0.002	-0.119	0.026	0.598	-0.188	-0.805	-0.005	-0.035	0.138

that market realized volatility is strongly correlated with average idiosyncratic volatility while the relation is weaker for the realized skewness and almost disappears in case of kurtosis.[17] This suggests that the average idiosyncratic and market parts of the third and fourth moments carry different information. More importantly, Panel B shows that short-run and long-run components of market higher moments are generally uncorrelated except strong negative correlation between long-run components of market volatility and kurtosis. Long-run components of market volatility and skewness display weak correlation of 0.314.

The pattern is not very different in the right part of Panel C where correlation matrix of horizon-specific components of average idiosyncratic moments is displayed. While most of the terms are not correlated, long-run components of average idiosyncratic volatil-

[17] Note the correlation matrix for ETFs is very similar and we report it in the Appendix D

ity and kurtosis show strong negative correlation. Finally, left part of Panel C displaying the cross-correlations between the horizon-specific components of market and average idiosyncratic moments confirms the previous findings. The values on the diagonal document that correlation between the corresponding components of the same moment is strongest in case of volatility, and becomes weaker in case of skewness and kurtosis. Moreover, we observe strong negative correlation between long-run components of average idiosyncratic volatility and market kurtosis as well as between the long-run components of average idiosyncratic kurtosis and market volatility.

This preliminary analysis documents that various types of the risks and information are hidden in the transitory and persistent components of higher moments. We will explore these in the next sections.

3 Frequency-Dependent Higher Moments and Future Returns

The previous discussion implies that individual assets exhibit different exposures to short-run and long-run components of higher moment risk, and that there are two relevant sources of such risk; market and idiosyncratic. Subsequently, different exposures to the corresponding risk factors yield different returns on average. In this section, we test the effects of exposure to each risk factor unconditionally using portfolio sorting exercises. We divide the factors to four "groups"; aggregate market factors, horizon-specific market factors, aggregate idiosyncratic factors, and horizon-specific idiosyncratic factors. When estimating the exposures used for sorting the portfolios we control for remaining factors from the corresponding group. The sorting procedure is described in detail in Appendix C.

It is not straightforward to connect our results to existing empirical evidence due to presence of different specifications of higher moments in the literature, although the existing empirical results provide us with certain guidance in terms of signing the individual relationships. Some researchers sort the portfolios directly based on the measures of idiosyncratic higher moments (Bollerslev et al., 2020; Amaya et al., 2015), some sort based on the exposures to innovations in market higher moments (Ang et al., 2006; Chang et al., 2013), we sort based on the exposures to market and average idiosyncratic higher moments.

Table 2. Cross-section of Stocks

We are sorting on the factor loadings of corresponding variables. The length of the rolling window is 3 months, the post-ranking returns are recorded for the next week. The complete sorting procedure is described in Apendix C. We report the average post-ranking raw returns (in bps) for each portfolio, the t-statistics are reported in parentheses. The last column displays the difference in returns of the highest quintile portfolio, and the lowest quintile portfolio, hence corresponds to the strategy of buying high exposure assets and selling low exposure assets.

Variable	1	2	3	4	5	High - Low
			Market			
$RVOL_t^{(m)}$	0.29	0.99	1.38	1.39	-0.08	-0.38
	(0.16)	(0.65)	(1)	(1.05)	(-0.06)	(-0.33)
$RS_t^{(m)}$	2.21	2.01	1.33	0.09	-1.71	-3.92
	(2)	(1.61)	(0.91)	(0.05)	(-0.8)	(-2.52)
$RK_t^{(m)}$	0.76	1.46	1.06	0.85	-0.17	-0.93
	(0.49)	(1.07)	(0.77)	(0.58)	(-0.09)	(-0.88)
$RVOL_t^{(s,m)}$	0.58	1.06	1.63	1.27	-0.57	-1.15
	(0.32)	(0.7)	(1.15)	(0.97)	(-0.39)	(-1.09)
$RVOL_t^{(l,m)}$	0.38	0.94	1.27	1.14	0.24	-0.14
	(0.23)	(0.66)	(0.9)	(0.82)	(0.15)	(-0.16)
$RS_t^{(s,m)}$	2.22	2.33	0.82	0.43	-1.88	-4.1
	(1.99)	(1.86)	(0.57)	(0.26)	(-0.88)	(-2.67)
$RS_t^{(l,m)}$	0.28	1.44	1.35	0.52	0.37	0.09
	(0.17)	(1.03)	(0.98)	(0.36)	(0.23)	(0.1)
$RK_t^{(s,m)}$	0.47	1.75	1.21	0.75	-0.21	-0.68
	(0.3)	(1.28)	(0.88)	(0.51)	(-0.12)	(-0.66)
$RK_t^{(l,m)}$	0.31	0.8	1.26	1.29	0.32	0.01
	(0.19)	(0.55)	(0.89)	(0.93)	(0.2)	(0.01)
			Idiosyncratic			
$RVOL_t^{(I)}$	0.22	1.4	1.57	1.18	-0.41	-0.64
	(0.13)	(0.93)	(1.13)	(0.88)	(-0.28)	(-0.66)
$RS_t^{(I)}$	2.06	2.22	0.96	0.24	-1.55	-3.61
	(1.79)	(1.77)	(0.66)	(0.14)	(-0.74)	(-2.36)
$RK_t^{(I)}$	1.49	1.15	1.31	0.81	-0.8	-2.29
	(1.03)	(0.86)	(0.95)	(0.52)	(-0.43)	(-1.96)
$RVOL_t^{(s,I)}$	0.44	0.91	2.03	1.26	-0.67	-1.1
	(0.25)	(0.63)	(1.46)	(0.92)	(-0.43)	(-1.16)
$RVOL_t^{(l,I)}$	0.68	1.06	1.37	0.91	-0.05	-0.73
	(0.4)	(0.75)	(0.98)	(0.66)	(-0.03)	(-0.81)
$RS_t^{(s,I)}$	2.09	2.02	1.19	0.27	-1.64	-3.72
	(1.81)	(1.61)	(0.81)	(0.17)	(-0.78)	(-2.46)
$RS_t^{(l,I)}$	0.17	0.94	1.24	1.24	0.39	0.21
	(0.1)	(0.67)	(0.91)	(0.89)	(0.23)	(0.23)
$RK_t^{(s,I)}$	1.4	1.48	1.13	0.87	-0.92	-2.32
	(0.97)	(1.1)	(0.81)	(0.57)	(-0.5)	(-2.01)
$RK_t^{(l,I)}$	1.19	1.23	1.37	0.87	-0.7	-1.89
	(0.71)	(0.87)	(0.98)	(0.61)	(-0.44)	(-2.01)

3.1 Sorts of Stock Portoflios

In this subsection we report the results of the portfolio sorting exercise applied on the stocks dataset. We are using daily returns and relatively short moving windows to capture the dynamics of the coefficients. We consider 3-months as the minimum window length to obtain precise estimates of the factor loadings. For the sake of robustness, we report results for window length of 3 months, and 6 months. We consider post-ranking returns to avoid any spurious effects (Harvey and Siddique, 2000; Ang et al., 2006; Agarwal et al., 2009; Chang et al., 2013).

Table 2 displays the results of sorts using rolling window length of 3 months, Table A2 in Appendix D displays the results of sorts using rolling window length of 6 months. Both Tables indicate a negative relationship between exposure to skewness and subsequent stock returns. The relationship is valid for market skewness as well as average idiosyncratic skewness. Exposure to market skewness displays a High-Low spread of -3.92 with a significant t-statistic of -2.52 in Table 2, and -3.31 with a significant t-statistics of -1.97 in Table A2. Exposure to average idiosyncratic skewness exhibits a High-Low spread that is slightly lower in magnitude; -3.61 and -2.78 respectively, with t-statistics of -2.36 and -1.62. Negative risk premia accepted for assets with higher exposure to skewness, and the dominant role of skewness among the three studied moments are in line with previous research (Ang et al., 2006; Agarwal et al., 2009; Chang et al., 2013; Amaya et al., 2015; Bollerslev et al., 2020). Moreover, the results in this section confirm that average idiosyncratic skewness is also priced in the cross-section of stocks (Jondeau et al., 2019), and that strategies based on exposure to both sources of skewness yield similar excess returns.

Decomposition into the frequency-specific components allows us to observe the term structure of skewness risk. The literature develops myriad of long-run risk models stressing the importance of the long-run volatility component in the decision making of investors (e.g., Bidder and Dew-Becker, 2016), however the empirical evidence suggests that the long-run component itself is not able to completely capture the dynamics of asset returns. To our knowledge, the literature does not provide any theoretical or empirical guidance towards the term structure of skewness risk. Table 2 and Table A2 indicate the short-run component of skewness risk as the dominant source of the skewness risk premia present in the cross-section of stock returns. The portfolios sorted based on exposure to the short-run component of market skewness display a High-Low spread of -4.1 (3-month rolling window), and -3.43 (6-month rolling window), with significant t-statistics of -2.57 and -2.06 respectively. On the contrary, the High-Low spread for portfolios sorted based on exposure to the long-run component of market skewness is not significantly different from 0 for either length of the rolling window. Portfolios sorted on horizon-specific components of average idiosyncratic skewness display qualitatively the same results in Table 2.

The evidence of downward-sloping term structure of average idiosyncratic skewness risk is weaker in Table A2 where portfolios sorted based on the long-run component of average idiosyncratic skewness display a High-Low spread of 2.79 with a significant t-statistic of 2.97. High-Low spread corresponding to the short-run component of average idiosyncratic skewness is -2.81 with a t-statistic of -1.65. However, positive risk-premia required for holding assets with high exposure to the long-run component of average idiosyncratic skewness do not correspond to any other results regarding exposure to skewness we obtain. It should be noted that all the results in this exercise are computed separately for market and idiosyncratic risk, hence we cannot conclude whether these elements are jointly relevant for investors. We take a closer look at the interrelations inside the pricing kernel in Section 4.

The relationship between exposure to skewness and subsequent average stock returns predominantly propagated through the short-run component of skewness is documented for both lengths of the rolling window, and it is stronger in case of market skewness. On the other hand, we fail to document any consistent relationship between exposure to volatility or kurtosis and subsequent stock returns on the aggregate level, while we document a negative relationship between exposure to the short-run component of average idiosyncratic kurtosis and subsequent stock returns both in Table 2 and Table A2.

3.2 Sorts of ETFs Portoflios

In this subsection we report the results of the portfolio sorting exercise applied on the ETFs dataset. Table 3 displays the results of sorts using rolling window length of 3 months, Table A3 in Appendix D displays the results of sorts using rolling window length of 6 months. Consistently with results from the previous subsection, high exposure to

Table 3. Cross-section of ETFs

We are sorting based on the factor loadings of corresponding variables. The length of the rolling window is 3 months, the post-ranking returns are recorded for the next week. The complete sorting procedure is described in Apendix C. We report the average post-ranking raw returns (in bps) for each portfolio, the t-statistics are reported in parentheses. The last column displays the difference in returns of the highest quintile portfolio, and the lowest quintile portfolio, hence corresponds to the strategy of buying high exposure assets and selling low exposure assets.

Variable	1	2	3	4	5	High - Low
Market						
$RVOL_t^{(m)}$	-1.22	-0.93	-0.53	-0.72	-1.18	0.03
	(-0.69)	(-0.63)	(-0.38)	(-0.52)	(-0.81)	(0.03)
$RS_t^{(m)}$	0.11	-0.89	-0.41	-0.64	-2.76	-2.87
	(0.09)	(-0.65)	(-0.28)	(-0.4)	(-1.47)	(-2.67)
$RK_t^{(m)}$	-1.39	-0.86	-0.09	-0.35	-1.89	-0.5
	(-0.9)	(-0.61)	(-0.06)	(-0.24)	(-1.14)	(-0.52)
$RVOL_t^{(s,m)}$	-1.43	-0.17	-0.64	-0.88	-1.45	-0.03
	(-0.83)	(-0.12)	(-0.45)	(-0.64)	(-0.97)	(-0.03)
$RVOL_t^{(l,m)}$	-0.87	-0.01	-0.45	-0.77	-2.47	-1.6
	(-0.56)	(-0.01)	(-0.32)	(-0.53)	(-1.53)	(-1.8)
$RS_t^{(s,m)}$	0.12	-0.83	-0.57	-0.68	-2.61	-2.73
	(0.1)	(-0.61)	(-0.39)	(-0.43)	(-1.4)	(-2.57)
$RS_t^{(l,m)}$	-1.33	-0.5	0.03	-0.6	-2.17	-0.84
	(-0.84)	(-0.35)	(0.02)	(-0.41)	(-1.36)	(-0.97)
$RK_t^{(s,m)}$	-1.12	-0.88	-0.3	-0.34	-1.94	-0.83
	(-0.71)	(-0.63)	(-0.21)	(-0.23)	(-1.15)	(-0.83)
$RK_t^{(l,m)}$	-1.12	-0.43	-0.62	-0.65	-1.76	-0.64
	(-0.7)	(-0.29)	(-0.43)	(-0.45)	(-1.16)	(-0.77)
Idiosyncratic						
$RVOL_t^{(I)}$	-1	-0.37	-0.61	-0.56	-2.03	-1.03
	(-0.58)	(-0.25)	(-0.43)	(-0.41)	(-1.38)	(-1.1)
$RS_t^{(I)}$	-0.06	0.01	-0.56	-1.68	-2.29	-2.23
	(-0.05)	(0.01)	(-0.38)	(-1.05)	(-1.2)	(-2.02)
$RK_t^{(I)}$	-0.81	-0.55	-0.29	-0.57	-2.35	-1.53
	(-0.53)	(-0.38)	(-0.21)	(-0.39)	(-1.42)	(-1.63)
$RVOL_t^{(s,I)}$	-1.11	-0.39	-0.72	-0.65	-1.71	-0.61
	(-0.64)	(-0.26)	(-0.5)	(-0.47)	(-1.14)	(-0.63)
$RVOL_t^{(l,I)}$	-0.63	-0.29	-0.59	-1.04	-2.02	-1.39
	(-0.39)	(-0.2)	(-0.42)	(-0.71)	(-1.3)	(-1.57)
$RS_t^{(s,I)}$	-0.01	0.01	-0.87	-1.45	-2.25	-2.24
	(-0.01)	(0)	(-0.59)	(-0.9)	(-1.18)	(-2.01)
$RS_t^{(l,I)}$	-1.98	-0.85	-0.25	-0.59	-0.91	1.07
	(-1.24)	(-0.59)	(-0.18)	(-0.41)	(-0.57)	(1.23)
$RK_t^{(s,I)}$	-1.07	-0.32	-0.36	-0.59	-2.24	-1.17
	(-0.7)	(-0.22)	(-0.25)	(-0.4)	(-1.36)	(-1.26)
$RK_t^{(l,I)}$	-0.4	-0.17	-0.62	-1.25	-2.13	-1.73
	(-0.25)	(-0.12)	(-0.43)	(-0.86)	(-1.36)	(-2.01)

both sources of skewness risk results in significantly lower subsequent average returns. The High-Low spreads corresponding to exposure to market skewness risk are -2.87 and -2.8 with significant t-statistics of -2.67 and -2.52 respectively, the High-Low spreads corresponding to exposure to average idiosyncratic skewness risk are -2.23 and -2.42 with significant t-statistics of -2.02 and -2.05 respectively. We detect no robust evidence that other factors apart from skewness are priced in the cross-section of ETFs returns except for the long-run component of average idiosyncratic kurtosis. Portfolios sorted based on exposure to the long-run component of average idiosyncratic kurtosis display High-Low spreads of -1.73 (3-month rolling window) and -3.32 (6-month rolling window) both with significant t-statistics.

The results in Tables 3 and A3 provide further evidence supporting the downward-sloping term structure of skewness risk. High-Low spreads associated with the portfolios sorted based on exposure to short-run component of market skewness are -2.73 and -3.01 with significant t-statistics of -2.57 and -2.75 respectively. High-Low spreads associated with the portfolios sorted based on exposure to short-run component of average idiosyncratic skewness are -2.24 and -2.6 with significant t-statistics of -2.01 and -2.2 respectively. There is no evidence that the long-run component of market or average idiosyncratic skewness is priced in the ETF returns.

4 Cross-Sectional Regressions

Portfolio sorting exercises consistently indicate that skewness is a dominant factor in pricing the stock and ETF returns, and it identifies average idiosyncratic skewness risk as an important factor in excess to market skewness risk. Most importantly, we observe downward-sloping term structure of skewness risk denoting that the transitory components of skewness risk play a major role in pricing the stock and ETF returns. Generally, portfolio sorts reveal the returns of investment strategies that form portfolios based on exposures to different sources of risk. While this illustrates the ability to make profit by trading based on the underlying risk factors, portfolio sorts ignore potentially important information by aggregating the stocks into quintile portfolios and consider market and average idiosyncratic moments separately. We expand these results using cross-sectional regressions that simultaneously control for multiple variables enabling us to inspect the implications of the interrelations between individual components of Equation (12). To understand how the risk factors affect the returns jointly, we evaluate the ability of frequency-dependent higher moments to predict subsequent returns.

Specifically, our estimation is based on the regression framework of Fama and MacBeth (1973). For a general set of K risk factors $X_t = \{x_{t,1}, x_{t,2}, \ldots, x_{t,K}\}$, where $\{x_{t,j}\}_{t=1}^{T}$ is a time-series of the j-th risk factor, $j \in [1, K]$, we estimate the following cross-sectional regressions

$$r_{t+1,i} = \alpha_i + \sum_{j=1}^{K} \beta_{i,j} x_{t,j} + \epsilon_{t,i}, \tag{15}$$

where $r_{t+1,i}$ denotes the return for stock i over the $t+1$ and K denotes the number of control variables $x_{t,j}$. Having estimated the slope coefficients $\beta_{i,j}$, we use the factor loadings $\widehat{\beta}_{i,j}$ to estimate the second stage regression in a following way

$$\bar{r}_i = \omega + \sum_{j=1}^{K} \lambda_j \widehat{\beta}_{i,j} + \eta_i, \tag{16}$$

where $\bar{r}_i$ is a time-series average of returns of asset i.

Empirical evidence suggests (see e.g. Ghysels, 1998), that the relationships estimated in Equation (15) evolve dynamically in time, hence we need to estimate a conditional model where coefficients will evolve over time as $\beta_{t,i,j}$. The time variation in state variable risk premia is recently discussed by Barroso et al. (2021), and it also is an important component of our model described by Proposition 1.

The time-varying parameter asset pricing regression then becomes

$$r_{t+1,i} = \alpha_{t,i} + \sum_{j=1}^{K} \beta_{t,i,j} x_{t,j} + \epsilon_{t,i}. \tag{17}$$

Similarly to rolling window approximation of the standard Fama-Macbeth framework, we obtain T regressions, and T sets of coefficients. However, our truly time-varying parameter estimates are able to fully capture the dynamics using localization of the regression. Our time-varying parameter (TVP) Fama-Macbeth estimates are estimated with help of the modified Quasi-Bayesian Local-Likelihood approach of Petrova (2019) detailed in Appendix B.

We consider two baseline models both estimated in the static and the dynamic specification. Firstly, we estimate the Static Four Moment Model (SFMM) as well as Dynamic Four Moment Model (DFMM) using market risk factors with subscript m and average idiosyncratic risk factors with subscript I:

$$X_t \in \{RVOL_t^{(m)}, RS_t^{(m)}, RK_t^{(m)}, RVOL_t^{(I)}, RS_t^{(I)}, RK_t^{(I)}\}$$

Secondly, we use the frequency-dependent higher moments in the Static Horizon-Specific Model (SHSM) as well as Dynamic Horizon-Specific Model (DHSM) with following regressors

$$\begin{aligned} X_t \in \{ & RVOL_t^{(s,m)}, RVOL_t^{(l,m)}, RS_t^{(s,m)}, RS_t^{(l,m)}, RK_t^{(s,m)}, RK_t^{(l,m)}, \\ & RVOL_t^{(s,I)}, RVOL_t^{(l,I)}, RS_t^{(s,I)}, RS_t^{(l,I)}, RK_t^{(s,I)}, RK_t^{(l,I)}\}. \end{aligned}$$

Above described models combine the effects of volatility, skewness, and kurtosis representing different aspects of risk on the financial markets. The results allow us to identify the aspects of risk dominant in pricing the cross-section of stock and ETF returns, and understand the signs of these effects and their implications. Generally, investors with sensible preferences over risk prefer portfolios with lower volatility, higher skewness, and lower kurtosis (Kimball, 1993), hence need to be compensated by higher returns for accepting portfolios with higher volatility, lower skewness, or higher kurtosis. However, it is not straightforward to connect these phenomena to the prices of risk. We are able to capture the conditions under which the price of volatility is positive/negative thanks to the Intertemporal CAPM (Merton, 1973; Campbell, 1996; Ang et al., 2006; Chen, 2002). Market volatility is priced because it serves as a hedge against future changes on the market. If high volatility is connected to downward price movements, then an asset whose return has a positive sensitivity to market volatility is a desirable hedging instrument, hence the negative price of market volatility in such case. If the opposite holds, an asset with positive sensitivity to market volatility is undesirable, hence investors should require compensation for holding such asset. The sign of market volatility risk price is therefore usually expected to be negative due to the presence of the leverage effect, but the theory does not rule out the opposite effect. The empirical results regarding the effect of volatility on asset returns are also mixed. There is evidence that innovations to market volatility are negatively priced in the cross-section of asset returns (Ang et al., 2006; Chang et al., 2013), and non-robust evidence that idiosyncratic volatility has a positive effect on subsequent returns on assets in the TAQ database (Bollerslev et al., 2020).

Price of higher moments cannot be determined by observing the empirical correlations, since such approach would ignore the individual investors' risk attitudes like skewness preference. Chabi-Yo (2012) concludes that the prices of market skewness risk, and market kurtosis risk depend on the fourth and fifth derivative of the utility function which are hard to sign. Hence, we shall perceive determining the prices of higher moment risks merely as an empirical exercise. Lastly, we expect that the prices of risk of the short-run and long-run components will retain the signs of the corresponding factors, but there is no force which would ensure that it will always be the case.

We describe the empirical results from several perspectives. Controlling for both sources of risk allows us to uncover the discrepancies in information contained in average idiosyncratic risk factors and market risk factors. Comparing the static models to their dynamic counterparts reveals the time variation in the risk factors and its consequences on model outcomes. Lastly, decomposing volatility, skewness and kurtosis to the short-run and long-run components helps disentangle the transitory and persistent nature of investors preferences towards different sources of higher moment risks. Distinguishing the transitory and persistent components of the individual risk sources, i.e. decomposing the risks into frequency-specific components should be a key feature of an asset pricing model (Neuhierl and Varneskov, 2021).

4.1 Stocks data

This section reports the results of the above described cross-sectional regressions using daily and weekly stocks data. In addition, the effects are controlled for the three Fama-French factors typically used in the literature (MKT, SMB, HML). Table 4 displays the results of SFMM (Panel A), and DFMM (Panel B) estimated using daily stock returns, Table 5 displays the results of SHSM (Panel A), and DHSM (Panel B) estimated using daily stock returns. Each Panel in the Tables throughout the rest of this section presents a model containing the group of market factors ([1], [4], [7], and [10]), a model containing the group of idiosyncratic factors ([2], [5], [8], and [11]), and a model combining these two groups together and controlling for the Fama-French factors ([3], [6], [9], and [12]).

The portfolio sorting exercises suggest that the most important indicator of subsequent returns is exposure to skewness. Both market and average idiosyncratic skewness risk prove to be relevant, and the frequency decomposition suggests that investors value mostly the transitory components of skewness risk. Results in Table 4 support the findings from the univariate exercise indicating negative effect of exposure to both market and average idiosyncratic skewness on subsequent asset returns. These effects are significant mostly in Panel B hence in the dynamic specification. The coefficients associated with market (model [4]) and average idiosyncratic (model [5]) skewness are -0.11 and -0.20 with significant t-statistics of -3.7 and -11.4 respectively, and both coefficients remain significant in model [6] controlling for market and average idiosyncratic moments as well as the Fama-French factors.

Evidence from both Panels suggests that neither market nor average idiosyncratic volatility is priced in the cross-section of daily stock returns. We observe certain degree of ambiguity regarding the prices of kurtosis risk. The coefficients associated with market kurtosis are negative while the coefficients associated with average idiosyncratic kurtosis are positive in Panel A. However, only the former are statistically significant on the 5%-level. On the other hand, all coefficients associated with market kurtosis and average idiosyncratic kurtosis are positive and highly significant in Panel B.

Table 5 displays the risk factors from Table 4 decomposed to the short-run and the long-run components. The decomposition into frequency-specific components shows us how different sources of risk are priced through their transitory and persistent components associated with high-frequency and low-frequency fluctuations on the financial markets. The portfolio sorting exercise indicates downward-sloping term structure of both market and average idiosyncratic skewness risk, and this result is generally confirmed by Table 5. The coefficients associated with the short-run component of market skewness are statistically significant in both Panels. The long-run components are significant as well, but they are much weaker economically. The results are qualitatively very similar in case of average idiosyncratic skewness with the exception of model [12] where the long-run component displays more significant effect on subsequent returns.

Figure 1 reveals that dynamics of skewness consists mostly of transitory fluctuations

Table 4. Cross-section of Stocks - Daily

We report estimated coefficients from the four moment (FMM) predictive regressions defined in Section 4. Prices of risk are estimated on daily stock data with market $(RVOL_t^{(m)}, RS_t^{(m)}, RK_t^{(m)})$ and average idiosyncratic $(RVOL_t^{(I)}, RS_t^{(I)}, RK_t^{(I)})$ moments as risk factors.

	Panel A: Static			Panel B: Dynamic		
	[1]	**[2]**	**[3]**	**[4]**	**[5]**	**[6]**
const	0.0005	0.0007	0.0007	0.0007	0.0017	0.0013
	(2.7673)	(3.7373)	(3.6225)	(6.2995)	(11.0043)	(10.431)
$RVOL_t^{(m)}$	-0.0001		-0.0001	0.0003		-0.0005
	(-0.4293)		(-0.5536)	(1.558)		(-2.5604)
$RS_t^{(m)}$	-0.0774		-0.1462	-0.1115		-0.1057
	(-1.4434)		(-2.3914)	(-3.6967)		(-3.6792)
$RK_t^{(m)}$	-1.2521		-1.3166	1.0201		1.764
	(-4.3359)		(-4.5113)	(9.2237)		(19.7774)
$RVOL_t^{(I)}$		0.0001	-0.0		0.0003	-0.0
		(0.5291)	(-0.0329)		(1.2392)	(-0.0169)
$RS_t^{(I)}$		-0.0408	-0.0265		-0.2047	-0.1456
		(-2.1499)	(-1.5138)		(-11.3823)	(-8.7436)
$RK_t^{(I)}$		0.137	0.1599		2.7028	2.5545
		(1.4439)	(1.5952)		(33.1023)	(35.6841)
R^2	0.1919	0.1236	0.2648			

Table 5. Cross-section of Stocks - Daily

We report estimated coefficients from the horizon-specific (HSM) predictive regressions defined in Section 4. Prices of risk are estimated on daily stock data with short- and long-term components of the market $(RVOL_t^{(s,m)}, RVOL_t^{(l,m)}, RS_t^{(s,m)}, RS_t^{(l,m)}, RK_t^{(s,m)}, RK_t^{(l,m)})$ and average idiosyncratic $(RVOL_t^{(s,I)}, RVOL_t^{(l,I)}, RS_t^{(s,I)}, RS_t^{(l,I)}, RK_t^{(s,I)}, RK_t^{(l,I)})$ moments as risk factors.

	Panel A: Static			Panel B: Dynamic		
	[7]	**[8]**	**[9]**	**[10]**	**[11]**	**[12]**
const	0.0007	0.0007	0.0008	0.0011	0.001	0.0008
	(3.3262)	(3.8432)	(4.0637)	(9.3069)	(5.9256)	(6.4268)
$RVOL_t^{(s,m)}$	-0.0001		-0.0001	0.0011		0.0007
	(-0.7655)		(-0.4715)	(6.0693)		(4.291)
$RVOL_t^{(l,m)}$	0.0001		0.0	-0.0017		-0.0013
	(0.4409)		(0.1529)	(-13.0928)		(-10.754)
$RS_t^{(s,m)}$	-0.0914		-0.1312	-0.0576		-0.0732
	(-1.7777)		(-2.207)	(-1.9567)		(-2.6463)
$RS_t^{(l,m)}$	-0.0177		-0.0179	-0.0302		-0.0303
	(-2.9919)		(-3.1058)	(-5.1788)		(-5.6145)
$RK_t^{(s,m)}$	-1.1635		-1.0454	-0.1411		-0.1171
	(-4.2631)		(-3.9092)	(-1.7178)		(-1.5133)
$RK_t^{(l,m)}$	0.0339		0.0129	0.6358		0.2998
	(0.9766)		(0.3899)	(19.8582)		(11.1084)
$RVOL_t^{(s,I)}$		0.0002	0.0001		0.0002	0.0004
		(0.9521)	(0.3589)		(1.4647)	(2.4549)
$RVOL_t^{(l,I)}$		0.0	-0.0		-0.0013	-0.0014
		(0.0721)	(-0.1758)		(-8.4728)	(-10.5486)
$RS_t^{(s,I)}$		-0.0335	-0.0275		-0.0381	-0.0029
		(-1.8849)	(-1.6464)		(-2.2737)	(-0.1798)
$RS_t^{(l,I)}$		-0.008	-0.0066		-0.0088	-0.0066
		(-4.7339)	(-4.3928)		(-5.4307)	(-4.6196)
$RK_t^{(s,I)}$		0.0714	0.0737		-0.0165	-0.0202
		(0.8661)	(0.8899)		(-0.4725)	(-0.6054)
$RK_t^{(l,I)}$		-0.0096	0.0128		0.282	0.2534
		(-0.37)	(0.5122)		(11.4221)	(11.8462)
R^2	0.2678	0.2156	0.363			

while the persistent component provides only a little contribution. Such notion is confirmed by the portfolio sorting exercises indicating a downward-sloping term structure of both market and average idiosyncratic skewness risk. The cross-sectional regressions consider the effects of all the risk factors in Equation (12) jointly, hence the results are influenced by the interrelations between the individual components of the pricing kernel. While the coefficients associated with the transitory components of skewness risk are economically more important, the results of the cross-sectional regressions suggest that the long-run component of skewness risk is priced in the daily stock returns as well. Hence the downward-sloping term structure of skewness risk is confirmed by Table 5, and the significance of the persistent component arises through interactions among the individual risk factors.

The results suggest that the short-run and the long-run components of volatility risk are connected to opposite effects on subsequent returns. Coefficient associated with the short-run component of market volatility is 0.0012 with a significant t-statistics of 5.5 in Panel B. The coefficients associated with the short-run component of average idiosyncratic volatility are also positive although not that statistically significant. On the other hand, the coefficients associated with the long-run component of market and average idiosyncratic volatility risk are -0.0014 and -0.0006 with significant t-statistics of -7.9 and -4.8 respectively in Panel B. Hence we observe offsetting effects of the short-run and the long-run components of volatility risk corresponding to no significant effect of exposure to either source of aggregate volatility risk on subsequent stock returns (Table 4).

Lastly, the evidence from Panel B of Table 5 indicates upward-sloping term structure of market as well as average idiosyncratic kurtosis risk. The coefficient associated with the long-run component of market kurtosis risk is 0.64 with a significant t-statistic of 19.9, the coefficient associated with the long-run component of average idiosyncratic kurtosis is 0.28 with a significant t-statistic of 11.4. Both effects remain significant in model [12] controlling for market frequency-specific factors, average idiosyncratic frequency-specific factors, and Fama-French factors.

Table 6 displays the results of SFMM (Panel A), and DFMM (Panel B) estimated using weekly stocks data. They very closely mimic what we observe in the daily data, i.e. signifi-

Table 6. Cross-section of Stocks - Weekly

We report estimated coefficients from the four moment (FMM) predictive regressions defined in Section 4. Prices of risk are estimated on weekly stock data with market $(RVOL_t^{(m)}, RS_t^{(m)}, RK_t^{(m)})$ and average idiosyncratic $(RVOL_t^{(I)}, RS_t^{(I)}, RK_t^{(I)})$ moments as risk factors.

	Panel A: Static			Panel B: Dynamic		
	[1]	[2]	[3]	[4]	[5]	[6]
const	0.0026	0.0027	0.0029	0.0037	0.0045	0.0044
	(3.1349)	(3.4573)	(3.5018)	(4.7208)	(6.0192)	(5.9532)
$RVOL_t^{(m)}$	-0.006		-0.0044	-0.0122		-0.024
	(-1.3199)		(-1.0251)	(-2.5406)		(-5.8391)
$RS_t^{(m)}$	-0.0593		-0.0623	-0.1838		-0.196
	(-1.2123)		(-1.4055)	(-3.6369)		(-3.8789)
$RK_t^{(m)}$	-0.2298		-0.2916	4.0725		0.903
	(-1.5517)		(-2.0751)	(28.2325)		(8.6208)
$RVOL_t^{(I)}$		-0.002	-0.0021		-0.0051	-0.0183
		(-0.4312)	(-0.446)		(-1.108)	(-4.1248)
$RS_t^{(I)}$		-0.0224	-0.0186		-0.1027	-0.0446
		(-1.2577)	(-1.2221)		(-5.7418)	(-3.2677)
$RK_t^{(I)}$		0.0947	0.1107		1.3736	1.3446
		(1.0216)	(1.1601)		(15.9887)	(17.0131)
R^2	0.0902	0.0666	0.1263			

Table 7. Cross-section of Stocks - Weekly

We report estimated coefficients from the horizon-specific (HSM) predictive regressions defined in Section 4. Prices of risk are estimated on weekly stock data with short- and long-term components of the market $(RVOL_t^{(s,m)}, RVOL_t^{(l,m)}, RS_t^{(s,m)}, RS_t^{(l,m)}, RK_t^{(s,m)}, RK_t^{(l,m)})$ and average idiosyncratic $(RVOL_t^{(s,I)}, RVOL_t^{(l,I)}, RS_t^{(s,I)}, RS_t^{(l,I)}, RK_t^{(s,I)}, RK_t^{(l,I)})$ moments as risk factors.

	Panel A: Static			Panel B: Dynamic		
	[7]	[8]	[9]	[10]	[11]	[12]
const	0.0029	0.0029	0.0031	0.0043	0.0038	0.0038
	(3.8041)	(3.7744)	(3.9637)	(5.7994)	(5.2712)	(5.4787)
$RVOL_t^{(s,m)}$	-0.0018		-0.0012	0.0137		0.0016
	(-0.5448)		(-0.3724)	(3.9143)		(0.5175)
$RVOL_t^{(l,m)}$	-0.0021		-0.0015	-0.0235		-0.0179
	(-0.8406)		(-0.6471)	(-10.5029)		(-8.9381)
$RS_t^{(s,m)}$	-0.058		-0.0638	-0.0783		-0.1013
	(-1.2604)		(-1.4347)	(-1.6486)		(-2.1209)
$RS_t^{(l,m)}$	-0.0156		-0.017	-0.0188		-0.0155
	(-2.483)		(-2.8848)	(-2.9598)		(-2.8876)
$RK_t^{(s,m)}$	-0.2804		-0.2951	-0.1082		-0.0981
	(-2.0657)		(-2.2832)	(-1.2455)		(-1.1557)
$RK_t^{(l,m)}$	0.05		0.03	0.5041		0.24
	(1.3733)		(0.9213)	(16.1614)		(8.1531)
$RVOL_t^{(s,I)}$		0.0011	0.0009		-0.0003	-0.0007
		(0.2957)	(0.2435)		(-0.0984)	(-0.2155)
$RVOL_t^{(l,I)}$		-0.0024	-0.0017		-0.0195	-0.0209
		(-0.8881)	(-0.6594)		(-7.1996)	(-8.6641)
$RS_t^{(s,I)}$		-0.023	-0.0207		-0.0387	-0.0108
		(-1.3472)	(-1.4188)		(-2.1972)	(-0.818)
$RS_t^{(l,I)}$		-0.0052	-0.0047		-0.0036	-0.0031
		(-3.4397)	(-3.1482)		(-2.5438)	(-2.3843)
$RK_t^{(s,I)}$		0.0332	0.0219		0.005	0.0042
		(0.4165)	(0.2771)		(0.0825)	(0.0701)
$RK_t^{(l,I)}$		0.0014	-0.0026		0.2267	0.1929
		(0.0516)	(-0.1099)		(8.7067)	(9.2873)
R^2	0.1673	0.1665	0.2312			

cant positive effect of exposure to market and average idiosyncratic kurtosis on subsequent stock returns, and significant negative effect of exposure to market and average idiosyncratic skewness on subsequent stock returns. In contrast to daily stocks data, we also observe evidence that market volatility is negatively priced in the weekly stock returns.

Table 7 displays the results of SHSM (Panel A), and DHSM (Panel B) estimated using weekly stocks data. Panel B confirms the evidence of upward-sloping term structure of market and average idiosyncratic kurtosis risk premia from daily stocks data. Table 7 provides evidence that the long-run component of market skewness and the long-run component of average idiosyncratic skewness are priced in the cross-section of weekly stocks data, while there is no robust evidence that the short-run components are priced. The coefficients associated with the long-run component of market skewness are -0.02 (in both Panel A and Panel B) with significant t-statistics of -2.48 and -2.96 respectively. The coefficients associated with the long-run component of average idiosyncratic skewness are -0.005 and -0.003 with significant t-statistics of -3.44 and -2.54 respectively. All these effects remain significants while controlling for market (average idiosyncratic) and Fama-French factors.

The evidence from weekly data does not confirm the downward-sloping term structure of skewness risk premia, although its presence is clearly indicated especially by the portfolio sorting exercises and cross-sectional regressions using daily data. Generally, the transitory component of risk consists of the high-frequency fluctuations propagating substantially into daily returns, however it is difficult to detect the high-frequency fluctuations in the weekly returns. Such notion is also supported by examining frequency-specific components of volatility risk. Table 7 indicates that the long-run components of market and average idiosyncratic volatility risk are priced in the weekly stock returns consistently with Table 5, while the short-run components display no significant effects contrary to the evidence from daily stock returns.

The cross-sectional regression analysis applied on the daily and weekly stocks data indicates that investors are willing to accept lower returns for holding assets with higher exposure to skewness risk. The relationship holds jointly for market skewness risk and average idiosyncratic skewness risk, thus we find strong evidence that market and average idiosyncratic skewness risk carry different information and that both channels of skewness risk are reflected separately in the preferences of investors (Jondeau et al., 2019). The downward-sloping term structure of skewness risk indicated by the portfolio sorting exercise is confirmed in the daily stocks data. While the daily data show that investors value predominantly the transitory component of skewness risk connected to the short-run fluctuations on the financial markets, we identify no such pattern in the weekly stock returns. We attribute this to the inability of weekly returns to capture the high-frequency fluctuations which is also documented by the coefficients associated with the frequency-specific components of market and average idiosyncratic volatility risk.

The results also indicate that investors require higher returns for holding assets with higher exposure to market kurtosis as well as average idiosyncratic kurtosis. Both patterns become highly pronounced in the dynamic models indicating high degree of time variation across the different sources of risk (Barroso et al., 2021). Both sources of kurtosis risk display upward-sloping term structure indicating investors' sensitivity to persistent fluctuations related to kurtosis risk premia. The long-run components corresponding to the persistent financial markets fluctuations also play a dominant role in pricing the volatility risk. We are able to distinguish a significant effect of exposure to volatility risk on subsequent stock returns mostly after decomposing the higher moment risks to individual frequency-specific components.

4.2 Exchange Traded Funds data

In this Section, we report the results of the cross-sectional regressions using the daily and weekly ETF returns. Table 8 reports the results of SFMM (Panel A), and DFMM (Panel B) estimated using daily ETFs data. Coefficients associated with market and average idiosyncratic skewness are all statistically significant with the exception of market skewness in model [3]. Hence, the evidence that investors are willing to accept lower returns for assets with higher exposure to market as well as average idiosyncratic skewness extends to daily ETFs data. The coefficient associated with average idiosyncratic kurtosis is 2.4 with significant t-statistics of 14.8 in Panel B and the effect remains significant after controlling for market and Fama-French factors. The coefficient associated with market kurtosis is 1.45 with significant t-statistic of 7.6 in Panel B and also remains significant in the model with all control variables. We find no robust evidence that market volatility is priced in the

Table 8. Cross-section of ETFs - Daily Data

We report estimated coefficients from the four moment (FMM) predictive regressions defined in Section 4. Prices of risk are estimated on daily ETF data with market $(RVOL_t^{(m)}, RS_t^{(m)}, RK_t^{(m)})$ and average idiosyncratic $(RVOL_t^{(I)}, RS_t^{(I)}, RK_t^{(I)})$ moments as risk factors.

	Panel A: Static			Panel B: Dynamic		
	[1]	**[2]**	**[3]**	**[4]**	**[5]**	**[6]**
const	0.0008	0.0007	0.0006	0.0008	0.0005	0.0005
	(3.7516)	(3.158)	(3.7296)	(5.1152)	(2.7333)	(3.8408)
$RVOL_t^{(m)}$	0.0002		0.0002	0.0004		-0.0007
	(0.4169)		(0.3325)	(1.0488)		(-1.9513)
$RS_t^{(m)}$	-0.1376		-0.0497	-0.1596		-0.1244
	(-2.5896)		(-0.5883)	(-4.1783)		(-3.1726)
$RK_t^{(m)}$	-0.332		0.1425	1.4505		1.8358
	(-0.4683)		(0.2319)	(7.6132)		(11.9096)
$RVOL_t^{(I)}$		0.0	0.0001		-0.0026	-0.001
		(0.0437)	(0.1879)		(-7.0209)	(-2.6316)
$RS_t^{(I)}$		-0.0685	-0.0616		-0.1292	-0.0695
		(-2.668)	(-2.3692)		(-5.2721)	(-3.1277)
$RK_t^{(I)}$		-0.0825	0.001		2.4072	2.1535
		(-0.3689)	(0.0041)		(14.8025)	(16.6763)
R^2	0.2616	0.2774	0.2825			

Table 9. Cross-section of ETFs - Daily Data

We report estimated coefficients from the horizon-specific (HSM) predictive regressions defined in Section 4. Prices of risk are estimated on daily ETF data with short- and long-term components of the market $(RVOL_t^{(s,m)}, RVOL_t^{(l,m)}, RS_t^{(s,m)}, RS_t^{(l,m)}, RK_t^{(s,m)}, RK_t^{(l,m)})$ and average idiosyncratic $(RVOL_t^{(s,I)}, RVOL_t^{(l,I)}, RS_t^{(s,I)}, RS_t^{(l,I)}, RK_t^{(s,I)}, RK_t^{(l,I)})$ moments as risk factors.

	Panel A: Static			Panel B: Dynamic		
	[7]	**[8]**	**[9]**	**[10]**	**[11]**	**[12]**
const	0.0008	0.0007	0.0006	0.0006	0.0007	0.0008
	(4.2794)	(3.7444)	(3.6681)	(4.3291)	(3.999)	(6.0274)
$RVOL_t^{(s,m)}$	-0.0002		-0.0002	0.0001		0.0004
	(-0.4742)		(-0.4181)	(0.4645)		(1.5568)
$RVOL_t^{(l,m)}$	0.0011		0.0011	0.0001		0.0003
	(3.502)		(3.6829)	(0.3598)		(1.1252)
$RS_t^{(s,m)}$	-0.1625		-0.0639	-0.1048		-0.0601
	(-3.0168)		(-0.7853)	(-2.8633)		(-1.6379)
$RS_t^{(l,m)}$	-0.0022		-0.001	-0.0107		-0.0195
	(-0.1403)		(-0.0763)	(-0.6656)		(-1.637)
$RK_t^{(s,m)}$	-1.149		-1.0168	-0.1477		-0.1304
	(-1.845)		(-1.9777)	(-1.2326)		(-1.232)
$RK_t^{(l,m)}$	-0.0406		-0.0338	0.3021		0.1971
	(-0.7324)		(-0.6273)	(5.7618)		(4.1915)
$RVOL_t^{(s,I)}$		-0.0001	-0.0001		-0.0	0.0004
		(-0.2727)	(-0.324)		(-0.1542)	(1.3991)
$RVOL_t^{(l,I)}$		0.0012	0.0013		-0.0001	0.0005
		(3.7773)	(3.9447)		(-0.3664)	(1.8496)
$RS_t^{(s,I)}$		-0.0745	-0.065		-0.0558	-0.0551
		(-3.079)	(-2.3616)		(-2.3634)	(-2.5503)
$RS_t^{(l,I)}$		0.0001	0.0013		-0.0149	-0.003
		(0.0135)	(0.233)		(-2.3741)	(-0.5668)
$RK_t^{(s,I)}$		-0.0168	-0.1656		0.0001	-0.0117
		(-0.0695)	(-0.8125)		(0.0012)	(-0.2214)
$RK_t^{(l,I)}$		-0.0958	-0.1327		0.2076	0.0926
		(-1.6461)	(-2.7192)		(3.8162)	(2.066)
R^2	0.6049	0.5948	0.6516			

cross-section of daily ETF data, while the coefficient associated with average idiosyncratic volatility is significant in both models in Panel B.

Table 9 reports the results of SHSM (Panel A), and DHSM (Panel B) estimated using daily ETF data. The decomposition into the short-run and long-run components confirms the downward-sloping term structure of market and average idiosyncratic skewness risk in the cross-section of daily ETFs. Table 9 also shows that the long-run components are dominant in terms of pricing market and average idiosyncratic kurtosis risk.

Coefficients associated with the long-run components of market and average idiosyncratic volatility display significant t-statistics in Panel A, and the effects remain significant in model [9] including market, average idiosyncratic and Fama-French factors. The positive sign of these relationships contradicts the evidence from stocks data regarding price of aggregate as well as long-run volatility risk. We find no evidence that the transitory nor persistent components of market and average idiosyncratic volatility risk are priced in Panel B of Table 9.

Table 10 reports the results of SFMM (Panel A), and DFMM (Panel B) estimated using weekly ETF data. In contrast to all the other results we find no evidence that market or average idiosyncratic skewness risk is priced in the weekly ETFs data. Although the coefficient associated with average idiosyncratic skewness in model [5] is -0.09 with a significant t-statistic of -3.1, the negative effect does not persist in model [6] controlling for market and Fama-French factors. Consistently to the daily ETFs data, Panel B of Table 10 suggests that market as well as average idiosyncratic kurtosis are positively priced, and that average idiosyncratic volatility is negatively priced in the cross-section of weekly ETF returns.

Table 11 reports the results of SHSM (Panel A), and DHSM (Panel B) estimated using weekly ETF data. The results indicate that the aversion to assets with higher exposure to market volatility highlighted by Panel A of Table 10 is associated with the persistent component of market volatility risk complementing the results from ETFs daily data. Apart from market volatility risk we find little evidence that the horizon-specific components of the various sources of risk are priced in the cross-section of weekly ETF returns. The only exception is average idiosyncratic volatility, however it displays different term structures

Table 10. Cross-section of ETFs - Weekly Data

We report estimated coefficients from the four moment (FMM) predictive regressions defined in Section 4. Prices of risk are estimated on weekly ETF data with market $(RVOL_t^{(m)}, RS_t^{(m)}, RK_t^{(m)})$ and average idiosyncratic $(RVOL_t^{(I)}, RS_t^{(I)}, RK_t^{(I)})$ moments as risk factors.

	Panel A: Static			Panel B: Dynamic		
	[1]	[2]	[3]	[4]	[5]	[6]
const	0.0031	0.0034	0.0032	-0.0009	0.0013	0.0041
	(2.5768)	(3.007)	(3.0587)	(-0.7688)	(1.1847)	(3.9772)
$RVOL_t^{(m)}$	0.0215		0.0197	0.0132		-0.006
	(1.9228)		(2.1515)	(1.2724)		(-0.7752)
$RS_t^{(m)}$	0.0079		0.0377	0.0577		0.05
	(0.1613)		(0.679)	(1.0468)		(0.9292)
$RK_t^{(m)}$	-0.702		-0.4446	2.4423		1.4444
	(-1.7733)		(-1.0618)	(9.8747)		(8.1348)
$RVOL_t^{(I)}$		0.0129	0.02		-0.0293	-0.0193
		(1.2076)	(2.0838)		(-2.8123)	(-2.3329)
$RS_t^{(I)}$		-0.0327	-0.0101		-0.0917	0.0619
		(-1.107)	(-0.3537)		(-3.0904)	(2.2881)
$RK_t^{(I)}$		-0.3234	-0.1738		0.9127	1.7564
		(-1.8637)	(-0.9262)		(5.828)	(11.5084)
R^2	0.3213	0.248	0.3872			

Table 11. Cross-section of ETFs - Weekly Data

We report estimated coefficients from the horizon-specific (HSM) predictive regressions defined in Section 4. Prices of risk are estimated on weekly ETF data with short- and long-term components of the market $(RVOL_t^{(s,m)}, RVOL_t^{(l,m)}, RS_t^{(s,m)}, RS_t^{(l,m)}, RK_t^{(s,m)}, RK_t^{(l,m)})$ and average idiosyncratic $(RVOL_t^{(s,I)}, RVOL_t^{(l,I)}, RS_t^{(s,I)}, RS_t^{(l,I)}, RK_t^{(s,I)}, RK_t^{(l,I)})$ moments as risk factors.

	Panel A: Static			Panel B: Dynamic		
	[7]	[8]	[9]	[10]	[11]	[12]
const	0.0014	0.0028	0.0025	0.0013	0.0029	0.0029
	(1.3797)	(2.4491)	(2.6895)	(1.1349)	(3.1754)	(3.3322)
$RVOL_t^{(s,m)}$	0.0011		0.0002	0.011		0.0101
	(0.1675)		(0.0236)	(1.4135)		(1.6098)
$RVOL_t^{(l,m)}$	0.0205		0.0238	0.0089		0.0078
	(2.9572)		(3.4992)	(1.3749)		(1.2781)
$RS_t^{(s,m)}$	0.0098		0.0135	0.0426		0.0083
	(0.204)		(0.2559)	(0.8835)		(0.1652)
$RS_t^{(l,m)}$	0.0419		0.0147	0.0195		0.0061
	(1.8926)		(1.0373)	(1.0157)		(0.4475)
$RK_t^{(s,m)}$	-0.3404		-0.2452	-0.0955		-0.0895
	(-0.9427)		(-0.715)	(-0.9259)		(-0.8946)
$RK_t^{(l,m)}$	-0.0948		-0.1707	0.1163		0.0481
	(-1.4729)		(-2.6536)	(1.8625)		(0.8511)
$RVOL_t^{(s,I)}$		0.0048	0.0026		0.0181	0.0126
		(0.6577)	(0.387)		(2.8836)	(1.9827)
$RVOL_t^{(l,I)}$		0.0243	0.0293		0.0023	0.0105
		(3.0256)	(3.7598)		(0.2905)	(1.5226)
$RS_t^{(s,I)}$		-0.0048	-0.0074		0.0282	0.0063
		(-0.1934)	(-0.2604)		(1.085)	(0.2446)
$RS_t^{(l,I)}$		0.0007	-0.0058		-0.0084	-0.0006
		(0.0739)	(-0.7207)		(-1.0147)	(-0.087)
$RK_t^{(s,I)}$		0.0257	0.1891		-0.116	-0.0912
		(0.1492)	(1.066)		(-1.35)	(-1.0479)
$RK_t^{(l,I)}$		-0.084	-0.1587		0.2394	0.097
		(-0.9955)	(-2.2218)		(3.3369)	(1.5208)
R^2	0.5466	0.7186	0.8491			

in Panel A and Panel B.

The predictive cross-sectional regressions applied on the daily ETFs data confirm the negative relationship between skewness and subsequent asset returns. Both market and average idiosyncratic skewness are jointly priced in the cross-section of daily ETF returns. Moreover, we confirm that market as well as average idiosyncratic skewness risk is priced mainly through the short-run components. Hence the high-frequency fluctuations comprised into the transitory component of skewness risk are most strongly reflected in subsequent ETF returns. As is the case in the stocks dataset, we do not confirm the downward-sloping term structure of skewness risk in the weekly data possibly due to the inability of weekly returns to capture the high-frequency financial markets fluctuations.

5 Conclusion

We show that frequency-dependent higher moment risk is important for the cross-section of stock and exchange traded fund returns. Short-term and long-term fluctuation of realized market and average idiosyncratic volatility, skewness and kurtosis are priced in the cross-section of asset returns differently. We show that market and average idiosyncratic higher moment risks carry different information and that they enter the decision making of investors separately. We also show that investors value the transitory and persistent components of financial markets fluctuations differently implicating heterogeneity in term structures over various sources of higher moment risks.

The portfolio sorting exercises uncover that skewness is a dominant factor in pricing the cross-section of stock returns and that investors are willing to accept lower returns for assets with higher exposure to market and average idiosyncratic skewness. Moreover, they clearly indicate a downward-sloping term structure of market and average idiosyncratic skewness risk denoting that investors value mostly the short-run components of skewness risk associated with transitory fluctuations on the financial markets.

Important role of both sources of skewness risk in pricing the cross-section of stocks and ETFs is confirmed by the predictive cross-sectional regressions. The fact that average idiosyncratic skewness risk is priced in excess to market skewness risk documents that both of these channels of skewness risk translate into investors' preferences and affect future market returns (Jondeau et al., 2019). Overall, the significant effect of average idiosyncratic risks on subsequent returns suggests that the asymmetric transmission channels exist on the financial markets (Elliott et al., 2014; Barunik and Ellington, 2020a), and the investors take their existence into consideration.

The cross-sectional regressions confirm the downward-sloping term structure of skewness risk especially in the daily data, while it seems generally difficult to capture the effects of high-frequency fluctuations in the weekly asset returns. Overall, the Fama-Macbeth type cross-sectional regressions indicate that volatility and especially kurtosis are mostly long-run phenomena, while skewness risk is priced predominantly through its transitory components. In other words, we document discrepancies in the term structures across different sources of higher moment risk, and we stress the necessity to consider the transitory and persistent components of higher moment risks separately.

Overall, the results of the paper provide new insights into the sources of asset's predictability with respect to frequency decomposition of higher moment risks. We attempt to rationalize our findings in a formal theoretical model.

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Appendix for

"Frequency-Dependent Higher Moment Risks"

Abstract

This appendix presents supplementary details not included in the main body of the paper.

Contents

A Proofs

Proposition 1. Hansen and Jagannathan (1991) show that solution to portfolio choice problem can be expressed in terms of Euler equation (Dittmar, 2002)

$$E_t(R_{t+1,i}M_{t+1} \mid \Omega_t) = 1, \tag{A.1}$$

where $R_{t+1,i}$ is return of asset i, and M_{t+1} is the pricing kernel.

We build on the approach of Maheu et al. (2013) and Chabi-Yo (2012), and derive the pricing kernel M_{t+1} without explicitly assuming any form of utility function. We begin by taking a Taylor expansion of an unspecified utility function $U(W_{t+1})$ depending on aggregate wealth W_t up to the fourth order.[18] Aggregate wealth in time $t+1$ is determined in a typical manner as $W_{t+1} = W_t(1+R^w_{t+1})$, where R^w_{t+1} is the net return on aggregate wealth. $U(W_{t+1})$ is expanded around $W_t(1+C_t)$, where C_t is an arbitrary return

$$\begin{aligned} U(W_{t+1}) &\approx \sum_{n=0}^{4} \frac{U^{(n)}(W_t(1+C_t))}{n!}(W_{t+1} - W_t(1+C_t))^n \\ &= \sum_{n=0}^{4} \frac{U^{(n)}(W_t(1+C_t))}{n!}(W_t(R^w_{t+1} - C_t))^n. \end{aligned} \tag{A.2}$$

Without loss of generality we can assume the initial wealth is equal to 1. Taking a derivative of the sum in Equation (A.2) yields

$$U'(W_{t+1}) \approx \sum_{n=0}^{3} \frac{U^{(n+1)}(1+C_t)}{n!}(R^w_{t+1} - C_t)^n. \tag{A.3}$$

Note that we can interpret $U'(W_{t+1})$ as the marginal utility of wealth at time $t+1$. The pricing kernel represents investors' discounting between subsequent periods, it corresponds to changes in marginal utility given the time period in which the wealth is received. The pricing kernel, $M_{t+1} \equiv U'(W_{t+1})/U'(W_t)$, can be approximated as

$$\begin{aligned} M_{t+1} &\approx \sum_{n=0}^{3} \frac{U^{(n+1)}(1+C_t)}{U'(1)n!}(R^w_{t+1} - C_t)^n \\ &= g_{0,t} + g_{1,t}(R^w_{t+1} - C_t) + g_{2,t}(R^w_{t+1} - C_t)^2 + g_{3,t}(R^w_{t+1} - C_t)^3, \end{aligned} \tag{A.4}$$

where $g_{n,t} = [U^{(n+1)}(1+C_t)/U'(1)][1/n!] = [U^{(n+1)}(1+C_t)/U'(1+C_t)n!][U'(1+C_t)/U'(1)]$.

When we assume investor decides between a pool of risky assets which yields the return on aggregate wealth R^w_{t+1}, and the risk-free asset yielding R^f_t, solution to the portfolio choice can be expressed as

$$E_t(R^w_{t+1}M_{t+1} \mid \Omega_t) = 1. \tag{A.5}$$

[18] Choice of $N = 4$ is justified in Dittmar (2002).

Using the formula $cov(X,Y) = E(XY) - E(X)E(Y)$, i.e. $E(XY) = cov(X,Y) + E(X)E(Y)$, we can decompose Equation (A.5) as

$$\begin{aligned} cov_t(R^w_{t+1}, M_{t+1}) + E_t(R^w_{t+1})E_t(M_{t+1}) &= 1, \\ E_t(R^w_{t+1})E_t(M_{t+1}) &= 1 - cov_t(R^w_{t+1}, M_{t+1}), \\ E_t(R^w_{t+1}) &= \frac{1}{E_t(M_{t+1})} - \frac{cov_t(R^w_{t+1}, M_{t+1})}{E_t(M_{t+1})}. \end{aligned} \tag{A.6}$$

Recall that $1/E_t(M_{t+1}) = R^f_t$, then Equation (A.6) becomes

$$E_t(R^w_{t+1}) - R^f_t = -R^f_t cov_t(R^w_{t+1}, M_{t+1}). \tag{A.7}$$

Substituting the pricing kernel M_{t+1} from Equation (A.4) into Equation (A.7) we obtain

$$\begin{aligned} E_t\left(R^w_{t+1}\right) - R^f_t =& \theta_{1,t} cov_t\left[R^w_{t+1}, R^w_{t+1} - C_t\right] + \theta_{2,t} cov_t\left[R^w_{t+1}, \left(R^w_{t+1} - C_t\right)^2\right] \\ &+ \theta_{3,t} cov_t\left[R^w_{t+1}, \left(R^w_{t+1} - C_t\right)^3\right], \end{aligned} \tag{A.8}$$

where R^f_t is the risk-free rate, and $\theta_{n,t} = -g_{n,t}R^f_t$.

At this point, it is convenient to specify the expansion point. A widely used choice is $C_t = 0$ (e.g., Harvey and Siddique, 2000; Dittmar, 2002), an alternative approach is to set $C_t = E_t(R^w_{t+1})$. We employ the latter specification also used by Chabi-Yo (2012) or Maheu et al. (2013). Chabi-Yo (2012) shows that this approach is equivalent to small noise expansion, if we write

$$R^w_{t+1} - E_t(R^w_{t+1}) = \epsilon Y_{t+1}, \tag{A.9}$$

then driving ϵ towards zero causes R^w_{t+1} to approach $E_t(R^w_{t+1})$. Let us simplify notation by denoting for each $i \in \{1, ..., N\}$

$$\begin{aligned} R^e_{t+1,i} &= R_{t+1,i} - R^f_t, \\ R_{t+1} &= R^w_{t+1} - R^f_t. \end{aligned} \tag{A.10}$$

Next, we can rewrite Equation (A.9) in terms of excess returns on aggregate wealth

$$\epsilon_{t+1} = R^w_{t+1} - E_t(R^w_{t+1}) = (R^w_{t+1} - R^f_t) - (E_t(R^w_{t+1}) - R^f_t) = R_{t+1} - E_t(R_{t+1}). \tag{A.11}$$

Thus far we have assumed that investors' decision making depends on the representation of returns aggregated across investment horizons R_{t+1}. As we show in Section 2.2 we are able to decompose returns to components with different levels of persistence as

$$R_{t+1} \equiv \sum_{j=1}^{N} R^{(j)}_{t+1} + R^{(\infty)}_{t+1} = R^{(short)}_{t+1} + R^{(long)}_{t+1}, \tag{A.12}$$

where $R^{(short)}_{t+1} = \sum_{j=1}^{N} R^{(j)}_{t+1}$, $R^{(long)}_{t+1} = R^{(\infty)}_{t+1} = R^{(n>N)}_{t+1}$.[19] Applying such decomposition to Equation (A.11), we obtain

$$\epsilon_{t+1} = \left[R^{(short)}_{t+1} - E_t\left(R^{(short)}_{t+1}\right)\right] + \left[R^{(long)}_{t+1} - E_t\left(R^{(long)}_{t+1}\right)\right] = \epsilon^{(s)}_{t+1} + \epsilon^{(l)}_{t+1}. \tag{A.13}$$

Let us assume, that for each i, investors invest their whole wealth into asset i, hence $R^e_{t+1,i}$ and R_{t+1} can be treated as interchangeable below. This allows us to express risk premium of asset i as

$$E_t\left(R^e_{t+1,i}\right) = \theta_{1,t,i} cov_t\left(R_{t+1}, \epsilon_{t+1}\right) + \theta_{2,t,i} cov_t\left(R_{t+1}, \epsilon^2_{t+1}\right) + \theta_{3,t,i} cov_t\left(R_{t+1}, \epsilon^3_{t+1}\right). \tag{A.14}$$

As we have shown, each term inside the covariances in Equation (A.14) can be decomposed to short-run and long-run components. Assuming orthogonality of components at individual scales; i.e. $\left(\epsilon^{(s)}_{t+1}\right)^n \left(\epsilon^{(l)}_{t+1}\right)^n = 0$ for $n \in \{1,2\}$, and $\left(\epsilon^{(s)}_{t+1}\right)^n R^{(l)}_{t+1} = 0$, $\left(\epsilon^{(l)}_{t+1}\right)^n R^{(s)}_{t+1} = 0$, for $n \in \{1,2,3\}$; we can rewrite Equation (A.14) as

$$\begin{aligned} E_t\left(R^e_{t+1,i}\right) =& \theta_{1,t,i} cov_t\left[R^{(s)}_{t+1} + R^{(l)}_{t+1}, \epsilon^{(s)}_{t+1} + \epsilon^{(l)}_{t+1}\right] + \theta_{2,t,i} cov_t\left[R^{(s)}_{t+1} + R^{(l)}_{t+1}, \left(\epsilon^{(s)}_{t+1} + \epsilon^{(l)}_{t+1}\right)^2\right] + \\ & \theta_{3,t,i} cov_t\left[R^{(s)}_{t+1} + R^{(l)}_{t+1}, \left(\epsilon^{(s)}_{t+1} + \epsilon^{(l)}_{t+1}\right)^3\right] = \sum_{h \in \{s,l\}} \sum_{k=1}^{3} \theta^{(h)}_{k,t,i} cov_t\left[R^{(h)}_{t+1}, \left(\epsilon^{(h)}_{t+1}\right)^n\right]. \end{aligned} \tag{A.15}$$

Recall that $R_{t+1} - E_t(R_{t+1}) = \epsilon_{t+1}$, hence we can rewrite $cov_t(R_{t+1}, \epsilon_{t+1})$ as

$$\begin{aligned} cov_t[R_{t+1}, \epsilon_{t+1}] =& E_t[R_{t+1}\epsilon_{t+1}] - E_t[R_{t+1}]E_t[\epsilon_{t+1}] \\ =& E_t\left[(\epsilon_{t+1} + E_t[R_{t+1}])\epsilon_{t+1}\right] - E_t[R_{t+1}]E_t[\epsilon_{t+1}] \\ =& E_t\left[\epsilon^2_{t+1}\right] + E_t\left[\epsilon_{t+1}E_t(R_{t+1})\right] - E_t[R_{t+1}]E_t[\epsilon_{t+1}] \\ =& E_t\left[\epsilon^2_{t+1}\right] = var_t[\epsilon_{t+1}], \end{aligned} \tag{A.16}$$

since $E_t[\epsilon_{t+1}E_t(R_{t+1})] = E_t(R_{t+1})E_t(\epsilon_{t+1})$, and $E_t(\epsilon_{t+1}) = 0$. Analogously, we can derive that

$$\begin{aligned} cov_t\left(R_{t+1}, \epsilon^2_{t+1}\right) &= E_t\left(\epsilon^3_{t+1}\right), \\ cov_t\left(R_{t+1}, \epsilon^3_{t+1}\right) &= E_t\left(\epsilon^4_{t+1}\right). \end{aligned} \tag{A.17}$$

We have shown that $cov_t\left(R_{t+1}, \epsilon^n_{t+1}\right) = E\left(\epsilon^{n+1}_{t+1}\right)$ for $n \in \{1,2,3\}$, which implies that

[19]Note that due to the equivalence in Equation (A.12), the decomposition is not restricted to two horizons. In fact, we are able to construct components from arbitrary number of horizons by splitting the sum in intermediate points.

$cov_t\left[R^{(h)}_{t+1}, \left(\epsilon^{(h)}_{t+1}\right)^n\right] = E\left[\left(\epsilon^{(h)}_{t+1}\right)^{n+1}\right]$ for $n \in \{1,2,3\}$ and $h \in \{s,l\}$. Hence, we can express Equation (A.15) as

$$E_t(R^e_{t+1,i}) = \sum_{h\in\{s,l\}} \theta^{(h)}_{1,t,i} E_t\left[\left(\epsilon^{(h)}_{t+1}\right)^2\right] + \sum_{h\in\{s,l\}} \theta^{(h)}_{2,t,i} E_t\left[\left(\epsilon^{(h)}_{t+1}\right)^3\right] + \sum_{h\in\{s,l\}} \theta^{(h)}_{3,t,i} E_t\left[\left(\epsilon^{(h)}_{t+1}\right)^4\right], \tag{A.18}$$

where $E_t\left[\left(\epsilon^{(h)}_{t+1}\right)^n\right]$ denotes n-th centralized moment of returns on aggregate wealth and

$$\theta^{(h)}_{1,t,i} = -\frac{U^{(2)}(1+C_t)}{U'(1+C_t)} \frac{U'(1+C_t)R^f_t}{U'(1)} w^{(h)}_{1,t,i}, \tag{A.19}$$

$$\theta^{(h)}_{2,t,i} = -\frac{U^{(3)}(1+C_t)}{U'(1+C_t)2!} \frac{U'(1+C_t)R^f_t}{U'(1)} w^{(h)}_{2,t,i}, \tag{A.20}$$

$$\theta^{(h)}_{3,t,i} = -\frac{U^{(4)}(1+C_t)}{U'(1+C_t)3!} \frac{U'(1+C_t)R^f_t}{U'(1)} w^{(h)}_{3,t,i}, \tag{A.21}$$

where $h \in \{s,l\}$ and $w^{(h)}_{k,t,i}$, $k \in \{1,2,3\}$, $i \in \{1,...,N\}$ are the spectral weights. For sake of clarity, we define $v^{(h)}_t = var_t\left(\epsilon^{(h)}_{t+1}\right)$, $s^{(h)}_t = E_t\left[\left(\epsilon^{(h)}_{t+1}\right)^3\right] / \left(v^{(h)}_t\right)^{3/2}$, $k^{(h)}_t = E_t\left[\left(\epsilon^{(h)}_{t+1}\right)^4\right] / \left(v^{(h)}_t\right)^2$. Then Equation (A.18) can be expressed in terms of volatility

$$E_t(R^e_{t+1,i}) = \sum_{h\in\{s,l\}} \beta^{(h)}_{t,i} \sqrt{v^{(h)}_t} + \sum_{h\in\{s,l\}} \delta^{(h)}_{t,i} s^{(h)}_t + \sum_{h\in\{s,l\}} \mathcal{K}^{(h)}_{t,i} k^{(h)}_t, \tag{A.22}$$

where $\sqrt{v^{(s)}_t}$ is the short-term component of volatility of returns on aggregate wealth, $\sqrt{v^{(l)}_t}$ is the long-term component of volatility of returns on aggregate wealth, $s^{(s)}_t$ is the short-term component of skewness of returns on aggregate wealth, $s^{(l)}_t$ is the long-term component of skewness of returns on aggregate wealth, $k^{(s)}_t$ is the short-term component of kurtosis of returns on aggregate wealth, $k^{(l)}_t$ is the long-term component of kurtosis of returns on aggregate wealth, and

$$\beta^{(h)}_{t,i} = \theta^{(h)}_{1,t,i}\left(v^{(h)}_t\right)^{1/2}, \tag{A.23}$$

$$\delta^{(h)}_{t,i} = \theta^{(h)}_{2,t,i}\left(v^{(h)}_t\right)^{3/2}, \tag{A.24}$$

$$\mathcal{K}^{(h)}_{t,i} = \theta^{(h)}_{3,t,i}\left(v^{(h)}_t\right)^{2}, \tag{A.25}$$

for $h \in \{s, l\}$. This completes the proof. $\square$

B Estimation of time-varying parameter regressions

We now consider a linear regression with time-varying parameters that has a Normal-Gamma quasi-posterior distribution. Specifically, this is a univariate version of Petrova (2019). Let

$$y_{t,T} = \beta_0(t/T) + x_{1,t,T}\beta_1(t/T) + \cdots + x_{l,t,T}\beta_l(t/T) + \epsilon_{t,T}, \quad \epsilon_{t,T} \backsim \mathcal{N}\left(0, \sigma^2_{t,T}\right) \tag{B.26}$$

$$y_{t,T} = x_{t,T}\beta(t/T) + \epsilon_{t,T} \tag{B.27}$$

where $x_{t,T} \equiv (1, x_{1,t,T}, \ldots, x_{l,t,T})$ and $\beta(t/T) \equiv (\beta_0(t/T), \beta_1(t/T), \ldots, \beta_l(t/T))^\top$.

Now let $\lambda_{t,T} \equiv \sigma^{-2}_{t,T}$. The weighted local likelihood function of the sample $Y \equiv (y_1, \ldots, y_T)$, using $\mathbf{X} \equiv \left(x_1^\top, \ldots, x_T^\top\right)^\top$ as a $T \times l$ matrix, at each discrete time point s, where we drop the double time index for notational convenience, is given by

$$L_s\left(Y|\beta_s, \lambda_s, \mathbf{X}\right) = (2\pi)^{-\mathrm{tr}(\mathbf{D}_s)/2}\lambda_s^{\mathrm{tr}(\mathbf{D}_s)/2}\exp\left\{-\frac{\lambda_s}{2}(Y - \mathbf{X}\beta_s)^\top\mathbf{D}_s(Y - \mathbf{X}\beta_s)\right\} \tag{B.28}$$

with

$$\mathbf{D}_s = \mathrm{diag}\left(\vartheta_{s,1}, \ldots, \vartheta_{s,T}\right) \tag{B.29}$$

$$\vartheta_{s,t} = \zeta_{T,s}w_{s,t} / \sum_{t=1}^{T} w_{s,t} \tag{B.30}$$

$$w_{s,t} = \left(1/\sqrt{2\pi}\right)\exp\left((-1/2)((k-t)/H)^2\right), \forall\, s, t \in \{1, \ldots, T\} \tag{B.31}$$

$$\zeta_{T,s} = \left(\sum_{t=1}^{T} w^2_{s,t}\right)^{-1} \tag{B.32}$$

Now assuming β_s, λ_s have a Normal-Gamma prior distribution $\forall s \in \{1, \ldots, T\}$

$$\beta_s|\lambda_s \backsim \mathcal{N}\left(\beta_{0,s}, \left(\lambda_s\kappa_{0,s}\right)^{-1}\right) \tag{B.33}$$

$$\lambda_s \backsim \mathcal{G}\left(\alpha_{0,s}, \gamma_{0,s}\right) \tag{B.34}$$

We can combine L_s with the above priors such that β_s, λ_s have Normal-Gamma quasi-posterior distribution $\forall k \in \{1, \ldots, T\}$

$$\beta_s|\lambda_s \backsim \mathcal{N}\left(\bar{\beta}_s, (\lambda_s \bar{\kappa}_s)^{-1}\right) \tag{B.35}$$

$$\lambda_s \backsim \mathcal{G}\left(\bar{\alpha}_s, \bar{\gamma}_s\right) \tag{B.36}$$

with (quasi) posterior parameters:

$$\bar{\beta}_s = \bar{\kappa}_s^{-1}\left(\mathbf{X}^\top \mathbf{D}_s \mathbf{X} \widehat{\beta}_s + \kappa_{0,s}\beta_{0,s}\right), \ \widehat{\beta}_s = \left(\mathbf{X}^\top \mathbf{D}_s \mathbf{X}\right)^{-1} \mathbf{X}^\top \mathbf{D}_s y \tag{B.37}$$

$$\bar{\kappa}_s = \kappa_{0,s} + \mathbf{X}^\top \mathbf{D}_s \mathbf{X} \tag{B.38}$$

$$\bar{\alpha}_s = \alpha_{0,s} + \sum_{t=1}^{T} \vartheta_s \tag{B.39}$$

$$\bar{\gamma}_s = \gamma_{0,s} + \frac{1}{2}\left(Y^\top \mathbf{D}_s Y - \bar{\beta}_s^\top \bar{\kappa}_s \bar{\beta}_s + \beta_{0,s}^\top \kappa_{0,s} \beta_{0,s}\right) \tag{B.40}$$

The Algorithm

1. Initialise $\beta_{0,s}$, $\kappa_{0,s}, \alpha_{0,s}, \gamma_{0,s}$ and compute kernel weights. Then repeat steps 2-3 $\iota = 1, 2, \ldots, \mathcal{I}$ times.

2. For every $s \in \{1, 2, \ldots, T\}$, draw $\lambda_s^s|\mathbf{X}, y, \beta_s^{s-1}$ from $\backsim \mathcal{G}\left(\bar{\alpha}_s, \bar{\gamma}_s\right)$.

3. For every $s \in \{1, 2, \ldots, T\}$, draw $\beta_s^s|y, \mathbf{X}, \lambda_s^s$ from $\backsim \mathcal{N}\left(\bar{\beta}_s, (\lambda_s \bar{\kappa}_s)^{-1}\right)$.

In our case, for each regression in Section 4 we set the parameter $H = \sqrt{T}$ to compute the kernel weights and initialize $\beta_{0,s}$, $\kappa_{0,s}, \alpha_{0,s}, \gamma_{0,s}$ using OLS parameters from a linear regression for the time-series in question. Then, we generate $\mathcal{I}$=1000 simulations for each time-period.

C Sorting procedure

For each 3-month (6-month) rolling window, we sort the assets into quintile portfolios based on the factor loadings of corresponding variable, i.e. Portfolio 1 contains the assets with the lowest values of the factor loading, Portfolio 5 contains the assets with the highest values of the factor loading. The portfolios we form are equal-weighted. Then, we record the average daily post-ranking portfolio returns over the next week. We repeat the procedure by rolling the beta estimation window forward by one week at a time. In the end, we have a time-series of post-ranking returns for each quintile portfolio, and we report the time-series mean for each portfolio. The factor loadings are obtained as a result of one of the following multivariate regressions

$$r_{t+1,i} = \alpha_i + \beta_i^{(m)} RVOL_t^{(m)} + \delta_i^{(m)} RS_t^{(m)} + \mathcal{K}_i^{(m)} RK_t,$$

$$r_{t+1,i} = \alpha_i + \sum_{h \in \{s,l\}} \beta_i^{(h,m)} RVOL_t^{(h,m)} + \sum_{h \in \{s,l\}} \delta_i^{(h,m)} RS_t^{(h,m)} + \sum_{h \in \{s,l\}} \mathcal{K}_i^{(h,m)} RK_t^{(h,m)},$$

$$r_{i,t+1} = \alpha_i + \beta_i^{(I)} RVOL_t^{(I)} + \delta_i^{(I)} RS_t^{(I)} + \mathcal{K}_i^{(I)} RK_t^{(I)},$$

$$r_{i,t+1} = \alpha_i + \sum_{h \in \{s,l\}} \beta_i^{(h,I)} RVOL_t^{(h,I)} + \sum_{h \in \{s,l\}} \delta_i^{(h,I)} RS_t^{(h,I)} + \sum_{h \in \{s,l\}} \mathcal{K}_i^{(h,I)} RK_t^{(h,I)}.$$

D Additional figures and tables

Table A1. Correlations: ETFs

This table provides the correlation matrix for the realized market and average idiosyncratic realized moments in the Panel A, correlation matrix for the frequency-decomposed realized market and average idiosyncratic moments in the Panel B, and Panel C.

Panel A	$RVOL_t^{(m)}$	$RS_t^{(m)}$	$RK_t^{(m)}$	$RVOL_t^{(I)}$	$RS_t^{(I)}$	$RK_t^{(I)}$
$RVOL_t^{(m)}$						
$RS_t^{(m)}$	-0.041					
$RK_t^{(m)}$	-0.049	0.174				
$RVOL_t^{(I)}$	0.949	-0.024	-0.076			
$RS_t^{(I)}$	-0.045	0.765	0.137	-0.016		
$RK_t^{(I)}$	-0.234	0.208	0.538	-0.151	0.283	

Panel B	$RVOL_t^{(s,m)}$	$RVOL_t^{(l,m)}$	$RS_t^{(s,m)}$	$RS_t^{(l,m)}$	$RK_t^{(s,m)}$	$RK_t^{(l,m)}$
$RVOL_t^{(s,m)}$						
$RVOL_t^{(l,m)}$	0.109					
$RS_t^{(s,m)}$	-0.07	0.004				
$RS_t^{(l,m)}$	0.03	0.315	0.029			
$RK_t^{(s,m)}$	0.003	-0.014	0.177	0.006		
$RK_t^{(l,m)}$	-0.059	-0.704	0.002	-0.049	0.044	

Panel C	$RVOL_t^{(s,m)}$	$RVOL_t^{(l,m)}$	$RS_t^{(s,m)}$	$RS_t^{(l,m)}$	$RK_t^{(s,m)}$	$RK_t^{(l,m)}$	$RVOL_t^{(s,I)}$	$RVOL_t^{(l,I)}$	$RS_t^{(s,I)}$	$RS_t^{(l,I)}$	$RK_t^{(s,I)}$
$RVOL_t^{(s,I)}$	0.93	0.138	-0.048	0.022	-0.032	-0.071					
$RVOL_t^{(l,I)}$	0.115	0.969	0.001	0.339	-0.015	-0.659	0.137				
$RS_t^{(s,I)}$	-0.068	0.006	0.766	0.019	0.138	0.007	-0.033	0.005			
$RS_t^{(l,I)}$	0.056	0.209	0.027	0.647	0.015	0.056	0.059	0.195	0.041		
$RK_t^{(s,I)}$	-0.13	-0.041	0.224	-0.009	0.532	0.057	-0.026	-0.039	0.297	0.005	
$RK_t^{(l,I)}$	-0.1	-0.879	-0.002	-0.316	0.029	0.796	-0.103	-0.876	0.001	-0.172	0.062

Table A2. Cross-section of Stocks

We are sorting on the factor loadings of corresponding variables. The length of the rolling window is 6 months, the post-ranking returns are recorded for the next week. The complete sorting procedure is described in Apendix C. We report the average post-ranking raw returns (in bps) for each portfolio, the t-statistics are reported in parentheses. The last column displays the difference in returns of the highest quintile portfolio, and the lowest quintile portfolio, hence corresponds to the strategy of buying high exposure assets, and selling low exposure assets.

Variable	1	2	3	4	5	High - Low
			Market			
$RVOL_t^{(m)}$	1.5	2.04	1.54	1.26	0.65	-0.84
	(0.79)	(1.35)	(1.1)	(0.96)	(0.48)	(-0.7)
$RS_t^{(m)}$	2.19	2.9	1.92	1.08	-1.12	-3.31
	(2.02)	(2.34)	(1.31)	(0.64)	(-0.52)	(-1.97)
$RK_t^{(m)}$	1.81	1.88	1.68	1.19	0.43	-1.38
	(1.24)	(1.38)	(1.21)	(0.81)	(0.24)	(-1.29)
$RVOL_t^{(s,m)}$	1.26	2.12	1.81	1.23	0.58	-0.68
	(0.68)	(1.4)	(1.32)	(0.92)	(0.41)	(-0.59)
$RVOL_t^{(l,m)}$	0.39	1.44	1.52	2.02	1.65	1.26
	(0.23)	(1)	(1.09)	(1.49)	(1.05)	(1.48)
$RS_t^{(s,m)}$	2.23	2.81	2.08	1.06	-1.2	-3.43
	(2.05)	(2.28)	(1.41)	(0.63)	(-0.56)	(-2.06)
$RS_t^{(l,m)}$	1.5	1.64	1.85	1.79	0.21	-1.29
	(0.92)	(1.18)	(1.35)	(1.26)	(0.13)	(-1.48)
$RK_t^{(s,m)}$	1.47	1.86	2.1	1.08	0.49	-0.99
	(1)	(1.37)	(1.52)	(0.73)	(0.27)	(-0.94)
$RK_t^{(l,m)}$	1.47	2.12	1.45	1.51	0.44	-1.02
	(0.92)	(1.52)	(1.05)	(1.07)	(0.28)	(-1.24)
			Idiosyncratic			
$RVOL_t^{(I)}$	1.28	2.49	1.84	1.22	0.16	-1.11
	(0.71)	(1.67)	(1.3)	(0.92)	(0.11)	(-1.04)
$RS_t^{(I)}$	2.03	2.77	1.94	1	-0.76	-2.78
	(1.89)	(2.17)	(1.34)	(0.6)	(-0.35)	(-1.62)
$RK_t^{(I)}$	1.67	2.04	1.76	1.5	0.02	-1.65
	(1.23)	(1.58)	(1.25)	(0.98)	(0.01)	(-1.32)
$RVOL_t^{(s,I)}$	1.54	1.79	1.59	1.37	0.7	-0.83
	(0.86)	(1.22)	(1.15)	(1.01)	(0.48)	(-0.76)
$RVOL_t^{(l,I)}$	0.35	1.42	1.88	2.38	0.98	0.63
	(0.22)	(1.01)	(1.37)	(1.68)	(0.62)	(0.74)
$RS_t^{(s,I)}$	2.18	2.66	2.02	0.75	-0.63	-2.81
	(2.03)	(2.07)	(1.4)	(0.45)	(-0.29)	(-1.65)
$RS_t^{(l,I)}$	-0.97	1.53	1.68	2.98	1.82	2.79
	(-0.55)	(1.07)	(1.22)	(2.25)	(1.18)	(2.97)
$RK_t^{(s,I)}$	1.85	1.97	2.02	1.39	-0.25	-2.1
	(1.36)	(1.53)	(1.44)	(0.9)	(-0.13)	(-1.72)
$RK_t^{(l,I)}$	0.41	1.6	2.19	2.55	0.27	-0.13
	(0.24)	(1.14)	(1.57)	(1.82)	(0.18)	(-0.15)

Table A3. Cross-section of ETFs

We are sorting based on the factor loadings of corresponding variables. The length of the rolling window is 6 months, the post-ranking returns are recorded for the next week. The complete sorting procedure is described in Apendix C. We report the average post-ranking raw returns (in bps) for each portfolio, the t-statistics are reported in parentheses. The last column displays the difference in returns of the highest quintile portfolio, and the lowest quintile portfolio, hence corresponds to the strategy of buying high exposure assets, and selling low exposure assets.

Variable	1	2	3	4	5	High - Low
			Market			
$RVOL_t^{(m)}$	-0.6	-0.27	-0.37	-0.37	-0.04	0.56
	(-0.35)	(-0.18)	(-0.26)	(-0.26)	(-0.03)	(0.61)
$RS_t^{(m)}$	0.59	0.39	-0.13	-0.27	-2.21	-2.8
	(0.5)	(0.29)	(-0.09)	(-0.17)	(-1.19)	(-2.52)
$RK_t^{(m)}$	-0.49	-0.05	-0.16	0.43	-1.38	-0.89
	(-0.32)	(-0.03)	(-0.11)	(0.3)	(-0.84)	(-0.91)
$RVOL_t^{(s,m)}$	-0.48	0	-0.22	-0.34	-0.6	-0.12
	(-0.28)	(0)	(-0.15)	(-0.25)	(-0.41)	(-0.13)
$RVOL_t^{(l,m)}$	-0.59	0.53	-0.15	0.3	-1.72	-1.13
	(-0.39)	(0.38)	(-0.11)	(0.2)	(-1.08)	(-1.25)
$RS_t^{(s,m)}$	0.74	0.27	-0.03	-0.35	-2.27	-3.01
	(0.63)	(0.2)	(-0.02)	(-0.22)	(-1.22)	(-2.75)
$RS_t^{(l,m)}$	-0.71	-0.07	0.05	0.02	-0.93	-0.22
	(-0.45)	(-0.05)	(0.04)	(0.01)	(-0.6)	(-0.25)
$RK_t^{(s,m)}$	-0.6	-0.07	0.01	0.19	-1.17	-0.57
	(-0.39)	(-0.05)	(0.01)	(0.13)	(-0.7)	(-0.6)
$RK_t^{(l,m)}$	0.1	0.16	0.17	-0.02	-2.06	-2.16
	(0.07)	(0.11)	(0.12)	(-0.01)	(-1.31)	(-2.59)
			Idiosyncratic			
$RVOL_t^{(I)}$	-0.49	0.23	0.34	-0.42	-1.3	-0.81
	(-0.29)	(0.15)	(0.24)	(-0.31)	(-0.9)	(-0.88)
$RS_t^{(I)}$	0.63	0.17	0.27	-0.91	-1.8	-2.42
	(0.56)	(0.12)	(0.18)	(-0.58)	(-0.92)	(-2.05)
$RK_t^{(I)}$	-0.78	-0.03	0.21	0.1	-1.14	-0.37
	(-0.5)	(-0.02)	(0.15)	(0.07)	(-0.7)	(-0.39)
$RVOL_t^{(s,I)}$	0.07	0.04	-0.22	-0.55	-0.98	-1.05
	(0.04)	(0.03)	(-0.16)	(-0.4)	(-0.65)	(-1.11)
$RVOL_t^{(l,I)}$	0.22	0.29	0.08	-0.37	-1.86	-2.08
	(0.14)	(0.2)	(0.05)	(-0.26)	(-1.16)	(-2.32)
$RS_t^{(s,I)}$	0.66	0.05	0.28	-0.69	-1.94	-2.6
	(0.59)	(0.03)	(0.19)	(-0.44)	(-1)	(-2.2)
$RS_t^{(l,I)}$	-1.38	0.06	0.05	-0.07	-0.31	1.08
	(-0.84)	(0.04)	(0.04)	(-0.05)	(-0.2)	(1.19)
$RK_t^{(s,I)}$	-0.84	-0.06	0.4	0.15	-1.29	-0.45
	(-0.54)	(-0.04)	(0.28)	(0.1)	(-0.79)	(-0.47)
$RK_t^{(l,I)}$	0.78	0.55	0.14	-0.57	-2.54	-3.32
	(0.51)	(0.38)	(0.1)	(-0.4)	(-1.59)	(-3.82)

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Univerzita Karlova v Praze, Fakulta sociálních věd
Institut ekonomických studií [UK FSV – IES] Praha 1, Opletalova 26
E-mail : ies@fsv.cuni.cz http://ies.fsv.cuni.cz